

NASA TM X-55434

# GO, NO-GO FOR APOLLO BASED ON ORBITAL LIFE TIME

FACILITY FORM 602

N66-23614	
(ACCESSION NUMBER)	(THRU)
17	1
(PAGES)	(CODE)
TMX-55434	30
(NASA CR OR TMX OR AD NUMBER)	(CATEGORY)

GPO PRICE \$ \_\_\_\_\_

CFSTI PRICE(S) \$ \_\_\_\_\_

Hard copy (HC) 1.00

Microfiche (MF) .50

BY  
# 653 July 65  
**F. O. VONBUN**

NOVEMBER 1965



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ABSTRACT

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A simple expression for the go, no-go criterion is derived in analytical form. This provides a better understanding of the problems involved which is lacking when large computer programs are used. A comparison between "slide rule" results and computer results is indicated on Figure 2.

An example is given using a 185 km near circular parking orbit. It is shown that a  $3\sigma$  speed error of 6 m/s and a  $3\sigma$  flight path angle error of 4.5 mrad result in a  $3\sigma$  perigee error of 30 km which is tolerable when a 5 day orbital life time of the Apollo (SIVB + Service + Command Module) is required.

Quaker

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# GO, NO-GO FOR APOLLO BASED ON ORBITAL LIFE TIME

## INTRODUCTION

The purpose of this paper is to derive, in simple form, an analytical expression of the go, no-go criterion for the insertion of the Apollo spacecraft into an earth parking orbit. This means in essence the determination of the maximum perigee height error ( $3\sigma h_p$ ) which can be allowed when a certain orbital lifetime is required after insertion. By doing so, a better understanding of the important physical parameters involved can be obtained, which is normally lacking when large computer programs are used.

Considering only the "orbital" safety of the astronauts\*, one can state that the inclination of the parking orbit is of no influence; and only the spacecraft orbital lifetime, which is independent of the inclination, is of importance. The lifetime, on the other hand, is directly related to the perigee height  $h_p$  as shown in References 1 and 2.

In brief, if the injection height  $h$  is assumed to be 185 km (100 nmi), the minimum perigee height  $h_p$  can be determined providing a certain orbital lifetime is assumed (see References 1 and 2). Please note that all the considerations here are made for near circular parking orbits, which are required for the Apollo spacecraft.

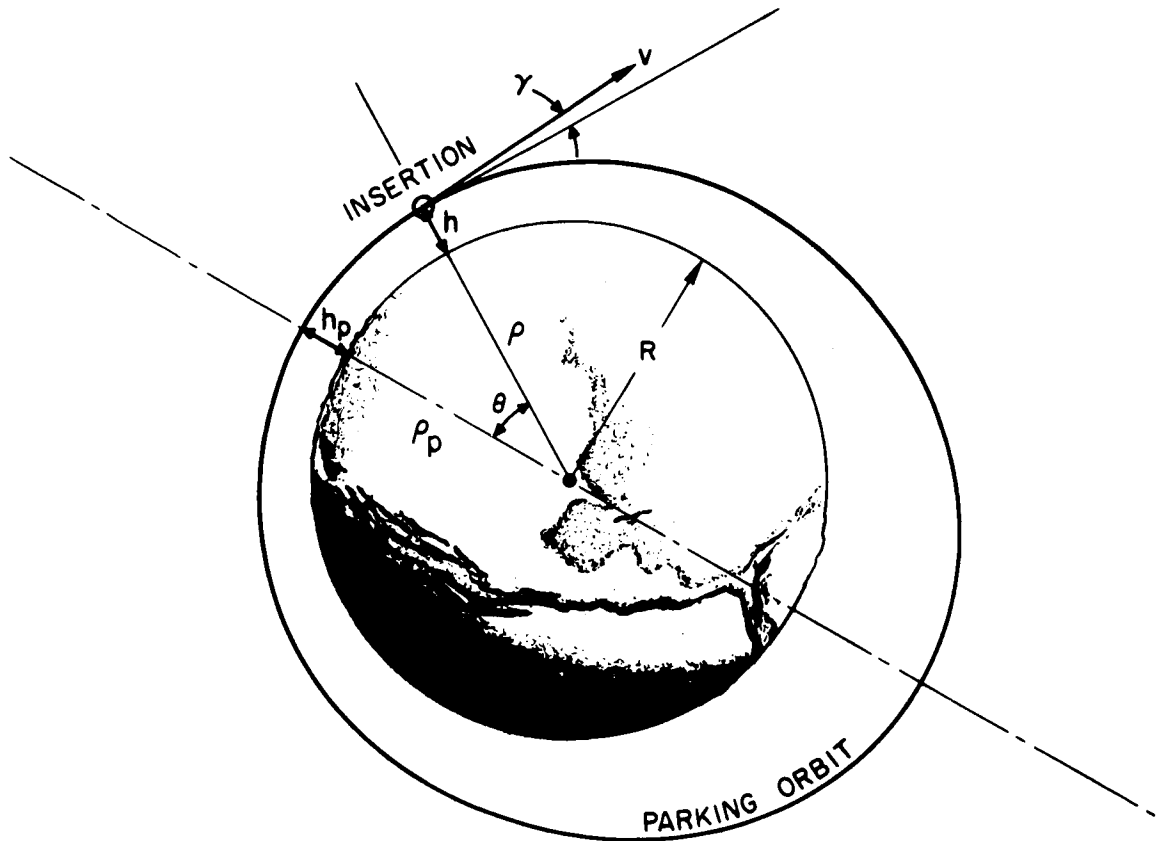
This analysis can be accomplished by deriving an expression for the variational equation of the perigee height (equation (10)), and in addition, it can be used to derive a simple error equation as expressed by (11).

