Parallel Circuits and Current Dividers

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Chapter 1

Introduction

This module explores the principles and properties of parallel electric circuits. You will note that the Full Tutorial begins with several pages of review from previous modules, on such topics as voltage, current, resistance, and simple circuit behavior. You may also note that this same review appears in other module Full Tutorial sections as well, and that it is no accident. In my experience teaching electric circuit theory to hundreds of students, I have found that a poor grasp of fundamentals leads to major conceptual problems later, and that the “basics” are always good to review.

Important concepts related to parallel circuits include Conservation of Energy, Conservation of Electric Charge, voltage, current, resistance, electric circuit, electrically common points, equipotential points, voltage polarity, sources versus loads, Ohm’s Law, Joule’s Law, voltmeters, bonding, current divider networks, loaded versus unloaded networks, and potentiometers.

A very important problem-solving strategy applied throughout the Tutorials is that of annotating diagrams with labels showing voltages, polarities, and currents (with direction). Mapping calculated values onto these diagrams is a helpful way to avoid confusion and to maintain proper context for all the quantities. Another important point about problem-solving is the need for patience, and the willingness to proceed with calculations even if the final strategy for solving the problem eludes one’s immediate grasp.

When reading any mathematically-based presentation, a useful habit for effective learning is to actually perform the mathematics being shown in the text. Don’t just passively read what the text tells you and trust that the math works – try the math for yourself. Not only will this serve to confirm what you are reading, but it is also an excellent way to practice those mathematical techniques.

Here are some good questions to ask of yourself while studying this subject:

• How might an experiment be designed and conducted to demonstrate the properties of parallel electrical networks? What hypotheses (i.e. predictions) might you pose for that experiment, and what result(s) would either support or disprove those hypotheses?

• How does a simple circuit respond to a switch being opened?
• How is a parallel network defined?
• How do sources differ from loads?
• How do individual component voltages relate to total (source) voltage in a parallel network?
• How do individual component currents relate to total (source) current in a parallel network?
• How do individual component resistances relate to total resistance in a parallel network?
• How do individual component power dissipations relate to total (source) power dissipation in a parallel network?
• How is the total parallel resistance formula derived from foundational principles?
• Which conservation laws apply to parallel networks?
• How do the properties of parallel networks result from more fundamental principles of electricity?
• How do the properties of parallel networks relate to electrical safety?
• What happens when multiple current sources are connected in parallel with each other?
• How do we properly connect a voltmeter to measure the voltage for any particular component in a circuit?
• What is the fundamental definition of an Ampere of current?
• How would you describe the meaning of Ohm’s Law, in your own words?
• How would you describe the meaning of Joule’s Law, in your own words?
• What does it mean to apply Ohm’s Law (and other formulae) in context when analyzing a circuit?
• What does it mean to “bond” a portion of an electrical circuit, and why might we wish to do this?
• What are some practical applications of current dividers?
• What happens when we “load” a current divider?
• How does a potentiometer function?

A useful resource for practicing circuit analysis is the Case Tutorial chapter, containing example circuits with voltage and current values provided to check your work.
Chapter 2

Case Tutorial

The idea behind a Case Tutorial is to explore new concepts by way of example. In this chapter you will read less presentation of theory compared to other Tutorial chapters, but by close observation and comparison of the given examples be able to discern patterns and principles much the same way as a scientific experimenter. Hopefully you will find these cases illuminating, and a good supplement to text-based tutorials.

These examples also serve well as challenges following your reading of the other Tutorial(s) in this module – can you explain why the circuits behave as they do?
2.1 Example: Battery, lamps, jumper wires, and meters

Here, a large 12 Volt battery and three 12 Volt-rated lamps are provided for experimentation, along with “jumper” wires consisting of plastic-clad stranded copper conductors terminated with spring-loaded “alligator” clip jaws at either end. A simple voltmeter and magnetic ammeter stand ready to take measurements:

**Current** is defined as the motion of subatomic “electric charge carriers” existing throughout the bulk of electrically conductive materials such as copper metal. The standard metric unit of measurement for electric current is the *Ampere*, with one Ampere (1 A) being equivalent to $6.2415 \times 10^{18}$ individual charge carriers drifting past a given location per second of time. The ammeter we will use senses electric current by the magnetic field produced around a current-carrying conductor. A positive indication on the ammeter’s display means charge carriers are moving (as interpreted by the “conventional flow” standard) in the direction of the meter’s arrow; a negative indication means motion in the opposite direction.

**Voltage** is defined as the amount of energy either gained or lost by an electric charge carrier between two different locations. The standard metric unit of measurement for voltage is the *Volt*, with one Volt (1 V) being equivalent to one Joule of energy difference per $6.2415 \times 10^{18}$ individual charge carriers. The voltmeter we will use has two copper-wire *test leads* that are touched to the two locations of interest. The red test lead is the “measurement” lead while the black test lead is the “reference” lead. A positive indication on the voltmeter’s display means charge carriers along
the red lead are at a higher state of energy than charge carriers along the black lead; a negative indication means the opposite.

Resistance converts some of the energy conveyed by moving electric charge carriers into other forms, most commonly heat. The standard metric unit of measurement for resistance is the Ohm, with one Ohm (1 Ω) being equivalent to one Volt of “drop” while carrying one Ampere of current.

Connecting the three lamps in parallel with each other results in each lamp glowing as brightly as it ordinarily would if connected to the battery by itself. However, the measured current at either battery terminal is three times greater than it would be for a single lamp. Each lamp drops the battery’s full voltage of 12 Volts:
Connecting multiple ammeters into the circuit reveals different amounts of current at different locations:
2.2 Example: equivalent parallel resistances

Let’s consider two circuits, each with the same 5-Volt (constant) voltage source and each with one 1 kΩ resistor. Naturally, they will both experience a current of 5 milliAmperes as predicted by Ohm’s Law \( I = \frac{V}{R} \):  

\[
\begin{align*}
5 \text{ V} & \quad 1 \text{ kΩ} & \quad 5 \text{ mA} \\
5 \text{ V} & \quad 1 \text{ kΩ} & \quad 5 \text{ mA}
\end{align*}
\]

Now, let’s add a second 1 kΩ resistor in parallel to the left-hand circuit, and then adjust the value of the resistor in the right-hand circuit to make the two source currents precisely match one another. Since each parallel-connected resistor in the left-hand circuit experiences the exact same voltage of 5 Volts (because all parallel-connected components must share the same voltage), each of those resistors passes 5 milliAmperes of current which makes the left-hand source supply a total of 10 milliAmperes. Adjusting the right-hand circuit’s single resistor to match that same 10 milliAmperes source current requires a resistor value of 500 Ω:  

\[
\begin{align*}
5 \text{ V} & \quad 1 \text{ kΩ} & \quad 5 \text{ mA} & \quad 1 \text{ kΩ} & \quad 5 \text{ mA} \\
5 \text{ V} & \quad 1 \text{ kΩ} & \quad 5 \text{ mA} & \quad 500 \text{ Ω} & \quad 5 \text{ mA}
\end{align*}
\]

Therefore, we can say that the 500 Ω resistor in the right-hand circuit is electrically equivalent to the parallel combination of two 1 kΩ resistors in the left-hand circuit. In other words, the two 5-Volt sources cannot tell the difference between a single 500 Ω resistor and two 1 kΩ resistors connected in parallel with each other.  

What we will do next is continue to add more 1 kΩ resistors to the left-hand circuit and adjust the single resistor in the right-hand circuit to be equivalent to that parallel network, making the right-hand resistor’s value such that it causes the 5 Volt source to drive the same amount of current. On the next page you will see a sequence of circuits, each successive circuit demonstrating the equivalent resistance of one more 1 kΩ resistor connected in parallel with the others.
Three 1 kΩ resistors connected in parallel are equivalent to a single 333.3 Ω resistor:

![Parallel resistor network](image1.png)

Four 1 kΩ resistors connected in parallel are equivalent to a single 250 Ω resistor:

![Parallel resistor network](image2.png)

Five 1 kΩ resistors connected in parallel are equivalent to a single 200 Ω resistor:

![Parallel resistor network](image3.png)

Each new 1 kΩ resistor stacked on to the parallel resistor network of the left-hand circuit provides a new pathway for current, and an additional 5 milliAmperes of draw from that 5-Volt source. In order for the single resistor of the right-hand circuit to be electrically equivalent to the growing parallel network of the left-hand circuit, that one resistor must decrease in value every time.
2.3 Example: Three-resistor circuit

In this scenario we will use terminal blocks to neatly organize all wire connections between a battery and three resistors:

Rather than visually show the placement of test equipment in the diagram, measurements will be documented in text form as $V$ and $I$ values$^1$. With the circuit constructed as shown in the above illustration, we obtain the following measurements:

- $V_{AB} = 12.00$ Volts
- $V_{CA} = 0.00$ Volts
- $V_{ED} = 12.00$ Volts
- $V_{DB} = 0.00$ Volts
- $V_{BF} = 0.00$ Volts
- $V_{FD} = 0.00$ Volts
- $V_{FE} = -12.00$ Volts
- $I_G = 14.35$ milliAmperes (conventional flow left to right)
- $I_H = 14.35$ milliAmperes (conventional flow right to left)

$^1$Double-lettered subscripts for $V$ denote the placement of voltmeter test leads, with the first and second letters always representing the red and black test leads respectively. For example, $V_{BC}$ means the voltage measured between terminals B and C with the red test lead touching B and the black test lead touching C. Single-lettered subscripts for $V$ represent the voltmeter’s red test lead, with the black test lead touching a defined “ground” point in the circuit. Current measurements ($I$) imply measurements made at a single location, and so there will only ever be single-lettered subscripts for $I$. Conventional flow notation will be used to specify current direction.
2.4 Example: Five-resistor circuit

In this scenario we will use terminal blocks to neatly organize all wire connections between a battery and five resistors:

Rather than visually show the placement of test equipment in the diagram, measurements will be documented in text form as \( V \) and \( I \) values\(^2\). With the circuit constructed as shown in the above illustration, we obtain the following measurements:

- \( V_{FE} = -12.00 \) Volts
- \( V_{ED} = 12.00 \) Volts
- \( V_{DC} = -12.00 \) Volts
- \( V_{CG} = 0.00 \) Volts
- \( V_{HC} = -12.00 \) Volts
- \( V_{AF} = 12.00 \) Volts
- \( V_{KB} = 0.00 \) Volts
- \( I_M = 31.93 \) milliAmperes (conventional flow right to left)
- \( I_L = 31.93 \) milliAmperes (conventional flow left to right)

\(^2\)Double-lettered subscripts for \( V \) denote the placement of voltmeter test leads, with the first and second letters always representing the red and black test leads respectively. For example, \( V_{BC} \) means the voltage measured between terminals B and C with the red test lead touching B and the black test lead touching C. Single-lettered subscripts for \( V \) represent the voltmeter’s red test lead, with the black test lead touching a defined “ground” point in the circuit. Current measurements (\( I \)) imply measurements made at a single location, and so there will only ever be single-lettered subscripts for \( I \). Conventional flow notation will be used to specify current direction.
Chapter 3
Simplified Tutorial

An electric circuit is a closed loop formed of electrically-conductive material for electric charge carriers to travel, in which at least one source raises the energy level of each charge carrier passing through and at least one load extracts energy from each passing charge carrier passing through, the result being that energy is transferred from source to load with electricity as the medium. Note the “+” and “−” symbols used to show relative energy states of charges moving in and out of each component. Voltage is the amount of energy gained or lost by charge carriers passing between two points, while current is the rate of charge carrier motion:

Fluid power systems are analogous to electric circuits: instead of electric charge carriers flowing through conductors, there is fluid flowing through pipes. A pump functions as an energy source by boosting the energy levels of fluid molecules with pressure, and an actuator or fluid motor functions as a load by allowing the pressurized fluid to do useful mechanical work. Pressure rise and pressure drop in a fluid power circuit are analogous to voltage in an electric circuit; fluid flow rate in a fluid power circuit is analogous to current in an electric circuit. Resistance in either type of circuit, is any restriction of flow causing energy to leave the circuit and never return. Power, in either type of circuit, is the rate at which energy transfers.
Switches may be incorporated into a circuit to provide control over the flow of current. An “open” switch prohibits current through it, and so an open switch is able to interrupt current throughout an entire circuit if connected in-line with the other components.

The mathematical relationship between voltage ($V$) and current ($I$) for any given resistance ($R$) is described by Ohm’s Law:

$$ V = IR \quad I = \frac{V}{R} \quad R = \frac{V}{I} $$

Power is mathematically related to voltage, current, and/or resistance by Joule’s Law:

$$ P = IV \quad P = \frac{V^2}{R} \quad P = I^2R $$

Components are said to be connected in parallel when they all span the same two sets of electrically common points. All parallel networks exhibit common properties, including common voltage, additive currents, and diminishing resistances:

**Parallel-connected components**

First, parallel-connected components experience the same amount of voltage. This is true because the Conservation of Energy makes it impossible for energy to arise from or disappear into nothing, and that is what would have to occur for a set of parallel-connected components to experience different amounts of voltage: two independent charge carriers starting at the same point, passing through different components, and ending up at the same destination, but somehow having different energy levels.

Second, total current for a set of parallel-connected components must be the mathematical sum of the individual component currents. This is true because the Conservation of Electric Charge makes it impossible for charge carriers to simply appear or vanish, and that is what would have to occur for the sum of the individual component currents in a parallel network to be different from the total current of that whole network.
Third, resistance for a collection of parallel-connected components must be less than any of the individual component resistances. Specifically, total resistance is equal to the reciprocal of the sum of the reciprocals of the individual resistances. This is really just a combination of the first and second properties with Ohm’s Law: if all component voltages are identical \( V_T = V_1 = V_2 = V_3 = \cdots = V_n \) and currents add \( I_T = I_1 + I_2 + I_3 + \cdots + I_n \) and \( I = \frac{V}{R_T} \), then \( \frac{V}{R_T} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} + \cdots + \frac{V}{R_n} \). Factoring out \( V \) from this equation yields \( \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n} \). Solving for total resistance yields \( R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n}} \).

This should also make intuitive sense: if charge carriers have multiple alternative routes to make it through a parallel network, then the overall resistance of that network must be less than the resistance of any one route.

Additionally, total power is equal to the sum of the individual component powers. This is true because of the Law of Energy Conservation, and applies to all types circuit networks, not just parallel.

A tactic many students find helpful for managing calculations during parallel circuit analysis is to draw a table listing voltage, current, and resistance as rows, and listing each component as a column, as shown here for the following parallel circuit:

\[
\begin{array}{c|c|c|c}
 & R_1 & R_2 & R_3 \\
V & 6 V & 6 V & 6 V \\
I & & & \\
R & 2200 \Omega & 4700 \Omega & 3300 \Omega \\
\end{array}
\]

Properties of parallel circuits apply to values in the same row of the table, while Ohm’s Law applies to values in the same column of the table. Any empty cell within a row becomes an obvious cue to apply the appropriate network property, and any empty cell within a column becomes an obvious cue to apply Ohm’s Law.

In this example, a clear first step is to apply the property of parallel voltages being equal in the top row:
The next clear step is to apply Ohm’s Law each resistor’s column, filling the empty “current” cells there:

<table>
<thead>
<tr>
<th></th>
<th>R1</th>
<th>R2</th>
<th>R3</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>V</td>
<td>6 V</td>
<td>6 V</td>
<td>6 V</td>
<td>6 V</td>
</tr>
<tr>
<td>I</td>
<td>2.7273 mA</td>
<td>1.2766 mA</td>
<td>1.8182 mA</td>
<td>5.8221 mA</td>
</tr>
<tr>
<td>R</td>
<td>2200 Ω</td>
<td>4700 Ω</td>
<td>3300 Ω</td>
<td>1030.56 Ω</td>
</tr>
</tbody>
</table>

Now we can apply the property of parallel currents adding up to equal total current to the middle row:

<table>
<thead>
<tr>
<th></th>
<th>R1</th>
<th>R2</th>
<th>R3</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>V</td>
<td>6 V</td>
<td>6 V</td>
<td>6 V</td>
<td>6 V</td>
</tr>
<tr>
<td>I</td>
<td>2.7273 mA</td>
<td>1.2766 mA</td>
<td>1.8182 mA</td>
<td>5.8221 mA</td>
</tr>
<tr>
<td>R</td>
<td>2200 Ω</td>
<td>4700 Ω</td>
<td>3300 Ω</td>
<td>1030.56 Ω</td>
</tr>
</tbody>
</table>

Lastly, we may apply Ohm’s Law to the “total” column to calculate total circuit resistance (or, alternatively, apply the property of parallel resistances diminishing to equal total resistance):

<table>
<thead>
<tr>
<th></th>
<th>R1</th>
<th>R2</th>
<th>R3</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>V</td>
<td>6 V</td>
<td>6 V</td>
<td>6 V</td>
<td>6 V</td>
</tr>
<tr>
<td>I</td>
<td>2.7273 mA</td>
<td>1.2766 mA</td>
<td>1.8182 mA</td>
<td>5.8221 mA</td>
</tr>
<tr>
<td>R</td>
<td>2200 Ω</td>
<td>4700 Ω</td>
<td>3300 Ω</td>
<td>1030.56 Ω</td>
</tr>
</tbody>
</table>

As evidenced by the multiple solutions in the last step, the tabular format helps us check our work by showing us more than one way to calculate (and/or verify) parameters.
Any network of parallel-connected resistors acts to divide the total (source) current into fractional portions, each fraction equal to the total (parallel network) resistance divided by that resistance. This is a very practical use for parallel resistor networks, reducing an applied current by a prescribed ratio:

\[
I_{\text{output}} = I_{\text{input}} \left( \frac{R_{\text{total}}}{R} \right)
\]

A potentiometer is an adjustable device intended to provide resistance ratios in one unit. It has three terminals: two connecting to the ends and the third serving as the adjustable “wiper” contacting some point in between. Some potentiometers are coarse, having just 3/4 turn from minimum to maximum, while others are designed to be multi-turn for fine precision adjustment:

This schematic diagram shows how a potentiometer may be used an an adjustable current divider:
Chapter 4

Full Tutorial

4.1 Concept review

First, a review of some foundational principles.

Energy is the ability to set matter into motion. Energy may take many different forms, but is broadly divided into two categories: kinetic energy and potential energy. “Kinetic” energy is that exhibited through motion: for example, a moving mass has energy because of its ability to set other matter into motion by collision, and its energy is deemed kinetic because it is moving. “Potential” energy is that which exists without motion: for example, a suspended mass has energy because it may potentially cause motion if allowed to fall, but until it does the energy is regarded as “potential” because it is not being put into action (yet).

An interesting property shared by both mass and energy is conservation, which means these quantities cannot be created from nothing nor can they be destroyed (annihilated), but are eternal. Both matter and energy can and do, however, change forms.

Just as any mass experience a force when influenced by a gravitational field, some types of matter possess another property called electric charge, which is the ability to experience a force when influenced by an electric field. Just as mass is a conserved quantity (i.e. eternal), so is electric charge: it cannot be created or destroyed. Electric charges exist in two different types, called positive and negative. Examples of electrically charged matter include electrons (negative) and protons (positive), both subatomic particles found in equal numbers within “electrically balanced” atoms.

In some forms of matter, electric charges are mobile. Metals, for example, have very loosely-bound electrons which are able to drift through the solid space of the metal object while the remaining portions of the atoms remain bound in place. In some liquids, molecules split into unbalanced atoms called ions, some carrying excess electrons (negative charge) and others missing electrons (positive charge), and these ions are free to drift within the liquid. Any mobile electric charges, whether they be single subatomic particles or ions, are generally referred to as charge carriers. Substances possessing free charge carriers are called electrical conductors, while substances lacking available charge carriers are called insulators. The degree of mobility experienced by charge carriers between any two locations is called electrical resistance, mathematically symbolized by the variable $R$ and
measured in the unit of the Ohm (Ω). A perfect conductor has zero resistance, and a perfect insulator has infinite resistance.

Free charge carriers within conductive substances are constantly moving in random directions, motivated by thermal (heat) energy. If charges are persuaded to move in a particular direction by the presence of an electric field, their net motion is called current which is mathematically symbolized by the variable $I$ and measured in the unit of the Ampere ($A$). Since electric charge is a conserved quantity, which means charge carriers cannot simply spring into existence or vanish, the only way to sustain a continuous motion of charge carriers is to provide a closed loop for those charge carriers to circulate, called a circuit.

Electric charges may act as carriers of energy, just as masses may possess either kinetic or potential energy. The amount of potential energy exhibited per electric charge carrier is simply called electric potential. Any gain or loss of potential between two locations is referred to as voltage or potential difference, mathematically symbolized by the variable $V$ or $E$ and measured in the unit of the Volt (one Volt defined as one Joule of energy transferred per Coulomb of electric charge). A very important feature of voltage, like all forms of potential energy, is that it is relative and not absolute. Just as the potential energy of a suspended weight is quantifiable only between the point of suspension and the point to which it may fall, the amount of potential carried by electric charges is quantifiable only between the location of the charge and another location of different energy, with voltage being the amount of energy per charge either gained or lost between those two points. There is no such thing as voltage existing at any single point. Voltage only exists between two points, between which charge carriers either gain or lose potential energy.

If charge carriers gain energy while moving through a device, that device is called a source. If charge carriers lose energy while moving through a device, that device is called a load:

---

1 One Ampere of electric current is defined as the passing of one Coulomb of electric charge carriers (equivalent to $6.2415 \times 10^{18}$ electrons) per second of time.

2 This concept often causes confusion for students, and so is good to explore in more detail. Try imagining a broken loop with current continuously flowing at all the unbroken points within the loop. What this would inevitably mean is that charge carriers would have to mysteriously vanish where they reach the break, and that other charge carriers would have to mysteriously appear on the other side of the break to enter the circuit. So long as charge carriers are eternal and cannot pass through an insulating (non-conductive) break, it is impossible for a continuous flow to occur in a broken loop.

