LUNAR FLIGHT STUDY SERIES VOLUME 2:
EARTH-MOON TRANSIT STUDY BASED ON EPHEMERIS DATA AND USING BEST AVAILABLE COMPUTER PROGRAM.
PART 2: RETURN FLIGHT TO EARTH FROM LUNAR ORBIT

By Arthur J. Schwaniger

OTS PRICE
XEROX $4.60 ph
MICROFILM $1.55 mf

FOR INTERNAL USE ONLY
ABSTRACT

The problem of return to earth from an orbit about the moon has been investigated. The orbit is assumed to be retrograde (opposite the moon's rotation) and inclined to the lunar equator by not more than 20 degrees.

The method of trajectory calculation used takes account of the ephemerides of earth, moon, and sun. The investigation is made for three times in March 1969.

Earth landing sites considered are at Woomera, Australia and San Antonio, Texas. The reentry conditions for landing at these places are chosen typical of the Apollo spacecraft.

Injection conditions for return from the orbit to the desired reentry conditions are determined and the use of this data in establishing nominal return conditions and "injection window" is illustrated.
LUNAR FLIGHT STUDY SERIES: VOLUME 2

EARTH - MOON TRANSIT STUDY BASED ON EPHEMERIS DATA
AND USING BEST AVAILABLE COMPUTER PROGRAM

PART 2: RETURN FLIGHT TO EARTH FROM LUNAR ORBIT

By

Arthur J. Schwaniger

FUTURE PROJECTS BRANCH
AEROBALLISTICS DIVISION
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>SECTION I. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>SECTION II. DISCUSSION</td>
<td>1</td>
</tr>
<tr>
<td>A. PROBLEM STATEMENT</td>
<td>1</td>
</tr>
<tr>
<td>B. REENTRY CRITERIA</td>
<td>1</td>
</tr>
<tr>
<td>C. COORDINATE SYSTEMS</td>
<td>3</td>
</tr>
<tr>
<td>D. METHODS USED AND ASSUMPTIONS</td>
<td>3</td>
</tr>
<tr>
<td>SECTION III. RESULTS</td>
<td>5</td>
</tr>
<tr>
<td>SECTION IV. CONCLUSIONS</td>
<td>8</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>9</td>
</tr>
<tr>
<td>APPROVAL</td>
<td>40</td>
</tr>
<tr>
<td>DISTRIBUTION</td>
<td>41</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
</tr>
<tr>
<td>-------</td>
<td>-------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>1</td>
<td>Reentry Longitude and Azimuth for Return to San Antonio, Texas as Function of Reentry Latitude</td>
</tr>
<tr>
<td>2</td>
<td>Reentry Longitude and Azimuth for Return to Woomera, Australia as Function of Reentry Latitude</td>
</tr>
<tr>
<td>3 - 11</td>
<td>Injection Conditions at Moon for Return to San Antonio from Moon at Minus 28.4° Declination</td>
</tr>
<tr>
<td>12 - 20</td>
<td>Injection Conditions at Moon for Return to Woomera from Moon at 28.4° Declination</td>
</tr>
<tr>
<td>21 - 23</td>
<td>Injection Conditions at Moon for Return to San Antonio from Moon at Ascending Node</td>
</tr>
<tr>
<td>24 - 26</td>
<td>Injection Conditions at Moon for Return to Woomera from Moon at Ascending Node</td>
</tr>
<tr>
<td>27</td>
<td>Optimum Injection Path Angle as a Function of Injection Velocity Magnitude for a Typical Lunar Return Vehicle</td>
</tr>
<tr>
<td>28</td>
<td>Velocity Penalty for Deviation from Optimum Velocity Direction</td>
</tr>
<tr>
<td>29</td>
<td>Characteristic Velocity Required for Return to Woomera, Australia from Circular Orbit about Lunar Equator</td>
</tr>
<tr>
<td>30</td>
<td>Velocity Penalty for Departure from Equatorial Moon Orbit at Non-Optimum Time</td>
</tr>
</tbody>
</table>
As part of a larger effort to establish the feasibility of lunar orbit rendezvous as a method for manned lunar exploration, a study has been made of the return injection conditions which might be required for such a mission. This report presents the results of the study.

SECTION II. DISCUSSION

A. PROBLEM STATEMENT

The problem at hand is to determine the injection conditions which might be utilized for a return flight to earth from an orbit about the moon with a view toward the determination also of what "injection window" is available. The orbit is assumed to be retrograde (opposite the moon's rotation) and inclined to the lunar equator by not more than 20 degrees.

