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RTCC REQUIREMENTS FOR MISSION H PROGRAM FOR COMPUTATION OF LM IMU TORQUING ANGLES WITH THE LM AND CSM IN THE DOCKED CONFIGURATION

Guidance and Performance Branch
MISSION PLANNING AND ANALYSIS DIVISION

MANNED SPACECRAFT CENTER
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PROJECT APOLLO

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RTCC REQUIREMENTS FOR MISSION H PROGRAM FOR COMPUTATION OF LM IMU TORQUING ANGLES WITH THE LM AND CSM IN THE DOCKED CONFIGURATION

By A. David Long

1.0 SUMMARY AND INTRODUCTION

The purpose of this document is to present the Real-Time Computer Complex (RTCC) requirements for a program to compute and display gyro torquing angles for the lunar module (LM) platform with the LM and command/service module (CSM) in the docked configuration. The philosophy of this method makes use of gimbal angle data from the docked configuration to compute torquing angles for the LM platform while the LM and CSM are docked. This method, the eigen vector method, differs from the present fine alinement procedure because its solution for torquing angles is completely independent of navigation base misalinements between the two vehicles (LM and CSM). The correct torquing angles can be computed by this method even if the LM and CSM navigation bases have nonnominal alinement in all three axes.

The eigen vector method is analogous to an Apollo optical telescope (AOT) alinement in procedure and accuracy. Two rotation vectors are computed from three attitudes of the docked configuration, and these rotation vectors are then used in the same way that optical sightings on celestial bodies would be used. The necessary data to determine two rotation vectors and subsequently to determine torquing angles required to aline the LM platform to the CSM platform are three sets of gimbal angles read simultaneously from the LM and CSM. The two rotation vectors are expressed in IMU coordinates and are used to construct an inertial reference system. This inertial reference system can then be used to locate the LM platform relative to the CSM platform and the resultant torquing angles.

It is not necessary that the CSM platform have the desired orientation of the LM platform to use this method. This program should be general in nature and should have the capability to compute torquing angles necessary to aline the LM platform to any desired REFSMMAT. This capability is noted in the equations.

2.0 MODE 1 METHOD AND EQUATIONS

To obtain two directions of rotation in space (S and P) with respect to the CSM and LM IMU, the following procedure is used. The stable member to navigation base matrix (SMNB) is computed for three sets of gimbal angles (three sets for the LM and three for the CSM).

$$Q_1 = \begin{bmatrix} \cos IGA & 0 & -\sin IGA \\ 0 & 1 & 0 \\ \sin IGA & 0 & \cos IGA \end{bmatrix} \qquad (1a)$$

$$Q_2 = \begin{bmatrix} \cos MGA & \sin MGA & 0 \\ -\sin MGA & \cos MGA & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (1b)$$

$$Q_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos OGA & \sin OGA \\ 0 & -\sin OGA & \cos OGA \end{bmatrix} \qquad (1c)$$

$$[SMNB] = [Q_3][Q_2][Q_1] \qquad (1d)$$

A total of six such computations give $[SMNB_1]$ . . . $[SMNB_6]$. After these six computations have been completed, the following computations are necessary both for the LM and the CSM.

Computations for the CSM are presented in equations (2), (3), and (4).

$$[R1] = [SMNB_1]_{CSM}{}^T[SMNB_2]_{CSM} \qquad (2a)$$

$$[R2] = [SMNB_2]_{CSM}{}^T[SMNB_3]_{CSM} \qquad (2b)$$

$$[S]_{CSM} = [R1]^T - [R1] \qquad (2c)$$

$$[P]_{CSM} = [R2]^T - [R2] \qquad (2d)$$

$$RV1_{CSM} \text{ (rotation vector 1 for CSM)} = \text{Unit } [S_{32}, S_{13}, S_{21}] \quad (3)$$

$$RV2_{CSM} \text{ (rotation vector 2 for CSM)} = \text{Unit } [P_{32}, P_{13}, P_{21}] \quad (4)$$

These vectors ($RV1_{CSM}$, $RV2_{CSM}$) are the rotation vectors or eigen vectors normal to the plane of rotation and are expressed in CSM IMU coordinates. These two vectors are analagous to AOT sightings on celestial bodies and are used in the same manner to construct a reference coordinate system.