3 A Coulomb of charge is equivalent to $6.2415 \times 10^{18}$ electrons.
Sources and loads usually appear together in circuits, with charge carriers acting as couriers of energy from the source to the load. Energy enters the circuit through the source from the outside (e.g. light, heat, sound, chemical reaction, mechanical, etc.) to boost the charge carriers’ energy, those charge carriers travel to the load where they release that energy in some form (e.g. light, heat, sound, chemical reaction, mechanical, etc.), and then the charge carriers circle back to the source for another round. An electrical circuit is analogous to a fluid power circuit where energy is transferred by means of a circulating fluid, with a pump (source) infusing potential energy into the fluid by means of pressure and an actuator or motor (load) extracting potential energy from the fluid to do useful work such as lifting an object or moving a machine:

The positive (+) and negative (−) symbols refer to the relative energy levels of charge carriers as they pass through each source and load, and in pairs represents the polarity for voltage across each component. These polarity symbols are analogous to “high” and “low” pressure notations for a fluid circuit, revealing the relative pressures of fluid entering and exiting a component. The direction of current shown by the arrows is a convention, based on the assumption that the carriers are all positively charged. This convention for marking the direction of current is used regardless of that actual type(s)\(^4\) of charge carriers involved, and is called conventional flow notation.

Metal wires, which are very good conductors of electricity (i.e. possess very little resistance \(R\)) extract negligible energy from charge carriers flowing through, which explains why the conductors shown in the above diagram have no + or − symbols drawn near. Every point along those metal wires are considered electrically common by virtue of their negligible resistance. This forces all charge carriers to be at the same energy level (i.e. equipotential) which means there is practically zero voltage between any two points along a good conductor.

\(^4\)In metallic conductors, charge carriers consist of free electrons which are negatively charged and move in the opposite direction from that shown in the illustration. In liquids and ionized gases, charge carriers consist of positive and negative ions with motion in both directions (depending on the sign of each carrier’s charge). Conventional flow is used as a convenience for representing current through any conductor and is not to be taken literally.
**Switches** are electrical components designed to deliberately establish or sever electrical connections between two or more conductors. Switches are usually designed such that a moving piece of metal either makes or breaks contact with a stationary piece of metal, typically by the actuation of a pushbutton, lever, or some other mechanism. When a switch is positioned to make contact and permit the flow of electric charge carriers through it, it is said to be *closed* or *shorted*. When a switch is positioned to break contact and forbid the flow of current, it is said to be *open*.

The following illustration shows a simple circuit with three switches, all closed:

![Simple Circuit with Three Switches](image)

If *any* of these switches is opened, the circuit will become “broken” and current halts throughout. Whichever switch is opened, charge carriers cannot pass through it. This, in turn, immediately ceases the circulation of charge carriers at all points in the circuit due to the Conservation of Charge⁵:

![Modified Circuit with Open Switch](image)

The effect of opening a switch is immediate throughout the circuit: all current ceases, full source voltage appears across the terminals of the open switch, and the load no longer exhibits a voltage across its terminals. All points between the negative pole of the source and the switch are equipotential to each other, and all points between the positive pole of the source and the switch are also equipotential to each other.

---

⁵Since charges are eternal and therefore cannot appear from nowhere or disappear into nothingness, the continuous circulation of charge carriers in a circuit demands a complete, unbroken path. If the path becomes broken by an open switch, charge carriers at the “incoming” pole of the switch must stop moving because they cannot vanish at the open air gap of the switch, and charges at the “outgoing” switch pole must also stop moving because no new charge carriers may appear out of the air gap. This situation is analogous to a fluid circuit where the total quantity of fluid is fixed and a valve somewhere shuts off the path of flow: the motion of fluid must immediately stop everywhere in the fluid circuit.
Much the same happens if we open a different switch in this circuit. Current everywhere immediately ceases, and the source’s full voltage appears across the terminals of the open switch:

![Circuit Diagram]

No energy is exchanged from source to load, because charges cannot move anywhere.

Note how there is a difference between this scenario and the previous one with a different open switch, and that is the absolute energy states of stationary charge carriers within the load. From the perspective of energy transfer, however, this is of no concern. With no charge carriers moving through the load, transitioning from high energy to low energy, there is no potential difference (i.e. voltage) across the load, and therefore the load receives no energy from the charge carriers.

Voltage ($V$), current ($I$), and resistance ($R$) are proportionately related to one another by a set of formulae called *Ohm’s Law*:

\[ V = IR \quad I = \frac{V}{R} \quad R = \frac{V}{I} \]

Where,
- $V =$ Voltage, in Volts (V)
- $I =$ Current, in Amperes (A)
- $R =$ Resistance, in Ohms (Ω)

The rate at which energy is exchanged from one form to another is called *power*, mathematically symbolized by the variable $P$ and measured in the unit of the Watt\(^6\) ($W$). A set of formulae called *Joule’s Law* relates power to voltage, current, and resistance:

\[ P = IV \quad P = \frac{V^2}{R} \quad P = I^2R \]

Where,
- $P =$ Power, in Watts (W)
- $V =$ Voltage, in Volts (V)
- $I =$ Current, in Amperes (A)
- $R =$ Resistance, in Ohms (Ω)

\(^6\)One Watt is defined as one Joule of energy transferred per second of time.
4.2 Parallel networks

Now that we have reviewed some fundamental terms and principles of electric circuits, let us move on to the topic at hand: parallel circuits. A “parallel” circuit, by definition, is one where all components are connected between the same two sets of points made equipotential by direct wire connection. The following illustration shows four components (each one represented as a nondescript rectangle) connected in parallel with each other, with lettered points for reference:


If we were to connect this parallel set of components in a circuit, and provide a suitable energy source to provide a continuous voltage, we would find that the amount of voltage across each of the four components, and indeed between any two points spanning those components, was identical. No component within a parallel circuit may have less voltage or more voltage than any other component.

The reason for this equality of voltage in any parallel configuration is the equipotential nature of the connecting wires. If the conductors bridging all the upper component terminals together are equipotential (meaning charge carriers neither lose nor gain energy moving through that conductor), then whatever potential difference exists between any two points between those equipotential sets must be the same for all points between those equipotential sets. For example, if points A through D are equipotential with each other, and points E through H are also equipotential with each other, then the voltage between any point A-D and any point E-F must be the same (e.g. $V_{AE} = V_{BF} = V_{CH} = V_{DE}$, etc.). Illustrating this mathematically, if one set of variables are equal to each other and another set of variables are also equal to each other, the difference between any member of one set and any member of the other set must be the same:

$$\text{If } A = B = C = D \quad \text{and if } E = F = G = H$$

Then

$$\text{Then } (A - E) = (B - F) = (C - H) = (D - E)$$

This is the defining characteristic of a parallel circuit, and the first logical consequence of that characteristic: all components share the same two sets of electrically common points, and as a result all components experience the same amount of voltage at any given time.

Another way of defining a parallel circuit is to say that only two electrically distinct surfaces exist among the connected components.

---

7 To review, electrically common points are those connected together by negligible resistance, and therefore must be equipotential to each other. Any set of electrically common points and the conductor bridging them together comprise one electrically common surface.

8 If electrically common points are defined as being connected together by a conductor of negligible resistance and therefore equipotential, then electrically distinct points must have substantial resistance between them and therefore be capable of existing at different potentials.
This simple principle of parallel circuits (all components experience the same voltage) has a range of practical applications. One such application is in the use of voltmeters to measure voltage. If we know voltage must be the same for all parallel-connected components, then we may ensure a voltmeter intercepts a particular component’s voltage by connecting that voltmeter in parallel with that component.

To see how this principle might be applied, consider the following schematic diagram showing a complex circuit containing nine resistors ($R_1$ through $R_9$) and a single voltage source ($V_1$):

If our task was to measure the amount of voltage dropped by resistor $R_2$, we must connect the voltmeter’s test leads to points common with the upper and lower terminals of $R_2$, respectively:

The voltmeter is connected in parallel with resistor $R_2$, and therefore must experience the same voltage dropped across $R_2$. However complicated and mysterious the rest of the circuit might be, this much we know to be true!
Another practical application of parallel connections is electrical safety, specifically a concept known as *bonding*. To “bond” a set of electrical conductors means to connect them together using another conductor in order to ensure a zero-voltage state between those conductors. In other words, *to ensure a zero-voltage condition between two points, we intentionally connect a wire (which by definition will have negligible voltage end-to-end) in parallel with those two points, so that the wire’s zero-voltage state will force the connected points to have zero voltage between them as well.*

The following photograph shows a work site at a 230 kV electrical substation, where electricians are busy performing maintenance work on a high-voltage component. In addition to opening large switches (called *disconnects*) to isolate this new component from any source of voltage, they have taken the additional step of *bonding* the high-voltage conductors to each other and to Earth ground by means of temporary wire cables. The cables on this work site happen to be yellow in color, and may be seen hanging down from C-shaped clamps attached to three horizontal metal tubes called *busbars* which serve as conductors for electricity in this substation:

Another way of looking at bonding is to consider it a means to force multiple points in a circuit to become equipotential to each other. The bonding conductor is a *short* (i.e. a low-resistance path) between points, thus forcing those points to have the same potential. Any person whose body comes into contact with those points will not experience any substantial voltage between those points to cause bodily harm. Furthermore, bonding the shorted power conductors to ground forces all of them to be at Earth potential, further protecting human life by ensuring no one standing on the ground will become shocked or burned if they happen to touch one of the conductors.
4.3 Properties of parallel networks

While it is true that parallel-connected components experience the same voltage, this is not the only unique property of parallel circuits. Other properties become apparent when we analyze the relationship between voltage and current for each component in a parallel circuit. Let us consider a three-resistor parallel circuit as a test case, powered by a 9 Volt voltage source. The letters A through H identify different points in the circuit we will later reference:

![Diagram of a three-resistor parallel circuit with labels A through H, a 9 Volt voltage source, and resistors R1, R2, and R3 with values of 1.5 kΩ, 2.2 kΩ, and 710 Ω respectively.]

A good first step in analyzing this circuit is tracing directions of current and marking polarities of voltage. The polarity of the voltage source is already given by the source’s + and − symbols, and we also know all three resistors must share this same voltage because they are connected in parallel with each other. Therefore, we may place + and − symbols near each resistor to match the source:

![Diagram showing voltage polarities marked on each resistor.]

Current directions may now be assigned according to the polarity of voltage and the identity of each component as either a source or a load, namely that current always exits the positive terminal of a source and enters the positive terminal of a load. Our one voltage source is a source by definition, and resistors are always loads⁵, so the current directions must be as follows:

![Diagram showing current directions marked on each resistor.]

⁵Recall that the function of a load is to extract energy from charge carriers as they pass through. This is what any resistance does by its inherent opposition to the passage of current: it depletes some of the energy of those passing charge carriers, converting that energy into heat.
At this point we are ready to apply Ohm’s Law to the calculation of each resistor’s current, since Ohm’s Law (like all physics formulae) requires its variables to be in the same context. That is to say, if we wish to use Ohm’s Law \( I = \frac{V}{R} \) to calculate the current through a resistor \( (I_R) \), we need to know the voltage across that same resistor \( (V_R) \) as well as the resistance of that resistor \( (R) \). With only one value of voltage in this circuit \( (V) \) shared in common with all three resistors, and three different resistor values \( (R_1, R_2, \text{ and } R_3) \), we will calculate three different current values \( (I_{R1}, I_{R2}, \text{ and } I_{R3}) \), each resistor with its own “branch” current:

\[
I = \frac{V}{R} \quad \text{(General form of Ohm’s Law)}
\]

\[
I_{R1} = \frac{V}{R_1} = \frac{9 \text{ V}}{1.5 \text{ k}\Omega} = 6 \text{ mA}
\]

\[
I_{R2} = \frac{V}{R_2} = \frac{9 \text{ V}}{2.2 \text{ k}\Omega} = 4.091 \text{ mA}
\]

\[
I_{R3} = \frac{V}{R_3} = \frac{9 \text{ V}}{710 \Omega} = 12.676 \text{ mA}
\]

At this juncture it is worth asking, “How much current is there flowing through the voltage source?” A voltage source is only specified for the amount of voltage it outputs, not current. The amount of current output by a voltage source depends on what load(s) the source is powering. In this case, the source is powering three resistors, each of those drawing a known amount of current.

Recalling the definition of current being rate of electric charge carrier motion, and knowing in particular that one Ampere is defined as one Coulomb of charge moving by per second of time, we may begin to answer the question of the source’s current by examining charge carrier motion through the three resistors. Over one second’s worth of time, 6 milliCoulombs of charge pass through resistor \( R_1 \) (from C to D). over that same time interval 4.091 milliCoulombs of charge pass through \( R_2 \) (from E to F). Lastly, during that same time period 12.676 milliCoulombs of charge move through \( R_3 \) (from G to H).

Electric charge, like energy, is a conserved quantity, which means every charge carrier passing through the resistors to deliver energy must also pass through the source to replenish their energy. Totaling the flow of charge carriers through all three resistors, we get 6 mA + 4.091 mA + 12.676 mA = 22.767 mA. This means the rate of charge carrier flow through the source must be equal to 22.767 mA.
Here we see another important property of parallel circuits: *currents add in a parallel circuit*, because the charge carriers passing through each component must be accounted for according to the Law of Electric Charge Conservation.

This is true for parallel-connected sources as well as parallel-connected loads. Consider the following parallel set of current sources:

Imagine the flow of charge carriers through each current source being modeled by the flow of liquid through three pumps. The first pump discharges liquid upward at a rate of 5 units per second, while the second pump does likewise at 11 units per second. The combined flow rates of these two pumps from C to E (and also from F to D) must therefore be 16 units per second of liquid. However, at junctions E and F, the third pump’s flow rate *subtracts* from the total of the first two, because that pump is moving liquid downward through its branch of the circuit rather than upward. This leaves a fluid flow rate of only 13 units per second moving from point E to G, through the resistance to H, and then to F. Just as fluid molecules cannot be created or destroyed (the Conservation of Mass), the electric charge carriers moving in this parallel circuit must likewise be accounted for (the Conservation of Electric Charge), leaving 13 mA of current flowing downward through the resistor. This solitary resistor would develop a voltage drop as predicted by Ohm’s Law \( V = IR \).

As a general rule, then, we find that the current resulting from a set of parallel-connected components is equal to the algebraic \(^{10}\) sum of each component’s current within that set.

So far we have drawn two important conclusions about parallel-connected electrical components:

1. Parallel-connected components experience the *same voltage* at any given time.

2. The total resultant current for a set of parallel-connected components is equal to the *algebraic sum* of the components’ currents.

\(^{10}\)The term “algebraic sum” simply means a total respecting the mathematical sign of each argument. In the case of the three parallel-connected current sources, their sum total current was 13 milliAmperes because \( 5 + 11 + (-3) = 13 \).
One more important conclusion regarding parallel circuits awaits our discovery, regarding resistance. If we return to our example circuit powered by the 9 V voltage source, we may ask the question “How great is this circuit’s total resistance?”

![Parallel Circuit Diagram]

Of course, Ohm’s Law is useful in calculating this ($R = \frac{V}{I}$), but as always we must consider all its variables in context. If it is total circuit resistance we wish to calculate, then we need to identify the circuit’s total voltage and its total current prior to performing the calculation. Total voltage is the same as the source voltage because this is a parallel circuit, and we have already calculated the total current to be 22.767 milliAmperes. Therefore the calculation for total resistance is:

$$I_{total} = I_{R1} + I_{R2} + I_{R3}$$

$$R_{total} = \frac{V}{I_{total}} = \frac{9 \text{ V}}{22.767 \text{ mA}} = 395.31 \Omega$$

If you compare this total resistance figure of 395.31 Ω against the individual resistor values given in the schematic diagram, you will notice that total resistance is substantially less than any of the individual resistances. This should make intuitive sense, as we would expect the combined effect of multiple pathways in parallel to offer less resistance (i.e. less opposition to charge carrier motion) than any one of the resistors on its own. The precise mathematical relationship between this total value of 395.31 Ω and the three resistors’ individual values, however, is not obvious from inspection.

We may derive a mathematical relation by beginning with the statement that total current is equal to the sum of the parallel-connected resistor currents:

$$I_{total} = I_{R1} + I_{R2} + I_{R3}$$

We may substitute Ohm’s Law into this statement of summed currents, knowing each resistor’s current is equal to the quotient of the common parallel voltage and that resistance ($I_R = \frac{V}{R}$):

$$I_{total} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

Factoring out the common voltage value $V$:

$$I_{total} = V \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)$$

Then, dividing both sides of the equation by $V$:

$$\frac{I_{total}}{V} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$
This doesn’t quite get us to the definition of total resistance we were looking for, since $R = \frac{V}{I}$ rather than $\frac{I}{V}$. However, if we reciprocate both sides of the equation, we will have what we are looking for:

$$\frac{V}{I_{\text{total}}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

From our previous calculation of total resistance we know that $\frac{V}{I_{\text{total}}} = R_{\text{total}}$, so what we have here is really a definition of total resistance for a parallel circuit:

$$R_{\text{total}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

In summary, we may state one definition of a parallel circuit followed by three important electrical properties unique to parallel circuits:

<table>
<thead>
<tr>
<th>Definition:</th>
<th>Parallel-connected electrical components share two sets of electrically common points.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Property #1</strong></td>
<td>Parallel-connected components experience the same voltage at any given time. $V_1 = V_2 = V_3 \cdots = V_n$</td>
</tr>
<tr>
<td><strong>Property #2</strong></td>
<td>Currents add in parallel: total current for a set of parallel-connected components is equal to the algebraic sum of the components’ currents. $I_{\text{total}} = I_1 + I_2 + I_3 \cdots + I_n$</td>
</tr>
<tr>
<td><strong>Property #3</strong></td>
<td>Resistances diminish in parallel: total resistance for a set of parallel-connected resistances is equal to the reciprocal of the sum of the reciprocated resistance values. $R_{\text{total}} = \frac{1}{(1/R_1 + 1/R_2 + 1/R_3 \cdots + 1/R_n)}$</td>
</tr>
</tbody>
</table>

It should be noted that the total power ($P$) of any parallel circuit is equal to the sum of all load powers. This is simply a consequence of the Law of Energy Conservation, since power is nothing more than the rate of energy transfer over time: the sum total of all energy transfer out of the circuit (at load components) must be equal to the sum total of all energy transfer into the circuit (at source components). However, this is not unique to parallel circuits, but applies to all electrical circuits because the Law of Energy Conservation is universal:

$$P_{\text{total}} = P_1 + P_2 + P_3 \cdots + P_n$$

\(^{11} \frac{1}{R} \) actually does have a physical definition, and it is conductance. Conductance is a measure of how easy it is for electrical charge carriers to pass through a component. Conductance is the reciprocal of resistance and is mathematically symbolized by the variable $G$, and measured in the unit of Siemens (S). The total amount of conductance in a parallel circuit is the sum of all resistors’ individual conductances.
4.4 Current dividers

An important application of parallel-connected resistors is a particular configuration known as a current divider, the purpose of this network\(^\text{12}\) being to divide an applied current into smaller proportions. A simple example of a current divider appears in this schematic diagram, shown by itself with no power source connected, but with terminals provided for input and output currents:

![Current divider schematic diagram]

The characteristic formula for a current divider, predicting the amount of current passing through any one of the parallel resistances \(R_i\), is as follows:

\[
I_{output} = I_{input} \left( \frac{R_{total}}{R} \right)
\]

In the case of the two-resistor current divider illustrated above, the formula could be written as:

\[
I_{output} = I_{input} \left( \frac{1}{\frac{R_1}{R} + \frac{R_2}{R}} \right)
\]

In any case, the ratio \(\frac{R_{output}}{R}\) is the proportion of \(I_{input}\) represented by \(I_{output}\), and so long as the resistor values remain fixed this current division ratio will remain fixed as well. For example, if \(R_1\) and \(R_2\) in the two-resistor divider happen to be equal in resistance, the proportion will be 50% (i.e. \(I_{output}\) will be equal to one-half the current of \(I_{input}\)). If \(R_2\) happens to be three times the value of \(R_1\), the proportion will be 25% (i.e. \(I_{output}\) will be one-quarter the current of \(I_{input}\)). The remainder of the input current is diverted through the current divider’s other resistor(s).

Any ratio desired (between 0 and 1, inclusive) is possible with a current divider, given the appropriate resistor values. This makes current dividers especially useful in measurement applications (e.g. reducing some high-current quantity to a precisely smaller proportion for easier and safer interpretation), computational applications (e.g. performing fixed-value division on a current representing some other quantity), and electric power applications (e.g. reducing a large source current to a smaller current more suitable for powering certain components and sub-circuits).

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\(^{12}\)A network simply refers to a collection of interconnected components, with or without a source.

\(^{13}\)In case you find this confusing, and were expecting a division ratio of one-third, consider the fact that the division ratio of this two-resistor divider is defined by \(R_2\) compared to \(R_{total}\), not \(R_2\) versus \(R_1\). If \(R_2\) is three times the resistance of \(R_1\), then it will draw one-third as much current as \(R_1\), but only one-quarter of the total current. If you still find this confusing, assign some numerical values to it and see for yourself how \(I_{R1}\), \(I_{R2}\) and \(I_{total}\) (\(I_{input}\)) relate.
4.4. CURRENT DIVIDERS

Tempting though it may be to treat the current divider formula as a new principle of parallel resistor circuits, it is really just an expression of Ohm’s Law and familiar parallel-circuit properties. All we need to do is re-associate some of the variables in the formula to make this apparent:

\[ I_{\text{output}} = I_{\text{input}} \left( \frac{R_{\text{total}}}{R} \right) \]  
(Original form)

\[ I_{\text{output}} = I_{\text{input}} R_{\text{total}} \left( \frac{1}{R} \right) \]  
(Modified form)

The product \( I_{\text{input}} R_{\text{total}} \) is equivalent to voltage (\( V \)) in a parallel circuit, following Ohm’s Law (\( V = IR \)): the one and only voltage in this parallel circuit is equal to the total applied current multiplied by the total resistance of the circuit. Substituting \( V \) for \( I_{\text{input}} R_{\text{total}} \):

\[ I_{\text{output}} = V \left( \frac{1}{R} \right) \]

\[ I_{\text{output}} = \frac{V}{R} \]

This final result, of course, is just another application of Ohm’s Law: the current through any one resistor in a parallel network is equal to the voltage applied across that resistor divided by that resistor’s value.

