



NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
MANNED SPACECRAFT CENTER
HOUSTON, TEXAS 77058

IN REPLY REFER TO: EG7-70-55

RECEIVED

JUL 15 1970

June 29, 1970

MEMORANDUM TO: FA/Chairman, Apollo Software Configuration
APOLLO DIRECTOR

FROM : EG/Chief, Guidance and Control Division

SUBJECT : Recommendations for LM DPS throttle fix

Reference is made to MSC memorandum EG7-70-46 dated June 9, 1970, subject, "LM DPS throttling problem."

Our analysis of the LM DPS throttle loop in LUMINARY P66 has been completed. The results of the final analysis are essentially those contained in the reference; namely, the throttling loop is marginally stable and requires gain stabilization for correction (the reference states phase stabilization). The reasons for this are given below.

In synthesizing the throttle loop originally, one of the feedback terms was omitted. The term was added to the loop and the equations checked against the FMES. The first check case was for the system and erasable load flown on Apollo 11 and Apollo 12. The second test case was same system having a LAG/TAU load of -0.41333. The response of the math model to a single ROD input matched both FMES runs almost exactly. Because of this, we have high confidence in the correctness of the model used for analysis.

The results of the analysis (see MSC Memorandum EG2-70-100 dated July 1, 1970, subject, "P-66 stability analysis" for complete results) show that with LAG/TAU set to 0.41333 (Apollo 11; 12 load), the root structure has a negative real root and a damped complex pair. The complex pair is well behaved with a damping factor of about 0.9. The negative real root, which causes the castellation, lies at -0.9 and thus has only a stability margin of 0.1. However, the system has a nonlinear feedback term proportional to the square of the change in commanded thrust. A change in thrust of about 1,000 lbs drives the root out of the unit circle, which means the throttle loop is unstable. In other words, both Apollo 11 and Apollo 12 flew with systems marginally stable and probably at times were at least neutrally stable. P66 auto can be expected to be less stable than P66 because of the effect of the automatic velocity nulling feature which is certain to add additional roots to the system.

	Action	Info
R. RAGAN		
D. HOAG		
D. LARSON		✓
CENTRAL FILES		
R. Larson	✓	
Control Board	✓	
B. McCoy	✓	
G. Levine	✓	
A. Klumpp	✓	
G. Kalan	✓	
D. EYLES	✓	
DUE DATE	9, 1970,	

The reference makes note of the fact that both Apollo 11 and 12 had throttle commands going from 0 to about 1,500 lbs at 1-second intervals after touchdown. The reason for that was simply that the initial command to zero thrust drove the negative real root to at least the edge of the unit circle making the system neutrally stable.

We can surmise why neither Apollo 11 or Apollo 12 had diverging throttle commands in the presence of the large commanded thrust changes noted on those flights. In developing the equations for the throttle, we noted the system response was extremely sensitive to the modeling of the PIPA reading mechanization. The acceleration feedback consists of a combination of acceleration two seconds old plus acceleration one second old. If too much of the one second old acceleration is present, the system will go unstable. The amount of one second old acceleration feedback is a function of the commanded acceleration and also a function of the time it takes to input the command to the throttle because of the 3,200 pulses/second output rate. Thus, for small commands, the relative ratio of one second data to two second old data is larger than it is for large commands. Therefore, large commands tend to add a stabilizing factor to the loop. We verified this fact by merely changing the PIPA read equations accordingly.

The fix for Apollo 14 is simple -- merely reduce the feedback gain by changing the erasable load LAG/TAU. Root locus plots show that setting LAG/TAU to zero provides the best overall system response. The negative real root is changed from -0.9 to -0.43 giving a stability margin of 0.57 on the real root and about 0.5 on the complex pair. However, because of the system nonlinearities, LAG/TAU should be -0.1 to provide additional protection against large commands. This moves the negative real root to -0.33 and gives a stability margin of 0.67 on the negative real root. The stability margin of the complex pair remains at about 0.5 for small commands and a margin of about 0.4 for large commands. This causes the complex pair to be slightly less damped as compared to LAG/TAU at zero, but the increase in stability margin warrants the negative number. We expect this change in gain to afford stability to the P66 auto program and prevent the occurrence of throttle commands after touchdown. The effect of changing LAG/TAU to -0.1 should be verified by GAC and MIT from both a stability and handling qualities viewpoint using systems more nearly akin to the real vehicle.

