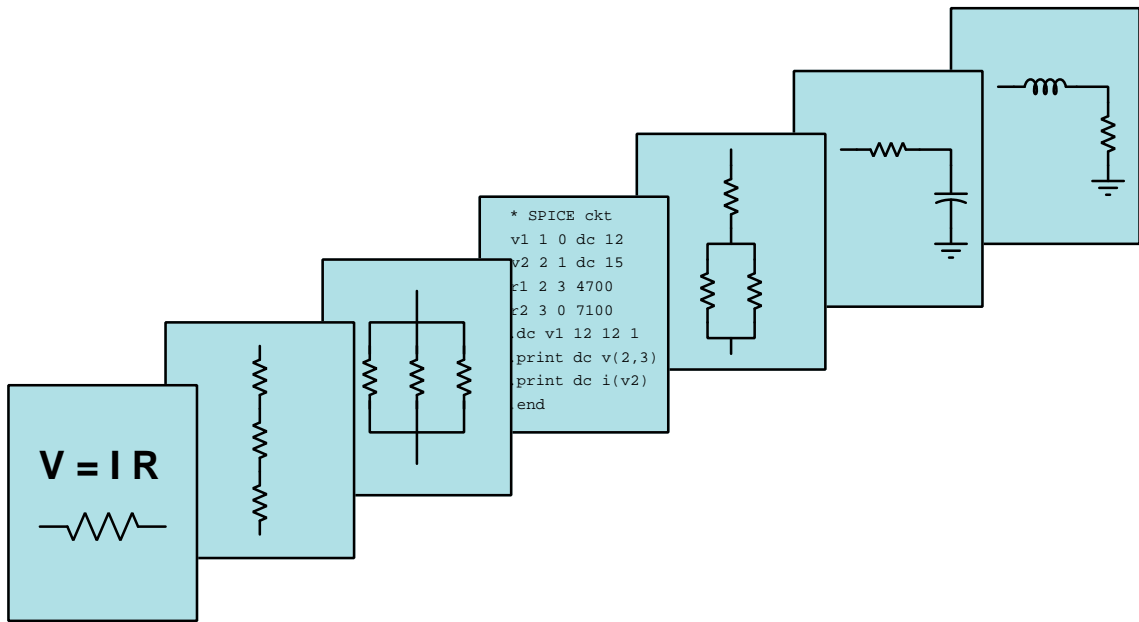


MODULAR ELECTRONICS LEARNING (MODEL) PROJECT



MANIPULATING ALGEBRAIC EQUATIONS

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Contents

1	Introduction	3
2	Case Tutorial	5
2.1	Example: manipulated equations	6
2.1.1	Parallel resistances	6
2.1.2	Voltages in a voltage divider circuit	7
2.1.3	Voltages in a series-parallel circuit	8
2.1.4	Torricelli’s Theorem	9
2.1.5	Satellite orbital velocity	9
2.1.6	Voltages in an AC resistor-capacitor circuit	10
2.1.7	Roller bearing life	11
2.1.8	Pipe size	11
2.1.9	Transformer winding turns and impedances	12
2.1.10	Ship propulsion	13
2.1.11	Broaching a hole	14
2.1.12	Brinell hardness	15
2.1.13	Binary states	15
3	Tutorial	17
3.1	What do all these letters mean?	17
3.2	The “equals” symbol	20
3.3	Addition and subtraction	23
3.4	Multiplication and division	25
3.5	Proper order of operations	28
3.6	Factoring	31
3.7	Reciprocation	32
3.8	Powers and roots	33
3.9	Exponents and logarithms	35
3.10	Substitution	40
4	Derivations and Technical References	43
4.1	Properties of real numbers	44
4.2	Dimensional analysis	45
4.3	The Ohm’s Law Circle and public education in America	46

CONTENTS	1
5 Questions	47
5.1 Conceptual reasoning	51
5.1.1 Reading outline and reflections	52
5.1.2 Foundational concepts	53
5.2 Quantitative reasoning	54
5.2.1 Miscellaneous physical constants	55
5.2.2 Introduction to spreadsheets	56
5.2.3 Solving for n	59
5.2.4 Manipulating the kinetic energy equation	60
5.3 Diagnostic reasoning	61
5.3.1 Identify the mistake, part 1	62
5.3.2 Identify the mistake, part 2	62
5.3.3 Identify the mistake, part 3	63
5.3.4 Identify the mistake, part 4	63
A Problem-Solving Strategies	65
B Instructional philosophy	67
C Tools used	73
D Creative Commons License	77
E References	85
F Version history	87
Index	88

Chapter 1

Introduction

If you poll technical college faculty about the algebraic skill they find most lacking in their new students, the answer you are likely to get more than any other is *manipulating equations*. This is arguably the biggest reason why technical students *need* to learn algebra – because equations necessary to calculate important quantities aren’t always found in the proper form – and paradoxically the point of greatest frustration for students and faculty alike. This module seeks to explain the logic and art of algebraic manipulation in a way that is both simple and practical. General principles are introduced section-by-section, with practical examples drawn from circuit analysis included for every principle.

Important concepts related to algebraic manipulation include the symbolic representation of numbers by non-numerical symbols called **variables**, the expression of general principles by **equations**, mathematics as a **language**, **subscripted** variables, mathematical **equality**, the **commutative** property of certain arithmetic operations, how equality remains unaffected by performing **identical operations** to both sides of an equation, **inverse operations**, **strategies** for algebraic manipulation, canceling terms when multiplying **fractions**, **order of operations**, **factoring**, **grouping symbols**, and **substitution**.

Here are some good questions to ask of yourself while studying this subject:

- How might you utilize an electronic calculator to perform experiments exploring inverse arithmetic operations? What hypothesis (i.e. prediction) might you pose for that experiment, and what result(s) would either support or disprove that hypothesis?
- What does it mean for two arithmetic operations to be the *inverse* of each other?
- How does a physical balance-beam help explain mathematical equality?
- What are some inverse-operation pairs in arithmetic?
- Why is it recommended to cultivate multiple methods for manipulating similar equations?
- What benefit is there in writing each and every step of an equation’s manipulation?
- What do “grouping” symbols represent in an equation?

- Why is the order of operations significant to manipulating equations?
- Where might you apply factoring when manipulating an equation?
- What does a *root symbol* (e.g. square-root) represent in mathematics?
- How do exponents relate to logarithms?
- Why do we have multiple types of logarithm functions (e.g. natural log, common log)?
- How may we derive all the forms of Joule's Law from the most basic form ($P = IV$) and Ohm's Law?

The Tutorial provides an excellent active-reading opportunity, in that you may perform all the same manipulations shown in the text. After reading how a particular manipulation was done, you may write the original equation on a separate sheet of paper and then (without referencing the text) challenge yourself to manipulate it all on your own. Then, when you think you have arrived at a solution you may refer to the text to check your answer. This requires self-discipline, as it is much easier to simply read the original explanation and tell yourself that it all makes sense, but challenging yourself in this way is vital to building independent problem-solving skill. Technical reading is not a spectator activity, but has maximum benefit to the reader who takes an active (participatory) approach.

Chapter 2

Case Tutorial

The idea behind a *Case Tutorial* is to explore new concepts by way of example. In this chapter you will read less presentation of theory compared to other Tutorial chapters, but by close observation and comparison of the given examples be able to discern patterns and principles much the same way as a scientific experimenter. Hopefully you will find these cases illuminating, and a good supplement to text-based tutorials.

These examples also serve well as challenges following your reading of the other Tutorial(s) in this module – can you explain *why* the circuits behave as they do?

2.1 Example: manipulated equations

2.1.1 Parallel resistances

The total resistance of a four-resistor parallel network is given by the following equation:

$$R_{total} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}}$$

Where,

R_{total} = Total parallel resistance (Ohms)

R_1 = Resistance of resistor #1 (Ohms)

R_2 = Resistance of resistor #2 (Ohms)

R_3 = Resistance of resistor #3 (Ohms)

R_4 = Resistance of resistor #4 (Ohms)

Equation manipulated to solve for R_1 :

$$R_1 = \frac{1}{\frac{1}{R_{total}} - \left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} \right)}$$

Equation manipulated to solve for R_2 :

$$R_2 = \frac{1}{\frac{1}{R_{total}} - \left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_4} \right)}$$

Equation manipulated to solve for R_3 :

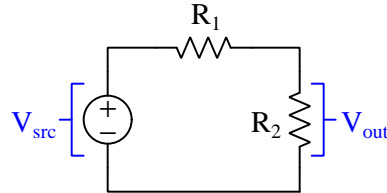
$$R_3 = \frac{1}{\frac{1}{R_{total}} - \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4} \right)}$$

Equation manipulated to solve for R_4 :

$$R_4 = \frac{1}{\frac{1}{R_{total}} - \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)}$$

2.1.2 Voltages in a voltage divider circuit

Voltages in the the following voltage divider circuit are related to each other by the equation shown below:



$$V_{out} = V_{src} \frac{R_2}{R_1 + R_2}$$

Where,

V_{src} = DC source voltage (Volts)

V_{out} = Voltage dropped across resistor R_2 (Volts)

R_1 = Resistance of resistor #1 (Ohms)

R_2 = Resistance of resistor #2 (Ohms)

Equation manipulated to solve for R_1 :

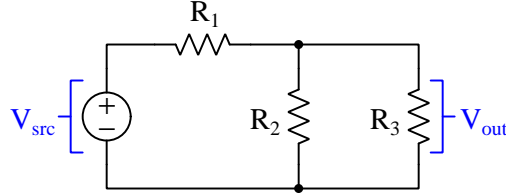
$$R_1 = \frac{V_{src} R_2}{V_{out}} - R_2$$

Equation manipulated to solve for R_2 :

$$R_2 = \frac{R_1}{\frac{V_{src}}{V_{out}} - 1}$$

2.1.3 Voltages in a series-parallel circuit

Voltages in the the following series-parallel circuit are related to each other by the equation shown below:



$$V_{out} = V_{src} \frac{\frac{R_2 R_3}{R_2 + R_3}}{R_1 + \frac{R_2 R_3}{R_2 + R_3}}$$

Where,

V_{src} = DC source voltage (Volts)

V_{out} = Voltage dropped across both resistor R_2 and resistor R_3 (Volts)

R_1 = Resistance of resistor #1 (Ohms)

R_2 = Resistance of resistor #2 (Ohms)

R_3 = Resistance of resistor #3 (Ohms)

Equation manipulated to solve for R_1 :

$$R_1 = \frac{\frac{V_{src}}{V_{out}} - 1}{\frac{1}{R_2} + \frac{1}{R_3}}$$

Equation manipulated to solve for R_2 :

$$R_2 = \frac{R_1}{\frac{V_{src}}{V_{out}} - \frac{R_1}{R_3} - 1}$$

Equation manipulated to solve for R_3 :

$$R_3 = \frac{R_1}{\frac{V_{src}}{V_{out}} - \frac{R_1}{R_2} - 1}$$

2.1.4 Torricelli's Theorem

An equation known as *Torricelli's Theorem* relates the velocity of water exiting a hole in a unpressurized container with the height of the liquid above the hole:

$$v = \sqrt{2gh}$$

Where,

v = Velocity of liquid exiting the hole

g = Acceleration of gravity (9.8 meters per second squared, or 32 feet per second squared)

h = Height of liquid column above the hole

Equation manipulated to solve for g :

$$g = \frac{v^2}{2h}$$

Equation manipulated to solve for h :

$$h = \frac{v^2}{2g}$$

2.1.5 Satellite orbital velocity

The velocity necessary for a satellite to maintain a circular orbit around Earth is given by this equation:

$$v_s = \sqrt{g_c + h}$$

Where,

v_s = Satellite velocity (feet per second)

g_c = Acceleration of Earth gravity at sea level (32 feet per second squared)

h = Orbit altitude, (feet)

Equation manipulated to solve for g_c :

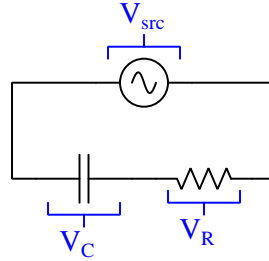
$$g_c = v_s^2 - h$$

Equation manipulated to solve for h :

$$h = v_s^2 - g_c$$

2.1.6 Voltages in an AC resistor-capacitor circuit

Voltages in the the following AC circuit are related to each other by the equation shown below:



$$V_{src} = \sqrt{V_C^2 + V_R^2}$$

Where,

V_{src} = AC source voltage (Volts)

V_C = Voltage dropped across capacitor (Volts)

V_R = Voltage dropped across resistor (Volts)

Equation manipulated to solve for V_C :

$$V_C = \sqrt{V_{src}^2 - V_R^2}$$

Equation manipulated to solve for V_R :

$$V_R = \sqrt{V_{src}^2 - V_C^2}$$

2.1.7 Roller bearing life

The expected operating life of a rolling-contact bearing may be predicted by the following equation:

$$R_L = \left(\frac{C}{L} \right)^{\frac{10}{3}}$$

Where,

R_L = Operating life (millions of shaft revolutions)

C = Dynamic capacity of bearing (pounds)

L = Radial load applied to bearing (pounds)

Equation manipulated to solve for C :

$$C = L(R_L)^{\frac{3}{10}}$$

Equation manipulated to solve for L :

$$L = \frac{C}{(R_L)^{\frac{3}{10}}}$$

2.1.8 Pipe size

Mechanical, chemical, and civil engineers must often calculate the size of piping necessary to transport fluids. One equation used to relate the water-carrying capacity of multiple, small pipes to the carrying capacity of one large pipe is as follows:

$$N = \left(\frac{d_2}{d_1} \right)^{2.5}$$

Where,

N = Number of small pipes

d_1 = Diameter of each small pipe

d_2 = Diameter of large pipe

Equation manipulated to solve for d_2 :

$$d_2 = d_1 N^{0.4}$$

Equation manipulated to solve for d_1 :

$$d_1 = \frac{d_2}{N^{0.4}}$$

2.1.9 Transformer winding turns and impedances

The ratios of electrical transformer winding turns and impedances are related to each other as such:

$$\frac{N_p}{N_s} = \sqrt{\frac{Z_p}{Z_s}}$$

Where,

N_p = Number of primary winding turns

N_s = Number of secondary winding turns

Z_p = Impedance of primary circuit

Z_s = Impedance of secondary circuit

Equation manipulated to solve for N_p :

$$N_p = N_s \sqrt{\frac{Z_p}{Z_s}}$$

Equation manipulated to solve for N_s :

$$N_s = \frac{N_p}{\sqrt{\frac{Z_p}{Z_s}}}$$

Equation manipulated to solve for Z_p :

$$Z_p = Z_s \left(\frac{N_p}{N_s} \right)^2$$

Equation manipulated to solve for Z_s :

$$Z_s = \frac{Z_p}{\left(\frac{N_p}{N_s} \right)^2}$$

2.1.10 Ship propulsion

The amount of power required to propel a ship is given by this equation:

$$P = \frac{D^{\frac{2}{3}}V^3}{K}$$

Where,

P = Power required to turn propeller(s) (horsepower)

D = Vessel displacement (long tons)

V = Velocity (nautical miles per hour)

K = Admiralty coefficient (approximately 70 for a 30 foot long ship, load waterline)

Equation manipulated to solve for D :

$$D = \left(\frac{PK}{V^3} \right)^{\frac{3}{2}}$$

Equation manipulated to solve for V :

$$V = \left(\frac{PK}{D^{\frac{2}{3}}} \right)^{\frac{1}{3}}$$

Equation manipulated to solve for K :

$$K = \frac{D^{\frac{2}{3}}V^3}{P}$$

2.1.11 Broaching a hole

In machining production technology, the time required to “broach” a hole in a plate may be estimated by the following equation:

$$T = \frac{L}{C} + \frac{L}{R_c}$$

Where,

T = Time to broach hole (minutes)