### I. VARIATIONAL EQUATION FOR THE PERIGEE HEIGHT $h_p$ .

Since the go, no-go decision depends on the perigee height variation  $\Delta h_p$  and ultimately its error  $\sigma h_p$ , this value will be derived. Knowing this quantity as a function of the insertion parameters,  $\rho$ ,  $v$ , and  $\gamma$  as shown in Figure 1, the go, no-go criterion can be established; or the errors in  $\rho$ ,  $v$ , and  $\gamma$ , can be determined for a certain error  $\sigma h_p$  of the perigee height  $h_p$  corresponding to a given or previously established orbital lifetime.

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\*Emergency landing areas are not included as they depend on the inclination of the orbits.



- $h_p$  --- PERIGEE HEIGHT
- $h$  --- INSERTION HEIGHT
- $v$  --- INSERTION SPEED
- $\gamma$  --- INSERTION FLIGHT PATH ANGLE
- $R$  --- EARTH RADIUS
- $\rho$  ---  $(R+h)$
- $\rho_p$  ---  $(R+h_p)$
- $\theta$  --- TRUE ANAMOLI

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## ORBITAL INSERTION GEOMETRY

Figure 1

Starting with the fundamental equations for the ellipse, which can be found in all basic textbooks (see References 1, 3, 4, and 5) on orbits, one can write:

a. The equation for the perigee radius

$$\rho_p = a(1 - e) \quad (1)$$

b. The vis viva integral

$$v^2 = \mu \left( \frac{2}{\rho} - \frac{1}{a} \right) \quad (2)$$

c. The equation for the eccentricity

$$(1 - e^2) = \frac{(v\rho \cos \gamma)^2}{a\mu} \quad (3)$$

d. The polar equation

$$\rho = \frac{a(1 - e^2)}{1 + e \cos \theta} \quad (4)$$

where  $\mu = 3.987 \times 10^5 \text{ km}^3/\text{sec}^2$  is the gravitational parameter of the earth (see References 1 and 6), and  $\theta$  is the true anomaly of the elliptic orbit under consideration.

A simple way of obtaining the variation (later the errors) in the perigee height is to vary equation (1), using  $\Delta\rho_p = \Delta h_p$  because of their linear relationship, that is

$$\Delta h_p = \Delta a (1 - e) - a\Delta e \quad (5)$$

The expressions for  $\Delta a$  and  $\Delta e$  can be obtained by varying equations (2) and (3) respectively.

