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GUIDANCE, NAVIGATION AND CONTROL

E-1973

STATISTICAL ESTIMATION IN
INERTIAL NAVIGATION SYSTEMS

by

Larry D. Brock and George T. Schmidt

June 1966

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Approved: Leonard S. Wilk Date: 27 May 1966
LEONARD S. WILK, DIRECTOR, SYSTEM TEST
APOLLO GUIDANCE AND NAVIGATION PROGRAM

Approved: David G. Hoag Date: 1 Jun 66
DAVID G. HOAG, DIRECTOR
APOLLO GUIDANCE AND NAVIGATION PROGRAM

Approved: Ralph R. Ragan Date: 1 Jun 66
RALPH R. RAGAN, DEPUTY DIRECTOR
INSTRUMENTATION LABORATORY

To be presented at the
AIAA Guidance and Control
Specialist Conference
Seattle, Washington
August 1966

E-1973

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ACKNOWLEDGEMENT

This report was prepared under DSR Project 55-238, sponsored by the Manned Spacecraft Center of the National Aeronautics and Space Administration through Contract NAS 9-4065.

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Published by the Instrumentation Laboratory, Massachusetts
Institute of Technology, Cambridge 39, Massachusetts.
Printed in the United States of America

STATISTICAL ESTIMATION IN INERTIAL NAVIGATION SYSTEMS

Larry D. Brock, Lt., U.S.A.F.
Central Inertial Guidance Test Facility
Holloman AFB, New Mexico

George T. Schmidt
Instrumentation Laboratory
Massachusetts Institute of Technology
Cambridge, Massachusetts

Abstract

Navigation systems have generally been developed using a deterministic approach; i.e. by solving exactly the dynamic equations of motion. In reality the navigation problem is a statistical one since there are unpredictable errors in the measurements. The advent of very powerful navigation computers makes it possible to retrieve more of the information contained in the navigation measurements using statistical filtering techniques. The navigation system is thus developed here as an information filtering problem rather than a simulation of a deterministic physical system. The Kalman optimum linear filter is found to apply to the navigation problem if some technique is used to account for possible non-gaussian maneuver accelerations. A major problem in the application of statistical techniques is the tremendous amount of computation required. Two methods are suggested which greatly reduce the amount of computation with a minimum degradation in the performance of the system. In one method, physical considerations are used to divide the total filter into smaller, simpler filters. In the other method, the optimum gains are precomputed and approximated by simple functions in the flight computer. The details are given for the alignment and calibration of the Apollo inertial platform using statistical filtering.

1. Introduction

Two recent developments are leading to changes in the way navigation systems are formulated. One of these developments is much smaller, more powerful, and cheaper flight computers. These computers make possible much more sophisticated techniques for reducing the navigation information. The other complementary development is new statistical filtering techniques which provide the methods for taking advantage of the increased computational capabilities. There are, however, several practical problems encountered when one attempts to apply these techniques to practical systems. One is the non-gaussian distributions of some of the signals encountered. The well-developed linear filter theory cannot always be applied directly in these cases. Another problem is obtaining an adequate statistical model of the physical systems involved in the navigation filter. A third problem is

the immense amount of computation that is usually required. The last two problems are interrelated in that the more accurately the system is statistically modeled, the more difficult the computational problem becomes. In general, the amount of computation increases with the square of the number of variables used to describe the system. This paper is concerned primarily with the last problem.

The plan for this paper is to first briefly show the form of a navigation system that uses statistical filtering. The basic Kalman linear filter is applied to a simple navigation system and the first problem of non-gaussian distributions is mentioned but not discussed in detail. There is much interest in applying Kalman filtering to navigation systems but in most cases the statistical filter is used only to apply external information such as position fixes and doppler radar velocities to the basic Schuler tuned system.^(1,2,3,4) Statistical filtering should be used to incorporate all measurements including the accelerometer inputs. This technique has been suggested by Åström for the vertical control of an inertial platform^(5,6). Two methods are then suggested for simplifying the problem in order that it can be more easily handled on a practical computer. Details are then given of the alignment and calibration of the inertial platform in a spacecraft that is on top of a swaying launch vehicle. One of the simplification techniques is used to obtain a very efficient computer program.

2. The Form of a Navigation System Using Statistical Filtering

The basic configuration of a navigation system is shown in functional form in Figure 1. The question with which we are concerned here is, what should the computer program be that converts the measurements to the desired outputs and generates the commands to the sensors?

The equations to be solved by the computer have usually been developed by first devising a hypothetical mechanical system that would solve the navigation problem. The physical system that has the proper characteristics is the Schuler tuned pendulum. This pendulum has the property that (assuming proper initial conditions) it will always indicate the true vertical regardless of the motions of the vehicle carrying the pendulum. This indicated true vertical can be compared with an inertial reference maintained by gyroscopes to produce the position of the vehicle on the earth's surface. It is virtually impossible to build a physical pendulum with a reasonable size that would have these characteristics. In order to obtain the proper characteristic frequency for the pendulum, the pivot point center of mass separation would have to be on the order of one atomic distance. However, it is possible to solve the proper equations in the computer shown in Figure 1 and produce the proper commands to the gyros so that an instrumented platform along with the computer simulates the characteristics of the hypothetical pendulum. This approach has been very successful particularly when it was necessary to use analog computing techniques. However, with the advent of very powerful digital computers, it is possible to improve the performance of a navigation system by using statistical methods to take advantage of more of the inherent information provided by the sensors.

The statistical method differs from the conventional approach in that the equations are not developed in order to simulate a physical system but are developed as a statistical information filtering problem. In other words, given a set of physical measurements and some knowledge of the statistical characteristic of all the components involved, find the best estimate of the desired output quantities. For a navigation system, the physical measurements are normally the output of accelerometers which are mounted on a gyro stabilized platform and the desired output is the position and velocity of the vehicle. The problem is statistical because there are always unpredictable noises and errors in the measurements. The path of the vehicle is also random. (If the position of the vehicle could be predicted, there would be no need for a navigation system.) Furthermore, it is usually necessary to incorporate redundant sources of information such

as position fixes and doppler velocities.

The maximum amount of information provided by the measurements in a system with unpredictable errors is the probability density function $p(\underline{x})$. The vector \underline{x} represents the state of the entire system involved including the sensors, the vehicle, and the environment. In other words, with the knowledge of all past measurements and the a priori assumptions of the statistical characteristics of the entire system, the most that can be known about the position of a vehicle is the probability that it is in each incremental volume of possible positions.

In most cases the entire probability density function is not desired. What is usually wanted is some set of values that is, in some respect, the "best" estimate of the state of the system. This estimate is symbolized by $\hat{\underline{x}}$. The meaning of "best" is somewhat arbitrary but, except for the possibility of very special situations, the estimate is usually chosen so as to minimize the average of some function of the error in the estimate $\underline{e} = \hat{\underline{x}} - \underline{x}$. The most obvious estimate and one that minimizes the mean-squared error is the expected or mean value which is given by

$$\hat{\underline{x}} = E(\underline{x} | \underline{m}) = \int_{-\infty}^{\infty} \underline{x} p(\underline{x} | \underline{m}_1, \dots, \underline{m}_n) d\underline{x} \quad (1)$$

where $p(\underline{x} | \underline{m}_1 \dots \underline{m}_n)$ is the probability density function conditioned by all past measurements.

