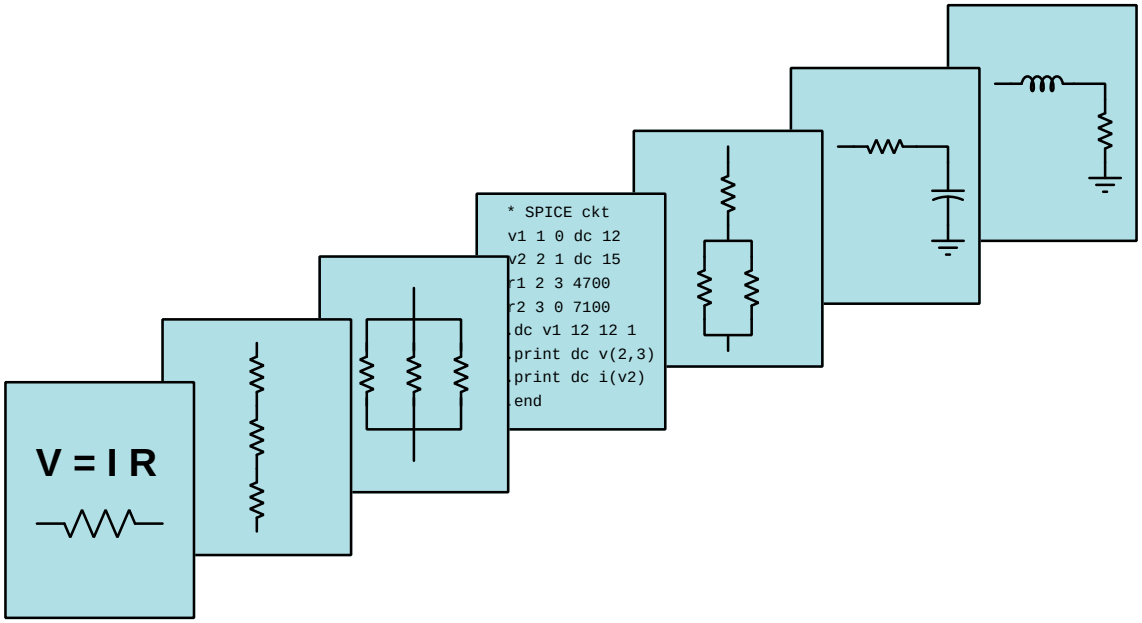
Modular Electronics Learning (ModEL) project



Closed-Loop Control

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Chapter 1

Introduction

This module introduces the field of closed-loop process control, which regards the automated control of certain "process variables" such as temperature, pressure, level, flow rate, etc. Closed-loop control actually pre-dates electronics as a profession, some of the earliest examples being mechanical *governors* used to regulate the speed of steam engines in the 18th and 19th centuries. Practical applications abound for closed-loop control, including vehicle speed regulation (i.e. "cruise control"), engine air/fuel mixture control, powerplant regulation, climate control systems for building interiors, and many, many others.

Important concepts related to closed-loop include **negative feedback**, **sensors**, **transmitters**, **control valves**, **setpoint**, **process variable**, **indicator**, **controller**, **automatic** versus **manual** mode, **process**, **P&ID**, and **loop diagram**.

Here are some good questions to ask of yourself while studying this subject:

- What does it mean if a controller is "direct" versus "reverse" acting?
- What types of details are typically documented in a P&ID?
- What types of details are typically documented in a loop diagram or loop sheet?
- Why are multiple types of diagrams generally necessary to fully document any control system?
- For what purpose might an instrument's calibrated range need to be reverse-acting?
- What is the purpose of a *transducer* in an industrial control system?
- How are instrument locations specified within P&IDs and/or loop diagrams?
- What do "zero" and "span" refer to for an instrument's calibrated range?
- What do "LRV" and "URV" refer to for an instrument's calibrated range?
- What does it mean for a compressor to *surge*, and how may we avoid this potentially destructive condition?

- How does a 4-20 mA electronic signal represent process variable information?
- How does a 3-15 PSI pneumatic signal represent process variable information?
- What does it mean if a control valve is *air-to-open* versus *air-to-close*?
- Why might it be a good idea to intentionally calibrate a process transmitter so that its LRV and URV do *not* align with the physical minimum and maximum limits of the process variable?
- Devise your own question based on the text, suitable for posing to someone encountering this subject for the first time

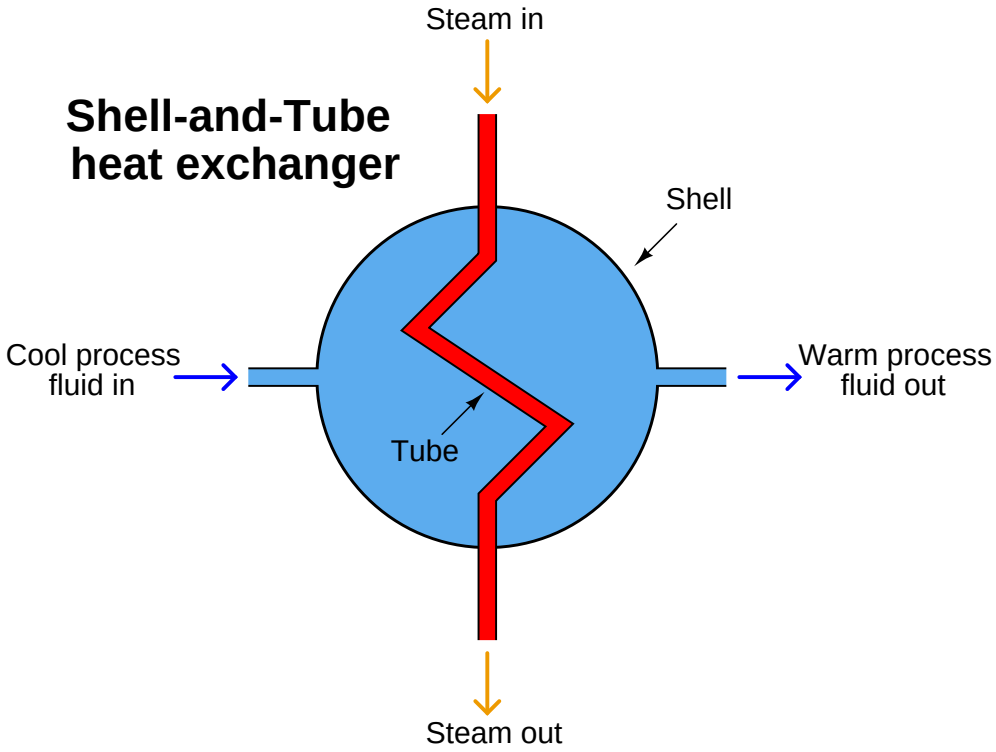
Chapter 2

Tutorial

Instrumentation is the science of automated measurement and control. Applications of this science abound in modern research, industry, and everyday living. From automobile engine control systems to home thermostats to aircraft autopilots to the manufacture of pharmaceutical drugs, automation surrounds us. This chapter explains some of the fundamental principles of automatic process control.

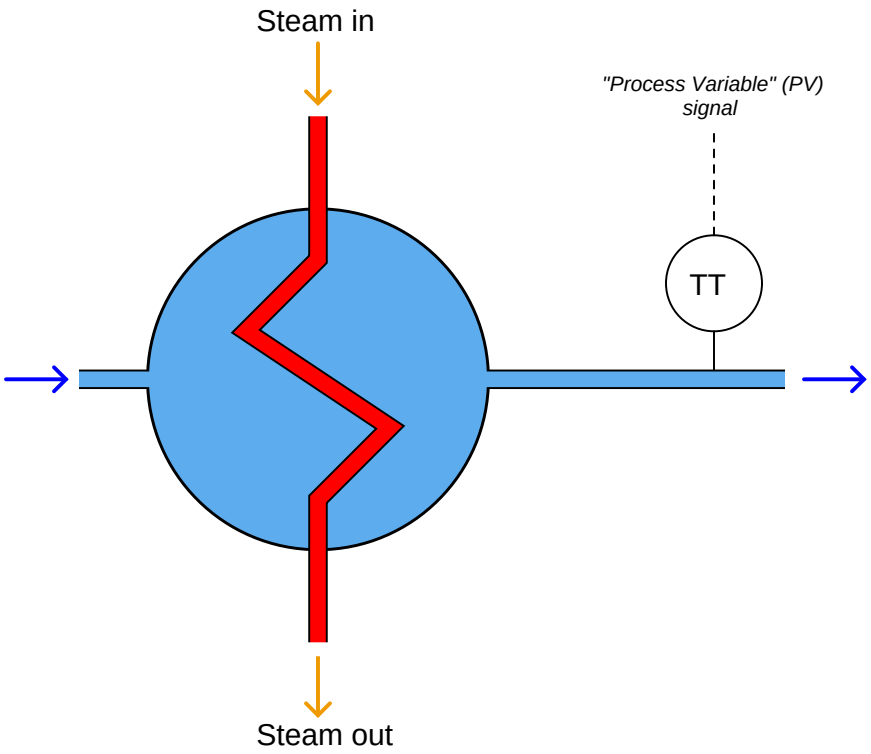
2.1 Basic feedback control principles

Before we begin our discussion on process control, we must define a few key terms. First, we have what is known as the *process*: the physical system we wish to monitor and control. For the sake of illustration, consider a heat exchanger that uses high-temperature steam to transfer heat to a lower-temperature liquid. Heat exchangers are used frequently in the chemical industries to maintain the necessary temperature of a chemical solution, so the desired blending, separation, or reactions can occur. A very common design of heat exchanger is the "shell-and-tube" style, where a metal shell serves as a conduit for the chemical solution to flow through, while a network of smaller tubes runs through the interior of the shell, carrying steam or some other heat-transfer fluid. The hotter steam flowing through the tubes transfers heat energy to the cooler process fluid surrounding the tubes, inside the shell of the heat exchanger:



In this case, the *process* is the entire heating system, consisting of the fluid we wish to heat, the heat exchanger, and the steam delivering the required heat energy. In order to maintain steady control of the process fluid's exiting temperature, we must find a way to measure it and represent that measurement in signal form so it may be interpreted by other instruments taking some form of control action. In instrumentation terms, the measuring device is known as a *transmitter*, because it *transmits* the process measurement in the form of a signal.

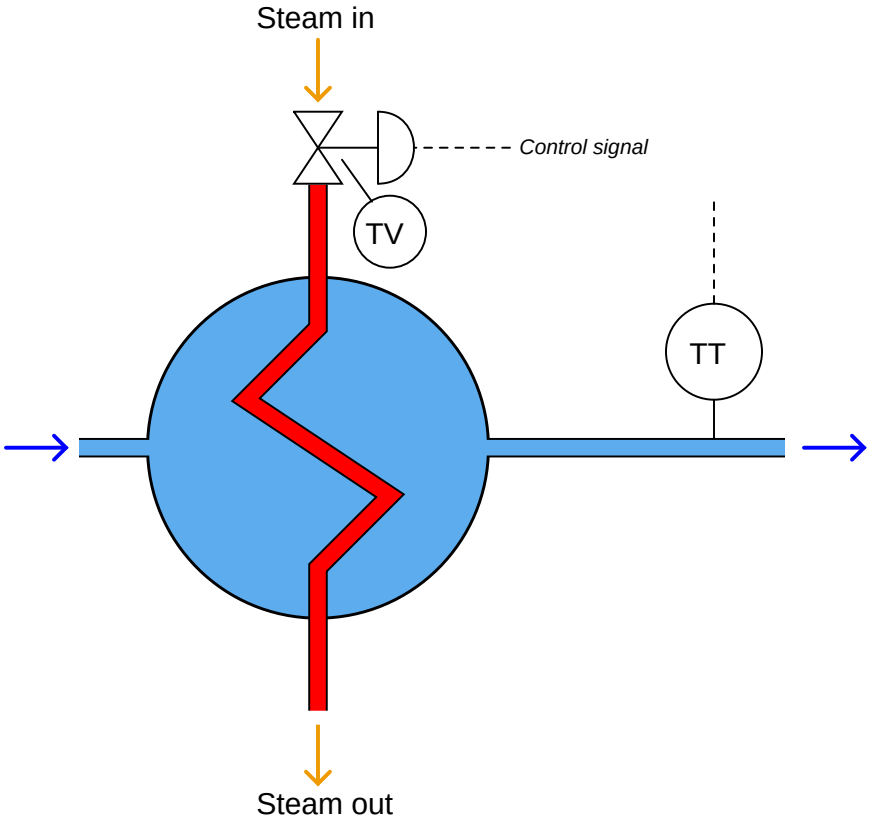
Transmitters are represented in process diagrams by small circles with identifying letters inside, in this case, "TT," which stands for **T**emperature **T**ransmitter:



The signal output by the transmitter (represented by the "PV" dashed line), representing the heated fluid's exiting temperature, is called the *process variable*. Like a variable in a mathematical equation that represents some story-problem quantity, this signal represents the measured quantity we wish to control in the process.

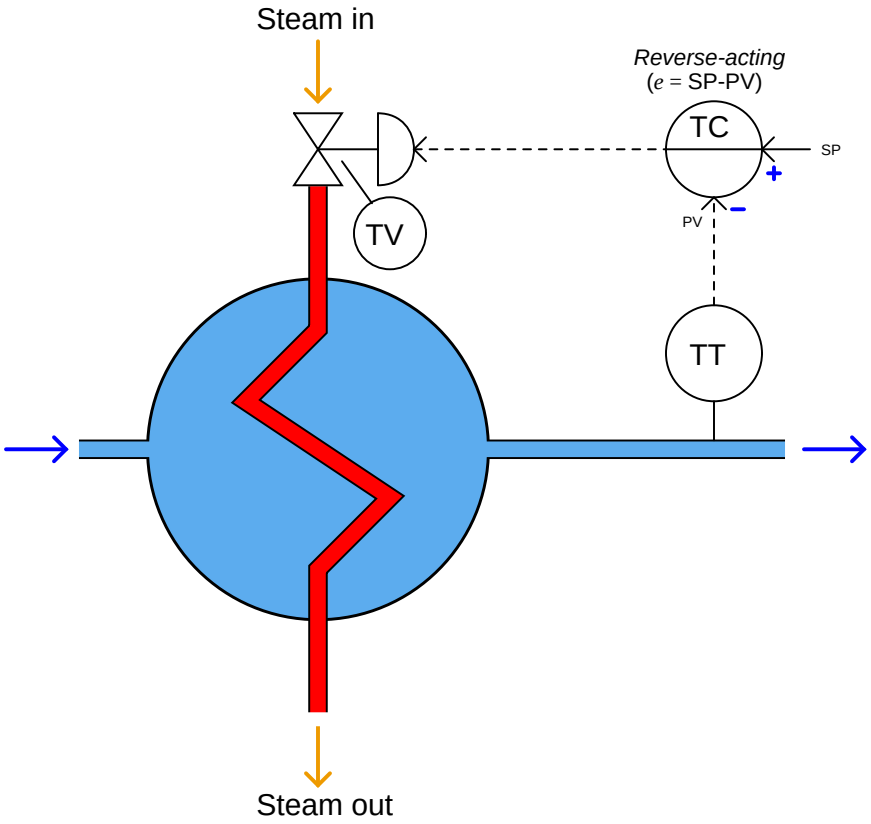
In order to exert control over the process variable, we must have some way of altering fluid flow through the heat exchanger, either of the process fluid, the steam, or both. Generally, it makes more sense to alter the flow of the heating medium (the steam), and let the process fluid flow rate be dictated by the demands of the larger process. If this heat exchanger were part of an oil refinery unit, for example, it would be far better to throttle steam flow to control oil temperature rather than to throttle the oil flow itself, since altering the oil's flow will also affect other process variables upstream and downstream of the exchanger. Ideally, the heat exchanger temperature control system would provide consistent temperature of the exiting oil, for any given incoming oil temperature and flow-rate of oil through it.

