

## Memorandum

DATE: 27 FEB 1968
68-FM47-60

FROM : FM/Mission Planning and Analysis Division
sUBJECT: Formulation for Ground Processing of Onboard Data to Determine Lunar Module attitude

The attached MSC Internal Note No. 68-FM-52 presents the basic requirements (equations and logic) for the RTCC processor to determine the Lunar Module attitude using telemetered data from LM radar systems.


The Flight Software Branch concurs with the above recommendation and requests IBM to proceed accordingly.


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UNITED STATES GOVERNMENT
Memorandum

DATE: 17 APR 1968
68-FM47-127

FROM : FM/Mission Planning and Analysis Division
subject: Change Sheet for MSC Internal Note 68-FM-52

1. Reference: MSC Internal Note 68-FM-52, "RTCC Requirements for Mission G: Lunar Module Attitude Determination Using Onboard Observations," by FM 4/B. F. Cockrell, February 23, 1968.
2. This memo specifies revisions of the referenced document to incorporate the time tag offset for downlinked data, and to correct errors in the flow charts.
3. The time tagging of the $R R$ data is done when the $C D U^{\prime} s$ are read which is 5 to 10 milliseconds after the doppler count is completed; however, the observation set will be time tagged in the middle of the doppler count. For this reason an offset of about 50 milliseconds will be added.
4. The attached pages from the original report were changed to reflect the revisions to the convergence processor flow charts.


James C. McPherson, Chief Mathematical Physics Branch

The Flight Software Branch consurs with the above recommendation and requests $I B M$ to proceed accordingly $\mathrm{y}_{\text {a }}$


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## CHANGE SHEET

FOR
MSC INTERNAL NOTE 68-FM-52, DATED FEBRUARY 23, 1968 RTCC REQUIREMENTS FOR MISSION G: LUNAR MODULE AITITIUDE DETERMINATION USING ONBOARD OBSERVATIONS

By B. F. Cockrell
Change 1
April 12, 1968

$\underset{\text { (with }}{\text { Page }} 1$ of $\frac{4}{4}$ osures)

After the attached enclosures, which are replacements, are inserted, insert this CHANGE SHEFT between the cover and title page and write on the cover "CHANGE 1 inserted".

1. Replace pages 12 and 13.


Flow chart 2.- Convergence processor.


Flow chart 2.- Convergence processor. - Continued

## PROJECT APOLLO

RTCC REQUIREMENTS FOR MISSION G: LUNAR MODULE ATTITUDE DETERMINATION USING ONBOARD OBSERVATIONS

By B. F. Cockrell
Mathematical Physics Branch

February 23, 1968

MISSION PLANNING AND ANALYSIS DIVISION

## NATIONAL AERONAUTICS AND SPACE ADMINISTRATION MANNED SPACECRAFT CENTER HOUSTON, TEXAS



# RTCC REQUIREMENTS FOR MISSION G: LUNAR MODULE ATTITUDE 

 DETERMINATION USING ONBOARD OBSERVATIONSBy B. F. Cockrell

SUMMARY AND INTRODUCTION


#### Abstract

The orientation of the LM is recorded just after lunar landing with respect to a mean fixed coordinate system and stored onboard. The preferred nominal mode of surface inertial measurement unit (IMU) alignment uses optical sightings on two stars with the alignment optical telescope. If it is found that an alignment cannot be made with the alignment optical telescope and if the stored alignment has changed due to LM settling, a separate LM attitude determination method must be available. This note presents a method for determining LM attitude on the lunar surface by processing rendezvous radar shaft and trunnion angle measurements. These angles relate the CSM-LM line of sight to the LM body axes. The ground Real-Time Computer Complex (RTCC) will process this data and a telemetered gravity vector, in body coordinates, to determine the attitude.

This note presents the formulation (basic requirements) for the RTCC program. This is a separate program from the Manned Space Flight Network (MSFN) data processor used for orbit determination. However, the data batching (preprocessor) is identical to the data batching for the Mission $G$ landing site determination program which is described in detail in reference 1 . The MSFN orbit determination processor and predictor (ref. 2) will be used to determine the $C M$ ephemeris over the landing site.


