## General Disclaimer One or more of the Following Statements may affect this Document

- This document has been reproduced from the best copy furnished by the organizational source. It is being released in the interest of making available as much information as possible.
- This document may contain data, which exceeds the sheet parameters. It was furnished in this condition by the organizational source and is the best copy available.
- This document may contain tone-on-tone or color graphs, charts and/or pictures, which have been reproduced in black and white.
- This document is paginated as submitted by the original source.
- Portions of this document are not fully legible due to the historical nature of some of the material. However, it is the best reproduction available from the original submission.

Produced by the NASA Center for Aerospace Information (CASI)

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION MSC INERNAL NOTE NO. C.F-R-69-3

JANUARY 22, 1969

## APOLLO MISSION D PERFORMANCE ANALYSIS OF RENDEZVOUS CHARTS



Flight Procedures Branch

## FLIGHT CREW SUPPORT DIVISION



## APOLLO MISSION D

## PERFORMANCE ANALYSIS

OF RENDEZVOUS CHARTS

Prepared by:

## $\frac{\text { Lanny B. } 27 \text { M ch }}{\substack{\text { Larry B Mort } \\ \text { Orbital Procedui Section }}}$

## Gore E. butch: <br> Jot E. Hutchins

Orbital Procedures Section

Approved by:


Part $C^{0}$ frame
Paul C. Kramer
Chief, Flight Procedures Branch


Chief, Flight Crew Support Division
Page
1.0 SUMMARY ..... 1-1
2.0 DISCUSSION ..... 2-1
2.1 Study Rules ..... 2-2
2.2 Digital Program ..... $2-3$
2.3 Charts ..... 2-4
2.4 Initial Conditions ..... 2-6
2.5 Errors in Sensors and Execution ..... $2-8$
2.6 Braking Schedule and LOS Angular Rate Deadbands ..... 2.-9
3.0 RESULTS ..... 3-1
3.1 Maneuver Values ..... 3-3
3.2 Miss Distance ..... 3-6
3.3 Translational $\Delta V$ Used ..... 3-7
3.4 Arrival Time at TPI ..... 3-8
$3.5 \Delta T$ from TPI to TPF ..... 3-9
4.0 REFERENCES ..... 4-1
5.0 FIGURES ..... 5-1
LIST OF ACRONYMS AND ARBREVIATIONS

| AGS | Abort Guidance System | TPI $\Delta \mathrm{V}_{\text {LOS }}$ | Along Line-of-Sight Component of TPI Solution |
| :---: | :---: | :---: | :---: |
| CDH | Constant Differential Height | TPI $\Delta V_{N}$ | Ncrmal to Line-of-Sight Component of TPI Solution |
| CES | Control Electronic System |  |  |
| CSI | Concentric Sequence Initiation | MCI $\Delta_{\text {LOS }}$ | Along Line-of-Sight Component of MCl Solution |
| CSM | Commrad and Service Module | MCl $\Delta V_{N}$ | Along Line-of-Sight Component of MC2 Solution |
| LM | Lunar Module | MC2 $\Delta V_{N}$ | Normal to Line-of-Sight Component of MC2 Solution |
| LOS | Line-of-Sight Between IM and CSM |  |  |
| MCl | First Mid-Course Correction | CSI $\Delta \Delta V_{H}$ | Error in Horizontal Component of CSI Solution |
| MC2 | Second Mid-Course Correction | CDH $\Delta \Delta \mathrm{V}_{V}$ | Error in Vertical Component of CJH Solution |
| TPI | Terminal Phase Initialization | $\mathrm{CDH} \quad \Delta \Delta \mathrm{~V}_{\mathrm{H}}$ | Error in Horizontal Component of CDH Solution |
| TPF | Terminal Phase Finalization | TPI $\Delta \Delta V_{\text {LOS }}$ | Error in Along Line-of-Sight Component of TPI Solution |
| $\Delta T$ | Change in Time | TPI $\Delta \Delta V_{N}$ | Error in Normal to Line-of-Sight Component of TPI Solution |
| $\Delta \mathrm{V}$ | Change in Velocity |  |  |
| $\sigma$ | Standard Deviation | MCl $\triangle \Delta V_{\text {LOS }}$ | Error in Along Line-of-Sight Component of MCl Solution. |
| $\operatorname{CSI} \Delta \mathrm{V}_{\mathrm{H}}$ | Horizontal Component of CSI Solution | MCl $\Delta \Delta \mathrm{V}_{\mathrm{N}}$ | Error in Normal to Line-of-Sight Component of MCl Solution |
| $\begin{aligned} & \text { CDH } \Delta V_{V} \\ & \text { CDH } \Delta V_{H} \end{aligned}$ | Vertical Component of CDH Solution <br> Horizontal Component of CDH Solution | $\text { MC2 } \Delta \Delta V_{\text {LOS }}$ | Error in Along Line-of-Sight Component of MC2 Solution |
|  |  | MC2 $\Delta \Delta V_{N}$ | Error is Normal to Line-of-Sight Component of MC2 Solution |