From (2) one obtains for  $\Delta a$ , using  $\Delta h = \Delta q$  in a straight forward manner:

$$\Delta a = \frac{2a^2}{\mu} v\Delta v + \frac{2a^2}{\rho^2} \Delta h \quad (6)$$

Similarly one obtains from equation (3) for  $\Delta e$  after some manipulation and updating equation (6), the following expression:

$$\Delta e = \left( \frac{1-e^2}{e} \right) \left[ \frac{a}{\mu} v\Delta v + \left( \frac{a}{\rho} - 1 \right) \left( \frac{\Delta h}{\rho} - \left( \frac{\Delta v}{v} \right) + \text{tg} \gamma \Delta \gamma \right] \quad (7)$$

Equations (6) and (7) can now be inserted into (5) yielding after proper collection of terms:

$$\begin{aligned} \Delta h_p = & (1-e) a \left( \frac{\Delta v}{v} \right) \left[ 2 \frac{av^2}{\mu} - \left( \frac{av^2}{\mu} - 1 \right) \cdot \left( \frac{1+e}{e} \right) \right] + \\ & + (1-e) \frac{a}{\rho} \Delta h \left[ 2 \frac{a}{\rho} - \left( \frac{a}{\rho} - 1 \right) \cdot \left( \frac{1+e}{e} \right) \right] + \\ & - (1-e^2) a \left( \frac{\text{tg} \gamma}{e} \right) \Delta \gamma \end{aligned} \quad (8)$$

The expressions in the brackets of the right side of equation (8) can be evaluated using the expression  $(a/\rho)$  from (4). Inserting these into (8) yields using  $e \ll 1$ , (say between 0 and 0.05):

$$\begin{aligned} \Delta h_p \doteq & \rho \left( \frac{\Delta v}{v} \right) 2 (1 - \cos \theta) + \Delta h (2 - \cos \theta) + \\ & - \rho \cdot \sin \theta \Delta \gamma \end{aligned} \quad (9)$$

where  $1/e \cdot \operatorname{tg} \gamma \doteq \sin \theta$  was assumed to be valued over the range of  $e$ .

From

$$\lim_{\substack{e \rightarrow 0 \\ \gamma \rightarrow 0}} \left( \frac{1}{e} \operatorname{tg} \gamma \right) \doteq 1$$

one obtains the values for  $\sin \theta = 1$  or  $\cos \theta = 0$ . The variational equation (9) for  $\Delta h_p$  thus can be written as:

$$\Delta h_p \doteq 2\Delta h + 2(R+h) \left( \frac{\Delta v}{v} \right) - (R+h) \Delta \gamma \quad (10)$$

## II. THE ERROR IN THE ORBITAL PERIGEE HEIGHT $\sigma_{h_p}$ .

Assuming zero covariances, the error in perigee height  $\sigma_{h_p}$  can be written as the sum of the squares of the variational expressions, (10), that is

$$\sigma_{h_p} = (R+h) \left[ 4 \left( \frac{\sigma_h}{R+h} \right)^2 + 4 \left( \frac{\sigma_v}{v} \right)^2 + \sigma_\gamma^2 \right]^{1/2} \quad (11)$$

This equation is presented in graph form in Figure 2, for a typical Apollo parking orbit,  $e \doteq 0$ ,  $h = 185$  km. It is shown in References 7 and 8 that the neglect of the covariances always gives conservative estimate of the errors volume.

