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# RTCC REQUIREMENTS FOR MISSIONS F AND G: SELENOGRAPHIC-MNBY VELOCITY TRANSFORMATION FORMULATION 

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# PROJECT APOLLO <br> RTCC REQUIREMENTS FOR MISSIONS F AND G: SELENOGRAPHIC-MNBY VELOCITY TRANSFORMA TION FORMULATION <br> By Paul F. Flanayan Mathematical Physic; Branch 

May 15, 1968

MISSION PLANNING AND ANALYSIS DIVISION NATIONAL AERONAUTICS AND SPACE ADMINISTRATION MANNED SPACECRAFT CENTER HOUSTON, TEXAS


# RTCC REQUIREMENTS FOR MISSIONS F AND G: <br> SELENOGRAPHIC-MNBY VELOCITY TRANSFORMATION FORMULATION 

By Paul F. Flanagan

## SUMMARY

This document presents the formulation and implementation procedures for use in the RTCC for transforming velocity vectors from the selenographic coordinate system to the mean nearest Besselian year (MNBY) system. The formulation includes equations for computing the lunar angular velocity vector.

## INTRODUCTION

The selenocentric MNBY system is moon centered and has its axes parallel to the following: the X-axis is in the mean earth equatorial plane and through the mean vernal equinox at the beginning of the nearest Besselian year; the Z-axis is north through the mean earth spin axis; and the Y -axis is on the mean earth equatorial plane and forms a righthanded orthogonal system.

The moon fixed selenographic system is defined as follows. The X-axis is fixed in the moon's equatorial plane and points approximately toward the center of the earth; the Z-axis is fixed in the moon through the moon spin axis; and the Y-axis is in the equatorial plane and completes the right-handed orthogonal system.

The interpolated precession-libration matrix (L) used to transform the velocity is also used to transform position vectors. If $R_{M}$ is a position vector in the selenocentric MNBY system and $R_{G}$ is a position vector in the selenographic system, then

$$
\mathrm{R}_{\mathrm{M}}=\mathrm{L}^{\mathrm{T}} \mathrm{R}_{\mathrm{G}}
$$

and

$$
R_{G}=L R_{M} .
$$

## VELOCITY VECTOR TRANSFORMATION

The inertial MNBY velocity, $\dot{R}_{M}$, of a vehicle with a fixed selenographic position, $R_{G}$, is computed using the following equation

$$
\dot{R}_{M}=L^{T}\left(W \times R_{G}\right)
$$

where $W$ is the moon's angular velocity expressed in the selenographic system.

The following general equations can be used to compute the inertial velocity for a vehicle moving relative to the selenographic system at velocity $\dot{R}_{G}$ or for the reverse computation.

$$
\begin{gathered}
\dot{R}_{M}=L^{T}\left(\dot{R}_{G}+W \times R_{G}\right) \\
\dot{R}_{G}=L \dot{R}_{M}-W \times R_{G}
\end{gathered}
$$

## COMPUTATION OF W

The following equations are used to compute the angular velocity vector, $W$. These equations account for the rate of change of $\Omega$, the longitude of the mean ascending node of the lunar orbit measured in the ecliptic plane from the mean equinox of date, and $M$, the geocentric mean longitude of the moon messured in the ecliptic plane from the mean equinox of date to the mean ascending node of the lunar orbit, and then along the orbit.

The computations neglect the small effect of the rates of precession, obliquity of the ecliptic, and physical libration. The inclination (i) of the lunar orbit to the ecliptic is constant and is $1^{\circ} 32^{\prime} 39^{\prime \prime}$ (ref. 1).

Prelaunch initialization:

1. Inplit year of launch, YOL, and day of launch, DOL.

Tomuary 1 is day 1.
2. Compute Julian centuries from January $0.5,1900$ to midnight prior to launch:

$$
\begin{aligned}
& \mathrm{TIL}=\text { integral part }[(\mathrm{YOL}-1901) / 4] \\
& \mathrm{TB}=(\mathrm{YOL}-1900) 365+\mathrm{TIL}+\mathrm{DOL}-0.5 \\
& \mathrm{TM}=\mathrm{TB} / 36525
\end{aligned}
$$

3. Compute time rates of angles, $\dot{\Omega}$ and $\dot{M}$ in rad/hr:
$\dot{\Omega}=\left[-1934.1420083+0.004155556 \mathrm{TM}+\left(0.6666667 \times 10^{-5}\right) \mathrm{TM}^{2}\right] \mathrm{RATE}$ $\dot{\mathrm{M}}=\left[481267.8831417-0.002266667 \mathrm{TM}+\left(0.5666667 \times 10^{-5}\right) \mathrm{TM}^{2}\right] \mathrm{RATE}$
where RATE $=\pi /(180 \cdot 36525 \cdot 24)=1.9110212776572292 \times 10^{-8}$ and converts $\dot{\Omega}$ and $\dot{M}$ from deg/Julian century to $\mathrm{rad} / \mathrm{hr}$.
4. Compute constant terms:

$$
\begin{aligned}
& K 1=\dot{\Omega} \cos 1 \\
& K 2=\dot{\Omega} \sin i
\end{aligned}
$$

