## APBLLO

## GUIDANCE, NAVIGATION

 AND CONTROLLARSON
G \& N DESIGN OF mechanization course NOTEBOOK

OCTOBER 17, 1966
MITTI INSTRUMENTATION
 GUIDANCE, NAVIGATION AND CONTROL

LARSON
G \& N DESIGN OF MECHANIZATION COURSE NOTEBOOK

OCTOBER 17, 1966



I G\&N SYSTEM DESIGN PHILOSOPHY AND MECHANIZATION (HOWS AND WHYS) (MIT I WEEK) 17-2l OCTOBER 1966
A. DEFINITION OF FUNDAMENTAL PROBLEMS OF GUIDANCE AND NAVIGATION AND THEIR SOLUTIONS AS MECHANIZED BY THE G\&N SYSTEM FOR 205.
B. G\&N SYSTEM CAPABILITIES AND LIMITATIONS AS MECHANIZED SOLUTIONS TO THESE PROBLEMS; EMPHASIZE REASONS FOR PARTICULAR SYSTEM MECHANIZATIONS.
C. DEFINITION OF SYSTEMS INTERFACING.
D. INTRODUCTION TO CREW PROCEDURES AND OPERATIONAL CONTROL.
E. INTRODUCTION TO SOFTWARE MODING AND PROGRAMMING LOGIC AS RELATED TO MISSION PHASING.

II INTRODUCTION TO HARDWARE (AC/G\&C LAB)
A. PHYSICAL EXPOSURE TO HARDWARE EQUIPMENT.
B. OPERATIONAL MODING AT G\&N WITH FLIGHT HARDWARE.

III DETAILED FUNCTIONAL DESCRIPTION (AC 40 HOURS)
A. INTRODUCTION TO G\&N REQUIREMENTS.
B. SYSTEMS FUNCTION (TO BLOCK DIAGRAM LEVEL) AND RELATION TO MISSION PHASES (BRIEF)
C. EQUIPMENT CONSTRUCTION, MECHANIZATION.
D. MODES OF OPERATION.
E. DISPLAYS AND CONTROLS.

REF: MIT/IL REPORT \#E-2000, "APOLLO OPERATIONS HANDBOOK G\&N SYSTEM CSM MISSION AS205A, SC01.4'

IV DETAILED PROCEDURES (MIT) (NOV 7-11 PRIME, NOV 14-18 BACKUP)
DETAILED CLASSROOM INSTRUCTION AND MOCKUP DRY RUNS OF THE FLIGHT PROGRAMS AND PROCEDURES AS DEVELOPED IN THE COMPUTER LOGIC-CHECKLIST INTERFACE.

REF: SECTION 4 OF MIT/IL REPORT \#R-507 (REV 1), "G\&N SYSTEM OPERATIONS PLAN FOR MISSIONS AS204A/AS205A"
V SIMULATION (MIT) (NOV $7-11$ PRIME, NOV 14-18 BACKUP)
A. REAL TIME PRACTICE (2 MAN) WITH HYBRID SIMULATION OF MAJOR MISSION PHASE BLOCKS.

1. THRUSTING MANEUVERS
2. NAVIGATION
3. ENTRY
B. SKILL PRACTICE OF NAVIGATION OPTICS SIGHTINGS (OPTICALLY AND PHOTOMETRICALLY CORRECT TARGETS). SPACE NAVIGATOR AND OPTICAL SIMULATIONS.

## CHAPTER II-4

OPERATION MODES OF GUIDANCE, NAVIGATION, AND CONTROL A POLLO COMMAND MODULE BLOCK I

The system that has been described so far can be seen to have a high degree of flexibility in performing the many tasks of concern. In this chapter a series of diagrams are used to show briefly the equipment involved and the infor mation flow in operations with these tasks.


Fig. II-47 Equipment Arrangement in the Command Module Lower Equipment Bay
The figure above shows the installed arrangement of the equipment in the Block I command module. In the following figures this equipment is shown separated in order to trace signal paths more easily. The Block I equipment configuration lends itself better to the tasks of this chapter than the later more integrated Block II equipment which will finally perform the lunar landing.


Fig. II-48 Subsystem Identification
This is the key figure of the series. Here the principal subsystems of the Block I command module guidance and navigation are arrayed and identified for use in the subsequent figures. The sensors of the system are shown in the top center: the two optical instruments, sextant and scanning telescope, and the inertial measurement unit (IMU) all mounted on the common rigid navigation base. At top left are the two sets of coupling data units (CDUs) to provide the communication of the optics and IMU angles with the computer shown at the center. The computer display and keyboard (DSKY) is at upper right. The whole vehicle - command and service modules - is represented by the figure center right. The separate stabilization and control system of the Block I system is bottom right. The astronaut navigator is shown bottom left surrounded by several of the important controls.


