MIT INSTRUMENTATION LABORATORY

DG MEMO NO. 145

A Manual Abort Guidance Procedure Based On Range and Altitude Measurements

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1. Summary

A guidance procedure is presented which can be used to guide the LEM from the injection point of a roughly circular orbit to intercept of the command module. The system depends only on measurement of altitude and of range to the command module. The system utilizes the principles of orbital mechanics (specifically, orbit perturbation theory) and therefore is not applicable to guidance during thrusting.

Rendezvous is accomplished in four phases. These are:

a. The initial LEM orbit is determined and corrected such as to produce a safe orbit which crosses the altitude of the Command Module. Phase and plane errors are ignored.

b. As the LEM approaches the altitude of the Command Module the range to the Command Module is measured and the LEM orbit is corrected at the equi-altitude point such as to eliminate the phase error in either one-half or a whole number of orbits, depending upon fuel and safety limitations. In either case plane error is ignored. The terminal LEM altitude is the Command Module altitude.

c. The LEM orbit is circularized. The resulting orbit has the same altitude as the Command Module and therefore the LEM will intercept the Command Module during the succeeding orbit.

d. The plane error is removed at the intercept point, which accomplishes rendezvous.

Orbit determination and phase correction are based on the theory of the propagation of perturbations from circular orbits. Orbit determination requires ascertaining the altitude rate and the velocity excess. (Velocity excess is defined as the difference between the horizontal component of velocity and that which would produce a circular orbit at the same altitude.) Altitude rate is obtained by dividing the change in altitude by the time interval. Velocity excess is obtained by measuring the apparent vertical acceleration. Thus altitude rate is found from two altitude measurements, and velocity excess from three. Phase correction is based upon producing at the altitude of the Command Module a LEM orbit perturbed from circularity by that vertical or horizontal velocity component which will eliminate the phase error in one-half or a whole number of orbits respectively.

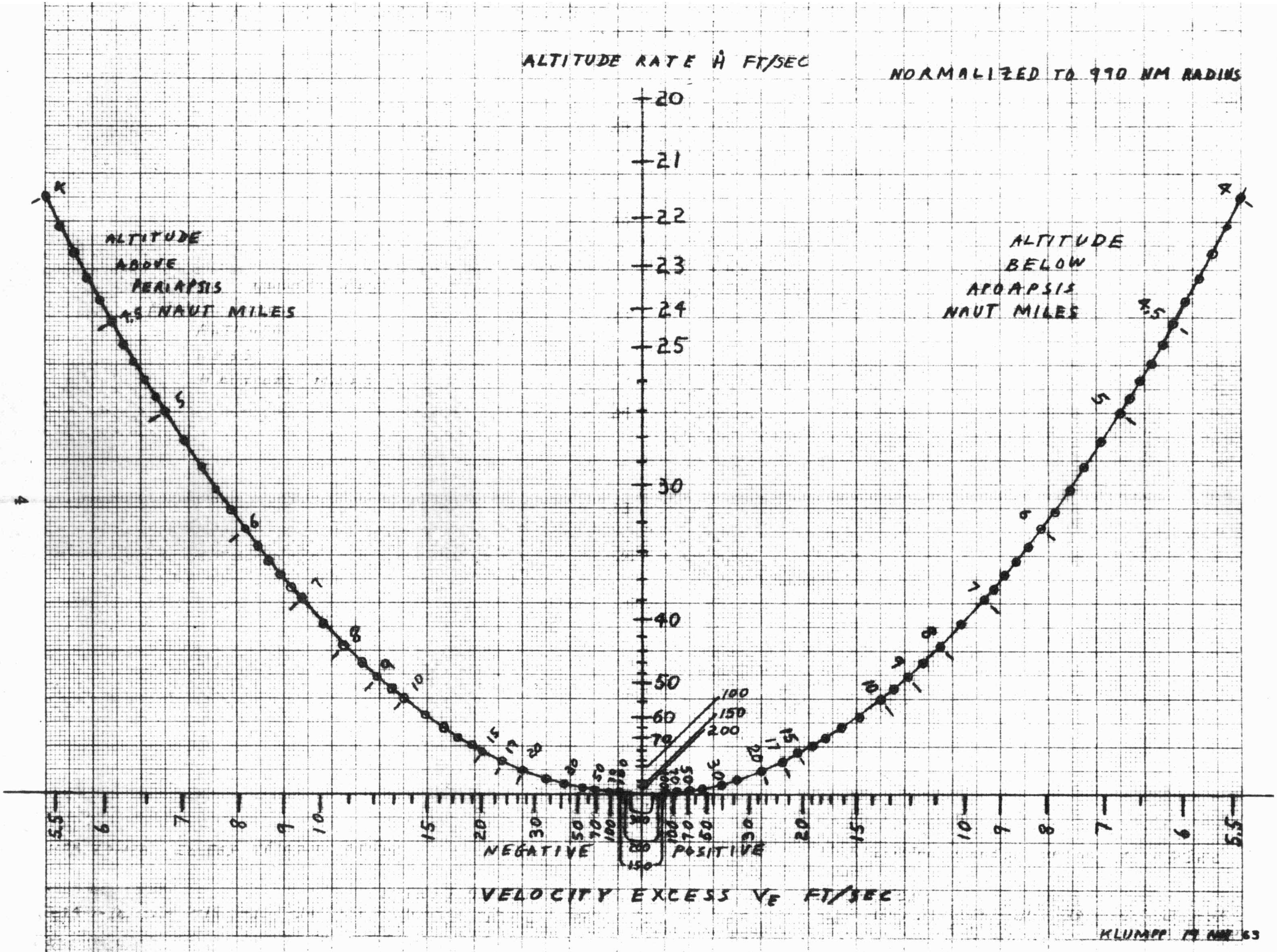
Altitude may be determined using a hand held sextant to measure the angle subtended by the lunar disk. 0.05 degree sextant accuracy is shown to be adequate in Section 2. Sextants with better precision are used aboard ships and may be built at a weight of about two pounds. The LEM has sufficient visibility to use a sextant, but because range is also required if the LEM is to intercept the command module, a hand held laser rangefinder to measure both altitude and range may be preferred. A rangefinder would also eliminate the visibility problem where the moon is not lighted. The characteristics of the rangefinder have not been investigated. A hand held calculator is proposed for solving the perturbation equations. This device, illustrated in Fig. 4-1, is estimated to weigh roughly two pounds. Alternatively to the calculator the solutions of the orbital equations may be tabulated or nomographed. A nomogram of the solution of the altitude equations is shown in Fig. 1-1.

From a closely spaced train of altitude measurements the vertical and horizontal velocity components can be determined with surprising precision. In lunar orbit, by using a sextant accurate to 0.05° to measure the angle subtended by the moon from 50,000 ft, and assuming 1000 ft uncertainty in the terrain at the horizon, the astronaut will obtain from three measurements at five minute intervals (ten minutes total time) the horizontal and vertical velocity components to an accuracy of 2.5 ft/sec and 16 ft/sec respectively.

In section 2 the theory underlying this approach to orbital and rendezvous guidance is developed. In section 3 thrust vector directional control is briefly discussed. In section 4 the hand held calculator is developed. In section 5 an example is presented in which the LEM is guided from the initial injection point to intercept of the command module. The example demonstrates the ease and the surprising precision with which this can be accomplished. Finally, in section 6 suggestions are made for further work.

2. Theory and Errors

The two objectives of the proposed guidance are to yield an orbit which is safe and ultimately to rendezvous with the Command Module. Safety depends upon timely prediction and control of altitude and



NOMOGRAM OF PERTURBATIONS FROM A CIRCULAR LUNAR ORBIT rendezvous depends primarily upon prediction and control of phase. Timely control of altitude means that the initial orbit must be determined and corrected in a small fraction of an orbital period. For lunar orbits, one-eighth of a period or 15 minutes has been found adequate for the errors estimated for a crude manual abort injection.

The fundamental principle of the proposed guidance system is that, with a nearly circular orbit, the effects of several perturbations occurring simultaneously or separately can be obtained by superposition of the effects of the individual perturbations. At any instant of time we consider the spacecraft to be in an orbit perturbed from a circular orbit at that instantaneous altitude by a vertical and a horizontal velocity component. The vertical component is the instantaneous altitude rate and the horizontal component is the velocity excess (or deficiency). The altitude rate and velocity excess required for safety and phase control are found by superposing the propagated effects of assumed perturbations. The existing perturbations are found from a train of altitude measurements. The difference between the required and the existing velocity perturbations are the components of the velocity to be gained.

2.1 Derivation

The altitude of an orbit with respect to initial altitude is given by*

$$H = \frac{\dot{H}_O}{\omega} \sin \omega t + \frac{2 V_{E0}}{\omega} (1 - \cos \omega t) \qquad (2-1)$$

*From Janusz Sciegienny "Summary of Error Propagation in an Inertial System" MIT Instrumentation Laboratory publication E-1388 August 1963.

where $\dot{H}_o$ is the initial altitude rate V_{E0} is the initial velocity excess, and ω is the orbital rate.

It is seen from Equation (2-1) that the altitude may be represented by the vertical component of a rotating vector. This representation is illustrated in Fig. (2-1). Thus if the initial altitude, altitude rate, and velocity excess can be determined, then it is possible to construct an analog device based on the concept of Fig. (2-1) for predicting the future course of altitude. The orbital guidance calculator of section 4 stems from this concept.

The equations given below for phase correction are special cases of perturbation equations found in the references.* The altitude rate required to advance in phase by the distance S in one-half orbit is given by

$$\dot{H} = -\frac{\omega}{4} S. \qquad (2\text{-}2)$$

If the phase is to be corrected in one orbit by imparting a horizontal velocity perturbation, it is a curious fact that the average apparent relative velocity of the perturbed orbit relative to the unperturbed orbit is minus three times the velocity perturbation. Thus in one revolution the distance the LEM will advance on the command module due to a horizontal velocity perturbation is given by

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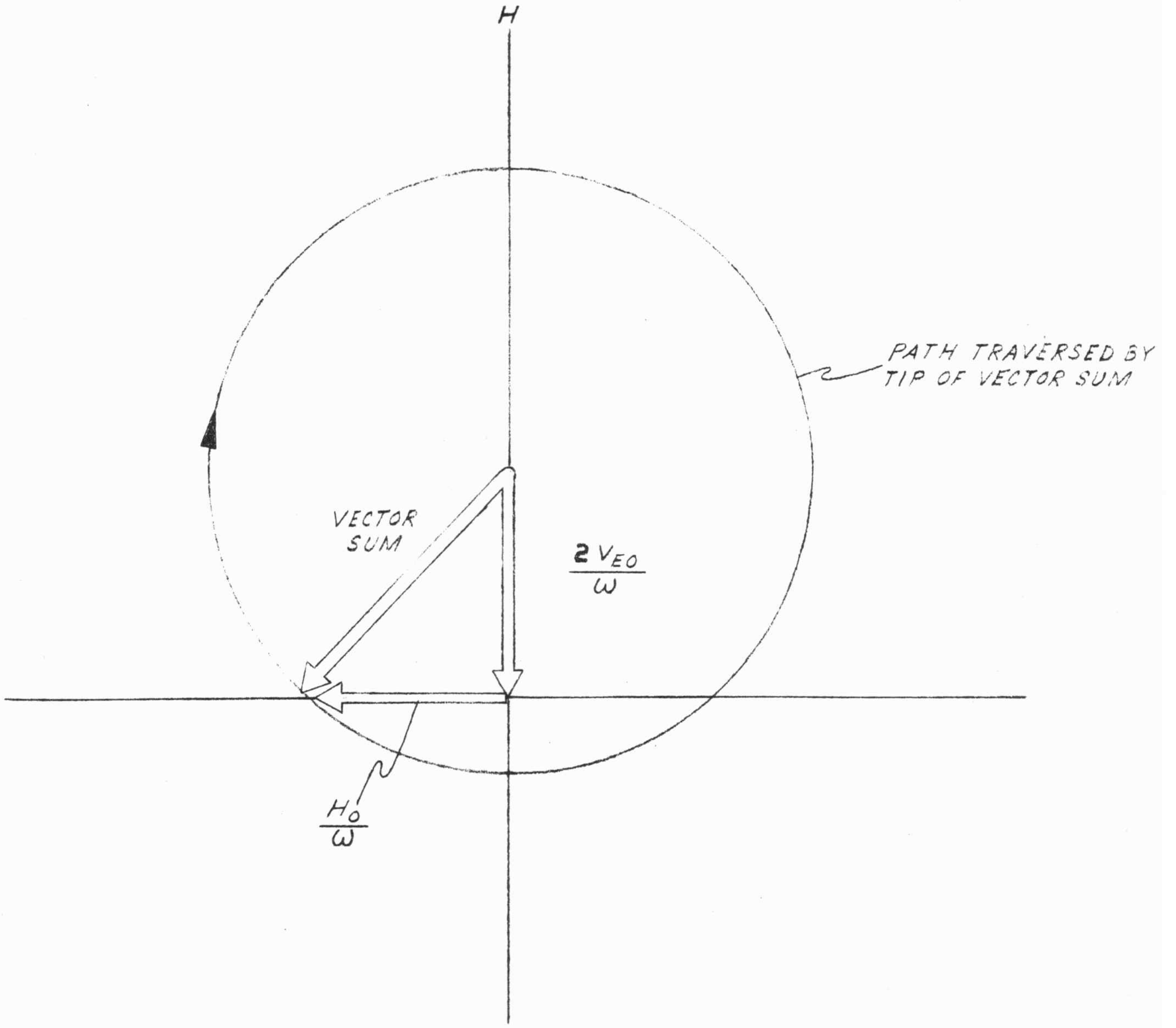
*Sciegienny, referenced previously, and W. S. Clohessy and R. S. Wiltshire "Terminal Guidance System for Satellite Rendezvous" Journal of the Aerospace Sciences, September, 1960.



