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PROJECT GEMINI

DETERMINATION OF ORBITAL ARRIVAL TIMES AND THE RELATIVE
CONDITIONS BETWEEN THE AGENA AND GEMINI SPACECRAFT
VEHICLES AT AN ORBITAL COMPARISON POINT

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1. Introduction

A basic requirement of the Gemini real time mission program has been the development of techniques for the rapid determination of arrival times of the Spacecraft and Agena vehicles at specific or relative orbital locations. A specific orbital location would be an apogee or perigee, a "counter" crossing, or some latitude or longitude; and a relative orbital location would be an Agena phase match with a specific Spacecraft location. A second requirement of the program, related to the first, is the determination of relative conditions between the two vehicles at an orbital location, such as phase angle (measured from the common node of the two orbital planes) and height difference (measured after the Agena has been "backed" to phase match with the Spacecraft).

The several subroutines presented in this note represent methods of computation of the above requirements. The fundamental mathematical tool used in determination of arrival times is the analytic ephemeris generation (AEG) subroutine, which lends itself readily to orbital iterations of this nature. Once the arrival times have been established, the relative conditions are extracted from the orbital elements of both vehicles at these times. Thus, the arrival time and relative conditions subroutines are necessarily presented together. The Subroutines THETR, TIMA, TAUA and TIMFA are working programs and are in constant employment by RAB personnel.

2. Subroutine THETR

This subroutine determines the phase angle between the Agena and Spacecraft, measured as the difference in central angles from the common node of their orbital planes. The equation employed by Subroutine THETR is:

$$\theta_R = U_1 - U_2 - 2 \tan^{-1} \left\{ \tan\left(\frac{h_1 - h_2}{2}\right) \left[\frac{\sin \frac{1}{2} (I_1 + I_2 - \pi)}{\sin \frac{1}{2} (I_1 - I_2 + \pi)} \right] \right\}$$

where the above symbols and subscripts are defined in the Appendix. The arc tangent term in the equation represents the correction in phase angle for the "wedge angle" between the two orbital planes.

3. Subroutine TIMA

This subroutine employs the AEG and THETR subroutines to determine three arrival times: (1) the time of arrival of the Spacecraft at a counter latitude (or maneuver point), (2) the time of arrival of the Spacecraft at any desired argument of latitude, and (3) the time of Agena phase match with a given Spacecraft argument of latitude.

The Spacecraft counter latitude is referenced to the argument of latitude of an early Spacecraft apogee, and advances in time as the secular rate of the argument of perigee, as given by the equation:

$$U_C = U_{OC} + \dot{g}\Delta t$$

where \dot{g} is the addition of the drag and oblateness terms for the secular advance of perigee. Time of arrival of the Spacecraft at the counter then is simply the time when the argument of latitude of the Spacecraft is equal to the argument of latitude of the counter.

In order to advance the Spacecraft to a desired argument of latitude, the increment of central angle between its present location and the future location must be known, or

$$\Delta U_1 = u_C - u_0$$

The increment of time required for the Spacecraft to advance is given by:

$$\Delta t = \frac{\Delta U_1}{\dot{U}}$$

where:

$$\dot{U} = n + \dot{g}$$

When the AEG Subroutine is advanced through Δt , a new argument of latitude, $U(t + \Delta t)$, is obtained and:

$$\Delta U_2 = U_C - U(t + \Delta t)$$

where ΔU_2 is smaller than ΔU_1 . Thus, the iterative process is to shrink ΔU to zero through successive approximations of Δt . To find the Agena time of phase match with the Spacecraft argument of latitude, the quantity Θ_R is similarly reduced to zero.

4. Subroutine TAUA

The purpose of this subroutine is to determine the relative conditions of height difference and time "lag" between the Agena and Spacecraft, where the Agena must be "backed" to phase match with the Spacecraft at a desired comparison point. First, the Spacecraft is advanced in time (as well as the Agena) using Subroutine TIMA, to some argument of latitude which is to be used as a comparison point. Subroutine THETR is called here to determine the phase angle between the two vehicles. Then, while holding the Spacecraft at the comparison point, the equation:

$$\Delta t = \theta_R / \dot{U}_1$$

is employed successively to determine the time increments through which Subroutine AEG will be called on to back the Agena to phase match with the Spacecraft.

