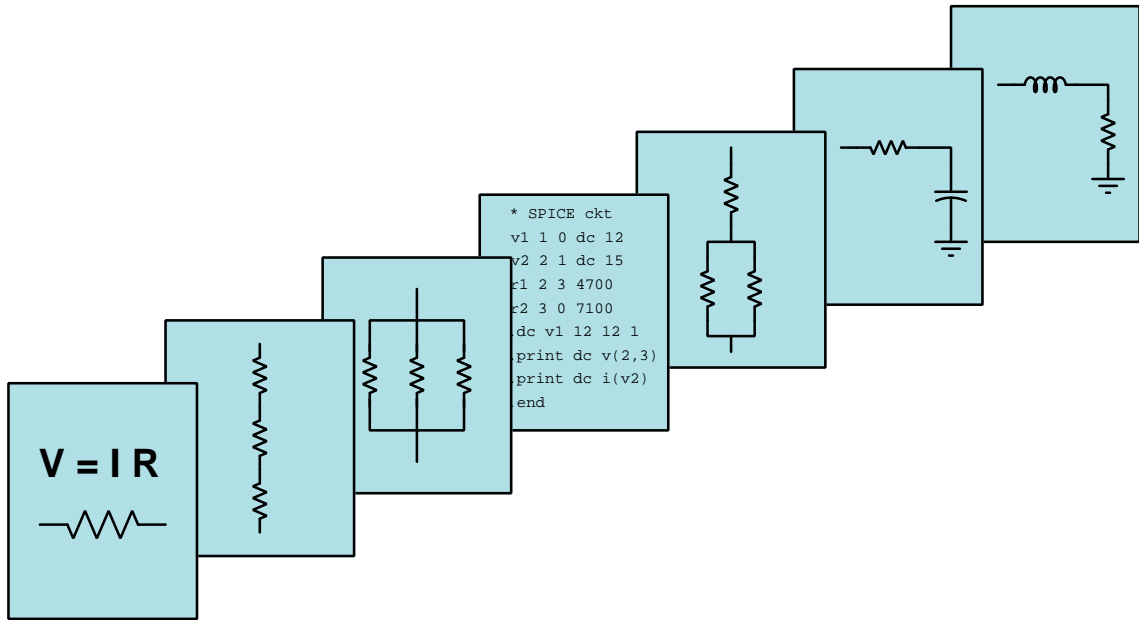


MODULAR ELECTRONICS LEARNING (MODEL) PROJECT



E AND H FIELD PROBING

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Chapter 1

Introduction

Chapter 2

Case Tutorial

The idea behind a *Case Tutorial* is to explore new concepts by way of example. In this chapter you will read less presentation of theory compared to other Tutorial chapters, but by close observation and comparison of the given examples be able to discern patterns and principles much the same way as a scientific experimenter. Hopefully you will find these cases illuminating, and a good supplement to text-based tutorials.

These examples also serve well as challenges following your reading of the other Tutorial(s) in this module – can you explain *why* the circuits behave as they do?

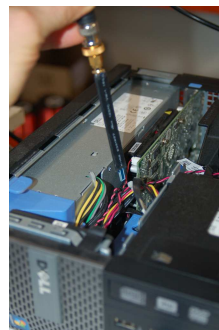
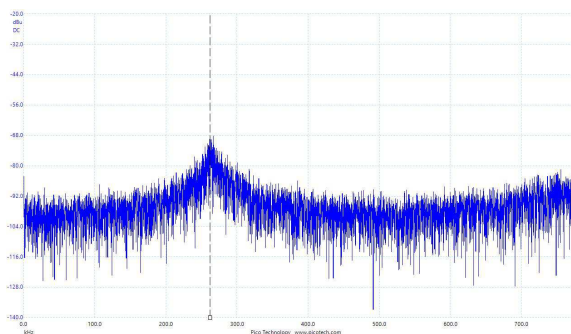
2.1 Example: field probing inside a PC

AC electric and magnetic fields may be qualitatively measured on printed circuit boards (PCBs) using special probes called *near-field probes*. These probes usually have BNC- or SMA-style coaxial cable connectors suitable for connection to the input of an oscilloscope or a spectrum analyzer.

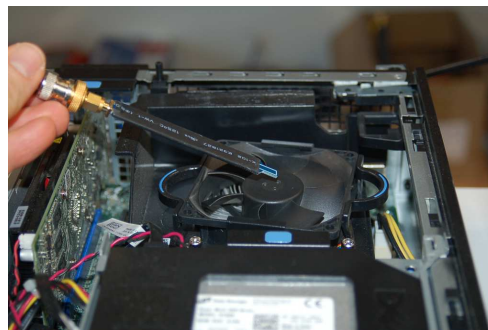
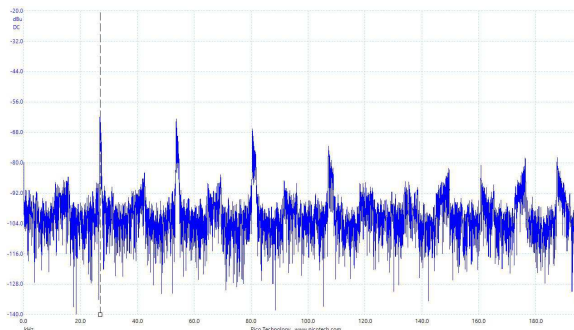
E-field probes sense electric fields with respect to ground, and consist of a metallic electrode coated in plastic (to avoid direct contact with circuit conductors) with a connector appropriate to the input of an oscilloscope or a spectrum analyzer. E-field probes tend to be omnidirectional in their sensing.

H-field probes sense magnetic fields created by current-carrying conductors, and consist of conductive loops (also insulated to avoid direct contact with circuit conductors) generating small AC voltages when exposed to AC magnetic fields. H-field probes tend to be planar in their sensing, which means they are maximally sensitive to magnetic fields caused by current through conductors parallel to the plane of the probe's loop(s) and minimally sensitive to magnetic fields perpendicular to that.

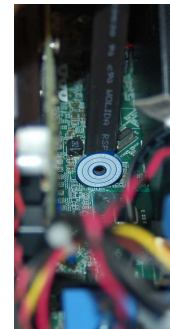
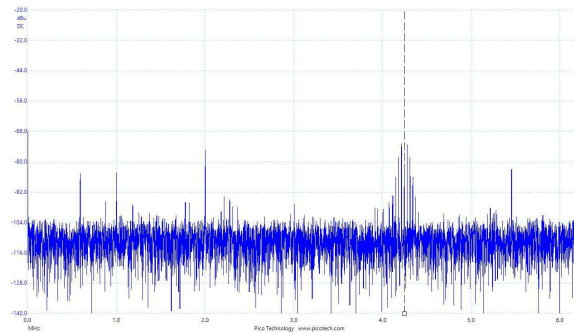
This first set of images shows the signal spectrum (left) and photograph (right) of an E-field probe held near a pair of wires connecting to a video processing PCB inside of a personal computer, picking up a spread of frequencies centered around 260 kHz:



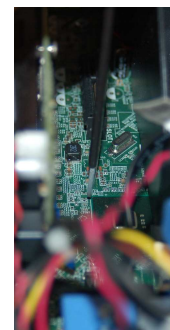
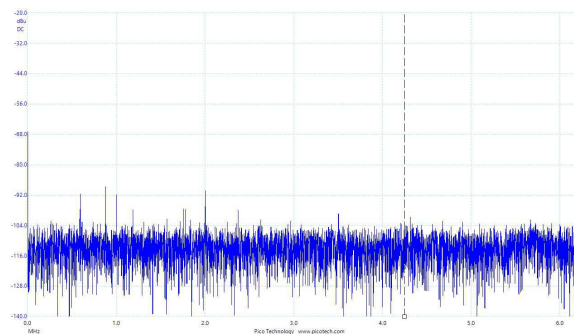
Next we see the same E-field probe held over the hub of a cooling fan motor, intercepting the electric field from a square-wave voltage signal with a fundamental frequency of approximately 27 kHz as well as its associated odd harmonics:



Switching to an H-field probe, the following two examples illustrate the polarization of this probe. In the first set of images we see the spectrum detected by placing the probe with its loop parallel to the current-carrying trace on the PC's motherboard, detecting a cluster of signals centered around 4.25 MHz with maximum sensitivity because parallel conductors ensure mutually perpendicularity with the magnetic field lines:



In the second image set we see the cluster of frequencies all but gone with the probe loop rotated to be perpendicular with the offending PCB trace. The pulsed current still flows through the PCB trace, but because the probe's loop is now perpendicular to that trace it means the magnetic field lines are now parallel to the probe loop, and therefore induction (from that signal) no longer occurs:

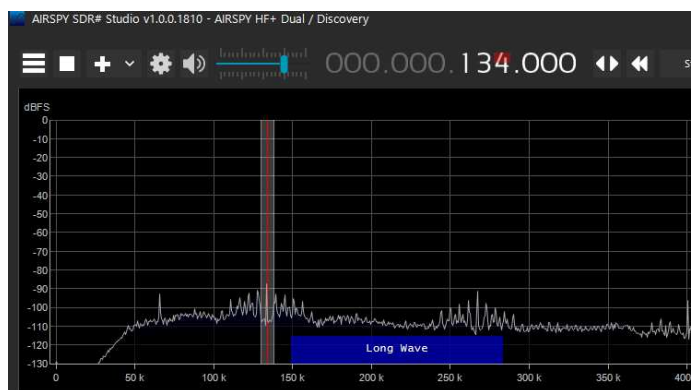


2.2 Example: SDR as a probe analyzer

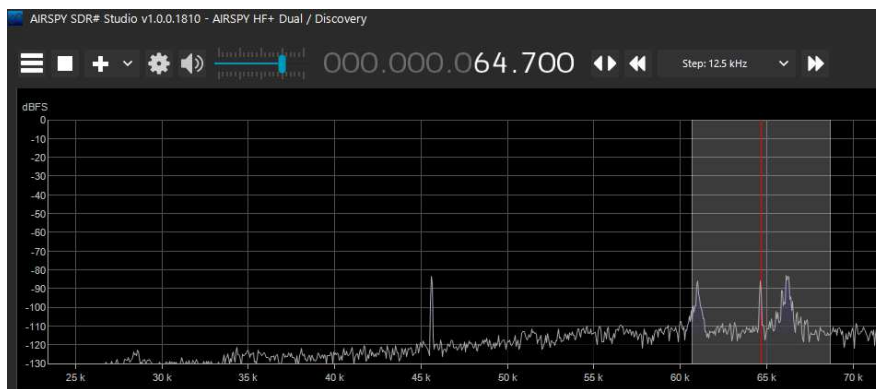
A spectrum analyzer is an ideal instrument to display signals intercepted by E-field and H-field probes, but spectrum analyzers are typically quite expensive. While even inexpensive digital oscilloscopes come equipped with Fast Fourier Transform (FFT) capability to display signal spectra, the bandwidth of an entry-level oscilloscope may be insufficient to reliably show frequencies of interest in near-field probing.

However, a viable alternative exists for affordable near-field probing, and that is the use of a Software-Defined Radio (SDR) receiver unit. An SDR receiver, paired with appropriate software to display the spectrum of any received signals, works quite well as a broadband spectrum analyzer, especially when precise measurement of signal amplitude is unimportant. When probing for electric and/or magnetic fields, we care about the frequencies of the intercepted signals and their *relative* amplitudes, but are generally uninterested in determining precisely how strong each signal is because the signal strength varies so much depending on how we hold the probe.

The first image here is of an AirSpy brand SDR receiving a signal from an E-field probe:



Next, we see the spectrum from an H-field probe:



Both spectra were detected outside the case of an uninterruptible power supply (UPS) unit for a personal computer.

2.3 Example: measuring signal rates of change

Capacitors and inductors relate voltage to current by rates of change. For a capacitor, the amount of current is proportional to how quickly voltage across that capacitor either rises or falls over time ($I = C \frac{dV}{dt}$). For an inductor, the amount of voltage is proportional to how quickly current through that inductor rises or falls over time ($V = L \frac{dI}{dt}$).

For example, a 330 microFarad capacitor experiencing a voltage increasing at a rate of 45 Volts per second will pass 14.85 milliAmperes. If the voltage happens to decrease at an equivalent rate (i.e. $\frac{dV}{dt} = -45$ Volts per second) then the 14.85 milliAmpere current will reverse direction through the capacitor compared to how it flowed with the increasing voltage.

Similarly, a 100 milliHenry inductor experiencing a current increasing at a rate of 5 Amperes per second will induce a voltage of 500 milliVolts. If the current happens to decrease at an equivalent rate (i.e. $\frac{dI}{dt} = -5$ Amperes per second) then the 500 milliVolt voltage induced across the inductor will reverse polarity from what it was during the period of increasing current.

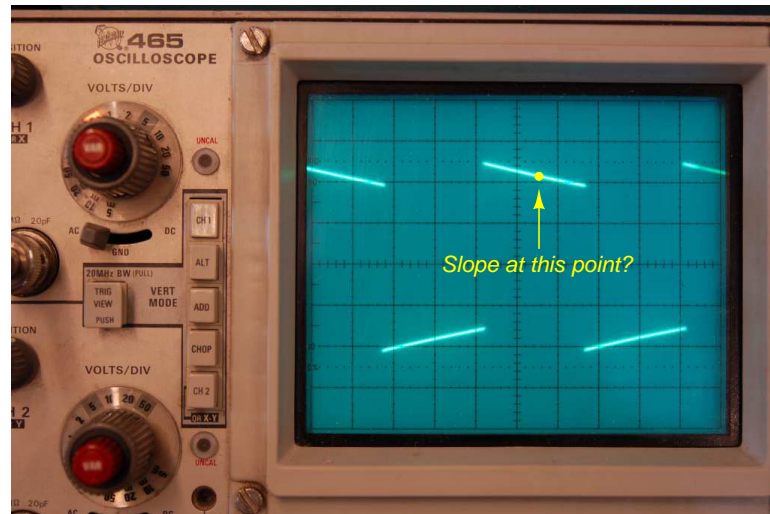
Not only are rates-of-change important in determining how energy-storing devices such as capacitors and inductors will respond in circuits, but rates-of-change are also important for determining how *parasitic* capacitances and inductances will affect intended circuit behavior. Parasitic capacitance exists between any two conducting surfaces separated by an electrically insulating medium, and parasitic inductance exists along any length of conductor. This means any rate-of-change of voltage over time between two separated conductors will cause some amount of current to “pass” between them, and that any rate-of-change of current over time through any single conductor will cause some amount of voltage to drop across its length. In many circuits these parasitic effects are negligible, but in circuits experiencing extremely fast rates of change for voltage and/or current the effects can be significant or even severe.

Oscilloscopes are ideal for performing empirical measurements of voltage rates-of-change, and of current rates-of-change given the proper accessories¹. Some skill is required to do this, though, and here we will explore practical examples to show how it is done.

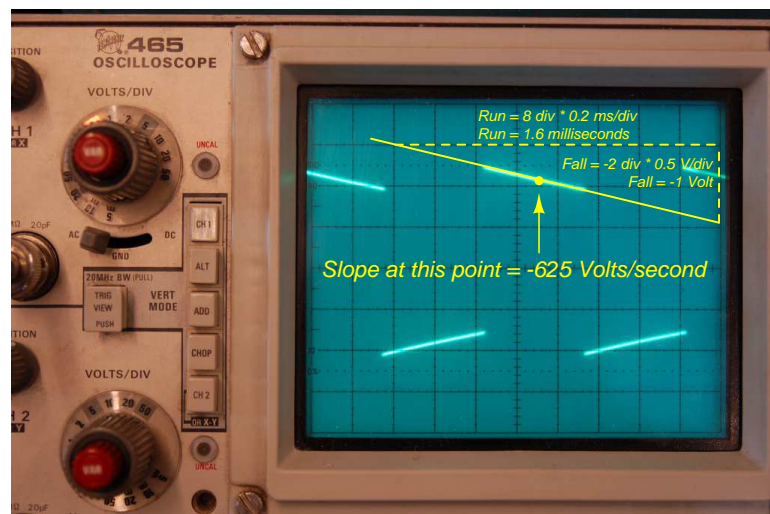
For any signal plotted in the time domain, where the horizontal axis of the plot is expressed in units of seconds, milliseconds, microseconds, etc., the signal’s rate of change at any given point will be the *slope* or *pitch* of the waveform, mathematically defined as its *rise over run*. A great aid to discerning slope at any location on a waveform is to sketch a straight line visually matching the wave’s slope at that point, then use locations along that straight line to more easily discern how far it rises (or falls) over some “run” of time. We call this straight line a *tangent line*.

¹For example, a *current probe* converting a sensed current into a voltage the oscilloscope may directly sense, or a *shunt resistor* placed in the circuit developing an oscilloscope-measurable voltage drop for any current passing through.

