## VARIATIONS

## OF THE LUNAR ORBITAL PARAMETERS

 OF THE APOLLO CSM-MODULEGPO PRICE \$ $\qquad$
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# VARIATIONS OF THE LUNAR ORBITAL PARAMETERS OF THE APOLLO CSM-MODULE 

by<br>T. P. Timer*

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Goddard Space Flight Center
Greenbelt, Maryland

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SUMMARY


This report investigates the variation of the orbital parameters for the forty hour period after insertion into two reference Apollo Lunar Parking Orbits ${ }^{1}$. It is found that while the variation of most of the parameters under the influence of a triaxial Moon and other perturbing forces is small, the change in pericynthion height is significant. In Table 1 the amount of change in the various elements during this time is shown.

The most pertinent piece of information derived from this investigation is with regard to the choice of hypothetical lunar landing sites on the surface of the Moon (see Figure 1). Choosing a point on the earthward side of the trace which satisfies the constraint that it has a longitude of between $315^{\circ}$ and $45^{\circ}$ and a latitude of $+5^{\circ}$ and $-5^{\circ}$ with respect to a selenographic coordinate system one has a hypothetical landing site. It is found that any subsequent trace of the sub-module point at the same longitude will always lie within 0.5 in latitude of the initial landing site chosen during the forty hours under consideration. Therefore, landing sites chosen in the above manner meet the two requirements as described in Apollo Navigation Technical Report ${ }^{2}$ which states that the two constrains on the lunar parking orbit mission are as follows:
(1) the plane of the trajectory must contain a landing site vector on the earth side of the Moon which has a longitude of between $315^{\circ}$ and $45^{\circ}$ and a latitude of between $+5^{\circ}$ and $-5^{\circ}$ in selenographic coordinates.
(2) the plane of the orbit must be oriented so that the lunar landing site doesn't move out of the orbital plane more than 0.5 during the period of 3 to 39 hours after lunar insertion.

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## INTRODUCTION

The basic concept behind this report was to see how the orbital parameters for the two reference lunar parking orbits behaved during the forty hour period after lunar insertion. To this end two 7090/7094 computer programs were used to generate this information which is presented as two series of graphs. Series A graphs are associated with the 1968 reference trajectory. Series B graphs are associated with the 1969 reference trajectory. It should be noted at the outset that several coordinate systems are in use throughout this report. The three coordinate systems in use are the following:

| center: | selenocentric |
| ---: | :--- |
| fundamental plane: | parallel to the earth's equatorial plane |
| x-axis: | mean equinox of date |
| center: | selenocentric |
| fundamental plane: | true equator (earth) of date |
| x-axis: | true equinox of date |
| center: | selenocentric |
| fundamental plane: | true equator of the moon |
| x-axis: | intersection of the prime meridian and the |
|  | equator of the moon. |

The two computer programs used as sources of data are the Jet Propulsion Laboratory's Space Trajectory program and the Goddard Interplanetary Trajectory Encke Method (Version 12-C) program. Both programs numerically integrate the orbit by the Encke Method hence one program serves as an independent check of the validity of the data generated by the other program.

## I. General Description of the Reference Trajectories

The lunar parking orbit is constrained to be near-circular with a semimajor axis of 1018.5 nautical miles ( 1886 km ) with an altitude above the lunar surface of 80 nautical miles $\pm 5$ nautical miles ( $148 \pm 9 \mathrm{~km}$ ). The duration of this trajectory is constrained to be between two to sixty hours ${ }^{3}$. In this study both cases meet these requirements. The insertion conditions for the two reference ${ }^{4}$ trajectories under consideration (study) are:

Date of Lunar Insertion
Reference Trajectory \#1
May 9, 1968
$15 \mathrm{hrs} / 54.0 \mathrm{mins} / 56.610 \mathrm{secs}$.