5. Subroutine TIMFA

The time of arrival of the Spacecraft at its next apogee is computed in Subroutine TIMFA. Since its mean anomaly at apogee will be π , the iterative process employs the equation:

$$\Delta t = \frac{\pi - l_2}{n_2}$$

where l_2 is the Spacecraft's current mean anomaly, and n_2 is its anomalistic mean motion. The AEG Subroutine is called upon to advance the vehicle through Δt until this time increment is satisfactorily small. It should be mentioned that in earth-orbits mean anomaly becomes practically undefined for orbits with near or below 0.005 eccentricity.

6. Employment of the Subroutines

The following symbols are those employed as FORTRAN variables in the working subroutines TIMFA, TIMA, THETR, and TAUA. Their algebraic symbol definitions are given in the Appendix.

A, E, AI, G, H, AL = a, e, I, g, h, l; orbital elements

AN, GN, HN, GD = n, \dot{e} , \dot{h} , $\dot{\xi}_D$, secular rates

R, RD, U = r, \dot{r} , u

T = t, current time

CD, ARE, WHT = C_D , A_L , W_L ; drag constants

K2, K3 = k_2 , k_3 ; AEG control numbers

HDP, AIDP = h'' , I'' ; AEG mean elements

L_1 , L_2 = L_1 , L_2 ; vehicle indices

ANI, ANF = N_A (initial), N_A (final); apsis counter numbers

TFA, UOC = t_{fa} , u_{oc} ; arrival time and argument of latitude of a counter crossing

TA = t_A ; time lag between Agena and Spacecraft

DELTH = ΔH ; height difference between Agena and Spacecraft

OR = θ_R , phase angle

DOS = $\delta\theta$; angular tolerance on iterations

DELT = ξt , time tolerance on iterations

All of the above quantities are subscripted (two locations) variables except T, L_1 , L_2 , ANI, ANF, TFA, UOC, TA, DELTH, OR, DOS, and DELT.

The argument of subroutine TIMFA is written:

TIMFA (A, E, AI, G, H, AL, AN, GN, HN, R, U, GD, ANI, DELT, T, K2, TFA, UOC, RD, HDP, CD, ARE, WHT, K3, AIDP)

This subroutine is employed as follows:

a. First, the AEG Subroutine must be called and initialized, and values are given to ANI, DELT, T, K2, CD, ARE, WHT, and K3.

b. Subroutine TIMFA may now be called, with output of time of arrival and orbital elements of the Spacecraft at apogee, and with the "reference" counter argument of latitude (u_{o1}). The Agena is not advanced by TIMFA.

The argument of Subroutine TIMA is written:

TIMA (A, F, AI, G, H, AL, AN, GN, HN, R, U, GD, UOC, T, TFA, ANF, ANI, DCS, L₂, L₁, K2, RD, HDP, CD, ARE, K3, AIDP).

The initial employment of TIMA for determining Spacecraft and Agena arrival times at counter latitudes must be preceded by either TIFA for the determination of u_{OC} for a given N_A , or these quantities must be input (along with the initialization of the AEG). Other quantities which must be always defined are GD, T, TFA, ANF, (ANI), DOS, L₂, L₁, K2, CD, ARE, and K3. The different manners in which TIMA may be used are as follows:

- a. Spacecraft arrival time at a counter latitude -- set $L_2 = 2$, $L_1 = 1$, $ANF = N_{Af}$, $ANI = N_A$ and call the subroutine. Both vehicles are advanced to the arrival time.
- b. Agena arrival time at phase match with a Spacecraft counter latitude -- set $L_2 = L_1 = 1$ and continue as above. Only the Agena is advanced.
- c. Spacecraft arrival time at any argument of latitude -- set $UOC = u$, $L_2 = L_1 = 2$, $ANI = ANF$, and continue as above. Only the Spacecraft is advanced.