## LIST OF TABLES

NO TITLE Page
2-1 Initial Conditions ..... 2-6
2-2 Covariance Matrix ..... 2-7
3-1 Run Summary ..... 3-1
3-2 Maneuver Values ..... 3-4
3-3 Differences Between Chart Solution with Errors ..... 3-5 and Without Errors
3-4 Coordinates at Closest Approach ..... 3-6
3-5 Translation $\Delta V$ ..... 3-7

## LIST OF FIGURES

| NO. | TITLE | Page |
| :--- | :--- | :---: |
| 1 | CSI Chart | $5-1$ |
| 2 | CDH Chart | $5-3$ |
| 3 | TPI Chart | $5-4$ |
| 4 | MCI Chart | $5-5$ |
| 5 | MC2 Chart | $5-6$ |
| 6 | Polar Plot | $5-7$ |
| 7 | LM Centered Relative Motion Plot | $5-8$ |
| 8 | Braking Schedule | $5-9$ |
| 9 | Time of TPI | $5-10$ |
| 10 | Total $\Delta V$ | $5-11$ |
| 11 | $\Delta T$ from TPI to TPF | $5-12$ |

APOLLO MISSION D
PERFURMANCE ANALYSIS
OF RENDEZVOUS CHARTS
Summary
A digital analysis of the D Mission backup rendezvous charts has been performed to verify their ability to predict CSI, CDH, TPI, and midcourse corrections in the presence of system and application errors and trajectory dispersions. Procedures for data acquisition were as defined in the current LM Rendezvous Procedures Document (Reference 1). The charts simulated were those from the LM 3 onboard data package. The mission situation simulated represented the PNGS inoperative, rendezvous radar information available from the tapemeter, attitude data from the AGS and control by the CES. However, use of the CSI and CDH charts is presently ground ruled out by Reference 2. The study showed that the stiandard deviation of the arrival time at TPI was 5.5 minutes. The mean $\Delta T$ from TPI to intercept with braking and line-of-sight control was 35.8 minutes with a standard deviation of 1.5 minutes. If no braking or line-of-sight control was executed after the second midcourse correction, the mean miss distance would have been 0.6 n.m. with a standard deviation of $.35 \mathrm{n} . \mathrm{m}$. The mean total translational $\Delta V$ required after insertion was $158 \mathrm{ft} / \mathrm{sec}$ with a standard deviation of $20 \mathrm{ft} / \mathrm{sec}$.

This anslysis was performed to determine the ability of the charts to predict all maneuvers after insertion required to complete the LM-active $D$ mission rendezvous. One hundred independent sets of initial conditions (IC's) were selected by adding random errors to the 6 components of the nominal relative state vector between the IM and CSM. Half of the rejative error was incorporated into the inertial state vector of each vehicle.

The 100 cases were run including system and execution errors with braking and line-of-sight (LOS) control. The runs were repeated omitting braking and LOS control to obtain miss distance. The results of the 100 Monte Carlo runs were processed to obtain statistical data for the parameters of interest.