Similar computations for the LM are presented in equations (5), (6), and (7).

$$[R1] = [SMNB_4]_{LM}^T [SMNB_5]_{LM} \quad (5a)$$

$$[R2] = [SMNB_5]_{LM} [SMNB_6]_{LM} \quad (5b)$$

$$[S]_{LM} = [R1]^T - [R1] \quad (5c)$$

$$[P]_{LM} = [R2]^T - [R2] \quad (5d)$$

$$RV1_{LM} \text{ (rotation vector 1 for LM)} = \text{Unit } [S_{32}, S_{13}, S_{21}] \quad (6)$$

$$RV2_{LM} \text{ (rotation vector 2 for LM)} = \text{Unit } [P_{32}, P_{13}, P_{21}] \quad (7)$$

These vectors ($RV1_{LM}$, $RV2_{LM}$) are the rotation vectors or eigen vectors normal to the plane of rotation and are expressed in LM IMU coordinates. These two vectors are analagous to AOT sightings on celestial bodies and are used in the same manner to construct a reference coordinate system.

Generate an orthogonal coordinate system by use of the four rotation vectors. The system for the CSM is given by equations (8) and (9). The system for the LM is given by equations (10) and (11).

$$UX = \text{Unit}\ [RV1_{CSM}] \tag{8a}$$

$$UY = \text{Unit}\left[\text{Unit}\ [RV1_{CSM}] \times \text{Unit}\ [RV2_{CSM}]\right] \tag{8b}$$

$$UZ = [UX] \times [UY] \tag{8c}$$

$$\begin{bmatrix} N \end{bmatrix} = \begin{bmatrix} UX \\ UY \\ UZ \end{bmatrix} \tag{9}$$

$$UX = \text{Unit}\ [RV1_{LM}] \tag{10a}$$

$$UY = \text{Unit}\left[\text{Unit}\ [RV1_{LM}] \times \text{Unit}\ [RV2_{LM}]\right] \tag{10b}$$

$$UZ = [UX] \times [UY] \tag{10c}$$

$$\begin{bmatrix} M \end{bmatrix} = \begin{bmatrix} UX \\ UY \\ UZ \end{bmatrix} \tag{11}$$

Determine the product according to equations (12) through (16) and the following program.

$$[A] = [N]^T[M] \tag{12}$$

$$Z'_D = \text{Unit}\ [-A\ (1,3),\ 0, A\ (1,1)] \tag{13}$$

$$\sin IGA = Z'_D\ (1) \tag{14a}$$

$$\cos IGA = Z'_D\ (3) \tag{14b}$$

$$IGA = \text{ARCTRIG}\ [\sin IGA,\ \cos IGA] * 57.2957795 \tag{14c}$$

$$\sin MGA = A(1,2) \tag{15a}$$

$$\cos MGA = Z'_D(3) * A(1,1) - Z'_D(1) * A(1,3) \tag{15b}$$

$$MGA = ARCTRIG\,[\sin MGA, \cos MGA] * 57.2957795 \tag{15c}$$

$$\sin OGA = Z'_D \cdot [A(2,1), A(2,2), A(2,3)] \tag{16a}$$

$$\cos OGA = Z'_D \cdot [A(3,1), A(3,2), A(3,3)] \tag{16b}$$

$$OGA = ARCTRIG\,[\sin OGA, \cos OGA] * 57.2957795 \tag{16c}$$

```
[a]FUNCTION ARCTRIG [S,C]
   SP = ABS [S]
   X = SIGN [1,S]
   IF (SP - 0.7072) 1, 2, 2
   2 ARCTRIG = X * ARCOS [C]
   GO TO 3
   1 ARCTRIG = ARSIN [S]
   IF (C) 4,3,3
   4 ARCTRIG = X * 3.141592654 - ARCTRIG
   3 CONTINUE
   RETURN
   END
```

[a]Output is in radians.

The following computations are necessary to determine the rotation angle and the angle between the eigen vectors. The rotation angle represents the amount of rotation (in degrees) of the docked configuration in the plane of rotation.

The matrix [B] shall be represented as follows.

$$[B] = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

This [B] matrix describes the orthogonal transformation from navigation base 1 orientation to navigation base 2 orientation or the first two attitudes of the docked configuration.

$$\phi = \text{rotation angle} = \text{ARCOS}\ [(\Sigma_i\ b_{ii} - 1.0)/2.0] \quad \text{(from attitude 1 to attitude 2)} \qquad (17)$$

The rotation angle will be computed for any two attitudes. There will be no sign associated with this angle.