It should be noted that current dividers will output current values faithful to these formulae only when unloaded: that is to say, only when negligible voltage is dropped across its output terminals. If any load (for example, another resistor) is connected across the output terminals of a current divider, its output current will “sag” below the value predicted by the current divider formula. Connecting a load to a current divider’s output terminals essentially modifies the current divider circuit, since the values of its constituent resistors are no longer the sole factors in determining the division ratio (\( \frac{R_{\text{total}}}{R} \)). A further exploration of this “loading” effect is beyond the scope of parallel circuits, and will be addressed in later modules after the introduction of series-parallel circuit analysis.

\(^{14}\)An extremely important pedagogical principle within the sciences is to connect concepts wherever possible, and to preferentially reason from foundational principles as opposed to accumulating new rules. This is just one example in action: showing how the division ratio of a current divider circuit is really just an application of Ohm’s Law and the concept of voltage being shared among all parallel-connected components. An unfortunate tendency among many students is to memorize new formulae for various circuit configurations without grasping the common origins of those formulae. I say this is “unfortunate” because the urge to memorize rather than reason represents an aversion to patient observation and logical thinking.

\(^{15}\)In this case, we are literally employing the associative property of multiplication which states that \( a(bc) = (ab)c \).
4.5 Potentiometers

A very important electrical component usable as a variable current divider is the potentiometer. This device has three terminals: two connecting to the ends of a resistive strip, and one connecting to a movable “wiper” that contacts the resistive strip at one point.

As the wiper forms a single point of contact with the resistive strip, it essentially makes the potentiometer appear as a pair of series-connected resistors having complementary values: when the wiper is in the exact center, each wiper-to-end resistance will be equal (each one half the total end-to-end resistance); when the wiper position is offset from center, one of those resistances will be greater than half and the other resistance will be more than half:

When the outer two terminals of a potentiometer are connected to a current source, the wiper provides a connection point to a node where current divides between two paths. Moving the wiper back and forth adjusts the current division ratio. In the example shown below, moving the wiper toward the right causes the right load to receive more of the source’s current, and the left load to receive less:

Potentiometers are extremely useful devices: as volume controls in audio circuits, calibration adjustments on electronic measuring instruments, sensors to detect angle or position, and many other applications.
Photographs of various potentiometers with *rotary* adjustments are shown here:

The left-hand image shows a 3/4 turn potentiometer with wires soldered to its three terminals. The black-colored wire connects to the potentiometer’s “wiper” terminal while the two red-colored wires each connect to opposite ends of the potentiometer’s resistive strip. The middle and right-hand images show “trimmer” potentiometers with multi-turn adjusting screws and leads designed for mounting to a printed circuit board (PCB).

*Linear* potentiometers are also manufactured, where the resistive strip and wiper motion range fall along a straight line. Linear potentiometers are commonly used for audio mixer controls (where each audio channel has a “slide” control for volume) and as machine position measurement sensors.

As with fixed-value resistors, the physical size of a potentiometer is no indication of resistance, but rather a reflection of its *power* rating. It is possible to design potentiometers for a wide range of resistance values in the same size package, but power dissipation requires surface area to shed heat which is why physical size is a reliable indication of power rating but not resistance.
Chapter 5

Derivations and Technical References

This chapter is where you will find mathematical derivations too detailed to include in the tutorial, and/or tables and other technical reference material.
5.1 Derivation of parallel resistance formulae

The parallel resistance formula \( R_{\text{total}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}} \) seems rather strange and unintuitive at first glance. While it may be derived from Ohm’s Law and the properties of parallel networks (specifically, branch currents adding to equal total current), there is another approach as well involving conductance.

Conductance is the reciprocal of resistance. It is symbolized by the variable \( G \) and measured in the unit of Siemens\(^1\) (S). If we were comparing two resistors, we could say that resistor A had more resistance than resistor B, or we could communicate the exact same idea by saying resistor B was more conductive than resistor A. For example, a resistance of 5 Ohms is equal to a conductance of \( \frac{1}{5} \) Siemens.

Just as series-connected resistances add to create a greater total resistance, parallel-connected conductances add to create a greater total conductance. This should make intuitive sense, as the more branches we provide in a parallel network for current to split and flow through, the easier it must be overall for current to flow. If we had a three-branch parallel network, we could express this idea of added conductances as follows:

\[
G_{\text{total}} = G_1 + G_2 + G_3
\]

Now, since we happen to know that conductance is merely the reciprocal of resistance (i.e. \( G = \frac{1}{R} \)), we may make this substitution in each term of the above equation:

\[
\frac{1}{R_{\text{total}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}
\]

Reciprocating both sides of this equation to solve for \( R_{\text{total}} \) in a parallel network:

\[
R_{\text{total}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}
\]

It should be easy to see that this general form of the equation holds true regardless of the number of branches in the parallel network, and so we may express this truth more generally:

\[
R_{\text{total}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}}
\]

\(^1\)An obsolete unit of measurement for conductance is the mho, which is just “ohm” spelled backwards. The symbol for the mho was an upside-down Greek Omega letter. And you didn’t think electrical engineers had a sense of humor?
5.1. DERIVATION OF PARALLEL RESISTANCE FORMULAE

For two-resistance parallel networks there exists an alternative formula for calculating total resistance, often called the product over sum formula. Like the first, this formula seems counter-intuitive at first sight:

\[ R_{\text{total}} = \frac{R_1 R_2}{R_1 + R_2} \]

Let us begin our derivation of this formula by starting with the other form which we have already derived:

\[ R_{\text{total}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \]

We may evaluate the sum within the denominator of this fraction by finding a common denominator for \( \frac{1}{R_1} \) and \( \frac{1}{R_2} \). One way to do this is to multiply \( \frac{1}{R_1} \) by \( \frac{R_2}{R_2} \), and to multiply \( \frac{1}{R_2} \) by \( \frac{R_1}{R_1} \), since both “unity fractions” have numerical values of 1 and as such do nothing to change the values of the terms they’re multiplied with:

\[ R_{\text{total}} = \frac{1}{\frac{R_2}{R_1 R_2} + \frac{R_1}{R_1 R_2}} \]

\[ R_{\text{total}} = \frac{1}{\frac{R_2 + R_1}{R_1 R_2}} \]

\[ R_{\text{total}} = \frac{R_1 R_2}{R_1 + R_2} \]

Now we just re-write the right-hand side of this equation so that it is no longer a compound fraction, and we will have derived our two-resistance parallel formula:

\[ R_{\text{total}} = \frac{R_1 R_2}{R_1 + R_2} \]
5.2 Metric prefixes

The metric system of measurement specifies certain units of measurement for various physical parameters, some of them listed here:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>Meter</td>
<td>m</td>
</tr>
<tr>
<td>Temperature</td>
<td>Celsius or Kelvin</td>
<td>C or K</td>
</tr>
<tr>
<td>Mass</td>
<td>Gram</td>
<td>g</td>
</tr>
<tr>
<td>Force</td>
<td>Newton</td>
<td>N</td>
</tr>
</tbody>
</table>

These units are useful for measuring any range imaginable, from subatomic to astronomical. In order to make the numerical values easier for human beings to manage, the metric system also includes prefixes representing power-of-ten multipliers, some of which are listed here:

<table>
<thead>
<tr>
<th>Prefix</th>
<th>Symbol</th>
<th>Power-of-ten</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peta</td>
<td>P</td>
<td>$10^{15} = 1,000,000,000,000,000$</td>
</tr>
<tr>
<td>Tera</td>
<td>T</td>
<td>$10^{12} = 1,000,000,000,000$</td>
</tr>
<tr>
<td>Giga</td>
<td>G</td>
<td>$10^9 = 1,000,000,000$</td>
</tr>
<tr>
<td>Mega</td>
<td>M</td>
<td>$10^6 = 1,000,000$</td>
</tr>
<tr>
<td>Kilo</td>
<td>k</td>
<td>$10^3 = 1,000$</td>
</tr>
<tr>
<td>Milli</td>
<td>m</td>
<td>$10^{-3} = \frac{1}{1000}$</td>
</tr>
<tr>
<td>Micro</td>
<td>µ</td>
<td>$10^{-6} = \frac{1}{1,000,000}$</td>
</tr>
<tr>
<td>Nano</td>
<td>n</td>
<td>$10^{-9} = \frac{1}{1,000,000,000}$</td>
</tr>
<tr>
<td>Pico</td>
<td>p</td>
<td>$10^{-12} = \frac{1}{1,000,000,000,000}$</td>
</tr>
<tr>
<td>Femto</td>
<td>f</td>
<td>$10^{-15} = \frac{1}{1,000,000,000,000,000}$</td>
</tr>
</tbody>
</table>

In order to represent very large or very small quantities all we need to do is combine the right prefix with the unit. For example, a distance of 12,509 meters could be written as 12.509 kilometers (12.509 km). Similarly, a mass of 0.000000421 grams could be written as 0.421 micrograms (0.421 µg) or 421 nanograms (421 ng).
Scientific hand calculators offer display modes and entry functions tailored for the purpose of expressing results and accepting input using standard metric prefix power-of-ten notation. Consider the following photograph of a Texas Instruments model TI-30X IIS calculator computing the current through a 4700 Ohm resistor energized by a 5 Volt source (using Ohm’s Law, $I = \frac{V}{R}$):

When the calculator has been set to the *Engineering* display mode, all results appear with a $\times 10^x$ label where $x$ is an integer multiple of three to match with the metric prefixes listed previously. Here, the quotient $\frac{5}{4700}$ results in $1.0638 \times 10^{-3}$, or 1.0638 milliAmperes (1.0638 mA).

A button marked EE (sometimes $\times 10^x$ or simply $E$) allows entry of numerical values in either scientific notation or “engineering” form. Here, the same current calculation is performed entering the 4700 Ohm resistor’s value as 4.7 kiloOhms (4.7 kΩ):
Another example shows the calculation of power dissipated by a 4 Ω resistor passing a current of 2.8 mA, using Joule’s Law \( P = I^2R \) and once again making use of the EE button when entering the current value:

Here, the resulting power dissipation is 31.36 microWatts (31.36 µW).

Some hand calculators provide the Normal/Scientific/Engineering display setting as an option within a general display menu, while others (like the TI-30X IIS) allow faster access by making those settings accessible directly on the front. Note the SCI/ENG function visible above one of the buttons in these photographs.

Engineering mode requires a knowledge of the powers-of-ten associated with common metric prefixes, but once those prefix multipliers are memorized it becomes very valuable in that it saves you from having to track numerical values with many leading or trailing zeroes. For example, without Engineering mode set, our calculated current of 1.0638 mA would be displayed by the calculator as 0.001638. It is surprisingly easy to mis-read sequential characters in a digital display, and so this value might easily be mistaken as 0.01638 (16.38 mA) or 0.0001638 (0.1638 mA or 163.8 µA).
5.3 Resistor labeling

A popular method of labeling resistor values utilizes colored bands\(^2\) to represent resistance values and tolerances. An example of this resistor style appears in the following photograph:

Four-band resistors are the most popular of the banded style, with the first and second bands representing significant digits, the third band representing a power-of-ten multiplier, and the fourth band representing tolerance.

<table>
<thead>
<tr>
<th>Band color</th>
<th>First digit value</th>
<th>Second digit value</th>
<th>Multiplier</th>
<th>Tolerance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Black</td>
<td>0</td>
<td>0</td>
<td>10(^0)</td>
<td>± 1%</td>
</tr>
<tr>
<td>Brown</td>
<td>1</td>
<td>1</td>
<td>10(^1)</td>
<td>± 2%</td>
</tr>
<tr>
<td>Red</td>
<td>2</td>
<td>2</td>
<td>10(^2)</td>
<td>± 2%</td>
</tr>
<tr>
<td>Orange</td>
<td>3</td>
<td>3</td>
<td>10(^3)</td>
<td>± 0.05%</td>
</tr>
<tr>
<td>Yellow</td>
<td>4</td>
<td>4</td>
<td>10(^4)</td>
<td>± 0.02%</td>
</tr>
<tr>
<td>Green</td>
<td>5</td>
<td>5</td>
<td>10(^5)</td>
<td>± 0.5%</td>
</tr>
<tr>
<td>Blue</td>
<td>6</td>
<td>6</td>
<td>10(^6)</td>
<td>± 0.25%</td>
</tr>
<tr>
<td>Violet</td>
<td>7</td>
<td>7</td>
<td>10(^7)</td>
<td>± 0.1%</td>
</tr>
<tr>
<td>Grey</td>
<td>8</td>
<td>8</td>
<td>10(^8)</td>
<td>± 0.01%</td>
</tr>
<tr>
<td>White</td>
<td>9</td>
<td>9</td>
<td>10(^9)</td>
<td></td>
</tr>
<tr>
<td>Gold</td>
<td></td>
<td></td>
<td>10(^{-1})</td>
<td>± 5%</td>
</tr>
<tr>
<td>Silver</td>
<td></td>
<td></td>
<td>10(^{-2})</td>
<td>± 10%</td>
</tr>
<tr>
<td>None</td>
<td></td>
<td></td>
<td></td>
<td>± 20%</td>
</tr>
</tbody>
</table>

Applying this color code to the resistor shown above, we see that yellow = 4, violet = 7, red = 2, and silver = ± 10%. Therefore, this resistor’s value is 47 \(\times\) 10\(^2\) Ohms (i.e. 4700 Ω) plus or minus 10%, which means it could be as low as 4230 Ω or as high as 5170 Ω.

Precision resistors require more significant digits to precisely quantify their values, and so use a five-band code where the first three bands represent digits, the fourth band represents the power-of-ten multiplier, and the fifth band represents tolerance. Both the four-band and five-band color codes are standardized as part of the international standard IEC 60062.

\(^2\)A useful mnemonic for associating these colors with decimal digits 0 through 9 and the percentages 5-10-20% is as follows: “Better Be Right Or Your Great Big Venture Goes Wrong. Get Started Now.”.
Many modern resistors are labeled with text rather than color-coded bands. For these resistors, a letter is used in place of a decimal point, the letter either being R (unit), K (kilo), or M (mega). This is commonly referred to as the RKM code. Some labeling examples are shown in the following list:

- R82 = 0.82 Ω
- 3R9 = 3.9 Ω
- 47R = 47 Ω
- 560R = 560 Ω
- 3K3 = 3.3 kΩ
- 27K = 27 kΩ
- 100K = 100 kΩ
- 2M2 = 2.2 MΩ
- 15M = 15 MΩ

Tolerances for text-labeled resistors use the following letter codes, the tolerance letter always being the last character of the resistor’s text label:

<table>
<thead>
<tr>
<th>Letter code</th>
<th>Tolerance</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>± 0.1%</td>
</tr>
<tr>
<td>C</td>
<td>± 0.25%</td>
</tr>
<tr>
<td>D</td>
<td>± 0.5%</td>
</tr>
<tr>
<td>F</td>
<td>± 1%</td>
</tr>
<tr>
<td>G</td>
<td>± 2%</td>
</tr>
<tr>
<td>J</td>
<td>± 5%</td>
</tr>
<tr>
<td>K</td>
<td>± 10%</td>
</tr>
<tr>
<td>M</td>
<td>± 20%</td>
</tr>
</tbody>
</table>

For example, a resistor labeled 6K8J would be 6.8 kΩ ± 5%.

Surface-mount device (SMD) resistors are often labeled with numbers only, either a three-digit numerical code or a four-digit numerical code. These codes follow the same pattern as four- and five-band color codes (except with no specified tolerance, just digit-digit-multiplier or digit-digit-digit-multiplier). For example, 473 would be 47 kΩ and 2711 would be 2.71 kΩ.
A powerful tool for mathematical modeling is text-based computer programming. This is where you type coded commands in text form which the computer is able to interpret. Many different text-based languages exist for this purpose, but we will focus here on just two of them, C++ and Python.
6.1 Programming in C++

One of the more popular text-based computer programming languages is called C++. This is a compiled language, which means you must create a plain-text file containing C++ code using a program called a text editor, then execute a software application called a compiler to translate your “source code” into instructions directly understandable to the computer. Here is an example of “source code” for a very simple C++ program intended to perform some basic arithmetic operations and print the results to the computer’s console:

```cpp
#include <iostream>
using namespace std;

int main (void)
{
    float x, y;
    x = 200;
    y = -560.5;

cout << "This simple program performs basic arithmetic on" << endl;
cout << "the two numbers " << x << " and " << y << " and then" << endl;
cout << "displays the results on the computer's console." << endl;

cout << endl;
cout << "Sum = " << x + y << endl;
cout << "Difference = " << x - y << endl;
cout << "Product = " << x * y << endl;
cout << "Quotient of " << x / y << endl;

    return 0;
}
```

Computer languages such as C++ are designed to make sense when read by human programmers. The general order of execution is left-to-right, top-to-bottom just the same as reading any text document written in English. Blank lines, indentation, and other “whitespace” is largely irrelevant in C++ code, and is included only to make the code more pleasing\(^1\) to view.

\(^1\) Although not included in this example, comments preceded by double-forward slash characters (//) may be added to source code as well to provide explanations of what the code is supposed to do, for the benefit of anyone reading it. The compiler application will ignore all comments.
Let’s examine the C++ source code to explain what it means:

- `#include <iostream>` and `using namespace std;` are set-up instructions to the compiler giving it some context in which to interpret your code. The code specific to your task is located between the brace symbols (`{` and `}`), often referred to as “curly-braces”).

- `int main (void)` labels the “Main” function for the computer: the instructions within this function (lying between the `{` and `}` symbols) it will be commanded to execute. Every complete C++ program contains a `main` function at minimum, and often additional functions as well, but the `main` function is where execution always begins. The `int` declares this function will return an `integer` number value when complete, which helps to explain the purpose of the `return 0;` statement at the end of the `main` function: providing a numerical value of zero at the program’s completion as promised by `int`. This returned value is rather incidental to our purpose here, but it is fairly standard practice in C++ programming.

- Grouping symbols such as (parentheses) and `{braces}` abound in C, C++, and other languages (e.g. Java). Parentheses typically group data to be processed by a function, called `arguments` to that function. Braces surround lines of executable code belonging to a particular function.

- The `float` declaration reserves places in the computer’s memory for two `floating-point` variables, in this case the variables’ names being `x` and `y`. In most text-based programming languages, variables may be named by single letters or by combinations of letters (e.g. `xyz` would be a single variable).

- The next two lines assign numerical values to the two variables. Note how each line terminates with a semicolon character (`;`) and how this pattern holds true for most of the lines in this program. In C++ semicolons are analogous to periods at the ends of English sentences. This demarcation of each line’s end is necessary because C++ ignores whitespace on the page and doesn’t “know” otherwise where one line ends and another begins.

- All the other instructions take the form of a `cout` command which prints characters to the “standard output” stream of the computer, which in this case will be text displayed on the console. The double-less-than symbols (`<<`) show data being sent toward the `cout` command. Note how verbatim text is enclosed in quotation marks, while variables such as `x` or mathematical expressions such as `x - y` are not enclosed in quotations because we want the computer to display the numerical values represented, not the literal text.

- Standard arithmetic operations (add, subtract, multiply, divide) are represented as `+`, `-`, `*`, and `/`, respectively.

- The `endl` found at the end of every `cout` statement marks the end of a line of text printed to the computer’s console display. If not for these `endl` inclusions, the displayed text would resemble a run-on sentence rather than a paragraph. Note the `cout << endl;` line, which does nothing but create a blank line on the screen, for no reason other than esthetics.
After saving this source code text to a file with its own name (e.g. myprogram.cpp), you would then compile the source code into an executable file which the computer may then run. If you are using a console-based compiler such as GCC (very popular within variants of the Unix operating system\(^2\), such as Linux and Apple’s OS X), you would type the following command and press the Enter key:

```
g++ -o myprogram.exe myprogram.cpp
```

This command instructs the GCC compiler to take your source code (myprogram.cpp) and create with it an executable file named myprogram.exe. Simply typing ./myprogram.exe at the command-line will then execute your program:

```
./myprogram.exe
```

If you are using a graphic-based C++ development system such as Microsoft Visual Studio\(^3\), you may simply create a new console application “project” using this software, then paste or type your code into the example template appearing in the editor window, and finally run your application to test its output.

As this program runs, it displays the following text to the console:

```
This simple program performs basic arithmetic on the two numbers 200 and -560.5 and then displays the results on the computer's console.

Sum = -360.5  
Difference = 760.5  
Product = -112100  
Quotient of -0.356824
```

As crude as this example program is, it serves the purpose of showing how easy it is to write and execute simple programs in a computer using the C++ language. As you encounter C++ example programs (shown as source code) in any of these modules, feel free to directly copy-and-paste the source code text into a text editor's screen, then follow the rest of the instructions given here (i.e. save to a file, compile, and finally run your program). You will find that it is generally easier to

\(^2\) A very functional option for users of Microsoft Windows is called Cygwin, which provides a Unix-like console environment complete with all the customary utility applications such as GCC!

\(^3\) Using Microsoft Visual Studio community version 2017 at the time of this writing to test this example, here are the steps I needed to follow in order to successfully compile and run a simple program such as this: (1) Start up Visual Studio and select the option to create a New Project; (2) Select the Windows Console Application template, as this will perform necessary set-up steps to generate a console-based program which will save you time and effort as well as avoid simple errors of omission; (3) When the editing screen appears, type or paste the C++ code within the `main()` function provided in the template, deleting the “Hello World” `cout` line that came with the template; (4) Type or paste any preprocessor directives (e.g. `#include` statements, `namespace` statements) necessary for your code that did not come with the template; (5) Lastly, under the Debug drop-down menu choose either Start Debugging (F5 hot-key) or Start Without Debugging (Ctrl-F5 hotkeys) to compile (“Build”) and run your new program. Upon execution a console window will appear showing the output of your program.
learn computer programming by closely examining others’ example programs and modifying them than it is to write your own programs starting from a blank screen.
6.2 Programming in Python

Another text-based computer programming language called Python allows you to type instructions at a terminal prompt and receive immediate results without having to compile that code. This is because Python is an interpreted language: a software application called an interpreter reads your source code, translates it into computer-understandable instructions, and then executes those instructions in one step.