One convenient way to throttle steam flow into the heat exchanger is to use a control valve (labeled "TV" because it is a **T**emperature **V**alve). In general terms, a control valve is known as a *final control element*. Other types of final control elements exist (servo motors, variable-flow pumps, and other mechanical devices used to vary some physical quantity at will), but valves are the most common, and probably the simplest to understand. With a final control element in place, the steam flow becomes known as the *manipulated variable*, because it is the quantity we will manipulate in order to gain control over the process variable:



Valves come in a wide variety of sizes and styles. Some valves are hand-operated: that is, they have a "wheel" or other form of manual control that may be moved to "pinch off" or "open up" the flow passage through the pipe. Other valves come equipped with signal receivers and positioner devices, which move the valve mechanism to various positions at the command of a signal (usually an electrical signal, like the type output by transmitter instruments). This feature allows for remote control, so a human operator or computer device may exert control over the manipulated variable from a distance. In the previous illustration, the steam control valve is equipped with such an electrical signal input, represented by the "control signal" dashed line.

This brings us to the final component of the heat exchanger temperature control system: the *controller*. This is a device designed to interpret the transmitter's process variable signal and decide how far open the control valve needs to be in order to maintain that process variable at the desired value.



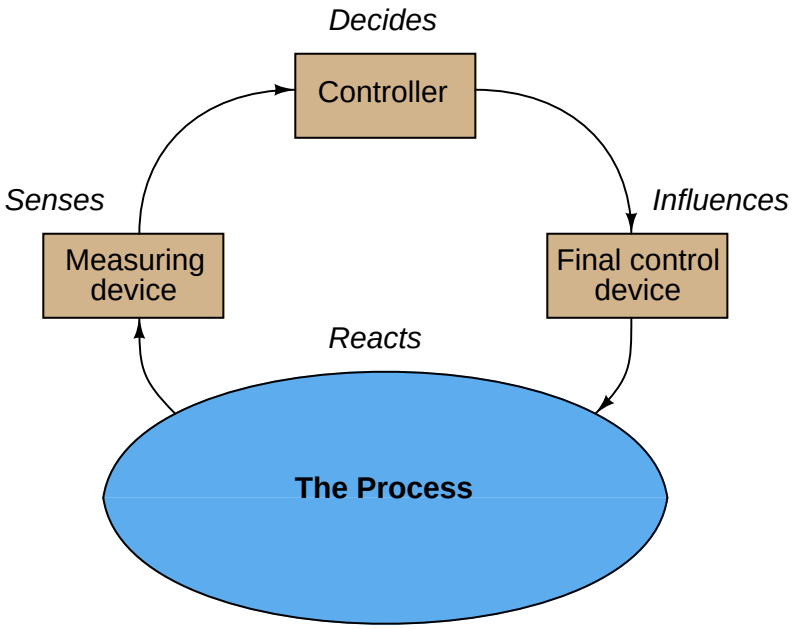
Here, the circle with the letters "TC" in the center represents the controller. Those letters stand for **T**emperature **C**ontroller, since the process variable being controlled is the process fluid's *temperature*. Usually, the controller consists of a computer making automatic decisions to open and close the valve as necessary to stabilize the process variable at some predetermined *setpoint*.

Note that the controller's circle has a solid line going through the center of it, while the transmitter and control valve circles are open. An open circle represents a field-mounted device according to the ISA standard for instrumentation symbols, and a single solid line through the middle of a circle tells us the device is located on the front of a control panel in a main control room location. So, even though the diagram might appear as though these three instruments are located close to one another, they in fact may be quite far apart. Both the transmitter and the valve must be located near the heat exchanger (out in the "field" area rather than inside a building), but the controller may be located a long distance away where human operators can adjust the setpoint from inside a safe and secure control room.

These elements comprise the essentials of a *feedback control system*: the *process* (the system

to be controlled), the *process variable* (the specific quantity to be measured and controlled), the *transmitter* (the device used to measure the process variable and output a corresponding signal), the *controller* (the device that decides what to do to bring the process variable as close to setpoint as possible), the *final control element* (the device that directly exerts control over the process), and the *manipulated variable* (the quantity to be directly altered to effect control over the process variable).

Feedback control may be viewed as a sort of information "loop," from the transmitter (measuring the process variable), to the controller, to the final control element, and through the process itself, back to the transmitter. Ideally, a process control "loop" not only holds the process variable at a steady level (the setpoint), but also maintains control over the process variable given changes in setpoint, and even changes in other variables of the process:



Specifically, the type of feedback we are employing here to control the process is *negative* or *degenerative* feedback. The term "negative" refers to the direction of action the control system takes in response to any measured change in the process variable. If something happens to drive the process variable up, the control system will automatically respond in such a way as to bring the process variable back down where it belongs. If the process variable happens to sag below setpoint, the control system will automatically act to drive the process variable back up to setpoint. Whatever the process variable does in relation to setpoint, the control system takes the opposite (inverse, or negative) action in an attempt to stabilize it at setpoint.

For example, if the unheated process fluid flow rate were to suddenly increase, the heat exchanger outlet temperature would fall due to the physics of heat transfer, but once this drop was detected by the transmitter and reported to the controller, the controller would automatically call for additional steam flow to compensate for the temperature drop, thus bringing the process variable back in agreement with the setpoint. Ideally, a well-designed and well-tuned control loop will sense and

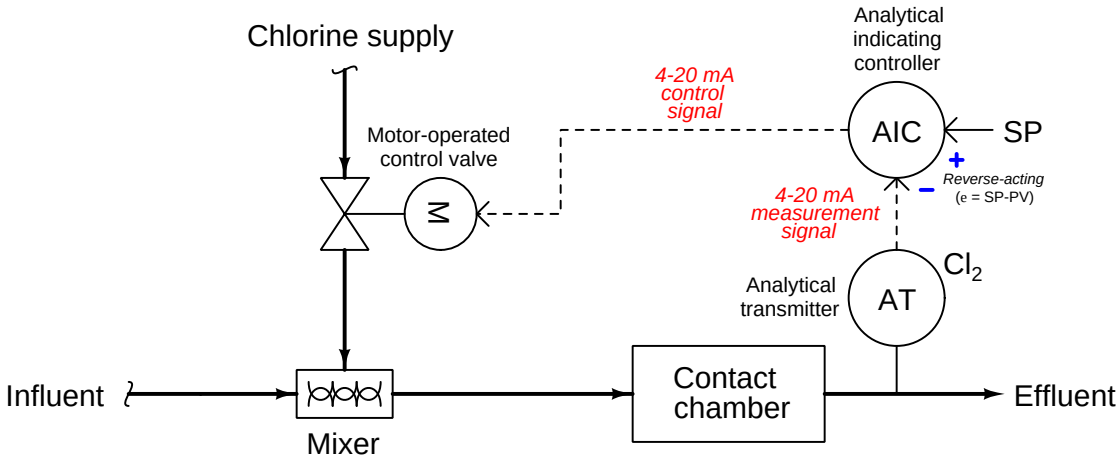
compensate for *any* change in the process or in the setpoint, the end result being a process variable value that always holds steady at the setpoint value.

The unheated fluid flow rate is an example of an uncontrolled, or *wild*, variable because our control system here has no ability to influence it. This flow is also referred to as a *load* because it "loads" or affects the process variable we are trying to stabilize. Loads are present in nearly every controlled system, and indeed are the primary factor necessitating a control system at all. Referring back to our heat exchanger process again, we could adequately control the operating temperature of it with just a manually-set steam control valve if only none of the other factors (steam temperature, fluid flow rate, incoming fluid temperature, etc.) ever changed!

Many types of processes lend themselves to feedback control. Consider an aircraft autopilot system, keeping an airplane on a steady course heading despite the effects of loads such as side-winds: reading the plane's heading (process variable) from an electronic compass and using the rudder as a final control element to change the plane's "yaw." An automobile's "cruise control" is another example of a feedback control system, with the process variable being the car's velocity, and the final control element being the engine's throttle. The purpose of a cruise control is to maintain constant driving speed despite the influence of loads such as hills, head-winds, tail-winds, and road roughness. Steam boilers with automatic pressure controls, electrical generators with automatic voltage and frequency controls, and water pumping systems with automatic flow controls are further examples of how feedback may be used to maintain control over certain process variables.

Modern technology makes it possible to control nearly anything that may be measured in an industrial process. This extends beyond the pale of simple pressure, level, temperature, and flow variables to include even certain chemical properties.

In municipal water and wastewater treatment systems, for example, numerous chemical quantities must be measured and controlled automatically to ensure maximum health and minimum environmental impact. Take for instance the chlorination of treated wastewater, before it leaves the wastewater treatment facility into a large body of water such as a river, bay, or ocean. Chlorine is added to the water to kill any residual bacteria so they do not consume oxygen in the body of water they are released to. Too little chlorine added, and not enough bacteria are killed, resulting in a high *biological oxygen demand* or *BOD* in the water which will asphyxiate the fish swimming in it. Too much chlorine added, and the chlorine itself poses a hazard to marine life. Thus, the chlorine content must be carefully controlled at a particular setpoint, and the control system must take aggressive action if the dissolved chlorine concentration strays too low or too high:



Now that we have seen the basic elements of a feedback control system, we will concentrate on the *algorithms* used in the controller to maintain a process variable at setpoint. For the scope of this topic, an "algorithm" is a mathematical relationship between the process variable and setpoint inputs of a controller, and the output (manipulated variable). Control algorithms determine *how* the manipulated variable quantity is deduced from PV and SP inputs, and range from the elementary to the very complex. In the most common form of control algorithm, the so-called "PID" algorithm, calculus is used to determine the proper final control element action for any combination of input signals.

2.2 Common control system terms and definitions

Industrial measurement and control systems have their own unique terms and standards, which is the primary focus of this lesson. Here are some common instrumentation terms and their definitions:

Process – The physical system we are attempting to control or measure. *Examples: water filtration system, molten metal casting system, steam boiler, oil refinery unit, power generation unit.*

Process Variable, or **PV** – The specific quantity we are measuring in a process. *Examples: pressure, level, temperature, flow, electrical conductivity, pH, position, speed, vibration.*

Setpoint, or **SP** – The value at which we desire the process variable to be maintained at. In other words, the "target" value for the process variable.

Primary Sensing Element, or **PSE** – A device directly sensing the process variable and translating that sensed quantity into an analog representation (electrical voltage, current, resistance; mechanical force, motion, etc.). *Examples: thermocouple, thermistor, bourdon tube, microphone, potentiometer, electrochemical cell, accelerometer.*

Transducer – A device converting one standardized instrumentation signal into another standardized instrumentation signal, and/or performing some sort of processing on that signal. Often referred to as a *converter* and sometimes as a "relay." *Examples: I/P converter (converts 4-20 mA electric signal into 3-15 PSI pneumatic signal), P/I converter (converts 3-15 PSI pneumatic signal into 4-20 mA electric signal), square-root extractor (calculates the square root of the input signal).*

Note: in general science parlance, a "transducer" is any device converting one form of energy into another, such as a microphone or a thermocouple. In industrial instrumentation, however, we generally use "primary sensing element" to describe this concept and reserve the word "transducer" to specifically refer to a conversion device for standardized instrumentation signals.

Transmitter – A device translating the signal produced by a primary sensing element (PSE) into a *standardized* instrumentation signal such as 3-15 PSI air pressure, 4-20 mA DC electric current, Fieldbus digital signal packet, etc., which may then be conveyed to an indicating device, a controlling device, or both.

Lower- and Upper-range values, abbreviated **LRV** and **URV**, respectively – the values of process measurement deemed to be 0% and 100% of a transmitter's calibrated range. For example, if a temperature transmitter is calibrated to measure a range of temperature starting at 300 degrees Celsius and ending at 500 degrees Celsius, its LRV would be 300 ^{o}C and its URV would be 500 ^{o}C.

Zero and **Span** – alternative descriptions to LRV and URV for the 0% and 100% points of an instrument's calibrated range. "Zero" refers to the beginning-point of an instrument's range (equivalent to LRV), while "span" refers to the width of its range (URV − LRV). For example, if a temperature transmitter is calibrated to measure a range of temperature starting at 300 degrees Celsius and ending at 500 degrees Celsius, its zero would be 300 ^{o}C and its span would be 200 ^{o}C.

Controller – A device receiving a process variable (PV) signal from a primary sensing element

(PSE) or transmitter, comparing that signal to the desired value (called the setpoint) for that process variable, and calculating an appropriate output signal value to be sent to a final control element (FCE) such as an electric motor or control valve.

Final Control Element, or **FCE** – A device receiving the signal output by a controller to directly influence the process. *Examples: variable-speed electric motor, control valve, electric heater.*

Manipulated Variable, or **MV** – The quantity in a process we adjust or otherwise manipulate in order to influence the process variable (PV). Also used to describe the output signal generated by a controller; i.e. the signal commanding ("manipulating") the final control element to influence the process.

Load – any uncontrolled factor affecting the process variable's value. *Example: a window opening in a room letting warm air out and cold air in, affecting that room's temperature in such a way that that thermostatic heating system must compensate to maintain the room's temperature at setpoint.*

Automatic mode – When the controller generates an output signal based on the relationship of process variable (PV) to the setpoint (SP).

