

# MODULAR ELECTRONICS LEARNING (MODEL) PROJECT



## MAXIMUM POWER TRANSFER THEOREM

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# Chapter 1

## Introduction

### 1.1 Recommendations for students

Ideal electrical sources maintain either output voltage or output current absolutely constant regardless of load. Real electrical sources do not behave ideally, though, and are commonly modeled as ideal sources together with some amount of internal resistance limiting their outputs. The essence of the Maximum Power Transfer Theorem is that for any given source, power delivery to a load will be maximized by adjusting the load's resistance to equal the source's.

The Maximum Power Transfer Theorem is extremely useful in any application where the purpose of using electricity is to transfer energy as much energy as possible in a given time from one location to another. Such examples include audio amplifier circuits where the goal is to be able to transfer the amplifier's maximum power output to loudspeakers, and also radio transmitting circuits where the goal is to transfer as much power from the transmitter circuit to the antenna. A set of loudspeakers mismatched to the amplifier will fail to convey the fullest possible sound levels to an audience, and an antenna mismatched to the transmitter will fail to convey the fullest possible radio field strength into space.

Important concepts related to the Maximum Power Transfer Theorem include **voltage sources**, **current sources**, **internal resistance**, the effects of **opens** versus **shorts**, **light** versus **heavy** load conditions, **Ohm's Law**, **Joule's Law**, **dependent** versus **independent** variables, and **sources** versus **loads**.

A useful problem-solving strategy applied in the Tutorial is that of a *thought experiment*, whereby we propose some conditions in a system and apply what we know of electrical circuit theory to determine the outcome(s) of those conditions. This is a very practical problem-solving technique and is worth applying to every new phenomenon you encounter in your studies. A related strategy is the so-called "cut-and-try" technique whereby we try different numerical values for some variable to see what the results are for each, and if there is any general pattern as a result.

When reading any mathematically-based presentation, a useful habit for effective learning is to actually perform the mathematics being shown in the text. Don't just passively read what the text tells you and trust that the math works – try the math for yourself. Not only will this serve

to confirm what you are reading, but it is also an excellent way to practice those mathematical techniques.

Here are some good questions to ask of yourself while studying this subject:

- How might an experiment be designed and conducted to demonstrate the Maximum Power Transfer Theorem? What hypothesis (i.e. prediction) might you pose for that experiment, and what result(s) would either support or disprove that hypothesis?
- What are some practical applications of the Maximum Power Transfer theorem?
- In what ways do voltage sources differ from current sources?
- Why do electrical sources provide different values of voltage/current when loaded versus when unloaded?
- What are some physical origins of internal source resistance?
- How much internal resistance does an ideal voltage source exhibit?
- How much internal resistance does an ideal current source exhibit?
- What makes a load either “heavy” or “light” from the perspective of the source supplying energy to it?
- How were we able to establish the truth of the Maximum Power Transfer Theorem by performing thought experiments?
- What does it mean to consider “limiting cases” for some variable in a problem you must solve?
- What does it mean to “cut-and-try” as a method of solving a problem?
- What factors directed our choice for values to “try” when determining the point of maximum load power?
- How would you recommend calculating load power for any of the “thought experiments” shown in the Tutorial, where a resistance value was varied in small increments to try to find its optimum value?
- What is one way that the Maximum Power Transfer Theorem is often misunderstood?
- How might you alter one of the example analyses shown in the text, and then determine the behavior of that altered circuit?

## 1.2 Challenging concepts related to the Maximum Power Transfer Theorem

The following list cites concepts related to this module's topic that are easily misunderstood, along with suggestions for properly understanding them:

- **Misapplication of the Maximum Power Transfer Theorem** – at first, this theorem appears to be very simple: maximum power will be dissipated at the load when the load resistance is equal to the source circuit's equivalent Thévenin or Norton resistance. However, many students misunderstand the scope of this theorem. What it is really saying is that power will be maximized at the load when  $R_{load} = R_{Thevenin}$ , *if  $R_{load}$  is the only resistance you have the freedom to change*. If  $R_{load}$  happens to be fixed, but you have the freedom to change the source circuit's internal resistance, then maximum load power will be achieved when  $R_{Thevenin}$  is equal to zero! Another misconception is that this theorem tells you when optimum power *efficiency* is reached. In fact, when  $R_{load} = R_{Thevenin}$ , the system efficiency is only 50%. Exactly half of the total source power will be wasted, with only half getting to the load! Maximum *efficiency* is actually reached when the load resistance is very large compared to the source resistance, but of course the actual amount of load power (measured in watts) will be much less than if the load resistance were closer in value to  $R_{Thevenin}$ .
- **Energy efficiency** – Energy efficiency is a ratio of energy directed out of a system toward useful work versus total energy put in to that system, and is not necessarily maximized when load power is maximized for all systems.
- **Schematic annotation as a problem-solving tool** – as an instructor you will find most students new to the study of electronics attempt to solve circuit-analysis problems by inspection, or by plugging given values into familiar equations until something resembling an answer emerges. Such efforts are usually fruitless because it is difficult to track the real-world context of every circuit parameter doing things this way. You will need to demonstrate, encourage, and at times *require* your students to apply the alternative strategy of first sketching a schematic diagram of the circuit and then annotating it with all known values. Once the given information gains context through this annotation, appropriate principles (e.g. Ohm's Law, KVL, KCL) are much easier to identify and then properly apply for that circuit. Then, as calculated values get annotated on the same schematic diagram the next logical steps to follow become easier and easier to identify. I recommend using different colors to label different parameters (e.g. blue for voltage, red for current, black for component values, green for power). Students should have blank "scratch" paper and writing instruments with them at all times, even during discussions, so as to neatly organize information and to articulate and focus their thoughts!

The best way to overcome the misapplication of Maximum Power Transfer Theorem is to analyze a two-resistor, single-source circuit for many different values of load resistance, seeing for yourself what happens to load power as the different resistor values change. Approach this scientifically: do a lot of calculations changing only one resistor at a time. Make sure you do enough calculations where you can see the trend of how that one resistor's value affects load power before you decide to



change the other resistor's value. Changing only one variable at a time in a scientific test is very important, otherwise it will be difficult (or impossible) to tell *what* what is causing what to happen.

## 1.3 Recommendations for instructors

This section lists realistic student learning outcomes supported by the content of the module as well as suggested means of assessing (measuring) student learning. The outcomes state what learners should be able to do, and the assessments are specific challenges to prove students have learned.

- **Outcome** – Demonstrate effective technical reading and writing

Assessment – Students present their outlines of this module’s instructional chapters (e.g. Case Tutorial, Tutorial, Historical References, etc.) ideally as an entry to a larger Journal document chronicling their learning. These outlines should exhibit good-faith effort at summarizing major concepts explained in the text.

Assessment – Students show how ??? quantitative results were obtained by the author in the Tutorial chapter’s examples.

- **Outcome** – Apply the concept of Maximum Power Transfer to practical scenarios

Assessment – Determine what parameter(s) would need to be adjusted to maximize the amount of power delivered to a load in a given system; e.g. pose problems in the form of the “Optimized electrolysis cell” Conceptual Reasoning question.

- **Outcome** – Prove the concept of Maximum Power Transfer by “thought experiment”

Assessment – Calculate load power in a simple two-resistor circuit for several different values of load resistance to see the effect changing that resistance has on load power dissipation; e.g. conduct an experiment similar to that shown in the “Series resistance” Quantitative Reasoning question.



## Chapter 2

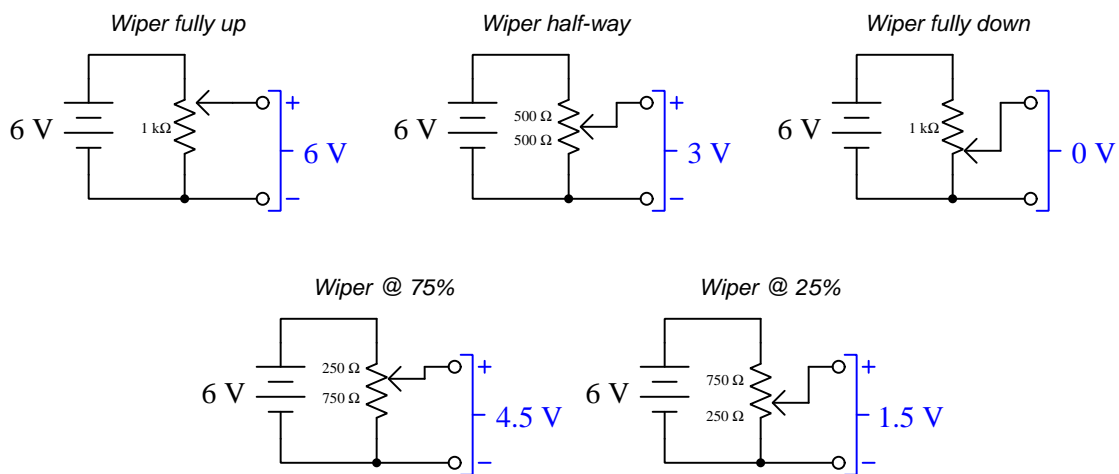
# Case Tutorial

The idea behind a *Case Tutorial* is to explore new concepts by way of example. In this chapter you will read less presentation of theory compared to other Tutorial chapters, but by close observation and comparison of the given examples be able to discern patterns and principles much the same way as a scientific experimenter. Hopefully you will find these cases illuminating, and a good supplement to text-based tutorials.

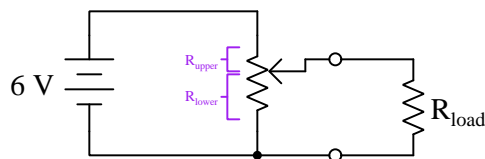
These examples also serve well as challenges following your reading of the other Tutorial(s) in this module – can you explain *why* the circuits behave as they do?

## 2.1 Example: variable-voltage source

A simple way to convert a fixed-voltage source such as an electrochemical battery into a variable-voltage “power supply” suitable for providing electrical energy to a test circuit is to connect a potentiometer as a variable voltage divider. The potentiometer’s movable wiper forms a voltage divider network from the upper and lower portions of the resistive strip. Assuming a battery voltage of exactly 6 Volts, the voltage obtained between the potentiometer’s wiper and bottom terminal will be a direct proportion of the battery’s voltage. The following illustrations show how this works for a 1 k $\Omega$  potentiometer:



When connected to a load resistance, the potentiometer forms a *series-parallel* network with the load:



In this “loaded” state the output voltage of the battery/potentiometer “power supply” will no longer be a simple proportion of the battery’s voltage based on potentiometer position, but will also depend on the load’s resistance. Thus, obtaining a certain amount of load voltage requires the user to measure voltage between the output terminals with the load connected while carefully turning the potentiometer’s shaft. That voltage will remain at the desired amount until the battery’s internal voltage changes (due to age) and/or the load resistance changes.