ROTATING VECTOR REPRESENTATION OF THE ALTITUDE OF A PERTURBED ORBIT WITH RESPECT TO AN UNDISTURBED CIRCULAR ORBIT

FIGURE (2-1)

$$S = -3 V_E T \tag{2-3}$$

where T is the orbital period. Solving for V_E and Substituting $2\pi/\omega$ for T yields

$$V_E = -\frac{\omega}{6\pi} S. \tag{2-4}$$

From Equations (2-1), (2-2) and (2-4), it is seen that the altitude and phase can be predicted and controlled if the initial altitude, altitude rate, and velocity excess can be determined, and if the altitude rate and velocity excess can be corrected.

Orbit determination requires finding the altitude rate and the velocity excess. We presume that the astronaut can measure altitude. The altitude rate $\dot{H}$ is found by dividing the difference between two measured altitudes by the time interval between the measurements. For reasons which will be apparent shortly, we assume three measurements spaced at time intervals Δt with the vertical velocity determined from the first and last measurement. $\dot{H}$ is then given by

$$\dot{H}_t = \frac{H_{t+\Delta t} - H_{t-\Delta t}}{2\,\Delta t}. \tag{2-5}$$

The velocity excess is found from the apparent vertical acceleration which it causes. The relationship is

$$\ddot{H} = \frac{V_H^2}{R+H} - g \tag{2-6}$$

where V_H is the horizontal component of velocity and g is the local acceleration of gravity. Substituting

$$V_H = V_C + V_E \ , \tag{2-7}$$

where V_C is circular velocity at the instantaneous altitude and V_E is the velocity excess, yields

$$\ddot{H} = \frac{V_C^2 + 2\, V_C\, V_E + V_E^2}{R + H} - g \ . \tag{2-8}$$

Ignoring V_E^2, cancelling g with its equal $V_C^2/(R + H)$, and defining the orbital rate ω as $V_C/(R + H)$ yields

$$\ddot{H} = 2\,\omega\, V_E \ . \tag{2-9}$$

To find V_E we must obtain an expression for $\ddot{H}$ in terms of altitude measurements. $\ddot{H}$ is approximated by

$$\ddot{H}_t = \frac{\dot{H}_{t + \frac{\Delta t}{2}} - \dot{H}_{t - \frac{\Delta t}{2}}}{\Delta t} \ . \tag{2-10}$$

Using Equation (2-5) yields

$$\ddot{H} = \frac{H_{t + \Delta t} - 2\, H_t + H_{t - \Delta t}}{(\Delta t)^2} \ . \tag{2-11}$$

With Equations (2-9) and (2-11) V_E is given in terms of altitude measurements by

$$V_E = \frac{H_{t+\Delta t} - 2\,H_t + H_{t-\Delta t}}{2\,\omega\,(\Delta t)^2} \,. \tag{2-12}$$

We have shown how the existing perturbations from a circular orbit may be calculated using only altitude measurements. Because both components of velocity are linearly related to the altitude measurements, the velocity components are easy to calculate with a gear driven analog calculator or with a nomogram.

2.2 Error Analysis

The following error analysis is based upon the use of a sextant for altitude determination. Altitude is determined by measuring the angle subtended by the lunar disk. Errors in altitude determination result both from angular errors in the sextant and from altitude errors in spotting the horizon. At the highest altitude of interest, (80 nautical miles) the angle subtended by the moon is 134°, (Fig. 2-2), and therefore it is safe to make the approximation that an altitude error in spotting the horizon results in an equal altitude error in locating the spacecraft. But the spacecraft altitude error due to a sextant angular error depends upon the distance to the horizon. In both cases, a fixed bias error is immaterial. Only the variations between one reading and the next are significant. Therefore refraction due to looking through a window may be neglected assuming it affects successive readings equally, and errors

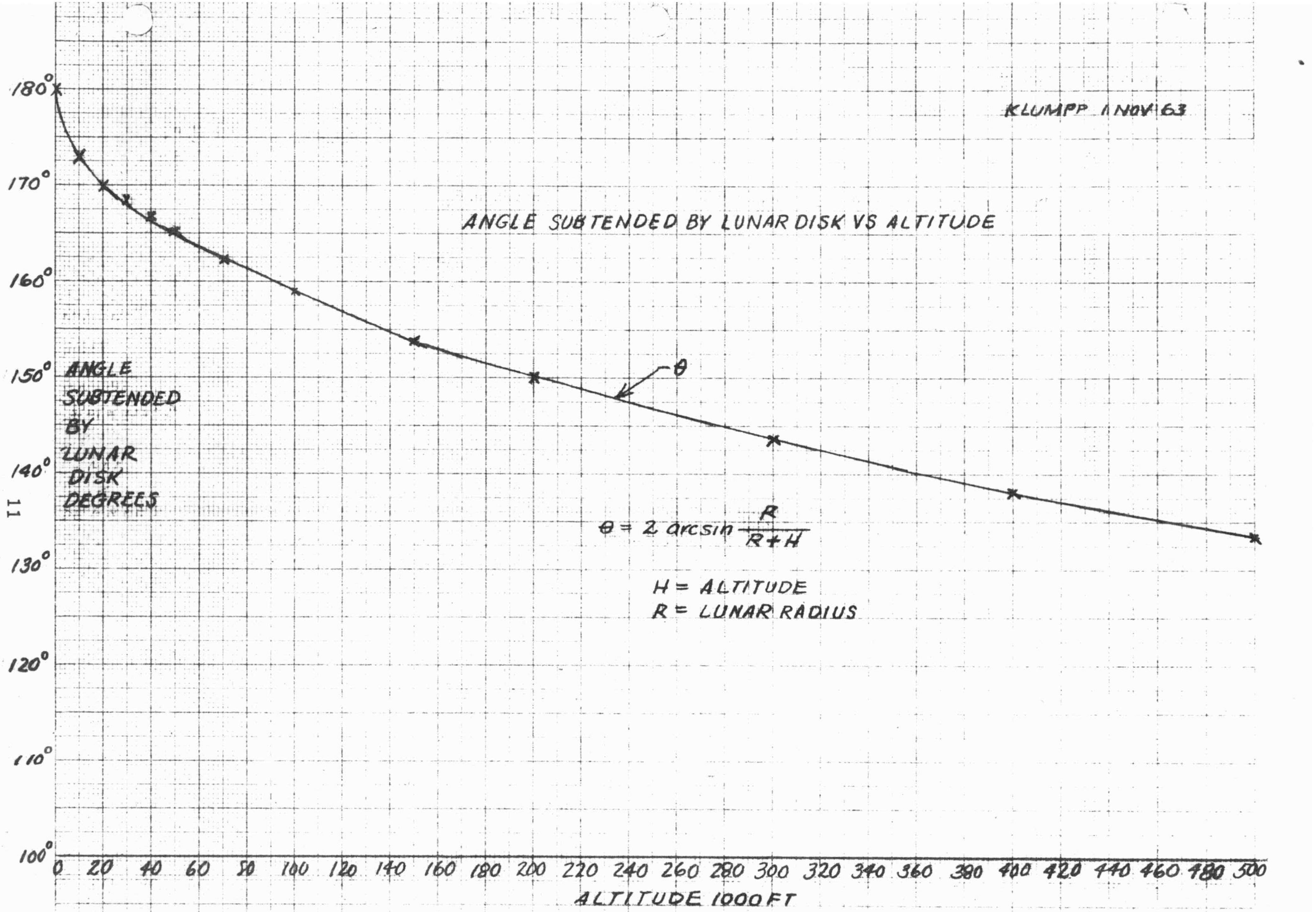

KLUMPP 1 NOV 63
ANGLE SUBTENDED BY LUNAR DISK VS ALTITUDE
ANGLE SUBTENDED BY LUNAR DISK DEGREES
180°
170°
160°
150°
140°
130°
120°
110°
100°
θ
$\theta = 2 \arcsin \frac{R}{R+H}$
H = ALTITUDE
R = LUNAR RADIUS
0 20 40 60 80 100 120 140 160 180 200 220 240 260 280 300 320 340 360 380 400 420 440 460 480 500
ALTITUDE 1000 FT

FIGURE (2-2)

in the knowledge of the diameter of the moon are of no significance because they affect successive horizon observations equally. Considering the random variations we assume 0. 05° sextant error and 1000 ft uncertainty in the elevation of the horizon. We also assume a five minute time interval between successive sextant fixes, and normalize the perturbation equations to a mean radius from the center of the moon of 990 nautical miles, (52 nautical miles altitude).

Assuming sextant angular errors and horizon altitude errors are uncorrelated, the error in spacecraft altitude is

$$\epsilon_H = \sqrt{\left(\epsilon_\theta \frac{dH}{d\theta}\right)^2 + \epsilon_{HOR}^2} \tag{2-13}$$

where ϵ_θ is the error in the sextant angle θ and ϵ_{HOR} is the error in the altitude of the horizon. To find $dH/d\theta$ we write the equation of the spacecraft-moon geometry

$$\frac{R}{R + H} = \sin \frac{\theta}{2} \tag{2-14}$$

where R is the lunar radius and H is the altitude. Taking differentials of both sides of Equation (2-14) and solving for $dH/d\theta$ yields

$$\frac{dH}{d\theta} = - \frac{(R + H)^2 \cos \frac{\theta}{2}}{2R} , \tag{2-15}$$

Solving (2-14) for $\cos \theta/2$, substituting this into Equation (2-15), and simplifying yields

$$\frac{dH}{d\theta} = - \frac{(R + H) \sqrt{2RH + H^2}}{2 R} , \tag{2-16}$$

Assuming H < < R yields

$$\frac{dH}{d\theta} = -\sqrt{\frac{RH}{2}} \quad , \tag{2-17}$$

Substituting (2-17) into (2-13) yields finally

$$\epsilon_H = \sqrt{\epsilon_\theta^2 \frac{RH}{2} + \epsilon_{HOR}^2} \quad , \tag{2-18}$$

Figure 2-3 is a plot of Equation (2-18).

The error in altitude rate may be found by taking the root-sum-square of the errors due to the individual altitude fixes assuming these are not correlated. Thus the altitude rate is given by Equation (2-5) repeated below,

$$\dot{H}_t = \frac{H_{t + \Delta t} - H_{t - \Delta t}}{2\, \Delta t} \quad , \tag{2-5}$$

and the error in altitude rate is

$$\epsilon_{\dot{H}} = \frac{\sqrt{2}\, \epsilon_H}{2\, \Delta t} \quad , \tag{2-19}$$

Figure 2-4 contains a plot of Equation (2-19).