The angle between the eigen vectors is the central angle (in degrees) between the two pointing directions in space (S and P). There is also no sign associated with this angle.

$$\theta = \text{ARCOS}[RV1_{CSM} \cdot RV2_{CSM}] \qquad (18)$$

3.0 MODE 2 METHOD AND EQUATIONS

For this option, the assumption must be made that the LM and CSM X-body axes are coincident. If this assumption can be made, then the X-axis of the docked configuration is considered a rotation vector and can replace the computation for one of the rotation vectors as explained in mode 1. The X-axis can be expressed in CSM and LM IMU coordinates by the direction cosines from the stable member to navigation base matrix.

To obtain two directions in space (S and P) with respect to the CSM and LM IMU, the following procedure is used. As in mode 1, the matrices Q_1, Q_2, and Q_3 are computed.

$$Q_1 = \begin{bmatrix} \cos IGA & 0 & -\sin IGA \\ 0 & 1 & 0 \\ \sin IGA & 0 & \cos IGA \end{bmatrix}$$

$$Q_2 = \begin{bmatrix} \cos MGA & \sin MGA & 0 \\ -\sin MGA & \cos MGA & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$Q_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos OGA & \sin OGA \\ 0 & -\sin OGA & \cos OGA \end{bmatrix}$$

$$[SMNB] = [Q_3][Q_2][Q_1]$$

Because there are only two attitudes for this option, a total of four computations give $[SMNB_1]\ldots[SMNB_4]$. Computations for the CSM are given by equations (2a), (2c), (19), and (20). Computations for the LM are given by equations (5a), (5c), (21), and (22).

$$[R1] = [SMNB_1]_{CSM}{}^T [SMNB_2]_{CSM} \quad (2a)$$

$$[S]_{CSM} = [R1]^T - [R1] \quad (2c)$$

$$RV1_{CSM}(\text{rotation vector 1 for CSM}) = \text{Unit } [S_{32}, S_{13}, S_{21}] \quad (19)$$

If $[SMNB_1]_{CSM} = [R2]$ and $[SMNB_2]_{CSM} = [R3]$, then

$$RV2_{CSM}(\text{rotation vector 2 for CSM}) = -\text{ Unit } [R2_{11}, R2_{12}, R2_{13}] \quad (20a)$$

or

$$RV2_{CSM} \text{ (rotation vector 2 for CSM)} = - \text{Unit } [R3_{11}, R3_{12}, R3_{13}] \qquad (20b)$$

$$[R1] = [SMNB_3]_{LM}{}^T [SMNB_4]_{LM} \qquad (5a)$$

$$[S]_{LM} = [R1]^T - [R1] \qquad (5c)$$

$$RV1_{LM} \text{ (rotation vector 1 for LM)} = \text{Unit } [S_{32}, S_{13}, S_{21}] \qquad (21)$$

Let $[SMNB_1]_{LM} = [R2]$ and $[SMNB_2]_{LM} = [R3]$.

$$RV2_{LM} \text{ (rotation vector 2 for LM)} = \text{Unit } [R2_{11}, R2_{12}, R2_{13}] \qquad (22a)$$

or

$$RV2_{LM} \text{ (rotation vector 2 for LM)} = \text{Unit } [R3_{11}, R3_{12}, R3_{13}] \qquad (22b)$$

The same procedure is now used as in mode 1 to construct a reference coordinate system and to solve for the torquing angles.

4.0 MODES 1 AND 2

Compute Q1, Q2, and Q3 six times for three attitudes of the CSM and three attitudes of the LM.

```
Q1 (1,1) = cos IGA
Q1 (1,2) = 0.0
Q1 (1,3) = -sin IGA
Q1 (2,1) = 0.0
Q1 (2,2) = 1.0
Q1 (2,3) = 0.0
Q1 (3,1) = sin IGA
Q1 (3,2) = 0.0
Q1 (3,3) = cos IGA
```

```
Q2 (1,1) = cos MGA
Q2 (1,2) = sin MGA
Q2 (1,3) = 0.0
Q2 (2,1) = -sin MGA
Q2 (2,2) = cos MGA
Q2 (2,3) = 0.0
Q2 (3,1) = 0.0
Q2 (3,2) = 0.0
Q2 (3,3) = 1.0
```

```
Q3 (1,1) = 1.0
Q3 (1,2) = 0.0
Q3 (1,3) = 0.0
Q3 (2,1) = 0.0
Q3 (2,2) = cos OGA
Q3 (2,3) = sin OGA
Q3 (3,1) = 0.0
Q3 (3,2) = - sin OGA
Q3 (3,3) = cos OGA
```