The following examples show values of voltage, current, and power dissipation for this circuit given a 6 Volt battery, a 1 kilo-Ohm potentiometer, and a 3.3 kilo-Ohm load resistance:

- Potentiometer position = 50%
- $V_{load} = 2.7887$  V
- $I_{load} = 845.07$   $\mu$ A
- $P_{load} = 2.3567$  mW
- $P_{upper} = 20.624$  mW
- $P_{lower} = 15.554$  mW
- $P_{pot(total)} = 36.179$  mW
- $I_{batt} = 6.4225$  mA
  
- Potentiometer position = 75%
- $V_{load} = 4.2581$  V
- $I_{load} = 1.2903$  mA
- $P_{load} = 5.4943$  mW
- $P_{upper} = 12.137$  mW
- $P_{lower} = 24.175$  mW
- $P_{pot(total)} = 36.312$  mW
- $I_{batt} = 6.9677$  mA
  
- Potentiometer position = 23%
- $V_{load} = 1.3097$  V
- $I_{load} = 396.88$   $\mu$ A
- $P_{load} = 519.80$   $\mu$ W
- $P_{upper} = 28.570$  mW
- $P_{lower} = 7.4580$  mW
- $P_{pot(total)} = 36.028$  mW
- $I_{batt} = 6.0913$  mA

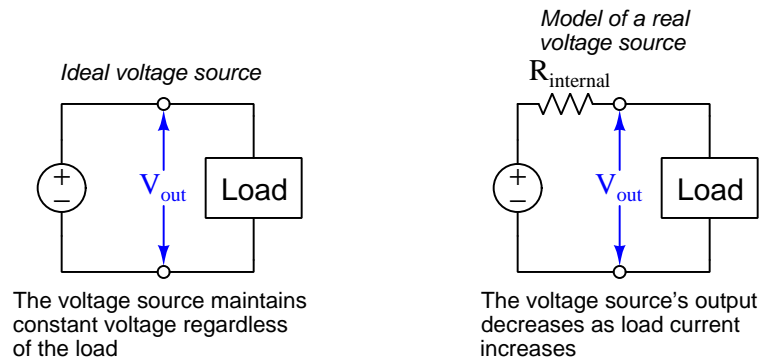


# Chapter 3

## Tutorial

### 3.1 Ideal versus real sources

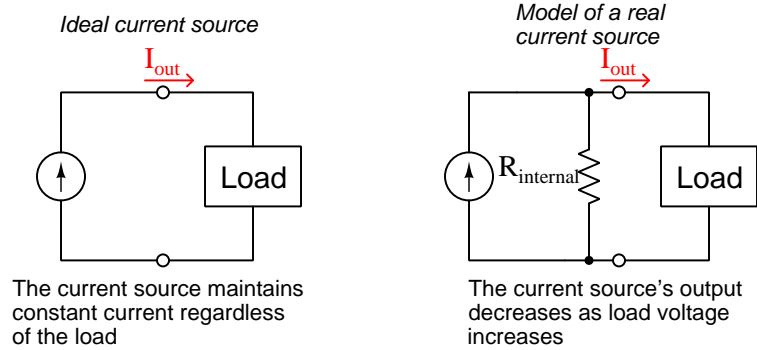
A perfect voltage source outputs constant voltage regardless of how much or little current passes through it: no matter the rate of electrical charge carriers flowing through a perfect voltage source, each charge carrier gains the exact same amount of energy from the source. Real voltage sources cannot maintain absolutely constant voltage over wide variations in current, and so we often model real voltage sources as ideal voltage sources connected in series with internal resistance:



The closer to ideal a voltage source is, the less its internal resistance. An ideal voltage source is considered to have zero internal resistance.



Similarly, a perfect current source outputs constant current regardless of how much or little voltage appears across its terminals: no matter how much energy is given to each electrical charge carrier as it passes through the source, the rate that charge carriers move through will remain constant. Real current sources cannot maintain absolutely constant current over wide variations in voltage, and so we often model real current sources as ideal current sources connected in parallel with internal resistance:



The closer to ideal a current source is, the more its internal resistance. An ideal current source is considered to have infinite internal resistance.

Real sources, both voltage and current, approach ideal behavior as their loads become lighter (i.e. dissipate less power) and deviate more from ideal behavior when their loads become heavier (i.e. dissipate more power). For a voltage source, a “light” load is one drawing little or no current, such as an open-circuit, because less current for any given voltage results in less power. For a current source, a “light” load is one dropping little to no voltage, such as a short-circuit, because less voltage for any given current also results in less power. If you examine the schematic models of real voltage and current sources previously shown, and imagine their respective minimum-power load conditions (i.e. open load for the voltage source and shorted load for the current source) you will see that in both cases the internal resistance becomes irrelevant. For the voltage source, a load drawing no current means that the source’s  $R_{internal}$  drops no voltage, leaving full source voltage at the output terminals. For the current source, a load dropping no voltage means that the source’s  $R_{internal}$  passes no current, leaving full source current at the output terminals.

While the internal resistances of real voltage and current sources have many practical implications, the important point to recognize for this tutorial is that *voltage sources ideally have no internal resistance while current sources ideally have infinite internal resistance*. In particular, some electrical network theorems require this knowledge to apply, as one must consider the internal resistance of each source as a factor in the theorem.

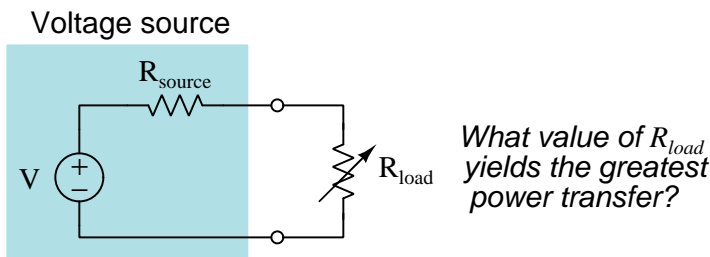
Ideal sources are unlimited in their ability to deliver energy to a load. This becomes apparent when you consider “heavy” load conditions for each source type. A “heavy” load for a voltage source is a condition of very low load resistance, where the source maintains its constant voltage at a high current value. The amount of power delivered by an ideal voltage source rises toward infinity as load resistance decreases toward zero. A “heavy” load for a current source, by contrast, is a condition of

very high load resistance, where the source maintains its constant current at a high voltage value. Ideal current source power rises toward infinity as load resistance increases toward infinity.

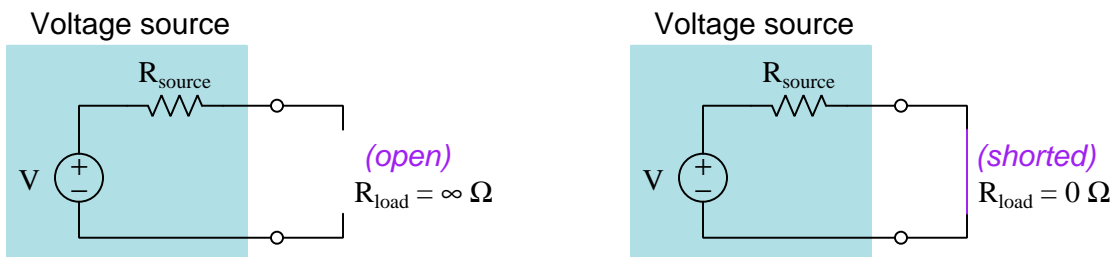
Of course, infinite electrical energy sources simply do not exist. Real voltage and current sources have intrinsic limitations to the amount of power they can deliver to any load, which is why real voltage sources can never output an infinite amount of current and real current sources can never output an infinite amount of voltage. The models shown for real voltage and current sources (with built-in resistances) help explain this practical behavior in ideal-component terms.

### 3.2 Load power with varying load resistance

When connecting resistive loads to real sources, is there a particular load resistance value extracting the greatest amount of power from the source? This is a very valid question when the source we are working with is meager for the task, and consequently where we must optimize its performance:



As a first step in answering this question, let us consider a real voltage source powering a short versus powering an open. From these two “thought experiments” we should be able to plot a path forward to a general answer:

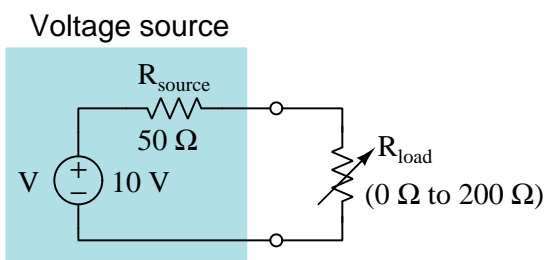


In the open-circuit experiment, load current is obviously zero. As such, there is no power dissipation anywhere in the circuit because no charge carriers are flowing. In the short-circuit experiment, load current reaches a maximum limited only by  $\frac{V}{R_{source}}$ , but load power is still zero; this time because the shorted load drops no voltage even as it carries maximum current (i.e. charge carriers passing through the shorted load lose no energy). In summary, load power dissipation is zero for these two limiting cases<sup>1</sup>.

<sup>1</sup>Interestingly, *source* power is not zero for both conditions, but rather goes from zero in the open-circuit condition to maximum in the short-circuit condition ( $P = \frac{V^2}{R_{source}}$ ). However, since the question we’re trying to answer pertains to load power we will ignore this source power phenomenon for now.

It should stand to reason that since load power is zero for an open-circuit condition due to lack of current, and load power is zero for a short-circuit condition due to lack of voltage, any condition exhibiting both voltage and current should yield a power dissipation greater than zero. Furthermore, if load power as a function of load resistance begins and ends at zero, there is likely some particular value of load resistance between zero and infinity which will maximize its power.

One way to approach this problem is to perform multiple calculations in “cut-and-try” style for different values of load resistance to determine what load resistance value maximizes its power. For this, we will need to arbitrarily choose  $V$  and  $R_{source}$  values as well as a range of  $R_{load}$  values. Let us choose a voltage value of 10 Volts, an internal source resistance value of 50 Ohms, and load resistance values ranging from 0 Ohms to 200 Ohms in 40 Ohm increments<sup>2</sup>:



| $R_{load}$ | $P_{load}$   |
|------------|--------------|
| 0 Ohms     | 0 Watts      |
| 40 Ohms    | 0.4938 Watts |
| 80 Ohms    | 0.4734 Watts |
| 120 Ohms   | 0.4152 Watts |
| 160 Ohms   | 0.3628 Watts |
| 200 Ohms   | 0.3200 Watts |

Note how the highest power value in this table is when  $R_{load} = 40$  Ohms. However, this might not be the actual peak power. From this limited data we cannot tell if the load’s power peak occurs below 40 Ohms, or if it occurs above 40 Ohms. More “thought experimentation” will be necessary.

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<sup>2</sup>The calculations for load power at each load resistance value are not shown, and are left as an exercise to the reader. All you need is to apply the properties of series networks, Ohm’s Law, and Joule’s Law to solve for load power dissipation in each case.

Applying the same exploratory “cut-and-try” technique will require finer increments of load resistance, and since there is no purpose in exceeding 80 Ohms in this second run, we will limit our load resistance range from 0 Ohms to 80 Ohms, this time taking 10 Ohm increments:

| $R_{load}$ | $P_{load}$   |
|------------|--------------|
| 0 Ohms     | 0 Watts      |
| 10 Ohms    | 0.2778 Watts |
| 20 Ohms    | 0.4082 Watts |
| 30 Ohms    | 0.4688 Watts |
| 40 Ohms    | 0.4938 Watts |
| 50 Ohms    | 0.5000 Watts |
| 60 Ohms    | 0.4959 Watts |
| 70 Ohms    | 0.4861 Watts |
| 80 Ohms    | 0.4734 Watts |

Now it would appear the load’s power reaches a maximum when  $R_{load}$  is equal to 50 Ohms, the same value as the source’s internal resistance. Is this a coincidence? More experimentation is necessary!