The solution of equation (1) would require the computation of the entire probability density function of all the state variables as a function of time. The computation would include the changes in the density function due to the dynamics of the system and also the changes due to the information gained by new measurements. The techniques for accomplishing this computation are not, to the authors' knowledge, well developed and furthermore would be extremely impractical for any computer in the foreseeable future. If certain assumptions can be made about the system, the computation can be simplified considerably. The most advantageous assumption is that the errors in the estimate can be represented by a linear system excited by uncorrelated noise. If this assumption is adequate, then the whole process represented by equation (1) can be stated concisely by equations developed by Kalman.^(7,8) The fol-

lowing section outlines the mathematical development of a navigation system using statistical filtering assuming a linear error model.

3. Mathematical Development

It is assumed that the state of the entire system including sensors, vehicle, and environment can be described by differential equations of the form

$$\dot{\underline{x}}(t) = \underline{f}(\underline{x}(t), \underline{c}(t), \underline{n}(t), t) \quad (2)$$

where \underline{c} are known control inputs and \underline{n} are white noises. For the assumptions to be made here, it can be shown that the control does not affect the form of the optimum filter.^(9,10) Thus the control variables \underline{c} will not be shown explicitly in the following discussion. It is assumed that measurements are made at discrete times according to the relation

$$\underline{m}(t_m) = \underline{h}(\underline{x}(t_m), \underline{u}(t_m)) \quad (3)$$

where $\underline{u}(t_m)$ are errors in the measurements that are uncorrelated between measurements. (Time correlated errors in the measurements would have to be included in the state vector \underline{x} .) It is further assumed that some initial estimate $\hat{\underline{x}}$ of the state variables is known where the error in the estimates of the state and the measurements are

$$\delta \underline{x} = \hat{\underline{x}} - \underline{x} \quad \text{and} \quad \delta \underline{m} = \hat{\underline{m}} - \underline{m}$$

By Taylor series expansion, equations (2) and (3) can be linearized around a nominal

$$\delta \dot{\underline{x}} = \left. \frac{\partial \underline{f}}{\partial \underline{x}} \right|_{\text{nom}} \delta \underline{x} + \left. \frac{\partial \underline{f}}{\partial \underline{n}} \right|_{\text{nom}} \delta \underline{n} + O(\delta^2)$$

and

$$\delta \underline{m} = \left. \frac{\partial \underline{h}}{\partial \underline{x}} \right|_{\text{nom}} \delta \underline{x} + \left. \frac{\partial \underline{h}}{\partial \underline{u}} \right|_{\text{nom}} \delta \underline{u} + O(\delta^2)$$

If the higher order terms can be neglected, then the optimum measurement process is given by Kalman's optimum linear filter. The derivation of these equations are given in references (7), (10) and (11). The basic equations are summarized as follows

$$\left. \begin{aligned} \hat{\underline{x}} &= \hat{\underline{x}}' + E'H^T (HE'H^T + U)^{-1} (\underline{m} - \underline{h}(\hat{\underline{x}}', t)) \\ E &= E' - E'H^T (HE'H^T + U)^{-1} HE' \end{aligned} \right\} \begin{array}{l} \text{at} \\ \text{a} \\ \text{measurement} \\ \text{time} \end{array}$$

$$\left. \begin{aligned} \dot{\hat{\underline{x}}} &= \underline{f}(\hat{\underline{x}}, t) \\ \dot{E} &= FE + EF^T + N \end{aligned} \right\} \begin{array}{l} \text{between} \\ \text{measurements} \end{array} \quad (4)$$

In these equations the prime indicates conditions that exist just before the measurement. The matrix E is called the covariance matrix of measurement errors and is defined by

$$E = \langle \delta \underline{x} \delta \underline{x}^T \rangle$$

where " $\langle \rangle$ " represents the expected or mean value. Other quantities in equation (4) are defined by

$$F = \frac{\partial \underline{f}(\hat{\underline{x}}, t)}{\partial \hat{\underline{x}}} \quad H = \frac{\partial \underline{h}(\hat{\underline{x}}, t)}{\partial \hat{\underline{x}}}$$

$$U = \frac{\partial \underline{h}}{\partial \underline{u}} R \frac{\partial \underline{h}^T}{\partial \underline{u}} \quad N = \frac{\partial \underline{f}}{\partial \underline{n}} Q \frac{\partial \underline{f}^T}{\partial \underline{n}}$$

The matrices R and Q are defined by

$$\langle \underline{u}(t) \underline{u}(\tau) \rangle = R \delta(t - \tau)$$

$$\langle \underline{n}(t) \underline{n}(\tau) \rangle = Q \delta(t - \tau)$$

The optimum filter for a linearized system is shown in diagram form in Figure 2 where, at the sample times, the estimate of the state \underline{x} is changed by an impulse through the weighing factor

$$K = E'H^T (HE'H^T + U)^{-1}$$

It has been found that navigation systems can usually be fitted into this linearized framework with one important exception - the accelerations due to the commanded maneuvers of the vehicle. The errors in the estimates of the state $\delta \underline{x}$ and measurements $\delta \underline{m}$ are assumed to have gaussian distributions. (This follows from the assumptions of linear systems with white noise inputs.) The maneuver acceleration is approximately zero except for certain random times when it is very large. The amplitude distribution of this acceleration is far from gaussian. Two methods are suggested for handling this problem. The simplest and probably best solution would be to provide a signal from the control system of the vehicle that would inform the navigation system when the vehicle is maneuvering. The vehicle maneuvering commands would be included in the control variables \underline{c} . The optimum

filter would then be affected only by the inaccuracy in predicting the vehicle motions. These inaccuracies are usually described adequately by gaussian distributions. If this signal cannot be provided, it is suggested that a nonlinear filter be developed that takes into account the actual amplitude distribution of the acceleration. A method for devising this nonlinear filter is suggested in reference (10). For the purposes of this paper, it is assumed that the problem of maneuvers is solved by some method and that statistical filtering takes place only during the steady-state cruise conditions between maneuvers. It is thus assumed that the disturbance accelerations are relatively small and have gaussian distributions.

With the assumptions that have been made above, it is possible to formulate the statistical filter for a navigation system using equation (4). A functional diagram of the computational part of the system is shown in Figure 3. Measurements such as position fixes which are not available at every basic measurement time are incorporated with a separate process but the equations are of the same form and are based on the same covariance matrix. With a navigation system formulated in this way the problems of velocity damping and reset are automatically included in an optimum manner. Furthermore, with minor changes in the content but not the form of the equations, the problems of pre-mission alignment, gyrocompassing, and component calibration can be accomplished in an optimum way.