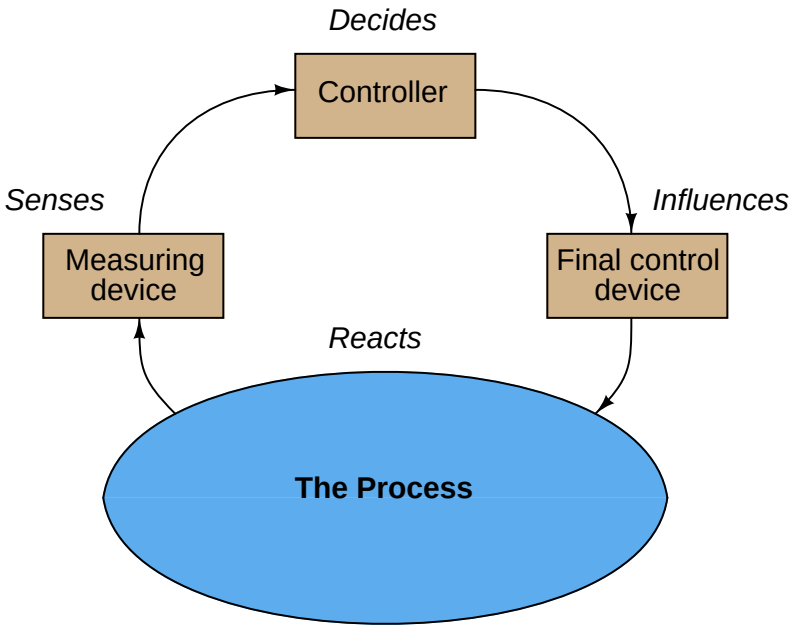
Manual mode – When the controller's decision-making ability is bypassed to let a human operator directly determine the output signal sent to the final control element.

Loop – This widely-used term unfortunately has multiple meanings. In one sense it refers to the complete electrical circuit comprising a 4-20 mA analog measurement or control signal. In another sense it refers to the circular flow of information in any negative feedback regulation system.

2.3 Diagnosing feedback control problems

Negative feedback systems, in general, tend to cause much confusion for those first learning their fundamental principles and behaviors. The closed-cycle "loop" formed by the interaction of sensing element, controller, final control element, and process means essentially that *everything affects everything else*. This is especially problematic when the feedback control system in question contains a fault and must be diagnosed. For example, if an operator happens to notice that the process variable (as indicated by a manual measurement or by some trusted indicating instrument) is not holding to setpoint, it could be the result of a fault in *any* portion of the system (sensor, controller, FCE, or even the process itself).

Recall that every feedback control loop consists of four basic elements: an element that *senses* the process variable (e.g. primary sensing element, transmitter), an element that *decides* what how to regulate this process variable (e.g. a PID controller), an element that *influences* the process variable (e.g. a control valve, motor drive, or some other final control device), and finally the process itself which *reacts* to the final control device's actions:



One of the basic diagnostic strategies for any instrumentation system is to assess whether the *input value(s)* and *output value(s)* correspond for each instrument. We may apply this same strategy to each of the four elements of a feedback control "loop" to identify where the problem might exist. If you encounter one of these four system portions whose output does not correspond with its input, you know that portion of the system is faulted.

You can check each element of your feedback control loop by comparing its input with its output to see if each element is doing what it should. I recommend beginning with the controller (the decision-making element) because typically those values are the most easily monitored:

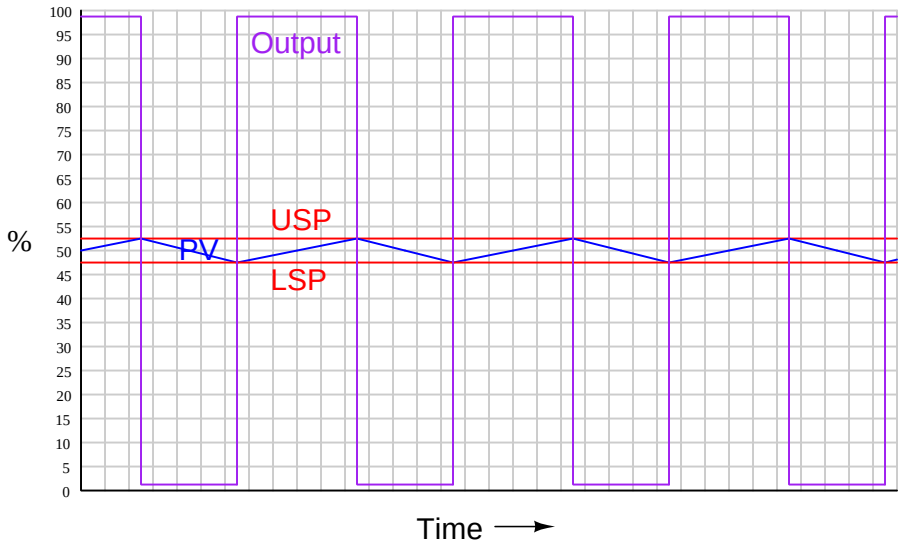
- **Decision-making:** Carefully examine the controller faceplate, looking at the values of PV, SP, and Output. Is the controller taking appropriate action to force PV equal to SP? In other words, is the Output signal at a value you would expect if the controller were functioning properly to regulate the process variable at setpoint? If so, then the controller's action and tuning are most likely not at fault. If not, then the problem definitely lies with the controller.
- **Sensing:** Compare the controller's displayed value for PV with the actual process variable value as indicated by local gauges, by feel, or by any other means of detection. If there is good correspondence between the controller's PV display and the real process variable, then there probably isn't anything wrong with the measurement portion of the control loop (e.g. transmitter, impulse lines, PV signal wiring, analog input of controller, etc.). If the displayed PV disagrees with the actual process variable value, then something is definitely wrong here.
- **Influencing:** Compare the controller's displayed value for Output with the actual status of the final control element. If there is good correspondence between the controller's Output display and the FCE's status, then there probably isn't anything wrong with the output portion of the control loop (e.g. FCE, output signal wiring, analog output of controller, etc.). If the controller Output value differs from the FCE's state, then something is definitely wrong here.
- **Reacting:** Compare the process variable value with the final control element's state. Is the process doing what you would expect it to? If so, the problem is most likely not within the process (e.g. manual valves, relief valves, pumps, compressors, motors, and other process equipment). If, however, the process is not reacting the way you would expect it to given the final control element's state, then something is definitely awry with the process itself.

2.4 On/off control

Once while working as an instrument technician in an aluminum foundry, a mechanic asked me what it was that I did. I began to explain my job, which was essentially to calibrate, maintain, troubleshoot, document, and modify (as needed) all automatic control systems in the facility. The mechanic seemed puzzled as I explained the task of "tuning" loop controllers, especially those controllers used to maintain the temperature of large, gas-fired industrial furnaces holding many tons of molten metal. "Why does a controller have to be 'tuned'?" he asked. "All a controller does is turn the burner on when the metal's too cold, and turn it off when it becomes too hot!"

In its most basic form, the mechanic's assessment of the control system was correct: to turn the burner on when the process variable (molten metal temperature) drops below setpoint, and turn it off when it rises above setpoint. However, the actual algorithm is much more complex than that, finely adjusting the burner intensity according to the amount of *error* between PV and SP, the amount of time the error has accumulated, and the rate-of-change of the error over time. In his casual observation of the furnace controllers, though, he had noticed nothing more than the full-on/full-off action of the controller.

The technical term for a control algorithm that merely checks for the process variable exceeding or falling below setpoint is *on/off control*. In colloquial terms, it is known as *bang-bang* control, since the manipulated variable output of the controller rapidly switches between fully "on" and fully "off" with no intermediate state. Control systems this crude usually provide very imprecise control of the process variable. Consider our example of the shell-and-tube heat exchanger, if we were to implement simple on/off control[1]:



As you can see, the degree of control is rather poor. The process variable "cycles" between the upper and lower setpoints (USP and LSP) without ever stabilizing at the setpoint, because that

[1] To be precise, this form of on/off control is known as *differential gap* because there are two setpoints with a gap in between. While on/off control is possible with a single setpoint (FCE on when below setpoint and off when above), it is usually not practical due to the frequent cycling of the final control element.

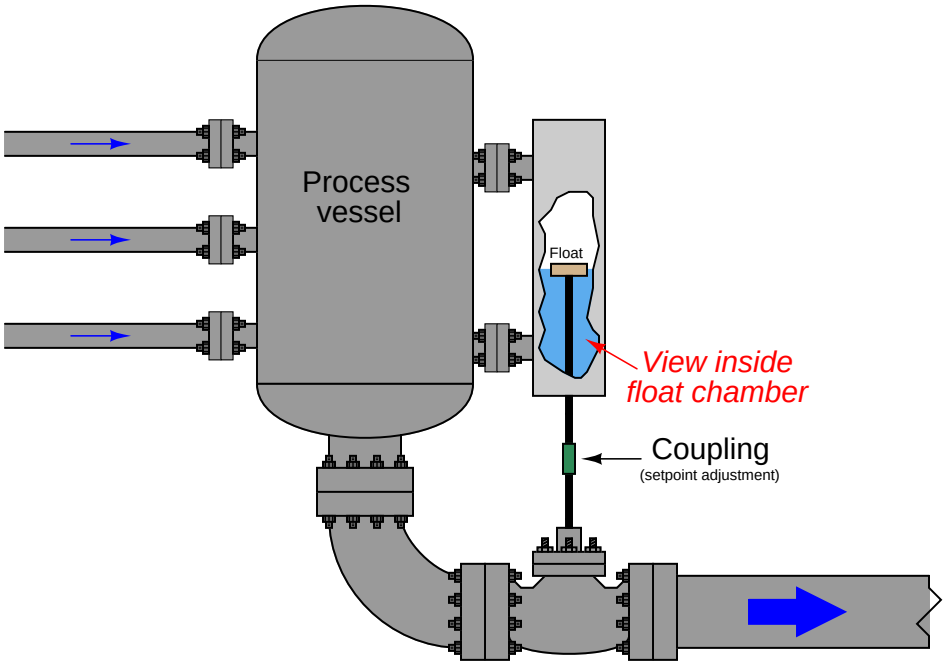
would require the steam valve to be position somewhere *between* fully closed and fully open.

This simple control algorithm may be adequate for temperature control in a house, but not for a sensitive chemical process! Can you imagine what it would be like if an automobile's cruise control system relied on this algorithm? Not only is the lack of precision a problem, but the frequent cycling of the final control element may contribute to premature failure due to mechanical wear. In the heat exchanger scenario, thermal cycling (hot-cold-hot-cold) will cause metal fatigue in the tubes, resulting in a shortened service life. Furthermore, every excursion of the process variable above setpoint is wasted energy, because the process fluid is being heated to a greater temperature than what is necessary.

Clearly, the only practical answer to this dilemma is a control algorithm able to *proportion* the final control element rather than just operate it at zero or full effect (the control valve fully closed or fully open). This, in its simplest form, is called *proportional control*.

2.5 Proportional-only control

Imagine a liquid-level control system for a vessel, where the position of a level-sensing float directly sets the stem position of a control valve. As the liquid level rises, the valve opens up proportionally:



Despite its crude mechanical nature, this *proportional* control system would in fact help regulate the level of liquid inside the process vessel. If an operator wished to change the "setpoint" value of this level control system, he or she would have to adjust the coupling between the float and valve stems for more or less distance between the two. Increasing this distance (lengthening the connection) would effectively raise the level setpoint, while decreasing this distance (shortening the connection) would lower the setpoint.

We may generalize the proportional action of this mechanism to describe *any* form of controller where the output is a direct function of process variable (PV) and setpoint (SP):

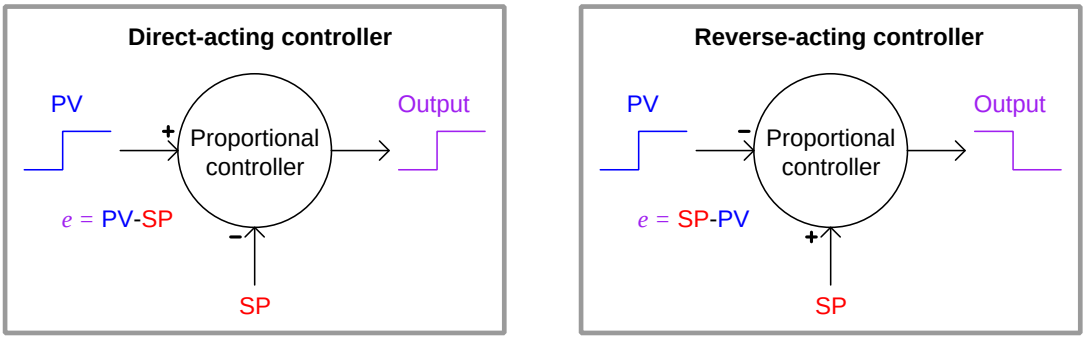
$$m = K_p e + b$$

Where,
m = Controller output
e = Error (difference between PV and SP)
K_p = Proportional gain
b = Bias

A new term introduced with this formula is e, the "error" or difference between process variable and setpoint. Error may be calculated as SP−PV or as PV−SP, depending on whether or not the controller must produce an *increasing* output signal in response to an increase in the process variable ("direct" acting), or output a *decreasing* signal in response to an increase in the process variable ("reverse" acting):

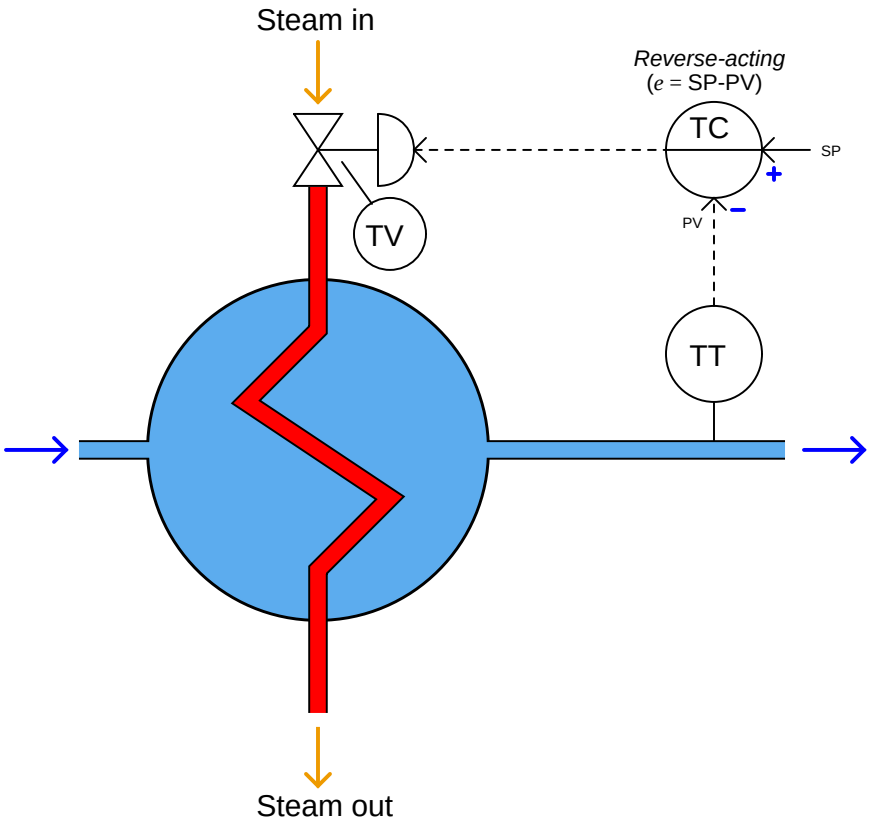
$$m = K_p(\text{PV} - \text{SP}) + b \qquad \text{(Direct-acting proportional controller)}$$

$$m = K_p(\text{SP} - \text{PV}) + b \qquad \text{(Reverse-acting proportional controller)}$$



The optional "+" and "−" symbols clarify the effect each input has on the controller output: a "−" symbol representing an *inverting* effect and a "+" symbol representing a *noninverting* effect. When we say that a controller is "direct-acting" or "reverse-acting" we are referring to it reaction to the PV signal, therefore the output signal from a "direct-acting" controller goes in the same direction as the PV signal and the output from a "reverse-acting" controller goes in the opposite direction of its PV signal. It is important to note, however, that the response to a change in setpoint (SP) will yield the *opposite* response as does a change in process variable (PV): a rising SP will drive the output of a direct-acting controller *down* while a rising SP drives the output of a reverse-acting controller *up*. "+" and "−" symbols explicitly show the effect both inputs have on the controller output, helping to avoid confusion when analyzing the effects of PV changes versus the effects of SP changes.