Fig. II-49 Guidance Steering Control
This first mode is that of powered flight guidance. The signals from the accelerometers on the IMU are processed within the computer where the steering equations develop a desired thrusting attitude of the vehicle to achieve the desired direction of acceleration. This is treated as a commanded attitude which is compared in the CDUs with actual attitude measured by the IMU. The difference is a steering error which is sent to the SCS to control the vehicle. Resulting vehicle motion is sensed by the IMU to complete the feedback. When the required velocity change is achieved, the computer sends a rocket engine shutdown signal. The crew can monitor the whole operation by the display of appropriate variables on the DSKY such as the components of velocity yet to be gained.

Before the IMU can be used for an operation such as this, it must be aligned to the desired spacial orientation. This process is described next.

IMU alignment is normally performed in two stages: "coarse" and "fine". Coarse alignment is described here in two steps using the figures on the opposite page.

The first step of coarse alignment is to give the computer a reasonably accurate knowledge of spacecraft attitude with respect to the celestial framework being used. Illustrated here, the navigator sights sequentially two stars using the scanning telescope (SCT). The star image is sensed by the navigator who uses his left hand optics controller to command the SCT prism such as to center the star on the reticle. He pushes the mark button when he achieves satisfactory tracking which signals the computer to read the $S C T$ angles being transmitted it by the optics CDUs. A second star direction at a reasonably large angle from the first is similarly measured. The navigator identifies which stars are being used to the computer through the keyboard of the DSKY. With these data the computer determines in three dimensions the spacecraft attitude which is held reasonably fixed by a gyro control attitude hold of the SCS during all of these coarse alignment operations.

In step two the computer determines desired IMU gimbal angles based upon its knowledge of spacecraft attitude and the guidance maneuver which will be next performed. These desired angle signals sent to the IMU through the CDU are quickly matched by the IMU gimbal servos in response to error signals developed on the angle transducers on each gimbal aixs. At this point the IMU gimbal servos are then switched over to the gyro stabilization error signals to hold the achieved orientation.


Fig. II-50 IMU Coarse Alignment Step 1


Fig. II-51 IMU Coarse Alignment Step 2

Fine IMU alignment will also be described in two steps using the figures on the opposite page.

In the first step two star directions are again measured by the navigator. This time he uses the high magnification of the sextant (SXT) with the precision readout on the star line in order to achieve necessary accuracy. The IMU is presumed to be under gyro stabilization control and to be reasonably close to the desired orientation. On each of two stars, which the navigator identifies to the computer, the navigator signals "mark" when he achieves precise alignment on the SXT crosshair. On these signals the computer simultaneously reads the SXT and IMU angles being transmitted through the CDUs. With these data the computer determines star directions in IMU stable member coordinates from which the spacial orientation of the IMU being held by gyro control can be computed. The spacecraft attitude need not be held fixed during these fine alignment operations as long as the angular velocity is small enough to permit accurate star tracking by the navigator.

In step two the computer determines the existing IMU attitude error based upon the desired attitude as determined from the next use of the IMU such as for a particular guided maneuver. The computer then meters out the necessary number of gyro torquing pulses necessary to precess the gyros and the IMU to correct the IMU alignment error.

The above two steps can be repeated if desired to obtain more precision in the fine alignment when the torquing precession angle is large.


Fig. II-52 Manual IMU Fine Alignment Step 1


Fig. II-53 Manual IMU Fine Alignment Step 2


Fig. II-54 Low Orbit Navigation - Landmark Tracking
Onboard navigation measurements in low orbit can be performed either using landmark references as shown on the above figure or using other references as described later. In the above figure, the navigator first aligns the IMU as previously described and then tracks identified landmarks as they pass beneath him using the SCT. When he is on target he signals "mark" and the computer records IMU and SCT angles and time so as to compute landmark direction in the coordinate frame of the aligned IMU. These direction measurements are then used to update the computer's estimate of position and velocity and the computer's estimate of error in these parameters. These data can be displayed to the astronauts if desired.

Although the above assumes identified landmarks of known coordinates, unidentified landmark features can be used as described in Part V.