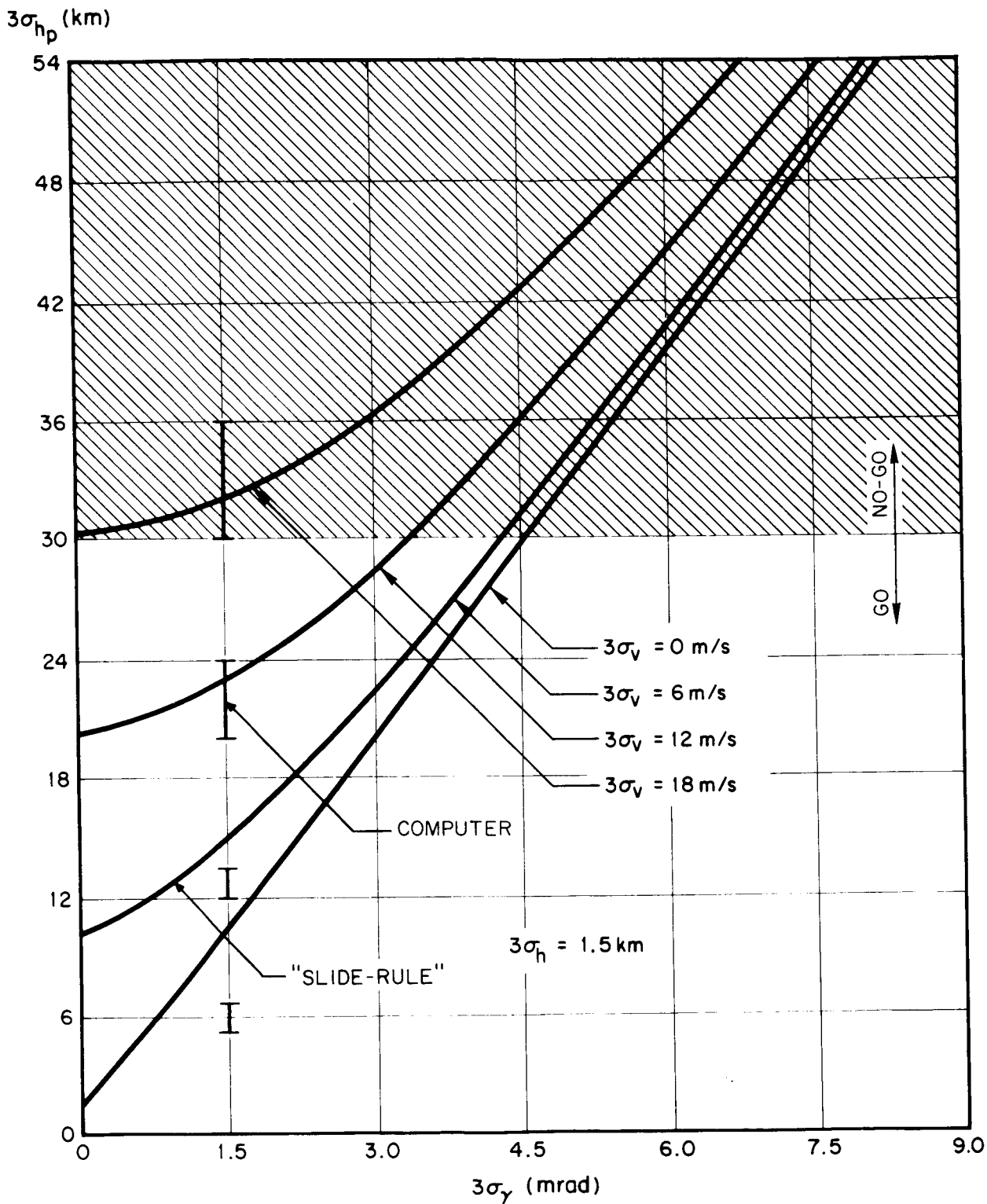
The corresponding values obtained with a computer program in conjunction with a Monte Carlo approach are shown in Figure 2. This substantiates what was mentioned in the previous paragraph.

## III. CRITERION FOR GO, NO-GO IN SIMPLE FORM.

The Apollo spacecraft has a weight of approximately 91,000 kg at the time of insertion into the earth parking orbit, and a frontal area of about 35 m<sup>2</sup>.

Using data as given in References 1 and 2, one obtains a lifetime of approximately 30 days for a 185 km (100 nmi) orbit, which corresponds to the area previously mentioned. For a 160 km (88 nmi) orbit, the lifetime is approximately 9 days (respectively). See Figures 3 and 3a.