L = Length of stroke (feet)

C = Cutting speed (feet per minute)

R_c = Return speed (feet per minute)

Equation manipulated to solve for L :

$$L = \frac{TCR_c}{R_c + C}$$

Equation manipulated to solve for C :

$$C = \frac{LR_c}{TR_c - L}$$

Equation manipulated to solve for R_c :

$$R_c = \frac{LC}{TC - L}$$

2.1.12 Brinell hardness

The *Brinell hardness* of a metal specimen is measured by pressing a ball into the specimen and measuring the size of the resulting indentation. To calculate Brinell hardness, this equation is used:

$$H = \frac{F}{\left(\frac{\pi d_1}{2}\right)(d_1 - \sqrt{d_1^2 - d_s^2})}$$

Where,

H = Hardness of specimen (Brinell units)

F = Force on ball (kg)

d_1 = Diameter of ball (mm)

d_s = Diameter of indentation (mm)

Equation manipulated to solve for F :

$$F = H \left(\frac{\pi d_1}{2} \right) \left(d_1 - \sqrt{d_1^2 - d_s^2} \right)$$

Equation manipulated to solve for d_s :

$$d_s = \sqrt{d_1^2 - \left(d_1 - \frac{2F}{H\pi d_1} \right)^2}$$

2.1.13 Binary states

In digital electronic systems based on binary numeration, the number of possible *states* representable by the system is given by the following equation:

$$n_s = 2^{n_b}$$

Where,

n_s = Number of possible states

n_b = Number of binary "bits"

Equation manipulated to solve for F :

$$n_b = \frac{\log n_s}{\log 2}$$

Chapter 3

Tutorial

3.1 What do all these letters mean?

Students new to algebra face something they never encountered with arithmetic: *letters* placed together with numbers. For example, any student familiar with multiplication will immediately be able to compute the following product:

$$3 \times 5$$

Three multiplied by five is equivalent to five added to itself three times: $5 + 5 + 5$, the product being fifteen (15). However, the following algebraic expression throws many a new student into confusion:

$$n \times R$$

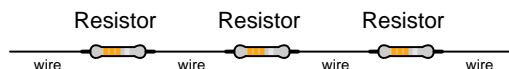
What in the world does n or R mean, any why would we be multiplying *letters* rather than *numbers*? The answer to this very good question is that these letters function as place-holders for number-values. For example, if we were told that n has a value of 3 and R has a value of 5, then $n \times R$ would be the same as saying 3×5 , the result once again being 15:

If n stands for 3 and R stands for 5 ,

Then $n \times R$ means the same as 3×5 which is 15

In other words, as place-holders for numbers n and R may be *substituted* for the actual numerical values. In a sense, we may view each of these letters – called *variables* – as a kind of alias for a number, subject to the same rules we follow when using numbers. Interesting though this may be, it is still a fair question to ask *why* we might ever want to do such a thing. After all, if 3×5 is good enough to signify the multiplication of three and five, what's the point of declaring a couple of letters (variables) to be place-holders for three and five, and then multiplying those variables? This just seems like so much pointless busy-work.

A practical example will show why variables are useful. Consider a set of electrical components called *resistors* connected in-line with one another by segments of metal wire. The purpose of a resistor is to dissipate energy from electrical charge carriers passing through it, that energy being converted into heat. Resistors may be used as electrical heaters because they become warm when electricity passes through, but they have many other uses too:



Every resistor has a *resistance* value measured in the unit of the Ohm (Ω), this value describing how effective the resistor is at extracting and dissipating energy from passing electric charge carriers. When multiple resistors are connected in-line like this (i.e. called a *series* network), the total amount of resistance from end-to-end is the sum of all the series-connected resistors.

If any series resistor network has a total resistance that is the sum of every resistance value in that network, then a series resistor network comprised of identical resistors must have a total resistance equal to the number of resistors multiplied by the resistance of any one of them. For example, if we built a network of three series-connected resistors, each resistor having 5 Ohms ($5\ \Omega$) of resistance, then the entire network would have a total resistance of $15\ \Omega$.

Here is an example of how variables may be useful. Instead of describing the electrical resistance of a series network comprised of identical resistors in verbal terms, we may simply write an algebraic *equation* or *formula* saying the same thing more efficiently:

$$\text{Total resistance} = n \times R$$

Where,

n = The number of identical series-connected resistors

R = The resistance of any one of those resistors

“*Total resistance equals $n \times R$* ” is much simpler to write and to say than “*total resistance is equal to the number of resistors multiplied by the resistance of any one of them*”. It is especially efficient if the reader happens to already know that the variable R always means “resistance” for electrical components and that n typically means some whole-numbered quantity of things.

Thus, variables give us the ability to write *general* mathematical statements applying to wide ranges of situations. The only resistor network described by the expression 3×5 is one where three $5\ \Omega$ resistors are connected in series, but $n \times R$ describes the total resistance for *any* number of series-connected resistors of *any* common value. “ 3×5 ” merely describes one particular network of resistors, but “ $n \times R$ ” is more powerful because it describes the *general principle*.

This is a very practical use of algebraic variables because it lets us easily say complicated things. In this way, mathematics plays the role of a *language* for describing principles. Like any language, algebra has rules of “grammar” we must master in order to use it as a tool of communication with other people, and we will soon see how these rules help us apply the described principles in ways that are not immediately apparent.

It is customary in algebra to use single letters (of any alphabet) as variables, and to interpret adjacent letters as being multiplied by one another. Thus, $n \times R = nR$. The use of single letters as variables is limiting, though, especially when we need to represent different quantities of the same *type*.

Suppose we wished to write an equation describing total resistance for any series resistor network, regardless of individual resistance values. In the previous example (nR) we were able to use a single R variable to represent individual resistance because all the resistors happened to be identical in value. But how may we represent multiple resistances of different resistance value when we only have one letter R in the English alphabet?

An answer to this practical problem is something called a *subscript*¹. These are additional characters written immediately below and to the right of the variable letter, to distinguish similar variables from one another. For example, if we have a three-resistor series network where each resistor has a different resistance value, we could write a total resistance equation using subscripts to keep all the different resistances distinct:

$$R_T = R_1 + R_2 + R_3$$

Where,

R_T = Total series resistance

R_1 = Resistance of resistor #1

R_2 = Resistance of resistor #2

R_3 = Resistance of resistor #3

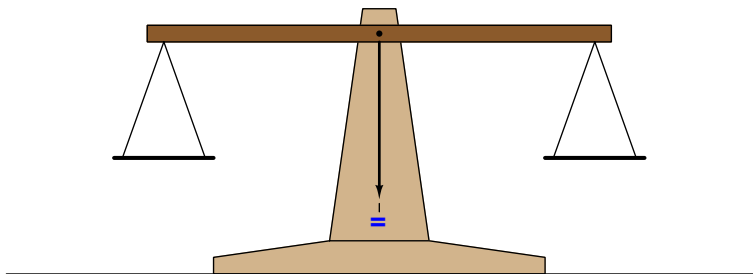
The subscript – be it a letter, number, or even a word – does not limit the numerical value of that variable in any way. It would be a mistake to think that R_3 must be larger than R_1 simply because 3 is larger than 1. These subscripts merely exist to distinguish each of the R variables in this question so we don't confuse one for another. While the letter chosen for any variable is often constrained by convention (e.g. R for electrical resistance, V for electrical voltage, I for electrical current), subscripts are entirely arbitrary.

Be aware as well that letters from non-English alphabets are fair to use as variables. This is often done in physics and engineering where the list of standardized variable letters exceeds the number of letters in the English alphabet. Greek letters are popular for standardized variables, including μ for magnetic permeability, ϵ for electric permittivity, ρ for mass density, γ for weight density, β for transistor current gain, λ for wavelength, θ for angle, ω for angular velocity, etc.

¹Other solutions to this problem exist in different mathematics-based disciplines such as computer programming. In programming, all variables must be described by plain-text characters where there is no provision for subscripts, and so the solution there is to allow variables to have multi-character *names*. For example, we might choose to create a variable named `R1` or `R_1` in lieu of R_1 to represent the amount of resistance for resistor #1 in a computer program. Similarly, we could create a variable named `Rtotal` to represent total resistance, since the inability to represent subscripts in plain-text prevents us from making a program variable called R_{total} . The disadvantage of allowing multi-character variable names is that we must give up the convention of adjacent letters implying multiplication. The algebraic statement nR means $n \times R$, but in computer programming `nR` is simply a single variable named “nR”. If we wish to instruct a computer to multiply two variables together, we must explicitly do so using a “star” character for the multiplication operator (i.e. `n * R`).

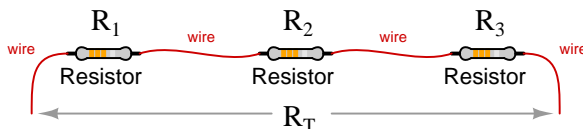
3.2 The “equals” symbol

The “equals” symbol ($=$) simply means two quantities share the same exact value, but many students find it helpful to visualize mathematical equality in physical terms. For this reason, I will relate the “equals” symbol to a balance-beam scale used to compare masses:

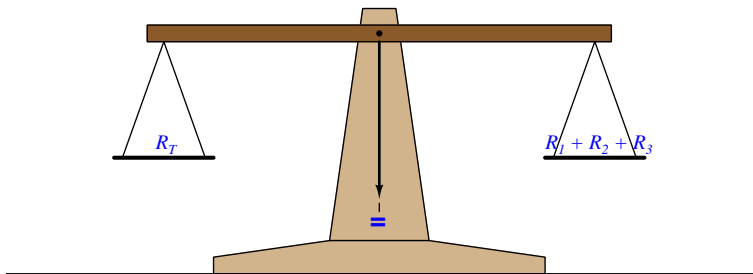


When the mass of each scale pan is equal to the other, the balance beam comes to rest in the center position, and we say that those two masses are equal to each other. As shown above with both pans empty, the scale may be said to embody the equation $0 = 0$.

Applying this concept to something previously discussed, we will show the balance beam as a metaphor for the total resistance equation of a three-resistor series network such as the three resistors shown below:

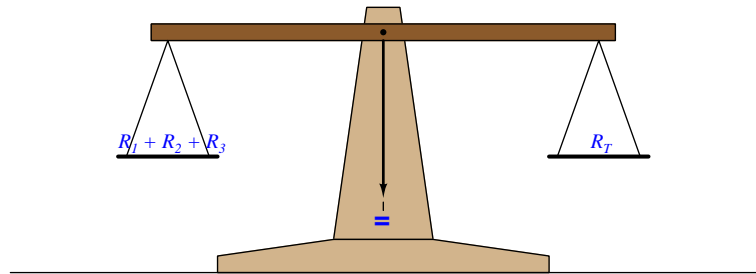


$$R_T = R_1 + R_2 + R_3$$



This equality remains just as true if we swap the written order. For example, if $R_1 + R_2 + R_3 = R_T$ then it must also be true to say $R_T = R_1 + R_2 + R_3$:

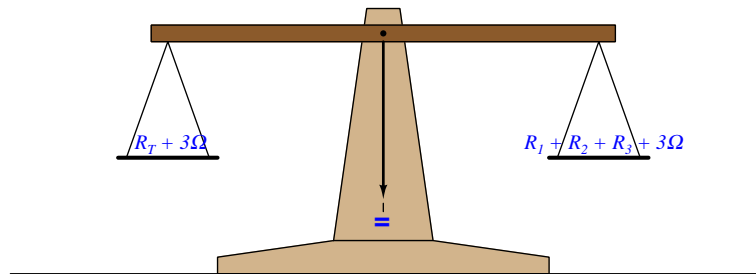
$$R_1 + R_2 + R_3 = R_T$$



A fundamental principle in algebra – which is rather obvious when related to the balance-beam analogy – is that we may arbitrarily manipulate any equation so long as we apply the exact same manipulation(s) to the terms on *both* sides of the “equals” symbol. If we imagine a balance-beam scale in a perfectly balanced condition, we ought to be able to add identical masses to both pans and still see the beam settle at a perfect balance condition.

For example, if we take our series resistor network equation and add 3 Ohms of resistance to both sides, it must remain balanced. This manipulation of the equation – so long as it is done *exactly the same* to both sides of it – will not invalidate the truth of that equation:

$$R_T + 3\Omega = R_1 + R_2 + R_3 + 3\Omega$$



It isn't just addition that works this way. It's possible for us to apply *any* mathematical operation to both sides of an equation and have that equation still remain true.

Consider the following possibilities of applying identical mathematical operations to both sides of this equation, where x represents any real-number quantity:

$$R_T - x = R_1 + R_2 + R_3 - x \quad (\text{Subtracting } x \text{ from both sides})$$

$$xR_T = x(R_1 + R_2 + R_3) \quad (\text{Multiplying both sides by } x)$$

$$\frac{R_T}{x} = \frac{R_1 + R_2 + R_3}{x} \quad (\text{Dividing both sides by } x)$$

$$-R_T = -(R_1 + R_2 + R_3) \quad (\text{Negating both sides})$$

$$\frac{1}{R_T} = \frac{1}{R_1 + R_2 + R_3} \quad (\text{Reciprocating both sides})$$

$$R_T^x = (R_1 + R_2 + R_3)^x \quad (\text{Raising both sides to the power of } x)$$

$$\sqrt{R_T} = \sqrt{R_1 + R_2 + R_3} \quad (\text{Taking the square-root of both sides})$$

$$x^{R_T} = x^{R_1 + R_2 + R_3} \quad (\text{Elevating both sides as powers of } x)$$

$$\log R_T = \log(R_1 + R_2 + R_3) \quad (\text{Taking the logarithm of both sides})$$

$$\sin R_T = \sin(R_1 + R_2 + R_3) \quad (\text{Applying trigonometric functions to both sides})$$

$$\frac{d}{dx} R_T = \frac{d}{dx} (R_1 + R_2 + R_3) \quad (\text{Differentiating both sides with respect to } x)$$

$$\int R_T dx = \int (R_1 + R_2 + R_3) dx \quad (\text{Integrating both sides with respect to } x)$$

Appealing to the balance-beam analogy once more, if we begin with a balance-beam containing precisely equal masses, even if those masses are comprised of physically different substances or different configurations of the same substance, we ought to be able to add, subtract, multiply, divide, etc. both of those masses identically and still end up with the system being in perfect balance.

As irresistible as this logic may seem, it quite possibly strikes you as being useless. So what if we can identically manipulate both sides of an equation and have it remain equal? Who cares? Of what possible use is this in any practical circumstance? If all of these examples seem as pointless to you as lettered variables once were, read on!

3.3 Addition and subtraction

The fact that we may freely manipulate algebraic equations so long as we apply the same operations equally to both sides turns out to be an incredibly useful mathematical tool for re-writing equations in different forms. Let us return to our three-resistor series resistance equation once more:

$$R_T = R_1 + R_2 + R_3$$

This equation is written to be useful if we happen to know the resistance value of each individual resistor (R_1 , R_2 , and R_3) and we wish to calculate the total resistance of the network (R_T), but what if our practical problem were different? Suppose we needed to build a three-resistor network with a certain total resistance (R_T) and had to use two other resistors of pre-determined value (R_1 and R_2), and were left to calculate the necessary value of the third resistor (R_3)? What we need is an equation written with R_3 all by itself on one side and all the other resistances (R_T , R_1 , and R_2) on the other side.