Here we see an example of a waveform with sloping sections. In the first image we see a specific location on the waveform where we wish to measure voltage rate-of-change ($\frac{dV}{dt}$):



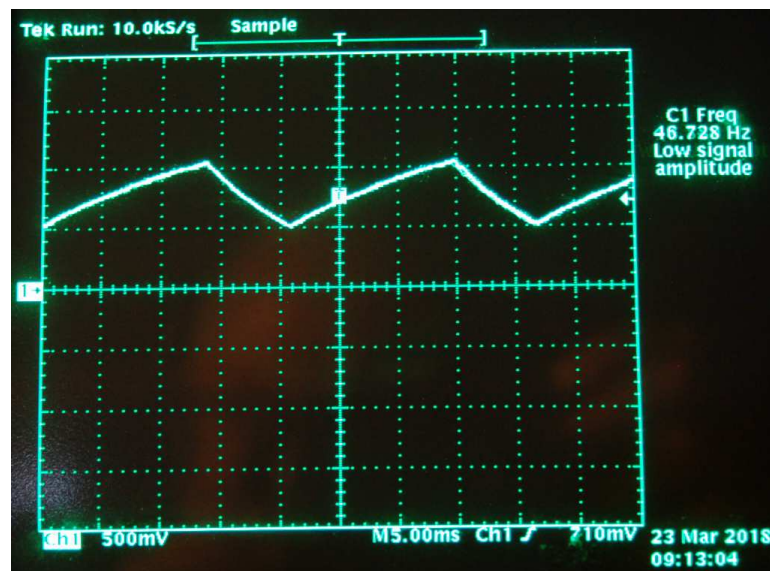
Next we see a tangent line drawn to match the slope of the waveform at the specified location, with convenient points for fall/run measurements taken on that line against the oscilloscope grid's major divisions. In this example, the oscilloscope's vertical sensitivity has been set for 0.5 Volts per division, and the horizontal timebase for 0.2 milliseconds per division:



As we can see, the tangent line falls 2 vertical divisions (-1 Volt) over a timespan of 8 horizontal divisions (1.6 milliseconds), yielding a $\frac{dV}{dt}$ quotient of -625 Volts per second, which may also be expressed as -0.625 Volts per millisecond. The negative sign is important, as it distinguishes this particular rate-of-change as *falling* rather than rising over time.

Such rate-of-change measurements are necessarily approximate, as they require us to visually gauge where a tangent line may be overlaid on the waveform's oscillograph, and also to visually assess the slope of that tangent line using the grid provided on the instrument's display screen. However, in most applications extremely precise rate-of-changes are not necessary, and such techniques suffice quite well.

Below is another example of a waveform with sloping sections, the oscilloscope configured for 500 milliVolts per division on the vertical axis and 5 milliseconds per division on the horizontal:

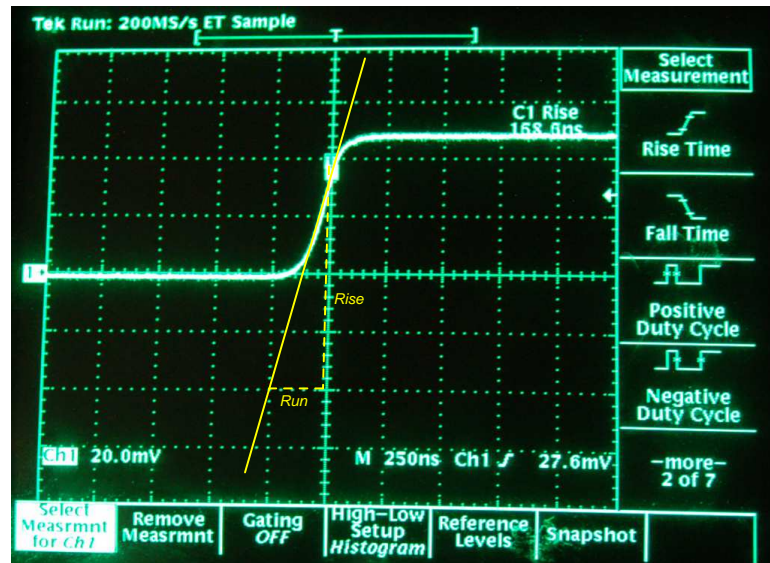


Approximating the slope for each rising section of this wave, we count one division of rise over 2.8 divisions of run, or 500 milliVolts rise over 14 milliseconds of run. This is a rate-of-rise of +35.7 Volts per second.

Approximating the slope for each falling section of this wave, we count one division of fall over 1.4 divisions of run, or -500 milliVolts fall over 7 milliseconds of run. This is a rate-of-fall of -71.4 Volts per second.

If these rates-of-change appear suspiciously large compared to the actual amplitude of the waveform, which barely crests over +1 Volt on the oscillograph, bear in mind that we are calculating *rates of change* for voltage and not absolute values of voltage itself. This is analogous to the distinction between *speed* and *distance*: traveling at a rate of 30 kilometers per hour does not necessarily mean you will travel 30 kilometers, as the actual distance traveled depends on how long that speed is sustained. A voltage rising at a rate of 35.7 Volts per second would indeed rise 35.7 Volts *if given a full second to do so*, but since each rising/falling portion of this waveform is so short in duration the actual amount of rise or fall in each case is only one-half of one Volt. It is therefore perfectly appropriate to consider any $\frac{dV}{dt}$ value as being the *speed* at which a voltage increases or decreases over time, distinct from the actual value of that voltage at any particular moment in time.

Here we see another oscillograph, this one zoomed into the rising edge of a square wave. For this measurement the oscilloscope was configured for 20 milliVolts per division of vertical sensitivity and a timebase of 250 nanoseconds per division on the horizontal:



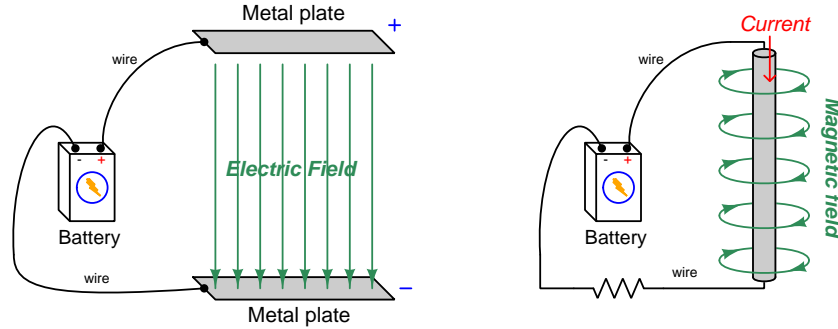
The tangent line overlaid on this screenshot for the purpose of measuring the pulse edge's rate-of-change rises approximately 4 vertical divisions over a run of 1 division, which is 80 milliVolts of rise over 250 nanoseconds of run. The pulse edge's rate-of-change, therefore, is approximately +320,000 Volts per second, or +320 Volts per millisecond, or +0.32 Volts per microsecond (all equivalent expressions of $\frac{dV}{dt}$).

Chapter 3

Tutorial

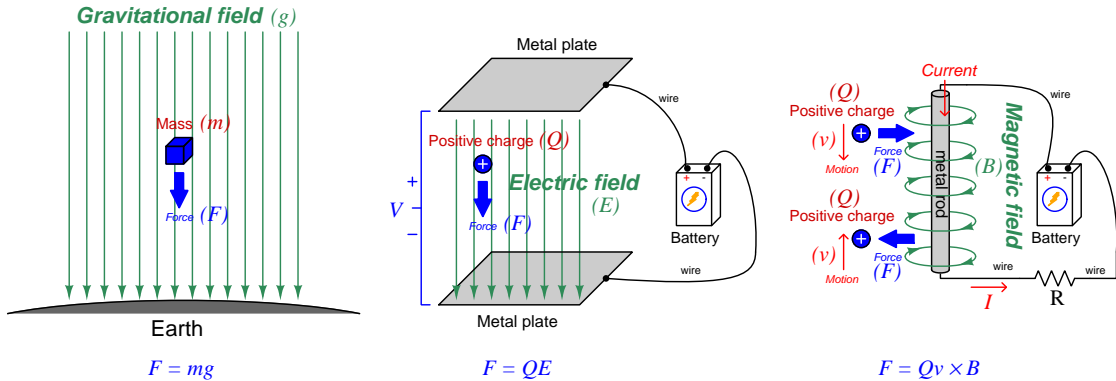
3.1 Electric versus magnetic fields

An electric field exists wherever voltage exists (i.e. wherever an imbalance exists between positive and negative electrical charges), parallel to the axis of that voltage. A magnetic field exists wherever an electric charge moves, perpendicular to the axis of that charge's motion. These fields may be represented as lines or curves in an illustration:



These two types of fields differ from each other in multiple ways. Electric fields terminate at conductors having opposite voltage polarity, while magnetic fields “loop” around current-carrying conductors and have no termination points at all. Electric field strength is proportional to the magnitude of the *voltage*, while magnetic field strength is proportional to the magnitude of the *current*.



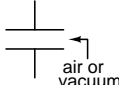



Fields are often defined and quantified in terms of the amount of *force* they exert on matter. This is true for gravitational, electric, and magnetic fields alike. Gravitational fields (g) act on mass (m) to produce force (F). Electric fields (E) act on electric charges (Q) to produce force (F). Magnetic fields (B) act on *moving* charges (charge Q with velocity v) to produce force (F):



Electric and magnetic fields alike are natural consequences of voltage and current, respectively. These fields are found *everywhere* in energized electric circuits, but are generally weak enough that we do not notice their forces.

3.2 Capacitance

Capacitance may be defined as the ability to store energy within an electric field as the result of an applied voltage. Although electrical components called *capacitors*¹ exist to exploit this phenomenon, any pair of electrically conductive surfaces separated by a layer of electrical insulation (called a *dielectric*) will exhibit capacitance, the amount of capacitance directly proportional to the overlapping area of the conductive surfaces (in square meters), directly proportional to the dielectric *permittivity*² (ϵ) of the dielectric material, and inversely proportional to the separation distance between the conductors (in meters):

	Less C	More C	
Area (A)			
Permittivity (ϵ)			$C = \frac{\epsilon A}{d}$
Distance (d)			

The mathematical relationship between voltage, current, and capacitance (i.e. the “Ohm’s Law” for capacitance) is quite different than Ohm’s Law for resistance, because it relates the magnitude of current to the *rate of change over time* of voltage:

$$I = C \frac{dV}{dt}$$

Where,

I = Current through capacitor, in Amperes

C = Capacitance, in Farads

$\frac{dV}{dt}$ = Rate-of-change of voltage across the capacitance over time, in Volts per second

One way to conceptualize this rate-of-change is to think of it as being the *speed* of the voltage. For example, if a voltage steadily rises from 150 Volts to 152 Volts over a period of half a second, the $\frac{dV}{dt}$ would be:

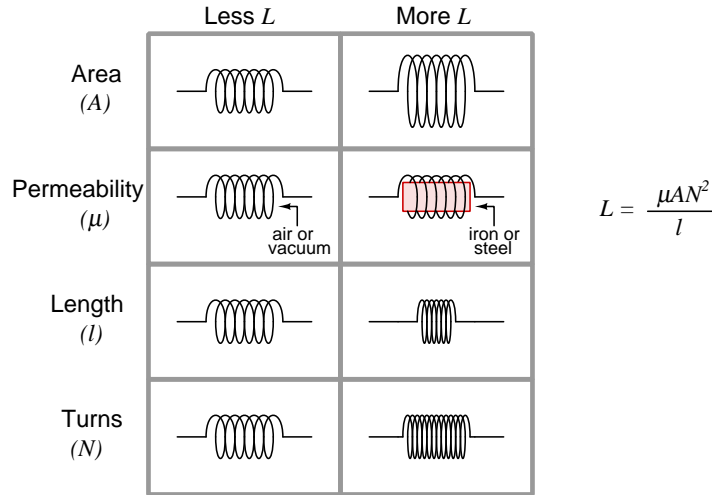
$$\frac{dV}{dt} = \frac{152 - 150}{0.5} = +4 \text{ Volts per second}$$

¹An obsolete term for capacitor is *condensor* or *condenser*.

²Permittivity, simply defined, is a measure of how effective a dielectric substance is at electrostatically storing energy for any given electric field strength. It may be measured in absolute terms, expressed in units of Farads per meter, or it may be expressed as a ratio relative to the permittivity of a perfect vacuum.

3.3 Inductance

Inductance may be defined as the ability to store energy within a magnetic field as the result of a current. Although electrical components called *inductors*³ exist to exploit this phenomenon, any conductor will exhibit inductance. Inductors are generally constructed with wire formed into a coil or loop, the amount of inductance directly proportional to the cross-sectional area enclosed by the coil, directly proportional to the square of the number of turns of wire in that coil, directly proportional to the magnetic permeability⁴ (μ) of any material enclosed by the coil, and inversely proportional to the axial length of the coil (in meters):



The mathematical relationship between voltage, current, and inductance (i.e. the “Ohm’s Law” for inductance) is quite different than Ohm’s Law for resistance, because it relates the magnitude of voltage to the *rate of change over time* of current:

$$V = L \frac{dI}{dt}$$

Where,

V = Voltage across inductor, in Volts

L = Inductance, in Henrys

$\frac{dI}{dt}$ = Rate-of-change of current through the inductor over time, in Amperes per second

One way to conceptualize this rate-of-change is to think of it as being the *speed* of the current. For example, if a current steadily falls from 10 milliAmperes to 7 milliAmperes over a period of 2 microseconds, the $\frac{dI}{dt}$ would be:

$$\frac{dI}{dt} = \frac{(7 \times 10^{-3}) - (10 \times 10^{-3})}{2 \times 10^{-6}} = -1500 \text{ Amperes per second}$$

³Also referred to as *reactors* in high-power electrical circuits, and sometimes as *chokes* in electronic circuits.

⁴Permeability, simply defined, is a measure of how effective a substance is at magnetically storing energy for any given amount of magnetomotive force (MMF). It may be measured in absolute terms, expressed in units of Henrys per meter, or it may be expressed as a ratio relative to the permeability of a perfect vacuum.

3.4 Electromagnetic induction

When an electric current passes through a conductor, the motion of those drifting charge carriers produces a magnetic field, and we call this phenomenon *electromagnetism*. A complementary phenomenon is *electromagnetic induction*, where magnetism may produce a voltage. Induction happens when a conductor experiences a perpendicular magnetic field varying in strength over time. This is how *generators* are constructed: by moving coils of wire past a stationary magnetic field, or vice-versa. The amount of voltage induced across the coil is proportional to the number of turns of wire in that coil and to the magnetic field's rate-of-change over time. This mathematical relationship is expressed in *Faraday's Law of Electromagnetic Induction*:

$$V = N \frac{d\Phi}{dt}$$

Where,

V = Induced voltage across the coil, in Volts

N = Number of “turns” or “wraps” in the coil

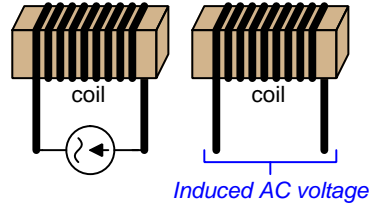
$\frac{d\Phi}{dt}$ = Rate-of-change of magnetic flux⁵, in Webers per second

Electromagnetic induction is proportional to the *rate* at which a magnetic field strengthens or weakens perpendicular to a conductive coil. This means a voltmeter connected to a wire coil will register nothing when a nearby magnet is stationary, but will register voltage every time the magnet *moves* (either toward the coil or away from it):



⁵Magnetic flux is related to magnetic field strength by cross-sectional area, with $B = \frac{\Phi}{A}$. Thus, it is valid to think of magnetic field strength as the degree to which magnetic flux is *concentrated* into an area. This holds true for the units of measurement used to express each: the unit of the *Tesla* used for magnetic field strength (B) is equivalent to *Webers of flux per square meter*.

Mutual induction is when a time-varying current through one conductor produces a time-varying magnetic field, which in turn induces voltage in an adjacent conductor:



We may quantify this effect by labeling it L_M (mutual inductance)

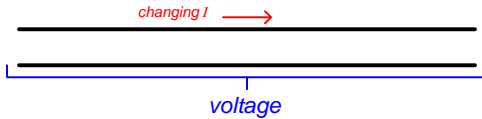
$$L_M = k\sqrt{L_1 L_2}$$

Where,

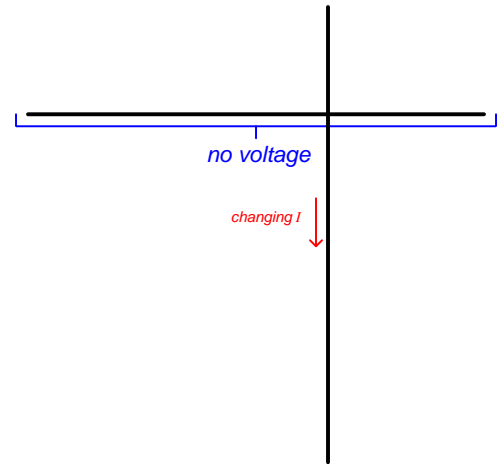
- L_M = Mutual inductance, in Henrys
- L_1 = Inductance of first coil, in Henrys
- L_2 = Inductance of second coil, in Henrys
- k = Coefficient of coupling between the two coils

The coupling coefficient (k) is a unitless value ranging from 0 to 1, inclusive. If $k = 1$ it means 100% of the magnetic field produced by the energized coil “links” properly to the other coil; if $k = 0$ it means none of the magnetism of one coil links to the second coil. This coupling factor k is not only related to how close the two coils lie near each other, but also their relative orientations. Remember that the magnetic field produced by a conductor will have its lines of flux perpendicular to that current, and that induction requires the varying magnetic field to also be perpendicular to the conductor experiencing induction. *Therefore, the mutual inductance is greatest when the two conductors lie parallel to each other, and will be least when the two conductors are perpendicular:*

Conductors parallel to each other



Conductors perpendicular to each other

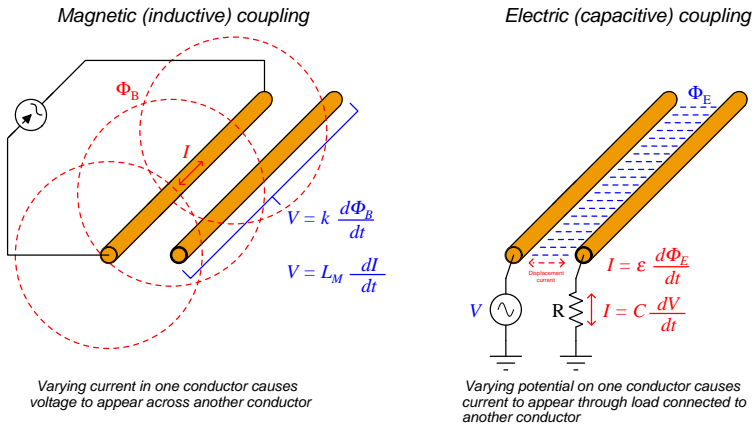


3.5 Signal coupling

If conductors lie too close to one another, AC or pulsing DC electrical signals may “couple” from one to the other(s). This can be especially detrimental to signal integrity when the coupling occurs between AC power conductors and low-voltage signal wiring, but it may also occur between sets of signal wires or between sets of traces on a printed circuit board (PCB). Terms commonly used to describe such “crosstalk” is to call the conductor generating the interference the *aggressor* and the conductor receiving the interference the *victim*.