5. Compute $W_{Z}$ in rad $/ \mathrm{hr}$ :

$$
W_{Z}=\dot{M}-\dot{\Omega}+K I
$$

Computation of $W_{X}$ and $W_{Y}$ at any time $t$ during mission:

1. Input hours from midnight prior launch, $H$.
2. Compute Julian centuries from January 0.5 , 1900 to anytime $t$ :

$$
T=(T B+H / 24) / 36525
$$

3. Compute $\mathrm{M}-\Omega$ in deg:

$$
\begin{aligned}
\mathrm{M}-\Omega & =11.2508889+483202.02515 \mathrm{~T} \\
& -0.003211111 \mathrm{~T}^{2}-\left(0.0333333 \times 10^{-5}\right) \mathrm{T}^{3}
\end{aligned}
$$

4. Compute $W_{X}$ and $W_{Y}$ in $\mathrm{rad} / \mathrm{hr}$ :

$$
\begin{aligned}
& W_{X}=K 2 \sin (M-\Omega) \\
& W_{Y}=K 2 \cos (M-\Omega)
\end{aligned}
$$

A listing of a single-precision checkout program to compute the angular velocity, $W$, is given in table I. A sample output for RTCC checkout is provided in table II for midnight January 1 through 10, 1965.

## REPRODUCIBILITY OF' THE ORIGINAL. PAGE IS POOR.

## TABLE I.- LISTING OF SINGLE-PRECISION CHECKOUT PROGRAM

 T'O COMPUTE THE ANGULAR VELOCITY VECTOR W

```
CCf:T[v01.,i\cdot]
l ACr:PT(:OL)
TVC!=v0l
IL=(IVNl - 10(1)/h
T=(YCL-1%f(.)*3C5.+TII + NOL -.5
T:=T`/35505.
N:3N\=3.1A15n265/100
*GLI=55,50 * *[ 2GनिA/3500.
NT=(-10.34.1420003+(.004155555+.6555567.-5.7..) T*)*i
    =(4*1007.8831417+(-.00225556+.5650667E-5*Ti)*T)*T S*
K1=0-*COC[A`CLI]
"*=OT*STN[ANGLI]
*(3)=(\ - - % +K|)
T=(TJ+H/24.)/35525.
O=(11.250880 + (483202.02515+(-.0032111111-.0333333E-5*T)*T)*T)*DEORA
U(1) = (K2*SIN[MMO])
N(2)=(K?*COS[MMO])
7% 1.0/(35525.0*24.0)
\because(1)=01(1)*7.7
'(2) = ''(?)*?, - %
"'(3)=1(3)*7.7
CISPLAY[4(1),名(?),4(3)]
DISPLAY[YOL.,ROL,!?
CO TO 1
```


## TABLE II.- ANGULAR VELOCITY VECTORS FOR

PRELAUNCH TNITIALIZATION FOR MIDNIGHT OF JANUARY 1 THROUGH 10, 1969

| Day | Angular velocity, rad hr |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{W}_{\mathrm{x}}$ | $\mathrm{W}_{\mathrm{y}}$ | $\mathrm{W}_{\mathrm{z}}$ |  |
| 1 | $-0.929147614 \times 10^{-6}$ | $-0.454106239 \times 10^{-6}$ | $0.958135087 \times 10^{-2}$ |  |
| 2 | $-0.100841172 \times 10^{-5}$ | $-0.229420041 \times 10^{-6}$ | $0.958135087 \times 10^{-2}$ |  |
| 3 | $-0.103415296 \times 10^{-5}$ | $0.744292239 \times 10^{-8}$ | $0.958135087 \times 10^{-2}$ |  |
| 4 | $-0.100500508 \times 10^{-5}$ | $0.24391089 \times 10^{-6}$ | $0.958135087 \times 10^{-2}$ |  |
| 5 | $-0.922515165 \times 10^{-6}$ | $0.467432877 \times 10^{-6}$ | $0.958135087 \times 10^{-2}$ |  |
| 6 | $-0.791061451 \times 10^{-6}$ | $0.666145255 \times 10^{-6}$ | $0.958135087 \times 10^{-2}$ |  |
| 7 | $-0.617621078 \times 10^{-6}$ | $0.829501023 \times 10^{-6}$ | $0.958135087 \times 10^{-2}$ |  |
| 8 | $-0.411399568 \times 10^{-6}$ | $0.948829879 \times 10^{-6}$ | $0.958135087 \times 10^{-2}$ |  |
| 9 | $-0.183342448 \times 10^{-6}$ | $0.101779825 \times 10^{-5}$ | $0.958135087 \times 10^{-2}$ |  |
| 10 | $0.544458047 \times 10^{-7}$ | $0.103274556 \times 10^{-5}$ | $0.958135087 \times 10^{-2}$ |  |
|  |  |  |  |  |

## 6

## REFERENCE

1. Roth, H.: Description of Two Modified Versions of the Selenographic Coordinate Transformation Program (Phase I). Report No. 3400-6059RUOOO, July 21, 1965.