Fig. II-55 Midcourse Navigation - Manual Star-Landmark Measurement
The use of the sextant to measure the angle between identified stars and landmarks for midcourse navigation is described with the above figure. The acquisition process using the scanning telescope is assumed already to have been performed so that the desired star and landmark images appear in the SXT field of view. With his right hand the navigator periodically commands jet impulses to hold the landmark in the field of view by controlling spacecraft attitude and the body-fixed landmark line. With his left hand he controls the sextant mirror to superimpose the star image onto the landmark. When this superposition is satisfactory he signals "mark" and the computer records the measured navigation angle and time. These numbers are then further processed in the computer navigation routines. The computer displays the correction to the state vector which would be caused by this sighting so that the navigator is given a basis to reject a faulty measurement before it is incorporated into the navigation.

The use for navigation of the automatic star tracker (AST) and photometer (PHO) is shown in two steps with the figures on the opposite page.

In the first step the navigator uses the scanning telescope (SCT) to acquire the navigation star with the automatic star tracker on the sextant. Acquisition is confirmed by a "star present" light signalled from the star tracker.

In step two the navigator maneuvers spacecraft attitude manually to point the bodyfixed horizon photometer line to the illuminated horizon by observing the geometry through the SCT. The SCT has a reticle pattern which permits the navigator to judge when the photometer is looking in the plane containing the star and the center of the planet. This puts the photometer sensitive area directly beneath the star. His task is then to sweep the photometer line in this plane through the horizon. When the sensed brightness drops to half the peak value, the photometer automatically sends a "mark" to the computer so that the resulting navigation angle and time can be recorded.

This operation can be performed using the sun illuminated limb of either the moon or earth. Operation with the earth depends upon the systematic brightness of the atmospheric scattered light with altitude described in Part V.

The navigation measurement process described above uses astronaut control in positioning the photometer line. If the IMU is on and aligned, this process could be completely automatic through computer control program.


Fig. II-56 Illuminated Horizon Manual Navigation Measurement - Step 1


Fig. II-57 Illuminated Horizon Manual Navigation Measurement - Step 2

The automatic star tracker on the sextant provides the capability of automatic IMU alignment as illustrated in two steps in the figures on the opposite page. Without astronaut help, however, the star tracker cannot acquire a known alignment star unless the IMU is already roughly aligned to provide a coarse direction reference. The automatic IMU alignment capability described here, then, is most useful to re-correct the IMU drift after a long period of IMU operation.

In the first step the computer points the sextant to the expected star direction through the optics CDUs based upon the vehicle attitude measured by the IMU. Presumably the star tracker now senses the desired star within its acquisition field of view and signals the computer that the star is detected.

The computer now changes equipment mode, step two, to send the star tracker error signals to the sextant drives so as to track the star automatically. The computer then reads simultaneously the sextant and IMU angles in order to determine two components of the actual IMU misalignment. This latter is corrected by computer torquing signals to the IMU gyros as described previously. Acquisition and tracking of a second star complete the automatic fine alignment in three degrees of freedom.


Fig. II-58 Automatic IMU Fine Alignment - Step 1


Fig. II-59 Automatic IMU Fine Alignment - Step 2


Fig. II-60 Star Occultation by Moon - Automatic Navigation Measurement
The automatic star tracker provides the means for making automatic star occultation navigation measurements with the moon, as shown above. An acquisition by the star tracker as shown in Fig. II-56 or Fig. II-58 is required, of course, as an initial step. While the star is being tracked, the instrument generates a "star present" signal for the computer which is based upon the detected star light energy. As the star sinks below the lunar horizon due to the orbital motion of the spacecraft, the star present signal disappears at the moment of occultation. The time of this event is measured by the computer as a point of the navigation data.

A similar process is possible using the earth's limb, but this requires a more elaborate star present detection. The star intensity diminishes gradually due to dispersion and scattering as the beam sinks into the earth's atmosphere.


Fig. II-61 Star Occultation by Moon - Manual Navigation Measurement
Besides the automatic occultation measurement just described, a manual detection is possible, of course, as shown above. This is of advantage since it does not require that the optics system electronics be turned on. In fact, the event can be observed by the astronaut through the window and timed with a separate stopwatch for transmission to the earth for use in aiding ground-based navigation measurement.

SINGLE AXIS SCHEMATIC - IMU, CDU AND AGC OPERATION


# Basic Concepts of Space Navigation and Guidance 

by

R. H. Battin

Of fundamental importance in the design of space guidance systems should be the creation of both flexible techniques and versatile instrumentation which have a wide range of applicability but neither compromise mission accuracy nor place an undue burden on propulsion requirements. Minimal constraints on the system and methods of its operation should be imposed by detailed mission objectives which are subject to frequent and last minute revision. In partial fulfillment of these goals, the development of explicit guidance techniques is warranted to reduce any dependence on precomputed reference orbits or specific rocket engine characteristics.