## Date of Lunar Insertion Reference Trajectory \#2

September 20, 1969 $5 \mathrm{hrs} / 10 \mathrm{mins} / 12.176 \mathrm{secs}$.

$$
\begin{aligned}
& \mathrm{X}=+165.639 \\
& \mathbf{Y}=-96 \mathrm{n} . \mathrm{mi} . \\
& \mathrm{Z}=-415.861 \\
& \\
& \\
& \mathbf{X}=-5208 \mathrm{n} . \mathrm{mi} \\
& \dot{\mathrm{X}}
\end{aligned}
$$

$$
\dot{X}=-\quad 1.587 \quad 513 \quad 284 \mathrm{~km} / \mathrm{sec}
$$

$$
\dot{\mathrm{Y}}=-\quad 0.249 \quad 035 \quad 253 \mathrm{~km} / \mathrm{sec}
$$

$$
\dot{\mathrm{Z}}=-\quad 0.081 \quad 751 \quad 788 \mathrm{~km} / \mathrm{sec}
$$

conversion factors: p4-8

## COORDINATE SYSTEM

center: selenocentric
fundamental plane: plane parallel to earth's mean equator of date
$x$-axis: mean equinox of date at insertion.

[^2]In terms of orbital elements, the insertion conditions, using the same coordinate system above, are:

| Reference Trajectory \#A$1968$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{a}=$ | 1878.569 | 2 |  | km |
| e | 0.000 | 406 | 737 | 41 |
| i $=$ | 158.183 | 342 |  |  |
| $\Omega=$ | 180.056 | 736 |  |  |
| $\omega=$ | 357.743 | 31 |  |  |
| $\mathrm{m}=$ | 359.196 | 573 |  |  |

## Reference Trajectory \#B <br> 1969

| $\mathrm{a}=$ | 1894.578 | 3 |  | km |
| :--- | ---: | :--- | :--- | :--- |
| $\mathrm{e}=$ | 0.000 | 419 | 706 | 10 |
| $\mathrm{i}=$ | 155.804 | 726 |  |  |
| $\Omega=$ | 182.414 | 087 |  |  |
| $\omega=$ | 262.877 | 900 |  |  |
| $\mathrm{~m}=$ | 0.000 | 824 |  | 8 |

## II. Discussion of the Graphical Results

The series of graphs A-1 through A-6, and B-1 through B-6 are related to the reference trajectories $A$ and $B$ respectively and show similar trends. Hence only the first set of graphs will be discussed in detail with much of the discussion being applicable to both sets.

By inspecting graphs A-1 to A-4 the graphs of the semi-major axis, eccentricity, inclination, and longitude of the ascending node it can be seen that the orbit chosen is relatively a very stable orbit. The semi-major axis varies at most about 0.5 kilometers from its original length during the 40 hours of investigation. The inclination changes less than 0.05 degrees. The eccentricity is plotted to a scale with a factor $10^{-3}$. Its fluctuations during the forty hour period are of the order of magnitude of $0.3 \times 10^{-3}$ as a maximum. The longitude of the ascending node for all practical purposes changes linearly with respect to time, varying less than 0.05 degrees during the time interval considered. The purpose of the preceding discussion was to show quantitatively how these various elements varied with time. Figure A-5 and Figure A-6, however, provide the most useful information directly affecting a lunar parking orbit. In Figure A-5, height (radius vector minus the mean lunar radius) versus hours from epoch, shows how the height varies periodically with the period being the time for one circumlunar orbit. This variation is less than 0.15 kilometers. Figure A-6 shows the limiting cases, the first and the last orbit data, of the trace of the command module on the lunar surface or the lunar sub-module point. It can be seen that for the worst case by choosing a particular point on the trace of the first trajectory and then choosing a point on the trace of the last trajectory at the same longitude, the variation in latitude would be at worst 0.2 . A considerable portion of the time, this variation is less than 0.2 . In graph B-5 a small secular difference is evident in the value of the ascending node as computed by the JPL
and Goddard Item programs. The reason for this slight difference has not been ascertained except that it is not due to numerical differences in the values of the physical parameters used by the program. (See Appendix A)