Two mechanisms of electrical “coupling” exist: *inductive* and *capacitive*. Both are capable of “coupling” signals from one conductor to another, and they usually exist in tandem.

- Inductance is a property intrinsic to any conductor, whereby energy is stored in the magnetic field formed by current through the wire. Mutual inductance existing between parallel wires forms another “bridge” whereby an AC current through one wire is able to induce an AC voltage along the length of another wire. Specifically, the amount of voltage induced across the length of the victim conductor is proportional to the mutual inductance between the victim and aggressor conductors as well as the rate-of-change of current through the aggressor conductor.
- Capacitance is a property intrinsic to any pair of conductors separated by a dielectric (an insulating substance), whereby energy is stored in the electric field formed by voltage between the wires. The natural capacitance existing between mutually insulated wires forms a “bridge” for AC signals to cross between those wires. Specifically, the amount of current induced in the victim conductor is proportional to the capacitance and to the rate-of-change of voltage between the victim and aggressor conductors.

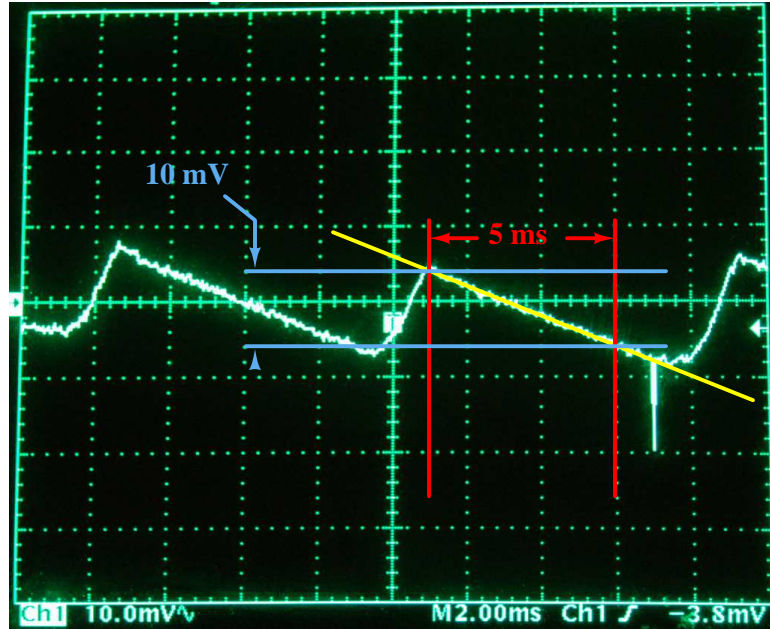


The fundamental “Ohm’s Law” formulae for inductance⁶ and capacitance is central to the phenomenon of signal coupling between conductors:

$$V = L \frac{dI}{dt} \quad I = C \frac{dV}{dt}$$

This is why signals “couple” more effectively between aggressor and victim conductors at higher frequency than at lower frequency. All other factors being equal, a greater frequency means the signal’s waveform will have steeper rise and fall rates ($\frac{dV}{dt}$ for voltages, $\frac{dI}{dt}$ for currents), which in turn results in greater influence from one conductor to the other.

An example of how $\frac{dV}{dt}$ may be estimated for a captured sawtooth-shaped voltage waveform appears in this next photograph, the annotations overlaid on the photograph after it was captured on camera from the display of an oscilloscope:



Here the yellow line follows the downward slope of the voltage waveform, while the red lines mark a time interval along that slope and the blue lines mark the voltage fall over that time interval. With the values shown here, -10 milliVolts of fall over 5 milliseconds of time, the estimated $\frac{dV}{dt}$ for this signal’s downward slope is -2 Volts per second.

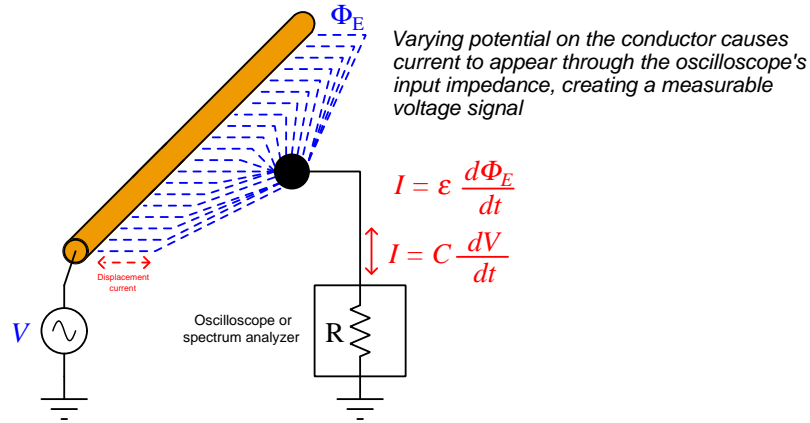
Interestingly, the downward “glitch” seen near the lower-right area of this oscillograph exhibits much larger rates-of-change than the relatively gentle slopes. Even though the vertical height of this transient is not very large, its extremely short timespan means its leading edge will have a large negative slope ($-\frac{dV}{dt}$) and its trailing edge will have a large positive slope ($+\frac{dV}{dt}$), possibly hundreds or even thousands of *Volts per second* each.

⁶For the particular case of coupling between conductors, the inductive “Ohm’s Law” formula could be written more precisely as $V_{victim} = L_M \frac{dI_{aggressor}}{dt}$, with L_M representing *mutual* inductance.

3.6 Electric field probing

In order to measure the electric field emanating from a conductor, whether a free wire or a trace on a printed circuit board (PCB), we need to establish a coupling capacitance between that conductor and one lead of a test instrument:

Electric field probing

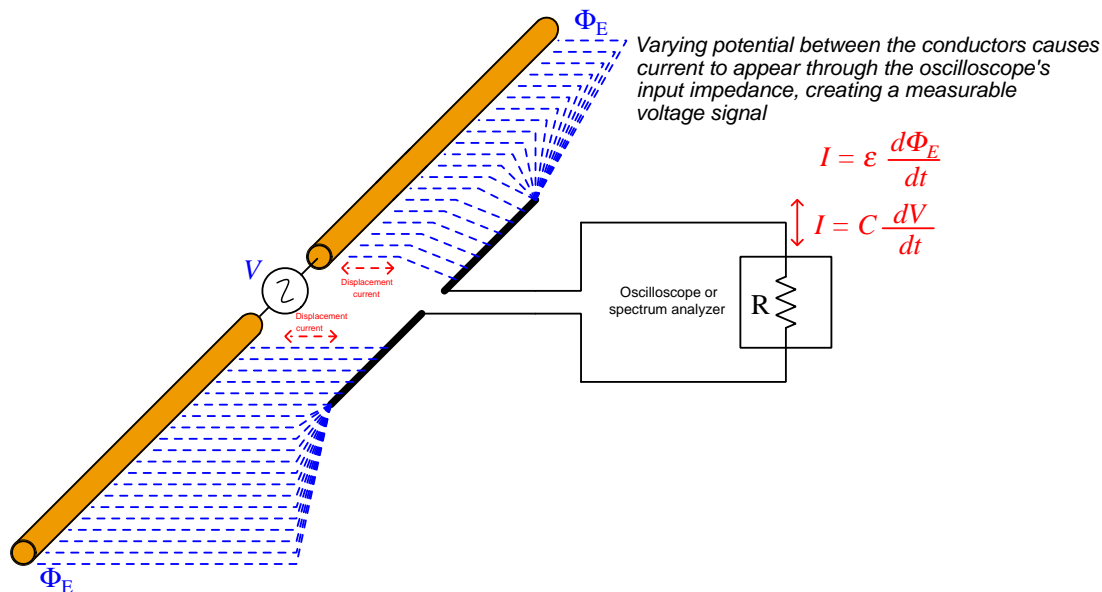


The ball-shaped electrode in the illustration is commonly referred to as an *E-field probe*⁷ because its purpose is to sense AC electric fields. Such probes are commonly used to locate sources of electric field emissions on the surface of a printed circuit board, moving the probe's tip to different locations on the PCB until the strongest offending signal is displayed on the test instrument.

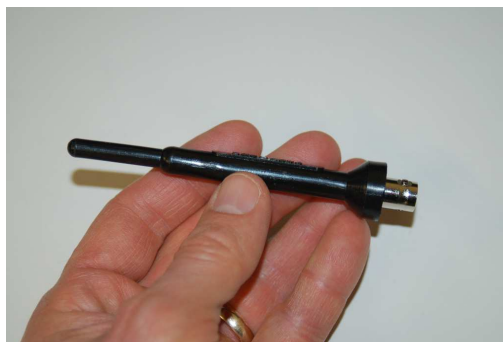
⁷In physics, E is often used to symbolize the strength of the electric field in units of Volts per meter.

A ball-tipped E-field probe is designed to sense ground-referenced electric fields and is completely insensitive to orientation, but some E-field probes utilize a *dipole* construction for differential field measurement. Orientation definitely matters in this case:

Differential electric field probing

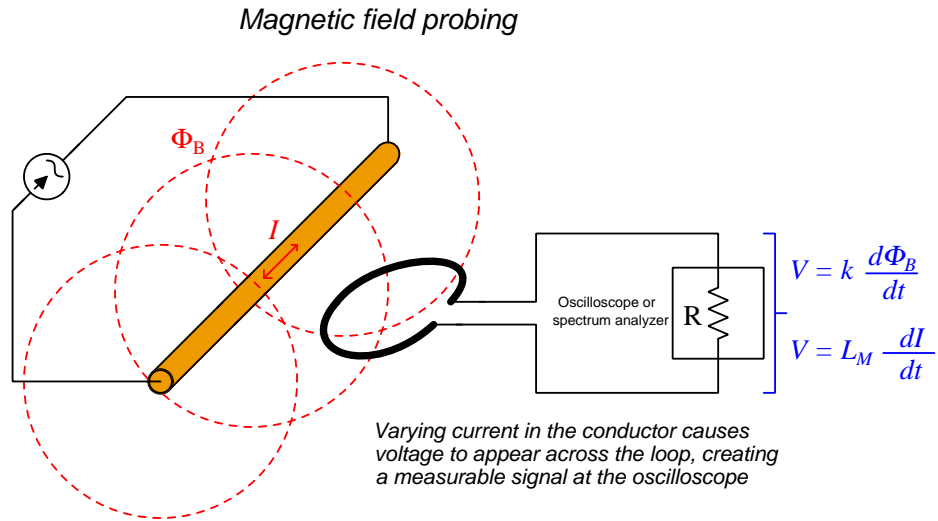


Different sizes of E-field probes exist. The larger the ball-shaped electrode, the more sensitive the probe will be to electric fields because the larger electrode intercepts more lines of electric flux than a smaller electrode. However, smaller-electrode E-field probes are superior for pinpointing exactly where an aggressor wire, trace, or component terminal is located in a circuit. Common practice is to start using a large-tip probe to locate the general area of the aggressor, then switch to a smaller-tip probe to pinpoint the exact location.



3.7 Magnetic field probing

In order to measure the magnetic field emanating from a conductor, whether a free wire or a trace on a printed circuit board (PCB), we need to establish a mutual inductance between that conductor and a test instrument:



The wire loop in the illustration is commonly referred to as an *H-field probe*⁸ because its purpose is to sense AC magnetic fields. Such probes are commonly used to locate sources of magnetic field emissions on the surface of a printed circuit board, moving the loop to different locations and in different directions on the PCB until the strongest offending signal is displayed on the test instrument.

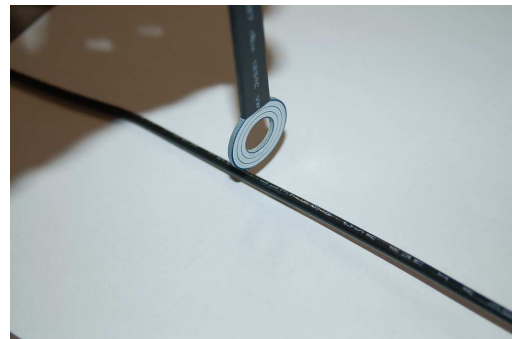
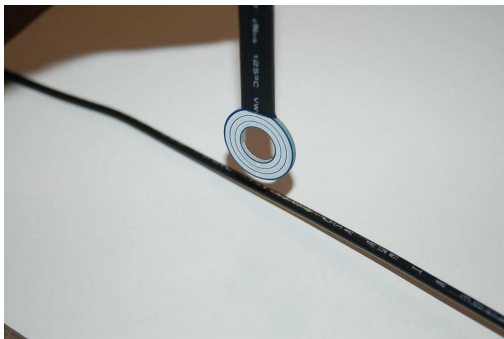
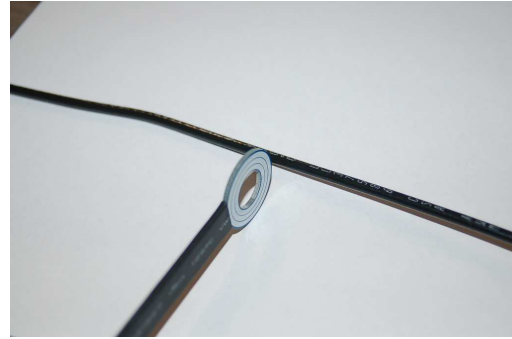
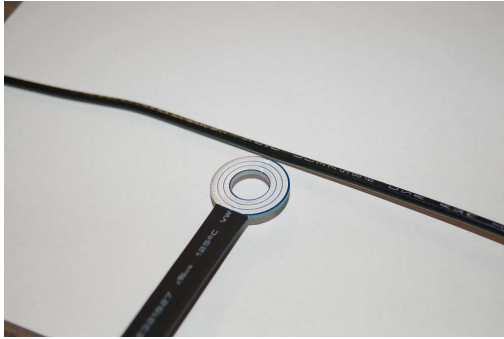
Like differential (dipole) E-field probes, the orientation of an H-field probe matters greatly. This is because the loop conductor must be *parallel* to the aggressor conductor in order for the aggressor's magnetic field to properly couple with the probe. Placing the probe's loop plane perpendicular to the aggressor conductor will result in little or no signal coupling.

⁸In physics, *H* is often used to symbolize magnetic field force in units of Amperes per meter.

Different sizes of H-field probes exist. The larger the loop diameter, the more sensitive the probe will be to magnetic fields because the larger loop will intercept more lines of magnetic flux than a smaller loop. However, smaller-loop H-field probes are superior for pinpointing exactly where an aggressor wire, trace, or component terminal is located in a circuit. Common practice is to start using a large-loop probe to locate the general area of the aggressor (being sure to try different orientations for each area tested), then switch to a smaller-loop probe to pinpoint the exact location.



The following photographs show proper (left) and improper (right) ways to hold an H-field probe to ensure coupling with the conductor (here, a black wire):



Strategically positioning an H-field probe above a printed circuit board is key to distinguishing the magnetic field emissions of different traces on that board. When the probe's loop is rotated for maximum signal, you then look for traces running parallel to the probe's loop to identify possible aggressors.

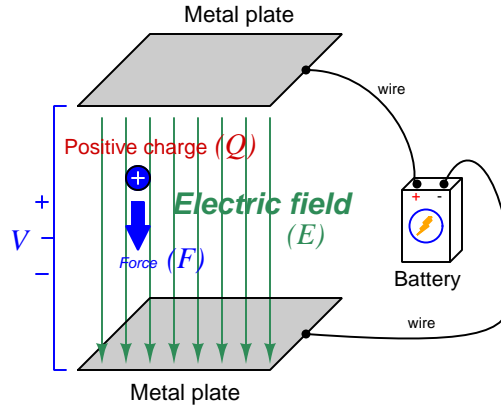
Chapter 4

Derivations and Technical References

This chapter is where you will find mathematical derivations too detailed to include in the tutorial, and/or tables and other technical reference material.