The argument of Subroutine THETR is written:

THETR (U, AI, H, OR)

Subroutine THETR is generally called after the Spacecraft and Agena have been advanced to a counter latitude by Subroutine TIMA. Values of U_i , AI_i and H_i are extracted for the computation of OR.

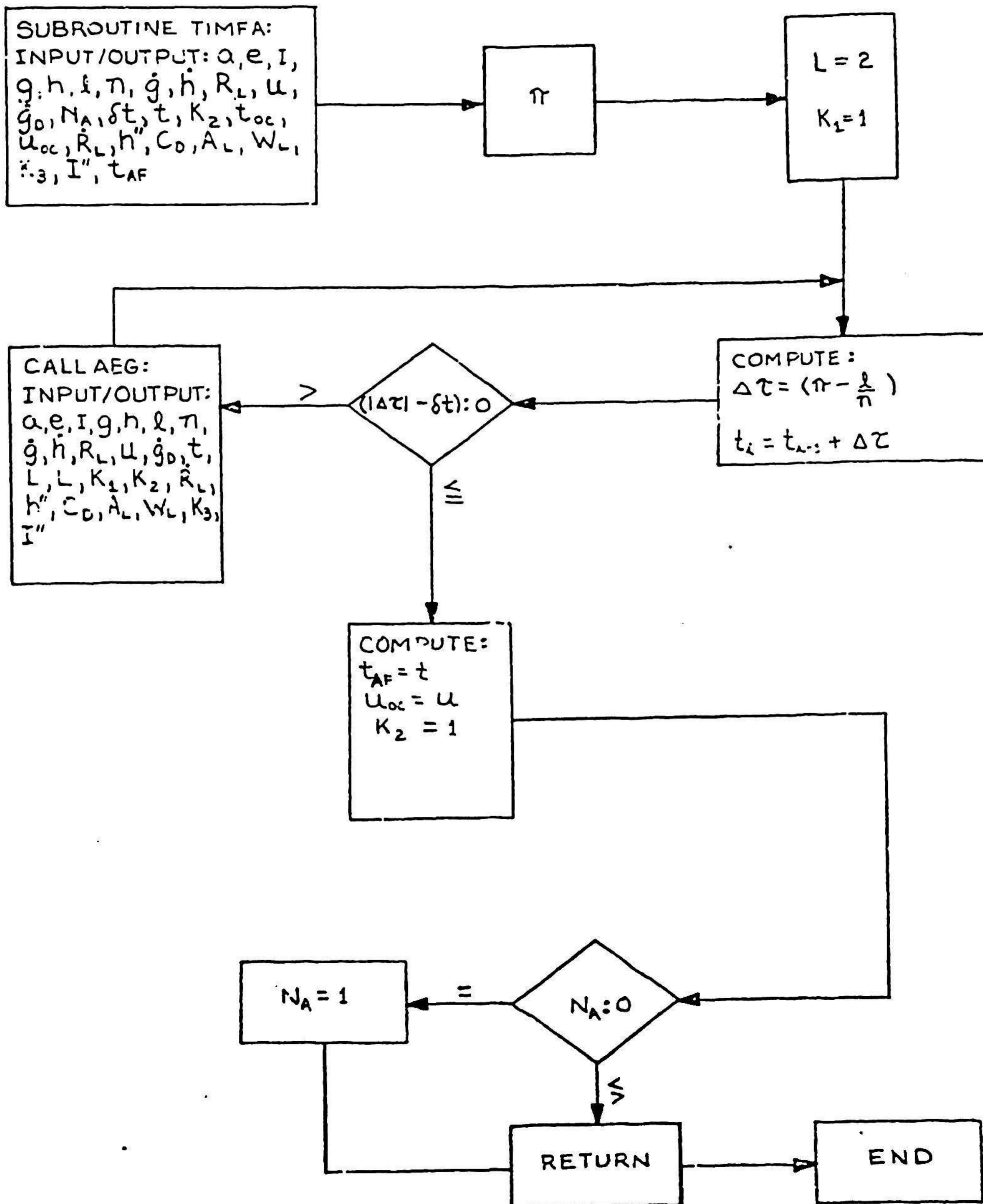
The argument of Subroutine TAUUA is written:

TAUUA (A, E, AI, G, H, AL, AN, GN, HN, R, U, GD, T, TFA, DOS, K2, TA, RD, HDP, DELTH, CD, ARE, WHT, K3, AIDP)

The required input values are the same as for TIMA except for UOC, L₁ and L₂. Once TAUUA has been called (usually immediately following TIMA), the subroutine will yield as output the relative quantities of TA and DELTH. The orbital element of both vehicles are returned as the same as those which came from TIMA.