4. Illustrative Example

These ideas can best be illustrated by applying filtering to a simple problem. For this example the vehicle is constrained to move only along a meridian on the surface of a spherical nonrotating earth. The vehicle is disturbed by random accelerations with a gaussian distribution and an exponential autocorrelation function. The navigation is to be performed on the basis of the measurements of a single-degree-of-freedom accelerometer that is mounted on a platform that rotates about an axis perpendicular to the meridian plane. The orientation of the platform is controlled by a single-degree-of-freedom gyro. The platform can be commanded to rotate at a given rate by a signal sent to the gyro. The platform will also rotate due to gyro drift. The gyro drift rate is assumed to be a random walk function beginning at

some initial value. The accelerometer measurements are fed into a computer which estimates the latitude and velocity of the vehicle and also computes the command signals to be sent to the gyro. A schematic drawing of the navigation system is shown in Figure 4.

The steps for applying a statistical filter are: (1) choose the state variables, the set of quantities that are needed to describe the complete system; (2) determine the dynamic equation that describes these state variables; (3) derive the equations that relate the measurements to the state variables; (4) develop the linearized equations that describe the errors in the estimates; and (5), if it can be assumed that the linear approximations of the errors are sufficient and that all signals and initial conditions have gaussian distributions, apply equation (4).

The state variables that describe this simplified navigation problem are vehicle acceleration a , velocity v , latitude L , platform misalignment from vertical α , and gyro drift rate d . The equations describing the dynamics of these variables are

$$\begin{aligned}\dot{a} &= -a/\tau + n_a \\ \dot{v} &= a \\ \dot{L} &= v/R \\ \dot{\alpha} &= v/R - \hat{v}/R + d - \hat{d} \\ \dot{d} &= n_d\end{aligned}\tag{5}$$

where τ is the correlation time of the vehicle acceleration, R is the radius of the earth and n_a and n_d are the white noise inputs into the models of vehicle acceleration and gyro drift rate respectively. The $\dot{\alpha}$ equation derives from the fact that the platform is rotated by \hat{v}/R in order to keep it aligned with the vertical while the true vertical rotates by v/R . The platform is also torqued by \hat{d} to compensate for the gyro drift rate d .

The measurement of the accelerometer is given by

$$m = a \cos \alpha + (g - v^2/R) \sin \alpha + n_m\tag{6}$$

where g is the acceleration of gravity and n_m is uncorrelated noise in the measurements.

The equations for the estimated state variables and measurement are the same as equations (5) and (6) with the actual state variables replaced by the

estimated variables and with the noise term deleted. The equations for the errors in the estimates and measurement are thus

$$\begin{bmatrix} \delta \dot{a} \\ \delta \dot{v} \\ \delta \dot{L} \\ \delta \dot{\alpha} \\ \delta \dot{d} \end{bmatrix} = \begin{bmatrix} -1/\tau & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1/R & 0 & 0 & 0 \\ 0 & 1/R & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta a \\ \delta v \\ \delta L \\ \delta \alpha \\ \delta d \end{bmatrix} - \begin{bmatrix} n_a \\ 0 \\ 0 \\ 0 \\ n_d \end{bmatrix} \quad (7)$$

and

$$\delta_m = \begin{bmatrix} 1 & 0 & 0 & (g - \hat{v}^2/R) & 0 \end{bmatrix} \begin{bmatrix} \delta a \\ \delta v \\ \delta L \\ \delta \alpha \\ \delta d \end{bmatrix} - n_m \quad (8)$$

The optimum filter for this navigation system is now completely defined by equation (4) where the F and H matrices are given by equations (7) and (8) respectively. The noise matrices are given by

$$N = \begin{bmatrix} A & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & D \end{bmatrix}$$

where A and D are defined by

$$\langle n_a(t) n_a(\tau) \rangle = A \delta(t - \tau)$$

$$\langle n_d(t) n_d(\tau) \rangle = D \delta(t - \tau)$$

and

$$U = \langle n_m^2 \rangle$$

This system is shown in diagram form in Figure 5. The numbers at the output of the gains K mean discrete changes are made at the corresponding points in the system at the sample times.

An example of the response of this meridian system to typical inputs is given in Figure 6. The three sources of disturbances in the system are the random walk gyro drift rate, the disturbance accelerations, and the random measurement error. The random walk gyro drift increases the RMS gyro drift rate by .05 meru/ $\sqrt{\text{hr}}$. The disturbance acceleration has an RMS value of .003 ft/sec² and a correlation time of one minute. The random error in the measurement has an RMS value of .001 ft/sec².

The sampling period for the measurement is one second. Also shown for the same inputs are the responses of a Schuler tuned undamped system and a system with lead-lag damping.

5. Physical Interpretation of Filtering

Some feeling for how a system with statistical filtering is different from a conventional system can be gained from the following arguments. The basic measurement of an inertial navigation system is the measurement of the accelerometer which is equal to the acceleration of the vehicle with respect to inertial space minus the gravitational acceleration at the position of the vehicle. For inertial navigation in general to be possible (say in space) the gravitational acceleration must be known as a function of position. However, in a terrestrial system an important piece of information is known a priori, the vehicle is constrained to be near the surface of the earth. It is thus known that there must be some force on the vehicle (lift, buoyancy) to counteract gravity. Thus the accelerometers measure the combination of two essentially different quantities, acceleration and gravity, which both give information about the vehicle's motion. The acceleration can be integrated to obtain the vehicle's change in velocity and position. The change in the direction of gravity measured with respect to the gyro reference also indicates the change in the position of the vehicle. The task of the statistical filter is to separate these two signals using their distinctive statistical characteristics. The vehicle accelerations are primarily high frequency while the acceleration of gravity is essentially a slowly changing bias. The filter compares the indications from these two sources in order to reduce the errors in each.

The improvement made possible by a statistical filter can be viewed in another way. It is known that most error sources in an inertial system excite an 84-minute mode of oscillation. In some conventional systems there is damping in order to reduce these 84-minute oscillations. A system with statistical filtering not only performs this 84-minute damping in an optimum way but makes some estimate of the sources of these errors. For example, in the simplified meridian navigation system from the previous section, if the statistical filter detects an 84-minute oscillation, it not only removes this oscillation but also determines that this oscillation was caused by

gyro drift. The filter thus changes the gyro compensation reducing the effective gyro drift rate.

6. Methods for Reducing Computation

It can be seen from equation (4) that the straightforward application of the optimum linear filter is not very practical because of the tremendous amount of computation required. The covariance matrix is $n \times n$ where n is the number of state variables used to describe the entire system. Of course, the more accurately the system is described, the more state variables are required. It has been found that most practical systems require at least 20 to 30 variables. Since the covariance matrix is symmetric, only half the off diagonal terms need be computed, but this still gives $n(n+1)/2$ terms. There are thus approximately 400 differential equations which must be updated for each measurement where the measurement period may need to be as short as 1 to 2 seconds. What is needed are techniques for reducing the amount of computation with a minimum degradation in performance of the system. Two methods are suggested in this section.