The velocity excess, given by Equation (2-12),

$$V_E = \frac{H_{t + \Delta t} - 2\, H_t + H_{t - \Delta t}}{2\, \omega\, (\Delta t)^2} \quad , \tag{2-12}$$

will be in error not only due to altitude errors. There will also be an error proportional to the product of the velocity excess and the altitude

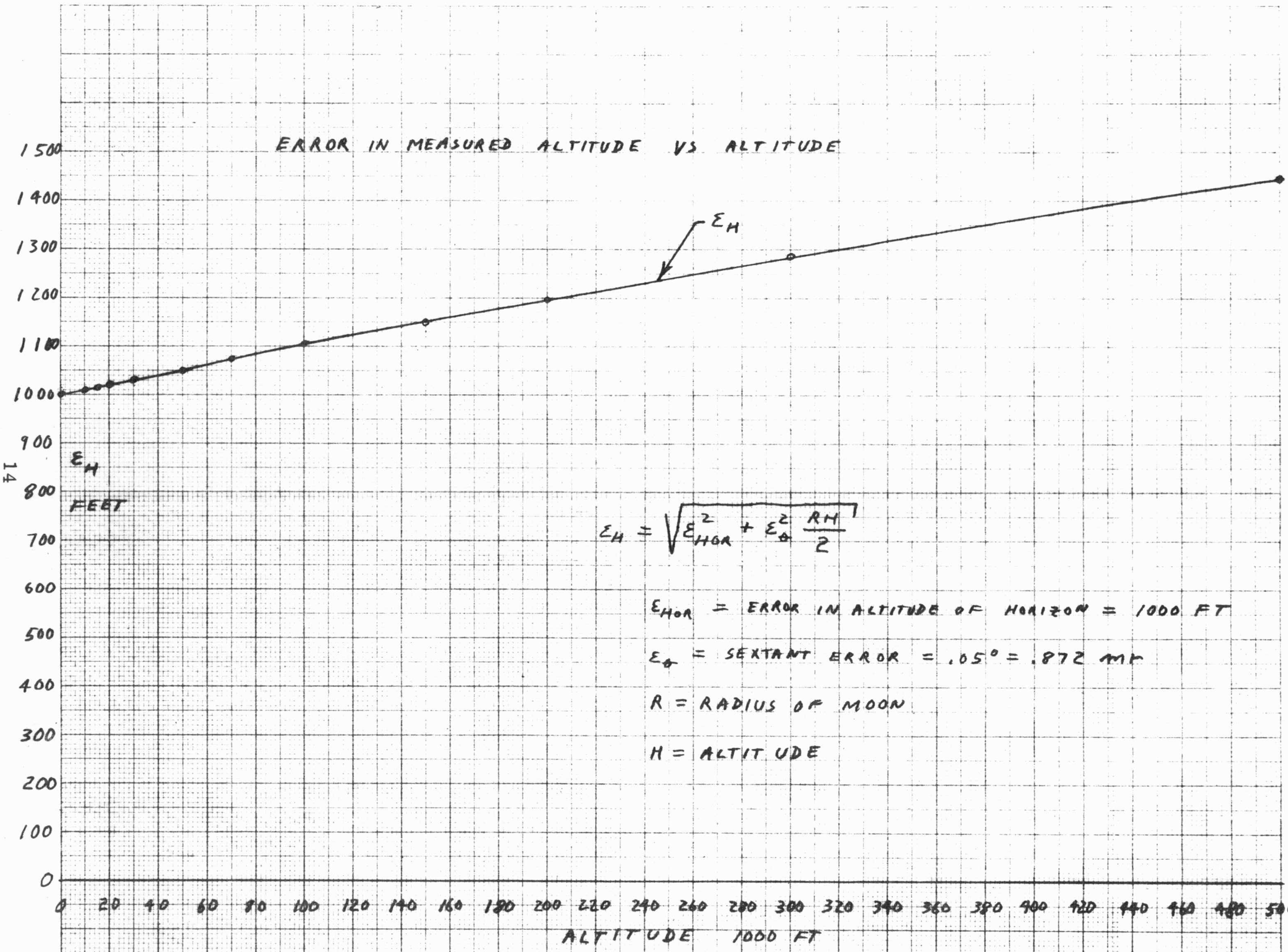

ERROR IN MEASURED ALTITUDE VS ALTITUDE
ε_H
ε_H FEET
1500
1400
1300
1200
1100
1000
900
800
700
600
500
400
300
200
100
0
0 20 40 60 80 100 120 140 160 180 200 220 240 260 280 300 320 340 360 380 400 420 440 460 480 500
ALTITUDE 1000 FT
$\varepsilon_H = \sqrt{\varepsilon_{HOR}^2 + \varepsilon_\theta^2 \frac{RH}{2}}$
ε_{HOR} = ERROR IN ALTITUDE OF HORIZON = 1000 FT
ε_θ = SEXTANT ERROR = .05° = .872 mr
R = RADIUS OF MOON
H = ALTITUDE

FIGURE (2-3)

of the spacecraft with respect to the normalizing altitude, 52 nautical miles. The error in V_E due to altitude errors is

$$\epsilon_{VE} = \frac{\sqrt{6}\,\epsilon_H}{2\omega\,(\Delta t)^2} \, . \tag{2-20}$$

Figure 2-4 contains a plot of Equation (2-20). V_E as calculated by Equation (2-12) may be corrected for altitude by multiplying by

$$\left(\frac{R + H}{\text{Normalizing Radius}}\right)^{3/2} .$$

Figure 2-5 is a plot of this correction factor.

The errors in periapsis altitude and in separation at the nominal time of intercept can be found by multiplying the velocity errors by the appropriate sensitivities. However, several runs have shown that at the point of closest approach on an intended intercept, the separation between the LEM and the Command Module is much less than one would calculate by multiplying the velocity errors by the sensitivity coefficients. The reason is that the sensitivity coefficients do truly give the separation at the nominal intercept time, but the closest approach occurs earlier or later. The sensitivity coefficients are:

0.185 nautical miles altitude per foot per second altitude rate.

0.74 nautical miles altitude per foot per second velocity excess.

0.74 nautical miles phase in one half orbit per foot per second altitude rate

3.5 nautical miles phase in one orbit per foot per second velocity excess.

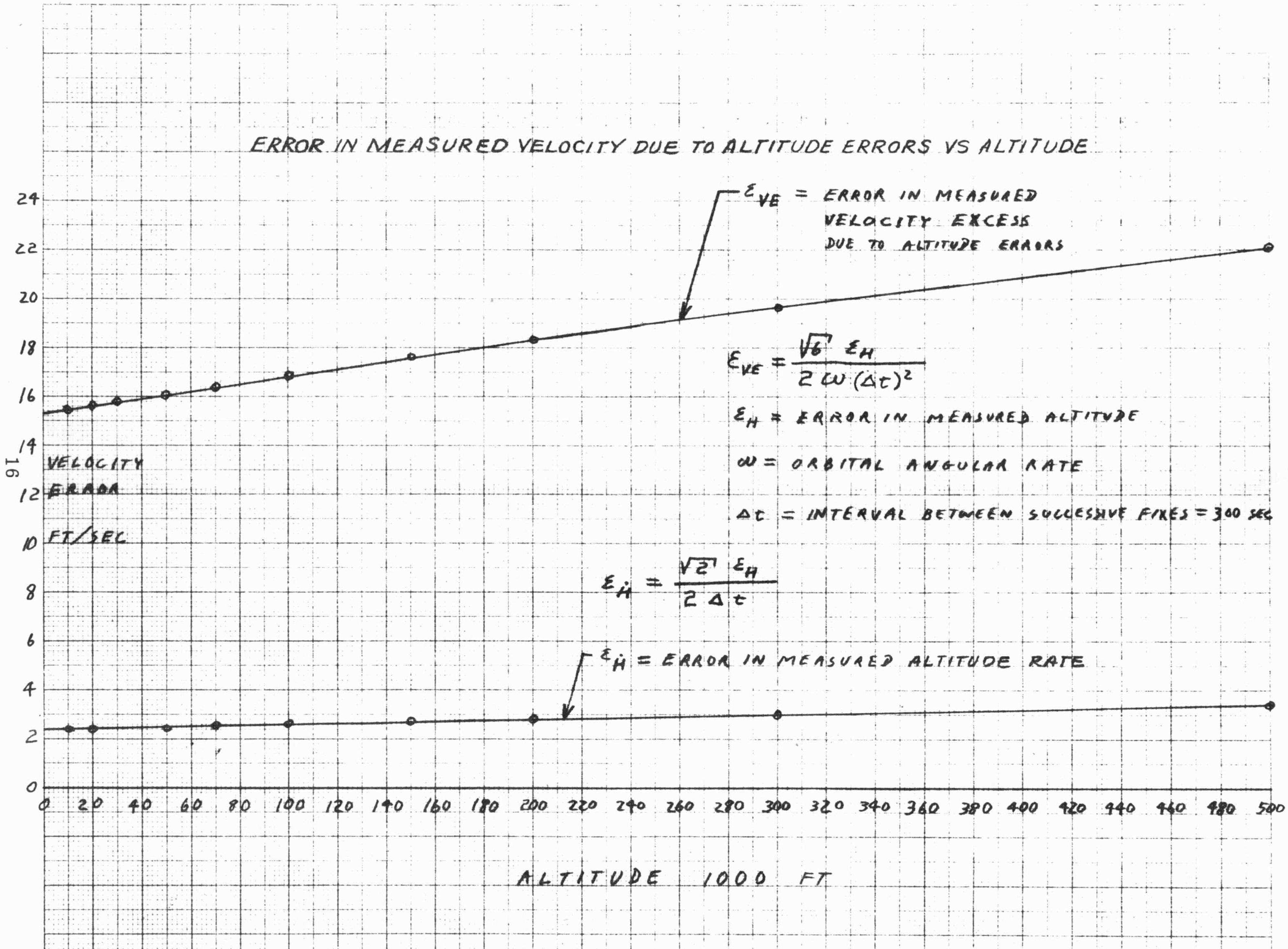

ERROR IN MEASURED VELOCITY DUE TO ALTITUDE ERRORS VS ALTITUDE
ε_{VE} = ERROR IN MEASURED VELOCITY EXCESS DUE TO ALTITUDE ERRORS
$\varepsilon_{VE} = \frac{\sqrt{6}\ \varepsilon_H}{2\,\omega\,(\Delta t)^2}$
ε_H = ERROR IN MEASURED ALTITUDE
ω = ORBITAL ANGULAR RATE
Δt = INTERVAL BETWEEN SUCCESSIVE FIXES = 300 SEC
$\varepsilon_{\dot{H}} = \frac{\sqrt{2}\ \varepsilon_H}{2\,\Delta t}$
$\varepsilon_{\dot{H}}$ = ERROR IN MEASURED ALTITUDE RATE
VELOCITY ERROR FT/SEC
0
2
4
6
8
10
12
14
16
18
20
22
24
0 20 40 60 80 100 120 140 160 180 200 220 240 260 280 300 320 340 360 380 400 420 440 460 480 500
ALTITUDE 1000 FT

FIGURE (2-4)

VELOCITY EXCESS CORRECTION FACTOR VERSUS ALTITUDE

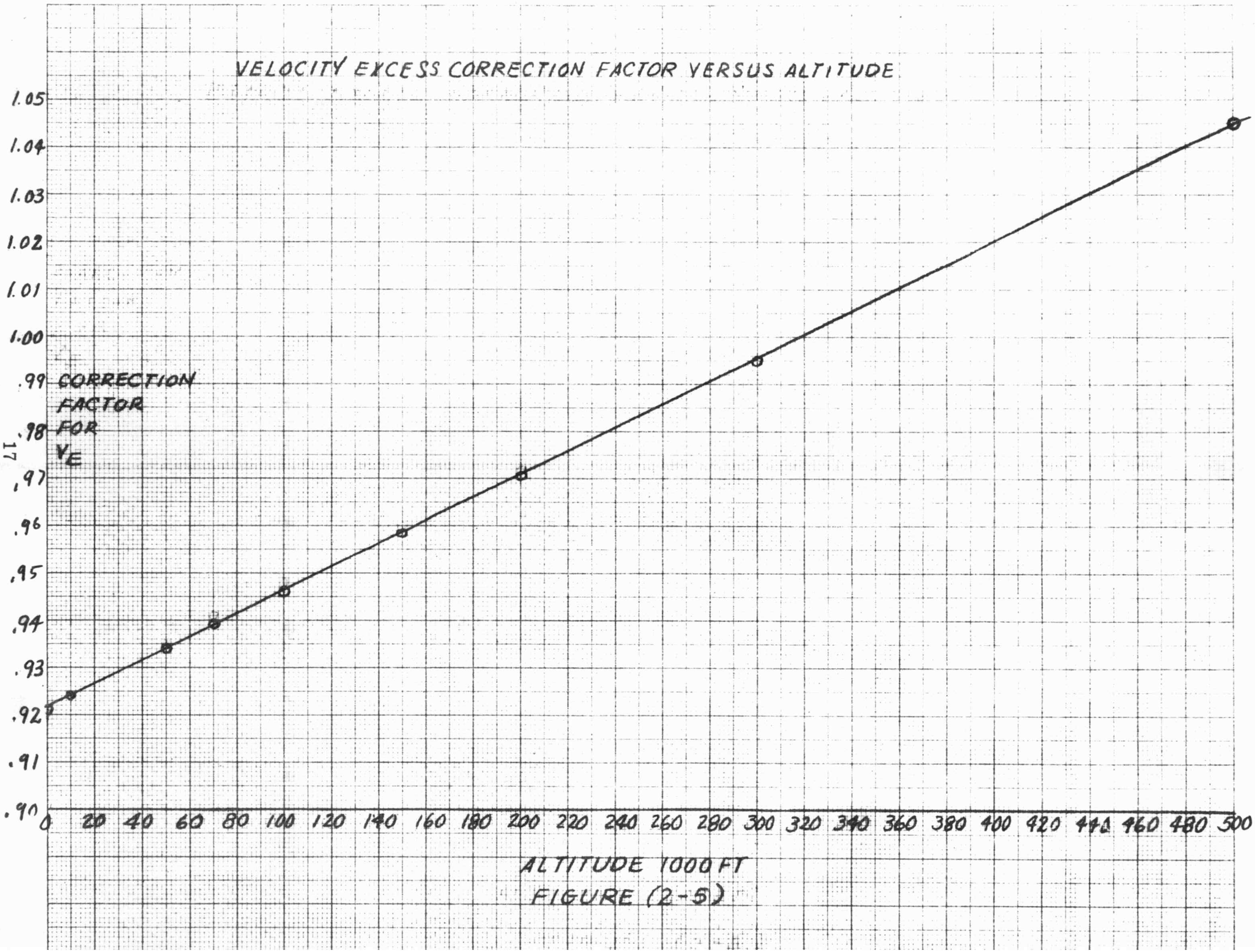


FIGURE (2-5)