The following shows what happens on my personal computer when I start up the Python interpreter on my personal computer, by typing `python3` and pressing the Enter key:

Python 3.7.2 (default, Feb 19 2019, 18:15:18)
GCC 4.1.2 on linux
Type "help", "copyright", "credits" or "license" for more information.
```
>>> 
```

The `>>>` symbols represent the prompt within the Python interpreter “shell”, signifying readiness to accept Python commands entered by the user.

Shown here is an example of the same arithmetic operations performed on the same quantities, using a Python interpreter. All lines shown preceded by the `>>>` prompt are entries typed by the human programmer, and all lines shown without the `>>>` prompt are responses from the Python interpreter software:

```python
>>> x = 200
>>> y = -560.5
>>> x + y
-360.5
>>> x - y
760.5
>>> x * y
-112100.0
>>> x / y
-0.35682426404995538
>>> quit()
```

---

4 Using version 3 of Python, which is the latest at the time of this writing.
6.2. PROGRAMMING IN PYTHON

More advanced mathematical functions are accessible in Python by first entering the line
\texttt{from math import *} which “imports” these functions from Python’s math \textit{library} (with functions
identical to those available for the C programming language, and included on any computer with
Python installed). Some examples show some of these functions in use, demonstrating how the
Python interpreter may be used as a scientific calculator:

\begin{verbatim}
>>> from math import *
>>> sin(30.0)
-0.98803162409286183
>>> sin(radians(30.0))
0.49999999999999994
>>> pow(2.0, 5.0)
32.0
>>> log10(10000.0)
4.0
>>> e
2.7182818284590451
>>> pi
3.1415926535897931
>>> log(pow(e,6.0))
6.0
>>> asin(0.7071068)
0.78539819000368838
>>> degrees(asin(0.7071068))
45.000001524425265
>>> quit()
\end{verbatim}

Note how trigonometric functions assume angles expressed in \textit{radians} rather than \textit{degrees}, and
how Python provides convenient functions for translating between the two. Logarithms assume a
base of $e$ unless otherwise stated (e.g. the $\log_{10}$ function for common logarithms).

The interpreted (versus compiled) nature of Python, as well as its relatively simple syntax, makes
it a good choice as a person's first programming language. For complex applications, interpreted
languages such as Python execute slower than compiled languages such as C++, but for the very
simple examples used in these learning modules speed is not a concern.
Another Python math library is `cmath`, giving Python the ability to perform arithmetic on complex numbers. This is very useful for AC circuit analysis using phasors\(^5\) as shown in the following example. Here we see Python’s interpreter used as a scientific calculator to show series and parallel impedances of a resistor, capacitor, and inductor in a 60 Hz AC circuit:

```python
>>> from math import *
>>> from cmath import *
>>> r = complex(400,0)
>>> f = 60.0
>>> xc = 1/(2*pi*f*4.7e-6)
>>> zc = complex(0,-xc)
>>> xl = 2*pi*f*1.0
>>> zl = complex(0,xl)
>>> r + zc + zl
(400-187.38811239154882j)
>>> 1/(1/r + 1/zc + 1/zl)
(355.837695813625+125.35793777619385j)
>>> polar(r + zc + zl)
(441.717448903332, -0.4381072059213295)
>>> abs(r + zc + zl)
441.717448903332
>>> phase(r + zc + zl)
-0.4381072059213295
>>> degrees(phase(r + zc + zl))
-25.10169387356105
```

When entering a value in rectangular form, we use the `complex()` function where the arguments are the real and imaginary quantities, respectively. If we had opted to enter the impedance values in polar form, we would have used the `rect()` function where the first argument is the magnitude and the second argument is the angle in radians. For example, we could have set the capacitor’s impedance \(zc\) as \(\angle -90^\circ\) with the command `zc = rect(xc,radians(-90))` rather than with the command `zc = complex(0,-xc)` and it would have worked the same.

Note how Python defaults to rectangular form for complex quantities. Here we defined a 400 Ohm resistance as a complex value in rectangular form \((400 +j0 \, \Omega)\), then computed capacitive and inductive reactances at 60 Hz and defined each of those as complex (phasor) values \((0 - jX_c \, \Omega\) and \(0 + jX_l \, \Omega\), respectively). After that we computed total impedance in series, then total impedance in parallel. Polar-form representation was then shown for the series impedance \((441.717 \, \Omega \angle -25.102^\circ)\).

Note the use of different functions to show the polar-form series impedance value: `polar()` takes the complex quantity and returns its polar magnitude and phase angle in radians; `abs()` returns just the polar magnitude; `phase()` returns just the polar angle, once again in radians. To find the polar phase angle in degrees, we nest the `degrees()` and `phase()` functions together.

The utility of Python’s interpreter environment as a scientific calculator should be clear from these examples. Not only does it offer a powerful array of mathematical functions, but also unlimited

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\(^5\)A “phasor” is a voltage, current, or impedance represented as a complex number, either in rectangular or polar form.
6.2. PROGRAMMING IN PYTHON

assignment of variables as well as a convenient text record\(^6\) of all calculations performed which may be easily copied and pasted into a text document for archival.

It is also possible to save a set of Python commands to a text file using a text editor application, and then instruct the Python interpreter to execute it at once rather than having to type it line-by-line in the interpreter’s shell. For example, consider the following Python program, saved under the filename `myprogram.py`:

\begin{verbatim}
x = 200
y = -560.5
print("Sum")
print(x + y)

print("Difference")
print(x - y)

print("Product")
print(x * y)

print("Quotient")
print(x / y)
\end{verbatim}

As with C++, the interpreter will read this source code from left-to-right, top-to-bottom, just the same as you or I would read a document written in English. Interestingly, whitespace \textit{is} significant in the Python language (unlike C++), but this simple example program makes no use of that.

To execute this Python program, I would need to type `python myprogram.py` and then press the Enter key at my computer console’s prompt, at which point it would display the following result:

\begin{verbatim}
Sum
-360.5
Difference
760.5
Product
-112100.0
Quotient
-0.35682426405
\end{verbatim}

As you can see, syntax within the Python programming language is simpler than C++, which is one reason why it is often a preferred language for beginning programmers.

\(^6\)Like many command-line computing environments, Python’s interpreter supports “up-arrow” recall of previous entries. This allows quick recall of previously typed commands for editing and re-evaluation.
If you are interested in learning more about computer programming in any language, you will find a wide variety of books and free tutorials available on those subjects. Otherwise, feel free to learn by the examples presented in these modules.
6.3 Modeling a parallel circuit using C++

Here is an example C++ program intended to display a crude representation of a three-resistor parallel circuit and then calculate all relevant voltages, currents, and powers:

```c++
#include <iostream>
#include <cmath>
using namespace std;

int main (void)
{
    float i1, r1, r2, r3, rtotal, v, ir1, ir2, ir3, pr1, pr2, pr3, ptotal;

    i1 = 90e-3;
    r1 = 2.2e3;
    r2 = 710;
    r3 = 1e3;

    cout << "| I1 R1 R2 R3 |
    ++-------+++---+
    | Amperes |
    | Ohms |
    | Ohms |
    ++-------+++---+

    rtotal = 1 / ((1 / r1) + (1 / r2) + (1 / r3));
    v = i1 * rtotal;

    cout << "V = " << v << " Volts" << endl;

    ir1 = v / r1;
    ir2 = v / r2;
    ir3 = v / r3;

    cout << "IR1 = " << ir1 << " Amperes" << endl;
    cout << "IR2 = " << ir2 << " Amperes" << endl;
    cout << "IR3 = " << ir3 << " Amperes" << endl;

    pr1 = pow(ir1,2) * r1;
    pr2 = pow(ir2,2) * r2;
    pr3 = pow(ir3,2) * r3;
    ptotal = pr1 + pr2 + pr3;

    cout << "PTotal = " << ptotal << " Watts" << endl;
}
```
pr2 = pow(ir2,2) * r2;
pr3 = pow(ir3,2) * r3;
ptotal = pr1 + pr2 + pr3;

    cout << "PR1 = " << pr1 << " Watts" << endl;
cout << "PR2 = " << pr2 << " Watts" << endl;
cout << "PR3 = " << pr3 << " Watts" << endl;
cout << "PTOTAL = " << ptotal << " Watts" << endl;

    return 0;
}

When compiled and executed, this program generates the following output:

+--------+
|        |
| I1     |
| R1     |
| R2     |
| R3     |
+--------+
I1 = 0.09 Amperes
R1 = 2200 Ohms
R2 = 710 Ohms
R3 = 1000 Ohms
V = 31.4356 Volts
IR1 = 0.0142889 Amperes
IR2 = 0.0442755 Amperes
IR3 = 0.0314356 Amperes
PR1 = 0.44918 Watts
PR2 = 1.39183 Watts
PR3 = 0.988197 Watts
PTOTAL = 2.8292 Watts
Chapter 7

Questions

This learning module, along with all others in the ModEL collection, is designed to be used in an inverted instructional environment where students independently read\(^1\) the tutorials and attempt to answer questions on their own \textit{prior} to the instructor’s interaction with them. In place of lecture\(^2\), the instructor engages with students in Socratic-style dialogue, probing and challenging their understanding of the subject matter through inquiry.

Answers are not provided for questions within this chapter, and this is by design. Solved problems may be found in the Tutorial and Derivation chapters, instead. The goal here is \textit{independence}, and this requires students to be challenged in ways where others cannot think for them. Remember that you always have the tools of \textit{experimentation} and \textit{computer simulation} (e.g. SPICE) to explore concepts!

The following lists contain ideas for Socratic-style questions and challenges. Upon inspection, one will notice a strong theme of \textit{metacognition} within these statements: they are designed to foster a regular habit of examining one’s own thoughts as a means toward clearer thinking. As such these sample questions are useful both for instructor-led discussions as well as for self-study.

\(^1\)Technical reading is an essential academic skill for any technical practitioner to possess for the simple reason that the most comprehensive, accurate, and useful information to be found for developing technical competence is in textual form. Technical careers in general are characterized by the need for continuous learning to remain current with standards and technology, and therefore any technical practitioner who cannot read well is handicapped in their professional development. An excellent resource for educators on improving students’ reading prowess through intentional effort and strategy is the book \textit{Reading For Understanding – How Reading Apprenticeship Improves Disciplinary Learning in Secondary and College Classrooms} by Ruth Schoenbach, Cynthia Greenleaf, and Lynn Murphy.

\(^2\)Lecture is popular as a teaching method because it is easy to implement: any reasonably articulate subject matter expert can talk to students, even with little preparation. However, it is also quite problematic. A good lecture always makes complicated concepts seem easier than they are, which is bad for students because it instills a false sense of confidence in their own understanding; reading and re-articulation requires more cognitive effort and serves to verify comprehension. A culture of teaching-by-lecture fosters a debilitating dependence upon direct personal instruction, whereas the challenges of modern life demand independent and critical thought made possible only by gathering information and perspectives from afar. Information presented in a lecture is ephemeral, easily lost to failures of memory and dictation; text is forever, and may be referenced at any time.
CHAPTER 7. QUESTIONS

General challenges following tutorial reading

- **Summarize** as much of the text as you can in one paragraph of your own words. A helpful strategy is to explain ideas as you would for an **intelligent child**: as simple as you can without compromising too much accuracy.

- **Simplify** a particular section of the text, for example a paragraph or even a single sentence, so as to capture the same fundamental idea in fewer words.

- Where did the text **make the most sense** to you? What was it about the text’s presentation that made it clear?

- Identify where it might be easy for someone to **misunderstand the text**, and explain why you think it could be confusing.

- Identify any **new concept(s)** presented in the text, and explain in your own words.

- Identify any **familiar concept(s)** such as physical laws or principles applied or referenced in the text.

- Devise a **proof of concept** experiment demonstrating an important principle, physical law, or technical innovation represented in the text.

- Devise an experiment to **disprove** a plausible misconception.

- Did the text reveal any **misconceptions** you might have harbored? If so, describe the misconception(s) and the reason(s) why you now know them to be incorrect.

- Describe any useful **problem-solving strategies** applied in the text.

- **Devise a question** of your own to challenge a reader’s comprehension of the text.
General follow-up challenges for assigned problems

• Identify where any fundamental laws or principles apply to the solution of this problem, especially before applying any mathematical techniques.

• Devise a thought experiment to explore the characteristics of the problem scenario, applying known laws and principles to mentally model its behavior.

• Describe in detail your own strategy for solving this problem. How did you identify and organized the given information? Did you sketch any diagrams to help frame the problem?

• Is there more than one way to solve this problem? Which method seems best to you?

• Show the work you did in solving this problem, even if the solution is incomplete or incorrect.

• What would you say was the most challenging part of this problem, and why was it so?

• Was any important information missing from the problem which you had to research or recall?

• Was there any extraneous information presented within this problem? If so, what was it and why did it not matter?

• Examine someone else’s solution to identify where they applied fundamental laws or principles.

• Simplify the problem from its given form and show how to solve this simpler version of it. Examples include eliminating certain variables or conditions, altering values to simpler (usually whole) numbers, applying a limiting case (i.e. altering a variable to some extreme or ultimate value).

• For quantitative problems, identify the real-world meaning of all intermediate calculations: their units of measurement, where they fit into the scenario at hand. Annotate any diagrams or illustrations with these calculated values.

• For quantitative problems, try approaching it qualitatively instead, thinking in terms of “increase” and “decrease” rather than definite values.

• For qualitative problems, try approaching it quantitatively instead, proposing simple numerical values for the variables.

• Were there any assumptions you made while solving this problem? Would your solution change if one of those assumptions were altered?

• Identify where it would be easy for someone to go astray in attempting to solve this problem.

• Formulate your own problem based on what you learned solving this one.

General follow-up challenges for experiments or projects

• In what way(s) was this experiment or project easy to complete?

• Identify some of the challenges you faced in completing this experiment or project.
• Show how thorough documentation assisted in the completion of this experiment or project.
• Which fundamental laws or principles are key to this system’s function?
• Identify any way(s) in which one might obtain false or otherwise misleading measurements from test equipment in this system.
• What will happen if (component X) fails (open/shorted/etc.)?
• What would have to occur to make this system unsafe?
7.1 Conceptual reasoning

These questions are designed to stimulate your analytic and synthetic thinking. In a Socratic discussion with your instructor, the goal is for these questions to prompt an extended dialogue where assumptions are revealed, conclusions are tested, and understanding is sharpened. Your instructor may also pose additional questions based on those assigned, in order to further probe and refine your conceptual understanding.

Questions that follow are presented to challenge and probe your understanding of various concepts presented in the tutorial. These questions are intended to serve as a guide for the Socratic dialogue between yourself and the instructor. Your instructor’s task is to ensure you have a sound grasp of these concepts, and the questions contained in this document are merely a means to this end. Your instructor may, at his or her discretion, alter or substitute questions for the benefit of tailoring the discussion to each student’s needs. The only absolute requirement is that each student is challenged and assessed at a level equal to or greater than that represented by the documented questions.

It is far more important that you convey your reasoning than it is to simply convey a correct answer. For this reason, you should refrain from researching other information sources to answer questions. What matters here is that you are doing the thinking. If the answer is incorrect, your instructor will work with you to correct it through proper reasoning. A correct answer without an adequate explanation of how you derived that answer is unacceptable, as it does not aid the learning or assessment process.

You will note a conspicuous lack of answers given for these conceptual questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your conceptual answers, where applicable, is to use circuit simulation software to explore the effects of changes made to circuits. For example, if one of these conceptual questions challenges you to predict the effects of altering some component parameter in a circuit, you may check the validity of your work by simulating that same parameter change within software and seeing if the results agree.

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3Analytical thinking involves the “disassembly” of an idea into its constituent parts, analogous to dissection. Synthetic thinking involves the “assembly” of a new idea comprised of multiple concepts, analogous to construction. Both activities are high-level cognitive skills, extremely important for effective problem-solving, necessitating frequent challenge and regular practice to fully develop.
7.1.1 Reading outline and reflections

“Reading maketh a full man; conference a ready man; and writing an exact man” – Francis Bacon

Francis Bacon’s advice is a blueprint for effective education: reading provides the learner with knowledge, writing focuses the learner’s thoughts, and critical dialogue equips the learner to confidently communicate and apply their learning. Independent acquisition and application of knowledge is a powerful skill, well worth the effort to cultivate. To this end, students should read these educational resources closely, write their own outline and reflections on the reading, and discuss in detail their findings with classmates and instructor(s). You should be able to do all of the following after reading any instructional text:

- **√** Briefly OUTLINE THE TEXT, as though you were writing a detailed Table of Contents. Feel free to rearrange the order if it makes more sense that way. Prepare to articulate these points in detail and to answer questions from your classmates and instructor. Outlining is a good self-test of thorough reading because you cannot outline what you have not read or do not comprehend.

- **√** Demonstrate ACTIVE READING STRATEGIES, including verbalizing your impressions as you read, simplifying long passages to convey the same ideas using fewer words, annotating text and illustrations with your own interpretations, working through mathematical examples shown in the text, cross-referencing passages with relevant illustrations and/or other passages, identifying problem-solving strategies applied by the author, etc. Technical reading is a special case of problem-solving, and so these strategies work precisely because they help solve any problem: paying attention to your own thoughts (metacognition), eliminating unnecessary complexities, identifying what makes sense, paying close attention to details, drawing connections between separated facts, and noting the successful strategies of others.

- **√** Identify IMPORTANT THEMES, especially GENERAL LAWS and PRINCIPLES, expounded in the text and express them in the simplest of terms as though you were teaching an intelligent child. This emphasizes connections between related topics and develops your ability to communicate complex ideas to anyone.

- **√** Form YOUR OWN QUESTIONS based on the reading, and then pose them to your instructor and classmates for their consideration. Anticipate both correct and incorrect answers, the incorrect answer(s) assuming one or more plausible misconceptions. This helps you view the subject from different perspectives to grasp it more fully.

- **√** Devise EXPERIMENTS to test claims presented in the reading, or to disprove misconceptions. Predict possible outcomes of these experiments, and evaluate their meanings: what result(s) would confirm, and what would constitute disproof? Running mental simulations and evaluating results is essential to scientific and diagnostic reasoning.

- **√** Specifically identify any points you found CONFUSING. The reason for doing this is to help diagnose misconceptions and overcome barriers to learning.
7.1. CONCEPTUAL REASONING

7.1.2 Foundational concepts

Correct analysis and diagnosis of electric circuits begins with a proper understanding of some basic concepts. The following is a list of some important concepts referenced in this module’s full tutorial. Define each of them in your own words, and be prepared to illustrate each of these concepts with a description of a practical example and/or a live demonstration.

- Energy
- Conservation of Energy
- Conservation of Electric Charge
- Voltage
- Conductors versus Insulators
- Resistance
- Current
- Electric circuit
- Electrical source
- Electrical load
- Equipotential points
- Electrically common points
CHAPTER 7. QUESTIONS

Open

Short

Switch

Ohm’s Law

Joule’s Law

Parallel

Bonding

Voltage source

Current source

Properties of parallel circuits

Current divider

Potentiometer
7.1.3 Identifying parallel circuits
Identify which of these circuits is a parallel circuit (there may be more than one shown!)

For every circuit that is not parallel, explain why.

How do component voltages relate to total (source) voltage in a parallel circuit, and why?
How do component currents relate to total (source) current in a parallel circuit, and why?
How do component resistances relate to total resistance in a parallel circuit, and why?

Challenges

• A common practice in industrial control system wiring is to label every wire with a number,
and to use common numbers for wires that are equipotential with each other. Apply this
principle to any of the circuits shown.

• Identify a place where a single fuse might be installed, which would interrupt power to all
loads when it blows.

7.1.4 Identifying parallel points and elements

Identify all labeled points as well as elements (i.e. components) in this circuit that are connected in
parallel with each other):

- If component 5 happens to fail open, will any of the parallel-connected relationships change?
7.1.5 Parallel misconceptions

A very common misconception among new students learning about parallel circuit networks is to try to identify parallel connections by the presence of multiple pathways for current. For example, you may hear a new student say something like, “parallel connections are where current splits and goes different directions.”

Examine the following parallel network, comprised of three switches (all in their open positions), and explain why that false definition of parallel fails:

Examine the following network comprised of five resistors, none of them being in parallel with one another, and explain why that false definition of parallel fails:

Finally, provide an accurate definition for parallel that cannot be mis-applied in either of these networks.

Challenges

- Sketch another circuit or network that serves to reveal the incorrect nature of the false parallel definition (“current splits”).
7.1.6 Battery bank connections

Secondary-cell batteries are electro-chemical devices capable of absorbing and releasing electrical energy, i.e. alternately acting as loads and as sources. They are often used as energy-storage media for off-grid electrical power systems where the dominant energy sources are solar (photovoltaic) and wind power, which of course do not reliably deliver energy at all times of the day.

One battery by itself does not store much energy, and so multiple batteries are typically connected to form a larger “bank” for practical energy storage purposes. Suppose we were designing such a battery bank where each battery was rated at 12.6 Volts and a maximum output current of 75 Amperes. In order to store enough energy to serve our needs, it has been decided that fourteen of these batteries are necessary. Determine how these batteries could be connected together to achieve maximum current for the bank as a whole, and then calculate how much that maximum current would be. A partial schematic diagram has been provided for you below, showing all fourteen secondary-cell batteries and the bank’s output terminals:

![Battery bank with 14 individual batteries](image)

**Challenges**

- With the bank’s batteries wired together for maximum output current, calculate the bank’s nominal output voltage value.
- With the bank’s batteries wired together for maximum output current, what will happen if any one of the batteries fails “open”?
- With the bank’s batteries wired together for maximum output current, what will happen if any one of the batteries fails “shorted”?
7.1.7 Measuring current in a parallel circuit

Identify appropriate ways to connect an ammeter to measure current through each of the three resistors in this parallel circuit, one resistor at a time. Be sure to specify which terminal the meter’s red lead should contact, versus which terminal the meter’s black lead should contact, in order to ensure a positive reading:

Note: multiple correct answers exist for every current measurement in this circuit!