Knowing we may subtract any quantity we want from any equation so long as we subtract equally from both sides is the key to solving this problem. What we may do is subtract R_1 and R_2 from both sides, one at a time, and see the equation take a different form:

$$R_T = R_1 + R_2 + R_3 \quad (\text{Equation in its original form})$$

$$R_T - R_1 = R_1 + R_2 + R_3 - R_1 \quad (\text{After subtracting } R_1 \text{ from both sides})$$

$$R_T - R_1 = R_2 + R_3 \quad (\text{After letting } R_1 - R_1 \text{ cancel to a difference of zero})$$

$$R_T - R_1 - R_2 = R_2 + R_3 - R_2 \quad (\text{After subtracting } R_2 \text{ from both sides})$$

$$R_T - R_1 - R_2 = R_3 \quad (\text{After letting } R_2 - R_2 \text{ cancel to a difference of zero})$$

Now we have what we want: an equation solved for R_3 . This tells us we need to take the total desired resistance value (R_T), subtract the known resistance values of R_1 and R_2 , and what remains must be the necessary resistance value of R_3 .

This example was performed in very explicit, step-by-step fashion for the purpose of avoiding assumptions. Usually when a person applies such manipulation to an equation they will do so in fewer steps:

$$R_T = R_1 + R_2 + R_3 \quad (\text{Equation in its original form})$$

$$R_T - (R_1 + R_2) = R_1 + R_2 + R_3 - (R_1 + R_2) \quad (\text{After subtracting } (R_1 + R_2) \text{ from both sides})$$

$$R_T - (R_1 + R_2) = R_3 \quad (\text{After letting } R_1 + R_2 - (R_1 + R_2) \text{ cancel to a difference of zero})$$

The result is the same, but with fewer steps than before.

We see how we may exploit the principle of identically manipulating both sides of an equation to change its form into something more useful to our immediate needs, but it is clear that this exploitation requires some strategic thinking. We must somehow examine the equation in its original form, and then decide which manipulations will re-format the equation to our liking.

Arithmetic operations are reversible, which means certain operations “un-do” other operations. Subtraction, for example, un-does addition. Division un-does multiplication. Square-rooting un-does squaring. These operation-pairs are called *inverse operations*, and we must know what they are in order to judiciously choose operations that will manipulate equations in the way we would prefer.

You will notice that we chose to *subtract* R_1 and R_2 from both sides of the series resistance equation to isolate R_3 because in its original form the equation showed R_1 and R_2 being *added* to R_3 . Subtraction “un-did” that addition, and allowed us to “cancel” R_1 and R_2 from the right-hand side of the equation to leave R_3 by itself.

Now we will return to our previous problem of solving for R_3 in the series resistance equation, in order to show an alternative method of solution. You will find there is usually more than one correct method for manipulating an equation to your liking, and that it is good training for your mind to practice multiple techniques rather than settle on a favorite. Observe the following sequence of steps, where our first manipulation will be to subtract R_3 from both sides rather than subtract R_1 and R_2 :

$$R_T = R_1 + R_2 + R_3 \quad (\text{Equation in its original form})$$

$$R_T - R_3 = R_1 + R_2 + R_3 - R_3 \quad (\text{After subtracting } R_3 \text{ from both sides})$$

$$R_T - R_3 = R_1 + R_2 \quad (\text{After letting } R_3 - R_3 \text{ cancel to a difference of zero})$$

$$R_T - R_3 - R_T = R_1 + R_2 - R_T \quad (\text{After subtracting } R_T \text{ from both sides})$$

$$-R_3 = R_1 + R_2 - R_T \quad (\text{After letting } R_T - R_T \text{ cancel to a difference of zero})$$

At this point we appear to be very close to our goal of solving for R_3 . However, note that we have not yet isolated R_3 , but rather have isolated $-R_3$. If we add R_1 and R_2 then subtract R_T , the result will be the resistance of R_3 expressed as a *negative* quantity rather than a *positive* quantity.

Something we may do next is *reverse the sign* of both sides of the equation, like this:

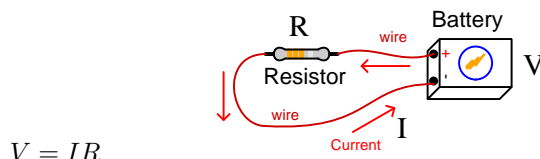
$$-(-R_3) = -(R_1 + R_2 - R_T) \quad (\text{After reversing the sign on both sides})$$

$$R_3 = -(R_1 + R_2 - R_T) \quad (\text{After replacing } -(-R_3) \text{ with } R_3)$$

Although this result appears different from what we obtained by subtracting R_1 and R_2 in our first step, it is equally valid.

3.4 Multiplication and division

Many equations used to describe the behavior of electricity involve multiplication and division. Perhaps the most familiar is *Ohm's Law*, relating voltage (V), current (I), and resistance (R) for any component or network of components:



Where,

V = Voltage (i.e. energy gained/lost by electric charge carriers)

I = Current (i.e. the rate at which electric charge carriers drift)

R = Resistance (i.e. voltage drop per drift rate)

The equation $V = IR$ works quite well to compute voltage if I already know the values of current and resistance. However, what if I happen to know the amount of voltage (V) across an electrical resistance (R) and wish to calculate the amount of current (I) through that resistance? I must manipulate $V = IR$ to solve for I :

$$V = IR \quad (\text{Equation in its original form})$$

$$\frac{V}{R} = \frac{IR}{R} \quad (\text{After dividing both sides by } R)$$

$$\frac{V}{R} = I \quad (\text{After letting } \frac{R}{R} \text{ cancel to a quotient of one})$$

Division was the correct operation to apply to both sides of the equation because this is what was necessary to “un-do” the *multiplication* existing between I and R in the original form. Like subtraction and addition are operationally inverse to each other, so are division and multiplication.

The same strategy works to solve for R , in case you happen to already know the values of V and I and need to calculate resistance from them:

$$V = IR \quad (\text{Equation in its original form})$$

$$\frac{V}{I} = \frac{IR}{I} \quad (\text{After dividing both sides by } I)$$

$$\frac{V}{I} = R \quad (\text{After letting } \frac{I}{I} \text{ cancel to a quotient of one})$$

Of course, we should be able to apply the same principle to manipulate Ohm's Law starting from any of its forms. For example, let's try to manipulate $I = \frac{V}{R}$ to solve for R . From our previous work we already know the result should be $R = \frac{V}{I}$, but practicing algebraic manipulation is good mental exercise and so we will do it:

$$I = \frac{V}{R} \quad (\text{Equation in its original form})$$

$$IR = \frac{VR}{R} \quad (\text{After multiplying both sides by } R)$$

$$IR = V \quad (\text{After letting } \frac{R}{R} \text{ cancel to a quotient of one})$$

$$\frac{IR}{I} = \frac{V}{I} \quad (\text{After dividing both sides by } I)$$

$$R = \frac{V}{I} \quad (\text{After letting } \frac{I}{I} \text{ cancel to a quotient of one})$$

An alternative solution exists, but it requires *reciprocation* of the equation:

$$I = \frac{V}{R} \quad (\text{Equation in its original form})$$

$$\frac{I}{V} = \frac{V}{RV} \quad (\text{After dividing both sides by } V)$$

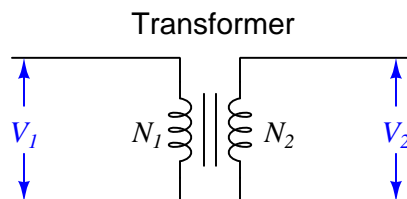
$$\frac{I}{V} = \frac{1}{R} \quad (\text{After letting } \frac{V}{V} \text{ cancel to a quotient of one})$$

$$\frac{V}{I} = \frac{R}{1} \quad (\text{After reciprocating both sides})$$

$$\frac{V}{I} = R \quad (\text{After simplifying } \frac{R}{1} \text{ to just } R)$$

A *very* common mistake committed by students new to algebraic manipulation is failing to realize $\frac{V}{RV}$ simplifies to $\frac{1}{R}$ and not to R .

Fractions tend to confuse many students, and so it is worth taking the time to explore manipulation of equations containing them. Consider the following equation relating the ratio (fraction) of voltages in a transformer to the number of turns in each of the transformer's wire coils:



$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

Suppose we wished to solve for N_1 in this equation. This is possible by multiplying each side by N_2 , but I find it makes more sense to represent this multiplying quantity as a fraction of its own, $\frac{N_2}{1}$:

$$\left(\frac{N_2}{1}\right) \left(\frac{V_1}{V_2}\right) = \left(\frac{N_1}{N_2}\right) \left(\frac{N_2}{1}\right)$$

When we do this, N_2 cancels out of the right-hand side of the equation, leaving us with N_1 by itself on that side which is what we wanted:

$$\frac{N_2 V_1}{V_2} = N_1$$

Suppose, though, we wished to solve for N_2 instead of N_1 . It's possible to do so by taking the last version of this equation (solved for N_1) and multiplying both sides of it by some quantity that will move the $\frac{V_1}{V_2}$ fraction to the right-hand side of the equation. That quantity will be the fraction $\frac{V_2}{V_1}$, designed to cancel out both V_1 and V_2 from the left-hand side:

$$\left(\frac{V_2}{V_1}\right) \left(\frac{N_2 V_1}{V_2}\right) = N_1 \left(\frac{V_2}{V_1}\right)$$

$$N_2 = \frac{N_1 V_2}{V_1}$$

With these examples we see the strategy of multiplication by strategically-chosen fractions in order to cancel out any variable(s) we wish to eliminate from one side of an equation. At the beginning of this section we saw how multiplication may be “un-done” by division, but when the equation in question contains fractions, dividing by more fractions tends to confuse matters by introducing *compound fractions* (i.e. a fraction comprised of one or more other fractions). If instead we *multiply* fractions (as shown here) rather than divide, we generally avoid the creation of these cumbersome compound fractions.

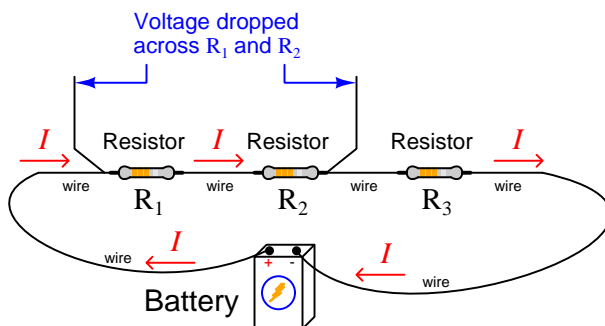
3.5 Proper order of operations

An important concept in arithmetic is *proper order of operations*, referring to the precedence some operations have over others². For example, the correct equivalence for the expression $4 + 3 \times 2$ is 10 rather than 14, because multiplication takes precedence over addition. Many students learn the mnemonic *PEMDAS* to help them remember proper order of operations:

1. **P**arentheses (and other grouping symbols)
2. **E**xponents and roots
3. **M**ultiplication and **D**ivision
4. **A**ddition and **S**ubtraction

When an equation contains more than one of the operation types listed by PEMDAS, proper order of operations becomes important to achieve the correct result. The same is true when manipulating equations containing more than one PEMDAS operation type: if we manipulate the equation in the wrong order, we will not obtain a correct result.

For example, consider the following circuit consisting of three resistors connected in series to a battery, while measuring the voltage across the span of resistors R_1 and R_2 . Electric current (I) of known magnitude circulates clockwise in the circuit:



Suppose we knew the desired voltage between the labeled points in this circuit, and we also knew the value of the current flowing through the resistors, and we also knew the resistance of R_1 , but we did not know the resistance of R_2 . The equation solving for voltage across both R_1 and R_2 with a known current is³:

$$V = I(R_1 + R_2)$$

²It is worth noting that there is no logical reason why some operations must take precedence over others. Just like the arbitrary conventions of any particular spoken language, PEMDAS is purely a mathematical convention.

³This equation comes from a recognition that series resistances *add*, and from Ohm's Law which tells us voltage is the product of current and resistance ($V = IR$).

Our task, then, is to manipulate this equation to solve for R_2 . Before we begin, we must determine the proper order of operations for evaluating the equation in its present form. Applying PEMDAS, we must evaluate the contents of the parentheses first (because those symbols *group* the terms R_1 and R_2 together as one), then multiply that by I . Documenting this proper order of steps:

1. Addition: $R_1 + R_2$
2. Multiplication: $I(R_1 + R_2)$

If our goal is to isolate R_2 by itself in the equation, we need to “un-do” all the operations surrounding R_2 in the equation, but we need to do so in the correct order. When manipulating an equation to isolate one variable “buried” amidst other variables and arithmetic operations, the proper order of steps must be *the reverse of the standard order of operations used to evaluate the original equation*. For this particular case it means first un-doing the multiplication with I , then un-doing the addition with R_1 . Here are the steps:

$$V = I(R_1 + R_2) \quad (\text{Equation in its original form})$$

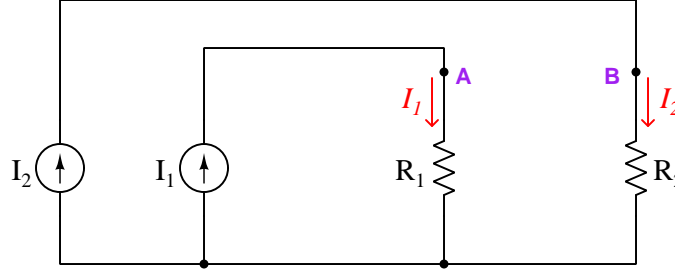
$$\frac{V}{I} = \frac{I(R_1 + R_2)}{I} \quad (\text{After dividing both sides by } I)$$

$$\frac{V}{I} = R_1 + R_2 \quad (\text{After letting } \frac{I}{I} \text{ cancel to a quotient of one})$$

$$\frac{V}{I} - R_1 = R_2 \quad (\text{After subtracting } R_1)$$

A useful analogy to help understand the importance of *order of operations* is to imagine the evaluation of an equation as being the assembly of a complex machine, where parts must be put together in a particular order. Manipulating the equation to solve for one variable is akin to removing one of the parts buried within that machine: to access that part, one must disassemble the machine in the reverse order of how it was assembled.