## CONCLUSIONS

(1) Based on the graphical data presented in this report, either the 1968 or 1969 reference trajectory could provide an orbit stable enough to meet the mission requirements of having suitable landing sites on the lunar surface which do not vary more than 0.5 in latitude from the original landing site. This information is presented in graphs A-6 and B-6 respectively where arbitrary hypothetical landing sites have been chosen.
(2) The use of two computer programs, one to serve as an independent check on the data generated by the other, is justified as the inter comparison of the data on the various graphs indicates.
(3) During this lunar parking orbit phase it would seem that the orbital path may be considered a constant with regard to three station USBS ground tracking. This statement is based on the knowledge that $3 \sigma$ errors in position including both measurement noise and station bias errors are 8,500 meters. 5 The change in the semi-major axis as found by this report is at most 500 meters which would be undiscernible since it is less than the accuracy of the position measurements made.

In Table 1 the maximum changes in the parameters for the two cases are shown. Essentially, the results are identical.

[^3]Table 1-Maximum Changes in Orbital Parameters

|  | $\Delta \mathrm{a}$ | $\Delta \mathrm{e}$ | $\Delta \mathrm{i}$ |
| :--- | :---: | :---: | :---: |
| Case \#1 | 0.5 km | $0.3 \times 10^{-3}$ | 0.05 |
| 1968 Data | 0.5 |  |  |
| Case \#2 <br> 1969 Data | 0.45 km | $0.3 \times 10^{-3}$ | 0.05 |


|  | $\Delta \Omega$ | $\Delta \mathrm{Hp} *$ | $\Delta$ Latitude |
| :--- | :---: | :---: | :---: |
| Case \#1 |  |  |  |
| 1968 Data | 0.05 | 0.15 km | 0.2 |
| Case \#2 <br> 1969 Data | 0.20 | 0.20 km | 0.5 |

*Hp = pericynthion height.


Figure 1

INJECTION CONDITIONS
COORDINATE SYSTEM

Fiwure A-1 Semi-major Axis (kilometers) Vs Hours from Epoch


Figure A-3 Inclination (degrees) Vs Hours from Epoch
2


Equatorial Plane $\}$| Item |
| :--- |
| Data |

PL Data

| GODDARD SPACE FLIGHT CENTER |
| :--- |
| MISSION ANALSII OFFICE |
| Feb 1966 |

Figure A-4 Longitude of the Ascending Node (degrees) Vs Hours from Epoch

COORDINATE SYSTEM
 Findamental Plane : Parallel to the Earth's Equatorial Plane $\}$ Data X-Axis
$\left.\begin{array}{ll}\text { Center } & : \text { Selenocentric } \\ \text { Fundamental Plane }: & \text { True Equator of Date }\end{array}\right\}$ JPL Data
X-Axis : True Equinox of Date
GODDARD SPACE FLIGHT CENTER
MISSION ANALYSIS OFFICE
Figure A-5 Height (kilometers) Vs Hours from Epoch