The direction of action required of the controller is determined by the nature of the process, transmitter, and final control element. In the case of the crude mechanical level controller, the action needs to be *direct* so that a greater liquid level will result in a further-open control valve to drain the vessel faster. In the case of the automated heat exchanger shown earlier, we are assuming that an increasing output signal sent to the control valve results in increased steam flow, and consequently higher temperature, so our controller will need to be reverse-acting (i.e. an increase in measured temperature results in a decrease in output signal; error calculated as SP−PV):



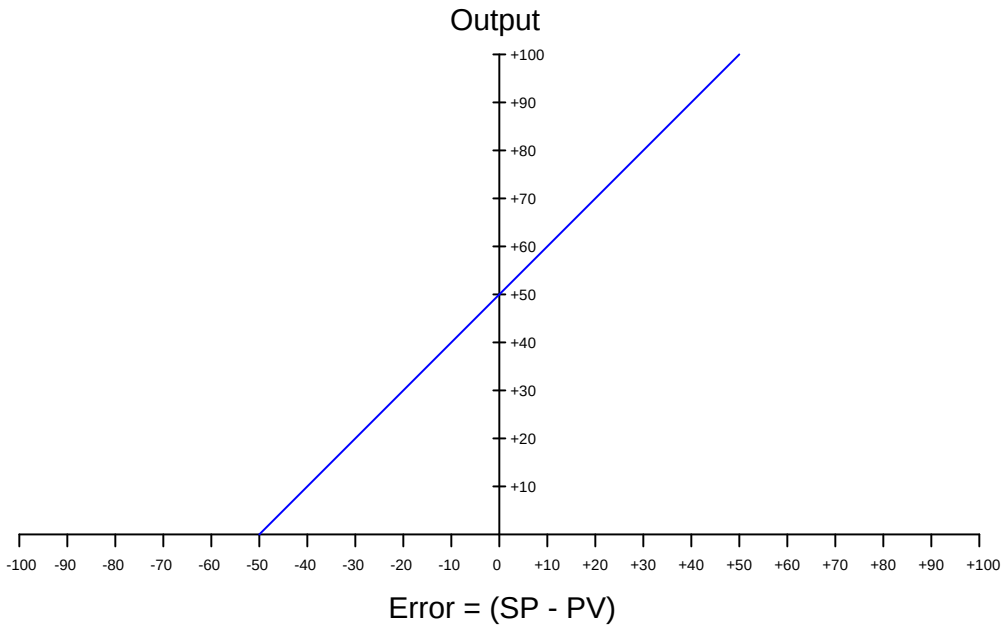
After the error has been calculated, the controller then multiplies the error signal by a constant value called the *gain*, which is programmed into the controller. The resulting figure, plus a "bias" quantity, becomes the output signal sent to the valve to proportion it. The "gain" value is exactly what it seems to be for anyone familiar with electronic amplifier circuits: a ratio of output to input. In this case, the gain of a proportional controller is the ratio of output signal change to input signal change, or how *aggressive* the controller reacts to changes in input (PV or SP).

To give a numerical example, a loop controller set to have a gain of 4 will change its output signal by 40% if it sees an input change of 10%: the ratio of output change to input change will be 4:1. Whether the input change comes in the form of a setpoint adjustment, a drift in the process variable, or some combination of the two does not matter to the magnitude of the output change.

The bias value of a proportional controller is simply the value of its output whenever process

variable happens to be equal to setpoint (i.e. a condition of zero *error*). Without a bias term in the proportional control formula, the valve would always return to a fully shut (0%) condition if ever the process variable reached the setpoint value. The bias term allows the final control element to achieve a non-zero state at setpoint.

If the $m = K_p e + b$ proportional controller formula resembles the standard slope-intercept form of linear equation ($y = mx + b$), it is more than coincidence. Often, the response of a proportional controller is shown graphically as a line, the slope of the line representing gain and the y-intercept of the line representing the output bias point, or what value the output signal will be when there is no error (PV precisely equals SP):

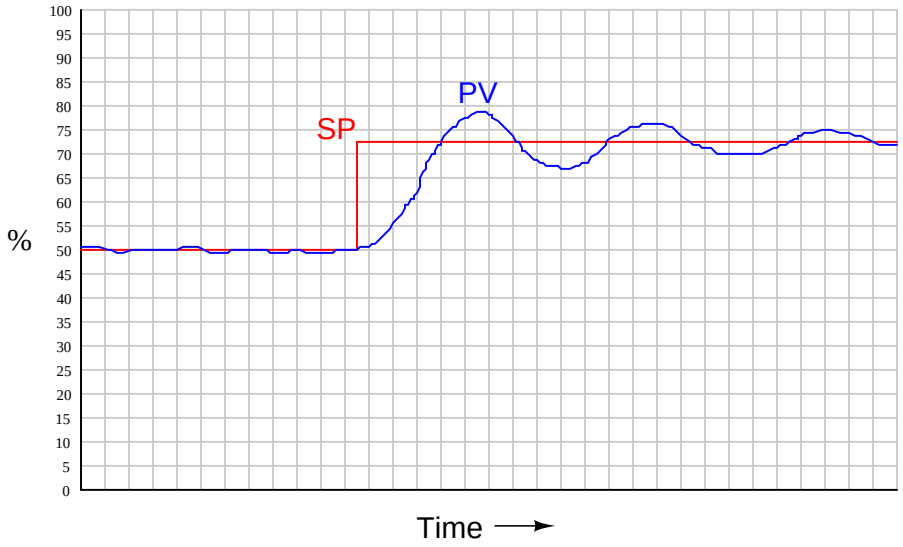


In this graph the bias value is 50% and the gain of the controller is 1. Changing the bias value (b) of the controller shifts the line up or down. Changing the gain value (K_p) alters the slope of the line for more or less aggressive control action.

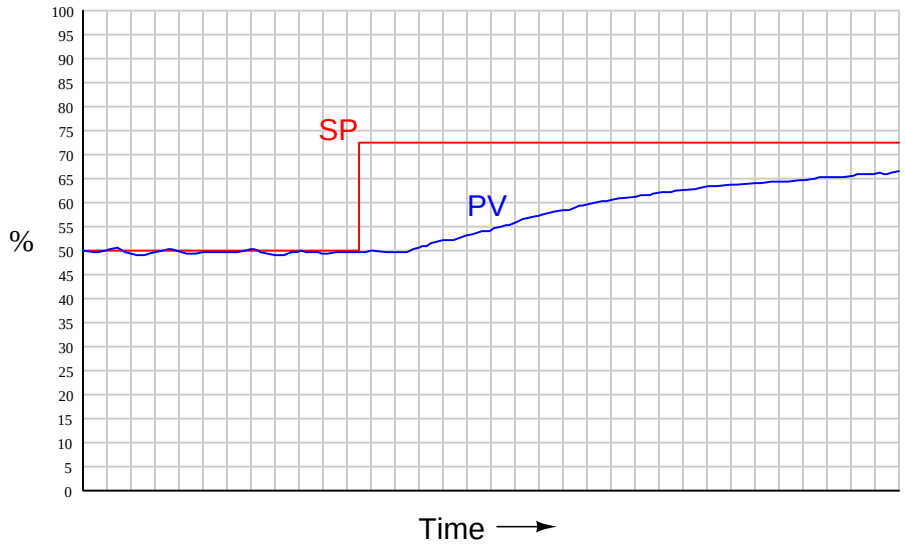
If the controller could be configured for infinite gain, its response would duplicate on/off control. That is, *any* amount of error will result in the output signal becoming "saturated" at either 0% or 100%, and the final control element will simply turn on fully when the process variable drops below setpoint and turn off fully when the process variable rises above setpoint. Conversely, if the controller is set for zero gain, it will become completely unresponsive to changes in either process variable *or* setpoint: the valve will hold its position at the bias point no matter what happens to the process.

Obviously, then, we must set the gain somewhere between infinity and zero in order for this algorithm to function any better than on/off control. Just how much gain a controller needs to have depends on the process and all the other instruments in the control loop.

If the gain is set too high, there will be oscillations as the PV converges on a new setpoint value:

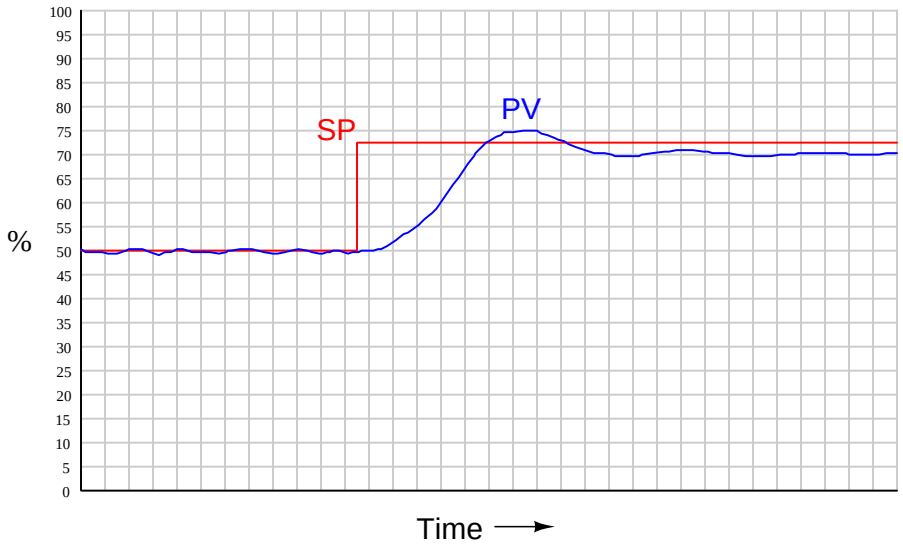

100
95
90
85
80
75
70
65
60
55
50
45
40
35
30
25
20
15
10
5
0
%
SP
PV
Time

If the gain is set too low, the process response will be stable under steady-state conditions but relatively slow to respond to changes in setpoint, as shown in the following trend recording:



A characteristic deficiency of proportional control action, exacerbated with low controller gain values, is a phenomenon known as *proportional-only offset* where the PV never fully reaches SP. A full explanation of proportional-only offset is too lengthy for this discussion and will be presented in a subsequent section of the book, but may be summarized here simply by drawing attention to the proportional controller equation which tells us the output always returns to the bias value when PV reaches SP (i.e. $m = b$ when PV = SP). If anything changes in the process to require a different output value than the bias (b) to stabilize the PV, an error between PV and SP *must* develop to drive the controller output to that necessary output value. This means it is only by chance that the PV will settle precisely at the SP value – most of the time, the PV will deviate from SP in order to generate an output value sufficient to stabilize the PV and prevent it from drifting. This persistent error, or offset, worsens as the controller gain is reduced. Increasing controller gain causes this offset to decrease, but at the expense of oscillations.

With proportional-only control, the choice of gain values is really a compromise between excessive oscillations and excessive offset. A well-tuned proportional controller response is shown here:



An unnecessarily confusing aspect of proportional control is the existence of two completely different ways to express controller proportionality. In the proportional-only equation shown earlier, the degree of proportional action was specified by the constant K_p, called *gain*. However, there is another way to express the sensitivity of proportional action, and that is to state the percentage of error change necessary to make the output (m) change by 100%. Mathematically, this is the inverse of gain, and it is called *proportional band* (PB):

$$K_p = \frac{1}{\text{PB}} \qquad \text{PB} = \frac{1}{K_p}$$

Gain is always specified as a unitless value[2], whereas proportional band is always specified as a percentage. For example, a gain value of 2.5 is equivalent to a proportional band value of 40%, because the error input to this controller must change by 40% in order to make the output change a full 100%.

[2] In electronics, the unit of *decibels* is commonly used to express gains. Thankfully, the world of process control was spared the introduction of decibels as a unit of measurement for controller gain. The last thing we need is a *third* way to express the degree of proportional action in a controller!