For the first method, it is proposed that the computation be reduced by eliminating elements of the covariance matrix that have a negligible contribution to the operation of the system. The linear filter given by equation (4) gives the "mathematically optimum" filter for the model that was assumed to represent the physical system. By "mathematically optimum" is meant that terms are included even if they would reduce the mean-squared uncertainty only in the ninth or tenth significant figure. The effect of these terms would be virtually undetectable in a typical mission. These terms are especially insignificant when it is realized that the assumed statistical model represents the actual system to no more than two or three significant figures. The problem now is to locate these negligible parts and then to simplify the system without significantly degrading the performance.

One possible method of simplifying the optimum filter is to neglect elements of the covariance matrix where the cross-correlation between terms is weak and thus will make little contribution to the weighting coefficients. With some physical insight into the nature of the navigation problem, some possible areas of weak correlation can be detected.

One area of possible weak correlation is

between the errors in the estimates of the acceleration of the vehicle in the two horizontal directions. It can be assumed that the accelerations in the two directions themselves are uncorrelated, but the errors in the estimates of the accelerations are correlated because of the filtering process itself. This correlation is due to a sequence of relatively weak correlations. Another area where the problem could possibly be decoupled is between variables with high and low frequency characteristics. For example, it might be suspected that the specific high frequency characteristics of oscillatory vehicle accelerations might have little effect on the estimate of gyro drift which is detected through 84-minute and 24-hour oscillations in the platform misalignment angles. Vice-versa, the specific low frequency characteristics of misalignment angles might have little effect on the estimate of high frequency acceleration terms.

A possible form for the navigation system which takes advantage of these weak correlations can be found by identifying and separating the principal functions of the filter in the navigation problem. The measurement of the horizontal accelerometers are primarily a function of vehicle accelerations and gravity times the vertical misalignment of the platform. The vehicle accelerations can be assumed to consist primarily of high frequency disturbances plus possible smaller magnitude terms at low frequencies. The vertical errors have characteristic frequencies near 84 minutes and 24 hours. Thus, one primary function of the filter is to separate high frequency accelerations from what are essentially slowly varying bias accelerations. Another major task is to separate this bias into actual vehicle accelerations and accelerations due to vertical misalignment and then to infer the cause of the estimated misalignment, e.g., gyro drift. The navigation system can be simplified if these functions can be decoupled, that is, if the fact that the bias acceleration is slowly varying does not affect the separation of the bias from the high frequency accelerations and, conversely, if the exact nature of the high frequency terms does not affect the separation of the low frequency terms.

If the above assumptions are valid, the optimum filter can be approximated by three separate filters. One filter in each of the two horizontal directions would separate high frequency accelerations from a

bias acceleration having a slow variation that is assumed to be known. The sampling rate for these two filters would be about the same as that for the original filter. The output estimates of the bias accelerations from the high frequency filters would be pseudo-measurements into a third filter. The third filter would use these pseudo-measurements to make corrections on the estimates of position and velocity that are due to low frequency characteristics and also to estimate misalignment angles, gyro drift rates, and long-term accelerations. The sampling period for the low frequency filter would be long enough that errors in the measurement of the bias acceleration can be assumed to be uncorrelated. The separated filter is shown in Figure 7. In this figure the state variables \underline{x}_x and \underline{x}_y include the bias acceleration, position, velocity, and all the variables that describe the high frequency accelerations in the two horizontal directions. The variables \underline{x} again include position and velocity and also long-term accelerations, misalignment angles, and gyro drift rates.

Since the statistics of the vehicle accelerations can usually be assumed to be stationary, the ability of the high frequency filters to separate a bias from the short correlation time accelerations quickly reaches a steady state. In other words, after two or three time constants of the principal high frequency terms, the mean-squared uncertainties in the estimates of the accelerations become constant. This means that the entire covariance matrix and also the weighting coefficients become constant. Thus, the covariance matrices for the high frequency filters do not have to be computed in real time in the navigation system if some satisfactory technique is used for the initial transient. If necessary, pre-computed gains could be used during the initial phase, but it has been found in most cases that the steady-state values are also sufficient for the transient. The covariance matrix for the third filter must be computed in real time since it has 84-minute and 24-hour modes and also depends on the position and velocity of the vehicle. For a typical case the low frequency filter would involve a 15 x 15 matrix. This gives 120 simultaneous differential equations, but the sample time can be as long as three minutes. This gives a reduction in the amount of computation over the straight forward application of the complete optimum filter by a factor of about 500. This computation problem can be easily handled by a

practical navigation computer.

This intuitive reasoning gives a possible form for a simplified system which might operate in a near optimum fashion, but the details of the design are not specified. For example, the mean-squared value of the errors in the pseudo-measurements must be determined. Also the gains in the high frequency filter for the estimation of the bias acceleration, position, and velocity need to be specified. The method proposed to specify these gains and also to determine how nearly the simplified system can be made to approach the optimum system is to match typical responses for the two systems. A complete optimum system is simulated on a computer for a typical navigation problem. The simplified system is then simulated for the same conditions, and the arbitrary gains on the system are adjusted so that the simplified system matches as nearly as possible the response of the optimum one. There are two sets of signals that should be matched. The most important signals are the actual discrete outputs of the two systems. The other set of signals is the estimated mean-squared errors in the estimates, i.e., the values of the common diagonal elements of the covariance matrices of the two systems.

Computer results are given in Figures 8 and 9 showing the comparison of a complete and simplified navigation system. In this example the navigation system is a simplified system that is essentially the same as the one described in section 4 except that a much more complete model of vehicle motion is used. The vehicle model included long and short correlation time and oscillatory accelerations. Only velocity is shown here as an example. It can be seen that the RMS results in Figure 8 can be made to match very closely and that the discrete results in Figure 9 are similar after the initial transient.

The second method of simplifying the optimum filter is through the use of precomputed gains. If the system is linear, the gains at each measurement are only functions of time and the a priori assumptions of the statistics of the noises and the initial state. By specifying the measurement schedule or rate, the gains may be precomputed and stored in the vehicle's computer. For a small number of measurements this is a practical solution to the problem involved in implementing an optimum filter. For a large number of measurements, the

precomputed gains are usually smoothly varying with respect to time and may be approximated by suitable curves (straight lines, exponentials, etc.) that give almost identical filter responses as the true gains. Some of the minor gains may even be neglected completely. This second technique for simplification of the computation will be applied to the pre-launch calibration and alignment of an inertial platform in a spacecraft on top of a swaying launch vehicle and is, in fact, the technique used in the MIT Apollo Guidance and Navigation System.