The error analysis shows that the velocity components of the initial orbit can be determined in two measurement periods, ten minutes total time, to an accuracy surprisingly close to the injection accuracy of the primary LEM ascent guidance. Figure 2-4 shows that the altitude rate can be determined in the first ten minutes (assuming 50, 000 ft altitude) to an accuracy of 2.5 ft/second, and the velocity excess to an accuracy of 16 ft/second. But, using the sensitivities listed above, the error in the predicted periapsis altitude due to the uncertainty in the velocity excess is just over 70, 000 ft, 20, 000 ft greater than the initial altitude. The error in the predicted periapsis altitude due to the error in the altitude rate is negligible, less than 3, 000 ft.

Because of the extreme sensitivity of the periapsis altitude to the velocity excess, it appears impractical to use this guidance procedure to rapidly obtain a circular orbit at a low altitude, i.e., under 100, 000 ft. One procedure which makes sense is to acquire a first estimate of altitude rate and velocity excess immediately after initial injection and make a first correction which nulls the altitude rate and produces a velocity excess several times as large as the uncertainty of 16 ft/second. As an example of this procedure, nulling the altitude rate and providing a velocity excess of 97 ft/sec at 50, 000 ft altitude will produce an apoapsis altitude of 80 nautical miles, the altitude of the orbiting command module. In section 5 it will be shown that this procedure can very easily be carried through, and that the resulting orbit can be corrected again at the equi-altitude point to yield an intercept orbit, (ignoring plane errors).

Two approaches might be taken to reduce the errors in determination of the altitude rate and velocity excess. First, the sextant accuracy could be made more on the order of the present day state of the art, which is far better than the 0.05° assumed here. But almost all of the error in orbit determination at low altitudes is due to variations in the altitude of the lunar terrain. Little gain is possible through increasing the precision of the sextant. Second, the time interval between successive fixes could be increased. The error in the measured altitude rate varies inversely with the time interval, and the error in the measured velocity excess varies inversely with the square of the time interval. The time interval of five minutes was chosen because it is convenient and because it meets the objective of providing rapid orbit determination with adequate accuracy. In section 5 it will be shown that it is possible to take advantage of extra time when available without resorting to statistical procedures, even though the hand calculator is geared for a time interval of five minutes.

3. Thrust Vector Directional Control

This study is not primarily concerned with thrust vector directional control, but token consideration of this vital element seems appropriate.

Considering the requirements for roll, pitch, and yaw control, roll control is relatively unimportant because the thrust vector lies fairly close to the roll axis. Pitch controls the direction of the thrust vector in the orbital plane. Yaw controls the angle between the thrust vector and the orbital plane, and is synonymous with azimuth control when the thrust vector is horizontal.

Fundamentally, it is only necessary to orient a line in space, the thrust vector, and rotations about that line are immaterial. But there does not appear to be any simple way of capitalizing on the fact that only two degrees of angular freedom need be controlled; a point on the window could be marked to identify the thrust line, but it would be difficult to mark the moving point in the heavens upon which the point on the window must lie. Two procedures for controlling all three degrees of angular freedom do appear feasible.

First, a mark is made on a side window (assuming a small window could be provided), to correspond with the spacecraft pitch axis. For any given mission, the point in the sky which lies along the normal to the orbital plane is identified by stellar geometry. The point on the side window is aligned with the celestial point. This controls the yaw angle such that the thrust vector lies in the orbital plane, and incidentally also controls the roll angle. A scale graduated in degrees is painted on the forward window, and the graduation corresponding to the required pitch angle is aligned with the horizon.

The second possibility is to identify the orbital plane by a line on a lunar map stored aboard the LEM. The pilot visualizes this line on the moon and aligns a line on the forward window with this imagined line. This produces crude control of yaw and roll. Pitch is controlled as described above.

4. Orbital Guidance Calculator

The fundamental tool of this guidance procedure is the orbital guidance calculator developed from an idea suggested by Russell Larson of A. C. Spark Plug Division of General Motors Corp. The calculator is a gear driven mechanical analog of the orbit perturbation equations. With periodic altitude data and a measurement of the range to the Command Module, the calculator can be used to guide the LEM from the injection point of an abort orbit to intercept of the Command Module.

The calculator is illustrated in Fig. 4-1. It has three display faces; left and right faces on the front, and one face on the rear. Each face consists of a fixed reticle and two or more moving films. The left face determines the present orbit and may be used to predict (extrapolate) the altitude and velocity profiles of the uncorrected orbit. The right face displays the required orbit from inputs by the astronaut. The rear face displays the velocity correction which will yield the required orbit.

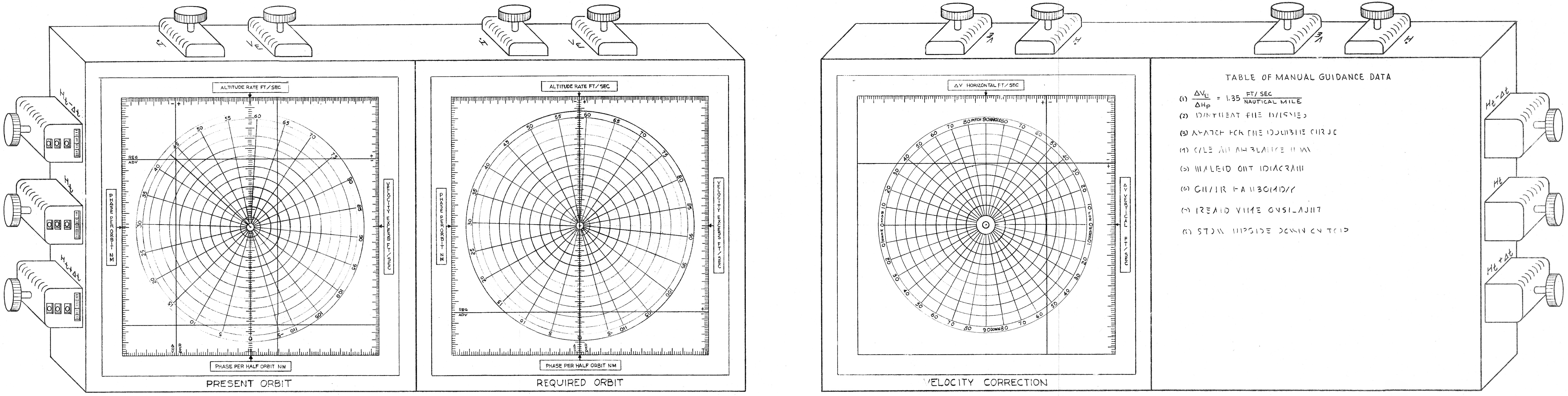
The input controls for the left face are the three counters through which are entered the three altitude measurements, and two knobs by which the present orbit is extrapolated. The input controls for the right face are two knobs by which the required orbit is entered. The films of the rear face are driven differentially by the inputs to the left and right faces. The rear face has no other inputs.

With appropriate inputs from the astronaut, the device calculates the following:

a. Altitude rate and velocity excess at the present point.

b. Periapsis and apoapsis altitudes of the present orbit.

c. Time to any point in the present orbit, e.g. time to periapsis, to apoapsis, to a given altitude, or to a given velocity state.



FRONT VIEW

REAR VIEW

FIGURE (4-1)
OBLIQUE VIEW OF THE ORBITAL GUIDANCE CALCULATOR

d. Phase advance or regress per orbit when the initial altitude rate is zero, and phase advance or regress per half orbit when the initial velocity excess is zero.

e. Items a, b, and d for the required orbit.

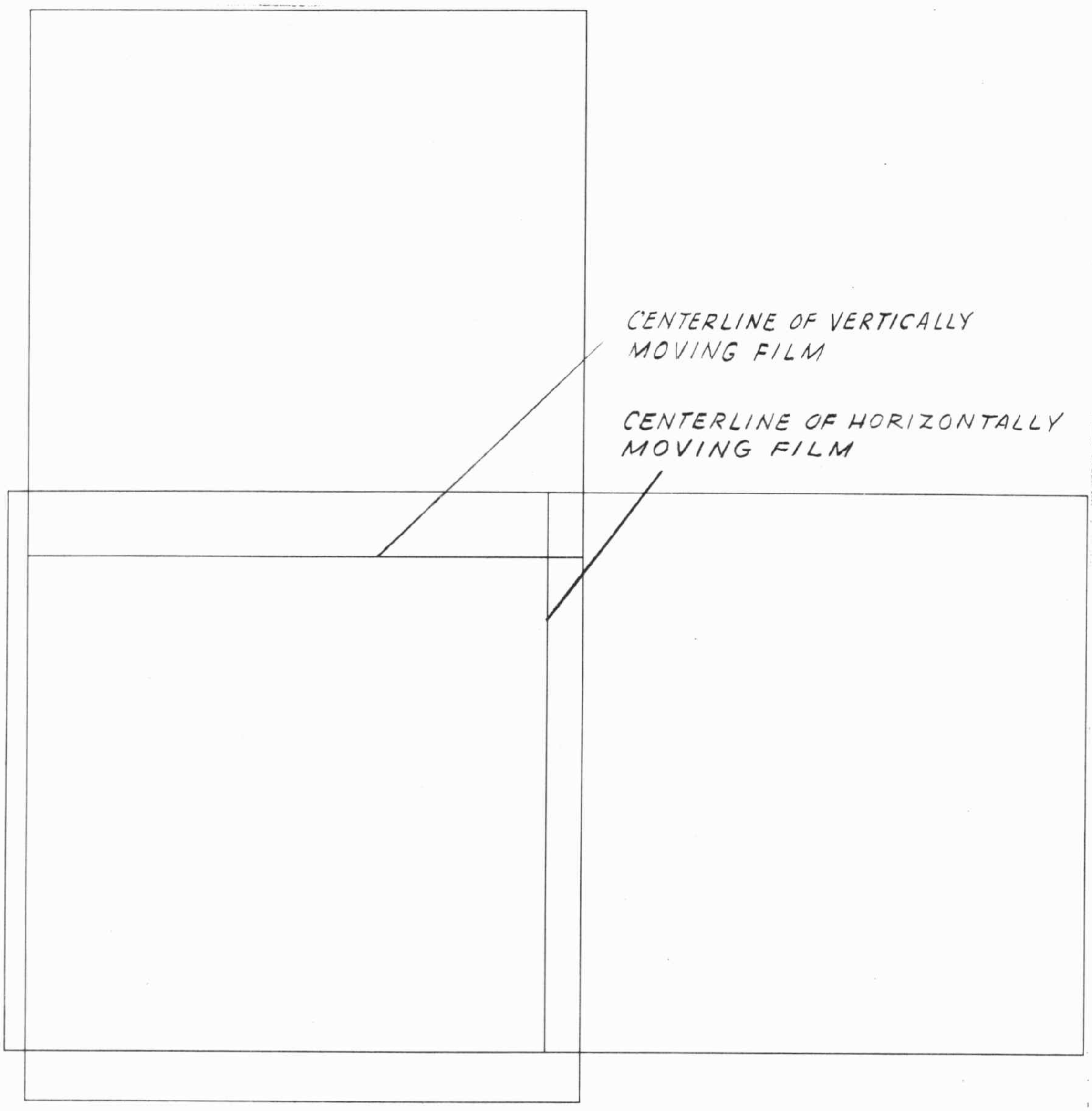
f. The correction which will produce the required orbit given the extrapolated present orbit. The correction is read in terms of the pitch angle and velocity magnitude.