For Apollo 15, we suggest you consider recoding the throttling circuitry. The system as now mechanized is entirely too complex for what needs to be done. For example, LAG/TAU could be set to zero (meaning that portion of the loop is not needed) if it were not for the nonlinear term being used as feedback in another part of the loop. There are also other coding changes that could be made to make the system somewhat more straightforward and less time consuming, which, if nothing else, would relieve the duty cycle problem accordingly.

Enclosed is MSC Memorandum EG2-70-100 containing the analysis upon which these recommendations are based. We are also prepared to give a presentation on the analysis and our results at your convenience.

R. G. Chilton

Robert G. Chilton

Enclosure

cc:

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 FM7/R. Nobles
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 FML3/GAC/G. Michos
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IN REPLY REFER TO: EG2-70-100

JUL 1 1970

MEMORANDUM TO: EG/Chief, Guidance and Control Division
FROM : EG2/Chief, Systems Analysis Branch
SUBJECT : P-66 stability analysis

INTRODUCTION

The purpose of this memo is to document a simplified stability analysis of P-66. The desirability of having such an analysis resulted from the observance of unexpected castellations in the throttle/throttle commands in flight test data and simulations of the P-66 phase of LM powered descent.

The analysis was conducted by deriving a simplified analytical model of the P-66 loop and applying the technique of Z-transforms. While the analytical model is a simplification of the actual system, it does predict the castellations and indicates the manner in which the loop gains should be changed to improve the system response.

The remainder of the memo is in the form of an analysis, and includes example calculations of the roots of the Z characteristic equation for several values of loop gains.

ANALYSIS

Problem Statement

The problem to be considered herein is to derive a simplified analytical model of the LM powered descent program P-66 and investigate the stability characteristics thereof.

Simplifying Assumptions

Vehicle Dynamics. It will be assumed that the LM is a point mass which is constrained to move only in the vertical direction. In other words, there are no vehicle rotations, no horizontal component of translation, no fuel slosh, etc.

Sensor Characteristics. It will be assumed that the IMU is at the C.G., perfectly aligned, and that the Pipas are perfect.

Throttle Command. In P-66, the change in throttle setting is commanded is a series of pulses having the appearance of a sort of "quantized ramp" input to the throttle. However, it will be assumed that the "ramp" is steep enough to be represented by a simple step input.

Throttle Response. It is assumed that the response of the throttle to an input command can be represented by a simple first order lag.

Computational Delay. The delay between reading the PIPAS and issuing the throttle command is simply a transport lag.

Basic Equations

Commanded Acceleration. Based on the above assumptions, and the description of P-66 given in Section 5 of the GSOP, the difference equation (see attachment for derivation) for the commanded acceleration is written as

$$a_{n+1} = a_n + \frac{V_D - V_{n+1}}{\tau} - K_1 (V_{n+1} - V_n) - K_2 (a_n - a_{n-1}) \quad (1)$$

where

a_n = commanded acceleration at time t_n

V_n = vertical velocity at time t_n

V_D = desired vertical velocity

τ = TAUROD (GSOP notation)

$$K_1 = \left(1 + \frac{\text{Lag}}{\text{TAUROD}} \right) \left(\frac{1}{\Delta t} \right)$$

$$K_2 = \frac{K_1}{\text{MASS}} \left[\frac{t_{fc} - t_n}{\Delta t} + \frac{\tau_{th}}{\Delta t} + \frac{|f - \hat{f}| / \text{FRATE}}{2 \Delta t} \right]$$

and it is noted that K_2 will be regarded as "constant" for purposes of analysis, whereas in reality it actually is a non-linearity.