The first 25 cases were also run without system or application errors to establish the theoretical chart capabilities.

### 2.1 Study Rules

The following ground rules consistent with D Mission procedures and planning were used in this study:

1. CST: occurs 40 min 56 sec after insertion.
2. CDH occurs 44 min 26 sec after CSI.
3. TPI was assumed to occur 8 minutes after the elevation angle of 19.73 degrees was reached.
4. The first midcourse correction (MCL) occurred 10 minutes after TPI and the second midcourse correction (MC2) occurred at 22 minutes after TPI.
5. CSI and CDH were burned in local vertical coordinates using impulsive thrust.
6. TPI, MC1, MC2, braking, and LOS control were executed along and normal to the line-of-sight using finite thrust, burning each component individually.
7. 130 degrees of CSM orbit travel between TPI and TPF.
8. No out-of-plane corrections were made prior to LOS control during the braking phase.

### 2.2 Digital Program

In the analysis, functions describing the backup rendezvous charts were programed into a digital routine which integrated the equations of motion of two particles about an oblate planet. These functions allowed simulation of the procedures for using the backup rendezvous charts by incorporating elevation angle (AG8 address 304), range, and range rate from the tapemeter at the times called for by the backup data sequence. System errors were included In the data taken at each point and an appropriate error of application was included in each maneuver.

The runs included effects of both bias and random errors as defined in Section 2.5. Bias errors were selected by the program at the beginning of each run and held constant for that run. Random errors were selected at each point data were taken.

## 2.3 <br> Charts

The charis modeled in the study were those which will be carried on the $D$ Mission, and are shown in Figures 1-6. The data sequence was obtained from Reference $l$ and is summarized on the relative motion plot of the nominal trajectory from CSI-40 to TPF (Figure 7). 2.3.1 CSI

The CSI chart solution is based on a Maclaurin's expansion of four variables for the delta $V$ at CSI (range rate at 30,20 , and 10 minutes prior to CSI and range 10 minutes prior to CSI). The coefficients are determined by the simultaneous solution of several expansions, each representing a dispersed trajectory prior to CSI.
2.3 .2

## CDH

The CDH chart solution utilizes the sinusoidal time history of range rate variations from coellipticity and relative velocity errors from coellipticity. Range rate data for the CDH chart are taken 29, 18, and 7 minutes prior to CDH.

### 2.3.3 TPI and MCC

The TPI charts solve for the relative position and velocity at TPI resolved into normal and along the line-of-sight (IOS) coordinates. The measured relative conditons are differenced from the required conditions for intercept in 130 degrees of orbit travel. Information required for the TPI charts is the elevation angle at 8 and 5 minutes prior to TPI and range and range rate at 5 minutes before

IPI. In a similar manner, the MCC charts maintain the time of TPF consistent with the TPI maneuver. Data are taken for the midcourses 5 and 8 minutes after TPI for $M C 1$ and at 17 and 20 minutes after TPI for MC2. The same measurement sequence as used at TPI is used for both midcourse corrfctions.

### 2.4 Initial Conditons

The initial conditions were generated by perturbing the nominal state vectors of the LM and CSM with errors supplied by a relative covariance matrix. Half the relative error was applied to the state vector of each vehicle.

The nominal vectors relative to the LM orbital plane were derived from Reference 3 and are summarized as follows:

Table 2-1
Initial Condit:ons
Insertion +56 sec
( 95 hrs 41 min 48 sec )

| LM Altitude | 863730 feet |
| :--- | :--- |
| CSM Altitude | 798403 feet |
| LM Total Velocity | $25431.2 \mathrm{ft} / \mathrm{sec}$ |
| CSM Total Velocity | $25468.7 \mathrm{ft} / \mathrm{sec}$ |
| LM Flight Path Angle | -.00563 deg |
| CSM Flight Path Angle | -.00652 deg |
| In Plane Central Angle | .401 deg |
| LM Latitude | 1.4 deg |
| Out of Plane Distance | -31.3 feet |
| Out of Plane Velocity | $-.02 \mathrm{ft} / \mathrm{sec}$ |
| Heading Angle (relative to equator) | 29 deg |