- It is surprisingly easy to blow the fuse inside of an ammeter by connecting it incorrectly to the circuit under test. Identify at least two different ways the ammeter might be connected to this circuit which would blow the fuse.

- Identify which terminals to touch with the ammeter probes to measure the combined current through $R_2$ and $R_3$.

- Predict the effects of resistor $R_1$ failing open.

- Predict the effects of resistor $R_2$ failing open.

- Predict the effects of resistor $R_3$ failing open.

- Predict the effects of resistor $R_1$ failing shorted.

- Predict the effects of resistor $R_2$ failing shorted.

- Predict the effects of resistor $R_3$ failing shorted.
7.1.8 Parallel lamp circuit with switches

Three incandescent lamps are wired in parallel with each other, each lamp having its own control switch wired in-line with it. With all three control switches in the “closed” state (as shown), the three lamps glow at normal brightness:

Describe the effects of opening switch 2, while leaving switches 1 and 3 closed.

Describe the effects of opening switches 1 and 2, while leaving switch 3 closed.

Describe the effects of opening all three switches.

Challenges

- Choose a good location for a fuse to protect all conductors within this circuit from overcurrent.
- Choose a good location for a fuse to protect just those conductors serving lamps B and C from overcurrent.
- Which combination of switch states results in the circuit exhibiting minimum resistance?
- Which combination of switch states results in the circuit exhibiting maximum resistance?
- Identify how a second battery could be installed in the circuit to make the lamps glow brighter than they do now.
7.1. CONCEPTUAL REASONING

7.1.9 Explaining the meaning of calculations

An unfortunate tendency among beginning students in any quantitative discipline is to perform calculations without regard for the real-world meanings of the values, and also to follow mathematical formulae without considering the general principles embodied in each. To ignore concepts while performing calculations is a serious error for a variety of reasons, not the least of which being an increased likelihood of computing results that turn out to be nonsense.

In the spirit of honoring concepts, I present to you a quantitative problem where all the calculations have been done for you, but all variable labels, units, and other identifying data have been stripped away. Your task is to assign proper meaning to each of the numbers, identifying the correct unit of measurement in each case, explaining the significance of each value by describing where it “fits” into the circuit being analyzed, and identifying the general principle employed at each step.

Here is the schematic diagram of the parallel circuit:

```
R_1  390 Ω  R_2  220 Ω  I_{source}  5 mA
```

Here are all the calculations performed in order from first to last:

1. \( \frac{1}{\frac{1}{390} + \frac{1}{220}} = 140.66 \)
2. \((5 \times 10^{-3}) \times (140.66) = 0.7033\)
3. \(\frac{0.7033}{390} = 1.803 \times 10^{-3}\)
4. \(\frac{0.7033}{220} = 3.197 \times 10^{-3}\)
5. \((1.803 \times 10^{-3}) + (3.197 \times 10^{-3}) = 5 \times 10^{-3}\)
6. \((1.803 \times 10^{-3}) \times (0.7033) = 1.268 \times 10^{-3}\)
7. \((3.197 \times 10^{-3}) \times (0.7033) = 2.248 \times 10^{-3}\)
8. \((5 \times 10^{-3})^2 \times (140.66) = 3.516 \times 10^{-3}\)
9. \((1.268 \times 10^{-3}) + (2.248 \times 10^{-3}) = 3.516 \times 10^{-3}\)

Explain what each value means in the circuit, identify its unit of measurement, and identify the general principle used to compute it!
Challenges

- Explain how you can check your own thinking as you solve quantitative problems, to avoid the dilemma of just “crunching numbers” to get an answer.

- For each calculated step shown, identify the physical Law or electric circuit principle being applied.

- Identify any of the calculations shown above whose order could be changed. In other words, can this problem be solved in a different order?

- Identify any of the calculations shown above that are not strictly necessary to the complete analysis of the circuit, explaining why it has any merit at all.

- Do you see any alternative paths to a solution, involving specific calculations not shown above?

- When performing addition or subtraction on physical quantities, all units of measurement must be identical. Identify where you see this principle being applied in the steps above.
7.2 Quantitative reasoning

These questions are designed to stimulate your computational thinking. In a Socratic discussion with your instructor, the goal is for these questions to reveal your mathematical approach(es) to problem-solving so that good technique and sound reasoning may be reinforced. Your instructor may also pose additional questions based on those assigned, in order to observe your problem-solving firsthand.

Mental arithmetic and estimations are strongly encouraged for all calculations, because without these abilities you will be unable to readily detect errors caused by calculator misuse (e.g. keystroke errors).

You will note a conspicuous lack of answers given for these quantitative questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. My advice is to use circuit simulation software such as SPICE to check the correctness of quantitative answers. Refer to those learning modules within this collection focusing on SPICE to see worked examples which you may use directly as practice problems for your own study, and/or as templates you may modify to run your own analyses and generate your own practice problems.

Completely worked example problems found in the Tutorial may also serve as “test cases” for gaining proficiency in the use of circuit simulation software, and then once that proficiency is gained you will never need to rely on an answer key!

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4In other words, set up the circuit simulation software to analyze the same circuit examples found in the Tutorial. If the simulated results match the answers shown in the Tutorial, it confirms the simulation has properly run. If the simulated results disagree with the Tutorial’s answers, something has been set up incorrectly in the simulation software. Using every Tutorial as practice in this way will quickly develop proficiency in the use of circuit simulation software.

5This approach is perfectly in keeping with the instructional philosophy of these learning modules: teaching students to be self-sufficient thinkers. Answer keys can be useful, but it is even more useful to your long-term success to have a set of tools on hand for checking your own work, because once you have left school and are on your own, there will no longer be “answer keys” available for the problems you will have to solve.
CHAPTER 7. QUESTIONS

7.2.1 Miscellaneous physical constants

Note: constants shown in **bold** type are *exact*, not approximations. Values inside of parentheses show one standard deviation ($\sigma$) of uncertainty in the final digits: for example, Avogadro’s number given as $6.02214179(30) \times 10^{23}$ means the center value ($6.02214179 \times 10^{23}$) plus or minus $0.00000030 \times 10^{23}$.

Avogadro’s number ($N_A$) = $6.02214179(30) \times 10^{23}$ per mole ($\text{mol}^{-1}$)

Boltzmann’s constant ($k$) = $1.3806504(24) \times 10^{-23}$ Joules per Kelvin ($\text{J/K}$)

Electronic charge ($e$) = $1.602176487(40) \times 10^{-19}$ Coulomb ($\text{C}$)

Faraday constant ($F$) = $9.64853399(24) \times 10^{4}$ Coulombs per mole ($\text{C/mol}$)

Magnetic permeability of free space ($\mu_0$) = $1.25663706212(19) \times 10^{-6}$ Henrys per meter ($\text{H/m}$)

Electric permittivity of free space ($\varepsilon_0$) = $8.8541878128(13) \times 10^{-12}$ Farads per meter ($\text{F/m}$)

Characteristic impedance of free space ($Z_0$) = $376.730313668(57)$ Ohms ($\text{Ω}$)

Gravitational constant ($G$) = $6.67428(67) \times 10^{-11}$ cubic meters per kilogram-seconds squared ($\text{m}^3/\text{kg-\text{s}^2}$)

Molar gas constant ($R$) = $8.314472(15)$ Joules per mole-Kelvin ($\text{J/mol-K}$) = $0.08205746(14)$ liters-atmospheres per mole-Kelvin

Planck constant ($h$) = $6.62606896(33) \times 10^{-34}$ joule-seconds ($\text{J-s}$)

Stefan-Boltzmann constant ($\sigma$) = $5.670400(40) \times 10^{-8}$ Watts per square meter-Kelvin$^4$ ($\text{W/m}^2\cdot\text{K}^4$)

Speed of light in a vacuum ($c$) = **299792458 meters per second** ($\text{m/s}$) = $186282.4$ miles per second ($\text{mi/s}$)

Note: All constants taken from NIST data “Fundamental Physical Constants – Extensive Listing”, from [http://physics.nist.gov/constans](http://physics.nist.gov/constans), National Institute of Standards and Technology (NIST), 2006; with the exception of the permeability of free space which was taken from NIST’s 2018 CODATA recommended values database.
7.2. INTRODUCTION TO SPREADSHEETS

A powerful computational tool you are encouraged to use in your work is a spreadsheet. Available on most personal computers (e.g. Microsoft Excel), spreadsheet software performs numerical calculations based on number values and formulae entered into cells of a grid. This grid is typically arranged as lettered columns and numbered rows, with each cell of the grid identified by its column/row coordinates (e.g. cell B3, cell A8). Each cell may contain a string of text, a number value, or a mathematical formula. The spreadsheet automatically updates the results of all mathematical formulae whenever the entered number values are changed. This means it is possible to set up a spreadsheet to perform a series of calculations on entered data, and those calculations will be re-done by the computer any time the data points are edited in any way.

For example, the following spreadsheet calculates average speed based on entered values of distance traveled and time elapsed:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance traveled</td>
<td>46.9</td>
<td>Kilometers</td>
<td></td>
</tr>
<tr>
<td>Time elapsed</td>
<td>1.18</td>
<td>Hours</td>
<td></td>
</tr>
<tr>
<td>Average speed</td>
<td>= B1 / B2</td>
<td>km/h</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Text labels contained in cells A1 through A3 and cells C1 through C3 exist solely for readability and are not involved in any calculations. Cell B1 contains a sample distance value while cell B2 contains a sample time value. The formula for computing speed is contained in cell B3. Note how this formula begins with an “equals” symbol (=), references the values for distance and speed by lettered column and numbered row coordinates (B1 and B2), and uses a forward slash symbol for division (/). The coordinates B1 and B2 function as variables\(^6\) would in an algebraic formula.

When this spreadsheet is executed, the numerical value 39.74576 will appear in cell B3 rather than the formula = B1 / B2, because 39.74576 is the computed speed value given 46.9 kilometers traveled over a period of 1.18 hours. If a different numerical value for distance is entered into cell B1 or a different value for time is entered into cell B2, cell B3’s value will automatically update. All you need to do is set up the given values and any formulae into the spreadsheet, and the computer will do all the calculations for you.

Cell B3 may be referenced by other formulae in the spreadsheet if desired, since it is a variable just like the given values contained in B1 and B2. This means it is possible to set up an entire chain of calculations, one dependent on the result of another, in order to arrive at a final value. The arrangement of the given data and formulae need not follow any pattern on the grid, which means you may place them anywhere.

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\(^6\)Spreadsheets may also provide means to attach text labels to cells for use as variable names (Microsoft Excel simply calls these labels “names”), but for simple spreadsheets such as those shown here it’s usually easier just to use the standard coordinate naming for each cell.
Common arithmetic operations available for your use in a spreadsheet include the following:

- Addition (+)
- Subtraction (-)
- Multiplication (*)
- Division (/)
- Powers (^)
- Square roots (sqrt())
- Logarithms (ln(), log10())

Parentheses may be used to ensure proper order of operations within a complex formula. Consider this example of a spreadsheet implementing the quadratic formula, used to solve for roots of a polynomial expression in the form of $ax^2 + bx + c$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>x_1 = (-B4 + sqrt((B4^2) - (4<em>B3</em>B5))) / (2*B3)</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>x_2 = (-B4 - sqrt((B4^2) - (4<em>B3</em>B5))) / (2*B3)</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>a = 9</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>b = 5</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>c = -2</td>
<td>-2</td>
</tr>
</tbody>
</table>

This example is configured to compute roots of the polynomial $9x^2 + 5x - 2$ because the values of 9, 5, and -2 have been inserted into cells B3, B4, and B5, respectively. Once this spreadsheet has been built, though, it may be used to calculate the roots of any second-degree polynomial expression simply by entering the new $a$, $b$, and $c$ coefficients into cells B3 through B5. The numerical values appearing in cells B1 and B2 will be automatically updated by the computer immediately following any changes made to the coefficients.

Modern spreadsheet software offers a bewildering array of mathematical functions you may use in your computations. I recommend you consult the documentation for your particular spreadsheet for information on operations other than those listed here.

Spreadsheet programs, like text-based programming languages, are designed to follow standard order of operations by default. However, my personal preference is to use parentheses even where strictly unnecessary just to make it clear to any other person viewing the formula what the intended order of operations is.

Reviewing some algebra here, a root is a value for $x$ that yields an overall value of zero for the polynomial. For this polynomial ($9x^2 + 5x - 2$) the two roots happen to be $x = 0.269381$ and $x = -0.82494$, with these values displayed in cells B1 and B2, respectively upon execution of the spreadsheet.
Alternatively, one could break up the long quadratic formula into smaller pieces like this:

\[ y = \sqrt{b^2 - 4ac} \quad z = 2a \]

\[ x = \frac{-b \pm y}{z} \]

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>x_1</td>
<td>= (-B4 + C1) / C2</td>
<td>= sqrt((B4^2) - (4<em>B3</em>B5))</td>
</tr>
<tr>
<td>2</td>
<td>x_2</td>
<td>= (-B4 - C1) / C2</td>
<td>= 2*B3</td>
</tr>
<tr>
<td>3</td>
<td>a</td>
<td>= 9</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>b</td>
<td>= 5</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>c</td>
<td>= -2</td>
<td></td>
</tr>
</tbody>
</table>

Note how the square-root term \( y \) is calculated in cell C1, and the denominator term \( z \) in cell C2. This makes the two final formulae (in cells B1 and B2) simpler to interpret. The positioning of all these cells on the grid is completely arbitrary\(^{10}\) – all that matters is that they properly reference each other in the formulae.

Spreadsheets are particularly useful for situations where the same set of calculations representing a circuit or other system must be repeated for different initial conditions. The power of a spreadsheet is that it automates what would otherwise be a tedious set of calculations. One specific application of this is to simulate the effects of various components within a circuit failing with abnormal values (e.g. a shorted resistor simulated by making its value nearly zero; an open resistor simulated by making its value extremely large). Another application is analyzing the behavior of a circuit design given new components that are out of specification, and/or aging components experiencing drift over time.

---

\(^{10}\)My personal preference is to locate all the “given” data in the upper-left cells of the spreadsheet grid (each data point flanked by a sensible name in the cell to the left and units of measurement in the cell to the right as illustrated in the first distance/time spreadsheet example), sometimes coloring them in order to clearly distinguish which cells contain entered data versus which cells contain computed results from formulae. I like to place all formulae in cells below the given data, and try to arrange them in logical order so that anyone examining my spreadsheet will be able to figure out how I constructed a solution. This is a general principle I believe all computer programmers should follow: document and arrange your code to make it easy for other people to learn from it.
7.2.3 Three-source, one-lamp circuit

How much current does the lamp experience in this circuit? Explain your answer.

Also, identify the polarity of the voltage across the lamp (mark with “+” and “−” signs), and sketch arrows showing the direction of current through the lamp.

Challenges

• Describe how to re-connect these three sources to achieve the brightest possible operation of the lamp.

• Describe how to re-connect these three sources to double the amount of current through the lamp.

• Describe how to re-connect these three sources to achieve the dimmest possible operation of the lamp.
7.2. QUANTITATIVE REASONING

7.2.4 VIRP table for a three-resistor parallel circuit

Complete the table of voltage, current, resistance, and power values \((V, I, R, \text{ and } P)\) for the circuit shown:

\[
\begin{array}{ccc}
\text{R}_1 & \text{80 Ω} & \text{R}_2 & \text{1.5 kΩ} & \text{R}_3 & \text{3.3 kΩ} \\
\hline
\text{V} & 14 \text{ V} \\
\text{I} & \\
\text{R} & 780 \text{ Ω} & 1.5 \text{ kΩ} & 3.3 \text{ kΩ} \\
\text{P} & \\
\end{array}
\]

Also, show how you may double-check your work without necessarily using a calculator (i.e. use “mental math” and estimation techniques).

**Challenges**

- Identify the effects of any single resistor failing open.
- Identify the effects of any single resistor failing shorted.
- Identify an alteration one could make to this circuit to increase the total current, without changing anything about the source.
- Identify an alteration one could make to this circuit to decrease resistor \(R_2\)’s voltage.
- Identify an alteration one could make to this circuit to increase resistor \(R_1\)’s voltage.
- Identify which data represented in this table illustrate a general principle of parallel circuits.
- Identify which data represented in this table illustrate the *Law of Energy Conservation* in action.
- Identify which data represented in this table illustrate the *Law of Electric Charge Conservation* in action.
7.2.5  Solving for a parallel branch resistance

A four-resistor parallel network contains a 10 kΩ resistor, a 5.1 kΩ resistor, a 3.9 kΩ resistor, and a fourth resistor with an undecided value. Calculate the necessary resistance value for this fourth resistor to give the entire parallel network a resistance value of 1.3 kΩ.
7.2. QUANTITATIVE REASONING

7.2.6 Current divider circuit

The circuit shown below is commonly called a current divider because it splits the source’s current into multiple, smaller currents of fixed proportion:

Calculate the current division percentage for each resistor (i.e. the proportion of source current that each branch current represents):

- \( I_{R1} \) percentage =
- \( I_{R2} \) percentage =
- \( I_{R3} \) percentage =
- \( I_{R4} \) percentage =

Challenges

- An appropriate problem-solving technique for portions of problems that are qualitative (i.e. no numerical figures) is to incorporate your own numerical values and use quantitative techniques to analyze. Identify a way you could incorporate a definite quantity into this problem to make it easier to solve.
- Predict the effects of resistor \( R_1 \) decreasing in value, on all the current division proportion (percentage) values.
- Predict the effects of resistor \( R_3 \) increasing in value, on all the current division proportion (percentage) values.
- Predict the effects of any one resistor failing open, on all the current division proportion (percentage) values.
7.2.7 Shunt resistor for an ammeter

Ammeters are designed to measure the amount of electric current passing through them, and in order to present minimal burden to the circuit being tested, ammeters are also designed to have as little internal resistance as possible. The following illustration is of an analog ammeter based on a moving coil of wire and a stationary magnet, with its full-scale ("F.S.") current rating and internal coil resistance values given:

\[
\text{F.S.} = 1 \text{ mA} \quad R_{\text{coil}} = 400 \Omega
\]

Analog ammeter

This means the ammeter’s needle will be deflected precisely to the full-scale (fully-right) position with exactly 1 milliAmpere of current passing through the 400 Ohm internal wire coil. In other words, this ammeter has a range of 0 to 1 milliAmpere (0-1 mA), and no more.

One milliAmpere is not a lot of electrical current, and so this ammeter will be of limited use to us in its present form. It would be greatly beneficial to our needs if we could figure out a way to extend the range of this ammeter to be able to measure higher currents. Fortunately, a simple technique exists for this, which requires the connection of a shunt resistor in parallel with the analog ammeter. The purpose of this “shunt” resistor is to bypass a precise fraction of current past the meter unit so that a larger (total) current is necessary to cause full-scale deflection of the needle.

---

11 An “analog” system is one where all quantities are continuous in nature, as opposed to a “digital” system where quantities exist in discrete units. An analog ammeter uses a needle (pointer) over a printed scale, representing current magnitude by how far the needle gets deflected over the range of the scale. Since the needle is free to move in a continuous manner up and down the scale (i.e. there is no minimum amount of motion it is restricted to), the analog ammeter can – at least in theory – represent vanishingly small increments in current. A digital ammeter, by contrast, uses decimal digits to display the measured current, and as such has limited resolution (i.e. a meter with four digits cannot show increments of current smaller than that least-significant digit).

12 The term shunt is a synonym for parallel.
For example, if we wanted to extend the range of this analog ammeter to 0–1 Ampere, we would have to connect a shunt resistor that bypassed 999 mA of current while letting 1 mA go through the analog meter unit at a measured current of 1 A (1000 mA):

\[
F.S. = 1 \text{ mA} \quad R_{coil} = 400 \Omega
\]

Calculate the value of shunt resistor for the range shown (0–1 A), and then calculate the necessary shunt resistor values for the following ranges as well, all assuming the use of the same 0–1 mA, 400 Ω analog ammeter unit:

- 0–5 A range: \( R_{shunt} = \)
- 0–250 mA range: \( R_{shunt} = \)
- 0–100 mA range: \( R_{shunt} = \)

**Challenges**

- What will happen if someone happens to connect this ammeter incorrectly to a circuit and cause current to go the opposite direction through it (i.e. conventional flow in the black test lead and out the red)?
- Predict the effects of the shunt resistor failing open.
- Predict the effects of the shunt resistor failing shorted.
- Explain why no \( R_{shunt} \) value is capable of giving this ammeter a full-scale measurement range of less than 1 mA.
7.2.8 Designing an electric heater

Suppose you need to design an electric heater for an industrial enclosure which will contain moisture-sensitive equipment. The purpose of this heater is to maintain an elevated temperature inside the enclosure so that moisture (i.e. dew) never condenses on this equipment.

A mechanical engineer determines that a heat output of 130 Watts should be sufficient for the task. The enclosure already contains a 25 Watt incandescent lamp that is always energized. Your task is to choose the necessary heating element resistance value for the electric heater such that the total heat dissipated inside this enclosure will be 130 Watts.

Assume an electrical power source of 115 Volts.

Challenges

- Does your calculated heater resistance value represent a minimum or a maximum resistance value in order to maintain a dew-free environment inside the enclosure?

- If a new series of enclosures destined for the European market were being designed, where the line voltage is typically 220 Volts instead of 115 Volts as it is in North America, would the heater need a greater or a lesser resistance value to dissipate the same amount of power?

- Identify any assumptions implicit in this design problem, that may not necessarily be true for a real enclosure housing real equipment in a real environment.
7.2. QUANTITATIVE REASONING

7.2.9 Interpreting a SPICE analysis

A computer program called SPICE was developed in the early 1970’s, whereby a text-based description of an electric circuit (called a netlist) could be entered into a computer, and then computer would be directed to apply fundamental laws of electric circuits to the netlist circuit description according to algorithms coded in SPICE.