Let's run more "thought experiments" over a range of 40 Ohms to 60 Ohms in 1 Ohm steps:

| $R_{load}$ | $P_{load}$    |
|------------|---------------|
| 40 Ohms    | 0.49383 Watts |
| 41 Ohms    | 0.49511 Watts |
| 42 Ohms    | 0.49622 Watts |
| 43 Ohms    | 0.49717 Watts |
| 44 Ohms    | 0.49796 Watts |
| 45 Ohms    | 0.49861 Watts |
| 46 Ohms    | 0.49913 Watts |
| 47 Ohms    | 0.49952 Watts |
| 48 Ohms    | 0.49979 Watts |
| 49 Ohms    | 0.49995 Watts |
| 50 Ohms    | 0.50000 Watts |
| 51 Ohms    | 0.49995 Watts |
| 52 Ohms    | 0.49981 Watts |
| 53 Ohms    | 0.49958 Watts |
| 54 Ohms    | 0.49926 Watts |
| 55 Ohms    | 0.49887 Watts |
| 56 Ohms    | 0.49840 Watts |
| 57 Ohms    | 0.49786 Watts |
| 58 Ohms    | 0.49726 Watts |
| 59 Ohms    | 0.49659 Watts |
| 60 Ohms    | 0.49587 Watts |

By now it should be clear that a load resistance value of 50 Ohms (being equal to the source's internal resistance) is not a coincidence<sup>3</sup>. This is the optimum load resistance for maximizing energy dissipation at the load. Thus, the Maximum Power Transfer Theorem may be stated as such:

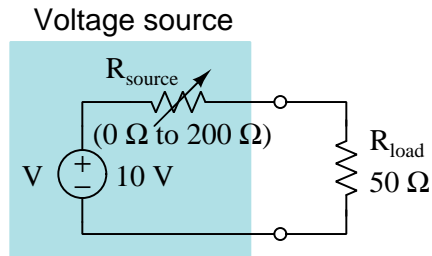
**Maximum Power Transfer Theorem:** power dissipated by a load resistance will be maximized when that load resistance is made equal to the source's internal resistance.

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<sup>3</sup>To see a mathematical proof of this fact, refer to section 4.1 on page 24.

### 3.3 Load power with varying source resistance

An important caveat of the Maximum Power Transfer Theorem is that load power is maximized only when the *load resistance* is adjusted to equal source resistance, not vice-versa. It is a common misconception to think load power would be maximized by adjusting the *source's* internal resistance to equal the load. All we need to do to disprove this misconception is run another thought experiment, this time holding the load resistor value at 50 Ohms and adjusting the source's internal resistance over a range of 0 Ohms to 200 Ohms:



| $R_{source}$ | $P_{load}$   |
|--------------|--------------|
| 0 Ohms       | 2.000 Watts  |
| 40 Ohms      | 0.6173 Watts |
| 80 Ohms      | 0.2959 Watts |
| 120 Ohms     | 0.1730 Watts |
| 160 Ohms     | 0.1134 Watts |
| 200 Ohms     | 0.0800 Watts |

Here we see a very different phenomenon: load power reaches its maximum value of 2 Watts with a source resistance of zero Ohms. Any source resistance value greater than zero results in less power delivered to the load. There is no “peaking” of power when  $R_{source} = R_{load}$ .

To anyone familiar with Ohm's and Joule's Laws but unfamiliar with the Maximum Power Transfer Theorem, this conclusion ought to be obvious. *Of course the source can deliver more power to the load when there is no extra resistance standing in the way!* In fact, the insertion of a series resistance between a voltage source and a load is one of the simplest methods existing for controlling power to that load: more series resistance, less load power; less series resistance, more load power.

This is a paradox only if you have misunderstood the Maximum Power Transfer Theorem, which states load power becomes maximized as the *load* resistance is adjusted to equal the source resistance. In mathematical terms, the function expressed by the Maximum Power Transfer Theorem is one where load power is the *dependent variable* and load resistance is the *independent variable*. If we try to substitute source resistance instead of load resistance for the independent variable, we obtain an entirely different mathematical function which is unrelated to the Maximum Power Transfer Theorem.

**Maximum Power Transfer Theorem:** (load power as a function of load resistance)

$$P_{load} = f(R_{load}) \quad \cdots \quad P_{load} \text{ maximized when } R_{load} = R_{source}$$

**Simple series-resistance load control:** (load power as a function of source resistance)

$$P_{load} = f(R_{source}) \quad \cdots \quad P_{load} \text{ maximized when } R_{source} = 0$$

### 3.4 Applications of the Maximum Power Transfer theorem

Any electrical system where we intend to direct as much power as possible from a source to a load, and where the source's internal resistance is fixed, means we must design the load to have that same amount of resistance in order to achieve our goal of maximizing load power. Many practical applications of this exist, of which a few are described here:

- **Solar electric power systems** – electrical systems powered by solar photovoltaic panels often benefit from optimizing the amount of power drawn from those panels, as sunlight is limited and panel array size is limited by practical concerns such as space and cost. One way to do this is to install a *maximum power point tracking* (MPPT) circuit between the solar panel array and the load, this circuit functioning in such a way as to make the load appear to be the same resistance as the panel array's internal resistance. MPPT circuits are often self-adjusting, which means they are able to automatically compensate for changing conditions of sunlight exposure and connection/disconnection of photovoltaic panels from the array.
- **Signal amplifier design** – an *amplifier* circuit takes in some weak electrical signal and uses that signal to steer or re-direct power from a strong DC source with the intent of creating a replica of the weak signal's characteristics that is boosted in voltage and/or current. If the source of that weak signal happens to have substantial internal resistance, then getting the most power out of it requires that the amplifier be designed to present that same amount of load resistance to the source. Thus, when designing an amplifier circuit we often choose its internal component values such to give it an “input resistance” which will be equal to the known internal resistance of our weak signal source.
- **Radio receiving antenna** – an *antenna* is any electrically-conductive structure designed to intercept a radio wave (which is a set of oscillating electric and magnetic fields radiating through space) and convert that intercepted radio wave into a small electrical signal which may be amplified and interpreted for the information it contains. Due to the nature of how antennas work, each one exhibits its own unique *impedance* value which for the purpose of this conversation is really just an internal resistance. Viewed from the perspective of the radio receiver circuitry, then, the antenna is a weak signal source with a known internal resistance. Therefore, in order for the receiver circuit to extract as much useful power (and also intelligible information) from the antenna its internal resistance at its input terminals should match the antenna's impedance value. This precise matching of resistance/impedance values becomes all the more important when the radio signals themselves are extremely weak, as is the case in radio astronomy or in receiving data signals from deep-space probes.





## Chapter 4

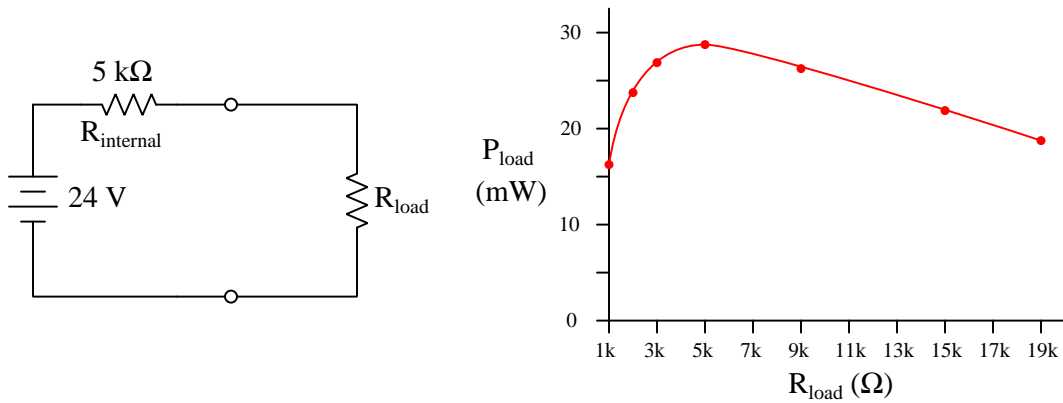
# Derivations and Technical References

This chapter is where you will find mathematical derivations too detailed to include in the tutorial, and/or tables and other technical reference material.

## 4.1 Derivation of the Maximum Power Transfer Theorem

The “trial-and-error” method used in the Tutorial to demonstrate load power peaking when  $R_{load}$  is set equal to  $R_{source}$  is fairly intuitive: you can see by the power values in the table that they reached their maximum at  $R_{load} = R_{source}$ . However, this is not the sort of proof that would satisfy a scientist. Surely there must be a more *mathematically rigorous* way to establish the Maximum Power Transfer Theorem, especially if we’re going to elevate it to the status of a Theorem!

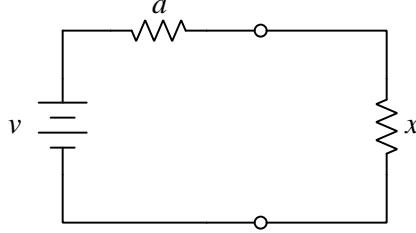
Yes, there is a way to do this, and it involves calculus. If we plot load power as a function of load resistance for any simple voltage-source circuit, we obtain a curve that is concave-down as shown below:



The point of maximum load point on this curve is the point at which its slope (i.e. its steepness) is zero. In other words, it reaches its highest point when it is no longer rising or falling, but is flat. As we can see on this graph, that high point happens to be a load resistance value of 5 k $\Omega$  for the circuit shown. To prove this mathematically, though, we will need to set up an equation describing that graph’s slope as a function of load resistance and then solve that equation for any value of load resistance resulting in a slope of zero.

Mathematically, the *slope* of any function is called the *derivative*. If we have an equation describing the graphed function (in this case, power as a function of load resistance), we can apply the calculus technique of differentiation to arrive at another equation describing the slope of the first equation. Therefore, what we need to do here is come up with an equation describing  $P_{load}$  as a function of  $R_{load}$ , and then we will differentiate that equation to arrive at a second equation which we will set equal to zero, and then solve for any  $R_{load}$  value satisfying the zero condition.