Velocity Equation. While the derivation of the difference equation for velocity is straightforward, it is quite tedious. In the interest of brevity only the result is given as follows:

$$V_{n+1} = V_n + (\tau_c + 1.1 \tau_e) a_n + (\tilde{\Delta} - 1.1 \tau_e) a_{n-1} - g \Delta \quad (2)$$

where

Δ = computational cycle time (1 sec)

τ_c = " delay time

$$\tilde{\Delta} = \Delta - \tau_c$$

τ_e = time constant for throttle response

g = lunar gravitational acceleration

The time at which the n-th throttle command is issued is $t_n + \tau_c$ because of the computational delay.

Stability Analysis

Characteristic Equation. The stability analysis is performed by taking the z-transforms of equations (1) and (2) and obtaining the characteristic equation for the commanded acceleration. Once again this is a straightforward but laborious process. The result is greatly simplified if numerical values are used for the time constants. Using the following values

$$\Delta = 1 \text{ sec}$$

$$\tau_c = .25 \text{ sec}$$

$$\tau_e = .1 \text{ sec}$$

The result was calculated to be

$$z^3 + [K_2 - 2 + .64 (K_1 + \frac{1}{\tau})] z^2 + (\frac{.36}{\tau} + 1 - .28K_1 - 2K_2) z + K_2 - .36K_1 = 0 \quad (3)$$

Present Gain Values. Thus far, in the Apollo Program, the values which have been used for τ and K_1 are given by

$$\tau = 1.5 \text{ sec}$$

$$K_1 = 1.4 \text{ sec}^{-1}$$

Since K_2 is not really a constant, one must estimate an approximate "average" value for this parameter. It appears that a reasonable value is given by

$$K_2 \approx \frac{1}{2} K_1 = .7065$$

Using these values for the gains the roots of (3) are found to be

$$z_1, z_2 = .44 \pm j(.165)$$

$$z_3 = -.897$$

which may be interpreted as follows:

a. The complex root, z_2 , indicates a heavily damped exponential that presents no problem at all.

b. The real root, z_3 , is wherein all the problems, or potential problems, lie. Being a negative real root immediately indicates the castellations which have been observed in the data. Moreover, being on the order of $-.897$ indicates relatively light damping. The real source of potential trouble is the fact that K_2 is not a constant gain so that the location of this "root" is really not well defined.

"Sensitivity" to K_2 . Since K_2 is not a constant, a quick check of the sensitivity to K_2 was made by solving for the roots using the gains

$$\tau = 1.5 \text{ sec}$$

$$K_1 = 1.413 \text{ sec}^{-1}$$

$$K_2 = .98$$

with the result being that z_1 and z_2 are not seriously effected but that

$$z_3 = -1.369$$


which is a completely unstable situation. Thus, there is some value of K_2 between .7 and 1.3 where the system goes unstable. However, it should be pointed out that due to the non-linear characteristic of the system, it is not clear whether or not the unstable situation would in reality be transient or catastrophic in nature. Of course, even the possibility of a transient instability is somewhat lacking in appeal.

Suggested Gains. The root locus of the system shown in the attached figure is for a system gain $\tau = 1.5$ and feedback gains $K_2 = 0.5K$. The system gain limit for stability is for $K = 1.5$. As the gain is decreased, the negative real root moves in while the complex pair moves upward and slightly to the right. The best gain is probably about 1.0 which gives a stability margin of about 0.57 on the real root and about 0.5 on the complex pair. However, because of the nonlinearity of the feedback, it would appear a gain of 0.9 would provide a larger safety factor without seriously degrading the complex pair response. Also, moving the real root in further will reduce the castellation which is a further advantage. The gain should be verified in the FMES at GAC and by the hybrid at MIT.

Concluding Remarks

This memorandum has presented the results of a simplified stability analysis of P-66. It was found that with the current Apollo gains, the program is marginally stable at best, and probably has a large signal instability due to non-linearities.

