

MODULAR ELECTRONICS LEARNING (MODEL) PROJECT



POLYPHASE TRANSFORMER CIRCUITS

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LAST UPDATE = 27 SEPTEMBER 2024

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Chapter 1

Introduction

1.1 Recommendations for students

Transformers are indispensable components in any large-scale electric power system due to their ability to step up AC voltage (and correspondingly step down AC current) for the sake of using smaller, cheaper transmission and distribution line conductors. While single-phase AC transformer circuits are simple enough, things become more complicated in three-phase AC circuits which are the standard for large-scale AC electric power systems. This module seeks to explore the fundamental principles of transformers applied to three-phase AC systems.

Important concepts related to polyphase transformers include **properties of wye networks**, **properties of delta networks**, **mutual inductance**, transformer **step ratio**, line versus phase quantities, phase **rotation** or **sequence**, **phasor diagrams**, **transformer polarity**, **Ohm's Law**, and polyphase **busses**.

Due to the heavy application of math to polyphase transformer networks, the reader is urged to apply the mathematical principles as soon as possible. The circuit examples contained in the tutorial serves this purpose well, as they allow you to apply the formulae discussed previously to an example and then check your work against the completed example to see if your application was correct. Do not simply read a quantitative example and assume you understand it just because nothing in the presentation seemed confusing. *Until you can perform the analysis yourself without assistance, you haven't mastered it!*

Here are some good questions to ask of yourself while studying this subject:

- How might an experiment be designed and conducted to explore the concept of phase versus line quantities in a three-phase AC system? What hypothesis (i.e. prediction) might you pose for that experiment, and what result(s) would either support or disprove that hypothesis?
- How might an experiment be designed and conducted to demonstrate the phenomenon of mutual inductance? What hypothesis (i.e. prediction) might you pose for that experiment, and what result(s) would either support or disprove that hypothesis?

- How might an experiment be designed and conducted to measure the turns ratio of a transformer? What hypothesis (i.e. prediction) might you pose for that experiment, and what result(s) would either support or disprove that hypothesis?
- What are some practical applications of polyphase AC circuits?
- Which line-versus-phase parameters must be equal in a Wye network, and *why* is this?
- Which line-versus-phase parameters must be equal in a Delta network, and *why* is this?
- How may Joule's Law be used to check our analytical work for any transformer-based circuit?
- How does the turns ratio of a transformer relate to its AC voltage ratio?
- How does the turns ratio of a transformer relate to its AC current ratio?
- What is a *bus* in a three-phase power system?
- How is "polarity" defined for an AC circuit?
- What are some of the ways polarity is represented in transformers?
- What do phasor diagrams represent, and how is this helpful in analyzing three-phase circuits?
- How may the windings of three transformers be wired together to form a wye network?
- How may the windings of three transformers be wired together to form a delta network?
- Which choices are arbitrary, and which are not, when connecting multiple transformers together to form a three-phase transformer bank?

1.2 Challenging concepts related to polyphase transformer banks

The following list cites concepts related to this module's topic that are easily misunderstood, along with suggestions for properly understanding them:

- **Delta versus Wye network properties** – a very common tendency among students is to memorize formulae for line and phase quantities in three-phase AC networks without actually understanding why those formulae are correct. Viewing line and phase elements as being either in series or in parallel with other elements is extremely helpful here, leveraging previously learned foundational concepts. For example, line and phase currents must be equal in a Wye network because each phase element of a Wye network is (obviously) connected in series with a respective line; line and phase voltages must be equal for Delta networks because each phase element of a Delta network is (obviously) connected in parallel with two lines. Once identical line/phase quantities are identified, unequal line/phase quantities must always differ by a factor of $\sqrt{3}$ (in a *balanced* three-phase system) with the line quantity always being larger than the phase quantity.
- **Phasors representing AC amplitudes and phase shifts** – a powerful tool used for understanding the operation of AC circuits is the *phasor diagram*, consisting of arrows pointing in different directions: the length of each arrow representing the amplitude of some AC quantity (voltage, current, or impedance), and the angle of each arrow representing the shift in phase relative to the other arrows. By representing each AC quantity thusly, we may more easily calculate their relationships to one another, with the phasors showing us how to apply trigonometry (Pythagorean Theorem, sine, cosine, and tangent functions) to the various calculations. An analytical parallel to the graphic tool of phasor diagrams is *complex numbers*, where we represent each phasor (arrow) by a pair of numbers: either a magnitude and angle (polar notation), or by “real” and “imaginary” magnitudes (rectangular notation). Where phasor diagrams are helpful is in applications where their respective AC quantities *add*: the resultant of two or more phasors stacked tip-to-tail being the mathematical sum of the phasors. Complex numbers, on the other hand, may be added, subtracted, multiplied, and divided; the last two operations being difficult to graphically represent with arrows.