Figure B-1 Semi-major Axis (kilometers) Vs Hours from Epoch

Figure B-3 Inclination (degrees) Vs Hours from Epoch

Figure B-5 Height (kilometers) Vs Hours from Epoch


APPENDIX A
constants

| $\begin{gathered} \text { JPL } \\ \text { NAME } \end{gathered}$ | QUANTITY | JPL VALUE | GSFC ITEM VALUE | ANWG APOLLO NAME | ANWG APOLLO VALUE \& QUANITTY |
| :---: | :---: | :---: | :---: | :---: | :---: |
| GME | gravitational coefficient for earth | $3.9860320 \times 10^{5} \mathrm{~km}^{3} / \mathrm{sec}^{2}$ | $19.9094165 \mathrm{er}^{3} / \mathrm{hr}^{2}$ | $\mu$ EARTH | $\begin{aligned} & \text { gravitational parameter for } \\ & \text { earth } \\ & 3.986032 \times 10^{4} \mathrm{~m}^{3} / \mathrm{sec}^{2} \\ & 1.99094165 \times 10 \mathrm{er}^{3} / \mathrm{hr}^{2} \end{aligned}$ |
| GMM | gravitational coefficient for moon | $4.9007588 \times 10^{3} \mathrm{~km}^{3} / \mathrm{sec}^{2}$ | $0.2447828913 \mathrm{er}^{3} / \mathrm{hr}^{2}$ | ${ }_{\sim} \mathrm{MOON}$ | $\begin{aligned} & \text { gravitational parameter for } \\ & \text { moon } \\ & 4.90278 \times 10^{12} \mathrm{~m}^{3} / \mathrm{sec}^{2} \\ & 0.244883757 \mathrm{er}^{3} / \mathrm{hr}^{2} \end{aligned}$ |
| GMS | gravitational coefficient for sun | $1.3271544 \times 10^{11} \mathrm{~km}^{3}-\mathrm{sec}^{2}$ | $0.513736472 \times 10^{-6} \mathrm{au}^{3} / \mathrm{hr}^{2}$ | $\mu$ SUN | gravitational parameters for $\begin{aligned} & \operatorname{sun} \\ & 1.32715445 \times 10^{20} \mathrm{~m}^{3} / \mathrm{sec}^{2} \\ & 6.62886568 \times 10^{6} \mathrm{er}^{3} / \mathrm{hr}^{2} \end{aligned}$ |
| GMA | gravitational coefficient for mars | $4.2977799 \times 10^{44} \mathrm{~km}^{3} / \mathrm{sec}^{2}$ | $0.1663654117 \times 10^{-12} \mathrm{au}^{3} / \mathrm{hr}^{2}$ | n.a. | n.a. |
| GMV | gravitational coefficient for venus | $3.2476950 \times 10^{5} \mathrm{~km}^{3} / \mathrm{sec}^{2}$ | $0.1257170343 \times 10^{-11} \mathrm{au}^{3} / \mathrm{hr}^{2}$ | n.a. | n.a. |
| GMB | gravitational coefficient for barycenter | $4.0350395 \times 10^{5} \mathrm{~km}^{3} / \mathrm{sec}^{2}$ | not in GSFC Item | n.a. | n.a. |
| GMJ | gravitational coefficient for jupiter | $1.2671060 \times 10^{8} \mathrm{~km}^{3} / \mathrm{sec}^{2}$ | $0.4904925907 \times 1 \sigma^{9} \mathrm{au}^{3} / \mathrm{hr}^{2}$ | n.a. | n.a. |
| J | coefficient of the second harmonic in earth's oblatness | $1.6234500 \times 10^{-3}$ <br> non-dimensional | $\begin{aligned} & 1082.3 \times 10^{-6} \\ & \text { nosi-dimensional } \end{aligned}$ | $\stackrel{\mathrm{J}}{2.0}$ | constants associated with the gravitational potential of the $\begin{aligned} & \text { earth } \\ & 1.623 \times 10^{-3} \\ & -1082.30 \times 10^{-6} \end{aligned}$ |
| H | coefficient of the third harmonic in earth's oblatness | $\begin{aligned} & -0.57499999 \times 10^{-5} \\ & \text { non-dimensional } \end{aligned}$ | $\begin{aligned} & -2.3 \times 10^{-6} \\ & \text { non-dimensional } \end{aligned}$ | $\stackrel{H}{\mathrm{C}_{3.0}}$ | constants associated with the gräitational potential for the earth $\begin{aligned} & \text { earch } \\ & -0.575 \times 10^{-5} \\ & 2.3 \times 10^{-6} \end{aligned}$ |