The covariance matrix used for initialization of the Monte Carlo runs is as follows:

Table 2-2
Covariance Matrix
9725775. $-358310.0 \quad 11.8853 \quad 799.127-205.801-.043464$
-358310.0 12124470. $-6.36828-887.559199 .080 \quad .0249741$
$11.8853-6.36828$ 6262300. . $0189843-.00404720 .2459$
$799.127-887.559 \quad .0189843 \quad 11.35113-.42248 \cdot-.0000708$
$-205.801 \quad 199.080 \quad-.004047 \quad-.42248 \quad 3.202925 \quad .00001599$
$-0434646 \quad .029741 \quad 20.2459-.0000708 .000015992 .394625$
It was obtained by increasing between 9 and 25 times the diagonal elements of a post insertion covarionce matrix provided by Math Physics Branch.

### 2.5 Errors in Sensors and Execution

The lo errors in sensor and maneuver executions were:

1. System Errors
A. Noise
1) Range . $333 \%$
2) Range Rate
$.433 \%$ or $.433 \mathrm{ft} / \mathrm{sec}$ which ever is larger
3) Elevation Angle . 12 degree.
B. Biases and Drifts (constant for a given run)
4) Range Rate . $333 \mathrm{ft} / \mathrm{sec}$
5) Initial Pitch Bias
.1 deg (assumes calibrated COAS)
6) Pitch Drift Rate
$.23 \mathrm{deg} / \mathrm{hr}$
2. Execution Errors
A. Reading Tapemeter
1) Range Rate $.25 \mathrm{ft} / \mathrm{sec}$
2) Range
a) Outer Scale
2400 ft
b) Middle Scale
100 ft
B. Application of Burns
$.5 \mathrm{ft} / \mathrm{sec}($ per exis)
C. Time Measurements
1.0 sec

### 2.6 Braking Schedule and LOS Angular Rate Deadbands

The braking schedule used in this simulation consisted of five gates and a lower limit on the range rate. The first gate was at 13500 feet. At this point only LOS control was execut,ed because the allowed range rate was $80 \mathrm{ft} / \mathrm{sec}$. The second gate was at 6000 ft with an allowed range rate of $30 \mathrm{ft} / \mathrm{sec}$. The nominal range rate at this range was $29 \mathrm{ft} / \mathrm{sec}$. The remaining gates were $20 \mathrm{ft} / \mathrm{sec}$ at 3000 ft , $10 \mathrm{ft} / \mathrm{sec}$ at 1500 ft and $5 \mathrm{ft} / \mathrm{sec}$ at 500 ft . The lower range rate limit consisted of a straight line connecting $20 \mathrm{ft} / \mathrm{sec}$ at 13500 ft and $0 \mathrm{ft} / \mathrm{sec}$ at intercept.

Both the upper and lower range rate limits are shown in Figure 8. LOS control procedures were simulated by sampling inertial drift of the LOS inplane and normal to the orbit plane every 15 seconds beginning at a range of 13500 ft . When the $L O S$ rates exceeded $.3 \mathrm{mr} / \mathrm{sec}$ at a sampling time, thrust was applied in the appropriate axis in increments of 1 second until the LOS rate was reduced below the threshold. The 15 seconds were allowed to elapse before sampling again.

### 3.0 Results

Several sets of Monte Carlo runs were made to obtain statistical data for determination of the effects of errors, trajectory dispersions, and braking on the size of maneuvers, arrival time at $T P I$, and total translational $\triangle V$. The sets of runs are identified in the following table:

Table 3-1<br>Run Summary

| Set | Number <br> Of Runs | Errors in <br> Maneuver <br> Solutions | Maneuvers <br> Applied | LOS Control <br> and Braking |
| :---: | :---: | :---: | :---: | :---: |
| A | 100 | Yes | With Errors | Yes |
| B | 100 | Yes | With Errors | No |
| C | 25 | No | No Errors | No |
| D | 25 | No | No Errors | Yes |

Solutions for the maneuvers in SET A were obtained with and without errors so that the effect of sensor and reading errors on the chart solutions could be determined. However, all maneuvers for SET A were made usi.ti the solutions with errors. The runs for SET B were identical to SET A, but with braking and LOS control ommitted to establish miss distance.