The reticle of the left face consists of circles scaled in nautical miles. Behind the reticle are two films which indicate the present orbit and are driven by the three altitude measurement counters. These are called the orbit determination films and are shown in Fig. 4-2. Each of these two films has a centerline, and the intersection point of the two centerlines represents the tip of the vector illustrated by the vector diagram of Fig. 2-1. The vector represents Equation (2-1) which reads

$$H = \frac{\dot{H}_0}{\omega} \sin \omega t + \frac{2\, V_{E0}}{\omega} (1 - \cos \omega t) . \qquad (2\text{-}1)$$

Thus the centerline of the horizontally moving film is driven off the center of the reticle a distance proportional to $-\dot{H}/\omega$, and the centerline of the vertically moving film a distance proportional to $-2\, V_E/\omega$. The orbit determination films are gear driven from the altitude measurement counters according to Equations (2-5) and (2-12) which read

$$\dot{H}_t = \frac{H_{t + \Delta t} - H_{t - \Delta t}}{2\, (\Delta t)} \, , \qquad (2\text{-}5)$$



ORBIT DETERMINATION FILMS

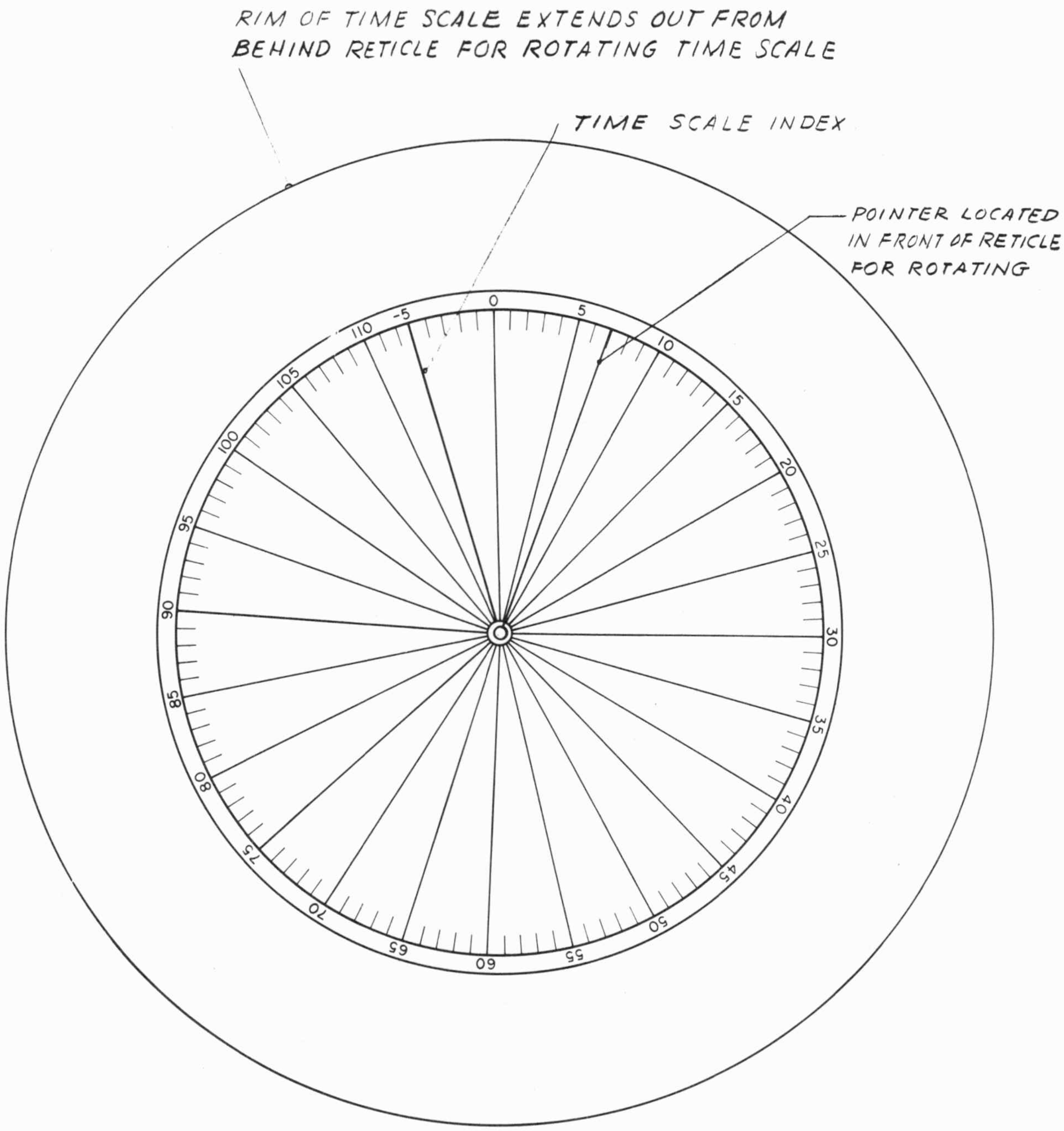
FIG (4-2)

$$V_E = \frac{H_{t + \Delta t} - 2 H_t + H_{t - \Delta t}}{2 \omega (\Delta t)^2} \qquad (2\text{-}12)$$

If the calculator were continuously updated, the point of intersection of the centerlines of the orbit determination films would move on a circular locus around the center of the reticle at the constant angular rate ω. Therefore it is possible to extrapolate along the circular path to predict any future state in the uncorrected orbit. For this purpose the left face also contains a time scale and a pointer (Fig. 4-3) and a second pair of films, called the extrapolation films, (Fig. 4-4), driven by the two knobs for the left face. The extrapolation films mark the extrapolated point, and are lined in a different color from the orbit determination films to avoid confusion. On the time scale, the index is displaced five minutes from the zero time line to correct for the five minutes delay inherent in orbit determination.

The right face, used for specifying the required orbit, is similar to the left face except that it has two instead of four translating films. The two translating films of the right face are called the required orbit films. These films are identical to the extrapolation films, and are driven by the two knobs for the right face.

The rear face consists of a reticle scaled in ft/second radially and pitch angle on the perimeter, (Fig. 4-1). It has two films which are driven to the horizontal and vertical components of the velocity to be gained. These are called the velocity correction films. Thus the magnitude of the velocity correction is read on the radial calibration of the reticle and the pitch angle on the angular calibrations.



TIME SCALE AND POINTER FOR FRONT FACES

FIG (4-3)

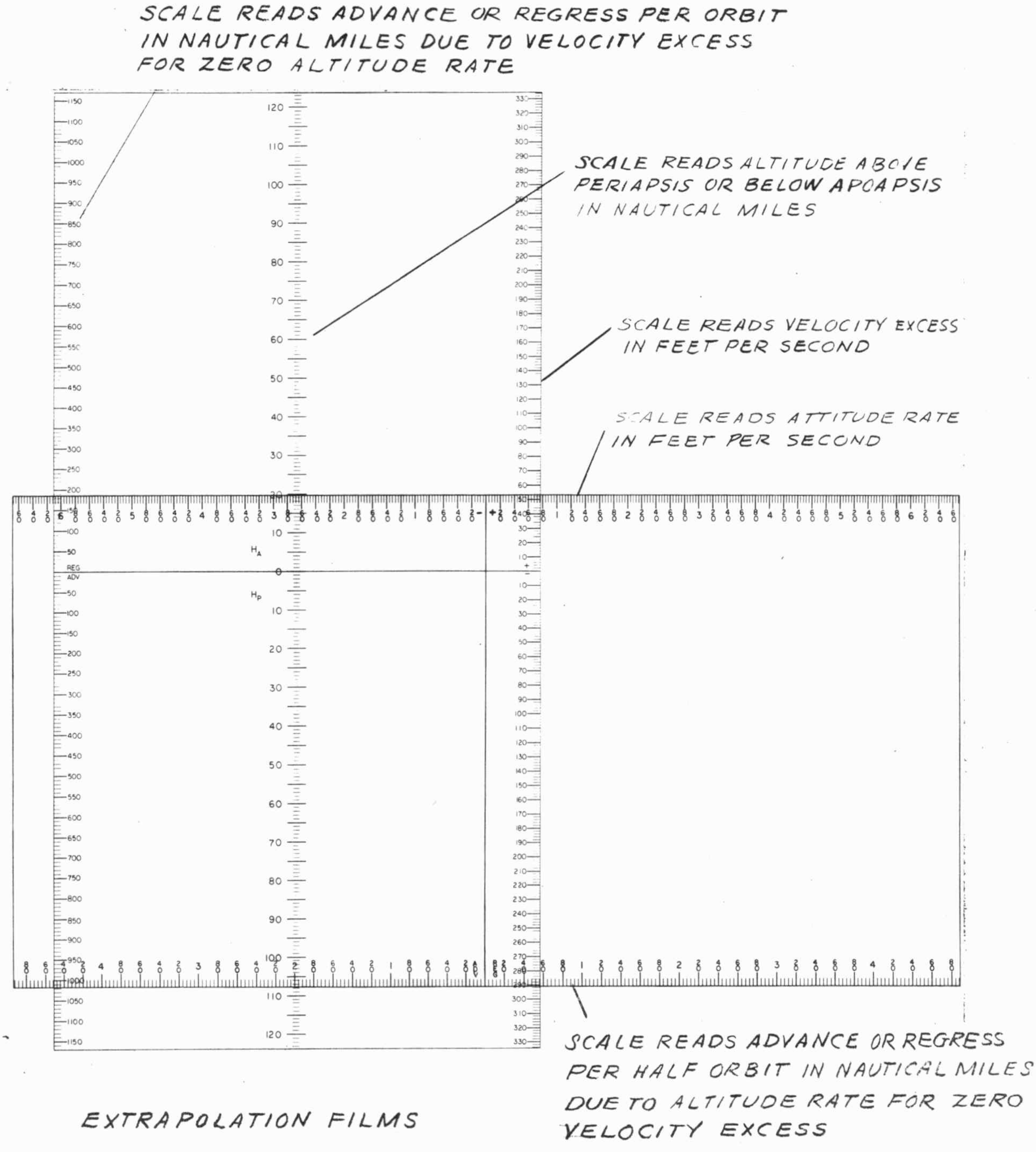

SCALE READS ADVANCE OR REGRESS PER ORBIT IN NAUTICAL MILES DUE TO VELOCITY EXCESS FOR ZERO ALTITUDE RATE
SCALE READS ALTITUDE ABOVE PERIAPSIS OR BELOW APOAPSIS IN NAUTICAL MILES
SCALE READS VELOCITY EXCESS IN FEET PER SECOND
SCALE READS ATTITUDE RATE IN FEET PER SECOND
SCALE READS ADVANCE OR REGRESS PER HALF ORBIT IN NAUTICAL MILES DUE TO ALTITUDE RATE FOR ZERO VELOCITY EXCESS
REG
ADV
H_A
H_p
EXTRAPOLATION FILMS

FIG (4-4)