During the evolution of a space flight program, the ultimate mission objective is attained progressively by a series of intermediate flights. Each successive flight is frequently planned as a direct extension of the previous one so that the need for special equipment and untried techniques can be minimized. The success of this approach is enhanced through the development of unified guidance methods. Then the guidance requirements for each new mission phase can be met as a specific application of a general guidance principle.

The two fundamental tasks of a guidance system are to maintain accurate knowledge of spacecraft position and velocity and to provide steering commands for required changes in course. It is the purpose here to review some of the current techniques for solving the guidance problem emphasizing those methods which are consistent with the explicit and unified philosophy of design.

## I. Navigation

The task of periodic determination of position and velocity, customarily referred to as navigation, divides naturally into two parts - accelerated flight and coasting flight. Except for vehicles propelled by low-thrust engines, the period of time of accelerated flight is usually of only a few minutes duration. On the other
hand, coasting flight times are measured in hours or days. Thus, it is to be expected that the physical sensors as well as the methods of processing the relevant data for these two flight phases will differ markedly from each other.

The vector equations of motion of the spacecraft are

$$
\frac{\mathrm{dr}}{\mathrm{dt}}=\underline{v} \quad \frac{\mathrm{~d} \underline{v}}{\mathrm{dt}}=\underline{a}
$$

where $\underline{r}$, $\underline{v}$, a are the position, velocity and acceleration vectors referred to as an inertially fixed frame of reference. The acceleration vector $\underline{a}$ is composed of gravitational forces $g$ and thrust acceleration $\underline{a}_{\mathrm{T}}$. Thus

$$
\underline{\mathrm{a}}=\underline{\mathrm{g}}+\underline{\mathrm{a}}_{\mathrm{T}}
$$

In earth orbit, the gravity vector is conveniently written as

$$
\underline{g}=-\frac{\mu}{r^{3}} \underline{r}+\underline{a}_{d}
$$

where the first term represents the central gravitational force field, with $\mu$ as the gravitational constant of the earth, and $a_{d}$ all other forces, other than thrust acceleration which tend to make the motion deviate from simple conic orbits.
A. Powered Flight

1. Gimballed sensors

For navigation during an accelerated maneuver, the system includes inertial instruments capable of measuring thrust acceleration along three mutually orthogonal axes which are nonrotating. The guidance computer is then required to perform accurate integrations and gravity calculations on a real-time basis.

A functional diagram of the basic computations required of the navigation sys tem is shown in Fig. 1. Incremental outputs from inertially stabilized integrating accelerometers, together with components of gravitational acceleration computed as functions of inertial position in a feedback loop, are summed to give the components of inertial velocity.

## a. Central gravitational force field

The gravity calculations may be performed by a simple algorithm. In Fig. 2, the equations of motion for a vehicle moving in a spherical gravitational field are given together with a simple computational algorithm by means of which position and velocity are obtained as a first order difference equation calculation. Since velocity is updated by means of the average effective gravity over the interval of one time step, this technique has been termed the "average g " method.


Fig. 1 Accelerated Flight Navigation System


Fig. 2 Accelerated Flight Navigation
b. Earth oblateness

The disturbing acceleration caused by the oblate shape of the earth must be taken into account partially to maintain accuracy during the powered maneuvers. The first term is of importance and is the only source of disturbing acceleration which we need include. We have

$$
\left.\underline{a} d=\frac{\mu}{r^{2}} J_{2}\left|\frac{r e q}{r}\right| \frac{15}{2} \cos ^{2} \phi-\frac{3}{2} \right\rvert\,{\underset{\mathrm{i}}{r}}^{-}-3 \cos \phi \frac{i_{z}}{}
$$

where $J_{2}$ is an empirical constant, $r_{e q}$ is the equatorial radius of the earth, $\underset{-}{i} r$ is a unit vector in the direction of the spacecraft, ${\underset{\mathrm{i}}{\mathrm{z}}}$ is a unit vector in the direction of the north pole, and $\phi$ is the colatitude of the vehicle

$$
\cos \phi=\underline{i}_{\mathrm{r}} \cdot \underline{\mathrm{i}}_{\mathrm{z}}
$$

2. Body-mounted sensors

During recent years increasing attention has been devoted to the so-called "gimballess inertial measurement unit" in which the inertial sensors are mounted directly to the spacecraft. Although many advantages might accrue in terms of system weight, volume, power, cost, packaging flexibility, reliability and maintainability, the realization of a satisfactory design is not without significant problems. Unlike the environment provided by a gimballed system, the body mounted inertial instruments are subjected to substantial angular velocity which tends to exaggerate performance errors. Also, the role of the guidance computer is expanded since the angular orientation of the vehicle must also be determined by integration of measured angular velocities. It is most convenient if the outputs of the body mounted accelerometers are immediately transformed into an inertially stabilized coordinate frame so that the navigation or guidance problem can be solved just as if a physically stabilized platform had been employed.