SEIS C and $\dot{U}$ were run to establish baseline data for chart performance. It was felt that a reduced number of runs would suffice to obtain statistically meaningfal results since
only initial conditions were varied. Examination of significant parameters such as maneuver solution and miss distances revealed nearly normal statistical distributions, confirming the adequacy of the 25 muns for those sets. The runs for SETS $C$ and $D$ were made with the same initial conditions as the first 25 muns of SET A.

### 3.1 Maneuver Values

The nominal chart solutions, average, mean, and standard deviation for each maneuver in SET $A$ and SET $D$ are shown in Table 3-2 on Page 3-4. The data given for SET A are the solutions with errors. The average, mean, and standard devjation for the difference between the error solution and the no error solution computed for each maneuver in SET A are shown in Table 3-3 on Page 3-5.

It can be noted from the data on Tables $3-2$ and $3-3$ that the chart solutions with errors progressively decrease in accuracy from CSI to CDH to TPI. The trend then reverses with MCCl more accurate than TPI and MCC2 being the most accurate of the maneuvers.


Table 3-2


$\begin{array}{ll}\text { Maneuvers } \\ \text { CSI } & \Delta V_{H} \\ \text { CDH } & \Delta V_{V} \\ \text { CDHI } & \Delta V_{H} \\ \text { TPI } & \Delta V_{\text {LOS }} \\ \text { TPI } & \Delta V_{N} \\ \text { MCCI } & \Delta V_{\text {IOS }} \\ \text { MCC1 } & \Delta V_{N} \\ M C C 2 . ~ & \Delta V_{\text {LOS }} \\ \text { MCC2 } & \Delta V_{N}\end{array}$

## Table 3-3

Differences Between Chart Solutions With Errors and Without Errors in SET A

| Maneuver | Average $\mathrm{ft} / \mathrm{sec}$ | $\begin{array}{r} \text { Mean } \\ \mathrm{ft} / \mathrm{sec} \\ \hline \end{array}$ | Standard Deviation $\qquad$ |
| :---: | :---: | :---: | :---: |
| CSI $1 . j \mathrm{~V}_{\mathrm{H}}$ | . 79 | . 01 | 1.02 |
| $\mathrm{CDH} \mu \Delta \mathrm{V}_{\mathrm{V}}$ | . 94 | . 07 | 1.24 |
| CDH Ars $\mathrm{V}_{\mathrm{H}}$ | . 48 | -. 06 | . 59 |
| $T P I \Delta \Delta V_{\text {LOS }}$ | 2.31 | -. 04 | 2.89 |
| TPI $\Delta \Delta V_{N}$ | 2.06 | . . 08 | 2.57 |
| MCCldAV ${ }_{\text {LOS }}$ | 2.50 | -. 24 | 3.07 |
| $\mathrm{MCCl} / \triangle \Delta \mathrm{V}_{\mathrm{N}}$ | 1.19 | -. 14 | 1.45 |
| $M_{\text {MCC2AAV }}$ LOS | . 47 | -. 00 | . 59 |
| MCCL $\mathrm{AV}_{\mathrm{N}}$ | . 54 | . 02 | . 66 |

The data of Table 3-2 indicates the amount of error directly attributable to the sensor and reading errors listed in Section 2.5.

### 3.2 Miss Distance

The miss distance was established by sets $B$ and $C$. The average in plane miss distance at the point of closest approach for the 25 cases without errors (SEI C) was 513 feet, and for the 100 cases with errors (SET B) was 2505 feet.