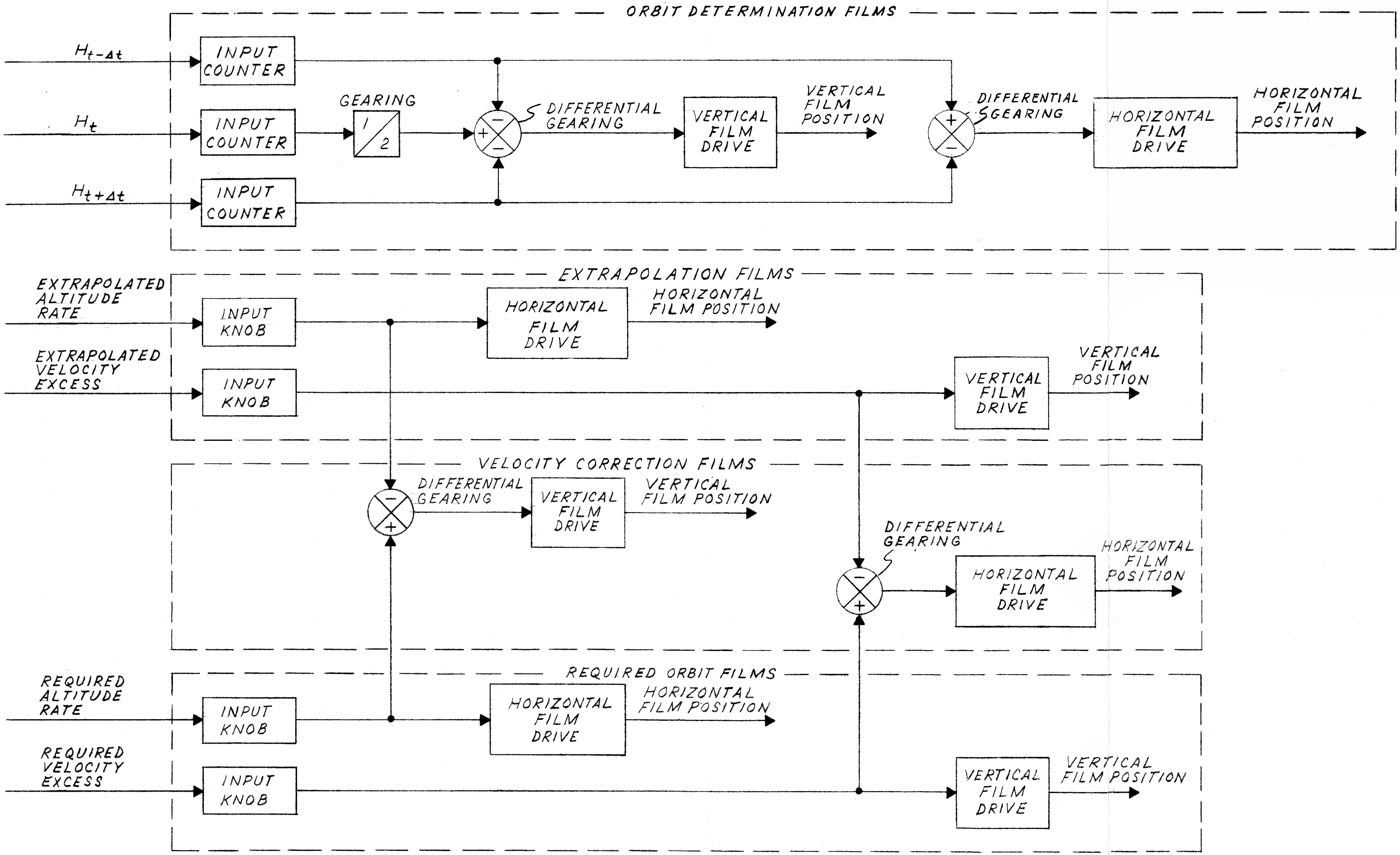


FIGURE (4-5) FILM DRIVE MECHANIZATION

The mechanization of the film drives is shown in Fig. 4-5. It should be noted that with this mechanization, the orbit correction to be made is indicated automatically by setting the required orbit films and the extrapolation films.

5. Application of Manual Guidance to an Abort

The manual guidance system will be demonstrated by application to a LEM abort. The Command Module is assumed to be in a circular orbit at 80 nautical miles altitude. The LEM is assumed to be initially at the injection point of an abort ascent, in a roughly circular orbit coplanar with the orbit of the Command Module, and 400 nautical miles in range behind the Command Module. The injection conditions are assumed to be 8 nautical miles altitude, 64 feet per second altitude rate, and -43 feet per second velocity excess. With these initial conditions, if no correction is made the LEM will impact in 32 minutes. In the example the initial orbit is determined and a correction is applied such as to yield a LEM orbit crossing the Command Module orbit. At the intersection point, where the range is still 180 nautical miles, a second correction is applied to yield a LEM orbit intercepting the Command Module in one orbital period. The minimum range without a terminal correction is 3.2 nautical miles. The procedure used by the LEM astronauts and the corresponding settings of the calculator faces are shown in Figs. 5-1 and 5-2. The resulting LEM orbit, from injection to intercept, is plotted in Command Module coordinates in Fig. 5-3 and tabulated in Table 5-1.

The example is as close to realistic as is possible at this state in the development of the guidance procedure. All calculations, manipulations of the calculator, and orbit corrections are based solely on the information which would be available to the astronaut. Thus, for the two orbit corrections, the velocity increment is calculated with the calculator using altitudes, the same as would be done in flight, and the true initial velocity is changed by this velocity increment to obtain the true corrected velocity. Therefore the true corrected velocity contains all errors inherent in computation with the calculator. However, to simplify the synthesis of this example, no errors are included in the altitude measurements. Ignoring altitude errors makes little difference in the outcome because the effects of altitude errors are almost completely eliminated by item 3 of the procedure of Fig. 5-2 when the altitude errors are of the order assumed in the error analysis of section 2.

The following criteria are used as a basis for this example:

a. 2-1/2 minutes time is required to orient the spacecraft for an altitude measurement, a range measurement, or a correction.

b. The first correction is such that the correction point is nominally the periapsis of the resulting orbit and the apoapsis is nominally at an altitude of 100 nautical miles. The 20 nautical mile difference between the Command Module orbital altitude and the nominal LEM apoapsis altitude assures that the LEM will cross the Command Module orbit even if the velocity excess of the corrected LEM trajectory is less than nominal by up to 27 feet per second.

c. The second correction is such as to intercept the Command Module in the smallest whole number of orbits consistent with a periapsis altitude of at least 25 nautical miles.

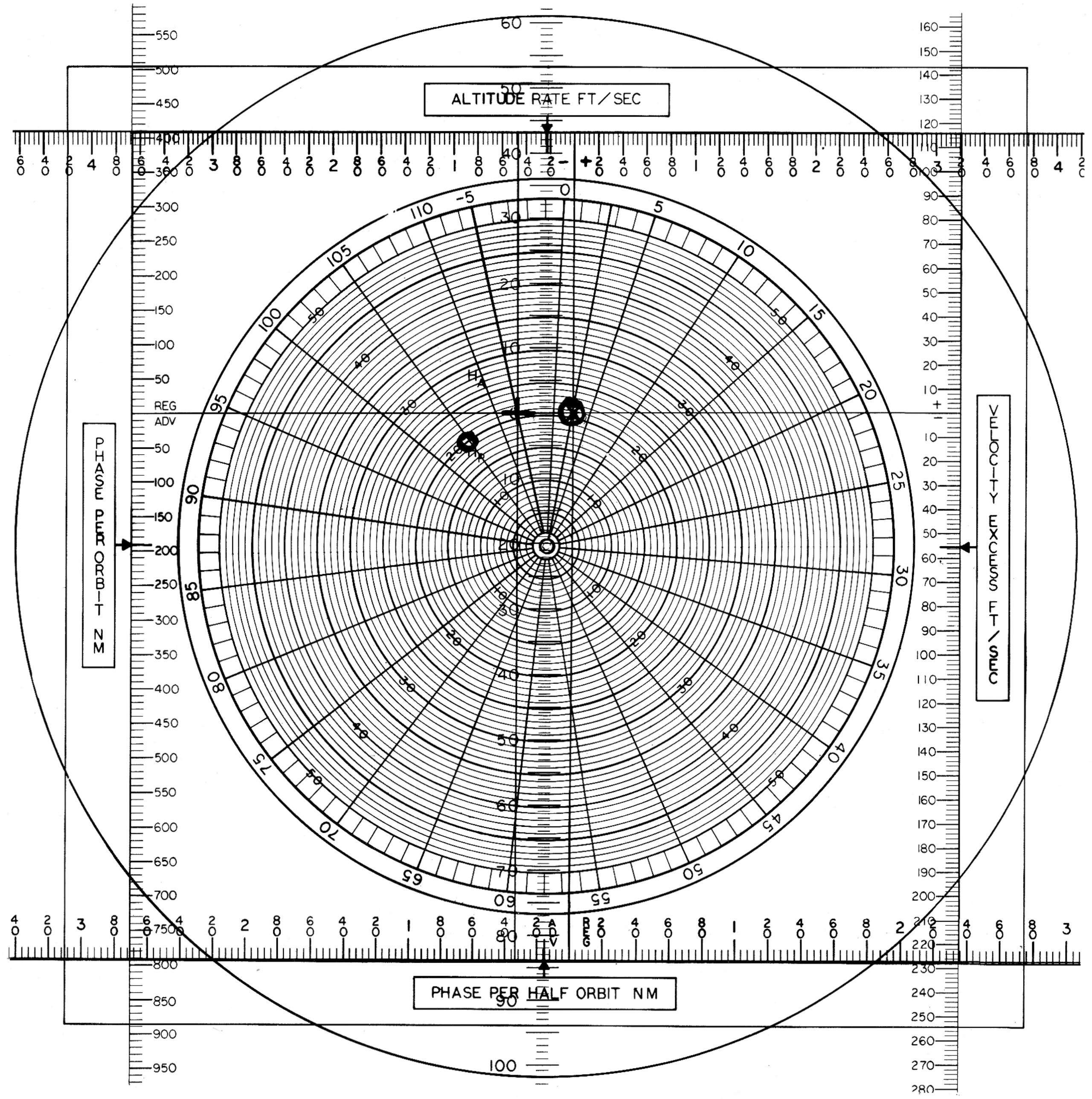
6. Recommendations for Further Work

The following are the next few steps which should be taken in development of this guidance procedure.

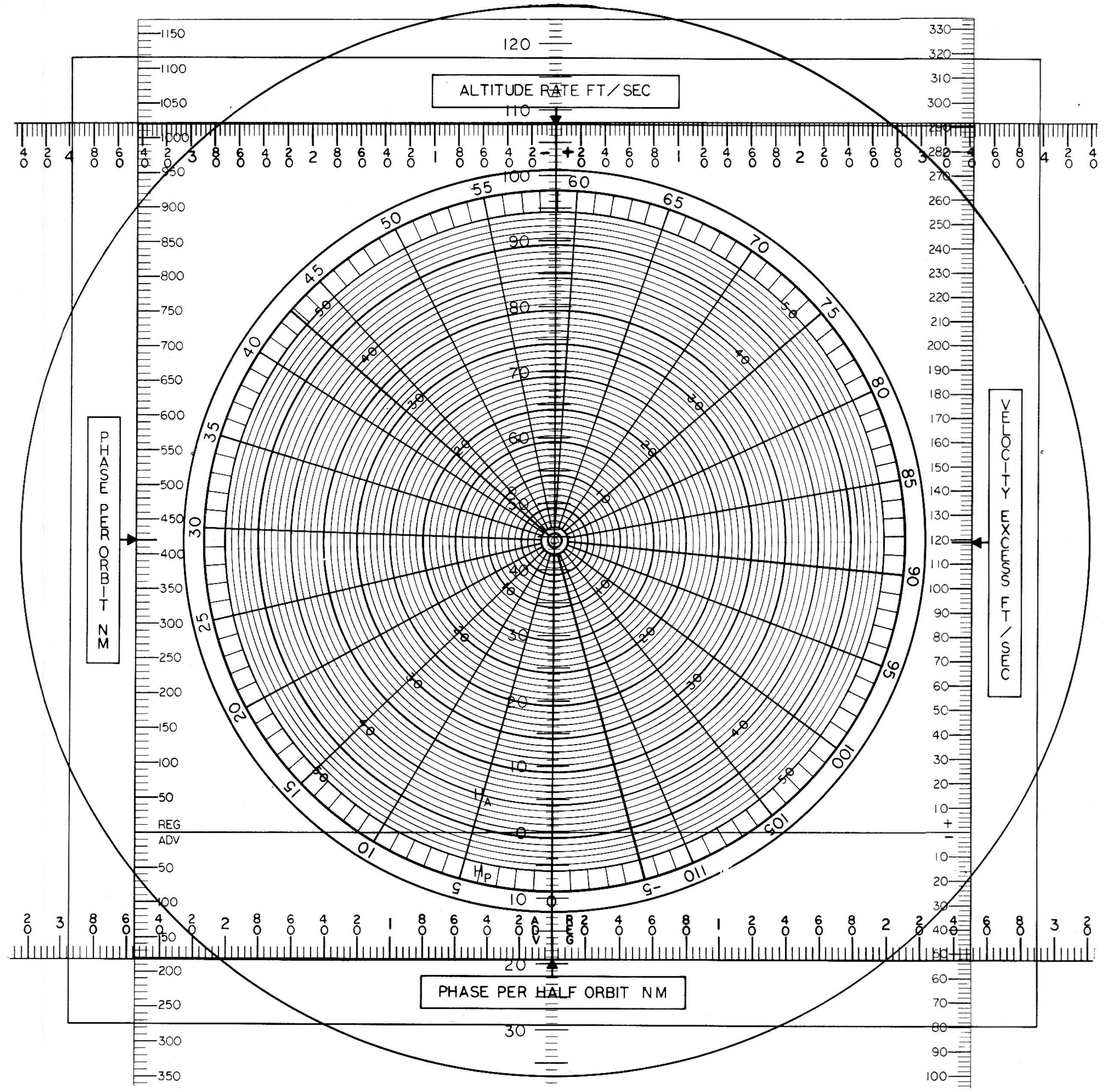
a. A Working calculator should be constructed. It is apparent that the scaling should be adjusted slightly to allow larger variations from circularity and larger velocity corrections.

b. Altimeters and range meters should be investigated to determine availability, physical characteristics, and performance capabilities.