APPENDIX

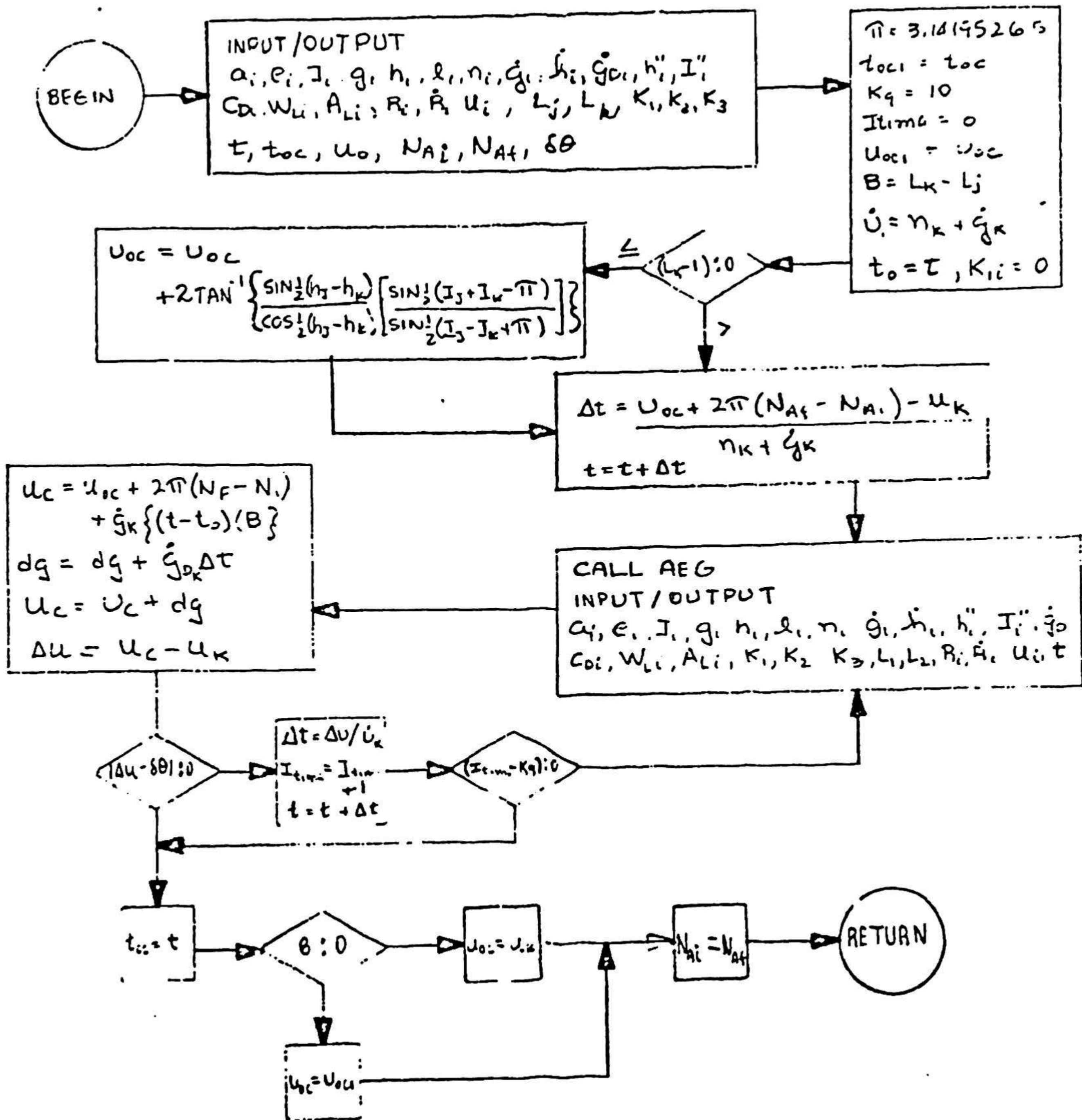
FLOW CHART OF SUBROUTINE TIMFA



LIST OF VARIABLES FOR SUBROUTINE TIMFA

VARIABLE	DEFINITION	UNIT
a	SEMI-MAJOR AXIS	FT
e	ECCENTRICITY	-
i	INCLINATION	RAD
g	ARGUMENT OF PERIGEE	
h	LONGITUDE OF ASCENDING NODE	RAD
l	MEAN ANOMALY	RAD
n	MEAN MOTION NUMBER	RAD/SEC
g-dot	ANGULAR RATE OF ARGUMENT OF PERIGEE	RAD/SEC
h-dot	ANGULAR RATE OF ASCENDING NODE	RAD/SEC
R _L	RADIUS OF VEHICLE	FT
u	ARGUMENT OF LATITUDE (g+f)	RAD
g-dot _D	RATE OF CHANGE OF ARGUMENT AT PERIGEE BECAUSE OF DRAG	RAD/SEC
N _A	COUNTER NUMBER	-
delta t	CONVERGENCE LIMIT ON TIME ITERATION	SEC
t	EPOCH	SEC
K ₂	DRAG CONTROL NUMBER (1 FOR DRAG , 0 FOR NO DRAG)	-
t _{oc}	TIME OF ARRIVAL AT COUNTER	SEC
u _{oc}	ARGUMENT OF LATITUDE OF COUNTER	RAD
R-dot _L	RADIUS RATE	FT/SEC
h ⁰	MEAN (SECULAR) LONGITUDE OF ASCENDING NODE	RAD
C _D	DRAG COEFFICIENT	-
A _L	FRONTAL AREA OF VEHICLE	FT ²
W _L	WEIGHT OF VEHICLE	LBS
K ₃	REDUCED AEG CONTROL NUMBER (0 IF COMPLETE AEG REQUIRED, 1 FOR REDUCED AEG)	-
I"	MEAN INCLINATION	RAD
t _{AF}	TIME OF SPACECRAFT ARRIVAL AT FIRST APOGEE	SEC