Due to the existence of these two completely opposite conventions for specifying proportional action, you may see the proportional term of the control equation written differently depending on whether the author assumes the use of gain or the use of proportional band:

$$K_p = \text{gain} \qquad \text{PB} = \text{proportional band}$$

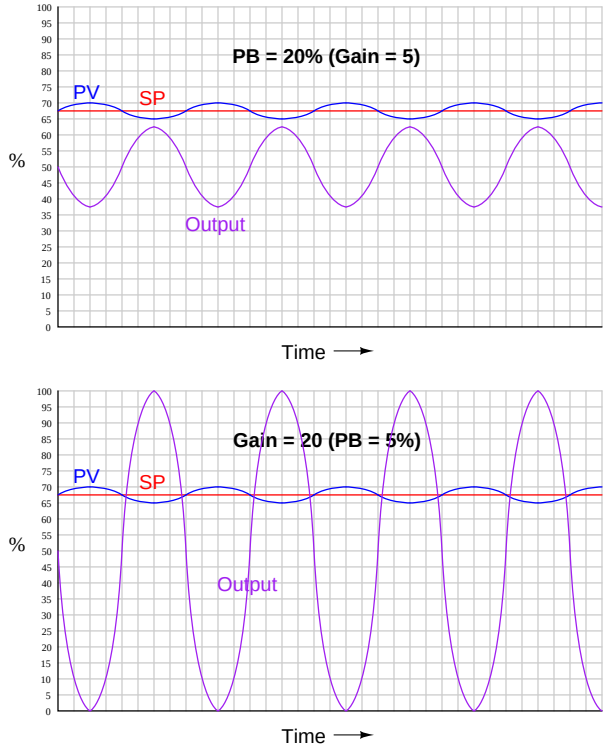
$$K_p e \qquad\qquad \frac{1}{\text{PB}} e$$

Many modern digital electronic controllers allow the user to conveniently select the unit they wish to use for proportional action. However, even with this ability, anyone tasked with adjusting a controller's "tuning" values may be required to translate between gain and proportional band, especially if certain values are documented in a way that does not match the unit configured for the controller.

When you communicate the proportional action setting of a process controller, you should always be careful to specify either "gain" or "proportional band" to avoid ambiguity. *Never* simply say something like, "The proportional setting is twenty," for this could mean either:

- Proportional band = 20%; Gain = 5 . . . *or* . . .
- Gain = 20; Proportional band = 5%

As you can see here, the real-life difference in controller response to an input disturbance (wave) depending on whether it has a proportional band of 20% or a gain of 20 is quite dramatic:



Chapter 3

Questions

This learning module, along with all others in the ModEL collection, is designed to be used in an inverted instructional environment where students independently read[1] the tutorials and attempt to answer questions on their own *prior* to the instructor's interaction with them. In place of lecture[2], the instructor engages with students in Socratic-style dialogue, probing and challenging their understanding of the subject matter through inquiry.

Answers are not provided for questions within this chapter, and this is by design. Solved problems may be found in the Tutorial and Derivation chapters, instead. The goal here is *independence*, and this requires students to be challenged in ways where others cannot think for them. Remember that you always have the tools of *experimentation* and *computer simulation* (e.g. SPICE) to explore concepts!

The following lists contain ideas for Socratic-style questions and challenges. Upon inspection, one will notice a strong theme of *metacognition* within these statements: they are designed to foster a regular habit of examining one's own thoughts as a means toward clearer thinking. As such these sample questions are useful both for instructor-led discussions as well as for self-study.

[1] Technical reading is an essential academic skill for any technical practitioner to possess for the simple reason that the most comprehensive, accurate, and useful information to be found for developing technical competence is in textual form. Technical careers in general are characterized by the need for continuous learning to remain current with standards and technology, and therefore any technical practitioner who cannot read well is handicapped in their professional development. An excellent resource for educators on improving students' reading prowess through intentional effort and strategy is the book textitReading For Understanding – How Reading Apprenticeship Improves Disciplinary Learning in Secondary and College Classrooms by Ruth Schoenbach, Cynthia Greenleaf, and Lynn Murphy.

[2] Lecture is popular as a teaching method because it is easy to implement: any reasonably articulate subject matter expert can talk to students, even with little preparation. However, it is also quite problematic. A good lecture always makes complicated concepts seem easier than they are, which is bad for students because it instills a false sense of confidence in their own understanding; reading and re-articulation requires more cognitive effort and serves to verify comprehension. A culture of teaching-by-lecture fosters a debilitating dependence upon direct personal instruction, whereas the challenges of modern life demand independent and critical thought made possible only by gathering information and perspectives from afar. Information presented in a lecture is ephemeral, easily lost to failures of memory and dictation; text is forever, and may be referenced at any time.

General challenges following tutorial reading

- Summarize as much of the text as you can in one paragraph of your own words. A helpful strategy is to explain ideas as you would for an intelligent child: as simple as you can without compromising too much accuracy.
- Simplify a particular section of the text, for example a paragraph or even a single sentence, so as to capture the same fundamental idea in fewer words.
- Where did the text make the most sense to you? What was it about the text's presentation that made it clear?
- Identify where it might be easy for someone to misunderstand the text, and explain why you think it could be confusing.
- Identify any new concept(s) presented in the text, and explain in your own words.
- Identify any familiar concept(s) such as physical laws or principles applied or referenced in the text.
- Devise a proof of concept experiment demonstrating an important principle, physical law, or technical innovation represented in the text.
- Devise an experiment to disprove a plausible misconception.
- Did the text reveal any misconceptions you might have harbored? If so, describe the misconception(s) and the reason(s) why you now know them to be incorrect.
- Describe any useful problem-solving strategies applied in the text.
- Devise a question of your own to challenge a reader's comprehension of the text.

General follow-up challenges for assigned problems

- Identify where any fundamental laws or principles apply to the solution of this problem, especially before applying any mathematical techniques.
- Devise a thought experiment to explore the characteristics of the problem scenario, applying known laws and principles to mentally model its behavior.
- Describe in detail your own strategy for solving this problem. How did you identify and organized the given information? Did you sketch any diagrams to help frame the problem?
- Is there more than one way to solve this problem? Which method seems best to you?
- Show the work you did in solving this problem, even if the solution is incomplete or incorrect.
- What would you say was the most challenging part of this problem, and why was it so?
- Was any important information missing from the problem which you had to research or recall?
- Was there any extraneous information presented within this problem? If so, what was it and why did it not matter?
- Examine someone else's solution to identify where they applied fundamental laws or principles.
- Simplify the problem from its given form and show how to solve this simpler version of it. Examples include eliminating certain variables or conditions, altering values to simpler (usually whole) numbers, applying a limiting case (i.e. altering a variable to some extreme or ultimate value).
- For quantitative problems, identify the real-world meaning of all intermediate calculations: their units of measurement, where they fit into the scenario at hand. Annotate any diagrams or illustrations with these calculated values.
- For quantitative problems, try approaching it qualitatively instead, thinking in terms of "increase" and "decrease" rather than definite values.
- For qualitative problems, try approaching it quantitatively instead, proposing simple numerical values for the variables.
- Were there any assumptions you made while solving this problem? Would your solution change if one of those assumptions were altered?
- Identify where it would be easy for someone to go astray in attempting to solve this problem.
- Formulate your own problem based on what you learned solving this one.

General follow-up challenges for experiments or projects

- In what way(s) was this experiment or project easy to complete?
- Identify some of the challenges you faced in completing this experiment or project.

- Show how thorough documentation assisted in the completion of this experiment or project.
- Which fundamental laws or principles are key to this system's function?
- Identify any way(s) in which one might obtain false or otherwise misleading measurements from test equipment in this system.
- What will happen if (component X) fails (open/shorted/etc.)?
- What would have to occur to make this system unsafe?

3.1 Conceptual reasoning

These questions are designed to stimulate your analytic and synthetic thinking[3]. In a Socratic discussion with your instructor, the goal is for these questions to prompt an extended dialogue where assumptions are revealed, conclusions are tested, and understanding is sharpened. Your instructor may also pose additional questions based on those assigned, in order to further probe and refine your conceptual understanding.

Questions that follow are presented to challenge and probe your understanding of various concepts presented in the tutorial. These questions are intended to serve as a guide for the Socratic dialogue between yourself and the instructor. Your instructor's task is to ensure you have a sound grasp of these concepts, and the questions contained in this document are merely a means to this end. Your instructor may, at his or her discretion, alter or substitute questions for the benefit of tailoring the discussion to each student's needs. The only absolute requirement is that each student is challenged and assessed at a level equal to or greater than that represented by the documented questions.

It is far more important that you convey your *reasoning* than it is to simply convey a correct answer. For this reason, you should refrain from researching other information sources to answer questions. What matters here is that *you* are doing the thinking. If the answer is incorrect, your instructor will work with you to correct it through proper reasoning. A correct answer without an adequate explanation of how you derived that answer is unacceptable, as it does not aid the learning or assessment process.

You will note a conspicuous lack of answers given for these conceptual questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your conceptual answers, where applicable, is to use circuit simulation software to explore the effects of changes made to circuits. For example, if one of these conceptual questions challenges you to predict the effects of altering some component parameter in a circuit, you may check the validity of your work by simulating that same parameter change within software and seeing if the results agree.

[3] *Analytical* thinking involves the "disassembly" of an idea into its constituent parts, analogous to dissection. *Synthetic* thinking involves the "assembly" of a new idea comprised of multiple concepts, analogous to construction. Both activities are high-level cognitive skills, extremely important for effective problem-solving, necessitating frequent challenge and regular practice to fully develop.

3.1.1 Reading outline and reflections

"Reading maketh a full man; conference a ready man; and writing an exact man" – Francis Bacon

Francis Bacon's advice is a blueprint for effective education: reading provides the learner with knowledge, writing focuses the learner's thoughts, and critical dialogue equips the learner to confidently communicate and apply their learning. Independent acquisition and application of knowledge is a powerful skill, well worth the effort to cultivate. To this end, students should read these educational resources closely, journal their own reflections on the reading, and discuss in detail their findings with classmates and instructor(s). You should be able to do all of the following after reading any instructional text:

☑ Briefly SUMMARIZE THE TEXT in the form of a journal entry documenting your learning as you progress through the course of study. Share this summary in dialogue with your classmates and instructor. Journaling is an excellent self-test of thorough reading because you cannot clearly express what you have not read or did not comprehend.

☑ Demonstrate ACTIVE READING STRATEGIES, including verbalizing your impressions as you read, simplifying long passages to convey the same ideas using fewer words, annotating text and illustrations with your own interpretations, working through mathematical examples shown in the text, cross-referencing passages with relevant illustrations and/or other passages, identifying problem-solving strategies applied by the author, etc. Technical reading is a special case of problem-solving, and so these strategies work precisely because they help solve any problem: paying attention to your own thoughts (metacognition), eliminating unnecessary complexities, identifying what makes sense, paying close attention to details, drawing connections between separated facts, and noting the successful strategies of others.

☑ Identify IMPORTANT THEMES, especially GENERAL LAWS and PRINCIPLES, expounded in the text and express them in the simplest of terms as though you were teaching an intelligent child. This emphasizes connections between related topics and develops your ability to communicate complex ideas to anyone.

☑ Form YOUR OWN QUESTIONS based on the reading, and then pose them to your instructor and classmates for their consideration. Anticipate both correct and incorrect answers, the incorrect answer(s) assuming one or more plausible misconceptions. This helps you view the subject from different perspectives to grasp it more fully.

☑ Devise EXPERIMENTS to test claims presented in the reading, or to disprove misconceptions. Predict possible outcomes of these experiments, and evaluate their meanings: what result(s) would confirm, and what would constitute disproof? Running mental simulations and evaluating results is essential to scientific and diagnostic reasoning.

☑ Specifically identify any points you found CONFUSING. The reason for doing this is to help diagnose misconceptions and overcome barriers to learning.

3.1.2 Foundational concepts

Correct analysis and diagnosis of electric circuits begins with a proper understanding of some basic concepts. The following is a list of some important concepts referenced in this module's full tutorial. Define each of them in your own words, and be prepared to illustrate each of these concepts with a description of a practical example and/or a live demonstration.

Energy

Conservation of Energy

Simplification as a problem-solving strategy

Thought experiments as a problem-solving strategy

Limiting cases as a problem-solving strategy

Annotating diagrams as a problem-solving strategy

Interpreting intermediate results as a problem-solving strategy

Graphing as a problem-solving strategy

Converting a qualitative problem into a quantitative problem

Converting a quantitative problem into a qualitative problem

Working "backwards" to validate calculated results

Reductio ad absurdum

Re-drawing schematics as a problem-solving strategy

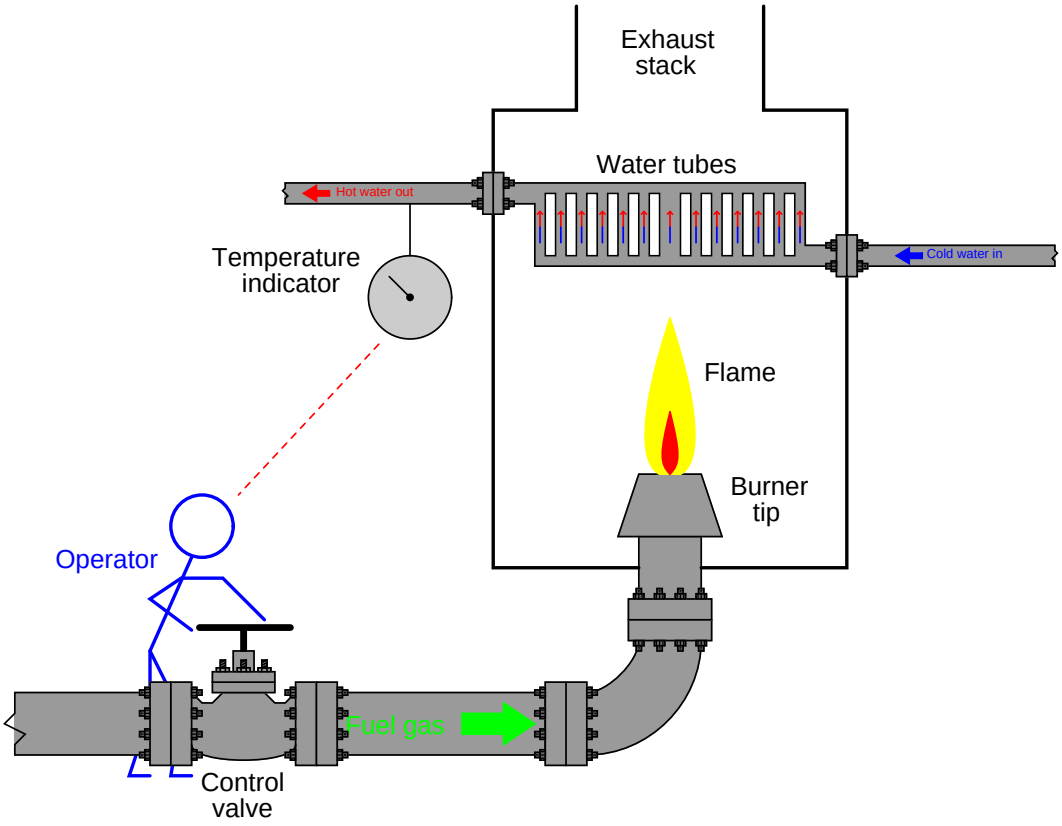
Cut-and-try problem-solving strategy

Algebraic substitution

???