## PROCEDURE FOR PROCESSING ONBOARD RENDEZVOUS RADAR OBSERVATIONS TO DETERMINE LM ATTITUDE

The LM body orientation will be defined with respect to a local vertical coordinate system by three Euler rotations about the local vertical system axes. A knowledge of both CSM and LM positions is assumed. The rendezvous radar must track the CSM, and the rendezvous radar shaft and trunnion angles must be transmitted via downlink to earth. In addition, the astronaut will determine a gravity vector in body coordinates by monitoring the IMU accelerometers at two special orientations of the stable
member. This, too, must be transmitted to earth. The three rotations will be determined using a weighted least squares, three-element state by solving the following basic equation:

$$
\Delta F=\left(\sum_{i=1}^{n} A^{T} W A\right)^{-1}\left(\sum_{i=1}^{n} A^{T} W \Delta y\right)
$$

where

$$
\begin{aligned}
F & =\left(\alpha_{1}, \alpha_{2}, \alpha_{3}\right) \text { three rotations about the local vertical system axes } \\
y & =\text { observation (shaft or trunnion) } \\
A & =\frac{\partial y}{\partial F} \\
W & =\text { observation weight matrix } \\
\Delta y & =\text { observation residual (observed - computed) } \\
i & =\text { observation frame index }
\end{aligned}
$$

Flow charts 1 and 2 present the detailed logic for the program supervisor and convergence processor, respectively.

## PREPROCESSOR TO HANDLE TELEMETERED DATA

A preprocessor is required to handle the telemetered data since this data will not be handled by the preprocessor program used for normal ground tracking. The function of this routine is to multiply the incoming telemetered rendezvous radar data by the correct granularity constants and store the data into batches suitable for subsequent use by the attitude processor. This preprocessor and these data batches are the same as used for the LM position determination and are explained in detail in reference l. From these data batches, working batches will be generated which will have the following format.

Working Data Batch

| Batch ID | No. of Obs. frames |
| :--- | :---: |
| Time of observations |  |
| Shaft | Trunnion | | Observation |
| :--- |
| frame no. I |

## ATTITUDE START ROUTINE

The operator must select one of two modes for this routine. In the first mode all three rotations will be determined from rendezvous radar data. To select this mode the operator enters a starting estimate of all three rotations determining LM attitude. These may come from the attitude at landing plus any pilot input from evidence that the grease pencil mark on the LM window has moved during lunar stay. The second mode determines only the first rotation (azimuth) from rendezvous radar data. The two other rotations are computed as direct functions of a gravity vector in LM body coordinates. This gravity vector is determined by the pilot and transmitted to earth by telemetry. To select this mode, the operator enters a starting estimate of only the azimuth and the gravity vector. The solutions of the second and third Euler angles from this gravity vector are:

$$
\begin{aligned}
& \alpha_{2}=\sin ^{-1}\left(-g_{z}\right) \\
& \alpha_{3}=\tan ^{-1}\left(-\frac{g_{y}}{g_{x}}\right)
\end{aligned}
$$

where $\left(g_{x}, g_{y}, g_{z}\right)=$ unit gravity vector in body coordinates.

INITIALIZATION

In setting up onboard data for processing a single pass of data the operator specifies the following:

1. Batch ID to be processed - must be rendezvous radar batches.
2. CSM vector used to generate ephemeris.
(a) ID of previously determined CSM vector (OD ephemeris).
(b) Current CSM anchor vector.
3. LM position vector.
(a) Computed estimate from landing site determination routine.
(b) Primary navigation and guidance system vector.
(c) Abort guidance system vector.
4. Initial attitude - must be entered as defined below:
(a) For mode $I$, enter $\alpha_{1}, \alpha_{2}, \alpha_{3}$.
(b) For mode II, enter $\bar{g}_{\mathrm{b}}, a_{1}$.

Reference 3 should be consulted for details on the above general input description.

The operator can process a maximum of two batches of data at one time under the following conditions:

1. The MSFN determination of the CSM orbit should be equally good for both passes.
2. The LM must not have moved during the time between the batches.

This will be checked by comparing and displaying gravity vectors and their differences. A minimum of three gravity vectors will probably need to be downlinked for the following times:
(a) Prior to the first batch.
(b) Prior to the second batch but following the first.
(c) Following the second batch.

For processing two batches together the operator selects:

1. The two batch ID's.
2. Two CSM vectors, one for each batch.
3. The LM vector.
4. The initial attitude:
(a) For mode $I$, enter $\alpha_{1}, \alpha_{2}, \alpha_{3}$.
(b) For mode II, enter $\alpha_{1}$ and one of the three gravity vectors.