First, a helpful step will be to replace subscripted variables such as  $R_{internal}$  and  $R_{load}$  with simpler variables such as  $a$  and  $x$ . This will make the equations more readable:



One way to calculate load power is by using the formula  $P = I^2R$ . In this case, the resistance we're referencing for power is our load resistance  $x$ . Current may be calculated by dividing voltage by total series resistance. In this case, current would be equal to  $\frac{v}{a+x}$ . Combining these two formulae to create a function for load power as a function of load resistance:

$$P_{load} = I^2 R_{load} \quad I = \frac{v}{a+b} \quad R_{load} = x$$

$$P_{load} = \left( \frac{v}{a+x} \right)^2 x$$

Computing the square and re-writing this as a polynomial quotient:

$$P_{load} = \left( \frac{v^2}{a^2 + 2ax + x^2} \right) x$$

$$P_{load} = \frac{v^2 x}{a^2 + 2ax + x^2}$$

Now we need to *differentiate* this function with respect to  $x$ . To do this, we need to apply the *Quotient Rule* for differentiation which is written in general form as follows:

$$\left( \frac{f(x)}{g(x)} \right)' = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

Where,

- $f(x)$  = The function of  $x$  in the numerator of the quotient
- $g(x)$  = The function of  $x$  in the denominator of the quotient
- $f'(x)$  = The first derivative of the function  $f(x)$
- $g'(x)$  = The first derivative of the function  $g(x)$

Differentiation of quotients can result in ugly formulae. One trick we will use to help ourselves is to summarize the numerator and denominator functions as well as their first derivatives:

$$\begin{aligned} f(x) &= v^2x \\ g(x) &= a^2 + 2ax + x^2 \\ f'(x) &= v^2 \\ g'(x) &= 2a + 2x \end{aligned}$$

Finally, putting it all together:

$$\begin{aligned} P_{load} &= \frac{v^2x}{a^2 + 2ax + x^2} & \left( \frac{f(x)}{g(x)} \right)' &= \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2} \\ (P_{load})' &= \frac{(a^2 + 2ax + x^2)(v^2) - (v^2x)(2a + 2x)}{(a^2 + 2ax + x^2)^2} \end{aligned}$$

Now that we have the first-derivative function of power in terms of load resistance  $x$ , we need to set it equal to zero. This will allow us to solve for that value of  $x$  where the curve peaks (i.e. reaches its “flat” portion):

$$0 = \frac{(a^2 + 2ax + x^2)(v^2) - (v^2x)(2a + 2x)}{(a^2 + 2ax + x^2)^2}$$

Knowing that the value of any quotient will be zero any time its numerator is equal to zero allows us to eliminate the denominator altogether and simply solve for the root<sup>1</sup> of the numerator:

$$0 = (a^2 + 2ax + x^2)(v^2) - (v^2x)(2a + 2x)$$

Step by step, solving for  $x$ :

$$0 = a^2v^2 + 2av^2x + v^2x^2 - 2av^2x - 2v^2x^2$$

$$0 = a^2v^2 + v^2x^2 - 2v^2x^2$$

$$0 = a^2v^2 - v^2x^2$$

$$0 = v^2(a^2 - x^2)$$

$$0 = a^2 - x^2$$

$$x^2 = a^2$$

$$x = a$$

With that, we have our proof of the Maximum Power Transfer Theorem: the power dissipation of load resistance  $x$  will be at its maximum when equal to the internal resistance of the source ( $a$ ).

<sup>1</sup>In algebra, a *root* of an equation is any value of its independent variable resulting in an over-all value of zero.

## Chapter 5

# Questions

This learning module, along with all others in the ModEL collection, is designed to be used in an inverted instructional environment where students independently read<sup>1</sup> the tutorials and attempt to answer questions on their own *prior* to the instructor's interaction with them. In place of lecture<sup>2</sup>, the instructor engages with students in Socratic-style dialogue, probing and challenging their understanding of the subject matter through inquiry.

Answers are not provided for questions within this chapter, and this is by design. Solved problems may be found in the Tutorial and Derivation chapters, instead. The goal here is *independence*, and this requires students to be challenged in ways where others cannot think for them. Remember that you always have the tools of *experimentation* and *computer simulation* (e.g. SPICE) to explore concepts!

The following lists contain ideas for Socratic-style questions and challenges. Upon inspection, one will notice a strong theme of *metacognition* within these statements: they are designed to foster a regular habit of examining one's own thoughts as a means toward clearer thinking. As such these sample questions are useful both for instructor-led discussions as well as for self-study.

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<sup>1</sup>Technical reading is an essential academic skill for any technical practitioner to possess for the simple reason that the most comprehensive, accurate, and useful information to be found for developing technical competence is in textual form. Technical careers in general are characterized by the need for continuous learning to remain current with standards and technology, and therefore any technical practitioner who cannot read well is handicapped in their professional development. An excellent resource for educators on improving students' reading prowess through intentional effort and strategy is the book *Reading For Understanding – How Reading Apprenticeship Improves Disciplinary Learning in Secondary and College Classrooms* by Ruth Schoenbach, Cynthia Greenleaf, and Lynn Murphy.

<sup>2</sup>Lecture is popular as a teaching method because it is easy to implement: any reasonably articulate subject matter expert can talk to students, even with little preparation. However, it is also quite problematic. A good lecture always makes complicated concepts seem easier than they are, which is bad for students because it instills a false sense of confidence in their own understanding; reading and re-articulation requires more cognitive effort and serves to verify comprehension. A culture of teaching-by-lecture fosters a debilitating dependence upon direct personal instruction, whereas the challenges of modern life demand independent and critical thought made possible only by gathering information and perspectives from afar. Information presented in a lecture is ephemeral, easily lost to failures of memory and dictation; text is forever, and may be referenced at any time.

## GENERAL CHALLENGES FOLLOWING TUTORIAL READING

- Summarize as much of the text as you can in one paragraph of your own words. A helpful strategy is to explain ideas as you would for an intelligent child: as simple as you can without compromising too much accuracy.
- Simplify a particular section of the text, for example a paragraph or even a single sentence, so as to capture the same fundamental idea in fewer words.
- Where did the text make the most sense to you? What was it about the text's presentation that made it clear?
- Identify where it might be easy for someone to misunderstand the text, and explain why you think it could be confusing.
- Identify any new concept(s) presented in the text, and explain in your own words.
- Identify any familiar concept(s) such as physical laws or principles applied or referenced in the text.
- Devise a proof of concept experiment demonstrating an important principle, physical law, or technical innovation represented in the text.
- Devise an experiment to disprove a plausible misconception.
- Did the text reveal any misconceptions you might have harbored? If so, describe the misconception(s) and the reason(s) why you now know them to be incorrect.
- Describe any useful problem-solving strategies applied in the text.
- Devise a question of your own to challenge a reader's comprehension of the text.

## GENERAL FOLLOW-UP CHALLENGES FOR ASSIGNED PROBLEMS

- Identify where any fundamental laws or principles apply to the solution of this problem, especially before applying any mathematical techniques.
- Devise a thought experiment to explore the characteristics of the problem scenario, applying known laws and principles to mentally model its behavior.
- Describe in detail your own strategy for solving this problem. How did you identify and organized the given information? Did you sketch any diagrams to help frame the problem?
- Is there more than one way to solve this problem? Which method seems best to you?
- Show the work you did in solving this problem, even if the solution is incomplete or incorrect.
- What would you say was the most challenging part of this problem, and why was it so?
- Was any important information missing from the problem which you had to research or recall?
- Was there any extraneous information presented within this problem? If so, what was it and why did it not matter?
- Examine someone else's solution to identify where they applied fundamental laws or principles.
- Simplify the problem from its given form and show how to solve this simpler version of it. Examples include eliminating certain variables or conditions, altering values to simpler (usually whole) numbers, applying a limiting case (i.e. altering a variable to some extreme or ultimate value).
- For quantitative problems, identify the real-world meaning of all intermediate calculations: their units of measurement, where they fit into the scenario at hand. Annotate any diagrams or illustrations with these calculated values.
- For quantitative problems, try approaching it qualitatively instead, thinking in terms of “increase” and “decrease” rather than definite values.
- For qualitative problems, try approaching it quantitatively instead, proposing simple numerical values for the variables.
- Were there any assumptions you made while solving this problem? Would your solution change if one of those assumptions were altered?
- Identify where it would be easy for someone to go astray in attempting to solve this problem.
- Formulate your own problem based on what you learned solving this one.

## GENERAL FOLLOW-UP CHALLENGES FOR EXPERIMENTS OR PROJECTS

- In what way(s) was this experiment or project easy to complete?
- Identify some of the challenges you faced in completing this experiment or project.



- Show how thorough documentation assisted in the completion of this experiment or project.
- Which fundamental laws or principles are key to this system's function?
- Identify any way(s) in which one might obtain false or otherwise misleading measurements from test equipment in this system.
- What will happen if (component  $X$ ) fails (open/shorted/etc.)?
- What would have to occur to make this system unsafe?

## 5.1 Conceptual reasoning

These questions are designed to stimulate your analytic and synthetic thinking<sup>3</sup>. In a Socratic discussion with your instructor, the goal is for these questions to prompt an extended dialogue where assumptions are revealed, conclusions are tested, and understanding is sharpened. Your instructor may also pose additional questions based on those assigned, in order to further probe and refine your conceptual understanding.

Questions that follow are presented to challenge and probe your understanding of various concepts presented in the tutorial. These questions are intended to serve as a guide for the Socratic dialogue between yourself and the instructor. Your instructor’s task is to ensure you have a sound grasp of these concepts, and the questions contained in this document are merely a means to this end. Your instructor may, at his or her discretion, alter or substitute questions for the benefit of tailoring the discussion to each student’s needs. The only absolute requirement is that each student is challenged and assessed at a level equal to or greater than that represented by the documented questions.

It is far more important that you convey your *reasoning* than it is to simply convey a correct answer. For this reason, you should refrain from researching other information sources to answer questions. What matters here is that *you* are doing the thinking. If the answer is incorrect, your instructor will work with you to correct it through proper reasoning. A correct answer without an adequate explanation of how you derived that answer is unacceptable, as it does not aid the learning or assessment process.

You will note a conspicuous lack of answers given for these conceptual questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your conceptual answers, where applicable, is to use circuit simulation software to explore the effects of changes made to circuits. For example, if one of these conceptual questions challenges you to predict the effects of altering some component parameter in a circuit, you may check the validity of your work by simulating that same parameter change within software and seeing if the results agree.

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<sup>3</sup>*Analytical* thinking involves the “disassembly” of an idea into its constituent parts, analogous to dissection. *Synthetic* thinking involves the “assembly” of a new idea comprised of multiple concepts, analogous to construction. Both activities are high-level cognitive skills, extremely important for effective problem-solving, necessitating frequent challenge and regular practice to fully develop.

### 5.1.1 Reading outline and reflections

*“Reading maketh a full man; conference a ready man; and writing an exact man”* – Francis Bacon

Francis Bacon’s advice is a blueprint for effective education: reading provides the learner with knowledge, writing focuses the learner’s thoughts, and critical dialogue equips the learner to confidently communicate and apply their learning. Independent acquisition and application of knowledge is a powerful skill, well worth the effort to cultivate. To this end, students should read these educational resources closely, journal their own reflections on the reading, and discuss in detail their findings with classmates and instructor(s). You should be able to do all of the following after reading any instructional text:

☑ Briefly **SUMMARIZE THE TEXT** in the form of a journal entry documenting your learning as you progress through the course of study. Share this summary in dialogue with your classmates and instructor. Journaling is an excellent self-test of thorough reading because you cannot clearly express what you have not read or did not comprehend.