7. Apollo Pre-Launch Calibration and Alignment

Physical Bases for Alignment and Calibration

The inertial platform we wish to calibrate and align is a three-gimbal stable member with three single-degree-of-freedom integrating gyroscopes and three pulsed integrating pendulous accelerometers defining three orthogonal stable member axes. The physical basis for alignment and calibration is the detection of the gravity reaction acceleration and the vector rotation of the gravity vector. The direction of these two vectors gives the information necessary to align the platform to any orientation. The known gravity reaction acceleration is used to calibrate the accelerometers; the known vector rotation of gravity (earth rate) is used to calibrate the gyros. The exact quantities to be measured are not considered at this point. We will first develop the general optimum method of platform alignment to a local vertical coordinate system and measurement of the south and vertical gyro drifts. In this procedure, the platform is approximately aligned to the local vertical coordinates then it goes inertial. The two horizontal accelerometer outputs (south and east) are used by the optimum filter to generate estimates of the relevant quantities.

The vertical gyro drift is the most difficult quantity to measure since it causes only a third-order effect on the measured acceleration. Gyro failures can be closely associated with changes in drift due to acceleration of gravity along the input axis so the pre-launch calibration of a gyro in a vertical position is highly desirable.

Since the estimates of the alignment and drift variables will depend on the measurement of the acceleration due to gravity, the primary disturbance factor is the wind-induced sway of the launch vehicle. The model of the system for the optimum filter

must, therefore, also include variables due to the wind-induced sway as well as the alignment angles and gyro drifts we wish to estimate.

We will first develop the models for the filter and then digitally simulate the complete filter. The filter will be linear so that by specifying the measurement schedule the optimum gains may be precomputed. The gains will be approximated by functions that will be stored in the flight computer. The method for using this simplified filter in other platform positions, as well as an illustration of a system test program, will then be presented.

Models

Launch Vehicle Sway Model. We approximate the launch vehicle bending dynamics in the north-south and east-west directions by identical second-order systems. The wind causing the vehicle sway is assumed to be exponentially correlated with a correlation time of $1/\lambda$ seconds. In Appendix A, we show the model and derive the correlation function of the white noise $n(t)$ which, if passed through a first-order lag, generates an exponentially correlated wind. The wind drives the model of the launch vehicle to generate an expected value of mean-squared sway. The state vector for the sway variables in the south direction may be written as

$$\begin{bmatrix} \dot{p}_s \\ \dot{v}_s \\ \dot{a}_s \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\lambda\omega_n^2 & -\omega_n^2 - 2\zeta\lambda\omega_n & -2\zeta\omega_n - \lambda \end{bmatrix} \begin{bmatrix} p_s \\ v_s \\ a_s \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ n(t) \end{bmatrix} \quad (9)$$

where

p_s, v_s, a_s are the horizontal displacement, velocity, and acceleration of the launch vehicle in the north-south direction,

ω_n and ζ are the natural frequency and damping ratio of the second-order approximation to the bending dynamics,

λ is the inverse correlation time of the wind, and

$n(t)$ is the white noise generating the exponentially correlated wind.

The model for the sway variables in the east direction (p_e, v_e, a_e) is the same.

Platform Dynamics. We describe the orientation of the platform with respect to a local vertical coordinate system (vertical, south, and east) by three angles (α, β, γ). If the platform axes (x, y, z) were rotated by $-\alpha, -\beta$, and $-\gamma$, the axes would coincide with the reference coordinates. The state vector equation for this substate is given by

$$\frac{d}{dt} \begin{bmatrix} \alpha \\ \beta \\ \gamma \\ d_x \\ d_y \end{bmatrix} = \begin{bmatrix} 0 & 0 & \Omega_h & 1 & 0 \\ 0 & 0 & \Omega_v & 0 & 1 \\ -\Omega_h & -\Omega_v & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \\ \gamma \\ d_x \\ d_y \end{bmatrix} + \begin{bmatrix} t_x - \Omega_v \\ t_y + \Omega_h \\ t_z + d_3 \\ 0 \\ 0 \end{bmatrix} \quad (10)$$

where it has been assumed that, for angle magnitudes of interest, the above equation is a valid representation of the general nonlinear platform dynamics.⁽¹¹⁾

Ω_h and Ω_v are the horizontal and vertical components of earth rate at the test site.

d_x, d_y, d_z are the constant drifts of the vertical, south, and east gyros.

t_x, t_y, t_z are the torquing rates (if any) applied to the gyros.

We will assume that the torquing rates, the components of earth rate, and the east gyro drift are known perfectly so that the vector on the right represents known forcing ($\underline{c}(t)$) and is independent of the state of the system.

Accelerometer Outputs. Again, using the small angle approximations, the south and east PIPA pulse rate outputs due to platform orientation in the gravity field may be written as

$$\begin{bmatrix} \dot{p}o_s \\ \dot{p}o_e \end{bmatrix} = g \begin{bmatrix} -\gamma \\ \beta \end{bmatrix} \quad (11)$$

where

p_o_s and p_o_e represent the total pulse counts at some instant of time, and

g is the local gravity (cm/sec^2)

We have assumed a one $\text{cm}/\text{sec}/\text{pulse}$ PIPA scale factor.

The Measurements. We cannot instantaneously measure accelerometer pulse rates but only the total pulse count which is made up of the output

due to sway velocity and orientation in the gravity field. Inherent in our measurements, then, are quantization errors. The measurements are

$$\underline{m} = \begin{bmatrix} m_s \\ m_e \end{bmatrix} = \begin{bmatrix} v_s \\ v_e \end{bmatrix} + \begin{bmatrix} p_o_s \\ p_o_e \end{bmatrix} + \begin{bmatrix} n_m \\ n_m \end{bmatrix} \quad (12)$$

The term " n_m " represents the quantization error at every sampling of the PIPA pulse count registers. Although the measurement noise is not normally distributed with zero mean (a requirement for the optimum filter), and is, in fact, uniformly distributed, we will view the quantization operation as a normally distributed error. It has been shown that this viewpoint does not hinder the optimum filter's estimates of the alignment and gyro drift variables.^(10,11) It does degrade the accuracy of the sway variable estimates which is of little concern. Also

$$\langle n_m(t) n_m(t + \tau) \rangle = r_a \delta(\tau) \quad (13)$$

We will assume that the measurements are made every second.

The State Vector. The state vector is thirteen dimensional; the state vector equation is

$$\frac{d\underline{x}}{dt} = F\underline{x} + \underline{c} + \underline{n} \quad (14)$$

(See Figure 10 for $\underline{x}, F, \underline{c}$ and \underline{n}).

The Complete Filter. With the derivation of the model for the system, the complete optimum linear filter is defined. The PIPA pulse count registers will be sampled at constant rates. The estimated state vector is extrapolated between measurements according to

$$\hat{\underline{x}}' = \Phi \hat{\underline{x}} + \underline{c} \quad (15)$$

and the covariance matrix according to

$$E' = \Phi E \Phi^T + S \quad (16)$$

where Φ and S are precomputed constant matrices for the time step between measurements. They satisfy the following differential equations:

$$\frac{dS}{dt} = FS + SF^T + N \quad S(0) = 0 \quad (17)$$

$$\frac{d\Phi}{dt} = F\Phi \quad \Phi(0) = I \quad (18)$$

which may be integrated on a digital computer for a time step between measurements of one second. (12)

At the time of a measurement, \hat{x} and E are changed according to equation (4).