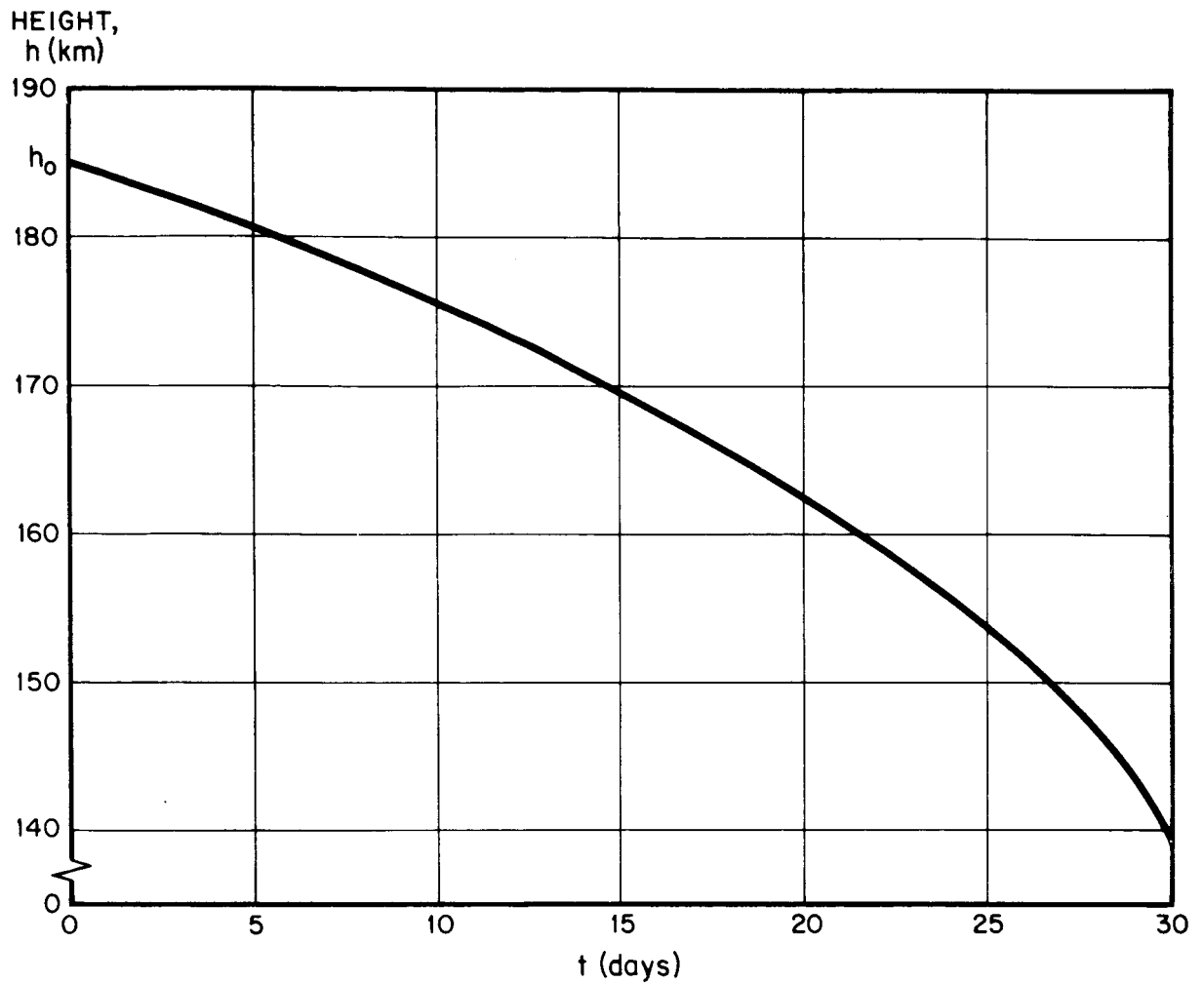




PERIGEE ERROR FOR APOLLO PARKING ORBITS  
 $(e \neq 0, h_0 = 185 \text{ km})$

Figure 2

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INITIAL HEIGHT,  $h_0 = 185$  km