Here is a netlist text for a three-resistor parallel circuit powered by a DC voltage source, each of the three resistors equipped with its own ammeter in the form of a zero-voltage source:

* Three-resistor parallel DC circuit
v1 1 0 dc 18
r1 1 2 7200
r2 1 3 4700
r3 1 4 10k
vamm1 2 0 dc 0
vamm2 3 0 dc 0
vamm3 4 0 dc 0
.dc v1 18 18 1
.print dc v(1,2) v(1,3) v(1,4)
.print dc i(vamm1) i(vamm2) i(vamm3)
.print dc i(v1)
.end

One of the fundamental concepts in SPICE programming is that connections between components in the virtual circuit to be analyzed are specified by identifying node numbers. All points in the circuit known to be equipotential must bear the same number. In this particular example, the only nodes in existence are 0 through 4. SPICE always assumes that node 0 is “ground” (i.e. a common point of reference for default voltage measurements).

Early versions\(^\text{13}\) of SPICE provided no means for displaying current through components other than voltage sources, and so three “dummy” voltage sources (each one having a DC voltage of zero Volts) have been included in this circuit as “ammeters” so we will be able to see the calculated value for current through each of the three resistors.

\(^\text{13}\)In the interest of my readers, my goal is to provide netlists compatible with all versions of SPICE, new and legacy, wherever possible.
The following schematic diagram shows one possible interpretation of this SPICE netlist, with each of the nodes numbered:

Identify how each component’s connecting nodes and essential parameters (i.e. values) are specified in the netlist. Based on the what you can identify in this netlist, perform your own calculations using Ohm’s Law to predict all other values in this circuit.

When SPICE (software version 2G6) processes the contents of this “netlist” file, it outputs a text description of the analysis. The following text has been edited for clarity (e.g. blank lines, extraneous characters, and statistical data removed):

```
  v1   v(1,2)   v(1,3)   v(1,4)
1.800E+01 1.800E+01 1.800E+01 1.800E+01

  v1   i(vamm1)   i(vamm2)   i(vamm3)
1.800E+01 2.500E-03 3.830E-03 1.800E-03

  v1   i(v1)
1.800E+01  -8.130E-03
```

Interpret the output of SPICE to the best of your ability. Identify all parameters that you can, and see if the results of the computer’s analysis agree with your own calculations.

Once you become familiar with the analysis data format of SPICE, you will be able to use the “Gallery” of SPICE simulations found in the “SPICE Modeling of Resistor Circuits” module as practice problems for developing your own circuit analysis skills.

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\(^{14}\)A recommended procedure to follow when generating your own SPICE netlist is to start with a schematic diagram, numbered each and every connecting wire in the circuit, with every equipotential point bearing the same node number. Once all nodes have been identified and numbered, it becomes easy to write each component line of the netlist.
7.2. QUANTITATIVE REASONING

Challenges

- SPICE utilizes power-of-ten notation for small and large values. The standard representation of this in a plain-text format is a capital letter E followed by the power value. Identify any such figures in the SPICE output listing, and express them in regular decimal notation.

- Explain the purpose of having a “shunt” resistor in this circuit, and also why its resistance value is so low compared to the other resistor values.

- Modify this SPICE netlist to utilize a voltage source with a value of 6 Volts instead of 24 Volts.

- Modify this SPICE netlist to specify resistor $R_1$’s resistance using scientific (power-of-ten) notation.

- Modify this SPICE netlist to specify resistor $R_1$’s resistance using metric prefix (“kilo”) notation.

7.2.10 Using SPICE to analyze a parallel circuit

A computer program called SPICE was developed in the early 1970’s, whereby a text-based description of an electric circuit (called a netlist) could be entered into a computer, and then computer would be directed to apply fundamental laws of electric circuits to the netlist circuit description according to algorithms coded in SPICE.

Write a netlist and perform a SPICE analysis on the following four-resistor circuit (this particular schematic utilizing European-style component symbols), instructing the computer to calculate current through each of the four resistors, as well as the voltage across the source:

If you are using a legacy version of SPICE such as version 2G6, the only way to instruct the computer to print a current value is through a voltage source. This means you will have to include “dummy” voltage sources (each one set to a voltage value of zero) in your circuit to function as ammeters.

Note: one of the most compelling reasons for beginning students of electronics to learn how to perform computer simulations of simple DC circuits is so they will be able to make their own practice problems for circuit analysis. If you learn to use SPICE as a tool, you will never lack for having practice problems!
7.3 Diagnostic reasoning

These questions are designed to stimulate your deductive and inductive thinking, where you must apply general principles to specific scenarios (deductive) and also derive conclusions about the failed circuit from specific details (inductive). In a Socratic discussion with your instructor, the goal is for these questions to reinforce your recall and use of general circuit principles and also challenge your ability to integrate multiple symptoms into a sensible explanation of what’s wrong in a circuit. Your instructor may also pose additional questions based on those assigned, in order to further challenge and sharpen your diagnostic abilities.

As always, your goal is to fully explain your analysis of each problem. Simply obtaining a correct answer is not good enough – you must also demonstrate sound reasoning in order to successfully complete the assignment. Your instructor’s responsibility is to probe and challenge your understanding of the relevant principles and analytical processes in order to ensure you have a strong foundation upon which to build further understanding.

You will note a conspicuous lack of answers given for these diagnostic questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your diagnostic answers, where applicable, is to use circuit simulation software to explore the effects of faults placed in circuits. For example, if one of these diagnostic questions requires that you predict the effect of an open or a short in a circuit, you may check the validity of your work by simulating that same fault (substituting a very high resistance in place of that component for an open, and substituting a very low resistance for a short) within software and seeing if the results agree.
7.3. DIAGONSTIC REASONING

7.3.1 Interpreting an ammeter measurement

Suppose a technician uses an ammeter to measure 0.00 milliAmperes between two test points in a circuit. Identify which (if any) of the following statements are true, based on this one measurement:

1. Those test points are directly connected together by low resistance (i.e. they are “electrically common” or “shorted” to each other).
2. Those test points are connected together by high resistance.
3. Those test points are connected directly to a voltage source.
4. The circuit’s power source is dead.

Now suppose you happened to know that those two test points in the circuit are supposed to have 33 milliAmperes of current passing between them through the ammeter, which is the normal operating current of the circuit’s only source. Does this information alter any of your conclusions about the 0.00 milliAmpere meter reading? Why or why not?

Challenges

• Sketch a complete diagram of a simple circuit, identifying two points within it that would normally have 33 milliAmperes flowing from one, through the ammeter, to the other but due to a fault that you propose now have no current flowing at all.


7.3.2 Faults in a three-resistor circuit

Examine this schematic diagram of a three-resistor circuit, drawn using European component symbols, and also using a four “Ground” symbols to represent one of the conductive connections to the battery:

A set of removable “jumpers” (J1 through J3) are provided to facilitate connecting an ammeter to measure branch currents in this parallel circuit. A single fuse protects all conductors and components from overcurrent.

Predict how all jumper currents in this circuit will be affected as a result of the following faults. Consider each fault independently (i.e. one at a time, no coincidental faults):

- Resistor $R_1$ fails open:
- Resistor $R_2$ fails open:
- Resistor $R_3$ fails open:
- Solder bridge (short) past resistor $R_1$:
- Solder bridge (short) past resistor $R_2$:
- Solder bridge (short) past resistor $R_3$:

Challenges

- An appropriate problem-solving technique for qualitative (i.e. no numerical figures) problems such as this is to incorporate your own component values and use quantitative techniques to analyze. Identify some “easy” values you could apply to this circuit to convert it from a qualitative problem into a quantitative problem.
- Suppose this circuit were built without a fuse. Would this alteration change any of your predictions for the effects of resistor faults?

\[15\] Here, “ground” does not literally refer to a connection to the Earth, but rather to a conductive surface or wire that is arbitrarily deemed the “reference” point or “ground-level” for electrical potential in this circuit. Any schematic diagram containing multiple “ground” symbols should interpreted as those symbols being connected directly together by a wire.
• Re-draw this circuit without using “ground” symbols.
7.3.3 Problem with a multiple-pump system

A set of electrically-powered water pumps is used to pump water out of the bilge of a large ship. Each of the four pumps is rated at 5 gallons per minute water flow when running, discharging that water overboard through a common pipe equipped with a water flowmeter. A schematic diagram shows how the four pumps (all located in the bilge, at the very bottom of the ship) receive power through four switches located in the engine control room above the bilge. Each of the “box” symbols in this schematic diagram represents a terminal block where wires connect with other wires:

One day there happens to be a major water spill into the ship’s bilge, and the engine room operator on shift turns on all four pumps to pump that water overboard. However, he happens to notice the flowmeter register only 14 gallons per minute with all switches turned “on”, when it normally registers a total flow rate of 20 gallons per minute. The electrician on shift uses a voltmeter to measure voltage across each of the fuses, and obtains 0 Volts for each one while all four switches remain in the “on” state.
Identify the likelihood of each specified fault for this bilge pump system. Consider each fault one at a time (i.e. no coincidental faults), determining whether or not each fault is compatible with all measurements and symptoms in this circuit.

<table>
<thead>
<tr>
<th>Fault</th>
<th>Possible</th>
<th>Impossible</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wire between TB15-5 and DC source failed open</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wire between TB20-1 and TB32-7 failed open</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wire between TB10-18 and TB32-4 failed open</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wire between TB15-1 and TB15-2 failed open</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wire between TB15-2 and TB15-3 failed open</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wire between TB15-3 and TB15-4 failed open</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Switch between TB10-12 and TB10-16 failed open</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wire between TB10-19 and TB32-2 shorted to ground</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fuse F1 blown</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fuse F2 blown</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fuse F3 blown</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fuse F4 blown</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fuse F5 blown</td>
<td></td>
<td></td>
</tr>
<tr>
<td>125 Volt DC source dead</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Challenges**

- For each of the proposed fault, explain *why* you believe it is either possible or impossible.
- Suppose all the switches had been turned “off” when the electrician performed the voltage measurements across each fuse. Would this affect your assessment of possible faults? Why or why not?
- A helpful problem-solving technique when given a circuit in pictorial form is to re-draw that circuit in *schematic* form so that it is easier to analyze. Do so for this circuit, showing the proper placement of all the terminal numbers in the circuit.
- Explain the purpose of fuse F5, given the existence of fuses F1 through F4.
### 7.3.4 Malfunctioning oven

This electric oven has a problem somewhere – with the switch “On” and the plug inserted into a 240 Volt power receptacle, it heats up at a slower rate than usual. The fuse, which is supposed to automatically “open” to stop current if it becomes excessive, is rated at 8 Amperes. Inside the oven’s heating chamber are three separate heater elements rated at 200 Watts, 400 Watts, and 800 Watts individually:

Suppose a technician uses a digital multimeter (DMM) to measure current, by connecting the ammeter across the switch terminals and then turning the switch to the “Off” position. The ammeter reads a current value of 4.2 Amperes.

First, explain why the technician connected the ammeter to the circuit as described. How, exactly, does this procedure ensure the meter will faithfully register the oven’s total current? Is this the only way to measure current in the circuit? If not, how else could the ammeter have been connected into this circuit?

Next, identify the most likely fault in the circuit which could explain the oven’s abnormally slow heating rate.

Finally, describe any subsequent diagnostic steps the technician could take to further isolate the identify and/or location of the fault.

**Challenges**

- What circumstances might lead to the fuse “blowing” open?
Suppose someone told you the fuse’s terminals would be safe to touch with your finger because there should be negligible voltage dropped across an intact fuse. Explain why this is a misguided conclusion.
Chapter 8

Projects and Experiments

The following project and experiment descriptions outline things you can build to help you understand circuits. With any real-world project or experiment there exists the potential for physical harm. *Electricity can be very dangerous in certain circumstances, and you should follow proper safety precautions at all times!*

8.1 Recommended practices

This section outlines some recommended practices for all circuits you design and construct.
8.1.1 Safety first!

Electricity, when passed through the human body, causes uncomfortable sensations and in large enough measures\(^1\) will cause muscles to involuntarily contract. The overriding of your nervous system by the passage of electrical current through your body is particularly dangerous in regard to your heart, which is a vital muscle. Very large amounts of current can produce serious internal burns in addition to all the other effects.

Cardio-pulmonary resuscitation (CPR) is the standard first-aid for any victim of electrical shock. This is a very good skill to acquire if you intend to work with others on dangerous electrical circuits. You should never perform tests or work on such circuits unless someone else is present who is proficient in CPR.

As a general rule, any voltage in excess of 30 Volts poses a definitive electric shock hazard, because beyond this level human skin does not have enough resistance to safely limit current through the body. “Live” work of any kind with circuits over 30 volts should be avoided, and if unavoidable should only be done using electrically insulated tools and other protective equipment (e.g. insulating shoes and gloves). If you are unsure of the hazards, or feel unsafe at any time, stop all work and distance yourself from the circuit!

A policy I strongly recommend for students learning about electricity is to never come into electrical contact\(^2\) with an energized conductor, no matter what the circuit’s voltage\(^3\) level! Enforcing this policy may seem ridiculous when the circuit in question is powered by a single battery smaller than the palm of your hand, but it is precisely this instilled habit which will save a person from bodily harm when working with more dangerous circuits. Experience has taught me that students who learn early on to be careless with safe circuits have a tendency to be careless later with dangerous circuits!

In addition to the electrical hazards of shock and burns, the construction of projects and running of experiments often poses other hazards such as working with hand and power tools, potential

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\(^1\) Professor Charles Dalziel published a research paper in 1961 called “The Deleterious Effects of Electric Shock” detailing the results of electric shock experiments with both human and animal subjects. The threshold of perception for human subjects holding a conductor in their hand was in the range of 1 milliampere of current (less than this for alternating current, and generally less for female subjects than for male). Loss of muscular control was exhibited by half of Dalziel’s subjects at less than 10 milliamperes alternating current. Extreme pain, difficulty breathing, and loss of all muscular control occurred for over 99% of his subjects at direct currents less than 100 milliamperes and alternating currents less than 30 milliamperes. In summary, it doesn’t require much electric current to induce painful and even life-threatening effects in the human body! Your first and best protection against electric shock is maintaining an insulating barrier between your body and the circuit in question, such that current from that circuit will be unable to flow through your body.

\(^2\) By “electrical contact” I mean either directly touching an energized conductor with any part of your body, or indirectly touching it through a conductive tool. The only physical contact you should ever make with an energized conductor is via an electrically insulated tool, for example a screwdriver with an electrically insulated handle, or an insulated test probe for some instrument.

\(^3\) Another reason for consistently enforcing this policy, even on low-voltage circuits, is due to the dangers that even some low-voltage circuits harbor. A single 12 Volt automobile battery, for example, can cause a surprising amount of damage if short-circuited simply due to the high current levels (i.e. very low internal resistance) it is capable of, even though the voltage level is too low to cause a shock through the skin. Mechanics wearing metal rings, for example, are at risk from severe burns if their rings happen to short-circuit such a battery! Furthermore, even when working on circuits that are simply too low-power (low voltage and low current) to cause any bodily harm, touching them while energized can pose a threat to the circuit components themselves. In summary, it generally wise (and always a good habit to build) to “power down” any circuit before making contact between it and your body.
contact with high temperatures, potential chemical exposure, etc. You should never proceed with a project or experiment if you are unaware of proper tool use or lack basic protective measures (e.g. personal protective equipment such as safety glasses) against such hazards.

Some other safety-related practices should be followed as well:

- All power conductors extending outward from the project must be firmly strain-relieved (e.g. “cord grips” used on line power cords), so that an accidental tug or drop will not compromise circuit integrity.

- All electrical connections must be sound and appropriately made (e.g. soldered wire joints rather than twisted-and-taped; terminal blocks rather than solderless breadboards for high-current or high-voltage circuits). Use “touch-safe” terminal connections with recessed metal parts to minimize risk of accidental contact.

- Always provide overcurrent protection in any circuit you build. Always. This may be in the form of a fuse, a circuit breaker, and/or an electronically current-limited power supply.

- Always ensure circuit conductors are rated for more current than the overcurrent protection limit. Always. A fuse does no good if the wire or printed circuit board trace will “blow” before it does!

- Always bond metal enclosures to Earth ground for any line-powered circuit. Always. Ensuring an equipotential state between the enclosure and Earth by making the enclosure electrically common with Earth ground ensures no electric shock can occur simply by one’s body bridging between the Earth and the enclosure.

- Avoid building a high-energy circuit when a low-energy circuit will suffice. For example, I always recommend beginning students power their first DC resistor circuits using small batteries rather than with line-powered DC power supplies. The intrinsic energy limitations of a dry-cell battery make accidents highly unlikely.

- Use line power receptacles that are GFCI (Ground Fault Current Interrupting) to help avoid electric shock from making accidental contact with a “hot” line conductor.

- Always wear eye protection when working with tools or live systems having the potential to eject material into the air. Examples of such activities include soldering, drilling, grinding, cutting, wire stripping, working on or near energized circuits, etc.

- Always use a step-stool or stepladder to reach high places. Never stand on something not designed to support a human load.

- When in doubt, ask an expert. If anything even seems remotely unsafe to you, do not proceed without consulting a trusted person fully knowledgeable in electrical safety.
8.1.2 Other helpful tips

Experience has shown the following practices to be very helpful, especially when students make their own component selections, to ensure the circuits will be well-behaved:

- Avoid resistor values less than 1 kΩ or greater than 100 kΩ, unless such values are definitely necessary\(^4\). Resistances below 1 kΩ may draw excessive current if directly connected to a voltage source of significant magnitude, and may also complicate the task of accurately measuring current since any ammeter’s non-zero resistance inserted in series with a low-value circuit resistor will significantly alter the total resistance and thereby skew the measurement. Resistances above 100 kΩ may complicate the task of measuring voltage since any voltmeter’s finite resistance connected in parallel with a high-value circuit resistor will significantly alter the total resistance and thereby skew the measurement. Similarly, AC circuit impedance values should be between 1 kΩ and 100 kΩ, and for all the same reasons.

- Ensure all electrical connections are low-resistance and physically rugged. For this reason, one should avoid compression splices (e.g. “butt” connectors), solderless breadboards\(^5\), and wires that are simply twisted together.

- Build your circuit with testing in mind. For example, provide convenient connection points for test equipment (e.g. multimeters, oscilloscopes, signal generators, logic probes).

- Design permanent projects with maintenance in mind. The more convenient you make maintenance tasks, the more likely they will get done.

- **Always document and save your work.** Circuits lacking schematic diagrams are more difficult to troubleshoot than documented circuits. Similarly, circuit construction is simpler when a schematic diagram precedes construction. Experimental results are easier to interpret when comprehensively recorded. Consider modern videorecording technology for this purpose where appropriate.

- **Record your steps** when troubleshooting. **Talk to yourself** when solving problems. These simple steps clarify thought and simplify identification of errors.

---

\(^4\) An example of a necessary resistor value much less than 1 kΩ is a **shunt resistor** used to produce a small voltage drop for the purpose of sensing current in a circuit. Such shunt resistors must be low-value in order not to impose an undue load on the rest of the circuit. An example of a necessary resistor value much greater than 100 kΩ is an electrostatic **drain resistor** used to dissipate stored electric charges from body capacitance for the sake of preventing damage to sensitive semiconductor components, while also preventing a path for current that could be dangerous to the person (i.e. shock).

\(^5\) Admittedly, solderless breadboards are very useful for constructing complex electronic circuits with many components, especially DIP-style integrated circuits (ICs), but they tend to give trouble with connection integrity after frequent use. An alternative for projects using low counts of ICs is to solder IC sockets into prototype printed circuit boards (PCBs) and run wires from the soldered pins of the IC sockets to terminal blocks where reliable temporary connections may be made.
8.1. RECOMMENDED PRACTICES

8.1.3 Terminal blocks for circuit construction

Terminal blocks are the standard means for making electric circuit connections in industrial systems. They are also quite useful as a learning tool, and so I highly recommend their use in lieu of solderless breadboards. Terminal blocks provide highly reliable connections capable of withstanding significant voltage and current magnitudes, and they force the builder to think very carefully about component layout which is an important mental practice. Terminal blocks that mount on standard 35 mm DIN rail are made in a wide range of types and sizes, some with built-in disconnecting switches, some with built-in components such as rectifying diodes and fuseholders, all of which facilitate practical circuit construction.

I recommend every student of electricity build their own terminal block array for use in constructing experimental circuits, consisting of several terminal blocks where each block has at least 4 connection points all electrically common to each other and at least one terminal block that is a fuse holder for overcurrent protection. A pair of anchoring blocks hold all terminal blocks securely on the DIN rail, preventing them from sliding off the rail. Each of the terminals should bear a number, starting from 0. An example is shown in the following photograph and illustration:

Screwless terminal blocks (using internal spring clips to clamp wire and component lead ends) are preferred over screw-based terminal blocks, as they reduce assembly and disassembly time, and also minimize repetitive wrist stress from twisting screwdrivers. Some screwless terminal blocks require the use of a special tool to release the spring clip, while others provide buttons for this task which may be pressed using the tip of any suitable tool.

---

6Solderless breadboards are preferable for complicated electronic circuits with multiple integrated “chip” components, but for simpler circuits I find terminal blocks much more practical. An alternative to solderless breadboards for “chip” circuits is to solder chip sockets onto a PCB and then use wires to connect the socket pins to terminal blocks. This also accommodates surface-mount components, which solderless breadboards do not.

7DIN rail is a metal rail designed to serve as a mounting point for a wide range of electrical and electronic devices such as terminal blocks, fuses, circuit breakers, relay sockets, power supplies, data acquisition hardware, etc.

8Sometimes referred to as equipotential, same-potential, or potential distribution terminal blocks.

9The small orange-colored squares seen in the above photograph are buttons for this purpose, and may be actuated by pressing with any tool of suitable size.
The following example shows how such a terminal block array might be used to construct a series-parallel resistor circuit consisting of four resistors and a battery:

Numbering on the terminal blocks provides a very natural translation to SPICE\textsuperscript{10} netlists, where component connections are identified by terminal number:

\begin{verbatim}
* Series-parallel resistor circuit
v1 1 0 dc 6
r1 2 5 7100
r2 5 8 2200
r3 2 8 3300
r4 8 11 4700
rjmp1 1 2 0.01
rjmp2 0 11 0.01
.op
.end
\end{verbatim}

Note the use of “jumper” resistances \texttt{rjmp1} and \texttt{rjmp2} to describe the wire connections between terminals 1 and 2 and between terminals 0 and 11, respectively. Being resistances, SPICE requires a resistance value for each, and here we see they have both been set to an arbitrarily low value of 0.01 Ohm realistic for short pieces of wire.