(See Figure 10 for H, U, and N).

Filter Design

Computer Simulations. A complete nonlinear simulation of the inertial platform in a swaying launch vehicle was made on a digital computer in order to simulate real PIPA outputs. The initial misalignments were one degree on all axes; drifts were ten meru for the vertical and south gyros and zero for the east gyro. The rms sway in both horizontal directions was 10 cm. The initial conditions for the correlation matrix were one degree² for the alignment angles and 100 meru² for the gyro drifts. All initial cross correlation terms were assumed zero. The initial estimate of the state was a zero vector.

The response of the filter was excellent for these conditions. A number of runs were first made in which the matrix U was varied so as to cause good agreement between the rms error as determined by the filter and the actual error. The errors between the estimates and the actual values of azimuth angle, vertical gyro drift and south gyro drift are shown in Figures 11a, b and c. The errors reach small values for the three cases in 15, 40 and 10 minutes respectively. The errors in the estimates of the two leveling angles (β and γ) are negligible after the first few measurements. The estimates of the sway variables are not particularly good but this is not important. In these simulation runs we have assumed perfect knowledge of east gyro drift for good reason; we could show by using the observability concept that east gyro drift cannot be identified from azimuth error; however, the mechanics of doing this are difficult because of the large state vector. It is generally easier to synthesize on a digital computer a series of filters in which we add one-by-one the state variables we wish to estimate. It becomes immediately clear which ones cannot be separated out. The fact that east gyro drift must be known does not hurt us; as we see from

Figure 11c, the south gyro can be calibrated in about 10 minutes and the error is independent of d_z . This gyro can then be placed east and a complete calibration and alignment made. (We discuss the question of other platform positions later.)

Design of the Simplified System. The gains for the optimum filter may be precomputed for all trials since the measurement times will be the same and the a priori assumption for the statistics of the initial state vector and noises will not change. The question is: how should these gains be stored in the flight vehicle's computer?

For the problem at hand, the implementation of the gains into the system involved, first, the design of a simplified optimum filter. The minor cross-coupling gains were completely neglected; i.e. the gains for each state variable depend on both accelerometer measurements and, in general, one gain is much smaller than the other and can be neglected. Typically the predominant gains vary as in Figure 12. The gains can be approximated by exponentials and straight-line segments where, at distinct intervals, the time constants and slopes are changed to continually fit the approximate gains to the true gains. The gains for the six sway variables quickly reach steady-state values and may be approximated by three constants.

The response of a simplified filter is shown in Figures 13a through 13c. The process of design enters since it required a number of runs using different slopes and time constants to get a good match with the response of the complete filter. In fact, in the end, the same exponential gains were used for both po_s and po_e ; the same exponential gains were used for β and γ . The total number of constants that were precomputed was:

- 3 sway variable gains
- 2 initial conditions for exponentials

and sets of five numbers which are changed at ten discrete times

- 2 time constants for exponentials (po_s and β)
- 3 slopes for straight-line segments (α, d_x, d_y)

Implementation

The simplified filter required about 800 double-precision computer words when it was implemented in the Apollo system. Part of the program

was concerned with initialization for platform positions other than the one we have considered. The optimum filter once implemented does not change for other platform positions; the measurements that the filter gets are made to simulate the standard platform configuration. For example, if the platform axes were vertical, north and east, the sign of the north PIPA output is changed to simulate a south PIPA. We need only interpret the filter output for the variables in the north direction as negative of their true values. Similarly, for some platform positions it is necessary to resolve the PIPA outputs to simulate south and east accelerometers.

For some applications, it may be desirable to torque the south gyro at negative horizontal earth rate. The form of the filter and the filter gains do not change because we assume perfect torquing; we just add negative horizontal earth rate to the extrapolation of the angle β . We have also found it convenient to extrapolate the alignment angles according to simple first order ($\alpha' = \alpha + \dot{\alpha} dt$, etc.). The sway variables are extrapolated according to a sway transition matrix whose elements can be changed to compensate for variations in launch vehicle parameters.

We close our discussion with the comment that once the optimum filter has been implemented according to the simple method outlined in the paper, it is usually quite simple to adapt it to various problems of alignment and calibration. For example, consider the following system test procedure in which we identify the platform axes as x, y, z.

1. Run a ten-minute test with x up, y south, z east to determine y gyro bias drift.
2. At ten minutes read out y drift and use the angle estimates to align the platform. Continue to torque the platform at earth rate for 90 seconds while counting pulses from the x PIPA (vertical).
3. Orient the platform to x down, y east, z south and run a 10 minute test to determine z bias drift.
4. Use the angle estimates at 10 minutes to align the platform and then torque for 90 seconds at earth rate while counting x PIPA pulses.

5. After 90 seconds continue to torque the south gyro at horizontal earth rate for 45 minutes while we determine the vertical drift (x gyro). The Y bias as determined in (1) is d_z for this run.

This procedure takes about 68 minutes after which we have enough information to determine y gyro bias drift, z gyro bias, the sum of x gyro bias and acceleration sensitive drift, x PIPA bias and scale factor. We can readily imagine how an automated system test procedure can be set up to completely calibrate the system in the swaying spacecraft. The last step in the program would be an alignment run to ready the system for launch. And through all of this, the basic simplified optimum filter does not change.

Errors in the Test. Perhaps the only disadvantage of optimum linear filtering is that it does not lend itself to easy analytical study of questions, such as, what happens for variations in the assumed natural frequency of the launch vehicle? Questions like this must be answered by digital simulation. Fortunately, the filter we have designed is, in fact, very insensitive to variations in most quantities that we must assume a priori. The assumptions of the statistics for the initial state and the noises may have variations without degrading the filter response. The primary source of azimuth error is the uncertainty in east gyro drift which comes about from errors in the calibration of the gyro in a south position or from variations in the drift during a test. If the east gyro has a large drift due to acceleration along its input axis, then it is desirable to keep the input axis almost horizontal during the test by torquing the south gyro at negative earth rate as was previously mentioned.

8. Conclusion

The possibility exists for significant reduction in the errors of an inertial navigation system by using statistical estimation techniques that make the most efficient use of all available information. It also appears likely that suitable simplifications can be made to adapt these techniques to practical navigation computers as we have illustrated with a real system. The amount of improvement obtained from using statistical filtering depends very strongly on the statistical characteristics of both the particular vehicle and components used. In

vehicles where the disturbance accelerations are high and the mission time short, the improvement may not be significant. But for the relatively stable vehicles and for long mission times, the improvement may be considerable.