B. REENTRY CRITERIA

At reentry into the earth's atmosphere the return trajectory must have the correct direction and velocity to assure that the vehicle arrives at the preselected recovery area and is not subjected to excessive thermodynamic or
deceleration loads. In a final analysis a reentry window could be established which would represent all sets of acceptable reentry conditions, and which would be determined by the exact aerodynamic characteristics of the vehicle and the flexibility of the guidance and control system. In this study, however, the reentry window is approximated by choosing most of the reentry parameters at a nominal value.

Work done by the author and others at this center, as well as information available in the literature, was considered in the choice of the reentry conditions. Reentry is assumed to take place at a radius of 6498 km from the earth's center. Since the trajectories were calculated for an ellipsoidal earth, this means reentry altitude was allowed to vary slightly with latitude of reentry. The range from reentry to touchdown is taken as 4300 n.m. or 72 degrees of central angle. This is about the average range of the Apollo type craft when returning from the moon. Path angle of 6 degrees below horizontal is approximately the direction of reentry which is satisfactory for most vehicles of the type likely to be used for the mission and is chosen as nominal here.

Reentry velocity was not restricted to a fixed value for the following reasons. For the velocities considered for the departure the reentry velocity measured in a non-rotating frame varies by only 50 m/s even though return transit times range from 55 to 100 hours. Thus, when the reentry velocity is measured with respect to the rotating earth, for all cases reentering with eastward direction (azimuth between 0 and 180 degrees), the velocity extremes encountered are 10,600 to 11,000 m/s. This variation of 400 m/s with respect to the atmosphere is not critical for those velocity magnitudes encountered.

Under these assumptions concerning reentry velocity we have, in effect, neglected the earth's rotation and assumed that all reentry situations will require the same range to touchdown. Thus, if we would also neglect the non-spherical shape of the earth, the reentry points would lie on a circle with the touchdown site at the center.

If the touchdown location is specified, spherical trigonometry can be used to determine what combinations of latitude and longitude lie at the correct angular distance from the touchdown point. Further, if reentry flight is assumed to occur in a plane, the correct azimuth for each reentry position is determined.
Two landing points on earth considered are near San Antonio, Texas, and Woomera, Australia. The latitude of these places is 29.48 degrees north and south, respectively. The northern location (near San Antonio) is taken as 261 degrees longitude (measured east from Greenwich) and the southern location (Woomera) is at 135 degrees east longitude. Figures 1 and 2 show the azimuth and longitude for the specified reentry conditions as functions of the reentry latitude. For convenience the conditions represented by these figures will be referred to as an "arrival circle."

C. COORDINATE SYSTEMS

At this point it is desirable to explain the system used to specify position and velocity coordinates at the moon. All of these coordinates are given in a selenographic system which is similar to that used to specify geocentric coordinates at the earth. The moon's equator plane is perpendicular to its axis of rotation. The zero longitude meridian crosses the equator on that principal axis of inertia which is assumed (by astronomers) to lie through the center of the bulge of the moon toward the earth. Since the moon's equator is inclined by 6.67 degrees to the plane of the moon's orbit about earth, and since also the orbit is slightly eccentric while the moon's rotation rate is essentially constant, the position of the zero latitude and longitude point seems to move in nearly a circle about the earth-moon line. In addition to this optical libration there is a small (always less than 0.40 degrees) physical libration.

In this moon-fixed system then, longitude is measured positive east from 0 to 360 degrees. East on the moon as on earth is the direction of sunrise. Azimuth is measured from 0 through 360 degrees starting eastward from north.

D. METHODS USED AND ASSUMPTIONS

Trajectories for return flights were computed by numerical integration of the equations of motion using a system programmed for high speed computers by the Jet Propulsion Laboratory. The equations of motion include the gravitational forces of the earth, moon, and sun with their coordinates given accurately according to the ephemeris. The oblateness effects of both earth and moon are also included.
Since the first manned lunar flights are expected to occur near the end of this decade, the time investigated was chosen to be March 1969. At that time the moon's orbit about the earth is inclined by 28.72 degrees, roughly the largest inclination ever occurring. Also the moon's perigee occurs at about the same time that it reaches its minimum (south) declination. (At perigee the moon is at 369,400 km distant from earth. At apogee, which in that month occurs obviously at maximum declination, the distance is 404,400 km.) Three times in the month were chosen for study. These are minimum declination, zero declination at ascending node, and maximum declination.