Listing all components and wires along with their numbered terminals happens to be a useful documentation method for any circuit built on terminal blocks, independent of SPICE. Such a “wiring sequence” may be thought of as a \textit{non-graphical description} of an electric circuit, and is exceptionally easy to follow.

\textsuperscript{10}SPICE is computer software designed to analyze electrical and electronic circuits. Circuits are described for the computer in the form of \textit{netlists} which are text files listing each component type, connection node numbers, and component values.
8.1. RECOMMENDED PRACTICES

An example of a more elaborate terminal block array is shown in the following photograph, with terminal blocks and “ice-cube” style electromechanical relays mounted to DIN rail, which is turn mounted to a perforated subpanel\textsuperscript{11}. This “terminal block board” hosts an array of thirty five undedicated terminal block sections, four SPDT toggle switches, four DPDT “ice-cube” relays, a step-down control power transformer, bridge rectifier and filtering capacitor, and several fuses for overcurrent protection:

Four plastic-bottomed “feet” support the subpanel above the benchtop surface, and an unused section of DIN rail stands ready to accept other components. Safety features include electrical bonding of the AC line power cord’s ground to the metal subpanel (and all metal DIN rails), mechanical strain relief for the power cord to isolate any cord tension from wire connections, clear plastic finger guards covering the transformer’s screw terminals, as well as fused overcurrent protection for the 120 Volt AC line power and the transformer’s 12 Volt AC output. The perforated holes happen to be on $\frac{1}{4}$ inch centers with a diameter suitable for tapping with 6-32 machine screw threads, their presence making it very easy to attach other sections of DIN rail, printed circuit boards, or specialized electrical components directly to the grounded metal subpanel. Such a “terminal block board” is an inexpensive\textsuperscript{12} yet highly flexible means to construct physically robust circuits using industrial wiring practices.

\textsuperscript{11} An electrical \textit{subpanel} is a thin metal plate intended for mounting inside an electrical enclosure. Components are attached to the subpanel, and the subpanel in turn bolts inside the enclosure. Subpanels allow circuit construction outside the confines of the enclosure, which speeds assembly. In this particular usage there is no enclosure, as the subpanel is intended to be used as an open platform for the convenient construction of circuits on a benchtop by students. In essence, this is a modern version of the traditional \textit{breadboard} which was literally a wooden board such as might be used for cutting loaves of bread, but which early electrical and electronic hobbyists used as platforms for the construction of circuits.

\textsuperscript{12} At the time of this writing (2019) the cost to build this board is approximately $250 US dollars.
8.1.4 Conducting experiments

An experiment is an exploratory act, a test performed for the purpose of assessing some proposition or principle. Experiments are the foundation of the scientific method, a process by which careful observation helps guard against errors of speculation. All good experiments begin with an hypothesis, defined by the American Heritage Dictionary of the English Language as:

An assertion subject to verification or proof, as (a) A proposition stated as a basis for argument or reasoning. (b) A premise from which a conclusion is drawn. (c) A conjecture that accounts, within a theory or ideational framework, for a set of facts and that can be used as a basis for further investigation.

Stated plainly, an hypothesis is an educated guess about cause and effect. The correctness of this initial guess matters little, because any well-designed experiment will reveal the truth of the matter. In fact, incorrect hypotheses are often the most valuable because the experiments they engender lead us to surprising discoveries. One of the beautiful aspects of science is that it is more focused on the process of learning than about the status of being correct. In order for an hypothesis to be valid, it must be testable, which means it must be a claim possible to refute given the right data. Hypotheses impossible to critique are useless.

Once an hypothesis has been formulated, an experiment must be designed to test that hypothesis. A well-designed experiment requires careful regulation of all relevant variables, both for personal safety and for prompting the hypothesized results. If the effects of one particular variable are to be tested, the experiment must be run multiple times with different values of (only) that particular variable. The experiment set up with the “baseline” variable set is called the control, while the experiment set up with different value(s) is called the test or experimental.

For some hypotheses a viable alternative to a physical experiment is a computer-simulated experiment or even a thought experiment. Simulations performed on a computer test the hypothesis against the physical laws encoded within the computer simulation software, and are particularly useful for students learning new principles for which simulation software is readily available.

13. Science is more about clarifying our view of the universe through a systematic process of error detection than it is about proving oneself to be right. Some scientists may happen to have large egos – and this may have more to do with the ways in which large-scale scientific research is funded than anything else – but scientific method itself is devoid of ego, and if embraced as a practical philosophy is quite an effective stimulant for humility. Within the education system, scientific method is particularly valuable for helping students break free of the crippling fear of being wrong. So much emphasis is placed in formal education on assessing correct retention of facts that many students are fearful of saying or doing anything that might be perceived as a mistake, and of course making mistakes (i.e. having one’s hypotheses disproven by experiment) is an indispensable tool for learning. Introducing science in the classroom – real science characterized by individuals forming actual hypotheses and testing those hypotheses by experiment – helps students become self-directed learners.

14. This is the principle of falsifiability: that a scientific statement has value only insofar as it is liable to disproof given the requisite experimental evidence. Any claim that is unfalsifiable – that is, a claim which can never be disproven by any evidence whatsoever – could be completely wrong and we could never know it.

15. A very pertinent example of this is learning how to analyze electric circuits using simulation software such as SPICE. A typical experimental cycle would proceed as follows: (1) Find or invent a circuit to analyze; (2) Apply your analytical knowledge to that circuit, predicting all voltages, currents, powers, etc. relevant to the concepts you are striving to master; (3) Run a simulation on that circuit, collecting “data” from the computer when complete; (4) Evaluate whether or not your hypotheses (i.e. predicted voltages, currents, etc.) agree with the computer-generated results; (5) If so, your analyses are (provisionally) correct – if not, examine your analyses and the computer simulation again to determine the source of error; (6) Repeat this process as many times as necessary until you achieve mastery.
8.1. RECOMMENDED PRACTICES

Thought experiments are useful for detecting inconsistencies within your own understanding of some subject, rather than testing your understanding against physical reality.

Here are some general guidelines for conducting experiments:

• The clearer and more specific the hypothesis, the better. Vague or unfalsifiable hypotheses are useless because they will fit any experimental results, and therefore the experiment cannot teach you anything about the hypothesis.

• Collect as much data (i.e. information, measurements, sensory experiences) generated by an experiment as is practical. This includes the time and date of the experiment, too!

• Never discard or modify data gathered from an experiment. If you have reason to believe the data is unreliable, write notes to that effect, but never throw away data just because you think it is untrustworthy. It is quite possible that even “bad” data holds useful information, and that someone else may be able to uncover its value even if you do not.

• Prioritize quantitative data over qualitative data wherever practical. Quantitative data is more specific than qualitative, less prone to subjective interpretation on the part of the experimenter, and amenable to an arsenal of analytical methods (e.g. statistics).

• Guard against your own bias(es) by making your experimental results available to others. This allows other people to scrutinize your experimental design and collected data, for the purpose of detecting and correcting errors you may have missed. Document your experiment such that others may independently replicate it.

• Always be looking for sources of error. No physical measurement is perfect, and so it is impossible to achieve exact values for any variable. Quantify the amount of uncertainty (i.e. the “tolerance” of errors) whenever possible, and be sure your hypothesis does not depend on precision better than this!

• Always remember that scientific confirmation is provisional – no number of “successful” experiments will prove an hypothesis true for all time, but a single experiment can disprove it. Put into simpler terms, truth is elusive but error is within reach.

• Remember that scientific method is about learning, first and foremost. An unfortunate consequence of scientific triumph in modern society is that science is often viewed by non-practitioners as an unerring source of truth, when in fact science is an ongoing process of challenging existing ideas to probe for errors and oversights. This is why it is perfectly acceptable to have a failed hypothesis, and why the only truly failed experiment is one where nothing was learned.
The following is an example of a well-planned and executed experiment, in this case a physical experiment demonstrating Ohm's Law.

Planning Time/Date = 09:30 on 12 February 2019

HYPOTHESIS: the current through any resistor should be exactly proportional to the voltage impressed across it.

PROCEDURE: connect a resistor rated 1 k Ohm and 1/4 Watt to a variable-voltage DC power supply. Use an ammeter in series to measure resistor current and a voltmeter in parallel to measure resistor voltage.

RISKS AND MITIGATION: excessive power dissipation may harm the resistor and/or pose a burn hazard, while excessive voltage poses an electric shock hazard. 30 Volts is a safe maximum voltage for laboratory practices, and according to Joule’s Law a 1000 Ohm resistor will dissipate 0.25 Watts at 15.81 Volts ($P = V^2 / R$), so I will remain below 15 Volts just to be safe.

Experiment Time/Date = 10:15 on 12 February 2019

DATA COLLECTED:

<table>
<thead>
<tr>
<th>Voltage (V)</th>
<th>Current (mA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000 V</td>
<td>0.000</td>
</tr>
<tr>
<td>2.700 V</td>
<td>2.603</td>
</tr>
<tr>
<td>5.400 V</td>
<td>5.206</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Voltage (V)</th>
<th>Current (mA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.100</td>
<td>7.812</td>
</tr>
<tr>
<td>10.00 V</td>
<td>9.643</td>
</tr>
<tr>
<td>14.00 V</td>
<td>13.49</td>
</tr>
</tbody>
</table>

Analysis Time/Date = 10:57 on 12 February 2019

ANALYSIS: current definitely increases with voltage, and although I expected exactly one milliAmpere per Volt the actual current was usually less than that. The voltage/current ratios ranged from a low of 1036.87 (at 8.1 Volts) to a high of 1037.81 (at 14 Volts), but this represents a variance of only -0.0365% to +0.0541% from the average, indicating a very consistent proportionality -- results consistent with Ohm’s Law.

ERROR SOURCES: one major source of error is the resistor’s value itself. I did not measure it, but simply assumed color bands of brown-black-red meant exactly 1000 Ohms. Based on the data I think the true resistance is closer to 1037 Ohms. Another possible explanation is multimeter calibration error. However, neither explains the small positive and negative variances from the average. This might be due to electrical noise, a good test being to repeat the same experiment to see if the variances are the same or different. Noise should generate slightly different results every time.
The following is an example of a well-planned and executed virtual experiment, in this case demonstrating Ohm’s Law using a computer (SPICE) simulation.

Planning Time/Date = 12:32 on 14 February 2019

HYPOTHESIS: for any given resistor, the current through that resistor should be exactly proportional to the voltage impressed across it.

PROCEDURE: write a SPICE netlist with a single DC voltage source and single 1000 Ohm resistor, then use NGSPICE version 26 to perform a "sweep" analysis from 0 Volts to 25 Volts in 5 Volt increments.

* SPICE circuit
  v1 1 0 dc
  r1 1 0 1000
  .dc v1 0 25 5
  .print dc v(1) i(v1)
  .end

RISKS AND MITIGATION: none.

DATA COLLECTED:

DC transfer characteristic Thu Feb 14 13:05:08 2019

<table>
<thead>
<tr>
<th>Index</th>
<th>v-sweep</th>
<th>v(1)</th>
<th>v1#branch</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.0000000e+00</td>
<td>0.0000000e+00</td>
<td>0.0000000e+00</td>
</tr>
<tr>
<td>1</td>
<td>5.0000000e+00</td>
<td>5.0000000e+00</td>
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</tr>
<tr>
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<td>1.0000000e+01</td>
<td>-1.000000e-02</td>
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<td>3</td>
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<td>1.5000000e+01</td>
<td>-1.500000e-02</td>
</tr>
<tr>
<td>4</td>
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<td>2.0000000e+01</td>
<td>-2.000000e-02</td>
</tr>
<tr>
<td>5</td>
<td>2.5000000e+01</td>
<td>2.5000000e+01</td>
<td>-2.500000e-02</td>
</tr>
</tbody>
</table>

Analysis Time/Date = 13:06 on 14 February 2019

ANALYSIS: perfect agreement between data and hypothesis -- current is precisely 1/1000 of the applied voltage for all values. Anything other than perfect agreement would have probably meant my netlist was incorrect. The negative current values surprised me, but it seems this is just how SPICE interprets normal current through a DC voltage source.

ERROR SOURCES: none.
As gratuitous as it may seem to perform experiments on a physical law as well-established as Ohm’s Law, even the examples listed previously demonstrate opportunity for real learning. In the physical experiment example, the student should identify and explain why their data does not perfectly agree with the hypothesis, and this leads them naturally to consider sources of error. In the computer-simulated experiment, the student is struck by SPICE’s convention of denoting regular current through a DC voltage source as being negative in sign, and this is also useful knowledge for future simulations. Scientific experiments are most interesting when things do not go as planned!

Aside from verifying well-established physical laws, simple experiments are extremely useful as educational tools for a wide range of purposes, including:

- Component familiarization (e.g. Which terminals of this switch connect to the NO versus NC contacts?)
- System testing (e.g. How heavy of a load can my AC-DC power supply source before the semiconductor components reach their thermal limits?)
- Learning programming languages (e.g. Let’s try to set up an “up” counter function in this PLC!)

Above all, the priority here is to inculcate the habit of hypothesizing, running experiments, and analyzing the results. This experimental cycle not only serves as an excellent method for self-directed learning, but it also works exceptionally well for troubleshooting faults in complex systems, and for these reasons should be a part of every technician’s and every engineer’s education.

### 8.1.5 Constructing projects

Designing, constructing, and testing projects is a very effective means of practical education. Within a formal educational setting, projects are generally chosen (or at least vetted) by an instructor to ensure they may be reasonably completed within the allotted time of a course or program of study, and that they sufficiently challenge the student to learn certain important principles. In a self-directed environment, projects are just as useful as a learning tool but there is some risk of unwittingly choosing a project beyond one’s abilities, which can lead to frustration.

Here are some general guidelines for managing projects:

- Define your goal(s) before beginning a project: what do you wish to achieve in building it? What, exactly, should the completed project do?
- Analyze your project prior to construction. Document it in appropriate forms (e.g. schematic diagrams), predict its functionality, anticipate all associated risks. In other words, plan ahead.
- Set a reasonable budget for your project, and stay within it.
- Identify any deadlines, and set reasonable goals to meet those deadlines.
- Beware of scope creep: the tendency to modify the project’s goals before it is complete.
- Document your progress! An easy way to do this is to use photography or videography: take photos and/or videos of your project as it progresses. Document failures as well as successes, because both are equally valuable from the perspective of learning.
8.2 Experiment: Ohm’s Law in a three-resistor parallel circuit

In this experiment you will confirm the validity of parallel circuit principles as it applies to carbon composition resistors powered by a battery. For this experiment you will need:

- A primary-cell battery (6 or 9 Volts is sufficient)
- An assortment of carbon composition resistors between 1 kΩ and 100 kΩ
- A digital multimeter (DMM) capable of measuring DC voltage and DC current (milliAmperes) and resistance
- Alligator-clip style jumper wires to connect the battery to a resistor
- (Optional) A solderless breadboard or set of terminal blocks suitable for holding the resistors

First and foremost, identify any potential hazards posed by this experiment. If any exist, identify how to mitigate each of those risks to ensure personal safety as well as ensure no components or equipment will be damaged.

Choose at least three resistors with different resistor values, all between 1 kΩ and 100 kΩ. Decode each resistor’s color-code bands to determine its nominal resistance value as well as its tolerance. Then, use your DMM (set to measure resistance) to verify each resistor’s actual resistance value. Document these nominal and actual values for all three resistors, labeling them $R_1$ through $R_3$.

Next, use your DMM to measure the battery’s voltage. Document this value as well. Based on this source voltage measurement, predict how much voltage should exist across each of the three resistors when connected in parallel with the source.

Sketch a complete schematic diagram of the circuit as you intend it to be built. Double-check the schematic to ensure there are no “opens” that will prohibit current where needed, and no “shorts” prohibiting necessary voltage drops and/or causing excessive current. After you are satisfied with your plan, construct the circuit.

Connect your DMM (set to measure voltage) and measure the voltage across each of the three resistors. Compare these measured voltage values with the predicted voltage values from the previous step.

Use Ohm’s Law to calculate the expected amount of current for a circuit where all three resistors are connected in parallel with each other, and with the battery. Document these predicted current values, as you will be comparing them with measured (empirical) values of current in the next step.

---

16 Avoiding resistor values below 1 kΩ and 100 kΩ helps avoid errors caused by meter burden and test lead connection resistance.
Connect your DMM (set to measure current) and measure the current at each resistor. Compare these measured current values with the predicted current values from the previous step. Notate on the schematic diagram where you took the three current measurements.

How well are the theoretical predictions for series components supported by the empirical (measured) data from this experiment? Are there any noteworthy sources of error in this experiment?

Be sure to document all data in a neat and well-organized format, easily understood by anyone viewing it. Be prepared to explain your reasoning at every step, and also to demonstrate the safe and proper use of all materials, components, and equipment. If a live demonstration is not practical, record your actions on video.

**Challenges**

- Identify unique properties of parallel-connected components, and then use your experiment to prove this property is true either by appealing to empirical data you have recorded or by demonstrating those measurements live.

- Describe the mathematical relationship of individual versus total resistances in a parallel circuit, and then demonstrate this relationship empirically using your DMM.

- Predict the effects of a particular resistor failing open.

- Predict the effects of a particular resistor failing shorted.

- A good self-test of an ohmmeter is to connect its test leads directly together (otherwise known as “shorting” the leads because a direct connection is the shortest, lowest resistance electrical path possible). What should an ohmmeter register when its leads are shorted together? Why?

- Explain why you cannot obtain a reliable resistance measurement using an ohmmeter when the component in question is energized by a source.

- Sometimes you will find your multimeter yields a non-zero resistance measurement when you would expect it to measure zero or very nearly zero. Explain why this is.

- Identify any data points (empirical) that disagree with theory (prediction), and suggest sources of error.

- Explain why connecting an ammeter directly to the terminals of a voltage source such as a primary-cell battery is a bad idea.

---

17 Exercise caution when connecting your ammeter to any circuit! If you connect it in parallel with a voltage source, you will create a short-circuit whose low resistance value will likely permit a large enough amount of current to blow the fuse inside your meter (and possibly damage the meter as well). One way to ensure your ammeter will sense the current going through a particular component is to (1) Disconnect the source so that you will not be touching an energized circuit, (2) Disconnect one of the component’s two leads from the rest of the circuit, (3) Connect your ammeter between that “lifted” component lead and the point where it used to attach to the circuit, (4) Re-connect the source and note the ammeter’s measurement.
8.2. EXPERIMENT: OHM’S LAW IN A THREE-RESISTOR PARALLEL CIRCUIT

• The manner in which you connect the DMM to a resistor to measure its resistance may introduce a measurement error. In particular, contacting each of the meter’s test probes with your skin while simultaneously contacting the resistor’s terminals places your body in “parallel” with the resistor, providing an alternate path for the meter’s test current to flow. Will this measurement error be positive (i.e. registering more than the resistor’s true value) or negative (registering less than the resistor’s true value)? Explain your reasoning.
Appendix A

Problem-Solving Strategies

The ability to solve complex problems is arguably one of the most valuable skills one can possess, and this skill is particularly important in any science-based discipline.

- **Study principles, not procedures.** Don’t be satisfied with merely knowing how to compute solutions – learn *why* those solutions work.

- **Identify** what it is you need to solve, identify all relevant data, identify all units of measurement, identify any general principles or formulae linking the given information to the solution, and then identify any “missing pieces” to a solution. **Annotate** all diagrams with this data.

- **Sketch a diagram** to help visualize the problem. When building a real system, always devise a plan for that system and analyze its function *before* constructing it.

- **Follow the units of measurement and meaning of every calculation.** If you are ever performing mathematical calculations as part of a problem-solving procedure, and you find yourself unable to apply each and every intermediate result to some aspect of the problem, it means you don’t understand what you are doing. Properly done, every mathematical result should have practical meaning for the problem, and not just be an abstract number. You should be able to identify the proper units of measurement for each and every calculated result, and show where that result fits into the problem.

- **Perform “thought experiments”** to explore the effects of different conditions for theoretical problems. When troubleshooting real systems, perform *diagnostic tests* rather than visually inspecting for faults, the best diagnostic test being the one giving you the most information about the nature and/or location of the fault with the fewest steps.

- **Simplify the problem** until the solution becomes obvious, and then use that obvious case as a model to follow in solving the more complex version of the problem.

- **Check for exceptions** to see if your solution is incorrect or incomplete. A good solution will work for *all* known conditions and criteria. A good example of this is the process of testing scientific hypotheses: the task of a scientist is not to find support for a new idea, but rather to *challenge* that new idea to see if it holds up under a battery of tests. The philosophical
principle of *reductio ad absurdum* (i.e. disproving a general idea by finding a specific case where it fails) is useful here.

- **Work “backward”** from a hypothetical solution to a new set of given conditions.

- **Add quantities** to problems that are qualitative in nature, because sometimes a little math helps illuminate the scenario.

- **Sketch graphs** illustrating how variables relate to each other. These may be quantitative (i.e. with realistic number values) or qualitative (i.e. simply showing increases and decreases).

- **Treat quantitative problems as qualitative** in order to discern the relative magnitudes and/or directions of change of the relevant variables. For example, try determining what happens if a certain variable were to increase or decrease before attempting to precisely calculate quantities: how will each of the dependent variables respond, by increasing, decreasing, or remaining the same as before?

- **Consider limiting cases.** This works especially well for qualitative problems where you need to determine which direction a variable will change. Take the given condition and magnify that condition to an extreme degree as a way of simplifying the direction of the system’s response.

- **Check your work.** This means regularly testing your conclusions to see if they make sense. This does *not* mean repeating the same steps originally used to obtain the conclusion(s), but rather to use some other means to check validity. Simply repeating procedures often leads to *repeating the same errors* if any were made, which is why alternative paths are better.
Appendix B

Instructional philosophy

“The unexamined circuit is not worth energizing” – Socrates (if he had taught electricity)

These learning modules, although useful for self-study, were designed to be used in a formal learning environment where a subject-matter expert challenges students to digest the content and exercise their critical thinking abilities in the answering of questions and in the construction and testing of working circuits.