As indicated in Fig. 3, the body fixed coordinates and the inertial coordinates of the thrust acceleration vector are related by a transformation matrix of direction cosines. The additional computations required of the guidance computer involve the updating of this matrix and using it to transform vectors from one frame of reference to the other. The transformation matrix $R$ is readily shown to satisfy a first order differential equation with a coefficient matrix $\Omega$ whose elements are the components of the angular velocity of the body fixed coordinate frame measured in body coordinates.


$$
\Omega=\left(\begin{array}{ccc}
0 & \omega_{3} & -\omega_{2} \\
-\omega_{3} & 0 & \omega_{1} \\
\omega_{2} & -\omega_{1} & 0
\end{array}\right)
$$

$\omega_{j}=$ ANGULAR VELOCITY SENSED BY $j^{\text {th }}$ GYRO


Fig. 3 Navigation Using Body-Mounted Sensors

Currently, pulse-torqued integrating gyros are the most promising candidates for angular velocity sensors. However, since their basic output consists of angular increments rather than angular velocity, the accuracy with which the transformation matrix differential equation can be integrated is adversely affected. The use of a higher order integration rule provides no advantage over simple rectangular integration since the basic data from the gyro has already an uncertainty of the order of the square of the gyro quantization error.

The accuracy attainable by a gimballess inertial system is limited primarily by the maximum angular velocities to which the vehicle is subjected. The required sampling time of the integrating gyros is inversely proportional to this maximum angular velocity. The time step needed for a numerical integration of the direction cosine differential equations is of the same order of magnitude as the gyro sampling time. If the sample time is very short, a digital differential analyzer may prove to be the best solution to the problem of selecting a guidance computer. On the other hand, if the sampling time is long enough to permit the use of a general purpose computer, there may be sufficient time remaining in which to process the navigation and/or steering equations. This is, of course, more satisfactory, for then one has the possibility of satisfying all or most of the complete system computation requirements with a single computer.

## B. Coasting Flight

1. Conic initial value problem

The only completely solvable problem in celestial mechanics is the relative motion of two bodies. The case occurs when the thrust acceleration $\underline{a}_{\mathrm{T}}$ is zero as well as all disturbing acceleration a $_{\mathrm{d}}$. The appropriate equation of motion is then

$$
\frac{\mathrm{d}^{2} \underline{r}}{\mathrm{dt}}+\frac{\mu}{\mathrm{r}^{3}} \underline{r}=0
$$

and is an excellent first approximation to the true motion.
For the earth orbit case, the trajectory is an ellipse as shown in Fig. 4. The major axis is of length $2 a$ and the eccentricity lies between 0 and 1 . If $\underline{r}_{0}$ and $\underline{v}_{0}$ are the position and velocity of the vehicle at time $t_{0}$, the position and velocity $\Delta t$ time units later may be calculated from

$$
\begin{aligned}
& \quad \Delta t=F\left(\underline{r}_{0}, \underline{v}_{0}, a, x\right) \\
& \underline{r}=f\left(r_{0}, a, x\right) \underline{r}_{0}+g(\Delta t, a, x) \underline{v}_{0} \\
& \underline{v}=h\left(r, r_{0}, a, x\right) \underline{r}_{0}+k(r, a, x) \underline{v}_{0}
\end{aligned}
$$



Fig. 4 Elliptical orbit
The first equation is solved iteratively for $x$ and the second two equations give $\underline{r}(t)$ and $\underline{v}(\mathrm{t})$. The semi-major axis is determined from

$$
a=\left(\frac{2}{r_{0}}-\frac{v_{0}^{2}}{\mu}\right)
$$

To locate the orbit in three dimensional space, it is customary to use the three Euler angles $\Omega, \omega$, i as defined in Fig. 5