Here is another example of manipulating an equation with a combination of operations that must be “disassembled” in a particular order. Suppose we are analyzing the following circuit, shown in standard electrical *schematic* form rather than as a pictorial illustration:



Using two important electrical principles – *Ohm’s Law* and *Kirchhoff’s Voltage Law*⁴ – it is possible to derive an equation predicting voltage between test points A and B, that equation being $V_{AB} = (I_1 R_1) - (I_2 R_2)$. The proper order of operations for evaluating this equation appears below:

1. Multiplications: $I_1 R_1$ and $I_2 R_2$
2. Subtraction: $(I_1 R_1) - (I_2 R_2)$

Therefore, in order to isolate any of the variables on the right-hand side of the equation, we must first un-do the subtraction, and then un-do multiplication. Suppose we needed to solve for current I_2 in this equation. Shown below is one possible solution:

$$V_{AB} = (I_1 R_1) - (I_2 R_2) \quad (\text{Equation in its original form})$$

$$V_{AB} + (I_2 R_2) = (I_1 R_1) - (I_2 R_2) + (I_2 R_2) \quad (\text{After adding } I_2 R_2 \text{ to both sides})$$

$$V_{AB} + (I_2 R_2) = I_1 R_1 \quad (\text{After letting } -(I_2 R_2) + (I_2 R_2) \text{ cancel to a sum of zero})$$

$$V_{AB} + (I_2 R_2) - V_{AB} = I_1 R_1 - V_{AB} \quad (\text{After subtracting } V_{AB} \text{ from both sides})$$

$$I_2 R_2 = I_1 R_1 - V_{AB} \quad (\text{After letting } V_{AB} - V_{AB} \text{ cancel to a difference of zero})$$

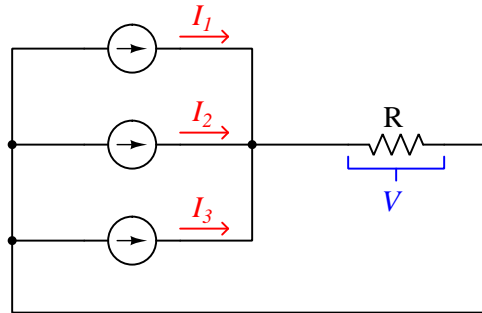
$$\frac{I_2 R_2}{R_2} = \frac{I_1 R_1 - V_{AB}}{R_2} \quad (\text{After dividing both sides by } R_2)$$

$$I_2 = \frac{I_1 R_1 - V_{AB}}{R_2} \quad (\text{After letting } \frac{R_2}{R_2} \text{ cancel to a quotient of one})$$

⁴Ohm’s Law predicts voltage across each resistance being the product of current and resistance (i.e. $V = IR$). Kirchhoff’s Voltage Law predicts the algebraic sum of all voltages in a loop must equal zero, and so the voltage between points A and B must be equal to the difference in voltages across the two resistors.

3.6 Factoring

An important mathematical tool for manipulating certain equations is *factoring*, which is where we identify and isolate one or more variables common to multiple (added) terms. To illustrate its use, we will explore a circuit multiple electric currents pass through a common resistor:



The amount of voltage across the single resistor in this circuit is given by the following equation:

$$V = I_1 R + I_2 R + I_3 R$$

Suppose we happened to know the value of all three currents in this circuit (I_1 , I_2 , I_3) as well as the desired voltage across the resistor (V), but did not know the necessary resistance value (R). Clearly, we must manipulate this equation to solve for R , but how can we isolate R when it appears *three times* in the equation rather than just once?

Factoring is key to the solution. By identifying the fact that R appears in all three terms on the right-hand side of the equation, we may re-write that side of the equation as a product between the common variable R and the sum of the three currents:

$$V = R(I_1 + I_2 + I_3)$$

If this equivalence is not clear to see, just *distribute*⁵ R to the three current terms inside of the parentheses and you will find it reverts back to the equation's original form.

Now that we have factored R out of the original three terms, solving for R is as simple as dividing both sides of the equation by $I_1 + I_2 + I_3$ to leave R by itself:

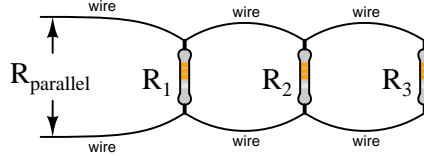
$$\frac{V}{I_1 + I_2 + I_3} = R$$

Note how the long fraction bar serves as a grouping symbol for the three current terms, just as those terms were enclosed by parentheses in the factored version of the equation. The fact that the fraction bar extends over all three current variables informs us that those three terms must be added *before* performing the division with voltage.

⁵*Distribution* is the repeated multiplication of each term inside the parentheses by the term outside the parentheses, resulting in a sum-of-products.

3.7 Reciprocation

When electrical resistors are connected together in *parallel* as shown below, the resistance of the parallel electrical network ends up being less than the lowest-valued resistance in that network. For nostalgia's sake we will return to pictorial illustrations instead of schematics:



The equation describing the network resistance of three resistors connected in parallel is ugly:

$$R_{parallel} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

For anyone new to algebraic manipulation, solving for R_2 in this equation is the stuff of nightmares. However, even an ungainly equation such as this will yield to our tool-set if we remember that it is permitted to *reciprocate* both sides of an equation.

To begin, we identify proper order-of-operation for this equation's evaluation:

1. Reciprocation: $\frac{1}{R_1}$ and $\frac{1}{R_2}$ and $\frac{1}{R_3}$
2. Addition: $\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$
3. Reciprocation: $\frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$

As a demonstration, we will now solve for R_2 :

$$R_{parallel} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \quad (\text{Equation in its original form})$$

$$\frac{1}{R_{parallel}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \quad (\text{After reciprocating both sides})$$

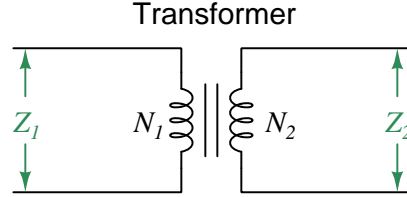
$$\frac{1}{R_{parallel}} - \frac{1}{R_1} - \frac{1}{R_3} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} - \frac{1}{R_1} - \frac{1}{R_3} \quad (\text{After subtracting } \frac{1}{R_1} \text{ and } \frac{1}{R_3} \text{ from both sides})$$

$$\frac{1}{R_{parallel}} - \frac{1}{R_1} - \frac{1}{R_3} = \frac{1}{R_2} \quad (\text{After letting } \frac{1}{R_1} - \frac{1}{R_1} \text{ and } \frac{1}{R_3} - \frac{1}{R_3} \text{ cancel to differences of zero})$$

$$\frac{1}{\frac{1}{R_{parallel}} - \frac{1}{R_1} - \frac{1}{R_3}} = R_2 \quad (\text{After reciprocating both sides})$$

3.8 Powers and roots

Powers are to roots as multiplication is to division, or addition is to subtraction: they are inverse operations, with one “un-doing” the other. One example of an electrical equation using a power is the relationship between the “turns ratio” of a *transformer* versus its “impedance ratio”. None of these quantities are important to grasp⁶ for the sake of learning how to apply manipulation techniques:



$$\frac{Z_1}{Z_2} = \left(\frac{N_1}{N_2} \right)^2$$

Suppose we needed to solve this equation for N_2 , so that we could determine the number of “turns” necessary on the right-hand side of the transformer. Assessing proper order-of-operations for evaluating the value of the impedance ratio ($\frac{Z_1}{Z_2}$) in this equation results in the following steps:

1. Division: $\frac{N_1}{N_2}$
2. Squaring: $\left(\frac{N_1}{N_2} \right)^2$

Therefore, to solve for N_2 we must un-do these operations in reverse order:

$$\frac{Z_1}{Z_2} = \left(\frac{N_1}{N_2} \right)^2 \quad (\text{Equation in its original form})$$

$$\sqrt{\frac{Z_1}{Z_2}} = \sqrt{\left(\frac{N_1}{N_2} \right)^2} \quad (\text{After square-rooting both sides})$$

$$\sqrt{\frac{Z_1}{Z_2}} = \frac{N_1}{N_2} \quad (\text{After square-rooting cancels squaring})$$

$$N_2 \sqrt{\frac{Z_1}{Z_2}} = \frac{N_1}{N_2} N_2 \quad (\text{After multiplying both sides by } N_2)$$

$$N_2 \sqrt{\frac{Z_1}{Z_2}} = N_1 \quad (\text{After letting } \frac{N_2}{N_2} \text{ cancel to a quotient of one})$$

⁶If you *must* know, a “transformer” is an electrical component made up of two or more wires coiled around a common form, the number of times each wire is wrapped around that form being called a *turn* (N). “Impedance” is very similar to the concept of resistance, and one of the properties of a transformer is that it makes the impedance of whatever it’s connected to appear to be different as sensed from the other coil, the ratio of apparent impedances being equal to the square of the ratio of turns in each coil.

$$\frac{N_2 \sqrt{\frac{Z_1}{Z_2}}}{\sqrt{\frac{Z_1}{Z_2}}} = \frac{N_1}{\sqrt{\frac{Z_1}{Z_2}}} \quad (\text{After dividing both sides by } \sqrt{\frac{Z_1}{Z_2}})$$

$$N_2 = \frac{N_1}{\sqrt{\frac{Z_1}{Z_2}}} \quad (\text{After letting } \frac{\sqrt{\frac{Z_1}{Z_2}}}{\sqrt{\frac{Z_1}{Z_2}}} \text{ cancel to a quotient of one})$$

Equations containing roots are just as easy to manipulate: we simply raise both sides of the equation to the appropriate power to cancel the root, at the appropriate step in the reverse-order-of-operations sequence. For example, let us consider the equation describing the velocity of a communications satellite orbiting the Earth:

$$v = \sqrt{g + h}$$

Where,

v = Satellite velocity

g = Acceleration of Earth gravity at sea level

h = Orbit altitude above sea level

In this equation, g is a well-known constant because it is fixed by the mass and radius from the center of the Earth at sea level – only v and h are truly variable. Thus, it makes sense that the practical manipulated form of this equation will be one where we solve for h .

Square-root symbols act as *grouping symbols* just like parentheses⁷. Therefore, proper order of operations requires we first add g and h before computing the square-root. Thus, solving for h demands we first un-do the square-root:

$$v = \sqrt{g + h} \quad (\text{Equation in its original form})$$

$$v^2 = \left(\sqrt{g + h} \right)^2 \quad (\text{After squaring both sides})$$

$$v^2 = g + h \quad (\text{After letting the square cancel the square-root})$$

$$v^2 - g = g + h - g \quad (\text{After subtracting } g \text{ from both sides})$$

$$v^2 - g = h \quad (\text{After letting } g - g \text{ cancel to a difference of zero})$$

⁷Another example of a mathematical operation which serves to group terms together is the *fraction bar*. For example, in the fraction $\frac{x+y+5}{z-8}$ the “bar” serves to group $x + y + 5$ together and also to group $z - 8$ together.

3.9 Exponents and logarithms

Exponents are to logarithms as powers are to roots, multiplication is to division, and addition is to subtraction. Once again, these are inverse operations. As such each one may be strategically applied to “un-do” the other in an equation to help solve for variables.

Logarithms tend to be confusing for many students, and so we will explore what logarithms mean before we try to apply them in equation manipulation. First, consider what it means to raise a number such as ten (10) to successively increasing powers:

- $10^{-3} = \frac{1}{1000} = 0.001$
- $10^{-2} = \frac{1}{100} = 0.01$
- $10^{-1} = \frac{1}{10} = 0.1$
- $10^0 = 1$
- $10^1 = 10$
- $10^2 = 100$
- $10^3 = 1000$

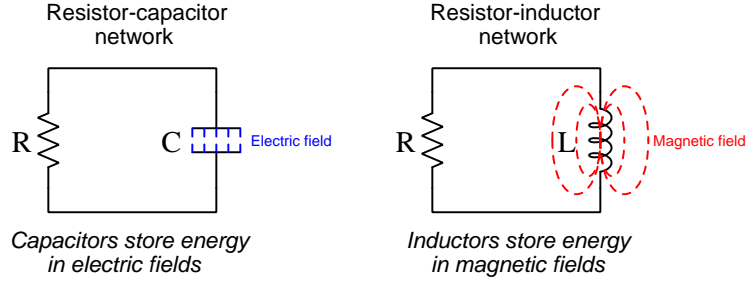
The exponent value simply declares how many times 10 is to be multiplied by itself. Negative exponents refer to reciprocated quantities, and an exponent of zero simply means 1.

A *logarithm* is the converse of an exponent: take the logarithm of a number, and it returns the exponent value you would need as a power of ten to give you that number. The following list illustrates this principle using the same numbers listed previously:

- $\log 0.001 = -3$
- $\log 0.01 = -2$
- $\log 0.1 = -1$
- $\log 1 = 0$
- $\log 10 = 1$
- $\log 100 = 2$
- $\log 1000 = 3$

The logarithms shown here are called *common logarithms* because they assume a “base” value of ten (i.e. $10^{\log x} = x$). Logarithms based on Euler’s constant (e) are called *natural logarithms*, and are written as \ln instead of \log (i.e. $e^{\ln x} = x$).

Exponential functions based on e and their corresponding natural logarithms are widely applied in physical sciences, electricity and electronics included. A prime example of this is inverse-exponential growth and decay in resistor-capacitor and resistor-inductor circuits, examples of each shown below in schematic form:



Capacitors consist of two conductive plates separated by an insulating material, with the electric field lines extending between the two plates. When “charged” with energy, capacitors manifest a voltage between their plates. If connected to a resistor, a “charged” capacitor gradually delivers its stored energy to the resistor which in turn dissipates it in the form of heat. Inductors consist of coils of wire, with magnetic field lines looping outside and through the center of the coil. When “charged” with energy, inductors manifest current through their wire coils. If connected to a resistor, a “charged” inductor gradually delivers its stored energy to the resistor which in turn dissipates it in the form of heat.

In either case, the transfer of energy from the capacitor or inductor to the resistor starts rapidly and then slows to a trickle over time, as described by these exponential equations:

$$V_C = V_0 e^{-\frac{t}{\tau}} \quad I_L = I_0 e^{-\frac{t}{\tau}}$$

Where,

V_C = Capacitor voltage at time t ; I_L = Inductor current at time t

V_0 = Initial capacitor voltage at time $t = 0$; I_0 = Initial inductor current at time $t = 0$

t = Time

τ = The “time constant” of the R-C or L-R network, $\tau = RC$ or $\tau = \frac{L}{R}$, respectively

Proper order-of-operations to evaluate either of these equations is as follows:

1. Divide: $\frac{t}{\tau}$
2. Negate: $-\frac{t}{\tau}$
3. Exponentiate: $e^{-\frac{t}{\tau}}$
4. Multiply: $V_0 e^{-\frac{t}{\tau}}$ or $I_0 e^{-\frac{t}{\tau}}$

As usual, if we wish to isolate any of the variables on the right-hand side of the equation, we will need to un-do each of these operations in reverse order.

Suppose we wished to manipulate the capacitor's inverse-exponential equation to solve for time (t). A practical application would be to determine how long it would take for the capacitor's voltage to decay given the initial voltage value, capacitance value, and resistance value for the network:

$$V_C = V_0 e^{-\frac{t}{\tau}} \quad (\text{Equation in its original form})$$

$$\frac{V_C}{V_0} = \frac{V_0 e^{-\frac{t}{\tau}}}{V_0} \quad (\text{After dividing both sides by } V_0)$$

$$\frac{V_C}{V_0} = e^{-\frac{t}{\tau}} \quad (\text{After letting } \frac{V_0}{V_0} \text{ cancel to a quotient of one})$$

$$\ln \frac{V_C}{V_0} = \ln \left(e^{-\frac{t}{\tau}} \right) \quad (\text{After taking the natural logarithm of both sides})$$

$$\ln \frac{V_C}{V_0} = -\frac{t}{\tau} \quad (\text{After letting the natural logarithm cancel the exponential})$$

$$-\tau \ln \frac{V_C}{V_0} = \left(-\frac{t}{\tau} \right) (-\tau) \quad (\text{After multiplying both sides by } -\tau)$$

$$-\tau \ln \frac{V_C}{V_0} = t \quad (\text{After letting } \left(-\frac{1}{\tau} \right) (-\tau) \text{ cancel to a product of one})$$

The procedure is similar when solving for τ . Consider this next example, using the resistor-inductor decay equation:

$$I_L = I_0 e^{-\frac{t}{\tau}} \quad (\text{Equation in its original form})$$

$$\frac{I_L}{I_0} = \frac{I_0 e^{-\frac{t}{\tau}}}{I_0} \quad (\text{After dividing both sides by } I_0)$$

$$\frac{I_L}{I_0} = e^{-\frac{t}{\tau}} \quad (\text{After letting } \frac{I_0}{I_0} \text{ cancel to a quotient of one})$$

$$\ln \frac{I_L}{I_0} = \ln \left(e^{-\frac{t}{\tau}} \right) \quad (\text{After taking the natural logarithm of both sides})$$

$$\ln \frac{I_L}{I_0} = -\frac{t}{\tau} \quad (\text{After letting the natural logarithm cancel the exponential})$$

$$\tau \ln \frac{I_L}{I_0} = \left(-\frac{t}{\tau} \right) (\tau) \quad (\text{After multiplying both sides by } \tau)$$

$$\tau \ln \frac{I_L}{I_0} = -t \quad (\text{After letting } \frac{\tau}{\tau} \text{ cancel to a quotient of one})$$

$$\frac{\tau \ln \frac{I_L}{I_0}}{\ln \frac{I_L}{I_0}} = \frac{-t}{\ln \frac{I_L}{I_0}} \quad (\text{After dividing both sides by } \ln \frac{I_L}{I_0})$$

$$\tau = \frac{-t}{\ln \frac{I_L}{I_0}} \quad (\text{After letting } \frac{\ln \frac{I_L}{I_0}}{\ln \frac{I_L}{I_0}} \text{ cancel to a quotient of one})$$

A different application of logarithms and exponents in electrical circuits is the use of *decibels*. This is a fundamentally logarithmic means for expressing ratios of power values at different points in a circuit. It is frequently used in radio communication circuit calculations because it's an easy way to represent large gains and losses in signal power over a relatively narrow range of numerical values⁸.