INITIAL ECCENTRICITY,  $e_0 \doteq 0$

INITIAL AIR DENSITY,  $\rho_0 = 2.42 \times 10^{-10}$  kg/m<sup>3</sup>

FRONTAL AREA,  $A_1 = 35$  m<sup>2</sup>

WEIGHT IN ORBIT,  $W = 91,000$  kg

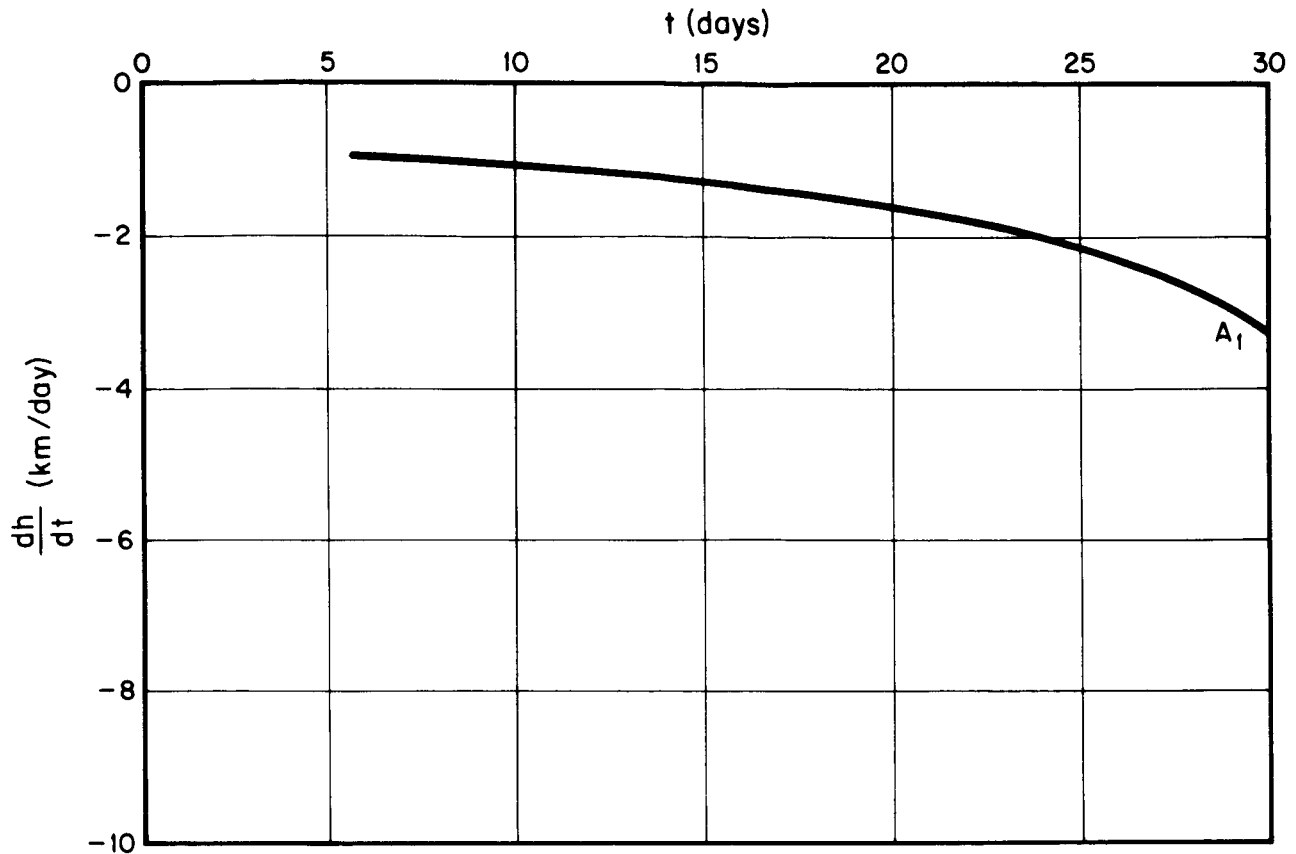
DRAG COEFFICIENT,  $C_D = 2$

$$\frac{C_D A_1}{W} = 0.769 \times 10^{-3} \text{ m}^2/\text{kg}$$

HEIGHT OF THE APOLLO PARKING ORBIT AFTER INSERTION  
(Perkins, Ref. 2)