| D | coefficient of the fourth harmonic in earth's oblatness | $0.78749999 \times 10^{-5}$ non-dimensional | $\begin{aligned} & -1.8 \times 10^{-6} \\ & \text { non-dimensional } \end{aligned}$ | $\stackrel{D}{C}_{4,0}$ | constants associated with the gravitational potential of the earth $1.8 \times 10^{-6} 10^{-5}$ |
| RE | earth's radius to be used in earth's oblate potential | 6378.165 km | 6378.165 km | Re | earth radius 6378.165 km |
| G | universal gravitational constant for lunar oblatness | $4.902643 \times 10^{12} \mathrm{~m}^{3} / \mathrm{sec}^{2}$ | $0.2447829 \mathrm{er}^{3} / \mathrm{hr}^{2}$ | $\begin{aligned} & \mu \text { MOON } \\ & \mu \text { MOON } \end{aligned}$ | gravitational parameters for the moon <br> $2.44883757 \times 10^{-1} \mathrm{er}^{3} / \mathrm{hr}^{2}$ <br> $4.902778 \times 10^{12} \mathrm{~m}^{3} / \mathrm{sec}^{2}$ |
| A | moment of inertia for the moon to be used in the lunar oblate potential | $0.88745998 \times 10^{29} \mathrm{kg-km}{ }^{2}$ | $0.88746 \times 10^{35} \mathrm{~kg}-\mathrm{m}^{2}$ | A | moment of inertia about principal rotational axis $0.8878179834 \times 10^{35} \mathrm{~kg}-\mathrm{m}^{2}$ |
| B | moment of inertia for the moon to be used in the lunar oblate potential | $0.88763998 \times 10^{29} \mathrm{~kg}-\mathrm{km}^{2}$ | $0.88764 \times 10^{35} \mathrm{~kg}-\mathrm{m}^{2}$ | B | moment of inertia about principal axis $0.8880019542 \times 10^{35} \mathrm{~kg}-\mathrm{m}^{2}$ |
| C | moment of inertia for the moon to be used in the lunar oblate potential | $0.88800998 \times 10^{29} \mathrm{~kg}-\mathrm{km}^{2}$ | $0.8801 \times 10^{35} \mathrm{~kg}-\mathrm{m}^{2}$ | C | moment of inertia about principal axis <br> $0.888 \quad 3697817 \times 10^{35} \mathrm{~kg}-\mathrm{m}^{2}$ |
| OME | rotational rate of the earth | $0.004178 \quad 0741 \mathrm{deg} / \mathrm{sec}$ | $0.26251617081 \mathrm{rads} / \mathrm{hr}$ | $\omega$ | angular velocity of the earth's rotation <br> $4.17807416 \times 10^{-3} \mathrm{deg} / \mathrm{sec}$ <br> $0.262516142 \mathrm{rad} / \mathrm{hr}$ |
| au | astronomical unit to convert planetary ephemerides to km | $1.4959900 \times 10^{8} \mathrm{~km} / \mathrm{au}$ | $1.49599 \times 10^{8} \mathrm{~km} / \mathrm{au}$ | au | astronomical unit $1.495990 \quad 00 \times 10^{11} \mathrm{~m}$ |
| REM | earth radius to convert lunar ephemerides to kilometers | 6378.165 km | 6378.165 km | n.a. | n.a. |

a.a. $=$ not applicuble


[^0]:    *This report prepared under the direction of the Mission Analysis Office, Code 507, by Mr. T. P. Timer of the Advanced Projects Branch, Data Systems Division, Code 543.2

[^1]:    ${ }^{1}$ pp. 3-10 Apollo Navigation Group Technical Report 65-AN-1.0, February 5, 1965.
    ${ }^{2}$ pp. 3-6, 3-7.

[^2]:    $\overline{3_{\mathrm{p}} \text { 3-7. }}$
    ${ }^{4}$ p 3-11.

[^3]:    5Apollo Navigation Working Group Technical Report No. 65-AN-2.0, pp. 6-5, September 1, 1965.