4.1 Electric field quantities

A useful definition of electric field (E) is in terms of the force (F) exerted on an electric charge (Q) influenced by that field:



$$\vec{F} = Q\vec{E}$$

Where,

\vec{F} = Force exerted on the charge (Newtons)

Q = Charge quantity (Coulombs¹)

\vec{E} = Electric field (Newtons per Coulomb)

The small “arrow” symbols above the variables for force and electric field in the equation denote those variables as *vector quantities*, having both magnitude and direction. Charge is a *scalar quantity* having only magnitude but no direction, and as a scalar quantity when multiplied by the electric field vector it simply magnifies the magnitude but does not alter the direction. Therefore, the force and electric field vectors always point in the same direction.

Alternatively electric field may be defined in terms of the voltage between the end-points and the distance separating them, in which case we may express the electric field in units of *Volts per meter* as an alternative to *Newtons per Coulomb*:

$$\vec{E} = \frac{V}{d}$$

This measurement of electric field strength is very important for quantifying the *breakdown* of electrical insulators: the point at which the electric field becomes so powerful that otherwise immobile charges within the insulating substance are torn free to constitute a current and that substance is no longer an insulator. For rating the dielectric strength of insulating materials, we often see electric fields expressed in units of *kiloVolts per millimeter* rather than Volts per meter just to make the numerical quantities easier to manage (1 kV/mm = 1 *million* V/m).

¹One Coulomb of electric charge is equal to 6.2415×10^{18} electrons.

The vector arrows shown in the previous illustration representing the electric field between two metal plates actually represent electric *flux* (Φ_E). The electric field (E) is related to electric flux by area (A), the field being a measurement of how densely-packed those flux lines are per unit area:

$$\vec{E} = \frac{\Phi_E}{\vec{A}}$$

Where,

\vec{E} = Electric field, or electric flux density (Newtons per Coulomb)

Φ_E = Electric flux (Newton-meter squared per Coulomb)

\vec{A} = Area over which flux is distributed (square meters)

The mere presence of an unbalanced electric charge at any point in space is sufficient to generate lines of electric flux, the total magnitude of that flux predicted by the following equation:

$$\Phi_E = \frac{Q}{\epsilon}$$

Where,

Φ_E = Electric flux (Newton-meter squared per Coulomb)

Q = Charge quantity (Coulombs)

ϵ = Electric permittivity of the surrounding space (Coulombs squared per Newton-meter squared, approximately 8.85×10^{-12} for empty space)

By convention, these flux vectors point *away* from positive charges and point *toward* negative charges, their direction indicating force exerted on any positively-charged particle influenced by that field². As the equation states, the amount of flux depends on how much charge exists at each particle as well as the permittivity of the surrounding space:

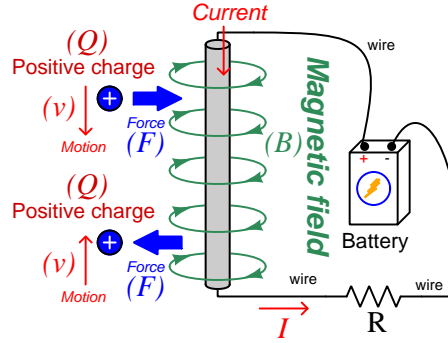


For example, identical charges suspended in a vacuum versus in a substance such as oil will have different amounts of flux associated with them as a result of oil and vacuum having different permittivity values. Perfectly empty space has the least amount of permittivity, which means anything else (gas, liquid, or solid matter) has greater ϵ which acts to diminish the amount of electric flux surrounding any charged particle. Superconducting materials have infinite permittivity, which means they forbid the existence of any electric field inside their bulk.

²Conversely, the flux vectors point exactly opposite the direction of force applied to any *negatively*-charged particle within that field. This makes sense of the aphorism that *like charges repel and opposite charges attract*. If you consider the two charges shown in this illustration, the positive charge will be pulled in the direction of the flux vectors pointing toward the negative charge, as the negative charge will also be pulled opposite the direction of the flux vectors pointing away from the positive charge (i.e. the negative charge will be pulled *toward* the positive charge) – thus the positive and negative charges feel mutual attraction.

4.2 Magnetic field quantities

A useful definition of magnetic field (B) is in terms of the force (F , called the *Lorentz force*) exerted on a moving electric charge (Q) influenced by that field:



$$\vec{F} = Q\vec{v} \times \vec{B}$$

Where,

\vec{F} = Force exerted on the charge (Newtons)

Q = Charge quantity (Coulombs³)

\vec{v} = Velocity of moving charge (meters per second)

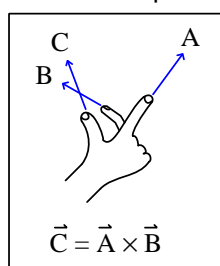
\vec{B} = Magnetic field (Tesla, Webers per square meter, or Newtons per Ampere-meter)

The small “arrow” symbols above the variables for force and velocity and magnetic field in the equation denote those variables as *vector quantities*, having both magnitude and direction. Charge is a *scalar quantity* having only magnitude but no direction, and as a scalar quantity when multiplied by the velocity vector it simply magnifies the magnitude but does not alter the direction. The “cross-product” (\times) is a specific form of vector multiplication, and it results in a product at right angles to the vector directions of both terms. Therefore, the force and velocity and electric field vectors never all point in the same direction.

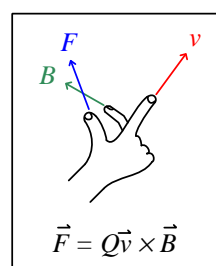
³One Coulomb of electric charge is equal to 6.2415×10^{18} electrons.

Vector cross-products conveniently relate to the fingers of the right hand, which is where the “right-hand rule” originates:

General "right-hand rule"
for vector cross-products

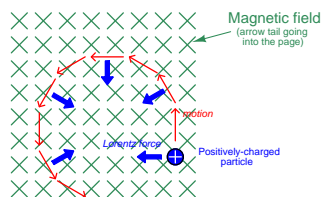


"Right-hand rule" specific
to the Lorentz force



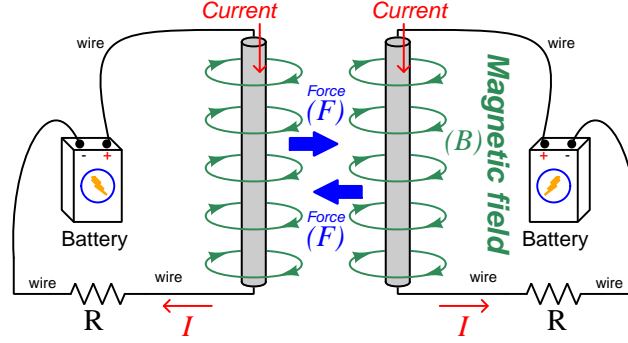
When holding the index finger, middle finger, and thumb of your right hand perpendicular to each other, your index finger points in the direction of the velocity vector (\vec{v}), your middle finger in the direction of the magnetic field vector (\vec{B}), and your thumb in the direction of the force vector (\vec{F}). A simple mnemonic I use to remember these relationships of fingers to vectors is that the **I**ndex finger points in the direction of current⁴ (I), the **M**iddle finger points in the direction of the **m**agnetic field (B), and the **T**humb points in the direction of the **t**hrust (i.e. force) acting upon the moving charge.

The Lorentz force’s effect on electrically-charged particles in motion has many applications, from redirecting the paths of charged-particle beams in particle accelerator machines, to bending the trajectory of electron beams in a cathode-ray tube (CRT), to forcing electrons to travel in spiral or circular trajectories inside of magnetron (microwave oscillator) tubes. An illustration of a positively-charged particle curving perpendicular to a magnetic field appears here:



⁴Some textbooks speak of a “left-hand rule” which is intended to make sense of electric charge motion (current) in terms of *electron flow*. As we know, electrons are the only real mobile charge carriers within metal conductors, and so technically “electron flow” notation is most physically accurate when describing the motion of electric charges in metallic circuits. However, the right-hand rule is a *mathematical definition* for vector cross products, the concept of the cross product arising in the late 18th century when electrical science was still in its infancy. Early explorers of electricity used the established mathematical tools of their time and applied it to their work with electric currents and magnetism. At that time, charge carriers in metal wires were assumed to be “positive” and this is how the motion of positively-charged carriers became associated with the first vector of the cross-product. As a result of this assumption which was later proven false, we have two different conventions for denoting the motion of electricity: electron-flow which is *physically* accurate (for metal wires, at least), and conventional flow which is *mathematically* accurate. This, perhaps more than any other reason, is why educational programs designed for mathematically rigorous fields (e.g. electrical engineering) exclusively use conventional flow notation rather than electron flow notation to denote the direction of current.

If the moving charge in question is not a single charged particle but rather part an electric *current* passing through a conductor parallel to the first, both conductors will experience a mutually-attracting force given by the following equation:



$$\vec{F} = I\vec{l} \times \vec{B}$$

Where,

\vec{F} = Force exerted on both conductors (Newtons)

I = Current (Amperes)

\vec{l} = Length of wire (meters)

\vec{B} = Magnetic field (Tesla, or Webers per square meter, or Newtons per Ampere-meter)

The point-charge Lorentz force equation and the two-conductor Lorentz force equation are not that different from one another. Dimensional analysis validates this: the Lorentz force on a moving charge uses that charge quantity (Coulombs) multiplied by the point-charge's velocity in meters per second to give Coulomb-meters per second for the first term:

$$Q\vec{v} = [\text{C}] \left[\frac{\text{m}}{\text{s}} \right] = \left[\frac{\text{C} \cdot \text{m}}{\text{s}} \right]$$

The Lorentz force on a current-carrying conductor uses the current (Amperes, which is Coulombs per second) multiplied by length in meters, for the same composite units of Coulomb-meters per second:

$$I\vec{l} = \left[\frac{\text{C}}{\text{s}} \right] [\text{m}] = \left[\frac{\text{C} \cdot \text{m}}{\text{s}} \right]$$

This dimensional equivalence makes conceptual sense as well: an electrically-charged particle moving through empty space is an electric current in its own right, and an electric current flowing through a conductor is just a collection of charged particles moving through space (just not *empty* space). In either case, the basis for the Lorentz force remains the same: the moving charge(s) create their own magnetic field, which reacts with the magnetic field of the original current-carrying wire to produce forces acting on both.

If the two currents flow in the same direction, their mutual forces *attract*. If the two currents flow in opposite directions, their mutual forces *repel*. This is the basis of electric motors: causing

mechanical motion by electro-magnetic attraction and repulsion. It also represents an interesting contrast with electric fields:

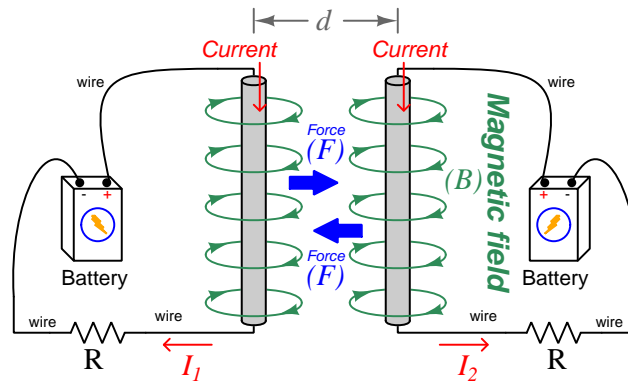
With electric fields, opposite *charges* attract and like *charges* repel.

With magnetic fields, opposite *poles* attract and like *poles* repel.

With parallel currents, opposite *directions* repel and like *directions* attract⁵.

Two parallel current-carrying conductors of length l and separated by a distance d will generate a mutual force proportional to both their currents:

$$F = l \frac{\mu I_1 I_2}{2\pi d}$$



⁵That is, assuming it's *like* charges moving in these directions! If the charges in question are opposite each other – for example electrons in one circuit and holes in another – then like directions will repel and opposite directions will attract!

The circular loops surrounding the current-carrying conductors in the previous illustrations represent the magnetic lines of *flux* (Φ_B) surrounding each of those conductors. The magnetic field (B) is related to magnetic flux by area (A), the field being a measurement of how densely-packed those flux lines are per unit area. For this reason, magnetic field (B) is more properly known as *magnetic flux density*:

$$\vec{B} = \frac{\Phi_B}{\vec{A}}$$

Where,

\vec{B} = Magnetic field or flux density (Tesla, Webers per square meter, or Newtons per Ampere-meter)

Φ_B = Magnetic flux (Webers)

\vec{A} = Area over which flux is distributed (square meters)

An older unit of measurement for magnetic flux density B is the *Gauss* which is much smaller than a Tesla, with one Tesla equivalent to 10,000 Gauss. To put things into perspective, the Earth's natural magnetic field has a strength of approximately one-half of one Gauss⁶.

Magnetic field strength is an inverse function of distance from any current-carrying wire, and also depends on the magnetic permeability of the space adjacent to the wire:

$$B = \frac{\mu I}{2\pi d}$$

Where,

B = Magnetic field or flux density (Tesla, Webers per square meter, or Newtons per Ampere-meter)

μ = Magnetic permeability of the surrounding space (Tesla-meters per Ampere, $4\pi \times 10^{-7}$ for empty space)

I = Current (Amperes)

d = Distance from conductor (meters)

⁶Using the online *Magnetic Field Calculator* application provided by NOAA (the National Oceanic and Atmospheric Administration) at <https://ngdc.noaa.gov/geomag/calculators/magcalc.shtml#igrfwmm>, applying the World Magnetic Model WMM modeling algorithm for years 2019-2024, the total magnetic field strength at my home is 53,584.4 nano-Tesla (53,584.4 nT or 0.535844 Gauss), and presently (May 2020) decaying at a rate of -104.1 nT per year.

The relation of magnetic *flux* to current through a conductor follows a similar equation:

$$\Phi = \frac{\mu AI}{2\pi d}$$

Where,

Φ = Magnetic flux (Webers)

μ = Magnetic permeability of the surrounding space (Tesla-meters per Ampere, $4\pi \times 10^{-7}$ for empty space)

A = Area over which flux is distributed (square meters)

I = Current (Amperes)

d = Distance from conductor (meters)

As this equation makes clear, the amount of magnetic flux surrounding a current-carrying conductor depends not only on the amount of current, but also on the sampled area, the distance from the wire, and also the surrounding material. Most⁷ substances (gas, liquid, solid) have permeability values greater than that of empty space, and so this means magnetic flux is usually *enhanced* by the presence of matter around the current-carrying conductor.

The total magnetic flux enclosed by a circular wire loop follows a similar equation:

$$\Phi = \frac{\pi\mu Ir}{2}$$

Where,

Φ = Magnetic flux (Webers)

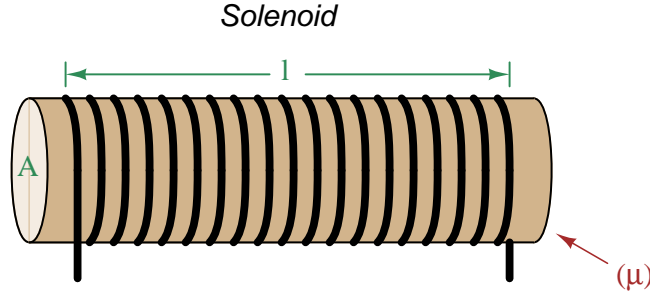
μ = Magnetic permeability of the surrounding space (Tesla-meters per Ampere, $4\pi \times 10^{-7}$ for empty space)

I = Current (Amperes)

r = Radius of circular loop (meters)

⁷Interestingly, superconducting materials *forbid* magnetic fields inside of their bulk, and so the permeability value of any superconductor must be zero!

A common form of electromagnet known as a *solenoid* takes the form of a wire coil wrapped in such a way as to form a long⁸ cylinder, often wrapped around a plastic frame, and often with a ferromagnetic material such as iron in the center:



The amount of magnetic flux, and the flux density, within the interior of a current-carrying solenoid are given by the following formulae:

$$\Phi = \frac{\mu N A I}{l} \qquad B = \frac{\mu N I}{l}$$

Where,

Φ = Magnetic flux (Webers)

B = Magnetic field or flux density (Tesla, Webers per square meter, or Newtons per Ampere-meter)

μ = Magnetic permeability of the surrounding space (Tesla-meters per Ampere, $4\pi \times 10^{-7}$ for empty space)

N = Number of turns of wire in the coil

A = Cross-sectional area of solenoid coil (square meters)

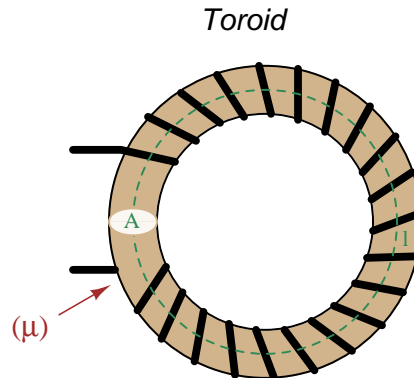
I = Current (Amperes)

l = Length of solenoid coil (meters)

These formulae have interesting implications for solenoid design. Note how a shorter (i.e. smaller length l) solenoid identical in all other respects will generate a stronger magnetic field for a given current. Note also how the flux density (B) remains constant with increasing cross-sectional area (A) if all other factors are equal, and that this necessarily means a greater amount of total magnetic flux (Φ) for a greater area A .