Figure 3

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INITIAL HEIGHT,  $h_0 = 185$  km

INITIAL ECCENTRICITY,  $e_0 \doteq 0$

INITIAL AIR DENSITY,  $\rho_0 = 2.42 \times 10^{-10}$  kg/m<sup>3</sup>

FRONTAL AREA,  $A_1 = 35$  m<sup>2</sup>

WEIGHT IN ORBIT,  $w = 91,000$  kg

DRAG COEFFICIENT,  $C_D = 2$

$$\frac{C_D A_1}{w} = 0.769 \times 10^{-3} \text{ m}^2/\text{kg}$$

HEIGHT RATES FOR THE APOLLO PARKING ORBIT  
(Perkins, Ref. 2)

Figure 3a

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As an example, assume that for a go, no-go decision a 5-day orbital lifetime is required. Using a 185 km (100 nmi) circular orbit as the nominal parking orbit, one could tolerate a  $3\sigma h_p$  error of approximately 30 km from the lifetime considerations. (See Figure 3.) Using Figure 2, one finds that a  $3\sigma$  speed error up to 6 m/s and a  $3\sigma$  flight path angle error up to 4.5 mrad are tolerable; or a  $3\sigma$  speed error of 12 m/s and a  $3\sigma$  flight path angle error of 3 mrad are also within the  $3\sigma h_p$  error bounds. In all the cases, the perigee height error of  $3\sigma h_p$  was assumed to be 1.5 km. The value for the insertion height can be easily obtained as shown in Reference 9 ( $1\sigma h \doteq 500$  m). Inspection of Figure 2 shows that the error in insertion height does not play an important role;  $3\sigma h = 1.5$  km or  $3\sigma h = 3$  km does not alter the conclusions reached. In Reference 9 the velocity errors  $1\sigma v$  are presented for different noise, bias and station locations (ships) using one tracking sample per second without a priori knowledge of the orbit.

Using the  $3\sigma$  error analysis results just mentioned, for speed and flight path angles given in Reference 10, for a 40 second tracking interval and 5 tracking samples per second, that is 4 m/s and 2 mrad, shows that the associated  $3\sigma$  error in perigee height is well within the assumed limits. Therefore, a safe go, no-go can be made using a tracking time of approximately 40 seconds with 5 tracking samples per second. The values stated are based upon radar measurements with a range noise error of  $1\sigma r = 10$  m and angular noise errors of  $1\sigma\alpha = 1\sigma\epsilon = 0.4$  mrad combined with bias errors twice the stated noise values respectively. A detailed table on the tracking errors used for Apollo is presented in Reference 11, the Apollo Navigation Working Group Report.

It is interesting to note that the location errors for the tracker (a ship in case of Apollo, see References 9 and 10) do not influence the velocity and flight path angle, which are the vital parameters for this decision. As clearly stated in Table 3.4 of Reference 10, speed, flight path angle and altitude errors are not affected much by the station location error. A numerical example is given for better understanding. Assume the insertion ship tracks the spacecraft with its C-band radar using the errors quoted before and in addition has a navigational error of  $3\sigma = 4$  km (in N and E direction). The corresponding error in spacecraft speed  $3\sigma_v \doteq 2$  m/s and that in flight path angle  $\sigma\gamma \doteq 1$  mrad after 2 minutes of tracking. This can also be seen from Figures 5 and 6 in Reference 9, which show that the navigation accuracies needed for the Apollo insertion ship (References 9 and 10) are only moderate for a go, no-go decision.

#### ACKNOWLEDGMENT

The author is indebted to Dr. B. Kruger for positive criticism and valuable suggestions.

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