A z-plane root locus was plotted and it was found that system stability margin can be increased by lowering the gain lag/TAUROD, which is an E-load parameter. The best value for this gain will be determined from simulations.

for 
Kenneth J. Cox

Enclosure

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THROTTLE LOOP EQUATION

The derivation of the ROD throttling loop is shown below. In the development, we have first defined the various quantities as given in Section 5 of the LUMINARY GSOP. The equations have then been reformed as difference equations for ease in Z-plane transformation.

From the GSOP:

$$\Delta f = (f - \tilde{f})$$

where Δf is the incremental throttle command and

$$f = m \cdot s = \text{MASS} \cdot (\text{AFCRAW} + \text{AFCNEW})$$

$$\tilde{f} = m \cdot (|AF|) + (\delta f_p)$$

where:

$$\text{AFCRAW} = \left(\frac{\dot{H}_D - \dot{H}}{\text{TAVPOD}} + g \right) / \cos \phi$$

$$\text{AFCNEW} = \frac{\text{LAG}}{\text{TAV}} \left[g / \cos \phi - |AF| - \delta f_p \right]$$

$$|AF| = | \text{PIPA READING OVER SAMPLE PERIOD} |$$

$$\delta f_p = (f - \tilde{f}) \left[\frac{t_c - t_n}{\Delta} + \frac{\tau_{EA}}{\Delta} + \frac{|f - \tilde{f}|}{2 \Delta \text{FRATE}} \right]$$

Δ = sampling period

FRATE = fixed constant

For purposes of analysis, the equations can be mathematically altered to the following form without loss of intent. We have also removed the $\cos \phi$ term since we assume a vertical vehicle; i.e., $\phi = 0$. Thus define:

$$\frac{\Delta f_n}{m} = a_n - a_{n-1} = (f - \tilde{f}) / m \quad (1)$$

$$f_{n/m} = \frac{V_D - V_n}{\tau} + g - \frac{L}{\tau} \left[\left(\frac{V_n - V_{n-1}}{\Delta} + g \right) - g + \frac{\delta f_{p,n-1}}{m} \right] \quad (2)$$

$$\tilde{f}_{n/m} = \left(\frac{V_n - V_{n-1}}{\Delta} + g \right) + \frac{\delta f_{p,n-1}}{m} \quad (3)$$

$$\delta f_{p,n} = (f - \tilde{f})_n \left[\frac{t_c - t_n}{\Delta} + \frac{\tau_{th}}{\Delta} + \frac{|f - \tilde{f}|_n}{2\Delta \text{FRATE}} \right] \quad (4)$$

$$\equiv K(f - \tilde{f})_n \quad (5)$$

We have assumed the effect of $|f - \tilde{f}|_n$ to be small compared to the first two terms as it is for small commanded thrust changes. The effect of large commands can be seen by perturbing K about the nominal value.

It is apparent from (4) and (1)

$$\frac{1}{m} \delta f_{p,n-1} = K(f - \tilde{f})_{n-1} / m \quad (6)$$

$$\equiv K(a_{n-1} - a_{n-2}) \quad (7)$$

Therefore, placing (2), (3), and (7) into (1), we obtain:

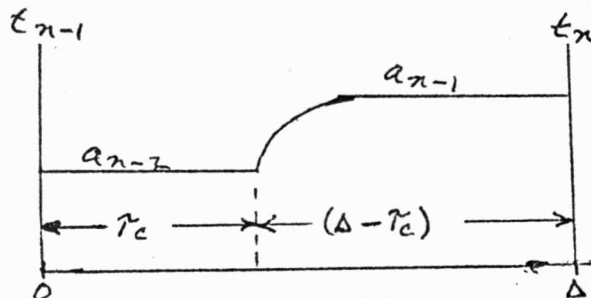
$$\begin{aligned} a_n - a_{n-1} = & \frac{V_D - V_n}{\tau} + g - \frac{L}{\tau} \left[\left(\frac{V_n - V_{n-1}}{\Delta} + g \right) - g \right. \\ & \left. + K(a_{n-1} - a_{n-2}) \right] - \left(\frac{V_n - V_{n-1}}{\Delta} + g \right) \\ & - K(a_{n-1} - a_{n-2}) \end{aligned} \quad (8)$$