The specification of a retrograde orbit of small inclination indicates injection from latitudes near the lunar equator, with westward azimuth, in order to avoid large performance penalties associated with large flight plane changes. Information available in the literature (References 1 and 2) concerning lunar return indicates that for such conditions the injection longitude will be in the region of 90 to 180 degrees. (This can also be seen from studies of earth to moon flight using the restricted three-body approach which has as a feature that for any trajectory from earth to moon there is a trajectory of the same shape reflected about the earth-moon line and which goes in the opposite direction, namely from moon to earth. A detailed explanation of this phenomenon is given in Reference 3.) Therefore, injection positions at 0, 10, and 20 degrees latitude for each of 90, 135, and 180 degrees longitude were chosen for investigation of injection velocity magnitude and direction requirements. The injection radius is held constant at 1938 km from the center of the moon or 200 km above the mean radius of the moon of 1738 km.

Southern latitudes were not considered since by proper rotation of coordinate systems to account for the previously mentioned tilt of the moon's axis, the results could be reflected about the earth-moon plane to give an approximation of conditions for those southern latitudes that happen to be symmetric about the earth-moon plane with the northern latitude used originally. The results would be approximate only because of the irregular gravity fields of the non-spherical moon and earth.

The procedure followed was to hold stepwise constant the position, velocity magnitude, and time of injection and to isolate for that combination of injection path angle and azimuth which results in reentry with a 6 degree path angle.
below horizontal at radius of 6498 km from earth. This isolation is done automatically by routines built into the trajectory computation program. For each position and velocity magnitude there are a family of trajectories which meet these conditions. Not all of these have the specified "arrival circle" conditions. The reentry latitude and azimuth which is produced is compared graphically with the acceptable values shown in Figures 1 and 2. The trajectories which produce desired latitude and azimuth combinations are considered without regard to longitude. This is explained by the following reasoning. A return flight arrived at by the above procedure may result in a longitude error the absolute value of which can be at most 180 degrees. Thus, if injection time is adjusted by at most 12 hours then except for the slight change in the trajectory shape due to the slightly changed positioning of moon with respect to earth at the corrected time, the reentry does occur at the correct longitude.

SECTION III. RESULTS

At maximum and minimum declination of the moon, injection conditions were established which produce flights to satisfy the specified reentry conditions for each of the nine injection locations. Solutions are found, however, only for the northern return location, if the moon is at southermost declination at departure and conversely solutions are only at the southern location for departure from the moon at maximum declination.

At present, data for the zero declination are available only for equatorial injection. In this case though, satisfactory return to either of the return latitudes is seen to be possible.

Injection path angle and azimuth which produce acceptable return flights and the associated flight times are shown in Figures 3 thru 26 as functions of injection velocity magnitude for each of the positions. Figures 3 thru 11 show cases for return from minimum declination to San Antonio, Texas. Figures 12 thru 20 present data for return from maximum declination to Woomera, Australia, and Figures 21 thru 26 indicate return conditions from zero declination to both San Antonio and Woomera.
For a fixed declination and distance of the moon, the location of the point of reentry on the "arrival circle" (which also defines the inclination of the reentry flight plane) is a function of the injection position and velocity. For the injection positions and the velocity vectors applicable in this study, however, the reentry occurs always near the same place for a given position of the moon. No attempt was made here to study the exact behavior of the reentry point on the "arrival circle." Such a study may be made eventually using a system referenced to the moon-earth orbit plane which makes easier the analysis of effects not related to the moon declination. The effects of changing declination can then be super-imposed on those due to injection position and velocity variation. The positions of reentry encountered in the study are given in the following.

For return to Woomera from the maximum declination, the reentry lies between 19 and 24 degrees south latitude having flight plane inclination of 32 to 34 degrees to the earth's equator while for return from zero declination, the reentry occurs between 11 and 15 degrees north latitude with inclination of 38 to 40 degrees. For return to San Antonio from minimum declination, the reentry latitudes are between 20 and 23 degrees north with inclination being 32 to 34 degrees. For return to San Antonio from zero declination, the latitudes are from 8 to 12 degrees south and inclinations from 36 to 38 degrees.

Since San Antonio and Woomera are positioned at symmetrical latitudes, the return latitudes could be expected to be the same for return from the extreme declination in either hemisphere to the recovery point in the opposite hemisphere if the moon were always at the same distance and orientation with respect to the earth. The above figures for reentry latitude reflect the changes in moon position and orientation which occur during the month studied.