SMNB1 = [Q3][Q2][Q1]	Set 1 CSM
SMNB2 = [Q3][Q2][Q1]	Set 2 CSM
SMNB3 = [Q3][Q2][Q1]	Set 3 CSM
SMNB4 = [Q3][Q2][Q1]	Set 1 LM
SMNB5 = [Q3][Q2][Q1]	Set 2 LM
SMNB6 = [Q3][Q2][Q1]	Set 3 LM

$$\downarrow$$

$$R1_{CSM} = [SMNB1]^T[SMNB2]$$
$$R2_{CSM} = [SMNB2]^T[SMNB3]$$
$$S_{CSM} = [R1]^T - [R1]$$
$$P_{CSM} = [R2]^T - [R2]$$

$$\downarrow$$

$$R1_{LM} = [SMNB4]^T[SMNB5]$$
$$R2_{LM} = [SMNB5]^T[SMNB6]$$
$$S_{LM} = [R1]^T - [R1]$$
$$P_{LM} = [R2]^T - [R2]$$

$$\downarrow$$

$$RV1_{CSM} = \text{Unit } [S_{32}, S_{13}, S_{21}]_{CSM}$$
$$^{a}RV2_{CSM} = \text{Unit } [P_{32}, P_{13}, P_{21}]_{CSM}$$
$$RV1_{LM} = \text{Unit } [S_{32}, S_{13}, S_{21}]_{LM}$$
$$^{a}RV2_{LM} = \text{Unit } [P_{32}, P_{13}, P_{21}]_{LM}$$

[a]For mode 2, $RV2_{CSM}$ = -Unit [SMNB1(1,1), SMNB1(1,2), SMNB1(1,3)]
$RV2_{LM}$ = Unit [SMNB4(1,1), SMNB4(1,2), SMNB4(1,3)]

$$UX_{CSM} = \text{Unit}\ [RV1_{CSM}]$$

$$UY_{CSM} = \text{Unit}\ \left\{\text{Unit}[RV1_{CSM}] \times \text{Unit}\ [RV2_{CSM}]\right\}$$

$$UZ_{CSM} = [UX] \times [UY]$$

$$UX_{LM} = \text{Unit}\ [RV1_{LM}]$$

$$UY_{LM} = \text{Unit}\ \left\{\text{Unit}[RV1_{LM}] \times \text{Unit}\ [RV2_{LM}]\right\}$$

$$UZ_{LM} = [UX] \times [UY]$$

$$[N] = \begin{bmatrix} UX_{CSM} \\ UY_{CSM} \\ UZ_{CSM} \end{bmatrix}$$

$$[M] = \begin{bmatrix} UX_{LM} \\ UY_{LM} \\ UZ_{LM} \end{bmatrix}$$

$$[A] = [N]^T[M]$$

[a] $$[A1] = \underbrace{[REFSMMAT]}_{\text{LM DESIRED}}\ \underbrace{[REFSMMAT]^T}_{\text{CSM ACTUAL}}[A]$$

[a]This additional computation is inserted to allow for torquing the LM platform to any desired REFSMMAT (not necessarily the CSM platform orientation).

Z'_D = Unit [-A1(1,3), 0, A1(1,1)]

Sin IGA = Z'_D (1)

Cos IGA = Z'_D (3)

Sin MGA = A1(1,2)

Cos MGA = Z'_D (3) * A1(1,1) - Z'_D (1) * A1(1,3)

Sin OGA = Z'_D · [A1(2,1), A1(2,2), A1(2,3)]

Cos OGA = Z'_D · [A1(3,1), A1(3,2), A1(3,3)]

↓

Call Arctrig
Output torquing angles[a]
IN, MI, OT in degrees

↓

[b] ϕ_1 = ARCOS $[(\Sigma_i \, R1_{ii} - 1.0)/2.0]$

[b] ϕ_2 = ARCOS $[(\Sigma_i \, R2_{ii} - 1.0)/2.0]$

θ = ARCOS $[RV1_{CSM} \cdot RV2_{CSM}]$

↓

Output angles ϕ_1, ϕ_2, θ in degrees

[a]In - inner, MI - middle, OT - outer.

[b]R1 and R2 are respectively $R1_{CSM}$ and $R2_{CSM}$.