3.1.3 Manually-controlled water heater

Suppose a gas-fired water heater is controlled manually, with a human operator observing a temperature indicator on the hot water outlet pipe and actuating a fuel gas control valve:



Does the operator play the part of a *diect-acting* controller, or a *reverse-acting* controller, in this process control scenario?

Also, identify the *process variable*, *setpoint*, and *manipulated variable* in this manual control system.

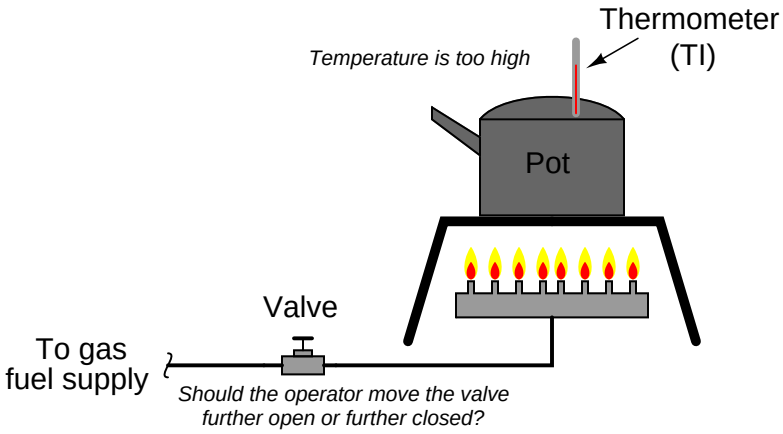
Challenges

- How would the operator need to respond if the incoming feed water's temperature were to increase for some reason?
- How would the operator need to respond if the fuel gas's heating value were to increase for some reason?

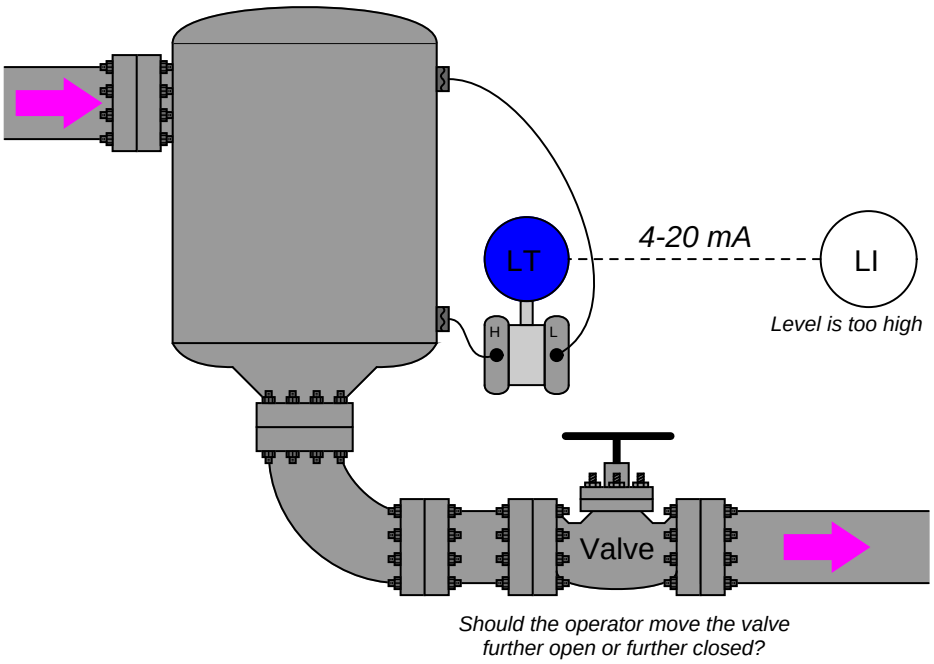
3.1.4 Identifying manual control actions and process loads

In any automated (controlled) system, there is a *process variable*, a *setpoint*, and a *manipulated variable*. There is also something called a *load*, which influences how well the control system is able to maintain setpoint. Provide a general description for a "load," and then identify the load(s) in each of the following manually-controlled processes:

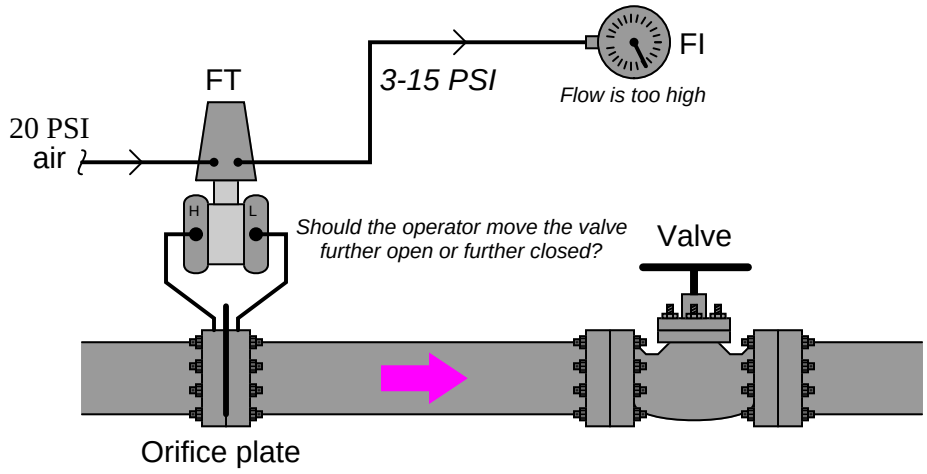
Example 1: Temperature control application



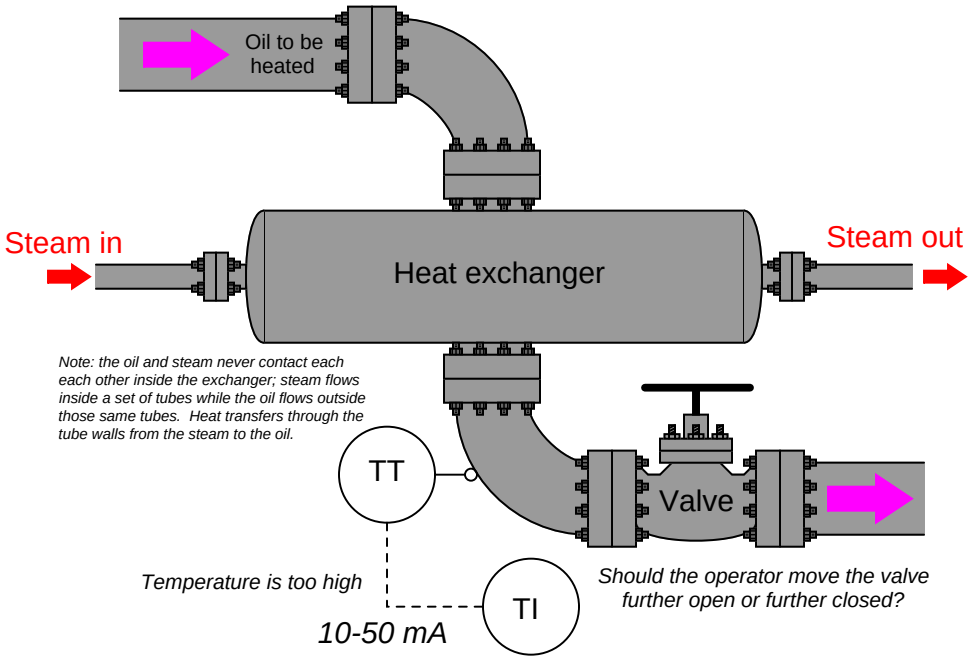
Example 2: Level control application



Example 3: Flow control application



Example 4: Temperature control application



In any automated (controlled) system, there is a *process variable*, a *setpoint*, and a *manipulated variable*. There is also something called a *load*, which influences how well the control system is able to maintain setpoint. Provide a general description for a "load," and then identify the load(s) in each of the these processes.

Challenges

- In which of these examples is the human operator functioning as a *direct-action controller* versus as a *reverse-action controller*?

3.1.5 Applying foundational concepts to ???

Identify which foundational concept(s) apply to each of the declarations shown below regarding the following circuit. If a declaration is true, then identify it as such and note which concept supports that declaration; if a declaration is false, then identify it as such and note which concept is violated by that declaration:

(Under development)

- ???
- ???
- ???
- ???

Here is a list of foundational concepts for your reference: **Conservation of Energy**, **Conservation of Electric Charge**, **behavior of sources vs. loads**, **Ohm's Law**, **Joule's Law**, **effects of open faults**, **effect of shorted faults**, **properties of series networks**, **properties of parallel networks**, **Kirchhoff's Voltage Law**, **Kirchhoff's Current Law**. More than one of these concepts may apply to a declaration, and some concepts may not apply to any listed declaration at all. Also, feel free to include foundational concepts not listed here.

Challenges

- ???.
- ???.
- ???.

3.1.6 Explaining the meaning of calculations

Below is a quantitative problem where all the calculations have been performed for you, but all variable labels, units, and other identifying data are unrevealed. *Assign proper meaning* to each of the numerical values, identify the correct unit of measurement for each value as well as any appropriate metric prefix(es), explain the significance of each value by describing where it "fits" into the circuit being analyzed, and identify the general principle employed at each step:

Schematic diagram of the ??? circuit:

(Under development)

Calculations performed in order from first to last:

1. $x + y = z$
2. $x + y = z$
3. $x + y = z$
4. $x + y = z$
5. $x + y = z$
6. $x + y = z$

Challenges

- ???.
- ???.
- ???.

3.1.7 Explaining the meaning of code

Shown below is a schematic diagram for a ??? circuit, and after that a source-code listing of a computer program written in the ??? language simulating that circuit. Explain the purpose of each line of code relating to the circuit being simulated, identify the correct unit of measurement for each computed value, and identify all foundational concepts of electric circuits (e.g. Ohm's Law, Kirchhoff's Laws, etc.) employed in the program:

Schematic diagram of the ??? circuit:

(Under development)

Code listing:

```
#include <stdio.h>

int main (void)
{

  return 0;
}
```

Challenges

- ???.
- ???.
- ???.

3.2 Quantitative reasoning

These questions are designed to stimulate your computational thinking. In a Socratic discussion with your instructor, the goal is for these questions to reveal your mathematical approach(es) to problem-solving so that good technique and sound reasoning may be reinforced. Your instructor may also pose additional questions based on those assigned, in order to observe your problem-solving firsthand.

Mental arithmetic and estimations are strongly encouraged for all calculations, because without these abilities you will be unable to readily detect errors caused by calculator misuse (e.g. keystroke errors).

You will note a conspicuous lack of answers given for these quantitative questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. My advice is to use circuit simulation software such as SPICE to check the correctness of quantitative answers. Refer to those learning modules within this collection focusing on SPICE to see worked examples which you may use directly as practice problems for your own study, and/or as templates you may modify to run your own analyses and generate your own practice problems.

Completely worked example problems found in the Tutorial may also serve as "test cases[4]" for gaining proficiency in the use of circuit simulation software, and then once that proficiency is gained you will never need to rely[5] on an answer key!

[4] In other words, set up the circuit simulation software to analyze the same circuit examples found in the Tutorial. If the simulated results match the answers shown in the Tutorial, it confirms the simulation has properly run. If the simulated results disagree with the Tutorial's answers, something has been set up incorrectly in the simulation software. Using every Tutorial as practice in this way will quickly develop proficiency in the use of circuit simulation software.

[5] This approach is perfectly in keeping with the instructional philosophy of these learning modules: *teaching students to be self-sufficient thinkers*. Answer keys can be useful, but it is even more useful to your long-term success to have a set of tools on hand for checking your own work, because once you have left school and are on your own, there will no longer be "answer keys" available for the problems you will have to solve.

3.2.1 Miscellaneous physical constants

Note: constants shown in **bold** type are *exact*, not approximations. Values inside of parentheses show one standard deviation (σ) of uncertainty in the final digits: for example, the magnetic permeability of free space value given as $1.25663706212(19) \times 10^{-6}$ H/m represents a center value (i.e. the location parameter) of $1.25663706212 \times 10^{-6}$ Henrys per meter with one standard deviation of uncertainty equal to $0.0000000000019 \times 10^{-6}$ Henrys per meter.

Avogadro's number (N_A) = **6.02214076** $\times 10^{23}$ **per mole** (mol^{-1})

Boltzmann's constant (k) = **1.380649** $\times 10^{-23}$ **Joules per Kelvin** (J/K)

Electronic charge (e) = **1.602176634** $\times 10^{-19}$ **Coulomb** (C)

Faraday constant (F) = **96,485.33212...** $\times 10^{4}$ **Coulombs per mole** (C/mol)

Magnetic permeability of free space (μ_0) = $1.25663706212(19) \times 10^{-6}$ Henrys per meter (H/m)

Electric permittivity of free space (ϵ_0) = $8.8541878128(13) \times 10^{-12}$ Farads per meter (F/m)

Characteristic impedance of free space (Z_0) = 376.730313668(57) Ohms (Ω)

Gravitational constant (G) = $6.67430(15) \times 10^{-11}$ cubic meters per kilogram-seconds squared ($m^3/kg\text{-}s^2$)

Molar gas constant (R) = **8.314462618... Joules per mole-Kelvin** (J/mol-K) = 0.08205746(14) liters-atmospheres per mole-Kelvin

Planck constant (h) = **6.62607015** $\times 10^{-34}$ **joule-seconds** (J-s)

Stefan-Boltzmann constant (σ) = **5.670374419...** $\times 10^{-8}$ **Watts per square meter-Kelvin**^{4} ($W/m^2 \cdot K^4$)

Speed of light in a vacuum (c) = **299,792,458 meters per second** (m/s) = 186282.4 miles per second (mi/s)

Note: All constants taken from NIST data "Fundamental Physical Constants – Complete Listing", from `http://physics.nist.gov/constants`, National Institute of Standards and Technology (NIST), 2018 CODATA Adjustment.

3.2.2 Introduction to spreadsheets

A powerful computational tool you are encouraged to use in your work is a *spreadsheet*. Available on most personal computers (e.g. Microsoft Excel), *spreadsheet* software performs numerical calculations based on number values and formulae entered into cells of a grid. This grid is typically arranged as lettered columns and numbered rows, with each cell of the grid identified by its column/row coordinates (e.g. cell `B3`, cell `A8`). Each cell may contain a string of text, a number value, or a mathematical formula. The spreadsheet automatically updates the results of all mathematical formulae whenever the entered number values are changed. This means it is possible to set up a spreadsheet to perform a series of calculations on entered data, and those calculations will be re-done by the computer any time the data points are edited in any way.