In the course of the study it was noticed that for short transit times (50 to 90 hours) the injection velocity magnitude required is greater at the moon's apogee than it is at the perigee while for long transit times (> 90 hours) the situation reverses. (A similar phenomenon occurs in flight from earth to moon, with a comparison of arrival velocity with respect to the moon as a function of flight time and distance to moon.) Due to this difference in velocity requirement, the curves are given for smaller velocities to about 2.5 km/sec for the minimum declination curves (coinciding here with smallest earth-moon distance) and to 2.4 km/sec for maximum declination.
From the information given in the above figures and certain performance data a nominal injection situation can be established and launch window determinations can be made. One simple arbitrary case is offered. This case is based on the assumption of an equatorial circular orbit of a 2 hr 7 minute period around the moon. In order to establish a nominal case we also assume that we want to return to a preselected point on earth in a specified transit time and to use a minimum of propellant in the departure maneuver.

Performance studies have been made by Space Projects Section II of this branch to determine the characteristic velocity required for these orbital maneuvers. The results of the studies will be given in detail in reports being prepared by that section; however, an example of the results is given here as part of the illustration of determination of nominal injection and launch window.

For injection at 1933 km radius the study showed that optimum transfer would be made from circular orbit radius of about 1932 km which is the orbit of the period specified above. (This optimum altitude change during the maneuver is of course also a function of the injection velocity magnitude and direction; however, the effect on performance is small.) The optimum path angle at injection is given then as a function of injection velocity in Figure 27 under the assumption of constant injection radius and no change in flight plane orientation. Figure 28 shows the characteristic velocity required for variation of velocity direction from optimum. This curve has been found to be representative of changes in either path angle or azimuth or of combined changes from the optimum. Thus, for each injection position on the moon the difference in required injection direction and the optimum direction can be determined, and the velocity penalty can be read from the figure.

The total velocity required for transfer is given as a function of flight time in Figure 29. Here the small change in latitude which takes place when azimuth changes are made during the injection maneuver has been neglected. The injection latitude is assumed to be always zero and longitude is given as the parameter. The time of maximum declination was used in this case. From this curve the nominal injection is determined and an approximation of the "injection window" is made as follows.
If the moon is near the maximum declination and we desire a return of 77 hr duration the injection longitude from which the least velocity increment is required is seen in Figure 29 to be 180 degrees. The nominal injection time is established to satisfy these longitude and flight time requirements. If we now consider early and late departures, we note that for the orbit period in consideration the longitude changes by about 3 degrees per minute so that for each minute early that injection occurs the velocity increment is read on the curve for 3 degrees greater longitude at the same flight time and likewise for each minute late the longitude decreases by 3 degrees. (More nearly exact would be to say, read the curve for flight time adjusted by the correct number of minutes, but obviously a few minutes are negligible here.) It is noticed that if the delay is to be greater than a few minutes, it may be advantageous to wait one more orbit of 2 hours and 7 minutes from nominal time and reduce flight time by that amount. (This assumed, the correct longitude is again achieved at reentry. This is not exactly true due to change of trajectory shape for smaller transit time as well as the change of the moon's position with respect to the earth, which takes place in the 2 hr 7 minute period. The assumption is, however, sufficiently accurate for this first approximation.) If this longer delay occurs, a second nominal injection may take place at longitude of 177 degrees and the velocity penalty for dispersions about the new nominal may be determined again as above.

The velocity penalty for departure at non-optimum time as determined in the above example is shown in Figure 30 plotted against the time deviation.

SECTION IV. CONCLUSIONS

The results presented may be used to determine injection conditions for return to earth from retrograde orbits about the moon inclined by as much as 20 degrees (and with certain location of the lines of nodes of the orbit and equator). Approximations of return injection windows can also be made.

The results indicate that return can be made to at least one of the two chosen sites regardless of the moon's position relative to earth and that reasonable injection windows are available.
REFERENCES


FIG. 1. REENTRY LONGITUDE AND AZIMUTH FOR RETURN TO SAN ANTONIO, TEXAS AS FUNCTION OF REENTRY LATITUDE
FIG. 2. REENTRY LONGITUDE AND AZIMUTH FOR RETURN TO WOOMERA, AUSTRALIA AS FUNCTION OF REENTRY LATITUDE
Lunar Azimuth, $AZ_M$, Clockwise from North (deg)
Path Angle from Lunar Horizon, $\theta_M$ (deg)
Transit Flight Time, $T$ (hr)

FIG. 3. INJECTION CONDITIONS AT MOON FOR RETURN TO SAN ANTONIO
Moon at -28.6° Declination at Injection
Injection from Latitude, $Lat_M$, 0°
and Longitude, $Long_M$, 90°
FIG. 4. SAME AS FIGURE 3

