Massachusetts Institute of Technology Instrumentation Laboratory Cambridge, Massachusetts

Space Guidance Analysis Memo #37-64

TO: SGA Distribution
FROM: Donald Fraser
DATE: September 25, 1964
SUBJECT: Computer Program Description - Root Locus Technique

This memo describes a MH-1800 computer program developed to calculate the points for a root locus diagram with gain as a variable. Up to 19 open loop poles may be processed.

The method of calculation used is to form the polynomial 1 + KG (assuming negative feedback) and then factoring it using a binary version of the ROOTFINDER program^{*} written by R. W. Baker.

Gain Control:

The polynomial KG is expressed in the form:

KG = K
$$\frac{(s^{m} + a_{m-1} s^{m-1} + a_{m-2} s^{m-2} + \dots + a_{1} s + a_{0})}{(s^{n} + b_{n-1} s^{n-1} + b_{n-2} s^{n-2} + \dots + b_{1} s + b_{0})}$$

Note that the highest order coefficient in both numerator and denominator is unity.

The gain K is incremented logarithmically according to the rule

$K = (\Delta K) K$

from a specified minimum gain to a specified maximum gain and may be either positive or negative. If negative gain is used (positive feedback) the minimum gain should be a negative number with a larger magnitude than the maximum gain.

Output Control:

Output from the program includes a printed list of the closed loop poles for each value of gain plus, if desired, a completely plotted root locus diagram.

See SGA Memo #32-64 for description of factoring technique

Provision is included to mark certain points on each branch to facilitate matching points on the plot with corresponding gains on the printout. The frequency of these points is controlled by the input variable "Space". If it is desired to mark every fifth point on each branch, space = 5.

Plotting of all points outside of a given radius can be suppressed. This radius is equal to the input variable "Limit".

Provision is included to plot either all the points or only those lying in the top half plane. This is controlled by the input variable "Plane" as follows:

Plane = 0 Both top and bottom half plane points are plotted Plane = 1 Only top half plane points are plotted

Input Cards Required

* Job Card

* Userfile = Apollo

* Run Rootlocus

F As many sets of data cards as desired

* Endjob

Data Cards

Card 1

Word 1	Gain Increment (ΔK)			
Word 2	Minimum Gain			
Word 3	Maximum	Gair	1	
Word 4	Root Flag	If =	0	Read poles and zeros in
				factored form
		If =	1	Read poles and zeros in
				polynomial form
Word 5	Space			
Word 6	Plot Flag	If =	0	No plotted output
		If =	1	Input control of plot for-
			m	at as described below is
			pc	ossible
		TC	0	Dist is done in 7 trans-

If = 2 Plot is done in Z transform format described below

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Card 2 (to be used only on the first run of any job when word 6 above is nonzero)

	Word 1	Limit
	Word 2	Plane
	Word 3	Number of runs in the job
If Root	Flag = 0	
Card 3	Word 1 = A	= number of real axis open loop poles
	Word $2 = E$	3 = number of complex open loop pole pairs
	Word $3 = C$	t = number of real axis open loop zeros
	Word 4 = D	= number of complex open loop zero pairs
Card 4	to Card 3 + 4	A
	Word 1 = p	osition of real poles (one per card)
Card 4	+ A to Card	3 + A + B
	Word $1 = r$	eal part of complex pole position
		maginary part of complex pole position one card for each complex pole pair)
Card 4	+ A + B to C	ard $3 + A + B + C$
	Word 1 = p	osition of real zeros (one per card)
Card 4	+A+B+C	to Card $3 + A + B + C + D$
	Word 1 = r	eal part of complex zero position
		maginary part of complex zero position one card for each complex zero pair)
If Root	Flag = 1	
Card 3	Word 1	order of denominator polynomial = number of open loop poles
	Word 2	order of numerator polynomial = number of open loop zeros
Card Gi	coup 4	Coefficients of denominator polynomial of G
		in increasing order (six per card). Continue using cards until all coefficients are entered.
Card G	coup 5	Coefficients of numerator polynomial of G in increasing order (six per card). Continue using cards until all coefficients are entered.
If Plot I	Flag = 0 or 2	
		NT

No additional input cards are required

If Plot Flag = 1

Control of the plot format with the following additional input cards is possible. For a description of the variables see the FILEPLOT memo (PR - 0007) published by the digital computation group. This group of cards should be included only on the first run of any job.

Card A1		
	Word 1	Point (for marked points only)
Card A2		
	Word 1	XMIN
	Word 2	XMAX
	Word 3	XINCH
	Word 4	XTICK
Card A3		
	Word 1	YMIN
	Word 2	YMAX
	Word 3	YINCH
	Word 4	YTICK

D1	TT 1
PIOU	Format

With Plot Flag non-zero, a point plot is made with open loop poles marked with X's, open loop zeros marked with small circles and locus points marked with small dots. If Plot Flag = 1.0, control of special point marking is possible, otherwise these points are marked with small squares.

The Z transform plot format is set up to plot only top half plane points in a frame small enough to fit in an $8 1/2 \times 11$ ring binder. The radius of the unit semicircle is four inches and tick spacing on both axes is 0.1. Limit is automatically reset to 1.25 if this plotting mode is selected; however it still must be read in.

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