## STATE VECTOR

The three-element state $F$ for this problem will be defined as three positive rotations about a local vertical coordinate system. The local vertical system is centered at the $L M$ and has axes along the local vertical, in the direction of lunar north, and in the direction of lunar east. The three positive rotations are ordered as follows.

1. About local vertical, $\alpha_{1}$.
2. About displaced east, $\alpha_{2}$.
3. About displaced north, $\alpha_{3}$.

The transformation from local vertical to LM body coordinates is then defined.

$$
\begin{aligned}
R_{b} & =\left(\begin{array}{l}
X_{b} \\
Y_{b} \\
z_{b} \\
b
\end{array}\right) \\
& =\left(\begin{array}{cccc}
\cos \alpha_{3} & \sin \alpha_{3} & 0 \\
-\sin \alpha_{3} & \cos \alpha_{3} & 0 \\
0 & 0 & 1
\end{array}\right)\left(\begin{array}{ccc}
\cos \alpha_{2} & 0 & -\sin \alpha_{2} \\
0 & 1 & 0 \\
\sin \alpha_{2} & 0 & \cos \alpha_{2}
\end{array}\right)\left(\begin{array}{ccc}
1 & 0 & 0 \\
0 & \cos \alpha_{1} & \sin \alpha_{1} \\
0 & -\sin \alpha_{1} & \cos \alpha_{1}
\end{array}\right)\left(\begin{array}{l}
X_{L V} \\
Y_{L V} \\
Z_{L V}
\end{array}\right) \\
\text { or } R_{b} & =F(\alpha) R_{L V}
\end{aligned}
$$

where

$$
\begin{aligned}
& F(\alpha)_{11}=\cos \alpha_{3} \cos \alpha_{2} \\
& F(\alpha)_{12}=\sin \alpha_{3} \cos \alpha_{1}+\cos \alpha_{3} \sin \alpha_{2} \sin \alpha_{1} \\
& F(\alpha)_{13}=\sin \alpha_{3} \sin \alpha_{1}-\cos \alpha_{3} \sin \alpha_{2} \cos \alpha_{1} \\
& F(\alpha)_{21}=-\sin \alpha_{3} \cos \alpha_{2} \\
& F(\alpha)_{22}=\cos \alpha_{3} \cos \alpha_{1}-\sin \alpha_{3} \sin \alpha_{2} \sin \alpha_{1} \\
& F(\alpha)_{23}=\cos \alpha_{3} \sin \alpha_{1}+\sin \alpha_{3} \sin \alpha_{2} \cos \alpha_{1} \\
& F(\alpha)_{31}=\sin \alpha_{2} \\
& F(\alpha)_{32}=-\cos \alpha_{2} \sin \alpha_{1} \\
& F(\alpha)_{33}=\cos \alpha_{2} \cos \alpha_{1} \\
& \text { and } R_{L V} \text { is the LM Iocal vertical state. }
\end{aligned}
$$

## OBSERVATION WEIGHTS

Shaft and trunnion weights will be computed by the program as functions of the computed observations. However, the operator may manually enter a two-element weight coefficient which adjusts the weights relative to each other (shaft and trunnion). Nominally these coefficients will be unity.

## Observation Computations

The following equations are used to compute values to compare with rendezvous radar raw observations for residual computations. This requires the availability of a six-point $C M$ ephemeris in selenographic coordinates. The procedure is as follows for each observation time.

1. Define the $L M\left(R_{L M}\right)$ state in a moon-centered local vertical system by

$$
R_{L M}=\left(\begin{array}{l}
X_{L V} \\
Y_{L V} \\
Z_{L V}
\end{array}\right)=\left(\begin{array}{l}
r_{L M} \\
0 \\
0
\end{array}\right)
$$

where $r=L M$ radius in the $\phi \lambda r$ system.
2. Compute the CM state $\left(\mathrm{R}_{\mathrm{CM}}\right)$ in this system by the following transformation.

$$
R_{C M}=\left(\begin{array}{c}
X_{L V} \\
Y_{L V} \\
Z_{L V}
\end{array}\right)=\left(\begin{array}{clc}
\cos \phi \cos \lambda & \cos \phi \sin \lambda & \sin \phi \\
-\sin \lambda & \cos \lambda & 0 \\
-\sin \phi \cos \lambda & -\sin \phi \sin \lambda & \cos \phi
\end{array}\right)\left(\begin{array}{c}
X_{\mathrm{SG}} \\
\mathrm{Y}_{\mathrm{SG}} \\
\mathrm{Z}_{\mathrm{SG}}
\end{array}\right)
$$