The average, mean, and standard deviation of the components of the miss distance in a local vertical coordinate system with $X$ along the radius vector of the $L M, Z$ along the argular momentum vector of the $L M$, and Y completing the right-handed system were as follows:

Table 3-4
Coordinates at Closest Approach

| Axis | Average |  | Mean |  | Standard Deviation |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{aligned} & \text { SET B B } \\ & \text { Feet } \end{aligned}$ | $\begin{aligned} & \text { SETC C } \\ & \text { Feet } \end{aligned}$ | $\begin{aligned} & \text { SET B } \\ & \text { Fect } \end{aligned}$ | SET C <br> Feet | $\begin{aligned} & \text { SET B } \\ & \text { Feet } \end{aligned}$ | SET C <br> Feet |
| X | 1482 | 375 | -915 | -375 | 1802 | 219 |
| Y | 2020 | 350 | -802 | -350 | 2710 | 188 |
| Z | 1844 | 1508 | -124 | - 95 | 2332 | 1879 |

The man total translation $\Delta V$ used in the 25 cases without errors (SEI D) was $138.2 \mathrm{ft} / \mathrm{sec}$ with a standard deviation of $13.4 \mathrm{ft} / \mathrm{sec}$ while the mean for the 100 cases with errors (SET A) was $158.7 \mathrm{ft} / \mathrm{sec}$ with a standard deviation of $20.5 \mathrm{ft} / \mathrm{sec}$.

The minimum and maximum $\Delta V$ cases without errors required $108.2 \mathrm{ft} / \mathrm{sec}$ and $170.8 \mathrm{ft} / \mathrm{sec}$, respectively, while with errors minimum and maximum $\Delta V$ cases required $100.0 \mathrm{ft} / \mathrm{sec}$ and $227.0 \mathrm{ft} / \mathrm{sec}$. Figure 9 shows the distribution of total $\Delta V$.

A breakdown of how the $\Delta V$ was used is shown in the following table:

Table 3-5
Translation $\Delta V$

| Maneuver | $\Delta V(S E T ~ D)$ <br> Average Without Errors <br> ft/sec | $\Delta V(S E T$ A) <br> Average With Errors <br> ft/sec |
| :--- | :---: | :---: |
| CSI | 37.4 | 36.8 |
| CDII | 40.9 | 40.2 |
| TPI | 22.1 | 24.0 |
| MCC1 | 1.9 | 6.8 |
| MCC2 | 1.3 | 10.8 |
| Braking and LOS <br> Control | 35.2 | 40.0 |

The mean arrival at TPI for the 25 cases without errors (SET C) was 5 seconds later than nominal with a standard deviation of 18 seconds, while the mean for the 100 cases with errors (SET A) was 12 seconds late with a standard deviation of 5 min and 35 seconds.

Figure 10 shows the distribution of arrival time at TPI over intervals of two minutes for the 100 cases with errors.

## 3.5 $\Delta T$ from TPI to TPF

The mean $\Delta T$ from TPI to close approach without braking for the 25 cases in SET C was 1980 ser:onds with a standard deviation of 18 seconds while the mean $\Delta 1$ of transfer without braking for the 100 cases in SET B was 1963 seconds with a standard deviation of 72 seconds.

The mean $\Delta T$ from TPI to intercept with braking and LOS control for the 25 cases in SET $D$ was 2121 seconds with a standard deviation of 22 seconds while the mean $\Delta T$ of transfer with braking for the 100 in SET A was 2148 seconds with a standard deviation of 87 seconds. The nominal case required 2115 seconds with braking and LOS control.

Figure 11 shows the distribution of the $\Delta T$ transfer with braking for the 100 cases with errors.
4.0 References

1. LM Rendezvous Procedures D Mission, Flight Crew Support Division, dated December 3, 1968.
2. S-PA-8M-036, Apollo Mission Techniques Mission D Rendezvous, Volume 1, dated December 20, 1968.
3. Apollo Mission D (AS-504/CSM-104/LM-3) Spacecraft Operational Trajectory, Volume 1, Mission Profile, dated December 2, 1968.





PAGE 2 of 2



 (NOM)

 CSI BACK-UP TABLE: MISSIO



"D" MISSION






FIGUFE 8
(\#\#\#