⁸These magnetic field formulae apply perfectly to a solenoid coil that is closely-packed (i.e. each turn adjacent to the next) and infinitely long. Therefore, they only approximate real solenoid behavior. This fact may be understood by performing a thought experiment where we decrease the solenoid coil's length to zero, in which case the formulae predict an *infinite* amount of magnetism for any amount of current at all, which of course cannot be true.

Another common form of electromagnet known as a *toroid* is really just a solenoid bent in a circle so that its two ends meet⁹ cylinder, often wrapped around a plastic frame, and often with a ferromagnetic material such as iron in the center. Toroids have the unusual property of *containing* their magnetic flux lines extremely well, unlike solenoids, wires, and simple coils which all radiate magnetic fields. They find application as energy-storage devices, or as electromagnets suitable for applying magnetic fields to specimens placed *inside* the toroid's cross-section:



The amount of magnetic flux, and the flux density, within the interior of a current-carrying toroid are identical to that within an otherwise identical solenoid having a length (l) equal to the toroid's circumference:

$$\Phi = \frac{\mu N A I}{l} \quad B = \frac{\mu N I}{l}$$

Where,

Φ = Magnetic flux (Webers)

B = Magnetic field or flux density (Tesla, Webers per square meter, or Newtons per Ampere-meter)

μ = Magnetic permeability of the surrounding space (Tesla-meters per Ampere, $4\pi \times 10^{-7}$ for empty space)

N = Number of turns of wire in the coil

A = Cross-sectional area of toroid (square meters)

I = Current (Amperes)

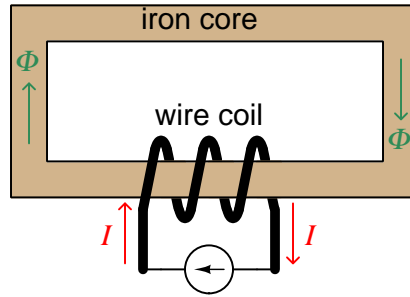
l = Circumference of toroid (meters)

If we wish to substitute toroid radius (r) for circumferential length (l), the formulae become the following:

$$\Phi = \frac{\mu N A I}{2\pi r} \quad B = \frac{\mu N I}{2\pi r}$$

⁹Again, the magnetic field formulae are only accurate for a toroidal coil that is closely-packed (i.e. each turn adjacent to the next) and infinitely long, and therefore only approximate real toroid behavior. This fact may be understood by performing an equivalent thought experiment as before where we decrease the toroid's circumference to zero and absurdly end up with *infinite* magnetism for a finite current.

Many applications of electromagnetism involve conductive *coils* wrapped around some form of ferromagnetic core material, the purpose of that core being to provide a higher-permeability pathway for the magnetic flux than would exist otherwise through air, and the purpose of the wire coil being to intensify the amount of magnetism developed by the electric current beyond what would be possible with a straight current-carrying wire. These magnetic cores typically form a closed loop, or *magnetic circuit* for the lines of magnetic flux to naturally form a closed path. A simple example appears here:



The amount of magnetic flux (Φ) present in the magnetic “circuit” formed by the iron core depends on many factors. First and foremost is the amount of electric current (in Amperes) passing through the wire coil and the number of turns that coil makes around the iron core. The product of this current and the number of turns is called the *magnetomotive force* or *mmf* of the magnetic circuit, analogous to “electromotive force” or “emf” often used as a synonym for voltage in an electric circuit. Not surprisingly, the standard metric unit of measurement for magnetomotive force is the *Ampere-turn*.

However, magnetomotive force alone does not fully describe the current’s effect on magnetism within the iron core. The total length of the magnetic circuit is also an important factor, since a longer path distributes that magnetomotive force over a greater distance. The quotient of magnetomotive force and magnetic circuit length is called *magnetic field intensity*, symbolized by the variable H and expressed in units of *Ampere-turns per meter*.

Magnetic permeability (μ) relates magnetic field intensity (H) to the magnetic flux density (B) within the core material, such that a greater permeability will result in higher flux density for any given amount of field intensity. Permeability is a property of the core material and not its geometry, mathematically defined as the ratio of flux density to field intensity: $\mu = \frac{B}{H}$

Magnetic reluctance (\mathfrak{R}) relates magnetomotive force (mmf) to magnetic flux (Φ), and is related not only to the core material’s permeability but also its geometry. It is mathematically defined as the ratio of magnetomotive force to magnetic flux: $\mathfrak{R} = \frac{\text{mmf}}{\Phi}$

If all this seems confusing, you are in good company. Not only are there many magnetic variables, some related to physical geometry and others not, but there are *two different sets of metric units* appropriate for expressing each! The older units were based on the centimeter-gram-second (CGS) version of the metric system, while the newer units are based on the meter-kilogram-second or SI (Système International) version of the metric system.

Quantity	Symbol	SI unit	CGS unit
Magnetomotive force	mmf	Ampere-turn (A-t)	Gilbert (Gb)
Flux	Φ	Weber (Wb)	Maxwell (Mx)
Field intensity	H	Ampere-turns per meter (A-t/m)	Oersted (Oe)
Flux density	B	Tesla (T)	Gauss (G)
Permeability	μ	Tesla-meters per Ampere-turn (T-m/A-t)	Gauss per Oersted (G/Oe)
Reluctance	\mathfrak{R}	Ampere-turns per Weber (A-t/Wb)	Gilberts per Maxwell (G/Mx)

Magnetomotive force (mmf) and magnetic flux (Φ) may be thought of as the “raw” measures of magnetism, with Ampere-turns and Webers being their respective SI metric units. Reluctance (\mathfrak{R}) is the ratio of the two for any given magnetic circuit with known dimensions and core material. Simply put, reluctance tells you how many Ampere-turns of magnetomotive force will be necessary to create one Weber of magnetic flux in a given space.

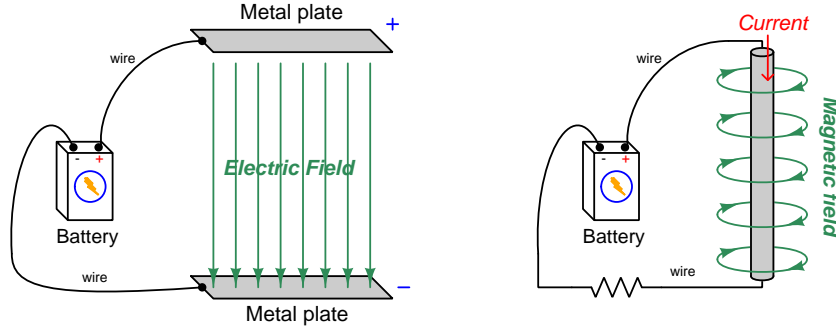
Magnetic field intensity (H) and magnetic flux density (B) may be thought of as the “normalized” measures of magnetism, with Ampere-turns per meter and Tesla being their respective SI metric units. H and B relate to mmf and flux by the physical dimensions of the magnetic circuit (length and cross-sectional area, respectively). Permeability is the ratio of the two for any given magnetic core material. Simply put, permeability tells you how many Tesla of magnetic field (i.e. flux density, or Webers of flux per square meter or cross-sectional core area) you will obtain for one Ampere-turn per meter of magnetic field intensity applied to a given core material.

Conversion between the newer SI and the older CGS metric units are as follows:

Quantity	Conversion equivalence
Magnetomotive force (mmf)	1 Ampere-turn = $\frac{4\pi}{10}$ Gilberts
Magnetic flux (Φ)	1 Weber = 10^8 Maxwells
Magnetic field intensity (H)	1 Ampere-turn/meter = $\frac{4\pi}{1000}$ Oersteds
Magnetic flux density (B)	1 Tesla = 10^4 Gauss
Permeability (μ)	1 Tesla-meter/Ampere-turn = $\frac{10^7}{4\pi}$ Gauss/Oersteds
Reluctance (\mathfrak{R})	1 Ampere-turn/Weber = $\frac{4\pi}{10^9}$ Gilberts/Maxwell

4.3 Near-field versus far-field regions

An electric field exists wherever voltage exists (i.e. wherever an imbalance exists between positive and negative electrical charges), parallel to the axis of that voltage. A magnetic field exists wherever an electric charge moves, perpendicular to the axis of that charge's motion. These phenomena exist for DC (direct) as well as AC (alternating) electricity, and are shown in the two following illustrations:



Electric and magnetic fields, however, are not simply manifestations of voltage and current, respectively. Mathematical discoveries made by the Scottish physicist James Clerk Maxwell (and later simplified by Oliver Heaviside) also relate electric fields and magnetic fields *directly to each other*. Consider two¹⁰ of Maxwell's equations shown below:

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

The first equation describes how an electric field (\vec{E}) is created by a varying magnetic flux ($\frac{d\Phi_B}{dt}$), which is otherwise known as *Faraday's Law of Electromagnetic Induction*. This is the law exploited in the function of electro-mechanical generators whereby coils of wire are subjected to changing magnetic fields, creating voltage between the coils' end-points. The negative sign in this equation is an embodiment of *Lenz's Law*, which states that any current resulting from the induced voltage will produce its own magnetic field opposing the first magnetic field's direction of change.

The second equation shown here describes *two* different ways to produce a magnetic field (\vec{B}). One way is to use a moving stream of electric charges known as a *current* (I), a fact also known as *Ampère's Law*. This is the law exploited in the function of electromagnets, where we produce a magnetic field by connecting a coil of wire to an electrical source. The second way is to vary an electric flux ($\frac{d\Phi_E}{dt}$) in empty space with surrounding magnetic permeability μ_0 and electric permittivity ϵ_0 .

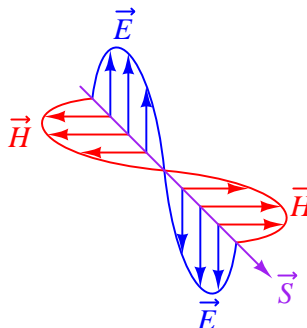
¹⁰Maxwell's equations of electromagnetism are typically presented as a set of *four*. Here we only list two of these, as the other two are not relevant to electromagnetic waves in particular.

Maxwell's equations suggest that in pulsed DC and AC circuits, where the resulting electric and/or magnetic fields vary over time ($\frac{d}{dt}$) rather than holding steady, those time-varying fields will mutually support one another over space and time. That is, a varying electric field will eventually produce a varying magnetic field, and vice-versa, those fields propagating one another as an *electromagnetic wave* radiating away from the circuit through space at the speed of light¹¹.

At close range, immediately adjacent to the circuit conductors, electric and magnetic fields are strictly functions of those conductors' voltages and currents, respectively. These fields' shapes follow the basic forms shown in the previous illustrations: magnetic field lines circling current-carrying conductors and electric field lines stretching between conductive surfaces. We generally refer to this region of space around an energized circuit as the *near-field region*, where electric and magnetic fields are distinct from one another in form and in relative magnitude.

At farther distances from the energized circuit, the electric and magnetic fields are found in pairs oscillating at right angles to one another in space, forming a single electromagnetic wave radiating away from the source circuit spherically in all directions. This form of wave is a direct consequence of Maxwell's equations describing how the rate-of-change of one field creates the other field, and vice-versa. Below we see a crude representation of an electromagnetic wave, with an electric \vec{E} field, a magnetic \vec{H} field, and the vector of propagation \vec{S} called the *Poynting vector*:

Electromagnetic wave



This region where the electric and magnetic fields exist as coupled pairs comprising a well-formed wave is referred to as the *far-field region*. In this region the electric and magnetic fields are always perpendicular to one another and their relative magnitudes are in a fixed proportion governed by the permeability and permittivity of free space.

The demarcation between near- and far-field regions depends on the physical dimensions of the radiating circuit as well as the *wavelength* of the signal (λ), wavelength being calculated by dividing the speed of light (2.9979×10^8 meters per second in empty space) by the signal frequency in Hertz. There is no simple rule for predicting where the near-field region ends and the far-field region begins, and in fact there is a gradual morphing of one to the other, but one may reliably consider any distance from the circuit in excess of multiple wavelengths to be far-field.

¹¹In fact, this is precisely what light is: an electromagnetic wave of exceptionally high frequency, well beyond the frequency range of contemporary electric circuits.

The following list compares and contrasts several near-field versus far-field characteristics:

- In the near-field region electric and magnetic field strengths depend greatly on the geometry of the energized circuit's conductors as well as on the specific voltage and current levels existing at each location. In the far-field region, however, the electric and magnetic field strengths are always in fixed proportion to one another.
- Far-field effects may be ignored for low-frequency AC circuits because these circuits' wavelengths are so long. At 60 Hz, for example, the wavelength is nearly 5 *million* meters. If we consider the far-field region of a circuit to begin at least multiple wavelengths from the source, this puts the far-field region of a 60 Hz circuit at least one-quarter of the Earth's circumference away from the circuit in question!
- In the near-field region the proportionality between electric and magnetic field strengths is a function of circuit impedance: high-impedance circuits will have stronger electric fields than magnetic fields, and low-impedance circuits will have stronger magnetic fields than electric fields, all other factors being equal. This is due to Ohm's Law establishing the relationship between voltage and current ($Z = \frac{V}{I}$), and how electric fields originate in a circuit from potential differences while magnetic fields originate in a circuit from currents. In the far-field region, however, where the electric and magnetic fields exist only through mutual support, their strengths are always in the same (fixed) proportion defined as the *characteristic impedance of free space* which is approximately 377 Ohms. This value is equal to the square root of the ratio of magnetic permeability to electric permittivity for free space ($Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$) because these parameters of space dictate just how strongly a varying electric field creates a magnetic field.
- The relationship between field strength and physical distance from the circuit is a very complex one within the near-field region, as this depends greatly on the geometry of the circuit conductors. In the far-field region, however, we find the *Inverse-Square Law* always holds true: as distance from the circuit increases, electromagnetic wave strength simply and reliably diminishes with the *square* of that distance as if the wave were radiating away in a spherical fashion from a point-source¹². For example, if we double the distance from the radiating circuit (from a point in the far-field region to another point in the far-field region twice as far away in empty space), the power conveyed by the electromagnetic radiation will always be *four times* less. Tripling the far-field distance always weakens the wave's power by a factor of *nine*. This is simply because the area over which the radiated energy spreads increases with the square of the distance from any point-source, viewing that point-source as the center of a sphere.
- In the far-field region every radiating circuit has a definite *radiation pattern* where the strength of the radiated electromagnetic wave as a function of the *angle* from the geometric centerline follows a consistent pattern regardless of distance. This is a very important feature of *antennas*, where different antenna designs feature unique far-field radiation patterns. By contrast, in the near-field region of an antenna the radiation pattern varies significantly with distance.

¹²A "point source" is a hypothetical point in space having no height nor width nor depth (i.e. it has *zero* physical dimensions) that emits radiation. Point-sources are a theoretical concept only. Real sources have spatial dimensions which makes their near-field strength/distance relationships complex, but the farther away you get from them the more their radiative behavior approaches that of a theoretical point-source.

- The spatial *polarization* of the electromagnetic wave (i.e. the orientation of its perpendicular electric and magnetic fields) is always well-defined in the far-field region but often ill-defined in the near-field region. This means, among other things, that two or more antennas will exchange electromagnetic energy efficiently only if they are appropriately oriented to one another over far-field distances, but may exchange energy fairly well regardless of orientation over near-field distances.
- Any “gain” specifications for an antenna structure apply only to apparent gains in signal power over the far-field range, because they refer to the degree to which an antenna focuses its energy in one direction more than another (i.e. its directionality), implying a distance over which the electromagnetic wave has become well-formed and the antenna’s radiation pattern is reliably established. At near-field distances these “gain” figures are meaningless.

4.4 The Poynting vector

We know that electric fields (E) are always associated with a voltage (V), and magnetic fields (field strength B and field force H) with a current (I). Specifically, electric field strength is the quotient of voltage and distance between two points which is why electric fields are measured in units of *Volts per meter*. Similarly, magnetic field force around a straight current-carrying conductor is the quotient of current and circumference of the circular magnetic path (equal to 2π times the distance from the conductor) which is why magnetic field force is measured in units of *Amperes per meter*.