For example, the following spreadsheet calculates average speed based on entered values of distance traveled and time elapsed:

	A	B	C	D
1	Distance traveled	46.9	Kilometers	
2	Time elapsed	1.18	Hours	
3	Average speed	= B1 / B2	km/h	
4				
5				

Text labels contained in cells `A1` through `A3` and cells `C1` through `C3` exist solely for readability and are not involved in any calculations. Cell `B1` contains a sample distance value while cell `B2` contains a sample time value. The formula for computing speed is contained in cell `B3`. Note how this formula begins with an "equals" symbol (`=`), references the values for distance and speed by lettered column and numbered row coordinates (`B1` and `B2`), and uses a forward slash symbol for division (`/`). The coordinates `B1` and `B2` function as *variables*[6] would in an algebraic formula.

When this spreadsheet is executed, the numerical value `39.74576` will appear in cell `B3` rather than the formula `= B1 / B2`, because 39.74576 is the computed speed value given 46.9 kilometers traveled over a period of 1.18 hours. If a different numerical value for distance is entered into cell `B1` or a different value for time is entered into cell `B2`, cell `B3`'s value will automatically update. All you need to do is set up the given values and any formulae into the spreadsheet, and the computer will do all the calculations for you.

Cell `B3` may be referenced by other formulae in the spreadsheet if desired, since it is a variable just like the given values contained in `B1` and `B2`. This means it is possible to set up an entire chain of calculations, one dependent on the result of another, in order to arrive at a final value. The arrangement of the given data and formulae need not follow any pattern on the grid, which means you may place them anywhere.

[6] Spreadsheets may also provide means to attach text labels to cells for use as variable names (Microsoft Excel simply calls these labels "names"), but for simple spreadsheets such as those shown here it's usually easier just to use the standard coordinate naming for each cell.

Common[7] arithmetic operations available for your use in a spreadsheet include the following:

- Addition (`+`)
- Subtraction (`-`)
- Multiplication (`*`)
- Division (`/`)
- Powers (`^`)
- Square roots (`sqrt()`)
- Logarithms (`ln()` , `log10()`)

Parentheses may be used to ensure[8] proper order of operations within a complex formula. Consider this example of a spreadsheet implementing the *quadratic formula*, used to solve for roots of a polynomial expression in the form of $ax^2 + bx + c$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

	A	B
1	x_1	= (-B4 + sqrt((B4^2) - (4*B3*B5))) / (2*B3)
2	x_2	= (-B4 - sqrt((B4^2) - (4*B3*B5))) / (2*B3)
3	a =	9
4	b =	5
5	c =	-2

This example is configured to compute roots[9] of the polynomial $9x^2 + 5x - 2$ because the values of 9, 5, and -2 have been inserted into cells `B3`, `B4`, and `B5`, respectively. Once this spreadsheet has been built, though, it may be used to calculate the roots of *any* second-degree polynomial expression simply by entering the new a, b, and c coefficients into cells `B3` through `B5`. The numerical values appearing in cells `B1` and `B2` will be automatically updated by the computer immediately following any changes made to the coefficients.

[7] Modern spreadsheet software offers a bewildering array of mathematical functions you may use in your computations. I recommend you consult the documentation for your particular spreadsheet for information on operations other than those listed here.

[8] Spreadsheet programs, like text-based programming languages, are designed to follow standard order of operations by default. However, my personal preference is to use parentheses even where strictly unnecessary just to make it clear to any other person viewing the formula what the intended order of operations is.

[9] Reviewing some algebra here, a *root* is a value for x that yields an overall value of zero for the polynomial. For this polynomial ($9x^2 + 5x - 2$) the two roots happen to be $x = 0.269381$ and $x = -0.82494$, with these values displayed in cells `B1` and `B2`, respectively upon execution of the spreadsheet.

Alternatively, one could break up the long quadratic formula into smaller pieces like this:

$$y = \sqrt{b^2 - 4ac} \qquad z = 2a$$

$$x = \frac{-b \pm y}{z}$$

	A	B	C
1	x_1	= (-B4 + C1) / C2	= sqrt((B4^2) - (4*B3*B5))
2	x_2	= (-B4 - C1) / C2	= 2*B3
3	a =	9	
4	b =	5	
5	c =	-2	

Note how the square-root term (y) is calculated in cell C1, and the denominator term (z) in cell C2. This makes the two final formulae (in cells B1 and B2) simpler to interpret. The positioning of all these cells on the grid is completely arbitrary[10] – all that matters is that they properly reference each other in the formulae.

Spreadsheets are particularly useful for situations where the same set of calculations representing a circuit or other system must be repeated for different initial conditions. The power of a spreadsheet is that it automates what would otherwise be a tedious set of calculations. One specific application of this is to simulate the effects of various components within a circuit failing with abnormal values (e.g. a shorted resistor simulated by making its value nearly zero; an open resistor simulated by making its value extremely large). Another application is analyzing the behavior of a circuit design given new components that are out of specification, and/or aging components experiencing drift over time.

[10] My personal preference is to locate all the "given" data in the upper-left cells of the spreadsheet grid (each data point flanked by a sensible name in the cell to the left and units of measurement in the cell to the right as illustrated in the first distance/time spreadsheet example), sometimes coloring them in order to clearly distinguish which cells contain entered data versus which cells contain computed results from formulae. I like to place all formulae in cells below the given data, and try to arrange them in logical order so that anyone examining my spreadsheet will be able to figure out *how* I constructed a solution. This is a general principle I believe all computer programmers should follow: *document and arrange your code to make it easy for other people to learn from it.*

3.2.3 First quantitative problem

Challenges

- ???.
- ???.
- ???.

3.2.4 Second quantitative problem

Challenges

- ???.
- ???.
- ???.

3.2.5 ??? simulation program

Write a text-based computer program (e.g. C, C++, Python) to calculate ???

Challenges

- ???.
- ???.
- ???.

3.3 Diagnostic reasoning

These questions are designed to stimulate your deductive and inductive thinking, where you must apply general principles to specific scenarios (deductive) and also derive conclusions about the failed circuit from specific details (inductive). In a Socratic discussion with your instructor, the goal is for these questions to reinforce your recall and use of general circuit principles and also challenge your ability to integrate multiple symptoms into a sensible explanation of what's wrong in a circuit. Your instructor may also pose additional questions based on those assigned, in order to further challenge and sharpen your diagnostic abilities.

As always, your goal is to fully *explain* your analysis of each problem. Simply obtaining a correct answer is not good enough – you must also demonstrate sound reasoning in order to successfully complete the assignment. Your instructor's responsibility is to probe and challenge your understanding of the relevant principles and analytical processes in order to ensure you have a strong foundation upon which to build further understanding.

You will note a conspicuous lack of answers given for these diagnostic questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your diagnostic answers, where applicable, is to use circuit simulation software to explore the effects of faults placed in circuits. For example, if one of these diagnostic questions requires that you predict the effect of an open or a short in a circuit, you may check the validity of your work by simulating that same fault (substituting a very high resistance in place of that component for an open, and substituting a very low resistance for a short) within software and seeing if the results agree.

3.3.1 First diagnostic scenario

Challenges

- ???.
- ???.
- ???.

3.3.2 Second diagnostic scenario

Challenges

- ???.
- ???.
- ???.

Appendix A

Problem-Solving Strategies

The ability to solve complex problems is arguably one of the most valuable skills one can possess, and this skill is particularly important in any science-based discipline.

- Study principles, not procedures. Don't be satisfied with merely knowing how to compute solutions – learn *why* those solutions work.
- Identify what it is you need to solve, identify all relevant data, identify all units of measurement, identify any general principles or formulae linking the given information to the solution, and then identify any "missing pieces" to a solution. Annotate all diagrams with this data.
- Sketch a diagram to help visualize the problem. When building a real system, always devise a plan for that system and analyze its function *before* constructing it.
- Follow the units of measurement and meaning of every calculation. If you are ever performing mathematical calculations as part of a problem-solving procedure, and you find yourself unable to apply each and every intermediate result to some aspect of the problem, it means you don't understand what you are doing. Properly done, every mathematical result should have practical meaning for the problem, and not just be an abstract number. You should be able to identify the proper units of measurement for each and every calculated result, and show where that result fits into the problem.
- Perform "thought experiments" to explore the effects of different conditions for theoretical problems. When troubleshooting real systems, perform *diagnostic tests* rather than visually inspecting for faults, the best diagnostic test being the one giving you the most information about the nature and/or location of the fault with the fewest steps.
- Simplify the problem until the solution becomes obvious, and then use that obvious case as a model to follow in solving the more complex version of the problem.
- Check for exceptions to see if your solution is incorrect or incomplete. A good solution will work for *all* known conditions and criteria. A good example of this is the process of testing scientific hypotheses: the task of a scientist is not to find support for a new idea, but rather to *challenge* that new idea to see if it holds up under a battery of tests. The philosophical

principle of *reductio ad absurdum* (i.e. disproving a general idea by finding a specific case where it fails) is useful here.

- Work "backward" from a hypothetical solution to a new set of given conditions.
- Add quantities to problems that are qualitative in nature, because sometimes a little math helps illuminate the scenario.
- Sketch graphs illustrating how variables relate to each other. These may be quantitative (i.e. with realistic number values) or qualitative (i.e. simply showing increases and decreases).
- Treat quantitative problems as qualitative in order to discern the relative magnitudes and/or directions of change of the relevant variables. For example, try determining what happens if a certain variable were to increase or decrease before attempting to precisely calculate quantities: how will each of the dependent variables respond, by increasing, decreasing, or remaining the same as before?
- Consider limiting cases. This works especially well for qualitative problems where you need to determine which direction a variable will change. Take the given condition and magnify that condition to an extreme degree as a way of simplifying the direction of the system's response.
- Check your work. This means regularly testing your conclusions to see if they make sense. This does *not* mean repeating the same steps originally used to obtain the conclusion(s), but rather to use some other means to check validity. Simply repeating procedures often leads to *repeating the same errors* if any were made, which is why alternative paths are better.

Appendix B

Instructional philosophy

"The unexamined circuit is not worth energizing" – Socrates (if he had taught electricity)

These learning modules, although useful for self-study, were designed to be used in a formal learning environment where a subject-matter expert challenges students to digest the content and exercise their critical thinking abilities in the answering of questions and in the construction and testing of working circuits.

The following principles inform the instructional and assessment philosophies embodied in these learning modules:

- The first goal of education is to enhance clear and independent thought, in order that every student reach their fullest potential in a highly complex and inter-dependent world. Robust reasoning is *always* more important than particulars of any subject matter, because its application is universal.
- Literacy is fundamental to independent learning and thought because text continues to be the most efficient way to communicate complex ideas over space and time. Those who cannot read with ease are limited in their ability to acquire knowledge and perspective.
- Articulate communication is fundamental to work that is complex and interdisciplinary.
- Faulty assumptions and poor reasoning are best corrected through challenge, not presentation. The rhetorical technique of *reductio ad absurdum* (disproving an assertion by exposing an absurdity) works well to discipline student's minds, not only to correct the problem at hand but also to learn how to detect and correct future errors.
- Important principles should be repeatedly explored and widely applied throughout a course of study, not only to reinforce their importance and help ensure their mastery, but also to showcase the interconnectedness and utility of knowledge.

These learning modules were expressly designed to be used in an "inverted" teaching environment[1] where students first read the introductory and tutorial chapters on their own, then individually attempt to answer the questions and construct working circuits according to the experiment and project guidelines. The instructor never lectures, but instead meets regularly with each individual student to review their progress, answer questions, identify misconceptions, and challenge the student to new depths of understanding through further questioning. Regular meetings between instructor and student should resemble a Socratic[2] dialogue, where questions serve as scalpels to dissect topics and expose assumptions. The student passes each module only after consistently demonstrating their ability to logically analyze and correctly apply all major concepts in each question or project/experiment. The instructor must be vigilant in probing each student's understanding to ensure they are truly *reasoning* and not just *memorizing*. This is why "Challenge" points appear throughout, as prompts for students to think deeper about topics and as starting points for instructor queries. Sometimes these challenge points require additional knowledge that hasn't been covered in the series to answer in full. This is okay, as the major purpose of the Challenges is to stimulate analysis and synthesis on the part of each student.

The instructor must possess enough mastery of the subject matter and awareness of students' reasoning to generate their own follow-up questions to practically any student response. Even completely correct answers given by the student should be challenged by the instructor for the purpose of having students practice articulating their thoughts and defending their reasoning. Conceptual errors committed by the student should be exposed and corrected not by direct instruction, but rather by reducing the errors to an absurdity[3] through well-chosen questions and thought experiments posed by the instructor. Becoming proficient at this style of instruction requires time and dedication, but the positive effects on critical thinking for both student and instructor are spectacular.

An inspection of these learning modules reveals certain unique characteristics. One of these is a bias toward thorough explanations in the tutorial chapters. Without a live instructor to explain concepts and applications to students, the text itself must fulfill this role. This philosophy results in lengthier explanations than what you might typically find in a textbook, each step of the reasoning process fully explained, including footnotes addressing common questions and concerns students raise while learning these concepts. Each tutorial seeks to not only explain each major concept in sufficient detail, but also to explain the logic of each concept and how each may be developed

[1] In a traditional teaching environment, students first encounter new information via *lecture* from an expert, and then independently apply that information via *homework*. In an "inverted" course of study, students first encounter new information via *homework*, and then independently apply that information under the scrutiny of an expert. The expert's role in lecture is to simply *explain*, but the expert's role in an inverted session is to *challenge*, *critique*, and if necessary *explain* where gaps in understanding still exist.

[2] Socrates is a figure in ancient Greek philosophy famous for his unflinching style of questioning. Although he authored no texts, he appears as a character in Plato's many writings. The essence of Socratic philosophy is to leave no question unexamined and no point of view unchallenged. While purists may argue a topic such as electric circuits is too narrow for a true Socratic-style dialogue, I would argue that the essential thought processes involved with scientific reasoning on *any* topic are not far removed from the Socratic ideal, and that students of electricity and electronics would do very well to challenge assumptions, pose thought experiments, identify fallacies, and otherwise employ the arsenal of critical thinking skills modeled by Socrates.