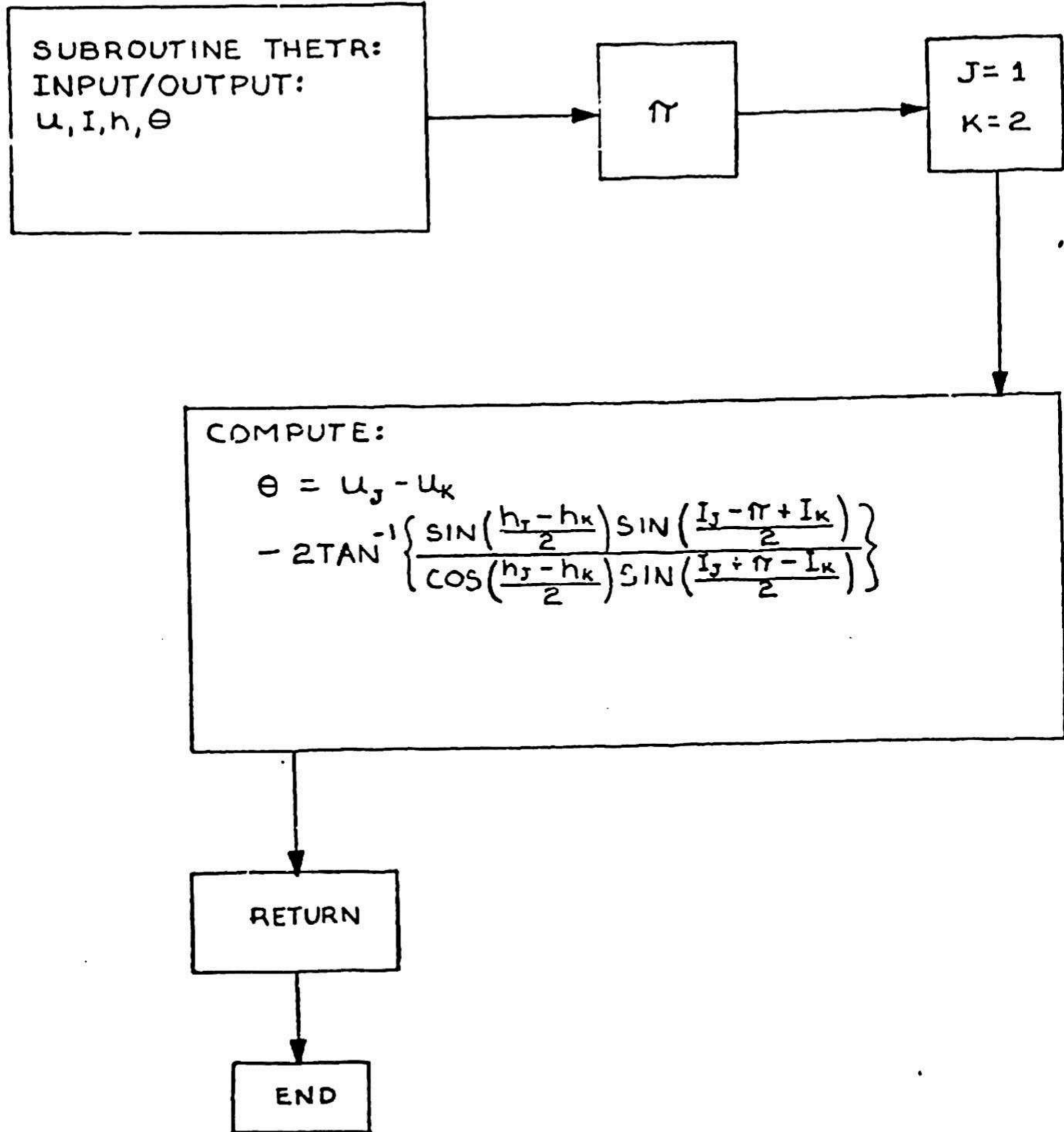
FLOW CHART OF SUBROUTINE TIMA



LIST OF VARIABLES FOR SUBROUTINE TIMA

VARIABLE	DEFINITION	UNIT
a	SEMI MAJOR AXIS	FT
e	ECCENTRICITY	
i	INCLINATION	RAD
g	ARGUMENT OF PERIGEE	RAD
h	LONGITUDE OF ASCENDING NODE	RAD
l	MEAN ANOMALY	RAD
n	MEAN MOTION NUMBER	RAD/SEC
g-dot	ANGULAR RATE OF ARGUMENT OF PERIGEE	RAD/SEC
h-dot	ANGULAR RATE OF ASCENDING NODE	RAD/SEC
R _L	RADIUS OF VEHICLE	FT
u	ARGUMENT OF LATITUDE (g+f)	RAD
g-dot-dot	RATE OF CHANGE OF ARGUMENT OF PERIGEE BECAUSE OF DRAG	RAD/SEC
u _α	ARGUMENT OF LATITUDE OF COUNTER EPOCH	SEC
t	EPOCH	SEC
t _α	TIME OF ARRIVAL AT COUNTER	SEC
N _F	FINAL COUNTER NUMBER	
N _I	INITIAL COUNTER NUMBER	
SE	CONVERGENCE LIMIT ON ANGULAR ITERATION	RAD
L ₁	VEHICLE NUMBER	
L ₂	VEHICLE NUMBER	
K ₂	DRAG CONTROL NUMBER (1, FOR DRAG; 0-NO DRAG)	F ² /SEC
R _L -dot	RADIUS RATE	
h''	MEAN (SECULAR) LONGITUDE OF ASCENDING NODE	RAD
C _D	DRAG COEFFICIENT	-
A _L	FRONTAL AREA OF VEHICLE	FT ²
W _L	WEIGHT OF VEHICLE	LBS
K ₃	REDUCED AEG CONTROL NUMBER (0 IF COMPLETE AEG REQUIRED, 1 FOR REDUCED AEG)	-
I''	MEAN INCLINATION	RAD