Lat$_M$ = 10°
Long$_M$ = 90°
FIG. 5. SAME AS FIGURE 3

$Lat_M = 20^\circ$

$Long_M = 90^\circ$
FIG. 6. SAME AS FIGURE 3

$\text{Lat}_M = 0^\circ$

$\text{Long}_M = 135^\circ$
FIG. 7. SAME AS FIGURE 3

\[ \text{Lat}_M = 10^\circ \]
\[ \text{Long}_M = 135^\circ \]
FIG. 8. SAME AS FIGURE 3

\[ Lat_M = 20^\circ \]
\[ Long_M = 135^\circ \]
FIG. 9. SAME AS FIGURE 3

$\text{Lat}_M = 0^\circ$

$\text{Long}_M = 180^\circ$
FIG. 10. SAME AS FIGURE 3

$$Lat_M = 10^\circ$$

$$Long_M = 180^\circ$$
FIG. 11. SAME AS FIGURE 3

$Lat_M = 20^\circ$

$Long_M = 180^\circ$
**Fig. 12. Injection Conditions at Moon for Return to Woomera**

Moon at 28.4° Declination at Injection

Injection from Latitude, $\text{Lat}_M$, 0° and Longitude, $\text{Long}_M$, 90°
FIG. 13. SAME AS FIGURE 12

Lat. = 10°
Long. = 90°
FIG. 14. SAME AS FIGURE 12

Lat$_M$ = 20°
Long$_M$ = 90°
FIG. 15. SAME AS FIGURE 12

\[ \text{Lat}_M = 0^\circ \]
\[ \text{Long}_M = 135^\circ \]
FIG. 16. SAME AS FIGURE 12

$Lat_M = 10^\circ$

$Long_M = 135^\circ$
FIG. 17. SAME AS FIGURE 12

Lat\(_M\) = 20°
Long\(_M\) = 135°
FIG. 18. SAME AS FIGURE 12

Lat_M = 0°
Long_M = 180°
FIG. 19. SAME AS FIGURE 12

\[ Lat_M = 10^\circ \]
\[ Long_M = 180^\circ \]
FIG. 20. SAME AS FIGURE 12

\[ \text{Lat}_M = 20^\circ \]
\[ \text{Long}_M = 180^\circ \]
Lunar Azimuth, \(A_{ZM}\), Clockwise from North (deg)
Path Angle from Lunar Horizon, \(S_M\), (deg)
Transit Flight Time, \(T\), (hr)

Injection Velocity, \(v_M\), (km/sec)

**FIG. 24 INJECTION CONDITIONS AT MOON**
FOR RETURN TO SAN ANTONIO
Moon at Ascending Node at Injection

Injection from Latitude, \(Lat_M\), 0°
and Longitude, \(Long_M\), 90°
FIG. 22. SAME AS FIGURE 21

\[ \text{Lat}_M = 0^\circ \]

\[ \text{Long}_M = 135^\circ \]
FIG. 23. SAME AS FIGURE 21

\[ \text{Lat}_M = 0^\circ \]
\[ \text{Long}_M = 180^\circ \]
FIG. 24. INJECTION CONDITIONS AT MOON FOR RETURN TO WOOMERA

Moon at Ascending Node at Injection
Injection from Latitude Lat_M 0° and Longitude Long_M 90°
FIG. 25. SAME AS FIGURE 24

$\text{Lat}_M = 0^\circ$
$\text{Long}_M = 135^\circ$
FIG. 26. SAME AS FIGURE 24

\[ \text{Lat}_M = 0^\circ \]
\[ \text{Long}_M = 180^\circ \]
Acceleration at Ignition = 4.1 m/s²
Acceleration at Injection = 5.3 to 5.7 m/s²

FIG. 27. OPTIMUM INJECTION PATH ANGLE AS A FUNCTION OF INJECTION VELOCITY MAGNITUDE FOR A TYPICAL LUNAR RETURN VEHICLE.
FIG. 28. VELOCITY PENALTY
FOR DEVIATION FROM
OPTIMUM VELOCITY DIRECTION
FIG. 29. CHARACTERISTIC VELOCITY REQUIRED FOR RETURN TO WOOMERA, AUSTRALIA, FROM CIRCULAR ORBIT ABOUT LUNAR EQUATOR.
For Orbit Period of 2 hr 8 min
Flight Time from Nominal Injection to Reentry = 77 hr
Nominal Injection Velocity = 24.7 km/sec

FIG. 30. VELOCITY PENALTY FOR DEPARTURE FROM EQUATORIAL MOON ORBIT AT NON-OPTIMUM TIME.
APPROVAL

RETURN FLIGHT TO EARTH
FROM LUNAR ORBIT

Arthur J. Schwaniger

The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

ORIGINATOR

Arthur J. Schwaniger
Ch, Space Projects Section III

APPROVAL

R. F. Hoelker
Ch, Future Projects Branch

E. D. Geissler
Director, Aeroballistics Division