☑ Demonstrate **ACTIVE READING STRATEGIES**, including verbalizing your impressions as you read, simplifying long passages to convey the same ideas using fewer words, annotating text and illustrations with your own interpretations, working through mathematical examples shown in the text, cross-referencing passages with relevant illustrations and/or other passages, identifying problem-solving strategies applied by the author, etc. Technical reading is a special case of problem-solving, and so these strategies work precisely because they help solve any problem: paying attention to your own thoughts (metacognition), eliminating unnecessary complexities, identifying what makes sense, paying close attention to details, drawing connections between separated facts, and noting the successful strategies of others.

☑ Identify **IMPORTANT THEMES**, especially **GENERAL LAWS** and **PRINCIPLES**, expounded in the text and express them in the simplest of terms as though you were teaching an intelligent child. This emphasizes connections between related topics and develops your ability to communicate complex ideas to anyone.

☑ Form **YOUR OWN QUESTIONS** based on the reading, and then pose them to your instructor and classmates for their consideration. Anticipate both correct and incorrect answers, the incorrect answer(s) assuming one or more plausible misconceptions. This helps you view the subject from different perspectives to grasp it more fully.

☑ Devise **EXPERIMENTS** to test claims presented in the reading, or to disprove misconceptions. Predict possible outcomes of these experiments, and evaluate their meanings: what result(s) would confirm, and what would constitute disproof? Running mental simulations and evaluating results is essential to scientific and diagnostic reasoning.

☑ Specifically identify any points you found **CONFUSING**. The reason for doing this is to help diagnose misconceptions and overcome barriers to learning.

### 5.1.2 Foundational concepts

Correct analysis and diagnosis of electric circuits begins with a proper understanding of some basic concepts. The following is a list of some important concepts referenced in this module's full tutorial. Define each of them in your own words, and be prepared to illustrate each of these concepts with a description of a practical example and/or a live demonstration.

Energy

Conservation of Energy

Thought experiments as a problem-solving strategy

Voltage

Current

Resistance

Electrical source

Electrical load

Ohm's Law

Series connection

Parallel connection

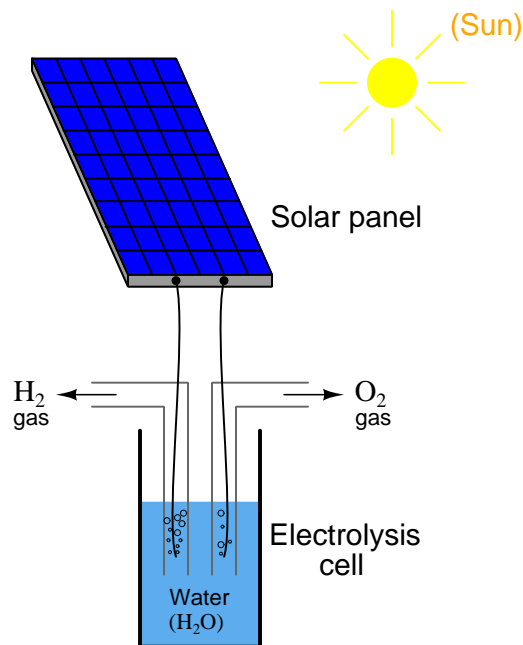
Joule's Law

Independent variable

Dependent variable

### 5.1.3 Optimized electrolysis cell

Suppose we were planning to use a photovoltaic panel to generate electricity and electrolyze water into hydrogen and oxygen gas, the purpose being to convert sunlight into usable chemical fuels which could later be converted back into electricity (using a “fuel cell”), run an engine, etc. The electrolysis cell is nothing more than two metal electrodes immersed in a bath of water, the electricity provided by the solar panel acting to break water molecules ( $\text{H}_2\text{O}$ ) into separate hydrogen and oxygen molecules, the hydrogen being useful as a fuel:



First, comment on how the Conservation of Energy applies to this system, specifically identifying source and load in this system.

Our goal is to electrolyze as much water as possible, and this means we must maximize the electrolysis cell’s power dissipation for the one photovoltaic panel we have. Explain how we could experimentally determine the optimum internal resistance of the electrolysis cell (i.e. how many Ohms of resistance it *ought* to exhibit), prior to actually building it, using nothing but the solar panel, a rheostat<sup>4</sup>, and a DMM (digital multimeter).

Furthermore, comment on what we could physically alter within the electrolysis cell to adjust its internal resistance.

Challenges

<sup>4</sup>“Rheostat” simply refers to a *variable* resistance.

- Assuming the open-circuit (unloaded) output voltage of this solar panel were high enough to pose an electric shock hazard, describe a procedure you might use to safely connect a “test load” to the panel.
- Suppose the electrolysis cell had *more* resistance than was optimal. What would be the consequence of this mismatch?
- Suppose the electrolysis cell had *less* resistance than was optimal. What would be the consequence of this mismatch?

#### 5.1.4 Audio amplifier loading

Most audio amplifiers (e.g. the electronic circuit used to boost signal strength for speech or music to be able to drive a loudspeaker) are work best when powering loudspeakers of a certain effective resistance value. For most audio entertainment systems the standard loudspeaker “impedance” value is 8 Ohms.

What would happen if someone wired two 8  $\Omega$  loudspeakers in *series* with each other, and to the output of an audio amplifier designed for an 8  $\Omega$  load?

What would happen if someone wired two 8  $\Omega$  loudspeakers in *parallel* with each other, and to the output of an audio amplifier designed for an 8  $\Omega$  load?

|            |
|------------|
| Challenges |
|------------|

- Loudspeakers are obviously *not* resistors. What happens to the electrical energy sent to a loudspeaker, versus the energy sent to a resistor?

## 5.2 Quantitative reasoning

These questions are designed to stimulate your computational thinking. In a Socratic discussion with your instructor, the goal is for these questions to reveal your mathematical approach(es) to problem-solving so that good technique and sound reasoning may be reinforced. Your instructor may also pose additional questions based on those assigned, in order to observe your problem-solving firsthand.

Mental arithmetic and estimations are strongly encouraged for all calculations, because without these abilities you will be unable to readily detect errors caused by calculator misuse (e.g. keystroke errors).

You will note a conspicuous lack of answers given for these quantitative questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. My advice is to use circuit simulation software such as SPICE to check the correctness of quantitative answers. Refer to those learning modules within this collection focusing on SPICE to see worked examples which you may use directly as practice problems for your own study, and/or as templates you may modify to run your own analyses and generate your own practice problems.

Completely worked example problems found in the Tutorial may also serve as “test cases<sup>5</sup>” for gaining proficiency in the use of circuit simulation software, and then once that proficiency is gained you will never need to rely<sup>6</sup> on an answer key!

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<sup>5</sup>In other words, set up the circuit simulation software to analyze the same circuit examples found in the Tutorial. If the simulated results match the answers shown in the Tutorial, it confirms the simulation has properly run. If the simulated results disagree with the Tutorial’s answers, something has been set up incorrectly in the simulation software. Using every Tutorial as practice in this way will quickly develop proficiency in the use of circuit simulation software.

<sup>6</sup>This approach is perfectly in keeping with the instructional philosophy of these learning modules: *teaching students to be self-sufficient thinkers*. Answer keys can be useful, but it is even more useful to your long-term success to have a set of tools on hand for checking your own work, because once you have left school and are on your own, there will no longer be “answer keys” available for the problems you will have to solve.



### 5.2.1 Miscellaneous physical constants

Note: constants shown in **bold** type are *exact*, not approximations. Values inside of parentheses show one standard deviation ( $\sigma$ ) of uncertainty in the final digits: for example, the magnetic permeability of free space value given as  $1.25663706212(19) \times 10^{-6}$  H/m represents a center value (i.e. the location parameter) of  $1.25663706212 \times 10^{-6}$  Henrys per meter with one standard deviation of uncertainty equal to  $0.0000000000019 \times 10^{-6}$  Henrys per meter.

Avogadro's number ( $N_A$ ) = **6.02214076**  $\times 10^{23}$  **per mole** (mol<sup>-1</sup>)

Boltzmann's constant ( $k$ ) = **1.380649**  $\times 10^{-23}$  **Joules per Kelvin** (J/K)

Electronic charge ( $e$ ) = **1.602176634**  $\times 10^{-19}$  **Coulomb** (C)

Faraday constant ( $F$ ) = **96,485.33212...**  $\times 10^4$  **Coulombs per mole** (C/mol)

Magnetic permeability of free space ( $\mu_0$ ) =  $1.25663706212(19) \times 10^{-6}$  Henrys per meter (H/m)

Electric permittivity of free space ( $\epsilon_0$ ) =  $8.8541878128(13) \times 10^{-12}$  Farads per meter (F/m)

Characteristic impedance of free space ( $Z_0$ ) =  $376.730313668(57)$  Ohms ( $\Omega$ )

Gravitational constant ( $G$ ) =  $6.67430(15) \times 10^{-11}$  cubic meters per kilogram-seconds squared (m<sup>3</sup>/kg-s<sup>2</sup>)

Molar gas constant ( $R$ ) = **8.314462618...** **Joules per mole-Kelvin** (J/mol-K) = 0.08205746(14) liters-atmospheres per mole-Kelvin

Planck constant ( $h$ ) = **6.62607015**  $\times 10^{-34}$  **joule-seconds** (J-s)

Stefan-Boltzmann constant ( $\sigma$ ) = **5.670374419...**  $\times 10^{-8}$  **Watts per square meter-Kelvin<sup>4</sup>** (W/m<sup>2</sup>·K<sup>4</sup>)

Speed of light in a vacuum ( $c$ ) = **299,792,458 meters per second** (m/s) = 186282.4 miles per second (mi/s)

Note: All constants taken from NIST data "Fundamental Physical Constants – Complete Listing", from <http://physics.nist.gov/constants>, National Institute of Standards and Technology (NIST), 2018 CODATA Adjustment.

### 5.2.2 Introduction to spreadsheets

A powerful computational tool you are encouraged to use in your work is a *spreadsheet*. Available on most personal computers (e.g. Microsoft Excel), *spreadsheet* software performs numerical calculations based on number values and formulae entered into cells of a grid. This grid is typically arranged as lettered columns and numbered rows, with each cell of the grid identified by its column/row coordinates (e.g. cell B3, cell A8). Each cell may contain a string of text, a number value, or a mathematical formula. The spreadsheet automatically updates the results of all mathematical formulae whenever the entered number values are changed. This means it is possible to set up a spreadsheet to perform a series of calculations on entered data, and those calculations will be re-done by the computer any time the data points are edited in any way.

For example, the following spreadsheet calculates average speed based on entered values of distance traveled and time elapsed:

|   | A                 | B         | C          | D |
|---|-------------------|-----------|------------|---|
| 1 | Distance traveled | 46.9      | Kilometers |   |
| 2 | Time elapsed      | 1.18      | Hours      |   |
| 3 | Average speed     | = B1 / B2 | km/h       |   |
| 4 |                   |           |            |   |
| 5 |                   |           |            |   |

Text labels contained in cells A1 through A3 and cells C1 through C3 exist solely for readability and are not involved in any calculations. Cell B1 contains a sample distance value while cell B2 contains a sample time value. The formula for computing speed is contained in cell B3. Note how this formula begins with an “equals” symbol (=), references the values for distance and speed by lettered column and numbered row coordinates (B1 and B2), and uses a forward slash symbol for division (/). The coordinates B1 and B2 function as *variables*<sup>7</sup> would in an algebraic formula.