3. Determine by interpolation the range vectors in this local vertical system for each LM rendezvous radar observation time.
4. Compute a unit range vector for each rendezvous radar observation time.

$$
\bar{\rho}_{L V}=\frac{\left(R_{C M}-R_{L M}\right)}{\mid R_{C M}-R_{L M}}
$$

5. Compute these vectors in the body system:

$$
\bar{\rho}_{b}=\left(\begin{array}{l}
X_{b} \\
Y_{b} \\
Z_{b}
\end{array}\right)=F(\alpha) \bar{\rho}_{L V}
$$

where $F(\alpha)$ is the transformation defined by the three Euler rotations $\alpha_{1}, \alpha_{2}, \alpha_{3}$.
6. With this body vector, the observations may be computed. The LM rendezvous radar rotates about two axes, the shaft ( $S$ ) axis and the trunnion ( $T$ ) axis. They are defined for a LM-CSM line-of-sight direction in the following manner.


$$
\begin{aligned}
& \tan S=\frac{X_{b}}{Z_{b}} \quad \text {, when } 40^{\circ} \leq S \leq 180^{\circ} \\
& \sin T=-Y_{b}, \text { when }|T| \leq 55^{\circ}
\end{aligned}
$$

## Partials for Onboard Data Processing

Earlier in the basic equation the matrix $A$ was defined as $\frac{\partial y}{\partial F}$, where $y$ is the observation and $F$ is the state. This matrix is a $2 \times 3$, and for observation of shaft and trunnion and a state of $\alpha_{1}, \alpha_{2}, \alpha_{3}$, the matrix is

$$
A=\left(\begin{array}{ccc}
\frac{\partial S}{\partial \alpha_{1}} & \frac{\partial S}{\partial \alpha_{2}} & \frac{\partial S}{\partial \alpha_{3}} \\
\frac{\partial T}{\partial \alpha_{1}} & \frac{\partial T}{\partial \alpha_{2}} & \frac{\partial T}{\partial \alpha_{3}}
\end{array}\right)
$$

The following equations are expressions for the six elements of this matrix, and $S$ and $T$ are computed values of the shaft and trunnion angles, respectively. The detailed derivation may be found in reference 4.

$$
\begin{aligned}
& \frac{\partial S}{\partial \alpha_{1}}=\sin \alpha_{3} \cos \alpha_{2}-\tan T \cos S \sin \alpha_{2}-\tan T \cos \alpha_{3} \cos \alpha_{2} \sin S \\
& \frac{\partial S}{\partial \alpha_{2}}=-\cos \alpha_{3}-\sin S \sin \alpha_{3} \tan T \\
& \frac{\partial S}{\partial \alpha_{3}}=-\tan T \cos S \\
& \frac{\partial T}{\partial \alpha_{1}}=\sin S \sin \alpha_{2}-\cos S \cos \alpha_{3} \cos \alpha_{2} \\
& \frac{\partial T}{\partial \alpha_{2}}=-\sin \alpha_{3} \cos S \\
& \frac{\partial T}{\partial \alpha_{3}}=\sin S
\end{aligned}
$$



Flow chart 1.-Supervisor logic.
Page 1 of 2


Flow chart 1.- Supervisor logic - Concluded.


Flow chart 2.-Convergence processor .
Page 1 of 3


Flow chart 2.- Convergence processor - Continued.


Flow chart 2.-Convergence processor - Concluded.

## REFERENCES

1. Flanagan, Paul F.; and Austin, George A.: RTCC Requirements for Mission G: Landing Site Determination Using Onboard Observations. MSC IN 68-FM-2, February 1, 1968.
2. Schiesser, Emil R.; Savely, Robert T.; deVezin, Howard G.; and Oles, Michael J.: Basic Equations and Logic for the Real-Time Ground Navigation Program for the Apollo Lunar Landing Mission. MSC IN 67-FM-51, May 31, 1967.
3. NASA: Real-Time Computer Program Requirements for Apollo C-V. NASA PHO-TR170A, March 17, 1967.
4. Cockrell, Bedford F.; and Pines, Samuel: Partial Derivatives Involving Rigid Rotations. MSC IN, to be published.