1.3 Recommendations for instructors

This section lists realistic student learning outcomes supported by the content of the module as well as suggested means of assessing (measuring) student learning. The outcomes state what learners should be able to do, and the assessments are specific challenges to prove students have learned.

- **Outcome** – Demonstrate effective technical reading and writing

Assessment – Students present their outlines of this module’s instructional chapters (e.g. Case Tutorial, Tutorial, Historical References, etc.) ideally as an entry to a larger Journal document chronicling their learning. These outlines should exhibit good-faith effort at summarizing major concepts explained in the text.

Assessment – Students show how quantitative results were obtained by the author in the Tutorial chapter’s examples.

- **Outcome** – Determine voltages, currents, and/or powers in three-phase transformer circuits

Assessment – Calculate various voltages, currents, and power values for balanced three-phase circuits containing transformers, given component values; e.g. pose problems in the form of the “Balanced Delta source and Wye load with transformers”, “Ground-referenced voltage calculations”, and “Load calculations” Quantitative Reasoning questions.

- **Outcome** – Determine proper winding connections to form three-phase transformer banks from multiple transformers

Assessment – Sketch connections between transformer windings and three-phase busses given phasor diagrams for primary and secondary; e.g. pose problems in the form of the “Sketching winding connections” Conceptual Reasoning question.

- **Outcome** – Independent research

Assessment – Locate three-phase transformer datasheets and properly interpret some of the information contained in those documents including winding configurations, phase shift, power rating, turns ratios, etc.

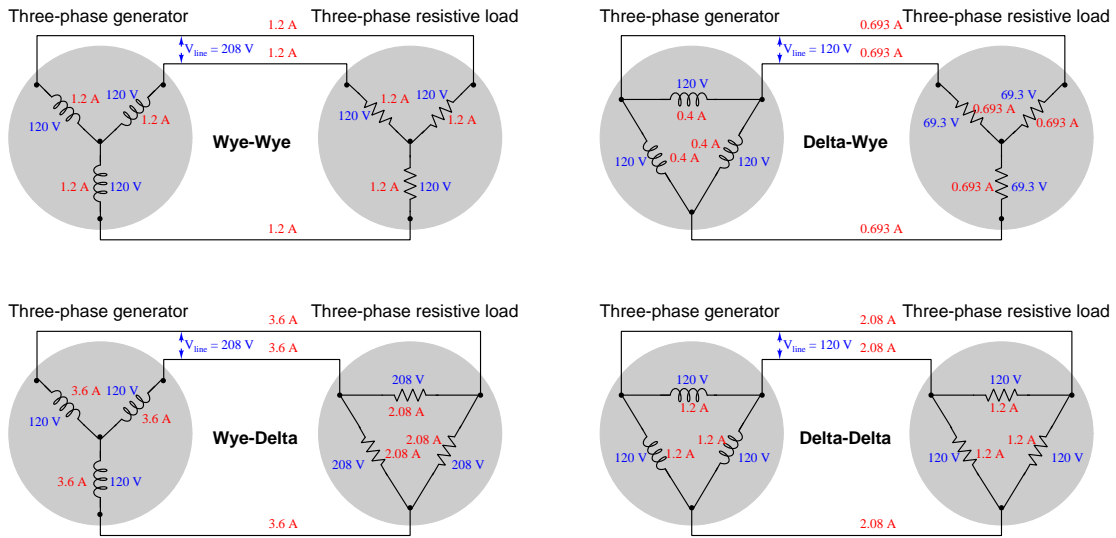
Chapter 2

Tutorial

2.1 Review of polyphase networks

A *polyphase* AC circuit has multiple sources shifted in phase from each other by a definite angle. In the case of *three-phase* AC circuits, that phase shift is 120° . The effect of this is to have an AC power system where the flow of energy is ceaseless despite the fact that each of the sources periodically passes through zero as it alternates positive and negative, much like a multi-cylinder piston engine delivers mechanical energy to a load more smoothly than a single-cylinder piston engine.