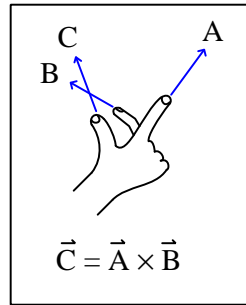
From Joule's Law we know that the rate of energy transfer (called power) whenever there is simultaneous voltage and current ($P = VI$). It stands to reason, then, that just as voltage multiplied by current yields power, an electric field multiplied by a magnetic field must similarly express power transferred by those two fields through space. That is, if $P = VI$, then the product of E and H should yield a power *intensity* which we shall label S :

$$P = VI \qquad [\text{Watts}] = [\text{Volts}][\text{Amperes}]$$

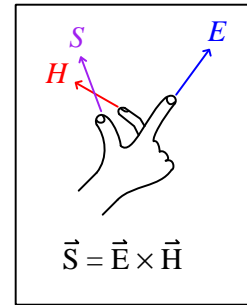
$$S = EH \qquad [\text{Watts/meter}^2] = [\text{Volts/meter}][\text{Amperes/meter}]$$

Notice how the Joule's Law equation $P = VI$ involves scalar quantities, while the $S = EH$ equation necessarily involves spatial dimensions of distance (meters) and area (square meters). This is because both electric and magnetic fields are *vector* quantities having both magnitude and spatial direction. Power intensity (S) is the measure of how much power is flowing per square meter of area in open space. As it so happens, power intensity in Watts per square meter relates to electric and magnetic fields by the right-hand rule of vector cross-products, the vector expression of that power intensity being called the *Poynting vector* (\vec{S}) after its formulator, the English physicist John H. Poynting:

General "right-hand rule"
for vector cross-products



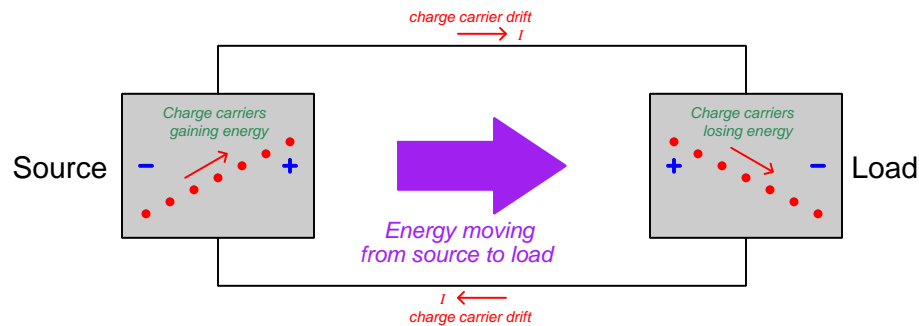
"Right-hand rule" specific
to the Poynting vector



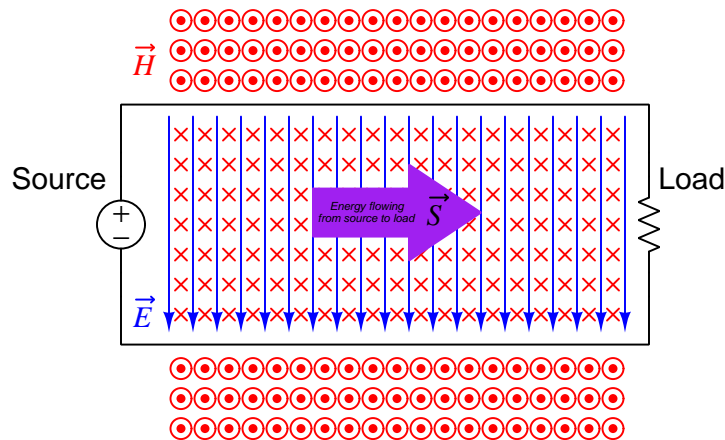
Poynting's contribution to the science of electromagnetism was expressing energy conservation and energy transfer in terms of electric and magnetic fields. That is, the electric and magnetic fields associated with any electrical *source* result in a flow of energy away from that source *through space*, and the electric and magnetic fields associated with any *load* result in a flow of energy toward that load *through space*. The direction of the Poynting vector shows the direction of that energy flow,

and the Poynting vector magnitude shows how many Watts of power flow per square meter of space.

This should be a rather startling conclusion to anyone familiar with voltage, current, and power in simple DC circuits. Beginning students of electricity regard energy transfer in simple circuits as the gain and loss of potential energy in mobile electric charges, those charges gaining energy as they pass through a source and losing energy as they pass through a load:

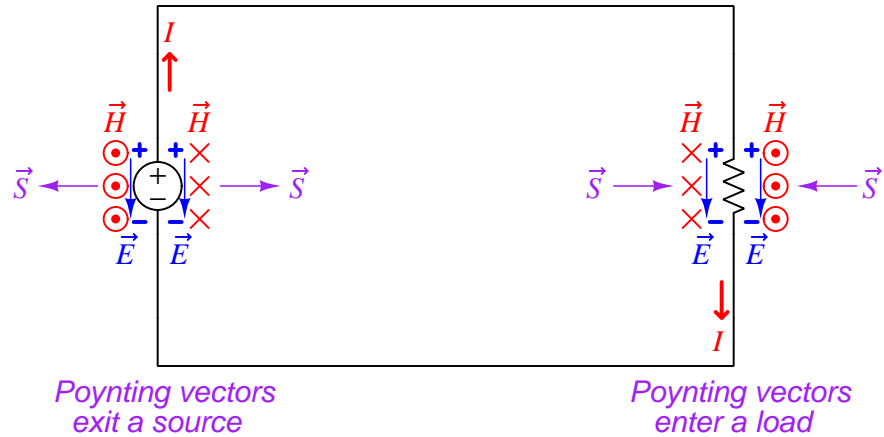


Though this model is conceptually correct, *so is Poynting's model of energy flowing through space*. In fact, if we re-draw this simple circuit illustration showing the electric and magnetic field vectors between the two wires, we can see the Poynting vector expressing that flow of energy from source to load¹³:



¹³Holding your right hand such that your index finger points downward (\vec{E}) and your middle finger into the page (\vec{H}), your thumb will point to the right (\vec{S}) from source to load.

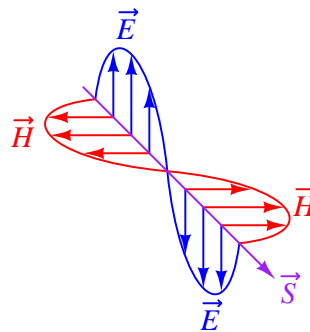
If we examine the source and load separately using the right-hand rule, we see how the Poynting vector in each case shows the proper direction of energy transfer:



It is well-worth your time to apply the right-hand rule to each of the four Poynting vectors shown above, to familiarize yourself with the use of this tool as a means to determine vector cross-product directions. For example, to the left of the source we see the electric field vector pointing down which means your index finger should point downward as well. The magnetic field vector on that side is coming out of the page (middle finger pointing toward you), and holding your right hand in that manner points your thumb to the left.

For simple DC circuits and circuit elements, Poynting's theory of power transfer may seem like a mere curiosity. It fails to explain anything we could not already explain using the model of potential energy gained and lost by mobile charge carriers, and it requires contortions of your right hand to properly align all the vectors. However, in some applications such as *radio* where energy radiates away from structures called *antennas* in the form of electromagnetic waves (oscillating electric and magnetic fields), Poynting's theory is the *only* suitable explanation for power transfer because there are no wires to convey mobile electric charges from source (transmitting antenna) to load (receiving antenna). Below we see a simplified representation of an electromagnetic wave consisting of two oscillating fields (shown here as an electric field oscillating vertically and a magnetic field oscillating horizontally), and if you use the right-hand rule in each half-cycle you will find that the Poynting vector consistently points in the same direction at the speed of light:

Electromagnetic wave



Based on the direction of the Poynting vector we see in this illustration, the source (transmitting antenna) would be located somewhere above and to the left of the drawn wave. There is no definite location for a load, as electromagnetic waves spread out in *all directions* away from the source. Our simple illustration merely shows a “cross-section” of a wave radiating *cylindrically* away from the transmitting antenna. The real electromagnetic wave (of which this illustration shows just a portion) would have an electric field resembling ripples on a pond oscillating vertically while radiating away in all horizontal directions from a single point, and have a magnetic field resembling a set of concentric circles oscillating rotationally in the horizontal plane while also radiating in all directions from that same point.

Chapter 5

Questions

This learning module, along with all others in the ModEL collection, is designed to be used in an inverted instructional environment where students independently read¹ the tutorials and attempt to answer questions on their own *prior* to the instructor's interaction with them. In place of lecture², the instructor engages with students in Socratic-style dialogue, probing and challenging their understanding of the subject matter through inquiry.

Answers are not provided for questions within this chapter, and this is by design. Solved problems may be found in the Tutorial and Derivation chapters, instead. The goal here is *independence*, and this requires students to be challenged in ways where others cannot think for them. Remember that you always have the tools of *experimentation* and *computer simulation* (e.g. SPICE) to explore concepts!

The following lists contain ideas for Socratic-style questions and challenges. Upon inspection, one will notice a strong theme of *metacognition* within these statements: they are designed to foster a regular habit of examining one's own thoughts as a means toward clearer thinking. As such these sample questions are useful both for instructor-led discussions as well as for self-study.

¹Technical reading is an essential academic skill for any technical practitioner to possess for the simple reason that the most comprehensive, accurate, and useful information to be found for developing technical competence is in textual form. Technical careers in general are characterized by the need for continuous learning to remain current with standards and technology, and therefore any technical practitioner who cannot read well is handicapped in their professional development. An excellent resource for educators on improving students' reading prowess through intentional effort and strategy is the book *Reading For Understanding – How Reading Apprenticeship Improves Disciplinary Learning in Secondary and College Classrooms* by Ruth Schoenbach, Cynthia Greenleaf, and Lynn Murphy.

²Lecture is popular as a teaching method because it is easy to implement: any reasonably articulate subject matter expert can talk to students, even with little preparation. However, it is also quite problematic. A good lecture always makes complicated concepts seem easier than they are, which is bad for students because it instills a false sense of confidence in their own understanding; reading and re-articulation requires more cognitive effort and serves to verify comprehension. A culture of teaching-by-lecture fosters a debilitating dependence upon direct personal instruction, whereas the challenges of modern life demand independent and critical thought made possible only by gathering information and perspectives from afar. Information presented in a lecture is ephemeral, easily lost to failures of memory and dictation; text is forever, and may be referenced at any time.

GENERAL CHALLENGES FOLLOWING TUTORIAL READING

- Summarize as much of the text as you can in one paragraph of your own words. A helpful strategy is to explain ideas as you would for an intelligent child: as simple as you can without compromising too much accuracy.
- Simplify a particular section of the text, for example a paragraph or even a single sentence, so as to capture the same fundamental idea in fewer words.
- Where did the text make the most sense to you? What was it about the text's presentation that made it clear?
- Identify where it might be easy for someone to misunderstand the text, and explain why you think it could be confusing.
- Identify any new concept(s) presented in the text, and explain in your own words.
- Identify any familiar concept(s) such as physical laws or principles applied or referenced in the text.
- Devise a proof of concept experiment demonstrating an important principle, physical law, or technical innovation represented in the text.
- Devise an experiment to disprove a plausible misconception.
- Did the text reveal any misconceptions you might have harbored? If so, describe the misconception(s) and the reason(s) why you now know them to be incorrect.
- Describe any useful problem-solving strategies applied in the text.
- Devise a question of your own to challenge a reader's comprehension of the text.

GENERAL FOLLOW-UP CHALLENGES FOR ASSIGNED PROBLEMS

- Identify where any fundamental laws or principles apply to the solution of this problem, especially before applying any mathematical techniques.
- Devise a thought experiment to explore the characteristics of the problem scenario, applying known laws and principles to mentally model its behavior.
- Describe in detail your own strategy for solving this problem. How did you identify and organized the given information? Did you sketch any diagrams to help frame the problem?
- Is there more than one way to solve this problem? Which method seems best to you?
- Show the work you did in solving this problem, even if the solution is incomplete or incorrect.
- What would you say was the most challenging part of this problem, and why was it so?
- Was any important information missing from the problem which you had to research or recall?
- Was there any extraneous information presented within this problem? If so, what was it and why did it not matter?
- Examine someone else's solution to identify where they applied fundamental laws or principles.
- Simplify the problem from its given form and show how to solve this simpler version of it. Examples include eliminating certain variables or conditions, altering values to simpler (usually whole) numbers, applying a limiting case (i.e. altering a variable to some extreme or ultimate value).
- For quantitative problems, identify the real-world meaning of all intermediate calculations: their units of measurement, where they fit into the scenario at hand. Annotate any diagrams or illustrations with these calculated values.
- For quantitative problems, try approaching it qualitatively instead, thinking in terms of “increase” and “decrease” rather than definite values.
- For qualitative problems, try approaching it quantitatively instead, proposing simple numerical values for the variables.
- Were there any assumptions you made while solving this problem? Would your solution change if one of those assumptions were altered?
- Identify where it would be easy for someone to go astray in attempting to solve this problem.
- Formulate your own problem based on what you learned solving this one.

GENERAL FOLLOW-UP CHALLENGES FOR EXPERIMENTS OR PROJECTS

- In what way(s) was this experiment or project easy to complete?
- Identify some of the challenges you faced in completing this experiment or project.

- Show how thorough documentation assisted in the completion of this experiment or project.
- Which fundamental laws or principles are key to this system's function?
- Identify any way(s) in which one might obtain false or otherwise misleading measurements from test equipment in this system.
- What will happen if (component X) fails (open/shorted/etc.)?
- What would have to occur to make this system unsafe?

5.1 Conceptual reasoning

These questions are designed to stimulate your analytic and synthetic thinking³. In a Socratic discussion with your instructor, the goal is for these questions to prompt an extended dialogue where assumptions are revealed, conclusions are tested, and understanding is sharpened. Your instructor may also pose additional questions based on those assigned, in order to further probe and refine your conceptual understanding.

Questions that follow are presented to challenge and probe your understanding of various concepts presented in the tutorial. These questions are intended to serve as a guide for the Socratic dialogue between yourself and the instructor. Your instructor's task is to ensure you have a sound grasp of these concepts, and the questions contained in this document are merely a means to this end. Your instructor may, at his or her discretion, alter or substitute questions for the benefit of tailoring the discussion to each student's needs. The only absolute requirement is that each student is challenged and assessed at a level equal to or greater than that represented by the documented questions.

It is far more important that you convey your *reasoning* than it is to simply convey a correct answer. For this reason, you should refrain from researching other information sources to answer questions. What matters here is that *you* are doing the thinking. If the answer is incorrect, your instructor will work with you to correct it through proper reasoning. A correct answer without an adequate explanation of how you derived that answer is unacceptable, as it does not aid the learning or assessment process.

You will note a conspicuous lack of answers given for these conceptual questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your conceptual answers, where applicable, is to use circuit simulation software to explore the effects of changes made to circuits. For example, if one of these conceptual questions challenges you to predict the effects of altering some component parameter in a circuit, you may check the validity of your work by simulating that same parameter change within software and seeing if the results agree.

³*Analytical* thinking involves the “disassembly” of an idea into its constituent parts, analogous to dissection. *Synthetic* thinking involves the “assembly” of a new idea comprised of multiple concepts, analogous to construction. Both activities are high-level cognitive skills, extremely important for effective problem-solving, necessitating frequent challenge and regular practice to fully develop.

5.1.1 Reading outline and reflections

“Reading maketh a full man; conference a ready man; and writing an exact man” – Francis Bacon

Francis Bacon’s advice is a blueprint for effective education: reading provides the learner with knowledge, writing focuses the learner’s thoughts, and critical dialogue equips the learner to confidently communicate and apply their learning. Independent acquisition and application of knowledge is a powerful skill, well worth the effort to cultivate. To this end, students should read these educational resources closely, write their own outline and reflections on the reading, and discuss in detail their findings with classmates and instructor(s). You should be able to do all of the following after reading any instructional text:

✓ Briefly **OUTLINE THE TEXT**, as though you were writing a detailed Table of Contents. Feel free to rearrange the order if it makes more sense that way. Prepare to articulate these points in detail and to answer questions from your classmates and instructor. Outlining is a good self-test of thorough reading because you cannot outline what you have not read or do not comprehend.

✓ Demonstrate **ACTIVE READING STRATEGIES**, including verbalizing your impressions as you read, simplifying long passages to convey the same ideas using fewer words, annotating text and illustrations with your own interpretations, working through mathematical examples shown in the text, cross-referencing passages with relevant illustrations and/or other passages, identifying problem-solving strategies applied by the author, etc. Technical reading is a special case of problem-solving, and so these strategies work precisely because they help solve any problem: paying attention to your own thoughts (metacognition), eliminating unnecessary complexities, identifying what makes sense, paying close attention to details, drawing connections between separated facts, and noting the successful strategies of others.

✓ Identify **IMPORTANT THEMES**, especially **GENERAL LAWS** and **PRINCIPLES**, expounded in the text and express them in the simplest of terms as though you were teaching an intelligent child. This emphasizes connections between related topics and develops your ability to communicate complex ideas to anyone.

✓ Form **YOUR OWN QUESTIONS** based on the reading, and then pose them to your instructor and classmates for their consideration. Anticipate both correct and incorrect answers, the incorrect answer(s) assuming one or more plausible misconceptions. This helps you view the subject from different perspectives to grasp it more fully.

✓ Devise **EXPERIMENTS** to test claims presented in the reading, or to disprove misconceptions. Predict possible outcomes of these experiments, and evaluate their meanings: what result(s) would confirm, and what would constitute disproof? Running mental simulations and evaluating results is essential to scientific and diagnostic reasoning.

✓ Specifically identify any points you found **CONFUSING**. The reason for doing this is to help diagnose misconceptions and overcome barriers to learning.

5.1.2 Foundational concepts

Correct analysis and diagnosis of electric circuits begins with a proper understanding of some basic concepts. The following is a list of some important concepts referenced in this module's full tutorial. Define each of them in your own words, and be prepared to illustrate each of these concepts with a description of a practical example and/or a live demonstration.

Energy

Conservation of Energy

Simplification as a problem-solving strategy

Thought experiments as a problem-solving strategy

Limiting cases as a problem-solving strategy

Annotating diagrams as a problem-solving strategy

Interpreting intermediate results as a problem-solving strategy

Graphing as a problem-solving strategy

Converting a qualitative problem into a quantitative problem

Converting a quantitative problem into a qualitative problem

Working “backwards” to validate calculated results

Reductio ad absurdum

Re-drawing schematics as a problem-solving strategy

Cut-and-try problem-solving strategy

Algebraic substitution

???