As an example of the long turn effects of earth oblateness on the spatial orientation of the orbit we note that the average change in $\Omega$ and $\omega$ is given approximately by

$$
\begin{aligned}
& \overline{\Delta \Omega} \cong-10 \cos \text { i deg/day } \\
& \overline{\Delta_{\omega}} \cong-5\left(1-5 \cos ^{2}\right. \text { i) deg/day }
\end{aligned}
$$

2. Disturbed two-body motion

The true motion of the spacecraft is governed by the equation

$$
\frac{\mathrm{d}^{2} \underline{r}}{\mathrm{dt}^{2}}+\frac{\mu}{\mathrm{r}^{3}} \underline{\mathrm{r}}=\underline{\mathrm{a}} \mathrm{~d}
$$



$$
\begin{aligned}
\Omega & =\text { longitude of the node } \\
\omega & =\text { argument of perigee } \\
\omega=\Omega+\omega & =\text { longitude of perigee } \\
\mathrm{i} & =\text { inclination }
\end{aligned}
$$

Fig. 5 Coordinate system geometry
If, at a particular instant of time $t_{0}$, all the effects embodied in the vector $\underline{a}_{d}$ ceased to exercise any influence on the motion, the resulting orbit would be a conic and the position and velocity would be exactly computable from the two-body formulae. Expressed differently, at any instant of time $t_{0}$, the position and velocity vectors may be used to define a conic orbit. The terminology osculating orbit is used to describe this instantaneous conic path associated with the time $t_{0}$. The body never actually moves along the osculating orbit; however, if the disturbing forces are small compared with the central body force, then over short intervals of time, the actual position will differ from the associated position in the osculating orbit by a correspondingly small amount.

If position in the osculating orbit is denoted by $\underline{\mathrm{r}}_{\mathrm{osc}}{ }^{(\mathrm{t})}$, then the vector difference

$$
\underline{\delta}(t)=\underline{r}(t)-\underline{r}_{o s c}(t)
$$

may be integrated instead of the total acceleration. This procedure of integrating this differential acceleration, due to Encke, permits an extremely efficient and accurate orbit extrapolation.

## C. Navigation Measurements

1. Geometry of the navigation fix

For a completely on-board determination of position, we consider, as an example, the simultaneous measurements of the angles between two stars and a known landmark and the angle between two landmarks. Let $\underline{r}$ be the unknown position vector of the spacecraft relative to the first landmark. Let $\underset{p}{r}$ be the known position of the second landmark with respect to the first and let ${\underset{1}{1}}_{1}$ and $\underline{i}_{2}$ be unit vectors in the direction of the two selected stars. The three measurements produce three angles $A_{1}, A_{2}, A_{3}$. The following three equations may then be solved simultaneously for the position vector $\underline{r}$.


An exact determination of position by this method has a number of distinct disadvantages. First of all, the resulting algebraic equations to be solved are always nonlinear. Second, the method requires simultaneous measurements which are almost certainly impractical. Finally, and perhaps most important of all, no satisfactory method of incorporating redundant measurements to compensate for instrumentation inaccuracies is known.

All these objections can be circumvented if the determination of spacecraft position is made relative to a known and nearby position.

## 2. Measurement geometry vector

Each measurement establishes a component of spacecraft position and/or velocity along some direction in space. If $q$ is the quantity to be measured and
$\delta q$ is the difference between the measured and known value, then the deviation in spacecraft position $\delta \underline{r}$ and velocity $\delta \underline{v}$ is linearly related to $\delta \mathrm{q}$ by

$$
\delta q=\underline{b} \cdot\left|\begin{array}{c}
\delta \underline{r} \\
\delta \underline{v}
\end{array}\right|
$$

The $\underline{\mathrm{b}}$ vector is, in general, six dimensional and characterizes the type of measurement.

For example, let $\underline{r}_{0}$ be the reference position of the spacecraft relative to a known landmark and $\underline{r}_{0}+\delta \underline{r}$ the true position. Measuring the angle between a star and the landmark will give the following relation between $\delta \underline{r}$ and the deviation in angle $\delta \mathrm{A}$.


The vector $\underline{n}$ is a unit vector in the plane of the measurement and perpendicular to the line-of-sight to the landmark.
3. Recursion method of navigation

Spacecraft navigation during prolonged coasting flight is performed by appropriate utilization of periodic measurements of convenient physical quantities such as (1) distance, velocity, elevation and azimuth from well-established reference points and (2) angles between lines of sight to known celestial objects. Since navigation measurements are more accurately made when the sensors are in proximity to the data source, vehicle-borne and ground-based instrumentation can serve in complementary roles.