The equation relating a decibel (dB) figure to a power ratio is as follows:

$$\text{dB} = 10 \log \frac{P_1}{P_2}$$

Correct order-of-operations for evaluating this expression is shown here:

1. Divide: $\frac{P_1}{P_2}$
2. Logarithm: $\log \frac{P_1}{P_2}$
3. Multiplication: $10 \log \frac{P_1}{P_2}$

Supposing we wished to solve for P_2 given known values for decibels (dB) and the other power (P_1), we could manipulate this equation using the following steps:

$$\text{dB} = 10 \log \frac{P_1}{P_2} \quad (\text{Equation in its original form})$$

$$\frac{\text{dB}}{10} = \log \frac{P_1}{P_2} \quad (\text{After dividing both sides by } 10)$$

$$10^{\frac{\text{dB}}{10}} = 10^{\log \frac{P_1}{P_2}} \quad (\text{After exponentiating both sides using } 10 \text{ as the base})$$

$$10^{\frac{\text{dB}}{10}} = \frac{P_1}{P_2} \quad (\text{After letting the exponent and logarithm cancel})$$

$$P_2 10^{\frac{\text{dB}}{10}} = \frac{P_1}{P_2} P_2 \quad (\text{After multiplying both sides by } P_2)$$

$$P_2 10^{\frac{\text{dB}}{10}} = P_1 \quad (\text{After letting } \frac{P_2}{P_2} \text{ cancel to a quotient of one})$$

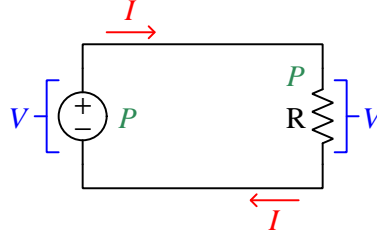
$$\frac{P_2 10^{\frac{\text{dB}}{10}}}{10^{\frac{\text{dB}}{10}}} = \frac{P_1}{10^{\frac{\text{dB}}{10}}} \quad (\text{After dividing both sides by } 10^{\frac{\text{dB}}{10}})$$

⁸This is the same advantage of the Richter scale used to describe earthquake energy, also a logarithmic system.

$$P_2 = \frac{P_1}{10^{\frac{\text{dB}}{10}}} \quad (\text{After letting } \frac{10^{\frac{\text{dB}}{10}}}{10^{\frac{\text{dB}}{10}}} \text{ cancel to a quotient of one})$$

3.10 Substitution

So far the algebraic manipulation techniques we've seen have been applied to a single equation at a time. Another powerful technique called *substitution* allows us to combine multiple equations to create wholly new equations useful for practical purposes. Our illustrative application will be a simple source and resistor circuit, shown below:



With this application we introduce a new electrical variable: *power* (P). This is a measure of energy transfer rate, measured in Joules per second, otherwise known as *Watts*.

Ohm's Law, which we have already seen, relates voltage and current for a resistance ($V = IR$). A new principle called *Joule's Law* relates power, voltage, and current:

$$P = IV$$

Each of these formulae are useful, but we may find ourselves in a situation where we know values of some of the variables from each, but not enough to use either equation by itself. For example, if we happened to know the value of R and the value of V in this circuit and wished to calculate power (P), neither the Ohm's Law nor Joule's Law equations as given to us here will suffice on their own. Of course, we could manipulate Ohm's Law to solve for current ($I = V \text{ over } R$) and then take that computed current value and multiply by the known voltage to arrive at power ($P = IV$), but it would be ideal if we just had one equation with power as a function of voltage and resistance.

Algebraic substitution lets us create new equations out of two or more other equations. Taking the Ohm's Law and Joule's Law equations just mentioned and placing them side-by-side:

$$I = \frac{V}{R} \quad P = IV$$

If we know from Ohm's Law that current is truly equal to voltage divided by resistance ($I = \frac{V}{R}$), then we may *substitute* $\frac{V}{R}$ in place of I wherever we see I in the Joule's Law equation. Showing this substitution below:

$$P = \left(\frac{V}{R} \right) V$$

Combining the two V variables to simplify this equation yields our desired result:

$$P = \frac{V^2}{R}$$

We may illustrate the practicality of substitution once more by creating an equation solving for power as a function of *current* and resistance. The key is to identify which variable(s) in Joule's Law we do *not* want as dependent variables for power, then substitute the variable(s) we wish to have in their place. Starting with the two simple forms of Ohm's Law and Joule's Law:

$$V = IR \qquad P = IV$$

If our goal is to have a power equation in terms of I and R , we must substitute some function of I and R for V in the original Joule's Law equation. As it turns out, Ohm's Law as given to us here is perfectly suited for that role, serving as a definition of V in terms of I and R . Substituting IR for V in Joule's Law:

$$P = I(IR)$$

Re-writing this to combine the two I variables:

$$P = I^2 R$$

Similar substitutions may be made to create equations solving for V , I , and R in terms of P . This is left as an exercise to the reader⁹.

⁹Hint: there are multiple ways to arrive at such equations. One way is to apply substitution once more to Ohm's and Joule's Laws, this time substituting parts of Joule's Law into Ohm's Law. Another way is to manipulate the two hybrid equations ($P = \frac{V^2}{R}$ and $P = I^2 R$) to solve for V , I , or R .

Chapter 4

Derivations and Technical References

This chapter is where you will find mathematical derivations too detailed to include in the tutorial, and/or tables and other technical reference material.

4.1 Properties of real numbers

The following equations are basic algebraic properties: rules that all real numbers adhere to.

Associative property:

$$a + (b + c) = (a + b) + c$$

$$a(bc) = (ab)c$$

Commutative property:

$$a + b = b + a$$

$$ab = ba$$

Distributive property:

$$a(b + c) = ab + bc$$

Properties of exponents:

$$a^x a^y = a^{x+y}$$

$$\frac{a^x}{a^y} = a^{x-y}$$

$$(ab)^x = a^x b^x$$

$$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$$

$$(a^x)^y = a^{xy}$$

Properties of roots:

$$(\sqrt[x]{a})^x = a$$

$$\sqrt[x]{a^x} = a \text{ if } a \geq 0$$

$$\sqrt[x]{ab} = \sqrt[x]{a} \sqrt[x]{b}$$

$$\sqrt[x]{\frac{a}{b}} = \frac{\sqrt[x]{a}}{\sqrt[x]{b}}$$

4.2 Dimensional analysis

A powerful technique called *dimensional analysis* helps us check for errors when evaluating formulae related to physical systems. This entails writing units of measurement in place of variables in physical formulae, to see how units cancel and combine to form the appropriate unit(s) of measurement for the result.

For example, let's take the familiar power formula used to calculate power in a simple DC electric circuit:

$$P = IV$$

Where,

P = Power (Watts)

I = Current (Amperes)

V = Voltage (Volts)

Each of the units of measurement in the above formula (Watt, Ampere, Volt) are actually comprised of more fundamental physical units. One Watt of power is one Joule of energy exchanged per second of time. One Ampere of current is one Coulomb of electric charge moving per second. One Volt of potential is one Joule of energy per Coulomb of electric charge. When we write the equation showing these units in their proper orientations, we see that the result (power in Watts, or Joules per second) actually does agree with the units for Amperes and Volts because the unit of electric charge (Coulombs) cancels out. In dimensional analysis we customarily distinguish unit symbols from variables by using non-italicized letters and surrounding each one with square brackets:

$$P = IV$$

$$[\text{Watts}] = [\text{Amperes}] \times [\text{Volts}] \quad \text{or} \quad [\text{W}] = [\text{A}][\text{V}]$$

$$\left[\frac{\text{Joules}}{\text{seconds}} \right] = \left[\frac{\text{Coulombs}}{\text{seconds}} \right] \times \left[\frac{\text{Joules}}{\text{Coulombs}} \right] \quad \text{or} \quad \left[\frac{\text{J}}{\text{s}} \right] = \left[\frac{\text{C}}{\text{s}} \right] \left[\frac{\text{J}}{\text{C}} \right]$$

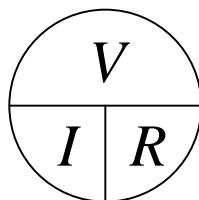
Dimensional analysis gives us a way to “check our work” when setting up new formulae for physics- and chemistry-type problems.

4.3 The Ohm's Law Circle and public education in America

In the United States of America it is widely understood that our public primary and secondary schools routinely graduate students who are barely (or not even) literate in either or both grammatical and mathematical senses of the word. Accounts of entry-level college students who cannot read effectively, cannot write a coherent paragraph, and cannot apply elementary principles of algebra are too numerous to mention, and college instructors everywhere lament this sad fact. To be fair, I am not saying *all* or even *most* public high school graduates are woefully incompetent, only enough to pose a major problem for colleges and employers of high school graduates¹.

As evidence of this fact, I present to you the “Ohm’s Law Circle” commonly seen on posters in electrical and electronic theory classrooms across the nation:

Ohm's Law "Circle"



This is a crutch for students who never learned elementary algebra, and it works like this:

- To solve for V , cover up that variable with your thumb and look at what remains: IR
- To solve for I , cover up that variable with your thumb and look at what remains: $\frac{V}{R}$
- To solve for R , cover up that variable with your thumb and look at what remains: $\frac{V}{I}$

Closely related to this is another variant of the “Circle” showing each and every variation of Ohm’s and Joule’s Laws solved for every variable in terms of all the others. All you have to do is find the manipulated version of the equation you’re looking for, no thumbs required.

As you can tell from reading the Tutorial contained in this module, the task of algebraically manipulating $V = IR$ is not difficult to learn. In fact, in over twenty years of teaching I have yet to encounter a student who cannot learn the elementary algebra necessary to solve for I or R given $V = IR$. In most cases this requires no more than a single day’s worth of instruction and practice. Yet, print shops continue to churn out posters of the “Circle” and instructors continue to hang them on their classroom walls, encouraging their students to hobble along with this crutch instead of learning the rudimentary algebra necessary to navigate Ohm’s Law without the aid of such gimmicks.

Attention educators: do not cripple your students by absolving them of the responsibility to *reason*. Throw out your Ohm’s Law Circle posters and *teach them to think through the math*.

¹Of course, this poses problems for society extending well beyond college and entry-level employment. For instance, a nation will struggle to maintain a functioning democracy if large portions of its electorate lack the literacy necessary to read and digest complex issues for themselves.

Chapter 5

Questions

This learning module, along with all others in the ModEL collection, is designed to be used in an inverted instructional environment where students independently read¹ the tutorials and attempt to answer questions on their own *prior* to the instructor's interaction with them. In place of lecture², the instructor engages with students in Socratic-style dialogue, probing and challenging their understanding of the subject matter through inquiry.

Answers are not provided for questions within this chapter, and this is by design. Solved problems may be found in the Tutorial and Derivation chapters, instead. The goal here is *independence*, and this requires students to be challenged in ways where others cannot think for them. Remember that you always have the tools of *experimentation* and *computer simulation* (e.g. SPICE) to explore concepts!

The following lists contain ideas for Socratic-style questions and challenges. Upon inspection, one will notice a strong theme of *metacognition* within these statements: they are designed to foster a regular habit of examining one's own thoughts as a means toward clearer thinking. As such these sample questions are useful both for instructor-led discussions as well as for self-study.

¹Technical reading is an essential academic skill for any technical practitioner to possess for the simple reason that the most comprehensive, accurate, and useful information to be found for developing technical competence is in textual form. Technical careers in general are characterized by the need for continuous learning to remain current with standards and technology, and therefore any technical practitioner who cannot read well is handicapped in their professional development. An excellent resource for educators on improving students' reading prowess through intentional effort and strategy is the book *Reading For Understanding – How Reading Apprenticeship Improves Disciplinary Learning in Secondary and College Classrooms* by Ruth Schoenbach, Cynthia Greenleaf, and Lynn Murphy.

²Lecture is popular as a teaching method because it is easy to implement: any reasonably articulate subject matter expert can talk to students, even with little preparation. However, it is also quite problematic. A good lecture always makes complicated concepts seem easier than they are, which is bad for students because it instills a false sense of confidence in their own understanding; reading and re-articulation requires more cognitive effort and serves to verify comprehension. A culture of teaching-by-lecture fosters a debilitating dependence upon direct personal instruction, whereas the challenges of modern life demand independent and critical thought made possible only by gathering information and perspectives from afar. Information presented in a lecture is ephemeral, easily lost to failures of memory and dictation; text is forever, and may be referenced at any time.

GENERAL CHALLENGES FOLLOWING TUTORIAL READING

- Summarize as much of the text as you can in one paragraph of your own words. A helpful strategy is to explain ideas as you would for an intelligent child: as simple as you can without compromising too much accuracy.
- Simplify a particular section of the text, for example a paragraph or even a single sentence, so as to capture the same fundamental idea in fewer words.
- Where did the text make the most sense to you? What was it about the text's presentation that made it clear?
- Identify where it might be easy for someone to misunderstand the text, and explain why you think it could be confusing.
- Identify any new concept(s) presented in the text, and explain in your own words.
- Identify any familiar concept(s) such as physical laws or principles applied or referenced in the text.
- Devise a proof of concept experiment demonstrating an important principle, physical law, or technical innovation represented in the text.
- Devise an experiment to disprove a plausible misconception.
- Did the text reveal any misconceptions you might have harbored? If so, describe the misconception(s) and the reason(s) why you now know them to be incorrect.
- Describe any useful problem-solving strategies applied in the text.
- Devise a question of your own to challenge a reader's comprehension of the text.