5.1.3 First conceptual question

Challenges

- ???.
- ???.
- ???.

5.1.4 Second conceptual question

Challenges

- ???.
- ???.
- ???.

5.1.5 Applying foundational concepts to ???

Identify which foundational concept(s) apply to each of the declarations shown below regarding the following circuit. If a declaration is true, then identify it as such and note which concept supports that declaration; if a declaration is false, then identify it as such and note which concept is violated by that declaration:

(Under development)

- ???
- ???
- ???
- ???

Here is a list of foundational concepts for your reference: **Conservation of Energy, Conservation of Electric Charge, behavior of sources vs. loads, Ohm's Law, Joule's Law, effects of open faults, effect of shorted faults, properties of series networks, properties of parallel networks, Kirchhoff's Voltage Law, Kirchhoff's Current Law.** More than one of these concepts may apply to a declaration, and some concepts may not apply to any listed declaration at all. Also, feel free to include foundational concepts not listed here.

Challenges

- ???.
- ???.
- ???.

5.1.6 Explaining the meaning of calculations

Challenges

- ???.
- ???.
- ???.

5.2 Quantitative reasoning

These questions are designed to stimulate your computational thinking. In a Socratic discussion with your instructor, the goal is for these questions to reveal your mathematical approach(es) to problem-solving so that good technique and sound reasoning may be reinforced. Your instructor may also pose additional questions based on those assigned, in order to observe your problem-solving firsthand.

Mental arithmetic and estimations are strongly encouraged for all calculations, because without these abilities you will be unable to readily detect errors caused by calculator misuse (e.g. keystroke errors).

You will note a conspicuous lack of answers given for these quantitative questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. My advice is to use circuit simulation software such as SPICE to check the correctness of quantitative answers. Refer to those learning modules within this collection focusing on SPICE to see worked examples which you may use directly as practice problems for your own study, and/or as templates you may modify to run your own analyses and generate your own practice problems.

Completely worked example problems found in the Tutorial may also serve as “test cases⁴” for gaining proficiency in the use of circuit simulation software, and then once that proficiency is gained you will never need to rely⁵ on an answer key!

⁴In other words, set up the circuit simulation software to analyze the same circuit examples found in the Tutorial. If the simulated results match the answers shown in the Tutorial, it confirms the simulation has properly run. If the simulated results disagree with the Tutorial’s answers, something has been set up incorrectly in the simulation software. Using every Tutorial as practice in this way will quickly develop proficiency in the use of circuit simulation software.

⁵This approach is perfectly in keeping with the instructional philosophy of these learning modules: *teaching students to be self-sufficient thinkers*. Answer keys can be useful, but it is even more useful to your long-term success to have a set of tools on hand for checking your own work, because once you have left school and are on your own, there will no longer be “answer keys” available for the problems you will have to solve.

5.2.1 Miscellaneous physical constants

Note: constants shown in **bold** type are *exact*, not approximations. Values inside of parentheses show one standard deviation (σ) of uncertainty in the final digits: for example, the magnetic permeability of free space value given as $1.25663706212(19) \times 10^{-6}$ H/m represents a center value (i.e. the location parameter) of $1.25663706212 \times 10^{-6}$ Henrys per meter with one standard deviation of uncertainty equal to $0.0000000000019 \times 10^{-6}$ Henrys per meter.

Avogadro's number (N_A) = **$6.02214076 \times 10^{23}$** per mole (mol^{-1})

Boltzmann's constant (k) = **1.380649×10^{-23}** Joules per Kelvin (J/K)

Electronic charge (e) = **$1.602176634 \times 10^{-19}$** Coulomb (C)

Faraday constant (F) = **$96,485.33212...$** $\times 10^4$ Coulombs per mole (C/mol)

Magnetic permeability of free space (μ_0) = $1.25663706212(19) \times 10^{-6}$ Henrys per meter (H/m)

Electric permittivity of free space (ϵ_0) = $8.8541878128(13) \times 10^{-12}$ Farads per meter (F/m)

Characteristic impedance of free space (Z_0) = $376.730313668(57)$ Ohms (Ω)

Gravitational constant (G) = $6.67430(15) \times 10^{-11}$ cubic meters per kilogram-seconds squared ($\text{m}^3/\text{kg}\cdot\text{s}^2$)

Molar gas constant (R) = **$8.314462618...$** Joules per mole-Kelvin (J/mol-K) = $0.08205746(14)$ liters-atmospheres per mole-Kelvin

Planck constant (h) = **$6.62607015 \times 10^{-34}$** joule-seconds (J-s)

Stefan-Boltzmann constant (σ) = **$5.670374419...$** $\times 10^{-8}$ Watts per square meter-Kelvin⁴ ($\text{W}/\text{m}^2\cdot\text{K}^4$)

Speed of light in a vacuum (c) = **$299,792,458$** meters per second (m/s) = 186282.4 miles per second (mi/s)

Note: All constants taken from NIST data "Fundamental Physical Constants – Complete Listing", from <http://physics.nist.gov/constants>, National Institute of Standards and Technology (NIST), 2018 CODATA Adjustment.

5.2.2 Introduction to spreadsheets

A powerful computational tool you are encouraged to use in your work is a *spreadsheet*. Available on most personal computers (e.g. Microsoft Excel), *spreadsheet* software performs numerical calculations based on number values and formulae entered into cells of a grid. This grid is typically arranged as lettered columns and numbered rows, with each cell of the grid identified by its column/row coordinates (e.g. cell B3, cell A8). Each cell may contain a string of text, a number value, or a mathematical formula. The spreadsheet automatically updates the results of all mathematical formulae whenever the entered number values are changed. This means it is possible to set up a spreadsheet to perform a series of calculations on entered data, and those calculations will be re-done by the computer any time the data points are edited in any way.

For example, the following spreadsheet calculates average speed based on entered values of distance traveled and time elapsed:

	A	B	C	D
1	Distance traveled	46.9	Kilometers	
2	Time elapsed	1.18	Hours	
3	Average speed	= B1 / B2	km/h	
4				
5				

Text labels contained in cells A1 through A3 and cells C1 through C3 exist solely for readability and are not involved in any calculations. Cell B1 contains a sample distance value while cell B2 contains a sample time value. The formula for computing speed is contained in cell B3. Note how this formula begins with an “equals” symbol (=), references the values for distance and speed by lettered column and numbered row coordinates (B1 and B2), and uses a forward slash symbol for division (/). The coordinates B1 and B2 function as *variables*⁶ would in an algebraic formula.

When this spreadsheet is executed, the numerical value 39.74576 will appear in cell B3 rather than the formula = B1 / B2, because 39.74576 is the computed speed value given 46.9 kilometers traveled over a period of 1.18 hours. If a different numerical value for distance is entered into cell B1 or a different value for time is entered into cell B2, cell B3’s value will automatically update. All you need to do is set up the given values and any formulae into the spreadsheet, and the computer will do all the calculations for you.

Cell B3 may be referenced by other formulae in the spreadsheet if desired, since it is a variable just like the given values contained in B1 and B2. This means it is possible to set up an entire chain of calculations, one dependent on the result of another, in order to arrive at a final value. The arrangement of the given data and formulae need not follow any pattern on the grid, which means you may place them anywhere.

⁶Spreadsheets may also provide means to attach text labels to cells for use as variable names (Microsoft Excel simply calls these labels “names”), but for simple spreadsheets such as those shown here it’s usually easier just to use the standard coordinate naming for each cell.

Common⁷ arithmetic operations available for your use in a spreadsheet include the following:

- Addition (+)
- Subtraction (-)
- Multiplication (*)
- Division (/)
- Powers (^)
- Square roots (sqrt())
- Logarithms (ln() , log10())

Parentheses may be used to ensure⁸ proper order of operations within a complex formula. Consider this example of a spreadsheet implementing the *quadratic formula*, used to solve for roots of a polynomial expression in the form of $ax^2 + bx + c$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

	A	B
1	x_1	= (-B4 + sqrt((B4^2) - (4*B3*B5))) / (2*B3)
2	x_2	= (-B4 - sqrt((B4^2) - (4*B3*B5))) / (2*B3)
3	a =	9
4	b =	5
5	c =	-2

This example is configured to compute roots⁹ of the polynomial $9x^2 + 5x - 2$ because the values of 9, 5, and -2 have been inserted into cells B3, B4, and B5, respectively. Once this spreadsheet has been built, though, it may be used to calculate the roots of *any* second-degree polynomial expression simply by entering the new a , b , and c coefficients into cells B3 through B5. The numerical values appearing in cells B1 and B2 will be automatically updated by the computer immediately following any changes made to the coefficients.

⁷Modern spreadsheet software offers a bewildering array of mathematical functions you may use in your computations. I recommend you consult the documentation for your particular spreadsheet for information on operations other than those listed here.

⁸Spreadsheet programs, like text-based programming languages, are designed to follow standard order of operations by default. However, my personal preference is to use parentheses even where strictly unnecessary just to make it clear to any other person viewing the formula what the intended order of operations is.

⁹Reviewing some algebra here, a *root* is a value for x that yields an overall value of zero for the polynomial. For this polynomial ($9x^2 + 5x - 2$) the two roots happen to be $x = 0.269381$ and $x = -0.82494$, with these values displayed in cells B1 and B2, respectively upon execution of the spreadsheet.

Alternatively, one could break up the long quadratic formula into smaller pieces like this:

$$y = \sqrt{b^2 - 4ac} \quad z = 2a$$

$$x = \frac{-b \pm y}{z}$$

	A	B	C
1	x_1	= (-B4 + C1) / C2	= sqrt ((B4^2) - (4*B3*B5))
2	x_2	= (-B4 - C1) / C2	= 2*B3
3	a =	9	
4	b =	5	
5	c =	-2	

Note how the square-root term (y) is calculated in cell C1, and the denominator term (z) in cell C2. This makes the two final formulae (in cells B1 and B2) simpler to interpret. The positioning of all these cells on the grid is completely arbitrary¹⁰ – all that matters is that they properly reference each other in the formulae.

Spreadsheets are particularly useful for situations where the same set of calculations representing a circuit or other system must be repeated for different initial conditions. The power of a spreadsheet is that it automates what would otherwise be a tedious set of calculations. One specific application of this is to simulate the effects of various components within a circuit failing with abnormal values (e.g. a shorted resistor simulated by making its value nearly zero; an open resistor simulated by making its value extremely large). Another application is analyzing the behavior of a circuit design given new components that are out of specification, and/or aging components experiencing drift over time.

¹⁰My personal preference is to locate all the “given” data in the upper-left cells of the spreadsheet grid (each data point flanked by a sensible name in the cell to the left and units of measurement in the cell to the right as illustrated in the first distance/time spreadsheet example), sometimes coloring them in order to clearly distinguish which cells contain entered data versus which cells contain computed results from formulae. I like to place all formulae in cells below the given data, and try to arrange them in logical order so that anyone examining my spreadsheet will be able to figure out *how* I constructed a solution. This is a general principle I believe all computer programmers should follow: *document and arrange your code to make it easy for other people to learn from it.*

5.2.3 First quantitative problem

Challenges

- ???.
- ???.
- ???.

5.2.4 Second quantitative problem

Challenges

- ???.
- ???.
- ???.

5.3 Diagnostic reasoning

These questions are designed to stimulate your deductive and inductive thinking, where you must apply general principles to specific scenarios (deductive) and also derive conclusions about the failed circuit from specific details (inductive). In a Socratic discussion with your instructor, the goal is for these questions to reinforce your recall and use of general circuit principles and also challenge your ability to integrate multiple symptoms into a sensible explanation of what's wrong in a circuit. Your instructor may also pose additional questions based on those assigned, in order to further challenge and sharpen your diagnostic abilities.

As always, your goal is to fully *explain* your analysis of each problem. Simply obtaining a correct answer is not good enough – you must also demonstrate sound reasoning in order to successfully complete the assignment. Your instructor's responsibility is to probe and challenge your understanding of the relevant principles and analytical processes in order to ensure you have a strong foundation upon which to build further understanding.

You will note a conspicuous lack of answers given for these diagnostic questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your diagnostic answers, where applicable, is to use circuit simulation software to explore the effects of faults placed in circuits. For example, if one of these diagnostic questions requires that you predict the effect of an open or a short in a circuit, you may check the validity of your work by simulating that same fault (substituting a very high resistance in place of that component for an open, and substituting a very low resistance for a short) within software and seeing if the results agree.

5.3.1 First diagnostic scenario

Challenges

- ???.
- ???.
- ???.

5.3.2 Second diagnostic scenario

Challenges

- ???.
- ???.
- ???.

Appendix A

Problem-Solving Strategies

The ability to solve complex problems is arguably one of the most valuable skills one can possess, and this skill is particularly important in any science-based discipline.

- Study principles, not procedures. Don't be satisfied with merely knowing how to compute solutions – learn *why* those solutions work.
- Identify what it is you need to solve, identify all relevant data, identify all units of measurement, identify any general principles or formulae linking the given information to the solution, and then identify any “missing pieces” to a solution. Annotate all diagrams with this data.
- Sketch a diagram to help visualize the problem. When building a real system, always devise a plan for that system and analyze its function *before* constructing it.
- Follow the units of measurement and meaning of every calculation. If you are ever performing mathematical calculations as part of a problem-solving procedure, and you find yourself unable to apply each and every intermediate result to some aspect of the problem, it means you don't understand what you are doing. Properly done, every mathematical result should have practical meaning for the problem, and not just be an abstract number. You should be able to identify the proper units of measurement for each and every calculated result, and show where that result fits into the problem.
- Perform “thought experiments” to explore the effects of different conditions for theoretical problems. When troubleshooting real systems, perform *diagnostic tests* rather than visually inspecting for faults, the best diagnostic test being the one giving you the most information about the nature and/or location of the fault with the fewest steps.
- Simplify the problem until the solution becomes obvious, and then use that obvious case as a model to follow in solving the more complex version of the problem.
- Check for exceptions to see if your solution is incorrect or incomplete. A good solution will work for *all* known conditions and criteria. A good example of this is the process of testing scientific hypotheses: the task of a scientist is not to find support for a new idea, but rather to *challenge* that new idea to see if it holds up under a battery of tests. The philosophical

principle of *reductio ad absurdum* (i.e. disproving a general idea by finding a specific case where it fails) is useful here.

- Work “backward” from a hypothetical solution to a new set of given conditions.
- Add quantities to problems that are qualitative in nature, because sometimes a little math helps illuminate the scenario.
- Sketch graphs illustrating how variables relate to each other. These may be quantitative (i.e. with realistic number values) or qualitative (i.e. simply showing increases and decreases).
- Treat quantitative problems as qualitative in order to discern the relative magnitudes and/or directions of change of the relevant variables. For example, try determining what happens if a certain variable were to increase or decrease before attempting to precisely calculate quantities: how will each of the dependent variables respond, by increasing, decreasing, or remaining the same as before?
- Consider limiting cases. This works especially well for qualitative problems where you need to determine which direction a variable will change. Take the given condition and magnify that condition to an extreme degree as a way of simplifying the direction of the system’s response.
- Check your work. This means regularly testing your conclusions to see if they make sense. This does *not* mean repeating the same steps originally used to obtain the conclusion(s), but rather to use some other means to check validity. Simply repeating procedures often leads to *repeating the same errors* if any were made, which is why alternative paths are better.

Appendix B

Instructional philosophy

“The unexamined circuit is not worth energizing” – Socrates (if he had taught electricity)

These learning modules, although useful for self-study, were designed to be used in a formal learning environment where a subject-matter expert challenges students to digest the content and exercise their critical thinking abilities in the answering of questions and in the construction and testing of working circuits.

The following principles inform the instructional and assessment philosophies embodied in these learning modules:

- The first goal of education is to enhance clear and independent thought, in order that every student reach their fullest potential in a highly complex and inter-dependent world. Robust reasoning is *always* more important than particulars of any subject matter, because its application is universal.
- Literacy is fundamental to independent learning and thought because text continues to be the most efficient way to communicate complex ideas over space and time. Those who cannot read with ease are limited in their ability to acquire knowledge and perspective.
- Articulate communication is fundamental to work that is complex and interdisciplinary.
- Faulty assumptions and poor reasoning are best corrected through challenge, not presentation. The rhetorical technique of *reductio ad absurdum* (disproving an assertion by exposing an absurdity) works well to discipline student’s minds, not only to correct the problem at hand but also to learn how to detect and correct future errors.
- Important principles should be repeatedly explored and widely applied throughout a course of study, not only to reinforce their importance and help ensure their mastery, but also to showcase the interconnectedness and utility of knowledge.

These learning modules were expressly designed to be used in an “inverted” teaching environment¹ where students first read the introductory and tutorial chapters on their own, then individually attempt to answer the questions and construct working circuits according to the experiment and project guidelines. The instructor never lectures, but instead meets regularly with each individual student to review their progress, answer questions, identify misconceptions, and challenge the student to new depths of understanding through further questioning. Regular meetings between instructor and student should resemble a Socratic² dialogue, where questions serve as scalpels to dissect topics and expose assumptions. The student passes each module only after consistently demonstrating their ability to logically analyze and correctly apply all major concepts in each question or project/experiment. The instructor must be vigilant in probing each student’s understanding to ensure they are truly *reasoning* and not just *memorizing*. This is why “Challenge” points appear throughout, as prompts for students to think deeper about topics and as starting points for instructor queries. Sometimes these challenge points require additional knowledge that hasn’t been covered in the series to answer in full. This is okay, as the major purpose of the Challenges is to stimulate analysis and synthesis on the part of each student.