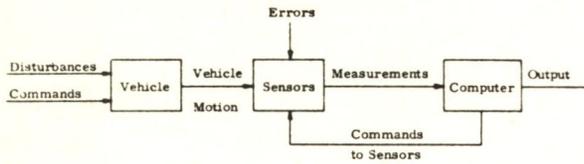


Fig. 1 Basic Configuration of a Navigation System

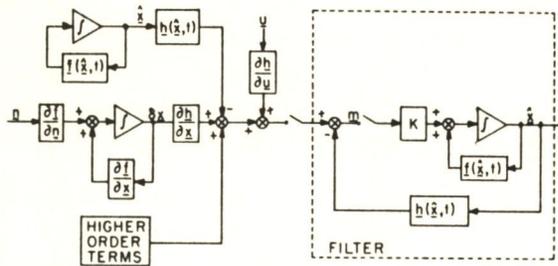


Fig. 2 Simplified Optimum Linear Filter for Linearized System

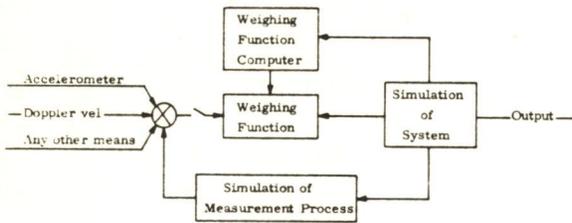


Fig. 3 Functional Diagram of the Computer

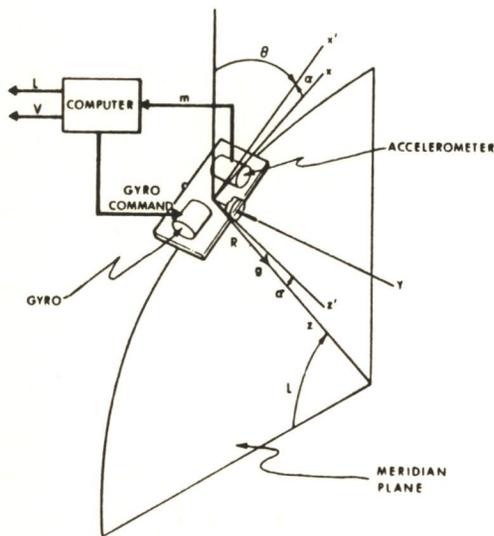


Fig. 4 Schematic of a Simplified Navigation System

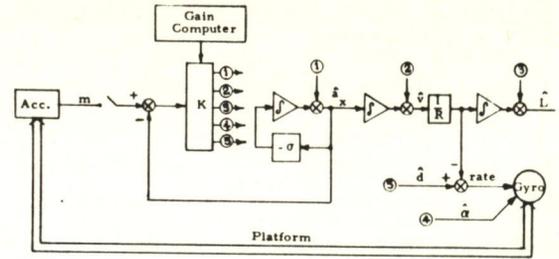


Fig. 5 Diagram of Simplified Navigation System

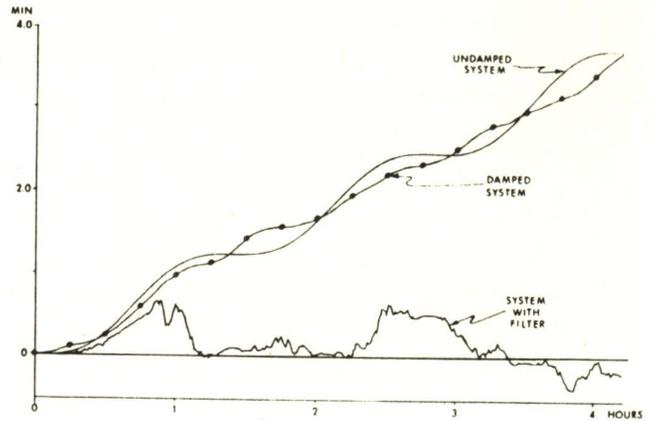


Fig. 6a Errors in Estimated Latitude

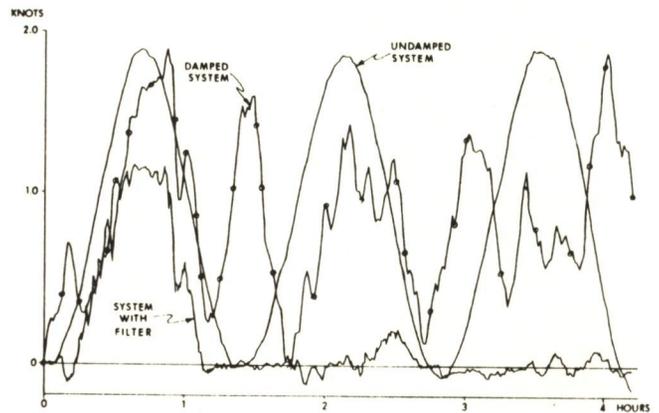


Fig. 6b Errors in Estimated Velocity

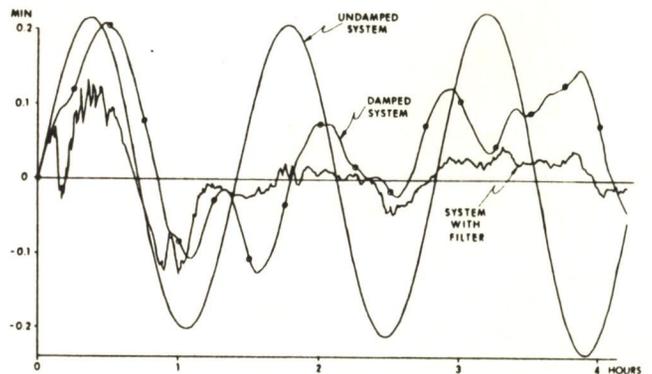


Fig. 6c Errors in Estimated Vertical

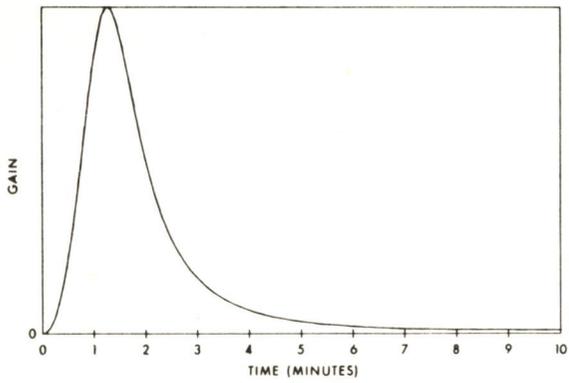


Fig. 12a α Gain

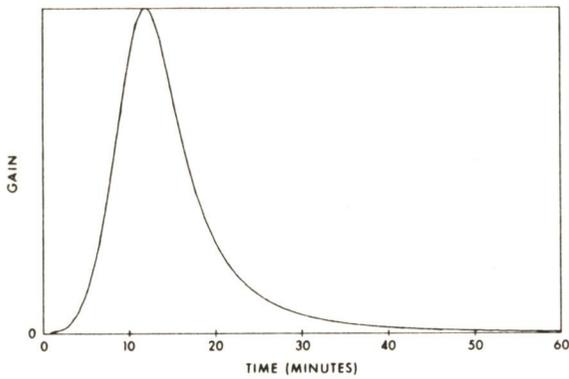


Fig. 12b d_x Gain

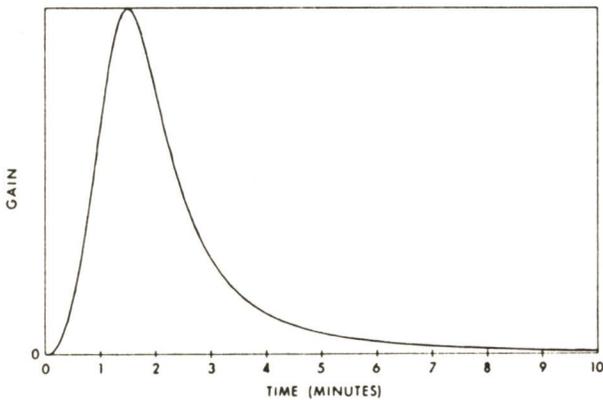


Fig. 12c d_y Gain

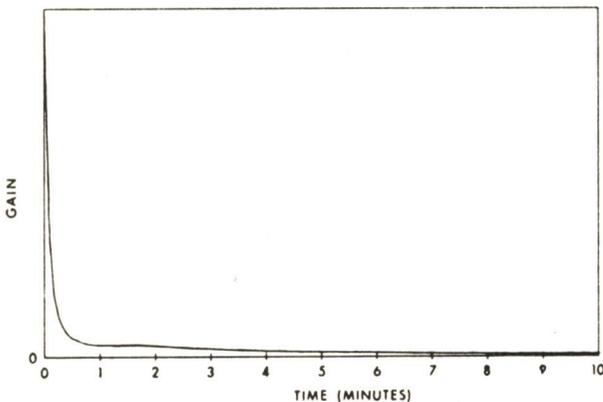


Fig. 12d p_{o_s} Gain

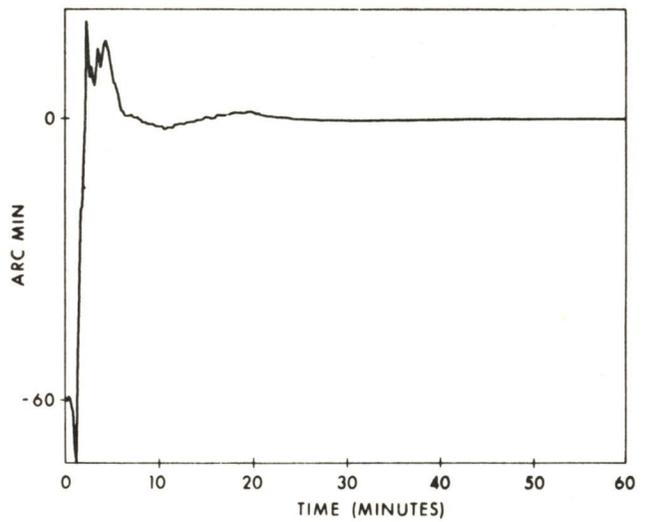


Fig. 13a Azimuth Error

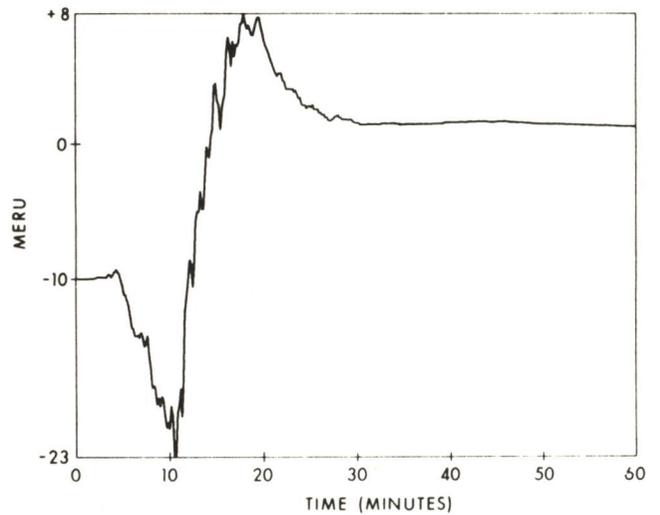


Fig. 13b Vertical Drift Error

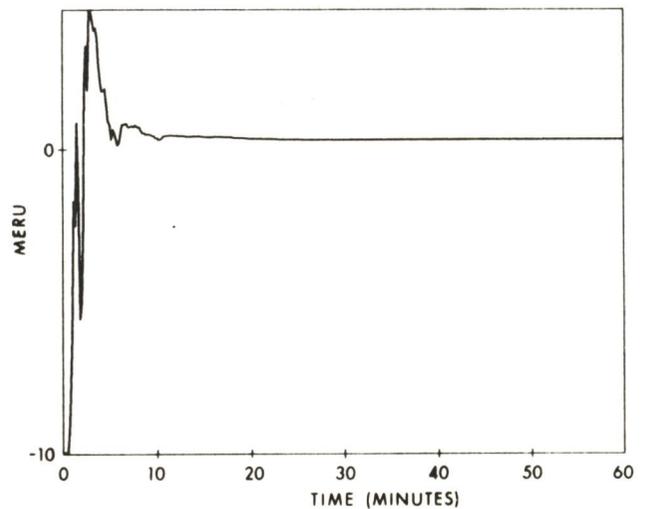
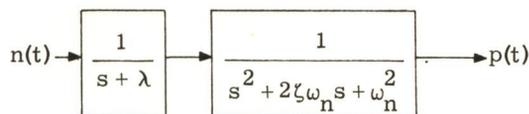


Fig. 13c South Drift Error

Appendix A Calculation of nw