The following principles inform the instructional and assessment philosophies embodied in these learning modules:

- The first goal of education is to enhance clear and independent thought, in order that every student reach their fullest potential in a highly complex and inter-dependent world. Robust reasoning is always more important than particulars of any subject matter, because its application is universal.

- Literacy is fundamental to independent learning and thought because text continues to be the most efficient way to communicate complex ideas over space and time. Those who cannot read with ease are limited in their ability to acquire knowledge and perspective.

- Articulate communication is fundamental to work that is complex and interdisciplinary.

- Faulty assumptions and poor reasoning are best corrected through challenge, not presentation. The rhetorical technique of reductio ad absurdum (disproving an assertion by exposing an absurdity) works well to discipline student’s minds, not only to correct the problem at hand but also to learn how to detect and correct future errors.

- Important principles should be repeatedly explored and widely applied throughout a course of study, not only to reinforce their importance and help ensure their mastery, but also to showcase the interconnectedness and utility of knowledge.
These learning modules were expressly designed to be used in an “inverted” teaching environment where students first read the introductory and tutorial chapters on their own, then individually attempt to answer the questions and construct working circuits according to the experiment and project guidelines. The instructor never lectures, but instead meets regularly with each individual student to review their progress, answer questions, identify misconceptions, and challenge the student to new depths of understanding through further questioning. Regular meetings between instructor and student should resemble a Socratic dialogue, where questions serve as scalpels to dissect topics and expose assumptions. The student passes each module only after consistently demonstrating their ability to logically analyze and correctly apply all major concepts in each question or project/experiment. The instructor must be vigilant in probing each student’s understanding to ensure they are truly reasoning and not just memorizing. This is why “Challenge” points appear throughout, as prompts for students to think deeper about topics and as starting points for instructor queries. Sometimes these challenge points require additional knowledge that hasn’t been covered in the series to answer in full. This is okay, as the major purpose of the Challenges is to stimulate analysis and synthesis on the part of each student.

The instructor must possess enough mastery of the subject matter and awareness of students' reasoning to generate their own follow-up questions to practically any student response. Even completely correct answers given by the student should be challenged by the instructor for the purpose of having students practice articulating their thoughts and defending their reasoning. Conceptual errors committed by the student should be exposed and corrected not by direct instruction, but rather by reducing the errors to an absurdity through well-chosen questions and thought experiments posed by the instructor. Becoming proficient at this style of instruction requires time and dedication, but the positive effects on critical thinking for both student and instructor are spectacular.

An inspection of these learning modules reveals certain unique characteristics. One of these is a bias toward thorough explanations in the tutorial chapters. Without a live instructor to explain concepts and applications to students, the text itself must fulfill this role. This philosophy results in lengthier explanations than what you might typically find in a textbook, each step of the reasoning process fully explained, including footnotes addressing common questions and concerns students raise while learning these concepts. Each tutorial seeks to not only explain each major concept in sufficient detail, but also to explain the logic of each concept and how each may be developed in sufficient detail.

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1In a traditional teaching environment, students first encounter new information via lecture from an expert, and then independently apply that information via homework. In an “inverted” course of study, students first encounter new information via homework, and then independently apply that information under the scrutiny of an expert. The expert’s role in lecture is to simply explain, but the expert’s role in an inverted session is to challenge, critique, and if necessary explain where gaps in understanding still exist.

2Socrates is a figure in ancient Greek philosophy famous for his unflinching style of questioning. Although he authored no texts, he appears as a character in Plato’s many writings. The essence of Socratic philosophy is to leave no question unexamined and no point of view unchallenged. While purists may argue a topic such as electric circuits is too narrow for a true Socratic-style dialogue, I would argue that the essential thought processes involved with scientific reasoning on any topic are not far removed from the Socratic ideal, and that students of electricity and electronics would do very well to challenge assumptions, pose thought experiments, identify fallacies, and otherwise employ the arsenal of critical thinking skills modeled by Socrates.

3This rhetorical technique is known by the Latin phrase reductio ad absurdum. The concept is to expose errors by counter-example, since only one solid counter-example is necessary to disprove a universal claim. As an example of this, consider the common misconception among beginning students of electricity that voltage cannot exist without current. One way to apply reductio ad absurdum to this statement is to ask how much current passes through a fully-charged battery connected to nothing (i.e. a clear example of voltage existing without current).
from “first principles”. Again, this reflects the goal of developing clear and independent thought in students’ minds, by showing how clear and logical thought was used to forge each concept. Students benefit from witnessing a model of clear thinking in action, and these tutorials strive to be just that.

Another characteristic of these learning modules is a lack of step-by-step instructions in the Project and Experiment chapters. Unlike many modern workbooks and laboratory guides where step-by-step instructions are prescribed for each experiment, these modules take the approach that students must learn to closely read the tutorials and apply their own reasoning to identify the appropriate experimental steps. Sometimes these steps are plainly declared in the text, just not as a set of enumerated points. At other times certain steps are implied, an example being assumed competence in test equipment use where the student should not need to be told again how to use their multimeter because that was thoroughly explained in previous lessons. In some circumstances no steps are given at all, leaving the entire procedure up to the student.

This lack of prescription is not a flaw, but rather a feature. Close reading and clear thinking are foundational principles of this learning series, and in keeping with this philosophy all activities are designed to require those behaviors. Some students may find the lack of prescription frustrating, because it demands more from them than what their previous educational experiences required. This frustration should be interpreted as an unfamiliarity with autonomous thinking, a problem which must be corrected if the student is ever to become a self-directed learner and effective problem-solver. Ultimately, the need for students to read closely and think clearly is more important both in the near-term and far-term than any specific facet of the subject matter at hand. If a student takes longer than expected to complete a module because they are forced to outline, digest, and reason on their own, so be it. The future gains enjoyed by developing this mental discipline will be well worth the additional effort and delay.

Another feature of these learning modules is that they do not treat topics in isolation. Rather, important concepts are introduced early in the series, and appear repeatedly as stepping-stones toward other concepts in subsequent modules. This helps to avoid the “compartmentalization” of knowledge, demonstrating the inter-connectedness of concepts and simultaneously reinforcing them. Each module is fairly complete in itself, reserving the beginning of its tutorial to a review of foundational concepts.

This methodology of assigning text-based modules to students for digestion and then using Socratic dialogue to assess progress and hone students’ thinking was developed over a period of several years by the author with his Electronics and Instrumentation students at the two-year college level. While decidedly unconventional and sometimes even unsettling for students accustomed to a more passive lecture environment, this instructional philosophy has proven its ability to convey conceptual mastery, foster careful analysis, and enhance employability so much better than lecture that the author refuses to ever teach by lecture again.

Problems which often go undiagnosed in a lecture environment are laid bare in this “inverted” format where students must articulate and logically defend their reasoning. This, too, may be unsettling for students accustomed to lecture sessions where the instructor cannot tell for sure who comprehends and who does not, and this vulnerability necessitates sensitivity on the part of the “inverted” session instructor in order that students never feel discouraged by having their errors exposed. Everyone makes mistakes from time to time, and learning is a lifelong process! Part of the instructor’s job is to build a culture of learning among the students where errors are not seen as shameful, but rather as opportunities for progress.
To this end, instructors managing courses based on these modules should adhere to the following principles:

- Student questions are always welcome and demand thorough, honest answers. The only type of question an instructor should refuse to answer is one the student should be able to easily answer on their own. Remember, the fundamental goal of education is for each student to learn to think clearly and independently. This requires hard work on the part of the student, which no instructor should ever circumvent. Anything done to bypass the student’s responsibility to do that hard work ultimately limits that student’s potential and thereby does real harm.

- It is not only permissible, but encouraged, to answer a student’s question by asking questions in return, these follow-up questions designed to guide the student to reach a correct answer through their own reasoning.

- All student answers demand to be challenged by the instructor and/or by other students. This includes both correct and incorrect answers – the goal is to practice the articulation and defense of one’s own reasoning.

- No reading assignment is deemed complete unless and until the student demonstrates their ability to accurately summarize the major points in their own terms. Recitation of the original text is unacceptable. This is why every module contains an “Outline and reflections” question as well as a “Foundational concepts” question in the Conceptual reasoning section, to prompt reflective reading.

- No assigned question is deemed answered unless and until the student demonstrates their ability to consistently and correctly apply the concepts to variations of that question. This is why module questions typically contain multiple “Challenges” suggesting different applications of the concept(s) as well as variations on the same theme(s). Instructors are encouraged to devise as many of their own “Challenges” as they are able, in order to have a multitude of ways ready to probe students’ understanding.

- No assigned experiment or project is deemed complete unless and until the student demonstrates the task in action. If this cannot be done “live” before the instructor, video-recordings showing the demonstration are acceptable. All relevant safety precautions must be followed, all test equipment must be used correctly, and the student must be able to properly explain all results. The student must also successfully answer all Challenges presented by the instructor for that experiment or project.
Students learning from these modules would do well to abide by the following principles:

- No text should be considered fully and adequately read unless and until you can express every idea in your own words, using your own examples.

- You should always articulate your thoughts as you read the text, noting points of agreement, confusion, and epiphanies. Feel free to print the text on paper and then write your notes in the margins. Alternatively, keep a journal for your own reflections as you read. This is truly a helpful tool when digesting complicated concepts.

- Never take the easy path of highlighting or underlining important text. Instead, summarize and/or comment on the text using your own words. This actively engages your mind, allowing you to more clearly perceive points of confusion or misunderstanding on your own.

- A very helpful strategy when learning new concepts is to place yourself in the role of a teacher, if only as a mental exercise. Either explain what you have recently learned to someone else, or at least imagine yourself explaining what you have learned to someone else. The simple act of having to articulate new knowledge and skill forces you to take on a different perspective, and will help reveal weaknesses in your understanding.

- Perform each and every mathematical calculation and thought experiment shown in the text on your own, referring back to the text to see that your results agree. This may seem trivial and unnecessary, but it is critically important to ensuring you actually understand what is presented, especially when the concepts at hand are complicated and easy to misunderstand. Apply this same strategy to become proficient in the use of circuit simulation software, checking to see if your simulated results agree with the results shown in the text.

- Above all, recognize that learning is hard work, and that a certain level of frustration is unavoidable. There are times when you will struggle to grasp some of these concepts, and that struggle is a natural thing. Take heart that it will yield with persistent and varied effort, and never give up!

Students interested in using these modules for self-study will also find them beneficial, although the onus of responsibility for thoroughly reading and answering questions will of course lie with that individual alone. If a qualified instructor is not available to challenge students, a workable alternative is for students to form study groups where they challenge one another.

To high standards of education,

Tony R. Kuphaldt

\footnote{As the old saying goes, “Insanity is trying the same thing over and over again, expecting different results.” If you find yourself stumped by something in the text, you should attempt a different approach. Alter the thought experiment, change the mathematical parameters, do whatever you can to see the problem in a slightly different light, and then the solution will often present itself more readily.}

\footnote{Avoid the temptation to simply share answers with study partners, as this is really counter-productive to learning. Always bear in mind that the answer to any question is far less important in the long run than the method(s) used to obtain that answer. The goal of education is to empower one’s life through the improvement of clear and independent thought, literacy, expression, and various practical skills.}
APPENDIX B. INSTRUCTIONAL PHILOSOPHY
Appendix C

Tools used

I am indebted to the developers of many open-source software applications in the creation of these learning modules. The following is a list of these applications with some commentary on each.

You will notice a theme common to many of these applications: a bias toward code. Although I am by no means an expert programmer in any computer language, I understand and appreciate the flexibility offered by code-based applications where the user (you) enters commands into a plain ASCII text file, which the software then reads and processes to create the final output. Code-based computer applications are by their very nature extensible, while WYSIWYG (What You See Is What You Get) applications are generally limited to whatever user interface the developer makes for you.

The GNU/Linux computer operating system

There is so much to be said about Linus Torvalds’ Linux and Richard Stallman’s GNU project. First, to credit just these two individuals is to fail to do justice to the mob of passionate volunteers who contributed to make this amazing software a reality. I first learned of Linux back in 1996, and have been using this operating system on my personal computers almost exclusively since then. It is free, it is completely configurable, and it permits the continued use of highly efficient Unix applications and scripting languages (e.g. shell scripts, Makefiles, sed, awk) developed over many decades. Linux not only provided me with a powerful computing platform, but its open design served to inspire my life’s work of creating open-source educational resources.

Bram Moolenaar’s Vim text editor

Writing code for any code-based computer application requires a text editor, which may be thought of as a word processor strictly limited to outputting plain-ASCII text files. Many good text editors exist, and one’s choice of text editor seems to be a deeply personal matter within the programming world. I prefer Vim because it operates very similarly to vi which is ubiquitous on Unix/Linux operating systems, and because it may be entirely operated via keyboard (i.e. no mouse required) which makes it fast to use.
Donald Knuth’s **\TeX** typesetting system

Developed in the late 1970’s and early 1980’s by computer scientist extraordinaire Donald Knuth to typeset his multi-volume magnum opus *The Art of Computer Programming*, this software allows the production of formatted text for screen-viewing or paper printing, all by writing plain-text code to describe how the formatted text is supposed to appear. \TeX is not just a markup language for documents, but it is also a Turing-complete programming language in and of itself, allowing useful algorithms to be created to control the production of documents. Simply put, \TeX is *a programmer’s approach to word processing*. Since \TeX is controlled by code written in a plain-text file, this means anyone may read that plain-text file to see exactly how the document was created. This openness afforded by the code-based nature of \TeX makes it relatively easy to learn how other people have created their own \TeX documents. By contrast, examining a beautiful document created in a conventional WYSIWYG word processor such as Microsoft Word suggests nothing to the reader about how that document was created, or what the user might do to create something similar. As Mr. Knuth himself once quipped, conventional word processing applications should be called WYSIAYG (What You See Is All You Get).

Leslie Lamport’s **\LaTeX** extensions to **\TeX**

Like all true programming languages, \TeX is inherently extensible. So, years after the release of \TeX to the public, Leslie Lamport decided to create a massive extension allowing easier compilation of book-length documents. The result was \LaTeX, which is the markup language used to create all ModEL module documents. You could say that \TeX is to \LaTeX as C is to C++. This means it is permissible to use any and all \TeX commands within \LaTeX source code, and it all still works. Some of the features offered by \LaTeX that would be challenging to implement in \TeX include automatic index and table-of-content creation.

Tim Edwards’ **\Xcircuit** drafting program

This wonderful program is what I use to create all the schematic diagrams and illustrations (but not photographic images or mathematical plots) throughout the ModEL project. It natively outputs PostScript format which is a true vector graphic format (this is why the images do not pixellate when you zoom in for a closer view), and it is so simple to use that I have never had to read the manual! Object libraries are easy to create for \Xcircuit, being plain-text files using PostScript programming conventions. Over the years I have collected a large set of object libraries useful for drawing electrical and electronic schematics, pictorial diagrams, and other technical illustrations.
Gimp graphic image manipulation program

Essentially an open-source clone of Adobe’s Photoshop, I use Gimp to resize, crop, and convert file formats for all of the photographic images appearing in the ModEL modules. Although Gimp does offer its own scripting language (called Script-Fu), I have never had occasion to use it. Thus, my utilization of Gimp to merely crop, resize, and convert graphic images is akin to using a sword to slice bread.

SPICE circuit simulation program

SPICE is to circuit analysis as TeX is to document creation: it is a form of markup language designed to describe a certain object to be processed in plain-ASCII text. When the plain-text “source file” is compiled by the software, it outputs the final result. More modern circuit analysis tools certainly exist, but I prefer SPICE for the following reasons: it is free, it is fast, it is reliable, and it is a fantastic tool for teaching students of electricity and electronics how to write simple code. I happen to use rather old versions of SPICE, version 2g6 being my “go to” application when I only require text-based output. NGSPICE (version 26), which is based on Berkeley SPICE version 3f5, is used when I require graphical output for such things as time-domain waveforms and Bode plots. In all SPICE example netlists I strive to use coding conventions compatible with all SPICE versions.

Andrew D. Hwang’s ePiX mathematical visualization programming library

This amazing project is a C++ library you may link to any C/C++ code for the purpose of generating PostScript graphic images of mathematical functions. As a completely free and open-source project, it does all the plotting I would otherwise use a Computer Algebra System (CAS) such as Mathematica or Maple to do. It should be said that ePiX is not a Computer Algebra System like Mathematica or Maple, but merely a mathematical visualization tool. In other words, it won’t determine integrals for you (you’ll have to implement that in your own C/C++ code!), but it can graph the results, and it does so beautifully. What I really admire about ePiX is that it is a C++ programming library, which means it builds on the existing power and toolset available with that programming language. Mr. Hwang could have probably developed his own stand-alone application for mathematical plotting, but by creating a C++ library to do the same thing he accomplished something much greater.
Another open-source tool for mathematical visualization is gnuplot. Interestingly, this tool is not part of Richard Stallman’s GNU project, its name being a coincidence. For this reason the authors prefer “gnu” not be capitalized at all to avoid confusion. This is a much “lighter-weight” alternative to a spreadsheet for plotting tabular data, and the fact that it easily outputs directly to an X11 console or a file in a number of different graphical formats (including PostScript) is very helpful. I typically set my gnuplot output format to default (X11 on my Linux PC) for quick viewing while I’m developing a visualization, then switch to PostScript file export once the visual is ready to include in the document(s) I’m writing. As with my use of Gimp to do rudimentary image editing, my use of gnuplot only scratches the surface of its capabilities, but the important points are that it’s free and that it works well.

Python programming language

Both Python and C++ find extensive use in these modules as instructional aids and exercises, but I’m listing Python here as a tool for myself because I use it almost daily as a calculator. If you open a Python interpreter console and type from math import * you can type mathematical expressions and have it return results just as you would on a hand calculator. Complex-number (i.e. phasor) arithmetic is similarly supported if you include the complex-math library (from cmath import *). Examples of this are shown in the Programming References chapter (if included) in each module. Of course, being a fully-featured programming language, Python also supports conditionals, loops, and other structures useful for calculation of quantities. Also, running in a console environment where all entries and returned values show as text in a chronologically-ordered list makes it easy to copy-and-paste those calculations to document exactly how they were performed.
Appendix D

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Appendix E

Version history

This is a list showing all significant additions, corrections, and other edits made to this learning module. Each entry is referenced by calendar date in reverse chronological order (newest version first), which appears on the front cover of every learning module for easy reference. Any contributors to this open-source document are listed here as well.

27 November 2022 – placed questions at the top of the itemized list in the Introduction chapter prompting students to devise experiments related to the tutorial content.

15 September 2022 – added a new Case Tutorial section showing how adding more resistors in parallel decreases total (equivalent) resistance.

13 September 2022 – replaced incorrect quotation marks in a few places.

7 June 2022 – added a conceptual definition of resistance to the first page of the Simplified Tutorial chapter.

10 February 2022 – divided the Full Tutorial chapter into sections.

7 February 2022 – added a new sub-question to “VIRP table...” Quantitative Reasoning problem challenging students to check their work using estimation.

15 September 2021 – added more challenge questions.

18 August 2021 – minor edits to a schematic diagram in the Quantitative Reasoning section.

23 July 2021 – added foundational concept review to the “Identifying series circuits” Conceptual Reasoning question.


22 June 2021 – added technical reference on resistor labeling.

28 May 2021 – minor edits to the Simplified Tutorial.
18 March 2021 – corrected one instance of “volts” that should have been capitalized “Volts”.

9-10 February 2021 – minor additions to the Introduction chapter, and also edited the schematic diagram for the potentiometer-based current divider (image_0445). Minor edits to the Full Tutorial as well.

7 January 2021 – typographical error correction (“carries” should have been “carriers” in the first sentence of the Simplified Tutorial), thanks to Brett Deaton.

16-17 September 2020 – fixed typographical error in the Tutorial: a portion of the text said \( R_2 \) when it should have said \( R_1 \). Also, fixed minor wiring error in the “Problem with a multiple-pump system” question, where the main fuse (F5) was directly feeding the Pump A circuit in addition to indirectly feeding the other pumps’ individual fuses.

15 September 2020 – minor additions to the Introduction chapter.

3 September 2020 – added Technical Reference section on metric prefixes.

29 August 2020 – significantly edited the Introduction chapter to make it more suitable as a pre-study guide and to provide cues useful to instructors leading “inverted” teaching sessions.

11 March 2020 – minor addition to the Derivation section on parallel resistance formulae.

2 January 2020 – removed from C++ code execution output, to clearly distinguish it from the source code listing which is still framed.

1 January 2020 – changed `main ()` to `main (void)` in C++ programming example.

19 September 2019 – minor edit to footnote in the Case Tutorial chapter.

14 September 2019 – added a Case Tutorial chapter.

26 August 2019 – added mention of difficult concepts to the Introduction.

16 June 2019 – minor edits to diagnostic questions, replacing “no multiple faults” with “no coincidental faults”.

27 February 2019 – minor edit, replacing double-quote characters with single-quote for cleaner \LaTeX\ formatting.

10 February 2019 – added requirement in “Explaining the meaning of calculations” Conceptual Question to identify the principle applied in each step.

17 December 2018 – added an example C++ program showing simple parallel circuit calculations.

October 2018 – minor edit to parallel properties frame, noting that parallel networks are defined by the sharing of two electrically common point sets, not just two equipotential point sets.

September 2018 – added mathematical derivations of parallel resistance formulae.
August 2018 – added content to the Introduction chapter.

May 2018 – minor edit to “parallel” illustration, annotating connected points as electrically common to each other, not just equipotential to each other.

April 2018 – added a Simplified Tutorial chapter. Also added photographs of rotary potentiometers, and a paragraph about linear potentiometers.

November 2017 – minor edits.

August 2017 – added a conceptual question.

July 2017 – substituted “property” for “characteristic” when describing parallel circuits, because this is a more mathematically conventional term. Also enhanced the definition of “parallel” in terms of electrically distinct points. Finally, capitalized all instances of the word “Earth”.

May 2017 – added discussion of potentiometers to the tutorial.

January 2017 – minor clarifications made to the definition of a parallel network.

October 2016 – created changelog for future use.

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