[3] This rhetorical technique is known by the Latin phrase *reductio ad absurdum*. The concept is to expose errors by counter-example, since only one solid counter-example is necessary to disprove a universal claim. As an example of this, consider the common misconception among beginning students of electricity that voltage cannot exist without current. One way to apply *reductio ad absurdum* to this statement is to ask how much current passes through a fully-charged battery connected to nothing (i.e. a clear example of voltage existing without current).

from "first principles". Again, this reflects the goal of developing clear and independent thought in students' minds, by showing how clear and logical thought was used to forge each concept. Students benefit from witnessing a model of clear thinking in action, and these tutorials strive to be just that.

Another characteristic of these learning modules is a lack of step-by-step instructions in the Project and Experiment chapters. Unlike many modern workbooks and laboratory guides where step-by-step instructions are prescribed for each experiment, these modules take the approach that students must learn to closely read the tutorials and apply their own reasoning to identify the appropriate experimental steps. Sometimes these steps are plainly declared in the text, just not as a set of enumerated points. At other times certain steps are implied, an example being assumed competence in test equipment use where the student should not need to be told *again* how to use their multimeter because that was thoroughly explained in previous lessons. In some circumstances no steps are given at all, leaving the entire procedure up to the student.

This lack of prescription is not a flaw, but rather a feature. Close reading and clear thinking are foundational principles of this learning series, and in keeping with this philosophy all activities are designed to *require* those behaviors. Some students may find the lack of prescription frustrating, because it demands more from them than what their previous educational experiences required. This frustration should be interpreted as an unfamiliarity with autonomous thinking, a problem which must be corrected if the student is ever to become a self-directed learner and effective problem-solver. Ultimately, the need for students to read closely and think clearly is more important both in the near-term and far-term than any specific facet of the subject matter at hand. If a student takes longer than expected to complete a module because they are forced to outline, digest, and reason on their own, so be it. The future gains enjoyed by developing this mental discipline will be well worth the additional effort and delay.

Another feature of these learning modules is that they do not treat topics in isolation. Rather, important concepts are introduced early in the series, and appear repeatedly as stepping-stones toward other concepts in subsequent modules. This helps to avoid the "compartmentalization" of knowledge, demonstrating the inter-connectedness of concepts and simultaneously reinforcing them. Each module is fairly complete in itself, reserving the beginning of its tutorial to a review of foundational concepts.

This methodology of assigning text-based modules to students for digestion and then using Socratic dialogue to assess progress and hone students' thinking was developed over a period of several years by the author with his Electronics and Instrumentation students at the two-year college level. While decidedly unconventional and sometimes even unsettling for students accustomed to a more passive lecture environment, this instructional philosophy has proven its ability to convey conceptual mastery, foster careful analysis, and enhance employability so much better than lecture that the author refuses to ever teach by lecture again.

Problems which often go undiagnosed in a lecture environment are laid bare in this "inverted" format where students must articulate and logically defend their reasoning. This, too, may be unsettling for students accustomed to lecture sessions where the instructor cannot tell for sure who comprehends and who does not, and this vulnerability necessitates sensitivity on the part of the "inverted" session instructor in order that students never feel discouraged by having their errors exposed. *Everyone* makes mistakes from time to time, and learning is a lifelong process! Part of the instructor's job is to build a culture of learning among the students where errors are not seen as shameful, but rather as opportunities for progress.

To this end, instructors managing courses based on these modules should adhere to the following principles:

- Student questions are always welcome and demand thorough, honest answers. The only type of question an instructor should refuse to answer is one the student should be able to easily answer on their own. Remember, *the fundamental goal of education is for each student to learn to think clearly and independently.* This requires hard work on the part of the student, which no instructor should ever circumvent. Anything done to bypass the student's responsibility to do that hard work ultimately limits that student's potential and thereby does real harm.

- It is not only permissible, but encouraged, to answer a student's question by asking questions in return, these follow-up questions designed to guide the student to reach a correct answer through their own reasoning.

- All student answers demand to be challenged by the instructor and/or by other students. This includes both correct and incorrect answers – the goal is to practice the articulation and defense of one's own reasoning.

- No reading assignment is deemed complete unless and until the student demonstrates their ability to accurately summarize the major points in their own terms. Recitation of the original text is unacceptable. This is why every module contains an "Outline and reflections" question as well as a "Foundational concepts" question in the Conceptual reasoning section, to prompt reflective reading.

- No assigned question is deemed answered unless and until the student demonstrates their ability to consistently and correctly apply the concepts to *variations* of that question. This is why module questions typically contain multiple "Challenges" suggesting different applications of the concept(s) as well as variations on the same theme(s). Instructors are encouraged to devise as many of their own "Challenges" as they are able, in order to have a multitude of ways ready to probe students' understanding.

- No assigned experiment or project is deemed complete unless and until the student demonstrates the task in action. If this cannot be done "live" before the instructor, video-recordings showing the demonstration are acceptable. All relevant safety precautions must be followed, all test equipment must be used correctly, and the student must be able to properly explain all results. The student must also successfully answer all Challenges presented by the instructor for that experiment or project.

Students learning from these modules would do well to abide by the following principles:

- No text should be considered fully and adequately read unless and until you can express every idea *in your own words, using your own examples.*
- You should always articulate your thoughts as you read the text, noting points of agreement, confusion, and epiphanies. Feel free to print the text on paper and then write your notes in the margins. Alternatively, keep a journal for your own reflections as you read. This is truly a helpful tool when digesting complicated concepts.
- Never take the easy path of highlighting or underlining important text. Instead, *summarize* and/or *comment* on the text using your own words. This actively engages your mind, allowing you to more clearly perceive points of confusion or misunderstanding on your own.
- A very helpful strategy when learning new concepts is to place yourself in the role of a teacher, if only as a mental exercise. Either explain what you have recently learned to someone else, or at least *imagine* yourself explaining what you have learned to someone else. The simple act of having to articulate new knowledge and skill forces you to take on a different perspective, and will help reveal weaknesses in your understanding.
- Perform each and every mathematical calculation and thought experiment shown in the text on your own, referring back to the text to see that your results agree. This may seem trivial and unnecessary, but it is critically important to ensuring you actually understand what is presented, especially when the concepts at hand are complicated and easy to misunderstand. Apply this same strategy to become proficient in the use of *circuit simulation software*, checking to see if your simulated results agree with the results shown in the text.
- Above all, recognize that learning is hard work, and that a certain level of frustration is unavoidable. There are times when you will struggle to grasp some of these concepts, and that struggle is a natural thing. Take heart that it will yield with persistent and varied[4] effort, and never give up!

Students interested in using these modules for self-study will also find them beneficial, although the onus of responsibility for thoroughly reading and answering questions will of course lie with that individual alone. If a qualified instructor is not available to challenge students, a workable alternative is for students to form study groups where they challenge[5] one another.

To high standards of education,

Tony R. Kuphaldt

[4] As the old saying goes, "Insanity is trying the same thing over and over again, expecting different results." If you find yourself stumped by something in the text, you should attempt a different approach. Alter the thought experiment, change the mathematical parameters, do whatever you can to see the problem in a slightly different light, and then the solution will often present itself more readily.

[5] Avoid the temptation to simply share answers with study partners, as this is really counter-productive to learning. Always bear in mind that the answer to any question is far less important in the long run than the method(s) used to obtain that answer. The goal of education is to empower one's life through the improvement of clear and independent thought, literacy, expression, and various practical skills.

Appendix C

Tools used

I am indebted to the developers of many open-source software applications in the creation of these learning modules. The following is a list of these applications with some commentary on each.

You will notice a theme common to many of these applications: a bias toward *code*. Although I am by no means an expert programmer in any computer language, I understand and appreciate the flexibility offered by code-based applications where the user (you) enters commands into a plain ASCII text file, which the software then reads and processes to create the final output. Code-based computer applications are by their very nature *extensible*, while WYSIWYG (What You See Is What You Get) applications are generally limited to whatever user interface the developer makes for you.

The `GNU/Linux` computer operating system

> There is so much to be said about Linus Torvalds' `Linux` and Richard Stallman's `GNU` project. First, to credit just these two individuals is to fail to do justice to the *mob* of passionate volunteers who contributed to make this amazing software a reality. I first learned of `Linux` back in 1996, and have been using this operating system on my personal computers almost exclusively since then. It is *free*, it is completely *configurable*, and it permits the continued use of highly efficient `Unix` applications and scripting languages (e.g. shell scripts, Makefiles, `sed`, `awk`) developed over many decades. `Linux` not only provided me with a powerful computing platform, but its open design served to inspire my life's work of creating open-source educational resources.

Bram Moolenaar's `Vim` text editor

> Writing code for any code-based computer application requires a *text editor*, which may be thought of as a word processor strictly limited to outputting plain-ASCII text files. Many good text editors exist, and one's choice of text editor seems to be a deeply personal matter within the programming world. I prefer `Vim` because it operates very similarly to `vi` which is ubiquitous on `Unix`/`Linux` operating systems, and because it may be entirely operated via keyboard (i.e. no mouse required) which makes it fast to use.

Donald Knuth's TEX typesetting system

Developed in the late 1970's and early 1980's by computer scientist extraordinaire Donald Knuth to typeset his multi-volume magnum opus *The Art of Computer Programming*, this software allows the production of formatted text for screen-viewing or paper printing, all by writing plain-text code to describe how the formatted text is supposed to appear. TEX is not just a markup language for documents, but it is also a Turing-complete programming language in and of itself, allowing useful algorithms to be created to control the production of documents. Simply put, TEX *is a programmer's approach to word processing*. Since TEX is controlled by code written in a plain-text file, this means anyone may read that plain-text file to see exactly how the document was created. This openness afforded by the code-based nature of TEX makes it relatively easy to learn how other people have created their own TEX documents. By contrast, examining a beautiful document created in a conventional WYSIWYG word processor such as Microsoft `Word` suggests nothing to the reader about *how* that document was created, or what the user might do to create something similar. As Mr. Knuth himself once quipped, conventional word processing applications should be called WYSIAYG (What You See Is *All* You Get).

Leslie Lamport's LATEX extensions to TEX

Like all true programming languages, TEX is inherently extensible. So, years after the release of TEX to the public, Leslie Lamport decided to create a massive extension allowing easier compilation of book-length documents. The result was LATEX, which is the markup language used to create all ModEL module documents. You could say that TEX is to LATEX as `C` is to `C++`. This means it is permissible to use any and all TEX commands within LATEX source code, and it all still works. Some of the features offered by LATEX that would be challenging to implement in TEX include automatic index and table-of-content creation.

Tim Edwards' `Xcircuit` drafting program

This wonderful program is what I use to create all the schematic diagrams and illustrations (but not photographic images or mathematical plots) throughout the ModEL project. It natively outputs PostScript format which is a true vector graphic format (this is why the images do not pixellate when you zoom in for a closer view), and it is so simple to use that I have never had to read the manual! Object libraries are easy to create for `Xcircuit`, being plain-text files using PostScript programming conventions. Over the years I have collected a large set of object libraries useful for drawing electrical and electronic schematics, pictorial diagrams, and other technical illustrations.

`Gimp` graphic image manipulation program

Essentially an open-source clone of Adobe's `PhotoShop`, I use `Gimp` to resize, crop, and convert file formats for all of the photographic images appearing in the ModEL modules. Although `Gimp` does offer its own scripting language (called `Script-Fu`), I have never had occasion to use it. Thus, my utilization of `Gimp` to merely crop, resize, and convert graphic images is akin to using a sword to slice bread.

`SPICE` circuit simulation program

`SPICE` is to circuit analysis as TEX is to document creation: it is a form of markup language designed to describe a certain object to be processed in plain-ASCII text. When the plain-text "source file" is compiled by the software, it outputs the final result. More modern circuit analysis tools certainly exist, but I prefer `SPICE` for the following reasons: it is *free*, it is *fast*, it is *reliable*, and it is a fantastic tool for *teaching* students of electricity and electronics how to write simple code. I happen to use rather old versions of `SPICE`, version 2g6 being my "go to" application when I only require text-based output. `NGSPICE` (version 26), which is based on Berkeley `SPICE` version 3f5, is used when I require graphical output for such things as time-domain waveforms and Bode plots. In all `SPICE` example netlists I strive to use coding conventions compatible with all `SPICE` versions.

Andrew D. Hwang's `ePiX` mathematical visualization programming library

This amazing project is a `C++` library you may link to any `C/C++` code for the purpose of generating PostScript graphic images of mathematical functions. As a completely free and open-source project, it does all the plotting I would otherwise use a Computer Algebra System (CAS) such as `Mathematica` or `Maple` to do. It should be said that `ePiX` is *not* a Computer Algebra System like `Mathematica` or `Maple`, but merely a mathematical *visualization* tool. In other words, it won't determine integrals for you (you'll have to implement that in your own `C/C++` code!), but it can graph the results, and it does so beautifully. What I really admire about `ePiX` is that it is a `C++` programming library, which means it builds on the existing power and toolset available with that programming language. Mr. Hwang could have probably developed his own stand-alone application for mathematical plotting, but by creating a `C++` library to do the same thing he accomplished something much greater.

`gnuplot` mathematical visualization software

Another open-source tool for mathematical visualization is `gnuplot`. Interestingly, this tool is *not* part of Richard Stallman's GNU project, its name being a coincidence. For this reason the authors prefer "gnu" *not* be capitalized at all to avoid confusion. This is a much "lighter-weight" alternative to a spreadsheet for plotting tabular data, and the fact that it easily outputs directly to an X11 console or a file in a number of different graphical formats (including PostScript) is very helpful. I typically set my `gnuplot` output format to default (X11 on my Linux PC) for quick viewing while I'm developing a visualization, then switch to PostScript file export once the visual is ready to include in the document(s) I'm writing. As with my use of `Gimp` to do rudimentary image editing, my use of `gnuplot` only scratches the surface of its capabilities, but the important points are that it's *free* and that it *works well*.

`Python` programming language

Both Python and C++ find extensive use in these modules as instructional aids and exercises, but I'm listing Python here as a *tool* for myself because I use it almost daily as a *calculator*. If you open a Python interpreter console and type `from math import *` you can type mathematical expressions and have it return results just as you would on a hand calculator. Complex-number (i.e. *phasor*) arithmetic is similarly supported if you include the complex-math library (`from cmath import *`). Examples of this are shown in the Programming References chapter (if included) in each module. Of course, being a fully-featured programming language, Python also supports conditionals, loops, and other structures useful for calculation of quantities. Also, running in a console environment where all entries and returned values show as text in a chronologically-ordered list makes it easy to copy-and-paste those calculations to document exactly how they were performed.

Appendix D

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Appendix E

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Appendix F

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2-3 February 2025 – removed content duplicated in other ModEL modules and introduced new content copied from the *Lessons In Industrial Instrumentation* textbook.

30 March 2024 – document first created.

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