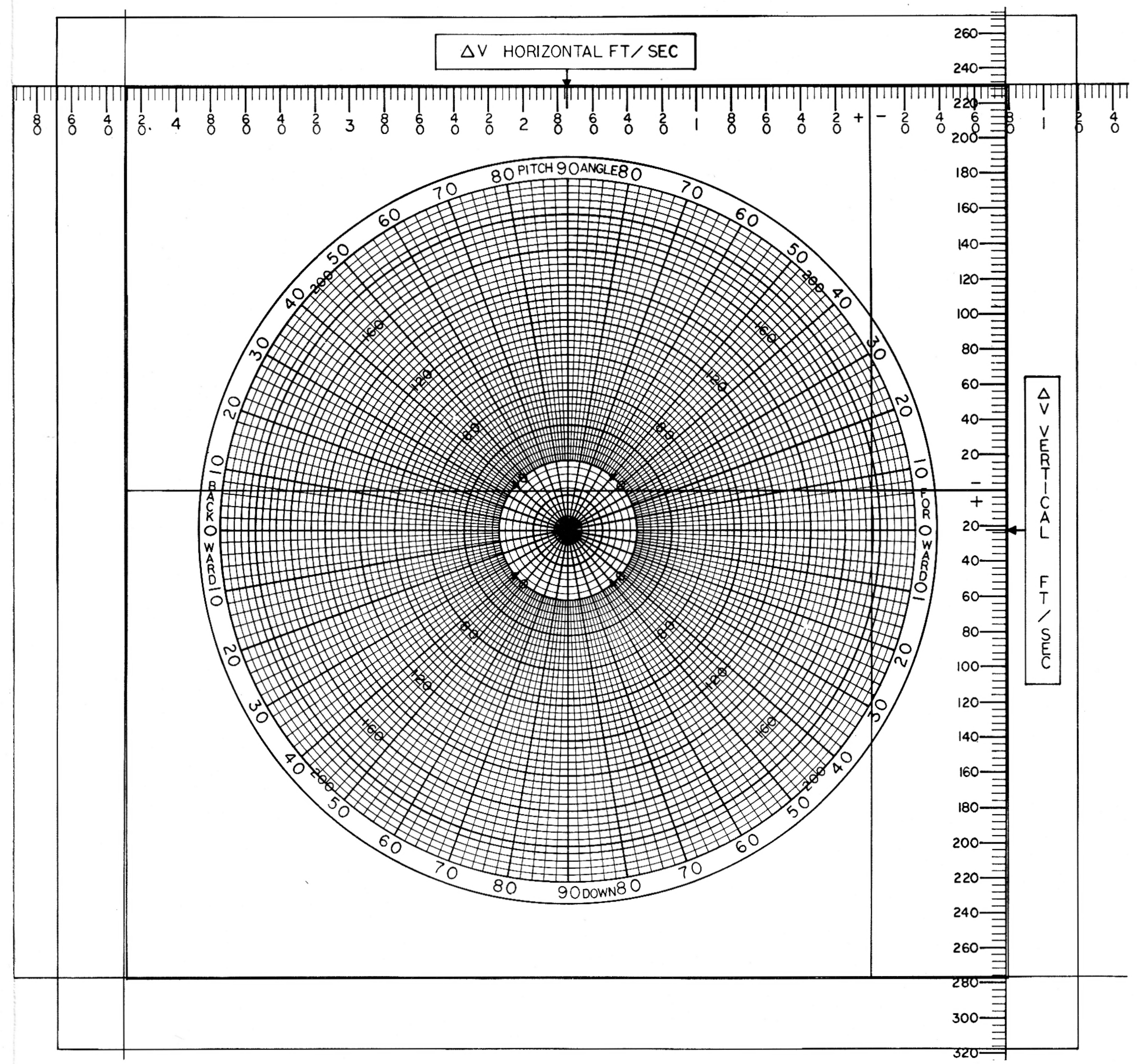
c. The operational procedure should be studied further and the actual performance limitations of the instruments should be included. Monte Carlo treatment of altitude errors appears necessary.



Left Face – Orbit Determination



Right Face – Required Orbit

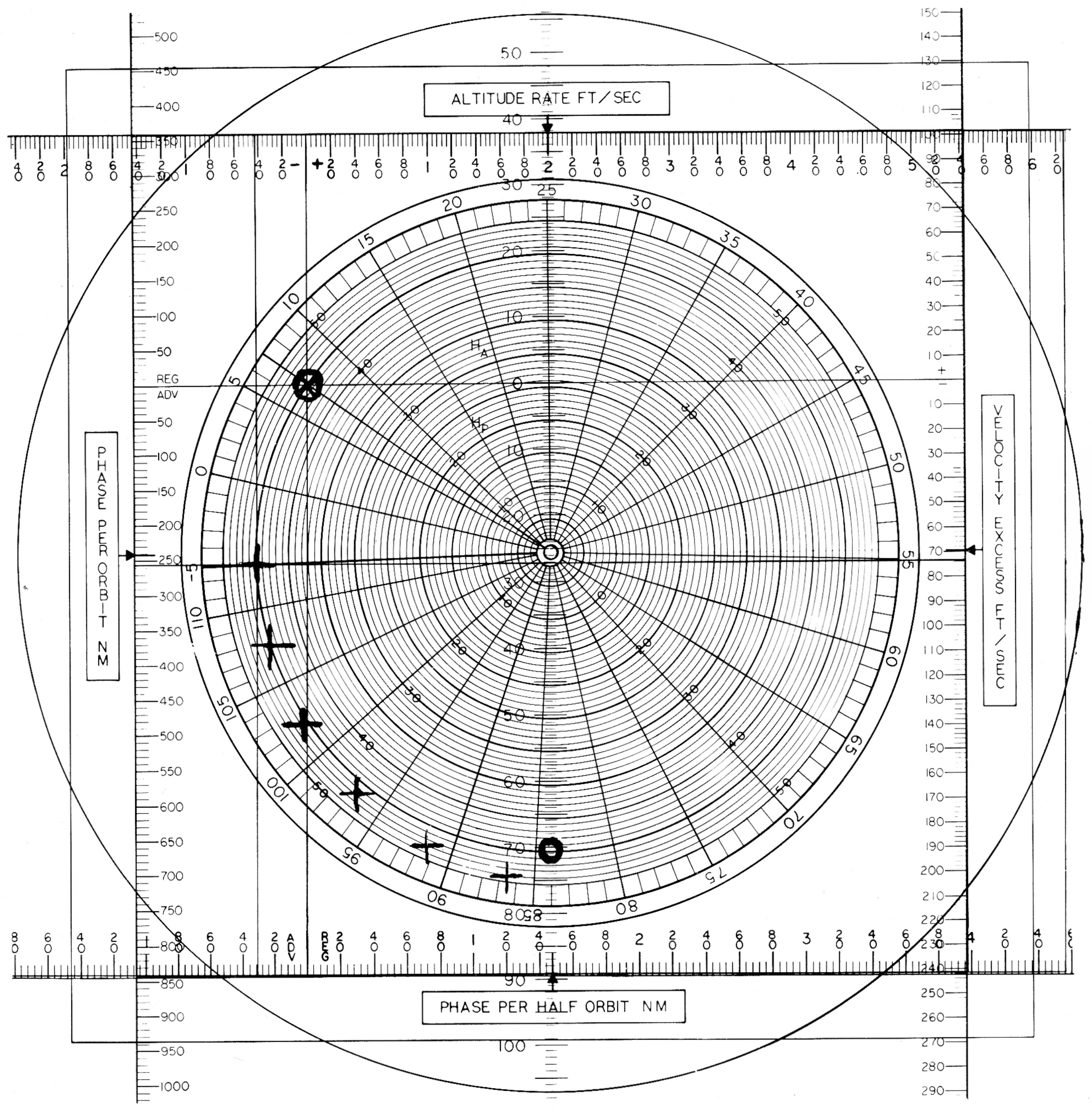


Rear Face – Velocity Correction

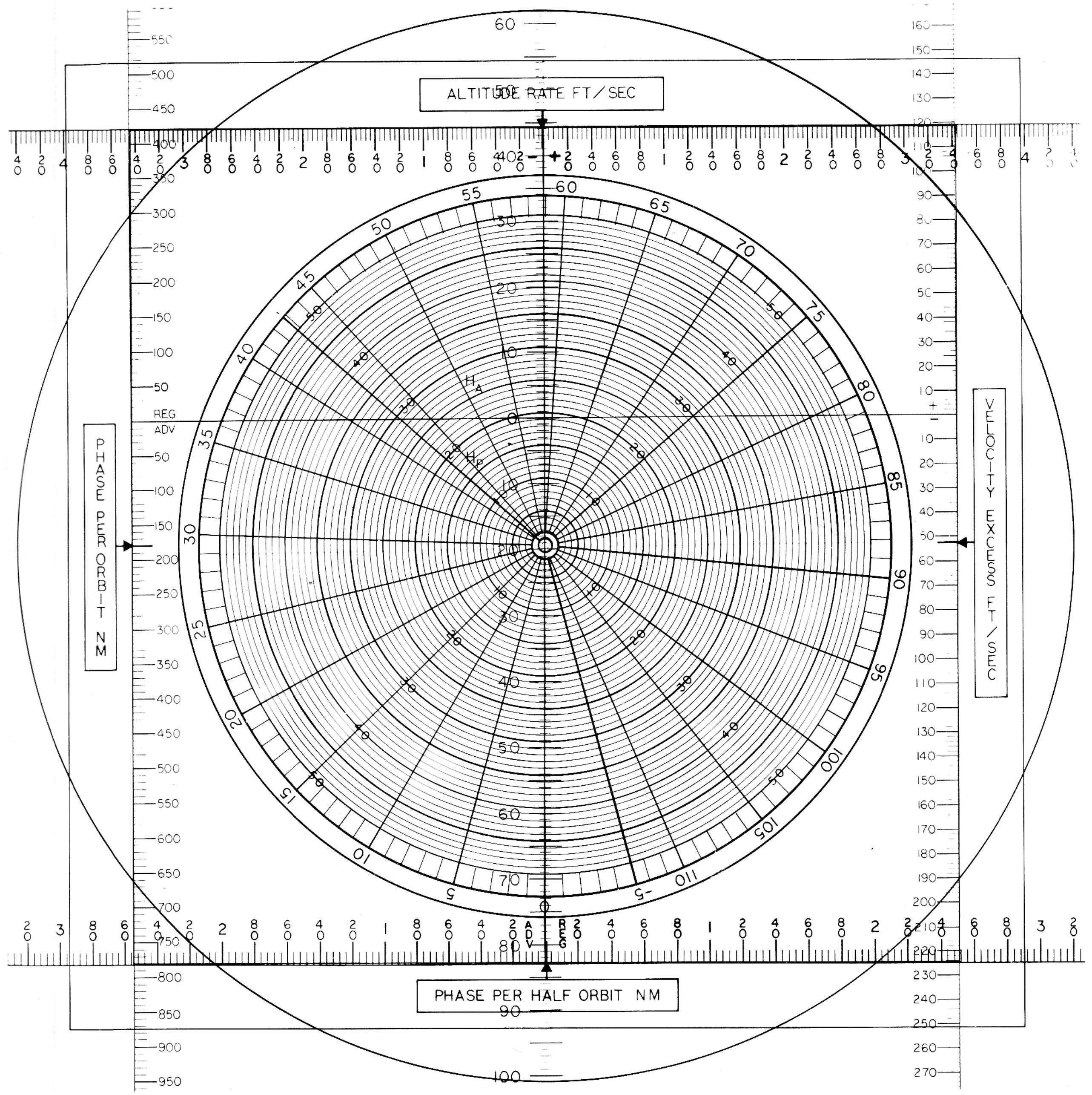
0. True initial state indicated by 0.
1. As soon after injection as possible make first altitude measurement (t = 2-1/2 min, H = 9.42 NM) and enter it into the $H_{t - \Delta t}$ counter.
2. Make and enter second altitude measurement (t = 7-1/2 min, H = 11.26 NM) and third altitude measurement (t = 12-1/2 min, H = 11.65 NM). Start stop watch. These three measurements determine point indicated by +.
3. Set up extrapolation films for making a correction at t = 15 mins, (2-1/2 minute extrapolation, point indicated by ⊗).
4. Set up required orbit films for zero altitude rate and 88.35 nautical miles below apoapsis (apoapsis altitude will be 88.35 + 11.65 = 100 nautical miles).
5. Read pitch angle (7° up and forward) and velocity magnitude (177 ft/sec).
6. Impart Δ V when stop watch reads 2-1/2 mins (t = 15 min).