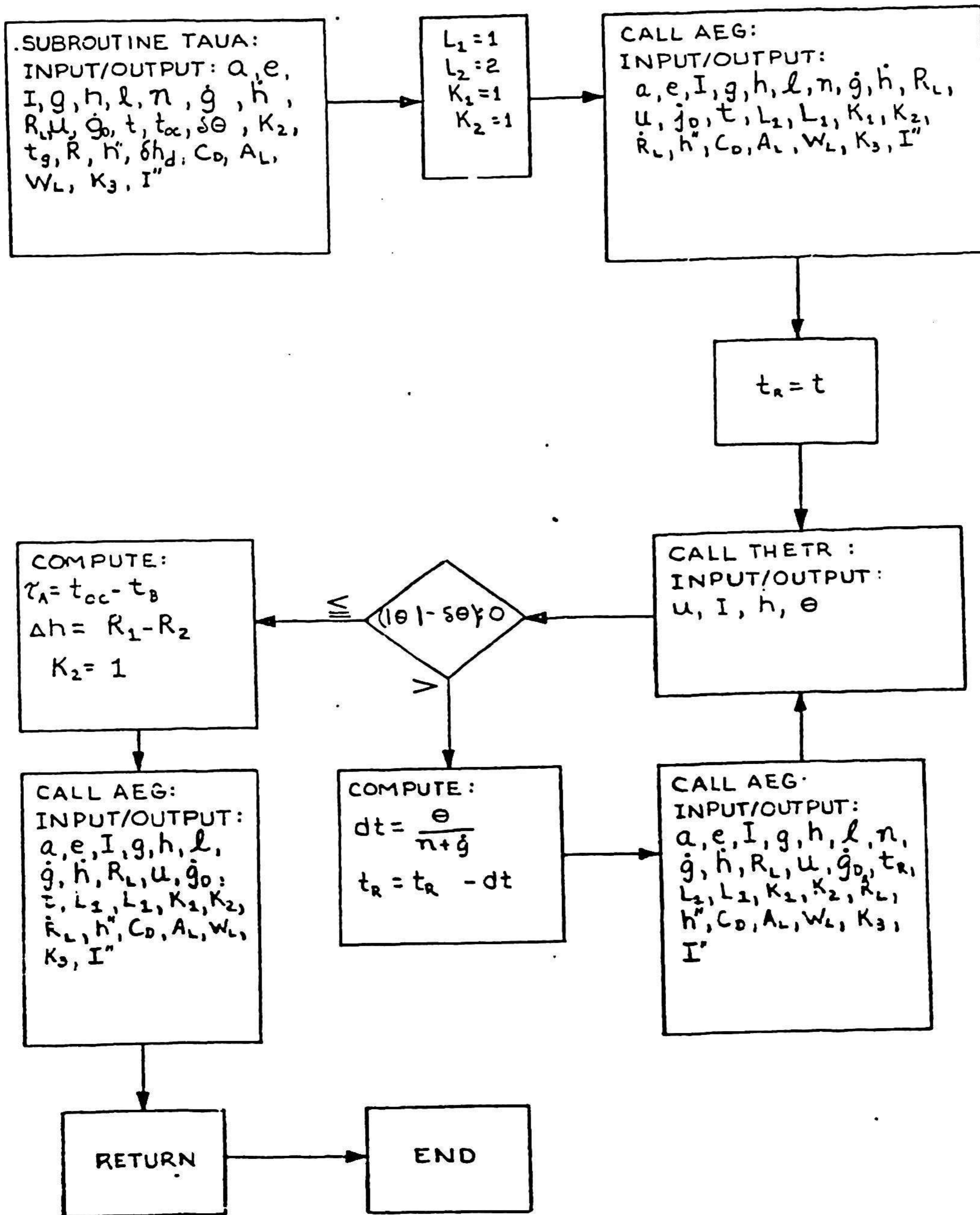
FLOW CHART OF SUBROUTINE THETR



LIST OF VARIABLES IN SUBROUTINE THETR

VARIABLE	DEFINITION	UNITS
u	ARGUMENT OF LATITUDE (g + f)	RAD
i	INCLINATION	RAD
h	LONGITUDE OF ASCENDING NODE	RAD
e	PHASE ANGLE	RAD