Given the expected value of mean-squared launch vehicle sway, it is necessary to calculate the correlation function of the white noise that is assumed to produce the sway. The model for the system is



The correlation function for white noise is

$$\phi(\tau) = \langle n(t)n(\tau) \rangle = nw \delta(t - \tau) \quad (A-1)$$

with a corresponding power spectral density

$$\phi_{nn}(\omega) = nw/\pi \quad (A-2)$$

The mean squared value of sway is then given by⁽¹³⁾

$$\langle p^2 \rangle = \frac{nw}{2\pi j} \int_{-j\infty}^{+j\infty} \left[\frac{1}{s + \lambda} \frac{1}{-s + \lambda} \frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2} \frac{1}{s^2 - 2\zeta\omega_n s + \omega_n^2} \right] ds \quad (A-3)$$

Using the integral tables we determine nw as⁽¹³⁾

$$nw = \langle n(t)n(\tau) \rangle = \frac{\langle p^2 \rangle 4\lambda\zeta\omega_n^3 (\omega_n^2 + 2\lambda\zeta\omega_n + \lambda^2)}{\lambda + 2\zeta\omega_n} \quad (A-4)$$

For the computer simulations, the above variables have the magnitude

$$\begin{aligned} \langle p^2 \rangle &= 100 \text{ cm}^2 \text{ (east and south)} \\ \lambda &= .1 \text{ sec}^{-1} \\ \omega_n &= 2.09 \text{ rad/sec} \\ \zeta &= .1 \end{aligned}$$

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