The estimate of position and velocity is maintained in the computer in non-rotating rectangular coordinates and is referenced to either the Earth or the Moon. An Earth centered equatorial coordinate system is used when the vehicle is outside
of the lunar sphere of influence. Inside of this sphere the center of coordinates coincides with the center of the Moon. The extrapolation of position and velocity is made by a direct numerical integration of the equations of motion employing the method of integration of differential accelerations suggested by Encke.

Periodically, the position and velocity of the spacecraft must be brought into accord with optical or radar observations made either with onboard of with ground based sensors. At the time a measurement is made, the best estimate of spacecraft position and velocity is the extrapolated estimate maintained in the computer and denoted by $\hat{\underline{r}}$ and $\hat{\underline{v}}$ as shown in Fig. 6. From this estimate, it is possible to determine an estimate of the quantity to be measured such as an angle, range from a tracking station or range rate. When the predicted value of this measurement is compared with the actual measured quantity, the difference is used to improve the estimated position and velocity vector.

The weighting vector $\underline{w}$ has six components and is determined so that the observational data is utilized in a statistically optimum manner. For this purpose, it is necessary to maintain statistical data in the guidance computer in the form of a six-dimensional correlation matrix of estimation errors.

Many advantages of the recursive navigation method are readily apparent. Although linear techniques are still employed, it has been possible to remove any dependence on a reference or pre-computed orbit. Within the framework of a single computational algorithm, measurement data from any source may be incorporated sequentially as obtained. Sensitive numerical computations, such as the inversion of matrices, are avoided.

## II. Guidance

The task of providing steering commands, frequently called guidance, separates naturally into two categories - major and minor maneuvers. Launch into parking orbit, transfer to lunar or interplanetary orbit, insertion into orbit, and landing are all examples of major thrusting maneuvers and differ markedly from the minor orbit changes typified by mid-course velocity corrections. In either case, the guidance problem is always a boundary value problem subject to a variety of constraints of which fuel conservation, vehicle maneuverability, and time are examples.

A function diagram of a powered flight guidance system is shown in Fig. 7.


Fig. 6 Coasting Flight Navigation Computation


Fig. 7 Powered Flight Guidance System

## A. Conic Boundary Value Problems

Conic orbits can be exploited to advantage in solving many guidance problems. For those major orbital transfer maneuvers which can be accomplished conceptually by a single impulsive velocity change, an instantaneous velocity-to-be-gained vector based on conic orbits can often be defined and the vehicle steered to null this vector. As examples of such problems we may consider the following

1. Constrained time-of-flight

Let $\underline{r}$ be the present position of the spacecraft and $\underline{r}_{T}$ a target position to be reached in a given time interval $\Delta t$. The required velocity vector

$\mathrm{v}_{\mathrm{r}}$ is a readily calculated quantity.
2. Constrained terminal flight path angle

Let $\underline{r}$ be the present position of the spacecraft and $\underline{r}_{T}$ a target vector to be reached on such a path that the terminal flight path angle $\gamma$ will be specified.


Again, the required velocity ${\underset{V}{r}}^{\text {is a readily calculated quantity. }}$
B. Compensation for Disturbing Forces

Consider the problem of establishing an orbit between two position vectors $\underline{r}_{1}$ and $\underline{r}_{2}$ requiring a time $\Delta t$ to traverse the arc. In absence of all except the single central force, the orbit will be a conic. We may now start at position $\underline{r}_{1}$
with the computed conic velocity vector $\mathrm{V}_{1}$ and integrate the equations of motion, subject to all relevant perturbations. At the end of the time interval $\Delta t$ the position vector will, of course, fail to coincide with $\underline{r}_{2}$. However, if the perturbation forces are, indeed, small, we may expect the difference $\delta \underline{r}_{2}{ }^{(1)}$, that is, the new terminus minus the original position $\underline{r}_{2}$, to be small.

The next step is to replace $\underline{\mathrm{r}}_{2}$ by $\underline{\mathrm{r}}_{2}{ }^{(1)}=\underline{\mathrm{r}}_{2}-\delta \underline{\mathrm{r}}_{2}{ }^{(1)}$ and repeat the process. A conic arc requiring a time $\Delta t$ for traversal is computed to connect $\underline{r}_{1}$ and the new location $\underline{r}_{2}{ }^{(1)}$. Using the new conic velocity at $\underline{r}_{1}$ as an initial condition, we again integrate the complete equations of motion for a time interval $\Delta t$. This time the difference $\delta \underline{r}_{2}{ }^{(2)}$ between the new terminus and the original $\underline{r}_{2}$ will be smaller in magnitude. If it is still too large to be acceptable, we set $\underline{r}_{2}^{(2)}=\underline{r}_{2}^{(1)}-\delta \underline{r}_{2}^{\text {(2) }}$ and continue the procedure until acceptable accuracy is attained. Usually, one iteration cycle is sufficient for most problems.
C. Velocity-to-be-Gained Methods