FLOW CHART FOR SUBROUTINE TAU A



LIST OF VARIABLES FOR SJBROUTINE TAU A

VARIABLE	DEFINITION	UNIT
q	SEMI-MAJOR AXIS	FT
e	ECCENTRICITY	-
i	INCLINATION	RAD
g	ARGUMENT OF PERIGEE	RAD
h	LONGITUDE OF ASCENDING NODE	RAD
l	MEAN ANOMALY	RAD
n	MEAN MOTION NUMBER	RAD/SEC
g-dot	ANGULAR RATE OF ARGUMENT OF PERIGEE	RAD/SEC
h-dot	ANGULAR RATE OF ASCENDING NODE	RAD/SEC
R _v	RADIUS OF VEHICLE	FT
u	ARGUMENT OF LATITUDE (g+f)	RAD
g-dot-dot	RATE OF CHANGE OF ARGUMENT OF PERIGEE BECAUSE OF DRAG	RAD/SEC
t	EPOCH	SEC
t _c	TIME OF ARRIVAL AT COUNTER	SEC
epsilon	CONVERGENCE LIMIT ON ANGULAR ITERATION	RAD
K ₂	DRAG CONTROL NUMBER (1 FOR DRAG, 0 FOR NO DRAG)	-
t _g	TIME LAG BETWEEN TWO VEHICLES	SEC
R-dot	RADIUS RATE	FT/SEC
h-dot-dot	MEAN (SECULAR) LONGITUDE OF ASCENDING NODE	RAD
delta h-dot	CONVERGENCE LIMIT ON HEIGHT ITERATION	RAD
C _D	DRAG COEFFICIENT	-
A _L	FRONTAL AREA OF VEHICLE	FT ²
W _L	WEIGHT OF VEHICLE	LBS
K _g	REDUCED AEG CONTROL NUMBER (0 IF COMPLETE AEG REQUIRED, 1 FOR REDUCED AEG)	-
I"	MEAN INCLINATION	RAD

INPUT FOR TIMFA

I. CONSTANT INPUT

a. Category

1. π

2. δt

II. VARIABLE INPUT

N_A, n_2, l_2

III. OUTPUT

$t_{FA}, u_{OC}, a_2, e_2, I_2, g_2, h_2, l_2, n_2, \dot{g}_2, \dot{h}_2, r_2, u_2, \dot{g}_{D2},$

$N_A, \dot{r}, \dot{h}''', I''$

IV. CONSTANT AND VARIABLE INPUT REQUIRED FOR:

AEG/DRAG

- A10 -

INPUT FOR THER

I. CONSTANT INPUT

a. Category

1. π

2. None

3. None

II. VARIABLE INPUT

u_i, I_i, h_i

III. OUTPUT

Θ_R

- All -
INPUT FOR TAU

I. CONSTANT INPUT

a. Category

1. None

2. $\delta\theta, \delta H_d$

3. None

II. VARIABLE INPUT

t_{OC}

III. OUTPUT

$t_g, \Delta H$

IV. CONSTANT AND VARIABLE INPUT REQUIRED FOR
AEG/DRAG and THETA

INPUT FOR TRA

I. CONSTANT INPUT

a. Category

1. π

2. K_g, θ

3. None

II. VARIABLE INPUT

$N_I, N_F, t, n_i, \dot{e}_i, L_1, L_2$

III. OUTPUT

$t_{OC}, u_{OC}, a_i, e_i, I_i, g_i, h_i, l_i, n_i, g_i, \dot{h}_i, r_i, u_i, g_{Di},$
 \dot{r}_i, h_i'', I_i''

IV. CONSTANT AND VARIABLE INPUT REQUIRED FOR

AEG/DRAG