When this spreadsheet is executed, the numerical value 39.74576 will appear in cell B3 rather than the formula = B1 / B2, because 39.74576 is the computed speed value given 46.9 kilometers traveled over a period of 1.18 hours. If a different numerical value for distance is entered into cell B1 or a different value for time is entered into cell B2, cell B3’s value will automatically update. All you need to do is set up the given values and any formulae into the spreadsheet, and the computer will do all the calculations for you.

Cell B3 may be referenced by other formulae in the spreadsheet if desired, since it is a variable just like the given values contained in B1 and B2. This means it is possible to set up an entire chain of calculations, one dependent on the result of another, in order to arrive at a final value. The arrangement of the given data and formulae need not follow any pattern on the grid, which means you may place them anywhere.

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<sup>7</sup>Spreadsheets may also provide means to attach text labels to cells for use as variable names (Microsoft Excel simply calls these labels “names”), but for simple spreadsheets such as those shown here it’s usually easier just to use the standard coordinate naming for each cell.

Common<sup>8</sup> arithmetic operations available for your use in a spreadsheet include the following:

- Addition (+)
- Subtraction (-)
- Multiplication (\*)
- Division (/)
- Powers (^)
- Square roots (sqrt())
- Logarithms (ln() , log10())

Parentheses may be used to ensure<sup>9</sup> proper order of operations within a complex formula. Consider this example of a spreadsheet implementing the *quadratic formula*, used to solve for roots of a polynomial expression in the form of  $ax^2 + bx + c$ :

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

|          | A   | B   |
|----------|-----|---|
| <b>1</b> | x_1 | = (-B4 + sqrt((B4^2) - (4*B3*B5))) / (2*B3) |
| <b>2</b> | x_2 | = (-B4 - sqrt((B4^2) - (4*B3*B5))) / (2*B3) |
| <b>3</b> | a = | 9   |
| <b>4</b> | b = | 5   |
| <b>5</b> | c = | -2  |

This example is configured to compute roots<sup>10</sup> of the polynomial  $9x^2 + 5x - 2$  because the values of 9, 5, and  $-2$  have been inserted into cells B3, B4, and B5, respectively. Once this spreadsheet has been built, though, it may be used to calculate the roots of *any* second-degree polynomial expression simply by entering the new  $a$ ,  $b$ , and  $c$  coefficients into cells B3 through B5. The numerical values appearing in cells B1 and B2 will be automatically updated by the computer immediately following any changes made to the coefficients.

<sup>8</sup>Modern spreadsheet software offers a bewildering array of mathematical functions you may use in your computations. I recommend you consult the documentation for your particular spreadsheet for information on operations other than those listed here.

<sup>9</sup>Spreadsheet programs, like text-based programming languages, are designed to follow standard order of operations by default. However, my personal preference is to use parentheses even where strictly unnecessary just to make it clear to any other person viewing the formula what the intended order of operations is.

<sup>10</sup>Reviewing some algebra here, a *root* is a value for  $x$  that yields an overall value of zero for the polynomial. For this polynomial ( $9x^2 + 5x - 2$ ) the two roots happen to be  $x = 0.269381$  and  $x = -0.82494$ , with these values displayed in cells B1 and B2, respectively upon execution of the spreadsheet.

Alternatively, one could break up the long quadratic formula into smaller pieces like this:

$$y = \sqrt{b^2 - 4ac} \quad z = 2a$$

$$x = \frac{-b \pm y}{z}$$

|          | <b>A</b> | <b>B</b>          | <b>C</b>                   |
|----------|----------|-------------------|----------------------------|
| <b>1</b> | x_1      | = (-B4 + C1) / C2 | = sqrt((B4^2) - (4*B3*B5)) |
| <b>2</b> | x_2      | = (-B4 - C1) / C2 | = 2*B3                     |
| <b>3</b> | a =      | 9                 |                            |
| <b>4</b> | b =      | 5                 |                            |
| <b>5</b> | c =      | -2                |                            |

Note how the square-root term ( $y$ ) is calculated in cell C1, and the denominator term ( $z$ ) in cell C2. This makes the two final formulae (in cells B1 and B2) simpler to interpret. The positioning of all these cells on the grid is completely arbitrary<sup>11</sup> – all that matters is that they properly reference each other in the formulae.

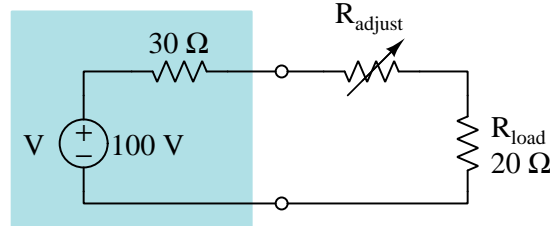
Spreadsheets are particularly useful for situations where the same set of calculations representing a circuit or other system must be repeated for different initial conditions. The power of a spreadsheet is that it automates what would otherwise be a tedious set of calculations. One specific application of this is to simulate the effects of various components within a circuit failing with abnormal values (e.g. a shorted resistor simulated by making its value nearly zero; an open resistor simulated by making its value extremely large). Another application is analyzing the behavior of a circuit design given new components that are out of specification, and/or aging components experiencing drift over time.

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<sup>11</sup>My personal preference is to locate all the “given” data in the upper-left cells of the spreadsheet grid (each data point flanked by a sensible name in the cell to the left and units of measurement in the cell to the right as illustrated in the first distance/time spreadsheet example), sometimes coloring them in order to clearly distinguish which cells contain entered data versus which cells contain computed results from formulae. I like to place all formulae in cells below the given data, and try to arrange them in logical order so that anyone examining my spreadsheet will be able to figure out *how* I constructed a solution. This is a general principle I believe all computer programmers should follow: *document and arrange your code to make it easy for other people to learn from it.*

### 5.2.3 Series resistance

Determine the optimal value for  $R_{adjust}$  that results in maximum power dissipated at  $R_{load}$ :



Now, determine the optimal value for  $R_{adjust}$  that results in maximum power dissipated for  $R_{adjust}$  and  $R_{load}$  together (i.e. *total* power between these two resistors).

Lastly, determine the optimal value for  $R_{adjust}$  that results in maximum power dissipated by itself ( $R_{adjust}$ ).

#### Challenges

- Explain why the correct answer for maximizing  $P_{load}$  is *not*  $R_{adjust} = 10 \Omega$ .

### 5.2.4 Battery bank short-circuit current

Suppose we have several identical batteries, each one having a nominal voltage of 3.7 Volts and an internal resistance of 60 m $\Omega$  (60 milli-Ohms).

Calculate the maximum short-circuit current that a single one of these batteries could output.

Calculate the maximum short-circuit current that a series array of three such batteries could output.

Calculate the maximum short-circuit current that a parallel array of three such batteries could output.

#### Challenges

- Based on your results, how should we connect multiple batteries together to achieve a greater current rating?
- Would real-life short-circuit current be more than these values, less than these values, or the same as these values?

## 5.3 Diagnostic reasoning

These questions are designed to stimulate your deductive and inductive thinking, where you must apply general principles to specific scenarios (deductive) and also derive conclusions about the failed circuit from specific details (inductive). In a Socratic discussion with your instructor, the goal is for these questions to reinforce your recall and use of general circuit principles and also challenge your ability to integrate multiple symptoms into a sensible explanation of what's wrong in a circuit. Your instructor may also pose additional questions based on those assigned, in order to further challenge and sharpen your diagnostic abilities.

As always, your goal is to fully *explain* your analysis of each problem. Simply obtaining a correct answer is not good enough – you must also demonstrate sound reasoning in order to successfully complete the assignment. Your instructor's responsibility is to probe and challenge your understanding of the relevant principles and analytical processes in order to ensure you have a strong foundation upon which to build further understanding.

You will note a conspicuous lack of answers given for these diagnostic questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your diagnostic answers, where applicable, is to use circuit simulation software to explore the effects of faults placed in circuits. For example, if one of these diagnostic questions requires that you predict the effect of an open or a short in a circuit, you may check the validity of your work by simulating that same fault (substituting a very high resistance in place of that component for an open, and substituting a very low resistance for a short) within software and seeing if the results agree.

### 5.3.1 Find the mistake(s)

A voltage source with an internal voltage of 15 Volts and internal resistance of  $120\ \Omega$  is connected to a  $100\ \Omega$  load resistor, and a student attempts to calculate load power dissipation by using the following steps. However, the student made a mistake somewhere:

1.  $120 + 100 = 220$
2.  $\frac{15}{220} = 68.182 \times 10^{-3}$
3.  $(100) \times (68.182 \times 10^{-3}) = 6.8182$
4.  $(120) \times (68.182 \times 10^{-3}) = 8.1818$
5.  $6.8182 + 8.1818 = 15$
6.  $(6.8182) \times (120) = 818.18$
7.  $(8.1818) \times (100) = 818.18$
8.  $(15) \times (68.182 \times 10^{-3}) = 1.0227$

Identify proper units of measurement for each calculation, identify the fundamental circuit principle applied in each step, identify the student's mistake, and make all corrections as needed.

|            |
|------------|
| Challenges |
|------------|

- Are there any steps in this sequence that are unnecessary?

# Appendix A

## Problem-Solving Strategies

The ability to solve complex problems is arguably one of the most valuable skills one can possess, and this skill is particularly important in any science-based discipline.

- Study principles, not procedures. Don't be satisfied with merely knowing how to compute solutions – learn *why* those solutions work.
- Identify what it is you need to solve, identify all relevant data, identify all units of measurement, identify any general principles or formulae linking the given information to the solution, and then identify any “missing pieces” to a solution. Annotate all diagrams with this data.
- Sketch a diagram to help visualize the problem. When building a real system, always devise a plan for that system and analyze its function *before* constructing it.
- Follow the units of measurement and meaning of every calculation. If you are ever performing mathematical calculations as part of a problem-solving procedure, and you find yourself unable to apply each and every intermediate result to some aspect of the problem, it means you don't understand what you are doing. Properly done, every mathematical result should have practical meaning for the problem, and not just be an abstract number. You should be able to identify the proper units of measurement for each and every calculated result, and show where that result fits into the problem.
- Perform “thought experiments” to explore the effects of different conditions for theoretical problems. When troubleshooting real systems, perform *diagnostic tests* rather than visually inspecting for faults, the best diagnostic test being the one giving you the most information about the nature and/or location of the fault with the fewest steps.
- Simplify the problem until the solution becomes obvious, and then use that obvious case as a model to follow in solving the more complex version of the problem.
- Check for exceptions to see if your solution is incorrect or incomplete. A good solution will work for *all* known conditions and criteria. A good example of this is the process of testing scientific hypotheses: the task of a scientist is not to find support for a new idea, but rather to *challenge* that new idea to see if it holds up under a battery of tests. The philosophical



principle of *reductio ad absurdum* (i.e. disproving a general idea by finding a specific case where it fails) is useful here.