Rearranging terms and solving for a_n , the commanded thrust at t_n is

681/4

$$a_n = \frac{V_D - V_n}{\tau} - \left(1 + \frac{L}{\tau}\right) \left(\frac{V_n - V_{n-1}}{\Delta}\right) - K \left(1 + \frac{L}{\tau}\right) (a_{n-1} - a_{n-2}) + a_{n-1} \quad (9)$$

The velocities V_n and V_{n-1} can be obtained by integrating the accelerations over the interval t_{n-1} to t_n . Diagrammatically, this is:



Now τ_c is the computation time, and the command a_{n-1} is through a ramp and a first order lag having a time constant τ_e . Mathematically,

$$V_n = V_{n-1} + \int_{t_{n-1}}^{t_n} a_{n-2} dt + \int_{t_{n-1}}^{t_n} (a_{n-1} - a_{n-2}) \left(1 - e^{-\frac{(t_{n-1} - \tau_c)}{\tau_e}}\right) u(t_{n-1} - \tau_c) dt \quad (10)^*$$

If the exponential and ramp are approximated, with $\tau_a = .25$ and $\tau_e = .1$

$$V_n \approx V_{n-1} + (\tau_c + 1.1\tau_e) a_{n-2} \Delta + (\Delta - \tau_c - 1.1\tau_e) a_{n-1} \Delta \quad (11)$$

* with the ramp included

$$V_n = V_{n-1} + \int_{t_{n-1}}^{t_n} a_{n-2} dt + \int_{\tau_e}^{t_n} \mathcal{L}^{-1} \left[\frac{K}{\tau_e s^2 (s + \frac{1}{\tau_e})} \left(e^{-\tau_e s} - e^{-(\tau_e + \frac{\Delta \tau_n}{F}) s} \right) \right] dt$$

$$\Delta a_n = a_{n-1} - a_{n-2}$$

$$K = \frac{FRATE \cdot BITPERF}{MASS}$$

BITPERF = FIXED MEMORY CONSTANT

In the Z-plane, equations (9) and (11) become:

$$\hat{a} = \frac{V_D}{r} - \frac{1}{r} \hat{V} - (1 + \frac{L}{r})(1 - \frac{1}{z}) \hat{V} - K(1 + \frac{L}{r})(\frac{1}{z} - \frac{1}{z^2}) \hat{a} + \frac{1}{z} \hat{a} \quad (12)$$

$$\hat{V} = \frac{1}{z} \hat{V} + \frac{(\tau_c + 1.1\tau_e)}{z^2} \hat{a} + \frac{(\Delta - \tau_c - \tau_e 1.1)}{z} \hat{a} \quad (13)$$

These may be combined to yield:

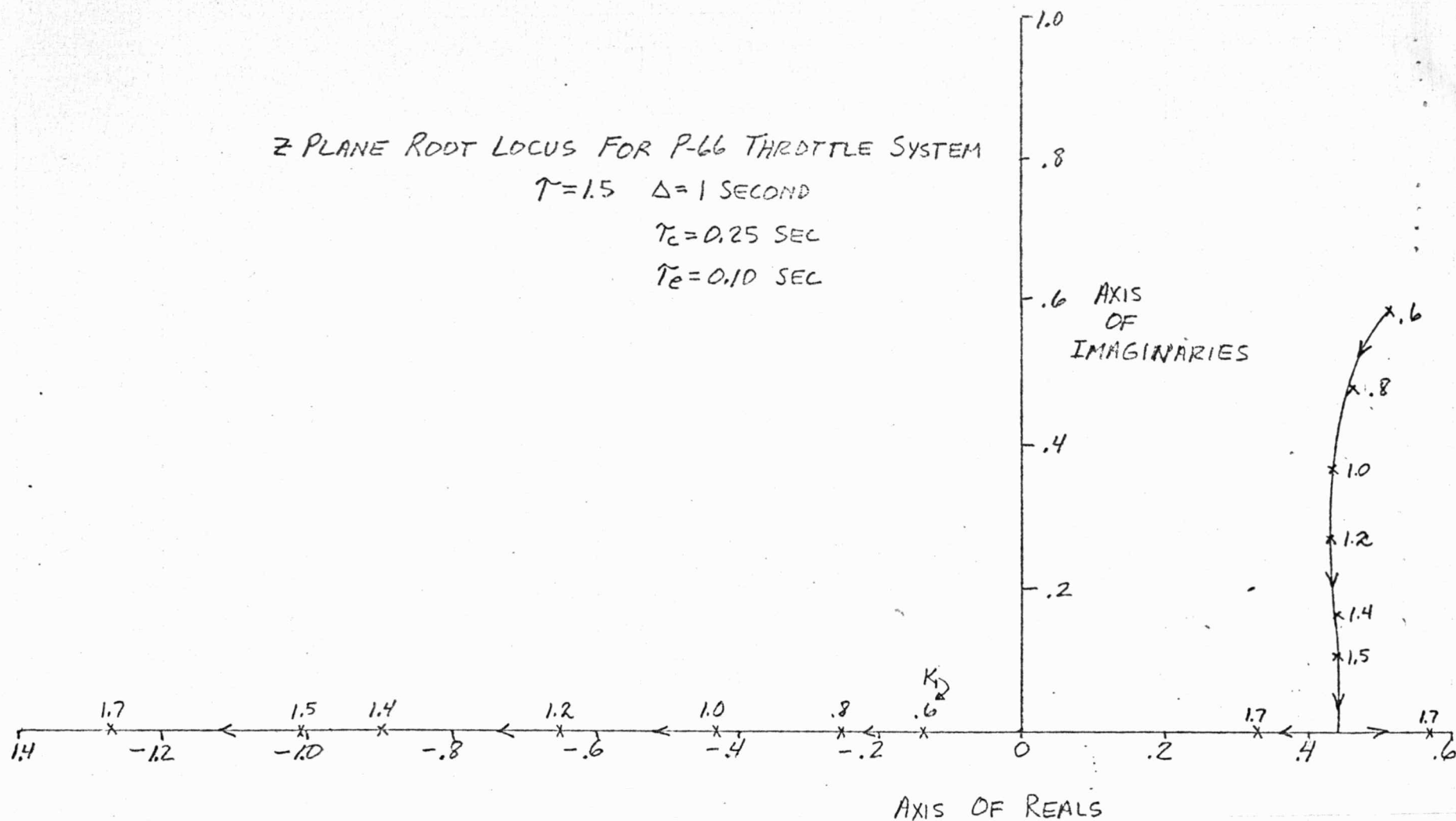
$$\left\{ z^3 + \left[K(1 + \frac{L}{r}) - z + (\Delta - \tau_c - 1.1\tau_e)(1 + \frac{L}{r} + \frac{1}{r}) \right] z^2 \right. \\ \left. \left[1 + \frac{\tau_c + 1.1\tau_e}{r} + (1 + \frac{L}{r})(2\tau_c + 2.2\tau_e - 1 - 2K) \right] z \right. \\ \left. + (K - \tau_c - 1.1\tau_e)(1 + \frac{L}{r}) \right\} \hat{a}(z) = z^2(z-1) \frac{V_D(z)}{r} \quad (14)$$

Z PLANE ROOT LOCUS FOR P-66 THROTTLE SYSTEM

$$T=1.5 \quad \Delta=1 \text{ SECOND}$$

$$\tau_c=0.25 \text{ SEC}$$

$$\tau_e=0.10 \text{ SEC}$$



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