The instructor must possess enough mastery of the subject matter and awareness of students’ reasoning to generate their own follow-up questions to practically any student response. Even completely correct answers given by the student should be challenged by the instructor for the purpose of having students practice articulating their thoughts and defending their reasoning. Conceptual errors committed by the student should be exposed and corrected not by direct instruction, but rather by reducing the errors to an absurdity³ through well-chosen questions and thought experiments posed by the instructor. Becoming proficient at this style of instruction requires time and dedication, but the positive effects on critical thinking for both student and instructor are spectacular.

An inspection of these learning modules reveals certain unique characteristics. One of these is a bias toward thorough explanations in the tutorial chapters. Without a live instructor to explain concepts and applications to students, the text itself must fulfill this role. This philosophy results in lengthier explanations than what you might typically find in a textbook, each step of the reasoning process fully explained, including footnotes addressing common questions and concerns students raise while learning these concepts. Each tutorial seeks to not only explain each major concept in sufficient detail, but also to explain the logic of each concept and how each may be developed

¹In a traditional teaching environment, students first encounter new information via *lecture* from an expert, and then independently apply that information via *homework*. In an “inverted” course of study, students first encounter new information via *homework*, and then independently apply that information under the scrutiny of an expert. The expert’s role in lecture is to simply *explain*, but the expert’s role in an inverted session is to *challenge*, *critique*, and if necessary *explain* where gaps in understanding still exist.

²Socrates is a figure in ancient Greek philosophy famous for his unflinching style of questioning. Although he authored no texts, he appears as a character in Plato’s many writings. The essence of Socratic philosophy is to leave no question unexamined and no point of view unchallenged. While purists may argue a topic such as electric circuits is too narrow for a true Socratic-style dialogue, I would argue that the essential thought processes involved with scientific reasoning on *any* topic are not far removed from the Socratic ideal, and that students of electricity and electronics would do very well to challenge assumptions, pose thought experiments, identify fallacies, and otherwise employ the arsenal of critical thinking skills modeled by Socrates.

³This rhetorical technique is known by the Latin phrase *reductio ad absurdum*. The concept is to expose errors by counter-example, since only one solid counter-example is necessary to disprove a universal claim. As an example of this, consider the common misconception among beginning students of electricity that voltage cannot exist without current. One way to apply *reductio ad absurdum* to this statement is to ask how much current passes through a fully-charged battery connected to nothing (i.e. a clear example of voltage existing without current).

from “first principles”. Again, this reflects the goal of developing clear and independent thought in students’ minds, by showing how clear and logical thought was used to forge each concept. Students benefit from witnessing a model of clear thinking in action, and these tutorials strive to be just that.

Another characteristic of these learning modules is a lack of step-by-step instructions in the Project and Experiment chapters. Unlike many modern workbooks and laboratory guides where step-by-step instructions are prescribed for each experiment, these modules take the approach that students must learn to closely read the tutorials and apply their own reasoning to identify the appropriate experimental steps. Sometimes these steps are plainly declared in the text, just not as a set of enumerated points. At other times certain steps are implied, an example being assumed competence in test equipment use where the student should not need to be told *again* how to use their multimeter because that was thoroughly explained in previous lessons. In some circumstances no steps are given at all, leaving the entire procedure up to the student.

This lack of prescription is not a flaw, but rather a feature. Close reading and clear thinking are foundational principles of this learning series, and in keeping with this philosophy all activities are designed to *require* those behaviors. Some students may find the lack of prescription frustrating, because it demands more from them than what their previous educational experiences required. This frustration should be interpreted as an unfamiliarity with autonomous thinking, a problem which must be corrected if the student is ever to become a self-directed learner and effective problem-solver. Ultimately, the need for students to read closely and think clearly is more important both in the near-term and far-term than any specific facet of the subject matter at hand. If a student takes longer than expected to complete a module because they are forced to outline, digest, and reason on their own, so be it. The future gains enjoyed by developing this mental discipline will be well worth the additional effort and delay.

Another feature of these learning modules is that they do not treat topics in isolation. Rather, important concepts are introduced early in the series, and appear repeatedly as stepping-stones toward other concepts in subsequent modules. This helps to avoid the “compartmentalization” of knowledge, demonstrating the inter-connectedness of concepts and simultaneously reinforcing them. Each module is fairly complete in itself, reserving the beginning of its tutorial to a review of foundational concepts.

This methodology of assigning text-based modules to students for digestion and then using Socratic dialogue to assess progress and hone students’ thinking was developed over a period of several years by the author with his Electronics and Instrumentation students at the two-year college level. While decidedly unconventional and sometimes even unsettling for students accustomed to a more passive lecture environment, this instructional philosophy has proven its ability to convey conceptual mastery, foster careful analysis, and enhance employability so much better than lecture that the author refuses to ever teach by lecture again.

Problems which often go undiagnosed in a lecture environment are laid bare in this “inverted” format where students must articulate and logically defend their reasoning. This, too, may be unsettling for students accustomed to lecture sessions where the instructor cannot tell for sure who comprehends and who does not, and this vulnerability necessitates sensitivity on the part of the “inverted” session instructor in order that students never feel discouraged by having their errors exposed. *Everyone* makes mistakes from time to time, and learning is a lifelong process! Part of the instructor’s job is to build a culture of learning among the students where errors are not seen as shameful, but rather as opportunities for progress.

To this end, instructors managing courses based on these modules should adhere to the following principles:

- Student questions are always welcome and demand thorough, honest answers. The only type of question an instructor should refuse to answer is one the student should be able to easily answer on their own. Remember, *the fundamental goal of education is for each student to learn to think clearly and independently*. This requires hard work on the part of the student, which no instructor should ever circumvent. Anything done to bypass the student's responsibility to do that hard work ultimately limits that student's potential and thereby does real harm.
- It is not only permissible, but encouraged, to answer a student's question by asking questions in return, these follow-up questions designed to guide the student to reach a correct answer through their own reasoning.
- All student answers demand to be challenged by the instructor and/or by other students. This includes both correct and incorrect answers – the goal is to practice the articulation and defense of one's own reasoning.
- No reading assignment is deemed complete unless and until the student demonstrates their ability to accurately summarize the major points in their own terms. Recitation of the original text is unacceptable. This is why every module contains an "Outline and reflections" question as well as a "Foundational concepts" question in the Conceptual reasoning section, to prompt reflective reading.
- No assigned question is deemed answered unless and until the student demonstrates their ability to consistently and correctly apply the concepts to *variations* of that question. This is why module questions typically contain multiple "Challenges" suggesting different applications of the concept(s) as well as variations on the same theme(s). Instructors are encouraged to devise as many of their own "Challenges" as they are able, in order to have a multitude of ways ready to probe students' understanding.
- No assigned experiment or project is deemed complete unless and until the student demonstrates the task in action. If this cannot be done "live" before the instructor, video-recordings showing the demonstration are acceptable. All relevant safety precautions must be followed, all test equipment must be used correctly, and the student must be able to properly explain all results. The student must also successfully answer all Challenges presented by the instructor for that experiment or project.

Students learning from these modules would do well to abide by the following principles:

- No text should be considered fully and adequately read unless and until you can express every idea *in your own words, using your own examples*.
- You should always articulate your thoughts as you read the text, noting points of agreement, confusion, and epiphanies. Feel free to print the text on paper and then write your notes in the margins. Alternatively, keep a journal for your own reflections as you read. This is truly a helpful tool when digesting complicated concepts.
- Never take the easy path of highlighting or underlining important text. Instead, *summarize* and/or *comment* on the text using your own words. This actively engages your mind, allowing you to more clearly perceive points of confusion or misunderstanding on your own.
- A very helpful strategy when learning new concepts is to place yourself in the role of a teacher, if only as a mental exercise. Either explain what you have recently learned to someone else, or at least *imagine* yourself explaining what you have learned to someone else. The simple act of having to articulate new knowledge and skill forces you to take on a different perspective, and will help reveal weaknesses in your understanding.
- Perform each and every mathematical calculation and thought experiment shown in the text on your own, referring back to the text to see that your results agree. This may seem trivial and unnecessary, but it is critically important to ensuring you actually understand what is presented, especially when the concepts at hand are complicated and easy to misunderstand. Apply this same strategy to become proficient in the use of *circuit simulation software*, checking to see if your simulated results agree with the results shown in the text.
- Above all, recognize that learning is hard work, and that a certain level of frustration is unavoidable. There are times when you will struggle to grasp some of these concepts, and that struggle is a natural thing. Take heart that it will yield with persistent and varied⁴ effort, and never give up!

Students interested in using these modules for self-study will also find them beneficial, although the onus of responsibility for thoroughly reading and answering questions will of course lie with that individual alone. If a qualified instructor is not available to challenge students, a workable alternative is for students to form study groups where they challenge⁵ one another.

To high standards of education,

Tony R. Kuphaldt

⁴As the old saying goes, “Insanity is trying the same thing over and over again, expecting different results.” If you find yourself stumped by something in the text, you should attempt a different approach. Alter the thought experiment, change the mathematical parameters, do whatever you can to see the problem in a slightly different light, and then the solution will often present itself more readily.

⁵Avoid the temptation to simply share answers with study partners, as this is really counter-productive to learning. Always bear in mind that the answer to any question is far less important in the long run than the method(s) used to obtain that answer. The goal of education is to empower one’s life through the improvement of clear and independent thought, literacy, expression, and various practical skills.

Appendix C

Tools used

I am indebted to the developers of many open-source software applications in the creation of these learning modules. The following is a list of these applications with some commentary on each.

You will notice a theme common to many of these applications: a bias toward *code*. Although I am by no means an expert programmer in any computer language, I understand and appreciate the flexibility offered by code-based applications where the user (you) enters commands into a plain ASCII text file, which the software then reads and processes to create the final output. Code-based computer applications are by their very nature *extensible*, while WYSIWYG (What You See Is What You Get) applications are generally limited to whatever user interface the developer makes for you.

The GNU/Linux computer operating system

There is so much to be said about Linus Torvalds' **Linux** and Richard Stallman's **GNU** project. First, to credit just these two individuals is to fail to do justice to the *mob* of passionate volunteers who contributed to make this amazing software a reality. I first learned of **Linux** back in 1996, and have been using this operating system on my personal computers almost exclusively since then. It is *free*, it is completely *configurable*, and it permits the continued use of highly efficient **Unix** applications and scripting languages (e.g. shell scripts, Makefiles, **sed**, **awk**) developed over many decades. **Linux** not only provided me with a powerful computing platform, but its open design served to inspire my life's work of creating open-source educational resources.

Bram Moolenaar's **Vim** text editor

Writing code for any code-based computer application requires a *text editor*, which may be thought of as a word processor strictly limited to outputting plain-ASCII text files. Many good text editors exist, and one's choice of text editor seems to be a deeply personal matter within the programming world. I prefer **Vim** because it operates very similarly to **vi** which is ubiquitous on **Unix/Linux** operating systems, and because it may be entirely operated via keyboard (i.e. no mouse required) which makes it fast to use.

Donald Knuth's \TeX typesetting system

Developed in the late 1970's and early 1980's by computer scientist extraordinaire Donald Knuth to typeset his multi-volume magnum opus *The Art of Computer Programming*, this software allows the production of formatted text for screen-viewing or paper printing, all by writing plain-text code to describe how the formatted text is supposed to appear. \TeX is not just a markup language for documents, but it is also a Turing-complete programming language in and of itself, allowing useful algorithms to be created to control the production of documents. Simply put, *\TeX is a programmer's approach to word processing*. Since \TeX is controlled by code written in a plain-text file, this means anyone may read that plain-text file to see exactly how the document was created. This openness afforded by the code-based nature of \TeX makes it relatively easy to learn how other people have created their own \TeX documents. By contrast, examining a beautiful document created in a conventional WYSIWYG word processor such as Microsoft **Word** suggests nothing to the reader about *how* that document was created, or what the user might do to create something similar. As Mr. Knuth himself once quipped, conventional word processing applications should be called WYSIAYG (What You See Is *All* You Get).

Leslie Lamport's \LaTeX extensions to \TeX

Like all true programming languages, \TeX is inherently extensible. So, years after the release of \TeX to the public, Leslie Lamport decided to create a massive extension allowing easier compilation of book-length documents. The result was \LaTeX , which is the markup language used to create all ModEL module documents. You could say that \TeX is to \LaTeX as **C** is to **C++**. This means it is permissible to use any and all \TeX commands within \LaTeX source code, and it all still works. Some of the features offered by \LaTeX that would be challenging to implement in \TeX include automatic index and table-of-content creation.

Tim Edwards' **Xcircuit** drafting program

This wonderful program is what I use to create all the schematic diagrams and illustrations (but not photographic images or mathematical plots) throughout the ModEL project. It natively outputs PostScript format which is a true vector graphic format (this is why the images do not pixellate when you zoom in for a closer view), and it is so simple to use that I have never had to read the manual! Object libraries are easy to create for **Xcircuit**, being plain-text files using PostScript programming conventions. Over the years I have collected a large set of object libraries useful for drawing electrical and electronic schematics, pictorial diagrams, and other technical illustrations.

Gimp graphic image manipulation program

Essentially an open-source clone of Adobe's **PhotoShop**, I use **Gimp** to resize, crop, and convert file formats for all of the photographic images appearing in the **ModEL** modules. Although **Gimp** does offer its own scripting language (called **Script-Fu**), I have never had occasion to use it. Thus, my utilization of **Gimp** to merely crop, resize, and convert graphic images is akin to using a sword to slice bread.

SPICE circuit simulation program

SPICE is to circuit analysis as **T_EX** is to document creation: it is a form of markup language designed to describe a certain object to be processed in plain-ASCII text. When the plain-text "source file" is compiled by the software, it outputs the final result. More modern circuit analysis tools certainly exist, but I prefer **SPICE** for the following reasons: it is *free*, it is *fast*, it is *reliable*, and it is a fantastic tool for *teaching* students of electricity and electronics how to write simple code. I happen to use rather old versions of **SPICE**, version 2g6 being my "go to" application when I only require text-based output. **NGSPICE** (version 26), which is based on Berkeley **SPICE** version 3f5, is used when I require graphical output for such things as time-domain waveforms and Bode plots. In all **SPICE** example netlists I strive to use coding conventions compatible with all **SPICE** versions.

Andrew D. Hwang's ePiX mathematical visualization programming library

This amazing project is a **C++** library you may link to any **C/C++** code for the purpose of generating PostScript graphic images of mathematical functions. As a completely free and open-source project, it does all the plotting I would otherwise use a Computer Algebra System (CAS) such as **Mathematica** or **Maple** to do. It should be said that **ePiX** is *not* a Computer Algebra System like **Mathematica** or **Maple**, but merely a mathematical *visualization* tool. In other words, it won't determine integrals for you (you'll have to implement that in your own **C/C++** code!), but it can graph the results, and it does so beautifully. What I really admire about **ePiX** is that it is a **C++** programming library, which means it builds on the existing power and toolset available with that programming language. Mr. Hwang could have probably developed his own stand-alone application for mathematical plotting, but by creating a **C++** library to do the same thing he accomplished something much greater.

`gnuplot` mathematical visualization software

Another open-source tool for mathematical visualization is `gnuplot`. Interestingly, this tool is *not* part of Richard Stallman’s GNU project, its name being a coincidence. For this reason the authors prefer “gnu” *not* be capitalized at all to avoid confusion. This is a much “lighter-weight” alternative to a spreadsheet for plotting tabular data, and the fact that it easily outputs directly to an X11 console or a file in a number of different graphical formats (including PostScript) is very helpful. I typically set my `gnuplot` output format to default (X11 on my Linux PC) for quick viewing while I’m developing a visualization, then switch to PostScript file export once the visual is ready to include in the document(s) I’m writing. As with my use of `Gimp` to do rudimentary image editing, my use of `gnuplot` only scratches the surface of its capabilities, but the important points are that it’s *free* and that it *works well*.

Python programming language

Both Python and C++ find extensive use in these modules as instructional aids and exercises, but I’m listing Python here as a *tool* for myself because I use it almost daily as a *calculator*. If you open a Python interpreter console and type `from math import *` you can type mathematical expressions and have it return results just as you would on a hand calculator. Complex-number (i.e. *phasor*) arithmetic is similarly supported if you include the complex-math library (`from cmath import *`). Examples of this are shown in the Programming References chapter (if included) in each module. Of course, being a fully-featured programming language, Python also supports conditionals, loops, and other structures useful for calculation of quantities. Also, running in a console environment where all entries and returned values show as text in a chronologically-ordered list makes it easy to copy-and-paste those calculations to document exactly how they were performed.

Appendix D

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Appendix E

References

Appendix F

Version history

This is a list showing all significant additions, corrections, and other edits made to this learning module. Each entry is referenced by calendar date in reverse chronological order (newest version first), which appears on the front cover of every learning module for easy reference. Any contributors to this open-source document are listed here as well.

19 February 2024 – added a new section to the Technical References chapter on near-field versus far-field effects.

14 November 2023 – added a new Case Tutorial section on empirically determining signal rates of change.

19 December 2021 – added Case Tutorial section showing the use of an SDR as a spectrum analyzer for E-field and H-field probing.

3 December 2021 – document first created.

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