GENERAL FOLLOW-UP CHALLENGES FOR ASSIGNED PROBLEMS

- Identify where any fundamental laws or principles apply to the solution of this problem, especially before applying any mathematical techniques.
- Devise a thought experiment to explore the characteristics of the problem scenario, applying known laws and principles to mentally model its behavior.
- Describe in detail your own strategy for solving this problem. How did you identify and organized the given information? Did you sketch any diagrams to help frame the problem?
- Is there more than one way to solve this problem? Which method seems best to you?
- Show the work you did in solving this problem, even if the solution is incomplete or incorrect.
- What would you say was the most challenging part of this problem, and why was it so?
- Was any important information missing from the problem which you had to research or recall?
- Was there any extraneous information presented within this problem? If so, what was it and why did it not matter?
- Examine someone else's solution to identify where they applied fundamental laws or principles.
- Simplify the problem from its given form and show how to solve this simpler version of it. Examples include eliminating certain variables or conditions, altering values to simpler (usually whole) numbers, applying a limiting case (i.e. altering a variable to some extreme or ultimate value).
- For quantitative problems, identify the real-world meaning of all intermediate calculations: their units of measurement, where they fit into the scenario at hand. Annotate any diagrams or illustrations with these calculated values.
- For quantitative problems, try approaching it qualitatively instead, thinking in terms of “increase” and “decrease” rather than definite values.
- For qualitative problems, try approaching it quantitatively instead, proposing simple numerical values for the variables.
- Were there any assumptions you made while solving this problem? Would your solution change if one of those assumptions were altered?
- Identify where it would be easy for someone to go astray in attempting to solve this problem.
- Formulate your own problem based on what you learned solving this one.

GENERAL FOLLOW-UP CHALLENGES FOR EXPERIMENTS OR PROJECTS

- In what way(s) was this experiment or project easy to complete?
- Identify some of the challenges you faced in completing this experiment or project.

- Show how thorough documentation assisted in the completion of this experiment or project.
- Which fundamental laws or principles are key to this system's function?
- Identify any way(s) in which one might obtain false or otherwise misleading measurements from test equipment in this system.
- What will happen if (component X) fails (open/shorted/etc.)?
- What would have to occur to make this system unsafe?

5.1 Conceptual reasoning

These questions are designed to stimulate your analytic and synthetic thinking³. In a Socratic discussion with your instructor, the goal is for these questions to prompt an extended dialogue where assumptions are revealed, conclusions are tested, and understanding is sharpened. Your instructor may also pose additional questions based on those assigned, in order to further probe and refine your conceptual understanding.

Questions that follow are presented to challenge and probe your understanding of various concepts presented in the tutorial. These questions are intended to serve as a guide for the Socratic dialogue between yourself and the instructor. Your instructor's task is to ensure you have a sound grasp of these concepts, and the questions contained in this document are merely a means to this end. Your instructor may, at his or her discretion, alter or substitute questions for the benefit of tailoring the discussion to each student's needs. The only absolute requirement is that each student is challenged and assessed at a level equal to or greater than that represented by the documented questions.

It is far more important that you convey your *reasoning* than it is to simply convey a correct answer. For this reason, you should refrain from researching other information sources to answer questions. What matters here is that *you* are doing the thinking. If the answer is incorrect, your instructor will work with you to correct it through proper reasoning. A correct answer without an adequate explanation of how you derived that answer is unacceptable, as it does not aid the learning or assessment process.

You will note a conspicuous lack of answers given for these conceptual questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your conceptual answers, where applicable, is to use circuit simulation software to explore the effects of changes made to circuits. For example, if one of these conceptual questions challenges you to predict the effects of altering some component parameter in a circuit, you may check the validity of your work by simulating that same parameter change within software and seeing if the results agree.

³*Analytical* thinking involves the “disassembly” of an idea into its constituent parts, analogous to dissection. *Synthetic* thinking involves the “assembly” of a new idea comprised of multiple concepts, analogous to construction. Both activities are high-level cognitive skills, extremely important for effective problem-solving, necessitating frequent challenge and regular practice to fully develop.

5.1.1 Reading outline and reflections

“Reading maketh a full man; conference a ready man; and writing an exact man” – Francis Bacon

Francis Bacon’s advice is a blueprint for effective education: reading provides the learner with knowledge, writing focuses the learner’s thoughts, and critical dialogue equips the learner to confidently communicate and apply their learning. Independent acquisition and application of knowledge is a powerful skill, well worth the effort to cultivate. To this end, students should read these educational resources closely, write their own outline and reflections on the reading, and discuss in detail their findings with classmates and instructor(s). You should be able to do all of the following after reading any instructional text:

✓ Briefly **OUTLINE THE TEXT**, as though you were writing a detailed Table of Contents. Feel free to rearrange the order if it makes more sense that way. Prepare to articulate these points in detail and to answer questions from your classmates and instructor. Outlining is a good self-test of thorough reading because you cannot outline what you have not read or do not comprehend.

✓ Demonstrate **ACTIVE READING STRATEGIES**, including verbalizing your impressions as you read, simplifying long passages to convey the same ideas using fewer words, annotating text and illustrations with your own interpretations, working through mathematical examples shown in the text, cross-referencing passages with relevant illustrations and/or other passages, identifying problem-solving strategies applied by the author, etc. Technical reading is a special case of problem-solving, and so these strategies work precisely because they help solve any problem: paying attention to your own thoughts (metacognition), eliminating unnecessary complexities, identifying what makes sense, paying close attention to details, drawing connections between separated facts, and noting the successful strategies of others.

✓ Identify **IMPORTANT THEMES**, especially **GENERAL LAWS** and **PRINCIPLES**, expounded in the text and express them in the simplest of terms as though you were teaching an intelligent child. This emphasizes connections between related topics and develops your ability to communicate complex ideas to anyone.

✓ Form **YOUR OWN QUESTIONS** based on the reading, and then pose them to your instructor and classmates for their consideration. Anticipate both correct and incorrect answers, the incorrect answer(s) assuming one or more plausible misconceptions. This helps you view the subject from different perspectives to grasp it more fully.

✓ Devise **EXPERIMENTS** to test claims presented in the reading, or to disprove misconceptions. Predict possible outcomes of these experiments, and evaluate their meanings: what result(s) would confirm, and what would constitute disproof? Running mental simulations and evaluating results is essential to scientific and diagnostic reasoning.

✓ Specifically identify any points you found **CONFUSING**. The reason for doing this is to help diagnose misconceptions and overcome barriers to learning.

5.1.2 Foundational concepts

Correct analysis and diagnosis of electric circuits begins with a proper understanding of some basic concepts. The following is a list of some important concepts referenced in this module's full tutorial. Define each of them in your own words, and be prepared to illustrate each of these concepts with a description of a practical example and/or a live demonstration.

Variable

Substitution

Subscript

Equality

Commutation

Manipulating both sides of an equation

Inverse operations

Order of operations

Reverse order of operations

Factoring

Distribution

Grouping symbol

5.2 Quantitative reasoning

These questions are designed to stimulate your computational thinking. In a Socratic discussion with your instructor, the goal is for these questions to reveal your mathematical approach(es) to problem-solving so that good technique and sound reasoning may be reinforced. Your instructor may also pose additional questions based on those assigned, in order to observe your problem-solving firsthand.

Mental arithmetic and estimations are strongly encouraged for all calculations, because without these abilities you will be unable to readily detect errors caused by calculator misuse (e.g. keystroke errors).

You will note a conspicuous lack of answers given for these quantitative questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. My advice is to use circuit simulation software such as SPICE to check the correctness of quantitative answers. Refer to those learning modules within this collection focusing on SPICE to see worked examples which you may use directly as practice problems for your own study, and/or as templates you may modify to run your own analyses and generate your own practice problems.

Completely worked example problems found in the Tutorial may also serve as “test cases⁴” for gaining proficiency in the use of circuit simulation software, and then once that proficiency is gained you will never need to rely⁵ on an answer key!

⁴In other words, set up the circuit simulation software to analyze the same circuit examples found in the Tutorial. If the simulated results match the answers shown in the Tutorial, it confirms the simulation has properly run. If the simulated results disagree with the Tutorial’s answers, something has been set up incorrectly in the simulation software. Using every Tutorial as practice in this way will quickly develop proficiency in the use of circuit simulation software.

⁵This approach is perfectly in keeping with the instructional philosophy of these learning modules: *teaching students to be self-sufficient thinkers*. Answer keys can be useful, but it is even more useful to your long-term success to have a set of tools on hand for checking your own work, because once you have left school and are on your own, there will no longer be “answer keys” available for the problems you will have to solve.

5.2.1 Miscellaneous physical constants

Note: constants shown in **bold** type are *exact*, not approximations. Values inside of parentheses show one standard deviation (σ) of uncertainty in the final digits: for example, the magnetic permeability of free space value given as $1.25663706212(19) \times 10^{-6}$ H/m represents a center value (i.e. the location parameter) of $1.25663706212 \times 10^{-6}$ Henrys per meter with one standard deviation of uncertainty equal to $0.0000000000019 \times 10^{-6}$ Henrys per meter.

Avogadro's number (N_A) = **$6.02214076 \times 10^{23}$** per mole (mol^{-1})

Boltzmann's constant (k) = **1.380649×10^{-23}** Joules per Kelvin (J/K)

Electronic charge (e) = **$1.602176634 \times 10^{-19}$** Coulomb (C)

Faraday constant (F) = **$96,485.33212...$** $\times 10^4$ Coulombs per mole (C/mol)

Magnetic permeability of free space (μ_0) = $1.25663706212(19) \times 10^{-6}$ Henrys per meter (H/m)

Electric permittivity of free space (ϵ_0) = $8.8541878128(13) \times 10^{-12}$ Farads per meter (F/m)

Characteristic impedance of free space (Z_0) = $376.730313668(57)$ Ohms (Ω)

Gravitational constant (G) = $6.67430(15) \times 10^{-11}$ cubic meters per kilogram-seconds squared ($\text{m}^3/\text{kg}\cdot\text{s}^2$)

Molar gas constant (R) = **$8.314462618...$** Joules per mole-Kelvin (J/mol-K) = $0.08205746(14)$ liters-atmospheres per mole-Kelvin

Planck constant (h) = **$6.62607015 \times 10^{-34}$** joule-seconds (J-s)

Stefan-Boltzmann constant (σ) = **$5.670374419...$** $\times 10^{-8}$ Watts per square meter-Kelvin⁴ ($\text{W}/\text{m}^2\cdot\text{K}^4$)

Speed of light in a vacuum (c) = **$299,792,458$** meters per second (m/s) = 186282.4 miles per second (mi/s)

Note: All constants taken from NIST data "Fundamental Physical Constants – Complete Listing", from <http://physics.nist.gov/constants>, National Institute of Standards and Technology (NIST), 2018 CODATA Adjustment.

5.2.2 Introduction to spreadsheets

A powerful computational tool you are encouraged to use in your work is a *spreadsheet*. Available on most personal computers (e.g. Microsoft Excel), *spreadsheet* software performs numerical calculations based on number values and formulae entered into cells of a grid. This grid is typically arranged as lettered columns and numbered rows, with each cell of the grid identified by its column/row coordinates (e.g. cell B3, cell A8). Each cell may contain a string of text, a number value, or a mathematical formula. The spreadsheet automatically updates the results of all mathematical formulae whenever the entered number values are changed. This means it is possible to set up a spreadsheet to perform a series of calculations on entered data, and those calculations will be re-done by the computer any time the data points are edited in any way.

For example, the following spreadsheet calculates average speed based on entered values of distance traveled and time elapsed:

	A	B	C	D
1	Distance traveled	46.9	Kilometers	
2	Time elapsed	1.18	Hours	
3	Average speed	= B1 / B2	km/h	
4				
5				

Text labels contained in cells A1 through A3 and cells C1 through C3 exist solely for readability and are not involved in any calculations. Cell B1 contains a sample distance value while cell B2 contains a sample time value. The formula for computing speed is contained in cell B3. Note how this formula begins with an “equals” symbol (=), references the values for distance and speed by lettered column and numbered row coordinates (B1 and B2), and uses a forward slash symbol for division (/). The coordinates B1 and B2 function as *variables*⁶ would in an algebraic formula.

When this spreadsheet is executed, the numerical value 39.74576 will appear in cell B3 rather than the formula = B1 / B2, because 39.74576 is the computed speed value given 46.9 kilometers traveled over a period of 1.18 hours. If a different numerical value for distance is entered into cell B1 or a different value for time is entered into cell B2, cell B3’s value will automatically update. All you need to do is set up the given values and any formulae into the spreadsheet, and the computer will do all the calculations for you.

Cell B3 may be referenced by other formulae in the spreadsheet if desired, since it is a variable just like the given values contained in B1 and B2. This means it is possible to set up an entire chain of calculations, one dependent on the result of another, in order to arrive at a final value. The arrangement of the given data and formulae need not follow any pattern on the grid, which means you may place them anywhere.

⁶Spreadsheets may also provide means to attach text labels to cells for use as variable names (Microsoft Excel simply calls these labels “names”), but for simple spreadsheets such as those shown here it’s usually easier just to use the standard coordinate naming for each cell.

Common⁷ arithmetic operations available for your use in a spreadsheet include the following:

- Addition (+)
- Subtraction (-)
- Multiplication (*)
- Division (/)
- Powers (^)
- Square roots (sqrt())
- Logarithms (ln() , log10())

Parentheses may be used to ensure⁸ proper order of operations within a complex formula. Consider this example of a spreadsheet implementing the *quadratic formula*, used to solve for roots of a polynomial expression in the form of $ax^2 + bx + c$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

	A	B
1	x_1	= (-B4 + sqrt((B4^2) - (4*B3*B5))) / (2*B3)
2	x_2	= (-B4 - sqrt((B4^2) - (4*B3*B5))) / (2*B3)
3	a =	9
4	b =	5
5	c =	-2

This example is configured to compute roots⁹ of the polynomial $9x^2 + 5x - 2$ because the values of 9, 5, and -2 have been inserted into cells B3, B4, and B5, respectively. Once this spreadsheet has been built, though, it may be used to calculate the roots of *any* second-degree polynomial expression simply by entering the new a , b , and c coefficients into cells B3 through B5. The numerical values appearing in cells B1 and B2 will be automatically updated by the computer immediately following any changes made to the coefficients.

⁷Modern spreadsheet software offers a bewildering array of mathematical functions you may use in your computations. I recommend you consult the documentation for your particular spreadsheet for information on operations other than those listed here.

⁸Spreadsheet programs, like text-based programming languages, are designed to follow standard order of operations by default. However, my personal preference is to use parentheses even where strictly unnecessary just to make it clear to any other person viewing the formula what the intended order of operations is.

⁹Reviewing some algebra here, a *root* is a value for x that yields an overall value of zero for the polynomial. For this polynomial ($9x^2 + 5x - 2$) the two roots happen to be $x = 0.269381$ and $x = -0.82494$, with these values displayed in cells B1 and B2, respectively upon execution of the spreadsheet.

Alternatively, one could break up the long quadratic formula into smaller pieces like this:

$$y = \sqrt{b^2 - 4ac} \quad z = 2a$$

$$x = \frac{-b \pm y}{z}$$

	A	B	C
1	x_1	= (-B4 + C1) / C2	= sqrt ((B4^2) - (4*B3*B5))
2	x_2	= (-B4 - C1) / C2	= 2*B3
3	a =	9	
4	b =	5	
5	c =	-2	

Note how the square-root term (y) is calculated in cell C1, and the denominator term (z) in cell C2. This makes the two final formulae (in cells B1 and B2) simpler to interpret. The positioning of all these cells on the grid is completely arbitrary¹⁰ – all that matters is that they properly reference each other in the formulae.

Spreadsheets are particularly useful for situations where the same set of calculations representing a circuit or other system must be repeated for different initial conditions. The power of a spreadsheet is that it automates what would otherwise be a tedious set of calculations. One specific application of this is to simulate the effects of various components within a circuit failing with abnormal values (e.g. a shorted resistor simulated by making its value nearly zero; an open resistor simulated by making its value extremely large). Another application is analyzing the behavior of a circuit design given new components that are out of specification, and/or aging components experiencing drift over time.