- Work “backward” from a hypothetical solution to a new set of given conditions.
- Add quantities to problems that are qualitative in nature, because sometimes a little math helps illuminate the scenario.
- Sketch graphs illustrating how variables relate to each other. These may be quantitative (i.e. with realistic number values) or qualitative (i.e. simply showing increases and decreases).
- Treat quantitative problems as qualitative in order to discern the relative magnitudes and/or directions of change of the relevant variables. For example, try determining what happens if a certain variable were to increase or decrease before attempting to precisely calculate quantities: how will each of the dependent variables respond, by increasing, decreasing, or remaining the same as before?
- Consider limiting cases. This works especially well for qualitative problems where you need to determine which direction a variable will change. Take the given condition and magnify that condition to an extreme degree as a way of simplifying the direction of the system’s response.
- Check your work. This means regularly testing your conclusions to see if they make sense. This does *not* mean repeating the same steps originally used to obtain the conclusion(s), but rather to use some other means to check validity. Simply repeating procedures often leads to *repeating the same errors* if any were made, which is why alternative paths are better.

## Appendix B

# Instructional philosophy

*“The unexamined circuit is not worth energizing”* – Socrates (if he had taught electricity)

These learning modules, although useful for self-study, were designed to be used in a formal learning environment where a subject-matter expert challenges students to digest the content and exercise their critical thinking abilities in the answering of questions and in the construction and testing of working circuits.

The following principles inform the instructional and assessment philosophies embodied in these learning modules:

- The first goal of education is to enhance clear and independent thought, in order that every student reach their fullest potential in a highly complex and inter-dependent world. Robust reasoning is *always* more important than particulars of any subject matter, because its application is universal.
- Literacy is fundamental to independent learning and thought because text continues to be the most efficient way to communicate complex ideas over space and time. Those who cannot read with ease are limited in their ability to acquire knowledge and perspective.
- Articulate communication is fundamental to work that is complex and interdisciplinary.
- Faulty assumptions and poor reasoning are best corrected through challenge, not presentation. The rhetorical technique of *reductio ad absurdum* (disproving an assertion by exposing an absurdity) works well to discipline student’s minds, not only to correct the problem at hand but also to learn how to detect and correct future errors.
- Important principles should be repeatedly explored and widely applied throughout a course of study, not only to reinforce their importance and help ensure their mastery, but also to showcase the interconnectedness and utility of knowledge.

These learning modules were expressly designed to be used in an “inverted” teaching environment<sup>1</sup> where students first read the introductory and tutorial chapters on their own, then individually attempt to answer the questions and construct working circuits according to the experiment and project guidelines. The instructor never lectures, but instead meets regularly with each individual student to review their progress, answer questions, identify misconceptions, and challenge the student to new depths of understanding through further questioning. Regular meetings between instructor and student should resemble a Socratic<sup>2</sup> dialogue, where questions serve as scalpels to dissect topics and expose assumptions. The student passes each module only after consistently demonstrating their ability to logically analyze and correctly apply all major concepts in each question or project/experiment. The instructor must be vigilant in probing each student’s understanding to ensure they are truly *reasoning* and not just *memorizing*. This is why “Challenge” points appear throughout, as prompts for students to think deeper about topics and as starting points for instructor queries. Sometimes these challenge points require additional knowledge that hasn’t been covered in the series to answer in full. This is okay, as the major purpose of the Challenges is to stimulate analysis and synthesis on the part of each student.

The instructor must possess enough mastery of the subject matter and awareness of students’ reasoning to generate their own follow-up questions to practically any student response. Even completely correct answers given by the student should be challenged by the instructor for the purpose of having students practice articulating their thoughts and defending their reasoning. Conceptual errors committed by the student should be exposed and corrected not by direct instruction, but rather by reducing the errors to an absurdity<sup>3</sup> through well-chosen questions and thought experiments posed by the instructor. Becoming proficient at this style of instruction requires time and dedication, but the positive effects on critical thinking for both student and instructor are spectacular.

An inspection of these learning modules reveals certain unique characteristics. One of these is a bias toward thorough explanations in the tutorial chapters. Without a live instructor to explain concepts and applications to students, the text itself must fulfill this role. This philosophy results in lengthier explanations than what you might typically find in a textbook, each step of the reasoning process fully explained, including footnotes addressing common questions and concerns students raise while learning these concepts. Each tutorial seeks to not only explain each major concept in sufficient detail, but also to explain the logic of each concept and how each may be developed

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<sup>1</sup>In a traditional teaching environment, students first encounter new information via *lecture* from an expert, and then independently apply that information via *homework*. In an “inverted” course of study, students first encounter new information via *homework*, and then independently apply that information under the scrutiny of an expert. The expert’s role in lecture is to simply *explain*, but the expert’s role in an inverted session is to *challenge, critique*, and if necessary *explain* where gaps in understanding still exist.

<sup>2</sup>Socrates is a figure in ancient Greek philosophy famous for his unflinching style of questioning. Although he authored no texts, he appears as a character in Plato’s many writings. The essence of Socratic philosophy is to leave no question unexamined and no point of view unchallenged. While purists may argue a topic such as electric circuits is too narrow for a true Socratic-style dialogue, I would argue that the essential thought processes involved with scientific reasoning on *any* topic are not far removed from the Socratic ideal, and that students of electricity and electronics would do very well to challenge assumptions, pose thought experiments, identify fallacies, and otherwise employ the arsenal of critical thinking skills modeled by Socrates.

<sup>3</sup>This rhetorical technique is known by the Latin phrase *reductio ad absurdum*. The concept is to expose errors by counter-example, since only one solid counter-example is necessary to disprove a universal claim. As an example of this, consider the common misconception among beginning students of electricity that voltage cannot exist without current. One way to apply *reductio ad absurdum* to this statement is to ask how much current passes through a fully-charged battery connected to nothing (i.e. a clear example of voltage existing without current).

from “first principles”. Again, this reflects the goal of developing clear and independent thought in students’ minds, by showing how clear and logical thought was used to forge each concept. Students benefit from witnessing a model of clear thinking in action, and these tutorials strive to be just that.

Another characteristic of these learning modules is a lack of step-by-step instructions in the Project and Experiment chapters. Unlike many modern workbooks and laboratory guides where step-by-step instructions are prescribed for each experiment, these modules take the approach that students must learn to closely read the tutorials and apply their own reasoning to identify the appropriate experimental steps. Sometimes these steps are plainly declared in the text, just not as a set of enumerated points. At other times certain steps are implied, an example being assumed competence in test equipment use where the student should not need to be told *again* how to use their multimeter because that was thoroughly explained in previous lessons. In some circumstances no steps are given at all, leaving the entire procedure up to the student.

This lack of prescription is not a flaw, but rather a feature. Close reading and clear thinking are foundational principles of this learning series, and in keeping with this philosophy all activities are designed to *require* those behaviors. Some students may find the lack of prescription frustrating, because it demands more from them than what their previous educational experiences required. This frustration should be interpreted as an unfamiliarity with autonomous thinking, a problem which must be corrected if the student is ever to become a self-directed learner and effective problem-solver. Ultimately, the need for students to read closely and think clearly is more important both in the near-term and far-term than any specific facet of the subject matter at hand. If a student takes longer than expected to complete a module because they are forced to outline, digest, and reason on their own, so be it. The future gains enjoyed by developing this mental discipline will be well worth the additional effort and delay.

Another feature of these learning modules is that they do not treat topics in isolation. Rather, important concepts are introduced early in the series, and appear repeatedly as stepping-stones toward other concepts in subsequent modules. This helps to avoid the “compartmentalization” of knowledge, demonstrating the inter-connectedness of concepts and simultaneously reinforcing them. Each module is fairly complete in itself, reserving the beginning of its tutorial to a review of foundational concepts.

This methodology of assigning text-based modules to students for digestion and then using Socratic dialogue to assess progress and hone students’ thinking was developed over a period of several years by the author with his Electronics and Instrumentation students at the two-year college level. While decidedly unconventional and sometimes even unsettling for students accustomed to a more passive lecture environment, this instructional philosophy has proven its ability to convey conceptual mastery, foster careful analysis, and enhance employability so much better than lecture that the author refuses to ever teach by lecture again.

Problems which often go undiagnosed in a lecture environment are laid bare in this “inverted” format where students must articulate and logically defend their reasoning. This, too, may be unsettling for students accustomed to lecture sessions where the instructor cannot tell for sure who comprehends and who does not, and this vulnerability necessitates sensitivity on the part of the “inverted” session instructor in order that students never feel discouraged by having their errors exposed. *Everyone* makes mistakes from time to time, and learning is a lifelong process! Part of the instructor’s job is to build a culture of learning among the students where errors are not seen as shameful, but rather as opportunities for progress.

To this end, instructors managing courses based on these modules should adhere to the following principles:

- Student questions are always welcome and demand thorough, honest answers. The only type of question an instructor should refuse to answer is one the student should be able to easily answer on their own. Remember, *the fundamental goal of education is for each student to learn to think clearly and independently*. This requires hard work on the part of the student, which no instructor should ever circumvent. Anything done to bypass the student's responsibility to do that hard work ultimately limits that student's potential and thereby does real harm.
- It is not only permissible, but encouraged, to answer a student's question by asking questions in return, these follow-up questions designed to guide the student to reach a correct answer through their own reasoning.
- All student answers demand to be challenged by the instructor and/or by other students. This includes both correct and incorrect answers – the goal is to practice the articulation and defense of one's own reasoning.
- No reading assignment is deemed complete unless and until the student demonstrates their ability to accurately summarize the major points in their own terms. Recitation of the original text is unacceptable. This is why every module contains an "Outline and reflections" question as well as a "Foundational concepts" question in the Conceptual reasoning section, to prompt reflective reading.
- No assigned question is deemed answered unless and until the student demonstrates their ability to consistently and correctly apply the concepts to *variations* of that question. This is why module questions typically contain multiple "Challenges" suggesting different applications of the concept(s) as well as variations on the same theme(s). Instructors are encouraged to devise as many of their own "Challenges" as they are able, in order to have a multitude of ways ready to probe students' understanding.
- No assigned experiment or project is deemed complete unless and until the student demonstrates the task in action. If this cannot be done "live" before the instructor, video-recordings showing the demonstration are acceptable. All relevant safety precautions must be followed, all test equipment must be used correctly, and the student must be able to properly explain all results. The student must also successfully answer all Challenges presented by the instructor for that experiment or project.

Students learning from these modules would do well to abide by the following principles:

- No text should be considered fully and adequately read unless and until you can express every idea *in your own words, using your own examples*.
- You should always articulate your thoughts as you read the text, noting points of agreement, confusion, and epiphanies. Feel free to print the text on paper and then write your notes in the margins. Alternatively, keep a journal for your own reflections as you read. This is truly a helpful tool when digesting complicated concepts.
- Never take the easy path of highlighting or underlining important text. Instead, *summarize* and/or *comment* on the text using your own words. This actively engages your mind, allowing you to more clearly perceive points of confusion or misunderstanding on your own.
- A very helpful strategy when learning new concepts is to place yourself in the role of a teacher, if only as a mental exercise. Either explain what you have recently learned to someone else, or at least *imagine* yourself explaining what you have learned to someone else. The simple act of having to articulate new knowledge and skill forces you to take on a different perspective, and will help reveal weaknesses in your understanding.
- Perform each and every mathematical calculation and thought experiment shown in the text on your own, referring back to the text to see that your results agree. This may seem trivial and unnecessary, but it is critically important to ensuring you actually understand what is presented, especially when the concepts at hand are complicated and easy to misunderstand. Apply this same strategy to become proficient in the use of *circuit simulation software*, checking to see if your simulated results agree with the results shown in the text.
- Above all, recognize that learning is hard work, and that a certain level of frustration is unavoidable. There are times when you will struggle to grasp some of these concepts, and that struggle is a natural thing. Take heart that it will yield with persistent and varied<sup>4</sup> effort, and never give up!