The Calculator at the Time of the First Correction

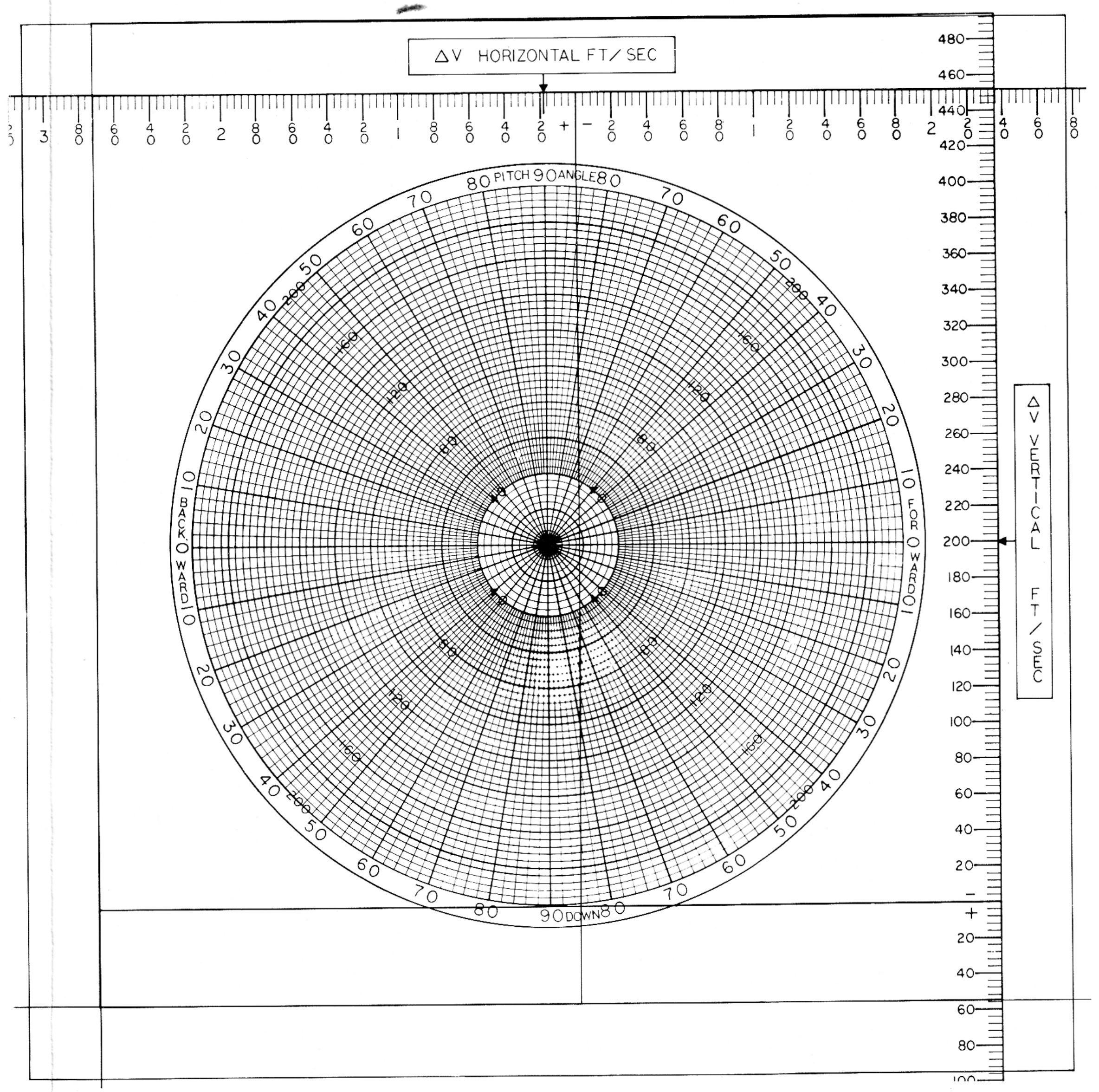
Fig. (5-1)



Left Face - Orbit Determination



Right Face - Required Orbit

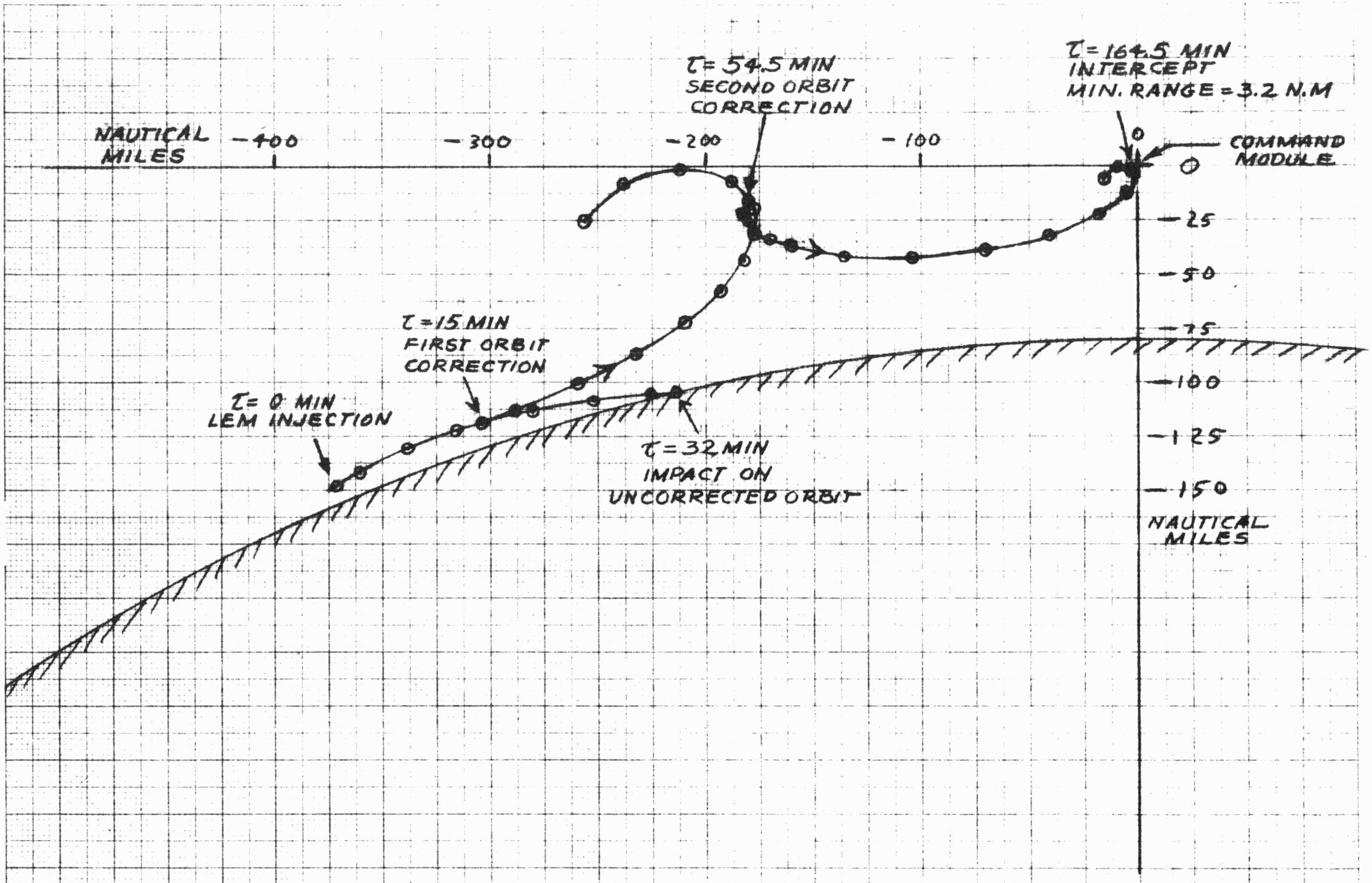


Rear Face - Velocity Correction

0. True initial state after first correction indicated by 0.
1. Make altitude measurements every five minutes starting 2-1/2 minutes after first correction (t = 17-1/2 mins.).
2. At each measurement starting with the third, mark the intersection point on the orbit determination face, (indicated by +), estimate the time to reach 80 NM, and prepare for the next measurement by setting $H_{t - \Delta t} = H_t$ and $H_t = H_{t + \Delta t}$.
3. Determine the radius of the circular locus being traced out as the radius of the point where the velocity excess is nearest zero.
4. Make the last altitude fix between five and ten minutes before reaching 80 NM. (t = 47.5 min in this case) Start stop watch.
5. Set extrapolation films for predicted 80 NM altitude (altitude to go = 27 NM determines point indicated by ⊗). Read time to correction from the time scale. (Time to correction = 7.0 min. correction to be made at t = 54.5 mins.).
6. Measure range to command module 2-1/2 mins. before correction. (t = 52 min, range = 178.5 NM). Set required orbit face to span range in the minimum whole number of orbits making sure that periapsis altitude will be at least 25 NM (intercept in 1 orbit, periapsis altitude prediction = 80 - 38 = 42 NM). Ignore range variation of last 2-1/2 minutes preceding correction.
7. Read pitch angle (85° down and forward) and velocity magnitude (202 ft/sec).
8. Impart Δ V

The Calculator at the Time of the Second Correction

Fig. (5-2)



ORBIT OF LEM MANUALLY GUIDED ON RETURN TO COMMAND MODULE
COORDINATES ARE LOCAL VERTICAL FIXED IN COMMAND MODULE

FIG (5-3)

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Table 5-1. LEM orbit.

Time (min)	Altitude (n. mi)	Altitude Rate (ft/sec)	Velocity Excess (ft/sec)	Range to CM (n. mi)	Range Rate (ft/sec)	Elevation of LEM (degrees)
0	8.000000000	64.00000000	-43.00000000	399.9999990	-525.9847929	-21.72336971
2.5	9.423082198	51.18347900	-47.04771300	387.2114279	-511.3298532	-21.47901500
7.5	11.26161288	22.86535700	-52.25458700	362.5110212	-492.2463563	-21.17950270
12.5	11.65085300	- 7.187509000	-53.35371800	338.4012224	-487.4306119	-21.18792820
15.0*	11.28836811	-22.15263600	-52.33019500	326.3480949	-490.2854781	-21.33082529
15.0	11.28836811	- 0.15000000	122.4700000	326.3480922**	-665.9162534	-21.33082515**
17.5	11.71800923	34.91458300	121.1778550	309.9416469	-662.7484429	-21.41798896
22.5	15.11859637	101.8856750	111.0015650	277.8833403	-631.1849735	-21.25672609
27.5	21.62293384	159.8328150	91.78621100	248.1881730	-567.3474362	-20.51088554
32.5	30.66579132	204.1539390	65.60300500	222.3772117	-474.4041189	-19.00866849
37.5	41.49637384	231.9712660	35.03328500	201.7720155	-357.3553791	-16.63863709
42.5	53.26644836	242.1940970	2.753684000	187.3838021	-224.0063308	-13.46447204
47.5	65.11354874	235.2789810	-28.78385000	179.7451627	- 86.01370280	- 9.817494223
52.0	75.17071626	215.7203015	-54.83333800	178.5573423	30.34598670	- 6.586016722
54.5*	80.30561389	200.0348756	-67.88483300	180.0296453	88.00110010	- 4.975983283
54.5	80.30561389	0.0300000	-49.88000000	180.0296417**	52.04774670	- 4.975982697**
60	79.55376422	-27.56275680	-47.966030000	182.6475547	41.91709220	- 5.288147162
70	74.58220720	-71.07785120	-35.220268000	184.4880396	- 11.35850710	- 6.887171773
80	66.11782733	-96.71218360	-13.15523100	179.3562376	- 96.32151470	- 9.495919993
90	56.34042846	-96.85648060	12.91903800	165.1312071	-191.7120989	-12.86940766
100	47.89715769	-70.07201510	35.95641900	141.9833794	-273.0937424	-16.99472877
110	43.19012132	-22.89271280	49.01449700	112.4245186	-318.5986853	-22.13093730
120	43.60687650	31.12082130	47.85203000	80.70233386	-316.0547458	-28.85119993
130	49.02312989	75.86445420	32.85569700	51.55799178	-268.1307844	-38.09742047
140	57.84888861	98.66134446	8.854408000	28.70220238	-192.6792419	-51.02843783
150	67.59106865	94.25175280	-17.029304000	13.49810657	-118.1515784	-66.97525751
160	75.63346249	65.20086640	-37.92869000	4.728494037	- 58.68060325	-67.48722710
164.5	78.11671652	46.16640850	-44.29811100	3.206948971	- 3.051699810	-36.03528004
165.0	78.33889136	43.88804960	-44.86608300	3.211125461	4.701481390	-31.22857769

*Orbit correction.

**The change in this number from the number above it is the result of computation error.