Refer to Fig. 8 and let a vector $\underline{V}_{\mathrm{r}}$ be defined, corresponding to the present vehicle location $\underline{r}$, as the instantaneous velocity required to satisfy a set of stated mission objectives. The velocity difference $\underline{v}_{\mathrm{g}}$ and $\underline{v}_{\mathrm{r}}$ and the present vehicle velocity $v$ is then the instantaneous velocity-to-be-gained.

Two convenient guidance laws are immediately apparent which will assure that all three components of the vector $\underline{v}_{\mathrm{g}}$ are simultaneously driven to zero. First, we may orient the vehicle to align the thrust acceleration vector $a_{T}$ with the direction of the velocity-to-be-gained vector. Alternatively, since a convenient expression can be developed for the time rate of change of the $\underline{v}_{\mathrm{g}}$ vector, we may direct the vector $\underline{\mathrm{a}}_{\mathrm{T}}$ to cause the vector $\dot{\mathrm{v}}_{\mathrm{g}}$ to be parallel to $\underline{\mathrm{v}}_{\mathrm{g}}$ and oppositely directed. Of course, if the thrust acceleration magnitude is not sufficiently large, it may not be possible to align the vector $\underline{v}_{\mathrm{g}}$ with its derivative. However, with typical chemical rockets for which the burning time is relatively short, no difficulty has been encountered with this guidance logic.

A combination of these two techniques leads to a highly efficient steering law which compares favorably with calculus of variations optimum solutions. The scalar mixing parameter $\gamma$ is chosen empirically to maximize fuel economy during the maneuver. A constant value of $\gamma$ is usually sufficient for a particular mission phse; however, if required, it may be allowed to vary as a function of some convenient system variable.

A functional diagram illustrating the computation of the error signal required for control purposes is shown in Fig. 9. The position, velocity and gravitation


$$
\begin{aligned}
& \underline{v}_{r}: \text { REQUIRED IMPULSIVE VELOCITY } \\
& \underline{v}: \text { ACTUAL VEHICLE VELOCITY } \\
& \underline{v}_{g}=\underline{v}_{r}-\underline{v}: \text { VELOCITY-TO-BE-GAINED } \\
& \underline{g} \text { : LOCAL GRAVITY VECTOR } \\
& \underline{\underline{a}}_{T} \text { THRUST ACCELERATION VECTOR } \\
& \underline{\dot{v}}_{g}=\underline{\dot{v}}_{r}-\underline{g}-\underline{a}_{T}=\underline{p}-\underline{a}_{T}
\end{aligned}
$$



Fig. 8 Velocity-to-be-Gained Methods


Fig. 9 Velocity-to-be-Gained Steering
vec ${ }^{2}$ ors are computed from the outputs of integrating accelerometers as described earlier in the section on navigation. The required impulsive velocity needed to achieve mission objectives is determined as a function of the position vector and used to calculate the velocity-to-be-gained. Numerical differentiation of the required velocity vector and the accelerometer outputs, using values stored from the previous sample time, provides two important ingredients of the error signal. When properly scaled, the system output is a vector rate of command whose mae nitude is proportional to the small angular differences between the actual and commanded thrust acceleration vectors and whose direction defines the direction of vehicle rotation required to null the error. Near the end of the maneuver, when the velocity-to-be-gained is small, cross-product steering is terminated, the vehicle holds a constant attitude and engine cut-off is made on the basis of the magnitude of the $\underline{v}_{\mathrm{g}}$ vector.

$$
\begin{aligned}
& \alpha=4 \delta c G \\
& D_{e y}=D_{\text {ez }} \text { in } \\
& \begin{array}{l}
\beta=4 \delta \\
S^{R S} \\
150 \sec 400 K \rightarrow 1056
\end{array} \\
& 440 \\
& T_{400 \mathrm{~K}}-105 \mathrm{G} \quad 150 \mathrm{sec} \quad 440 \\
& T_{\text {Hook }} \rightarrow 25^{\circ} \mathrm{K} \text { 625 } 1080 \\
& \gamma_{\text {400k }}-1.57-.64 \\
& \begin{array}{rl} 
\\
400 \mathrm{~K} & 25467 \quad 25414
\end{array}
\end{aligned}
$$