¹⁰My personal preference is to locate all the “given” data in the upper-left cells of the spreadsheet grid (each data point flanked by a sensible name in the cell to the left and units of measurement in the cell to the right as illustrated in the first distance/time spreadsheet example), sometimes coloring them in order to clearly distinguish which cells contain entered data versus which cells contain computed results from formulae. I like to place all formulae in cells below the given data, and try to arrange them in logical order so that anyone examining my spreadsheet will be able to figure out *how* I constructed a solution. This is a general principle I believe all computer programmers should follow: *document and arrange your code to make it easy for other people to learn from it.*

5.2.3 Solving for n

Solve for n in the following equations:

Equation 1: $-56 = -14n$

Equation 2: $54 - n = 10$

Equation 3: $\frac{4}{n} = 12$

Equation 4: $28 = 2 - n$

Equation 5: $\frac{n}{3} = 12$

Equation 6: $n^3 = 50$

Equation 7: $\frac{10}{n-4} = 5$

Equation 8: $\frac{3n}{11} = 6$

Equation 9: $\sqrt{\frac{10-n}{2}} = 4$

Equation 10: $5n + 8 - 2n = 10$

Next, describe a strategy to *test* your answers to make sure they are correct.

Challenges

- For each of these problems, show more than one procedure for manipulating the equation to arrive at the same answer.

5.2.4 Manipulating the kinetic energy equation

A common equation used in physics relates the kinetic energy, velocity, and mass of a moving object:

$$E_k = \frac{1}{2}mv^2$$

Where,

E_k = Kinetic energy (Joules)

m = Mass (kilograms)

v = Velocity (meters per second)

Manipulate this equation as many times as necessary to express it in terms of all its variables.

Challenges

- Explain how kinetic energy relates to but is distinct from *potential energy*.

5.3 Diagnostic reasoning

These questions are designed to stimulate your deductive and inductive thinking, where you must apply general principles to specific scenarios (deductive) and also derive conclusions about the failed circuit from specific details (inductive). In a Socratic discussion with your instructor, the goal is for these questions to reinforce your recall and use of general circuit principles and also challenge your ability to integrate multiple symptoms into a sensible explanation of what's wrong in a circuit. Your instructor may also pose additional questions based on those assigned, in order to further challenge and sharpen your diagnostic abilities.

As always, your goal is to fully *explain* your analysis of each problem. Simply obtaining a correct answer is not good enough – you must also demonstrate sound reasoning in order to successfully complete the assignment. Your instructor's responsibility is to probe and challenge your understanding of the relevant principles and analytical processes in order to ensure you have a strong foundation upon which to build further understanding.

You will note a conspicuous lack of answers given for these diagnostic questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your diagnostic answers, where applicable, is to use circuit simulation software to explore the effects of faults placed in circuits. For example, if one of these diagnostic questions requires that you predict the effect of an open or a short in a circuit, you may check the validity of your work by simulating that same fault (substituting a very high resistance in place of that component for an open, and substituting a very low resistance for a short) within software and seeing if the results agree.

5.3.1 Identify the mistake, part 1

In this series of algebraic steps there is a **mistake**. Find the mistake and correct it (as well as all subsequent steps):

$$5a - 6ac = 1 - 3a^2b$$

$$\frac{5a - 6ac}{a} = \frac{1 - 3a^2b}{a}$$

$$5 - 6c = 1 - 3ab$$

$$3ab + 5 - 6c = 1$$

Challenges

- Explain why this mistake is a common one for students new to algebraic manipulation.

5.3.2 Identify the mistake, part 2

In this series of algebraic steps there is a **mistake**. Find the mistake and correct it (as well as all subsequent steps):

$$[(xy + x)y + x]y = z$$

$$[x^2y + xy + x]y = z$$

$$x^2y^2 + xy^2 + xy = z$$

Challenges

- Explain why this mistake is a common one for students new to algebraic manipulation.

5.3.3 Identify the mistake, part 3

In this series of algebraic steps there is a **mistake**. Find the mistake and correct it (as well as all subsequent steps):

$$a^2 - 3a = 0$$

$$a^2 = 3a$$

$$a = 3$$

Note: this is a difficult problem to see. Although the answer, 3, does satisfy the original equation, there is still a mistake in the solution!

Challenges

- Demonstrate how a *division-by-zero* error is being committed in the original sequence.

5.3.4 Identify the mistake, part 4

In this series of algebraic steps there is a **mistake**. Find the mistake and correct it (as well as all subsequent steps):

$$a = \frac{b}{c}$$

$$(b)(a) = \frac{b}{c}(b)$$

$$ab = \frac{1}{c}$$

$$c = \frac{1}{ab}$$

Challenges

- There is more than one sequence of steps to solve for c in this equation. Demonstrate at least two mathematically correct sequences that could be used.

Appendix A

Problem-Solving Strategies

The ability to solve complex problems is arguably one of the most valuable skills one can possess, and this skill is particularly important in any science-based discipline.

- Study principles, not procedures. Don't be satisfied with merely knowing how to compute solutions – learn *why* those solutions work.
- Identify what it is you need to solve, identify all relevant data, identify all units of measurement, identify any general principles or formulae linking the given information to the solution, and then identify any “missing pieces” to a solution. Annotate all diagrams with this data.
- Sketch a diagram to help visualize the problem. When building a real system, always devise a plan for that system and analyze its function *before* constructing it.
- Follow the units of measurement and meaning of every calculation. If you are ever performing mathematical calculations as part of a problem-solving procedure, and you find yourself unable to apply each and every intermediate result to some aspect of the problem, it means you don't understand what you are doing. Properly done, every mathematical result should have practical meaning for the problem, and not just be an abstract number. You should be able to identify the proper units of measurement for each and every calculated result, and show where that result fits into the problem.
- Perform “thought experiments” to explore the effects of different conditions for theoretical problems. When troubleshooting real systems, perform *diagnostic tests* rather than visually inspecting for faults, the best diagnostic test being the one giving you the most information about the nature and/or location of the fault with the fewest steps.
- Simplify the problem until the solution becomes obvious, and then use that obvious case as a model to follow in solving the more complex version of the problem.
- Check for exceptions to see if your solution is incorrect or incomplete. A good solution will work for *all* known conditions and criteria. A good example of this is the process of testing scientific hypotheses: the task of a scientist is not to find support for a new idea, but rather to *challenge* that new idea to see if it holds up under a battery of tests. The philosophical

principle of *reductio ad absurdum* (i.e. disproving a general idea by finding a specific case where it fails) is useful here.

- Work “backward” from a hypothetical solution to a new set of given conditions.
- Add quantities to problems that are qualitative in nature, because sometimes a little math helps illuminate the scenario.
- Sketch graphs illustrating how variables relate to each other. These may be quantitative (i.e. with realistic number values) or qualitative (i.e. simply showing increases and decreases).
- Treat quantitative problems as qualitative in order to discern the relative magnitudes and/or directions of change of the relevant variables. For example, try determining what happens if a certain variable were to increase or decrease before attempting to precisely calculate quantities: how will each of the dependent variables respond, by increasing, decreasing, or remaining the same as before?
- Consider limiting cases. This works especially well for qualitative problems where you need to determine which direction a variable will change. Take the given condition and magnify that condition to an extreme degree as a way of simplifying the direction of the system’s response.
- Check your work. This means regularly testing your conclusions to see if they make sense. This does *not* mean repeating the same steps originally used to obtain the conclusion(s), but rather to use some other means to check validity. Simply repeating procedures often leads to *repeating the same errors* if any were made, which is why alternative paths are better.

Appendix B

Instructional philosophy

“The unexamined circuit is not worth energizing” – Socrates (if he had taught electricity)

These learning modules, although useful for self-study, were designed to be used in a formal learning environment where a subject-matter expert challenges students to digest the content and exercise their critical thinking abilities in the answering of questions and in the construction and testing of working circuits.

The following principles inform the instructional and assessment philosophies embodied in these learning modules:

- The first goal of education is to enhance clear and independent thought, in order that every student reach their fullest potential in a highly complex and inter-dependent world. Robust reasoning is *always* more important than particulars of any subject matter, because its application is universal.
- Literacy is fundamental to independent learning and thought because text continues to be the most efficient way to communicate complex ideas over space and time. Those who cannot read with ease are limited in their ability to acquire knowledge and perspective.
- Articulate communication is fundamental to work that is complex and interdisciplinary.
- Faulty assumptions and poor reasoning are best corrected through challenge, not presentation. The rhetorical technique of *reductio ad absurdum* (disproving an assertion by exposing an absurdity) works well to discipline student’s minds, not only to correct the problem at hand but also to learn how to detect and correct future errors.
- Important principles should be repeatedly explored and widely applied throughout a course of study, not only to reinforce their importance and help ensure their mastery, but also to showcase the interconnectedness and utility of knowledge.

These learning modules were expressly designed to be used in an “inverted” teaching environment¹ where students first read the introductory and tutorial chapters on their own, then individually attempt to answer the questions and construct working circuits according to the experiment and project guidelines. The instructor never lectures, but instead meets regularly with each individual student to review their progress, answer questions, identify misconceptions, and challenge the student to new depths of understanding through further questioning. Regular meetings between instructor and student should resemble a Socratic² dialogue, where questions serve as scalpels to dissect topics and expose assumptions. The student passes each module only after consistently demonstrating their ability to logically analyze and correctly apply all major concepts in each question or project/experiment. The instructor must be vigilant in probing each student’s understanding to ensure they are truly *reasoning* and not just *memorizing*. This is why “Challenge” points appear throughout, as prompts for students to think deeper about topics and as starting points for instructor queries. Sometimes these challenge points require additional knowledge that hasn’t been covered in the series to answer in full. This is okay, as the major purpose of the Challenges is to stimulate analysis and synthesis on the part of each student.

The instructor must possess enough mastery of the subject matter and awareness of students’ reasoning to generate their own follow-up questions to practically any student response. Even completely correct answers given by the student should be challenged by the instructor for the purpose of having students practice articulating their thoughts and defending their reasoning. Conceptual errors committed by the student should be exposed and corrected not by direct instruction, but rather by reducing the errors to an absurdity³ through well-chosen questions and thought experiments posed by the instructor. Becoming proficient at this style of instruction requires time and dedication, but the positive effects on critical thinking for both student and instructor are spectacular.

An inspection of these learning modules reveals certain unique characteristics. One of these is a bias toward thorough explanations in the tutorial chapters. Without a live instructor to explain concepts and applications to students, the text itself must fulfill this role. This philosophy results in lengthier explanations than what you might typically find in a textbook, each step of the reasoning process fully explained, including footnotes addressing common questions and concerns students raise while learning these concepts. Each tutorial seeks to not only explain each major concept in sufficient detail, but also to explain the logic of each concept and how each may be developed

¹In a traditional teaching environment, students first encounter new information via *lecture* from an expert, and then independently apply that information via *homework*. In an “inverted” course of study, students first encounter new information via *homework*, and then independently apply that information under the scrutiny of an expert. The expert’s role in lecture is to simply *explain*, but the expert’s role in an inverted session is to *challenge*, *critique*, and if necessary *explain* where gaps in understanding still exist.

²Socrates is a figure in ancient Greek philosophy famous for his unflinching style of questioning. Although he authored no texts, he appears as a character in Plato’s many writings. The essence of Socratic philosophy is to leave no question unexamined and no point of view unchallenged. While purists may argue a topic such as electric circuits is too narrow for a true Socratic-style dialogue, I would argue that the essential thought processes involved with scientific reasoning on *any* topic are not far removed from the Socratic ideal, and that students of electricity and electronics would do very well to challenge assumptions, pose thought experiments, identify fallacies, and otherwise employ the arsenal of critical thinking skills modeled by Socrates.

³This rhetorical technique is known by the Latin phrase *reductio ad absurdum*. The concept is to expose errors by counter-example, since only one solid counter-example is necessary to disprove a universal claim. As an example of this, consider the common misconception among beginning students of electricity that voltage cannot exist without current. One way to apply *reductio ad absurdum* to this statement is to ask how much current passes through a fully-charged battery connected to nothing (i.e. a clear example of voltage existing without current).

from “first principles”. Again, this reflects the goal of developing clear and independent thought in students’ minds, by showing how clear and logical thought was used to forge each concept. Students benefit from witnessing a model of clear thinking in action, and these tutorials strive to be just that.

Another characteristic of these learning modules is a lack of step-by-step instructions in the Project and Experiment chapters. Unlike many modern workbooks and laboratory guides where step-by-step instructions are prescribed for each experiment, these modules take the approach that students must learn to closely read the tutorials and apply their own reasoning to identify the appropriate experimental steps. Sometimes these steps are plainly declared in the text, just not as a set of enumerated points. At other times certain steps are implied, an example being assumed competence in test equipment use where the student should not need to be told *again* how to use their multimeter because that was thoroughly explained in previous lessons. In some circumstances no steps are given at all, leaving the entire procedure up to the student.

This lack of prescription is not a flaw, but rather a feature. Close reading and clear thinking are foundational principles of this learning series, and in keeping with this philosophy all activities are designed to *require* those behaviors. Some students may find the lack of prescription frustrating, because it demands more from them than what their previous educational experiences required. This frustration should be interpreted as an unfamiliarity with autonomous thinking, a problem which must be corrected if the student is ever to become a self-directed learner and effective problem-solver. Ultimately, the need for students to read closely and think clearly is more important both in the near-term and far-term than any specific facet of the subject matter at hand. If a student takes longer than expected to complete a module because they are forced to outline, digest, and reason on their own, so be it. The future gains enjoyed by developing this mental discipline will be well worth the additional effort and delay.

Another feature of these learning modules is that they do not treat topics in isolation. Rather, important concepts are introduced early in the series, and appear repeatedly as stepping-stones toward other concepts in subsequent modules. This helps to avoid the “compartmentalization” of knowledge, demonstrating the inter-connectedness of concepts and simultaneously reinforcing them. Each module is fairly complete in itself, reserving the beginning of its tutorial to a review of foundational concepts.

This methodology of assigning text-based modules to students for digestion and then using Socratic dialogue to assess progress and hone students’ thinking was developed over a period of several years by the author with his Electronics and Instrumentation students at the two-year college level. While decidedly unconventional and sometimes even unsettling for students accustomed to a more passive lecture environment, this instructional philosophy has proven its ability to convey conceptual mastery, foster careful analysis, and enhance employability so much better than lecture that the author refuses to ever teach by lecture again.

Problems which often go undiagnosed in a lecture environment are laid bare in this “inverted” format where students must articulate and logically defend their reasoning. This, too, may be unsettling for students accustomed to lecture sessions where the instructor cannot tell for sure who comprehends and who does not, and this vulnerability necessitates sensitivity on the part of the “inverted” session instructor in order that students never feel discouraged by having their errors exposed. *Everyone* makes mistakes from time to time, and learning is a lifelong process! Part of the instructor’s job is to build a culture of learning among the students where errors are not seen as shameful, but rather as opportunities for progress.