Students interested in using these modules for self-study will also find them beneficial, although the onus of responsibility for thoroughly reading and answering questions will of course lie with that individual alone. If a qualified instructor is not available to challenge students, a workable alternative is for students to form study groups where they challenge<sup>5</sup> one another.

To high standards of education,

Tony R. Kuphaldt

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<sup>4</sup>As the old saying goes, “Insanity is trying the same thing over and over again, expecting different results.” If you find yourself stumped by something in the text, you should attempt a different approach. Alter the thought experiment, change the mathematical parameters, do whatever you can to see the problem in a slightly different light, and then the solution will often present itself more readily.

<sup>5</sup>Avoid the temptation to simply share answers with study partners, as this is really counter-productive to learning. Always bear in mind that the answer to any question is far less important in the long run than the method(s) used to obtain that answer. The goal of education is to empower one’s life through the improvement of clear and independent thought, literacy, expression, and various practical skills.



# Appendix C

## Tools used

I am indebted to the developers of many open-source software applications in the creation of these learning modules. The following is a list of these applications with some commentary on each.

You will notice a theme common to many of these applications: a bias toward *code*. Although I am by no means an expert programmer in any computer language, I understand and appreciate the flexibility offered by code-based applications where the user (you) enters commands into a plain ASCII text file, which the software then reads and processes to create the final output. Code-based computer applications are by their very nature *extensible*, while WYSIWYG (What You See Is What You Get) applications are generally limited to whatever user interface the developer makes for you.

### The GNU/Linux computer operating system

There is so much to be said about Linus Torvalds' `Linux` and Richard Stallman's `GNU` project. First, to credit just these two individuals is to fail to do justice to the *mob* of passionate volunteers who contributed to make this amazing software a reality. I first learned of `Linux` back in 1996, and have been using this operating system on my personal computers almost exclusively since then. It is *free*, it is completely *configurable*, and it permits the continued use of highly efficient `Unix` applications and scripting languages (e.g. shell scripts, Makefiles, `sed`, `awk`) developed over many decades. `Linux` not only provided me with a powerful computing platform, but its open design served to inspire my life's work of creating open-source educational resources.

### Bram Moolenaar's Vim text editor

Writing code for any code-based computer application requires a *text editor*, which may be thought of as a word processor strictly limited to outputting plain-ASCII text files. Many good text editors exist, and one's choice of text editor seems to be a deeply personal matter within the programming world. I prefer `Vim` because it operates very similarly to `vi` which is ubiquitous on `Unix/Linux` operating systems, and because it may be entirely operated via keyboard (i.e. no mouse required) which makes it fast to use.



### Donald Knuth's $\text{\TeX}$ typesetting system

Developed in the late 1970's and early 1980's by computer scientist extraordinaire Donald Knuth to typeset his multi-volume magnum opus *The Art of Computer Programming*, this software allows the production of formatted text for screen-viewing or paper printing, all by writing plain-text code to describe how the formatted text is supposed to appear.  $\text{\TeX}$  is not just a markup language for documents, but it is also a Turing-complete programming language in and of itself, allowing useful algorithms to be created to control the production of documents. Simply put,  *$\text{\TeX}$  is a programmer's approach to word processing*. Since  $\text{\TeX}$  is controlled by code written in a plain-text file, this means anyone may read that plain-text file to see exactly how the document was created. This openness afforded by the code-based nature of  $\text{\TeX}$  makes it relatively easy to learn how other people have created their own  $\text{\TeX}$  documents. By contrast, examining a beautiful document created in a conventional WYSIWYG word processor such as Microsoft **Word** suggests nothing to the reader about *how* that document was created, or what the user might do to create something similar. As Mr. Knuth himself once quipped, conventional word processing applications should be called WYSIAYG (What You See Is *All* You Get).

### Leslie Lamport's $\text{\LaTeX}$ extensions to $\text{\TeX}$

Like all true programming languages,  $\text{\TeX}$  is inherently extensible. So, years after the release of  $\text{\TeX}$  to the public, Leslie Lamport decided to create a massive extension allowing easier compilation of book-length documents. The result was  $\text{\LaTeX}$ , which is the markup language used to create all ModEL module documents. You could say that  $\text{\TeX}$  is to  $\text{\LaTeX}$  as **C** is to **C++**. This means it is permissible to use any and all  $\text{\TeX}$  commands within  $\text{\LaTeX}$  source code, and it all still works. Some of the features offered by  $\text{\LaTeX}$  that would be challenging to implement in  $\text{\TeX}$  include automatic index and table-of-content creation.

### Tim Edwards' **Xcircuit** drafting program

This wonderful program is what I use to create all the schematic diagrams and illustrations (but not photographic images or mathematical plots) throughout the ModEL project. It natively outputs PostScript format which is a true vector graphic format (this is why the images do not pixellate when you zoom in for a closer view), and it is so simple to use that I have never had to read the manual! Object libraries are easy to create for **Xcircuit**, being plain-text files using PostScript programming conventions. Over the years I have collected a large set of object libraries useful for drawing electrical and electronic schematics, pictorial diagrams, and other technical illustrations.

### Gimp graphic image manipulation program

Essentially an open-source clone of Adobe's `PhotoShop`, I use `Gimp` to resize, crop, and convert file formats for all of the photographic images appearing in the `MODEL` modules. Although `Gimp` does offer its own scripting language (called `Script-Fu`), I have never had occasion to use it. Thus, my utilization of `Gimp` to merely crop, resize, and convert graphic images is akin to using a sword to slice bread.

### SPICE circuit simulation program

`SPICE` is to circuit analysis as `TEX` is to document creation: it is a form of markup language designed to describe a certain object to be processed in plain-ASCII text. When the plain-text "source file" is compiled by the software, it outputs the final result. More modern circuit analysis tools certainly exist, but I prefer `SPICE` for the following reasons: it is *free*, it is *fast*, it is *reliable*, and it is a fantastic tool for *teaching* students of electricity and electronics how to write simple code. I happen to use rather old versions of `SPICE`, version 2g6 being my "go to" application when I only require text-based output. `NGSPICE` (version 26), which is based on Berkeley `SPICE` version 3f5, is used when I require graphical output for such things as time-domain waveforms and Bode plots. In all `SPICE` example netlists I strive to use coding conventions compatible with all `SPICE` versions.

### Andrew D. Hwang's ePiX mathematical visualization programming library

This amazing project is a `C++` library you may link to any `C/C++` code for the purpose of generating PostScript graphic images of mathematical functions. As a completely free and open-source project, it does all the plotting I would otherwise use a Computer Algebra System (CAS) such as `Mathematica` or `Maple` to do. It should be said that `ePiX` is *not* a Computer Algebra System like `Mathematica` or `Maple`, but merely a mathematical *visualization* tool. In other words, it won't determine integrals for you (you'll have to implement that in your own `C/C++` code!), but it can graph the results, and it does so beautifully. What I really admire about `ePiX` is that it is a `C++` programming library, which means it builds on the existing power and toolset available with that programming language. Mr. Hwang could have probably developed his own stand-alone application for mathematical plotting, but by creating a `C++` library to do the same thing he accomplished something much greater.

### gnuplot mathematical visualization software

Another open-source tool for mathematical visualization is `gnuplot`. Interestingly, this tool is *not* part of Richard Stallman’s GNU project, its name being a coincidence. For this reason the authors prefer “gnu” *not* be capitalized at all to avoid confusion. This is a much “lighter-weight” alternative to a spreadsheet for plotting tabular data, and the fact that it easily outputs directly to an X11 console or a file in a number of different graphical formats (including PostScript) is very helpful. I typically set my `gnuplot` output format to default (X11 on my Linux PC) for quick viewing while I’m developing a visualization, then switch to PostScript file export once the visual is ready to include in the document(s) I’m writing. As with my use of `Gimp` to do rudimentary image editing, my use of `gnuplot` only scratches the surface of its capabilities, but the important points are that it’s *free* and that it *works well*.

### Python programming language

Both Python and C++ find extensive use in these modules as instructional aids and exercises, but I’m listing Python here as a *tool* for myself because I use it almost daily as a *calculator*. If you open a Python interpreter console and type `from math import *` you can type mathematical expressions and have it return results just as you would on a hand calculator. Complex-number (i.e. *phasor*) arithmetic is similarly supported if you include the complex-math library (`from cmath import *`). Examples of this are shown in the Programming References chapter (if included) in each module. Of course, being a fully-featured programming language, Python also supports conditionals, loops, and other structures useful for calculation of quantities. Also, running in a console environment where all entries and returned values show as text in a chronologically-ordered list makes it easy to copy-and-paste those calculations to document exactly how they were performed.

# Appendix D

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## Appendix E

## References



# Appendix F

## Version history

This is a list showing all significant additions, corrections, and other edits made to this learning module. Each entry is referenced by calendar date in reverse chronological order (newest version first), which appears on the front cover of every learning module for easy reference. Any contributors to this open-source document are listed here as well.

**27 February 2025** – included clearer definitions of “heavy” loads within the Tutorial.

**2 October 2024** – minor edits to the “Optimized electrolysis cell” Conceptual Reasoning question. Also added another “Challenging concepts” bullet-item to the list in the Introduction chapter.

**9 September 2024** – divided the Introduction chapter into sections, one with recommendations for students, one with a listing of challenging concepts, and one with recommendations for instructors.

**29 February 2024** – clarified one of the sub-questions in the “Optimized electrolysis cell” Conceptual Reasoning question.

**17 October 2023** – added a new section to the Tutorial on applications of the Maximum Power Transfer theorem.

**5 October 2023** – added some more questions to the Introduction chapter.

**27 November 2022** – placed questions at the top of the itemized list in the Introduction chapter prompting students to devise experiments related to the tutorial content.

**10 October 2021** – added a footnote defining a “rheostat” as a variable resistance.

**6 October 2021** – minor additions to the Tutorial and Introduction chapters.

**3 March 2021** – divided Tutorial chapter into sections.

**2 October 2021** – minor changes made to the “Optimized electrolysis cell” Conceptual Reasoning question.

**8 May 2021** – commented out or deleted empty chapters.

**3 March 2021** – re-named Diagnostic question from “Find the mistake” to “Find the mistake(s)”.

**7 October 2020** – minor additions to the Introduction.

**29 September 2020** – significantly edited the Introduction chapter to make it more suitable as a pre-study guide and to provide cues useful to instructors leading “inverted” teaching sessions. Also replaced “trial-and-error” with “cut-and-try” as the name for the problem-solving strategy applied within the Tutorial.

**26 September 2020** – added a Case Tutorial chapter.

**17 April 2020** – added questions.

**September 2019** – document first created.

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