To this end, instructors managing courses based on these modules should adhere to the following principles:

- Student questions are always welcome and demand thorough, honest answers. The only type of question an instructor should refuse to answer is one the student should be able to easily answer on their own. Remember, *the fundamental goal of education is for each student to learn to think clearly and independently*. This requires hard work on the part of the student, which no instructor should ever circumvent. Anything done to bypass the student's responsibility to do that hard work ultimately limits that student's potential and thereby does real harm.
- It is not only permissible, but encouraged, to answer a student's question by asking questions in return, these follow-up questions designed to guide the student to reach a correct answer through their own reasoning.
- All student answers demand to be challenged by the instructor and/or by other students. This includes both correct and incorrect answers – the goal is to practice the articulation and defense of one's own reasoning.
- No reading assignment is deemed complete unless and until the student demonstrates their ability to accurately summarize the major points in their own terms. Recitation of the original text is unacceptable. This is why every module contains an "Outline and reflections" question as well as a "Foundational concepts" question in the Conceptual reasoning section, to prompt reflective reading.
- No assigned question is deemed answered unless and until the student demonstrates their ability to consistently and correctly apply the concepts to *variations* of that question. This is why module questions typically contain multiple "Challenges" suggesting different applications of the concept(s) as well as variations on the same theme(s). Instructors are encouraged to devise as many of their own "Challenges" as they are able, in order to have a multitude of ways ready to probe students' understanding.
- No assigned experiment or project is deemed complete unless and until the student demonstrates the task in action. If this cannot be done "live" before the instructor, video-recordings showing the demonstration are acceptable. All relevant safety precautions must be followed, all test equipment must be used correctly, and the student must be able to properly explain all results. The student must also successfully answer all Challenges presented by the instructor for that experiment or project.

Students learning from these modules would do well to abide by the following principles:

- No text should be considered fully and adequately read unless and until you can express every idea *in your own words, using your own examples*.
- You should always articulate your thoughts as you read the text, noting points of agreement, confusion, and epiphanies. Feel free to print the text on paper and then write your notes in the margins. Alternatively, keep a journal for your own reflections as you read. This is truly a helpful tool when digesting complicated concepts.
- Never take the easy path of highlighting or underlining important text. Instead, *summarize* and/or *comment* on the text using your own words. This actively engages your mind, allowing you to more clearly perceive points of confusion or misunderstanding on your own.
- A very helpful strategy when learning new concepts is to place yourself in the role of a teacher, if only as a mental exercise. Either explain what you have recently learned to someone else, or at least *imagine* yourself explaining what you have learned to someone else. The simple act of having to articulate new knowledge and skill forces you to take on a different perspective, and will help reveal weaknesses in your understanding.
- Perform each and every mathematical calculation and thought experiment shown in the text on your own, referring back to the text to see that your results agree. This may seem trivial and unnecessary, but it is critically important to ensuring you actually understand what is presented, especially when the concepts at hand are complicated and easy to misunderstand. Apply this same strategy to become proficient in the use of *circuit simulation software*, checking to see if your simulated results agree with the results shown in the text.
- Above all, recognize that learning is hard work, and that a certain level of frustration is unavoidable. There are times when you will struggle to grasp some of these concepts, and that struggle is a natural thing. Take heart that it will yield with persistent and varied⁴ effort, and never give up!

Students interested in using these modules for self-study will also find them beneficial, although the onus of responsibility for thoroughly reading and answering questions will of course lie with that individual alone. If a qualified instructor is not available to challenge students, a workable alternative is for students to form study groups where they challenge⁵ one another.

To high standards of education,

Tony R. Kuphaldt

⁴As the old saying goes, “Insanity is trying the same thing over and over again, expecting different results.” If you find yourself stumped by something in the text, you should attempt a different approach. Alter the thought experiment, change the mathematical parameters, do whatever you can to see the problem in a slightly different light, and then the solution will often present itself more readily.

⁵Avoid the temptation to simply share answers with study partners, as this is really counter-productive to learning. Always bear in mind that the answer to any question is far less important in the long run than the method(s) used to obtain that answer. The goal of education is to empower one’s life through the improvement of clear and independent thought, literacy, expression, and various practical skills.

Appendix C

Tools used

I am indebted to the developers of many open-source software applications in the creation of these learning modules. The following is a list of these applications with some commentary on each.

You will notice a theme common to many of these applications: a bias toward *code*. Although I am by no means an expert programmer in any computer language, I understand and appreciate the flexibility offered by code-based applications where the user (you) enters commands into a plain ASCII text file, which the software then reads and processes to create the final output. Code-based computer applications are by their very nature *extensible*, while WYSIWYG (What You See Is What You Get) applications are generally limited to whatever user interface the developer makes for you.

The GNU/Linux computer operating system

There is so much to be said about Linus Torvalds' **Linux** and Richard Stallman's **GNU** project. First, to credit just these two individuals is to fail to do justice to the *mob* of passionate volunteers who contributed to make this amazing software a reality. I first learned of **Linux** back in 1996, and have been using this operating system on my personal computers almost exclusively since then. It is *free*, it is completely *configurable*, and it permits the continued use of highly efficient **Unix** applications and scripting languages (e.g. shell scripts, Makefiles, **sed**, **awk**) developed over many decades. **Linux** not only provided me with a powerful computing platform, but its open design served to inspire my life's work of creating open-source educational resources.

Bram Moolenaar's **Vim** text editor

Writing code for any code-based computer application requires a *text editor*, which may be thought of as a word processor strictly limited to outputting plain-ASCII text files. Many good text editors exist, and one's choice of text editor seems to be a deeply personal matter within the programming world. I prefer **Vim** because it operates very similarly to **vi** which is ubiquitous on **Unix/Linux** operating systems, and because it may be entirely operated via keyboard (i.e. no mouse required) which makes it fast to use.

Donald Knuth's \TeX typesetting system

Developed in the late 1970's and early 1980's by computer scientist extraordinaire Donald Knuth to typeset his multi-volume magnum opus *The Art of Computer Programming*, this software allows the production of formatted text for screen-viewing or paper printing, all by writing plain-text code to describe how the formatted text is supposed to appear. \TeX is not just a markup language for documents, but it is also a Turing-complete programming language in and of itself, allowing useful algorithms to be created to control the production of documents. Simply put, *\TeX is a programmer's approach to word processing*. Since \TeX is controlled by code written in a plain-text file, this means anyone may read that plain-text file to see exactly how the document was created. This openness afforded by the code-based nature of \TeX makes it relatively easy to learn how other people have created their own \TeX documents. By contrast, examining a beautiful document created in a conventional WYSIWYG word processor such as Microsoft **Word** suggests nothing to the reader about *how* that document was created, or what the user might do to create something similar. As Mr. Knuth himself once quipped, conventional word processing applications should be called WYSIAYG (What You See Is *All* You Get).

Leslie Lamport's \LaTeX extensions to \TeX

Like all true programming languages, \TeX is inherently extensible. So, years after the release of \TeX to the public, Leslie Lamport decided to create a massive extension allowing easier compilation of book-length documents. The result was \LaTeX , which is the markup language used to create all ModEL module documents. You could say that \TeX is to \LaTeX as **C** is to **C++**. This means it is permissible to use any and all \TeX commands within \LaTeX source code, and it all still works. Some of the features offered by \LaTeX that would be challenging to implement in \TeX include automatic index and table-of-content creation.

Tim Edwards' **Xcircuit** drafting program

This wonderful program is what I use to create all the schematic diagrams and illustrations (but not photographic images or mathematical plots) throughout the ModEL project. It natively outputs PostScript format which is a true vector graphic format (this is why the images do not pixellate when you zoom in for a closer view), and it is so simple to use that I have never had to read the manual! Object libraries are easy to create for **Xcircuit**, being plain-text files using PostScript programming conventions. Over the years I have collected a large set of object libraries useful for drawing electrical and electronic schematics, pictorial diagrams, and other technical illustrations.

Gimp graphic image manipulation program

Essentially an open-source clone of Adobe's **PhotoShop**, I use **Gimp** to resize, crop, and convert file formats for all of the photographic images appearing in the **Model** modules. Although **Gimp** does offer its own scripting language (called **Script-Fu**), I have never had occasion to use it. Thus, my utilization of **Gimp** to merely crop, resize, and convert graphic images is akin to using a sword to slice bread.

SPICE circuit simulation program

SPICE is to circuit analysis as **T_EX** is to document creation: it is a form of markup language designed to describe a certain object to be processed in plain-ASCII text. When the plain-text "source file" is compiled by the software, it outputs the final result. More modern circuit analysis tools certainly exist, but I prefer **SPICE** for the following reasons: it is *free*, it is *fast*, it is *reliable*, and it is a fantastic tool for *teaching* students of electricity and electronics how to write simple code. I happen to use rather old versions of **SPICE**, version 2g6 being my "go to" application when I only require text-based output. **NGSPICE** (version 26), which is based on Berkeley **SPICE** version 3f5, is used when I require graphical output for such things as time-domain waveforms and Bode plots. In all **SPICE** example netlists I strive to use coding conventions compatible with all **SPICE** versions.

Andrew D. Hwang's ePiX mathematical visualization programming library

This amazing project is a **C++** library you may link to any **C/C++** code for the purpose of generating PostScript graphic images of mathematical functions. As a completely free and open-source project, it does all the plotting I would otherwise use a Computer Algebra System (CAS) such as **Mathematica** or **Maple** to do. It should be said that **ePiX** is *not* a Computer Algebra System like **Mathematica** or **Maple**, but merely a mathematical *visualization* tool. In other words, it won't determine integrals for you (you'll have to implement that in your own **C/C++** code!), but it can graph the results, and it does so beautifully. What I really admire about **ePiX** is that it is a **C++** programming library, which means it builds on the existing power and toolset available with that programming language. Mr. Hwang could have probably developed his own stand-alone application for mathematical plotting, but by creating a **C++** library to do the same thing he accomplished something much greater.

`gnuplot` mathematical visualization software

Another open-source tool for mathematical visualization is `gnuplot`. Interestingly, this tool is *not* part of Richard Stallman’s GNU project, its name being a coincidence. For this reason the authors prefer “gnu” *not* be capitalized at all to avoid confusion. This is a much “lighter-weight” alternative to a spreadsheet for plotting tabular data, and the fact that it easily outputs directly to an X11 console or a file in a number of different graphical formats (including PostScript) is very helpful. I typically set my `gnuplot` output format to default (X11 on my Linux PC) for quick viewing while I’m developing a visualization, then switch to PostScript file export once the visual is ready to include in the document(s) I’m writing. As with my use of `Gimp` to do rudimentary image editing, my use of `gnuplot` only scratches the surface of its capabilities, but the important points are that it’s *free* and that it *works well*.

Python programming language

Both Python and C++ find extensive use in these modules as instructional aids and exercises, but I’m listing Python here as a *tool* for myself because I use it almost daily as a *calculator*. If you open a Python interpreter console and type `from math import *` you can type mathematical expressions and have it return results just as you would on a hand calculator. Complex-number (i.e. *phasor*) arithmetic is similarly supported if you include the complex-math library (`from cmath import *`). Examples of this are shown in the Programming References chapter (if included) in each module. Of course, being a fully-featured programming language, Python also supports conditionals, loops, and other structures useful for calculation of quantities. Also, running in a console environment where all entries and returned values show as text in a chronologically-ordered list makes it easy to copy-and-paste those calculations to document exactly how they were performed.

Appendix D

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Appendix E

References

Appendix F

Version history

This is a list showing all significant additions, corrections, and other edits made to this learning module. Each entry is referenced by calendar date in reverse chronological order (newest version first), which appears on the front cover of every learning module for easy reference. Any contributors to this open-source document are listed here as well.

13 June 2023 – edited some of the wording where I was using “function” as a synonym for “operation”. Also expanded on grouping symbols.

27 November 2022 – placed questions at the top of the itemized list in the Introduction chapter prompting students to devise experiments related to the tutorial content.

12-13 September 2022 – added more equations to the “Solve for n ” Quantitative Reasoning question. Also added another page to the Tutorial section on multiplication and division discussing fractions.

22 June 2022 – added more examples of algebraic manipulation (circuit-related) to the Case Tutorial chapter.

14 June 2022 – minor edits to the Tutorial section on multiplication and division, pointing out that these are inverse functions too. Also added index entries for inverse functions on several pages.

4 February 2022 – added some challenges in various questions to have students demonstrate strategies for checking their work.

30 August 2021 – minor edits to the Tutorial, among other details adding pictorial diagrams of circuits to accompany electrical formulae.

9 July 2021 – replaced some TeX-style italicizing markup with LaTeX-style.

27 January 2021 – added a Case Tutorial.

23 August 2020 – significantly edited the Introduction chapter to make it more suitable as a

pre-study guide and to provide cues useful to instructors leading “inverted” teaching sessions. Also, added questions to the Quantitative and Diagnostic reasoning sections.

1-4 March 2020 – continued writing the Tutorial.

29 February 2020 – document first created (hey, it’s a *leap year!*).

Index

- Adding quantities to a qualitative problem, 66
- Annotating diagrams, 65
- Capacitor, 36
- Carrier, electric charge, 18
- Charge carrier, 18
- Checking for exceptions, 66
- Checking your work, 66
- Code, computer, 73
- Common logarithm, 35
- Computer programming variables, 19
- Current, 19
- Decay, 36
- Dimensional analysis, 45, 65
- Edwards, Tim, 74
- Electric charge carrier, 18
- Energy, 18
- Euler's constant, 35
- Graph values to solve a problem, 66
- Greek letters, 19
- Greenleaf, Cynthia, 47
- Grouping symbol, 28, 29, 31, 34
- Growth, 36
- How to teach with these modules, 68
- Hwang, Andrew D., 75
- Identify given data, 65
- Identify relevant principles, 65
- Inductor, 36
- Instructions for projects and experiments, 69
- Intermediate results, 65
- Inverse exponential function, 36
- Inverse operations, 24, 25, 33, 35
- Inverted instruction, 68
- Joule's Law, 40
- Kirchhoff's Voltage Law, 30
- Knuth, Donald, 74
- Lamport, Leslie, 74
- Language, mathematics as, 18
- Limiting cases, 66
- Logarithm, 35
- Metacognition, 52
- Moolenaar, Bram, 73
- Murphy, Lynn, 47
- Natural logarithm, 35
- Ohm, 18
- Ohm's Law, 28, 30
- Open-source, 73
- Operations, inverse, 24, 25, 33, 35
- Order of operations, 28
- PEMDAS, 28
- Problem-solving: annotate diagrams, 65
- Problem-solving: check for exceptions, 66
- Problem-solving: checking work, 66
- Problem-solving: dimensional analysis, 65
- Problem-solving: graph values, 66
- Problem-solving: identify given data, 65
- Problem-solving: identify relevant principles, 65
- Problem-solving: interpret intermediate results, 65
- Problem-solving: limiting cases, 66
- Problem-solving: qualitative to quantitative, 66
- Problem-solving: quantitative to qualitative, 66
- Problem-solving: reductio ad absurdum, 66
- Problem-solving: simplify the system, 65
- Problem-solving: thought experiment, 65

- Problem-solving: track units of measurement, 65
- Problem-solving: visually represent the system, 65
- Problem-solving: work in reverse, 66
- Qualitatively approaching a quantitative problem, 66
- Reading Apprenticeship, 47
- Reductio ad absurdum, 66–68
- Resistance, 18
- Resistor, 18
- Richter scale, 38
- Schoenbach, Ruth, 47
- Scientific method, 52
- Series, 18
- Simplifying a system, 65
- Socrates, 67
- Socratic dialogue, 68
- SPICE, 47
- Stallman, Richard, 73
- Subscript, 19
- Substitution, 17
- Symbol, grouping, 28, 29, 31, 34
- Thought experiment, 65
- Torvalds, Linus, 73
- Transformer, 27, 33
- Units of measurement, 65
- Variable, 17
- Variables, computer programming, 19
- Visualizing a system, 65
- Voltage, 19
- Work in reverse to solve a problem, 66
- WYSIWYG, 73, 74