The following diagrams show four combinations of three-phase source and load networks. Each of these networks is balanced, with a generator phase voltage of 120 Volts AC and a load phase resistance of $100\ \Omega$. All other values have been computed for your review:



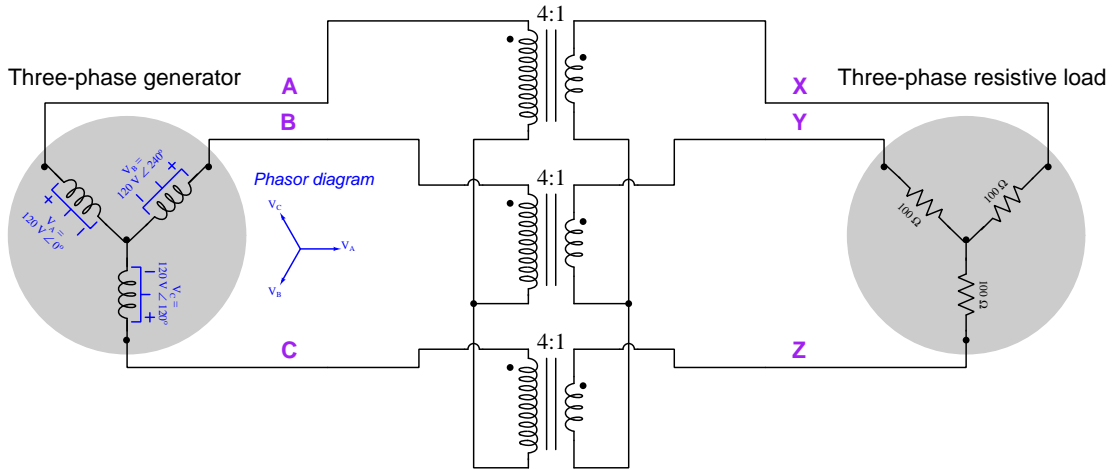
Note how phase and line currents are equal for all Wye networks because those quantities are in series with each other, and how phase and line voltages are equal for all Delta networks because those quantities are in parallel with each other. When currents combine at a node, or when voltages stack together, the result is a line quantity $\sqrt{3}$ times greater than the phase quantity.

Transformers are inductive components consisting of multiple coils of wire (called *windings*) wrapped around a common core so as to share the same magnetic flux, the purpose being to transfer energy from an energized winding (called the “primary”) to one or more other windings (called “secondary”) using magnetism as the link. A transformer intended for use in a three-phase AC power system typically has three primary windings and three secondary windings, one set of windings for each phase. Transformers in three-phase power systems may consist of single units with all windings wrapped around a common core, or as three separate single-phase transformers wired together to function as a three-phase assembly. The complexities of polyphase transformer circuits are best understood from the perspective of separate single-phase transformers wired together, and so that will be the focus of this tutorial.

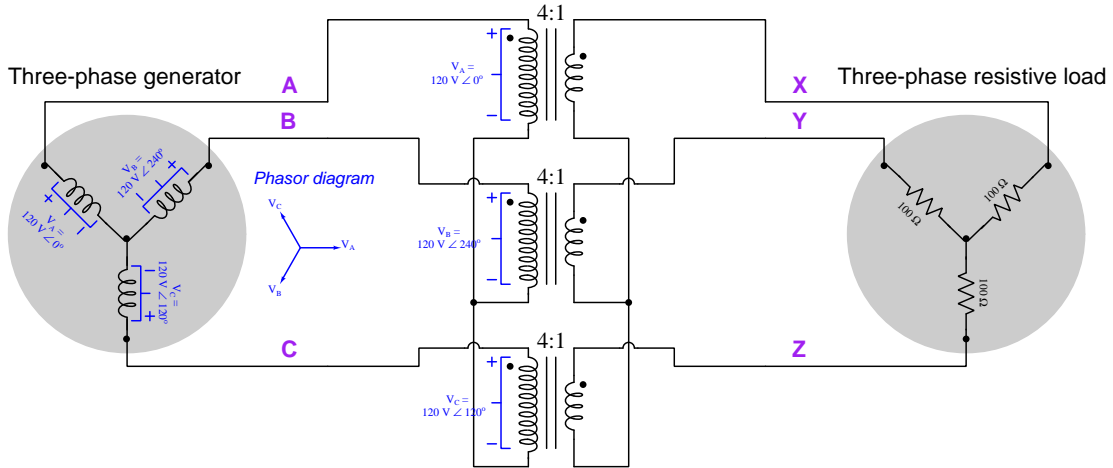
Like the generator/load combinations shown previously, three-phase transformers may have their primary and secondary windings arranged as Wye-Wye, Wye-Delta, Delta-Wye, or Delta-Delta. As with the four possible generator/load combinations, each transformer configuration has its own unique properties. We will explore each one as its own example, using the same Wye-connected generator with $V_{phase} = 120$ VAC, the same Wye-connected load with $R_{phase} = 100\ \Omega$, and the same primary:secondary transformer turns ratio of 4:1. In each example we will demonstrate the calculation of all voltages and currents (as polar-form phasor quantities) and also show phasor diagrams of each.

The relationships between phase angles, phasor diagrams, transformer winding polarity, and wire connections can be dauntingly complex at first, and so you are encouraged to engage with these examples by performing the calculations yourself as the text explains each step.

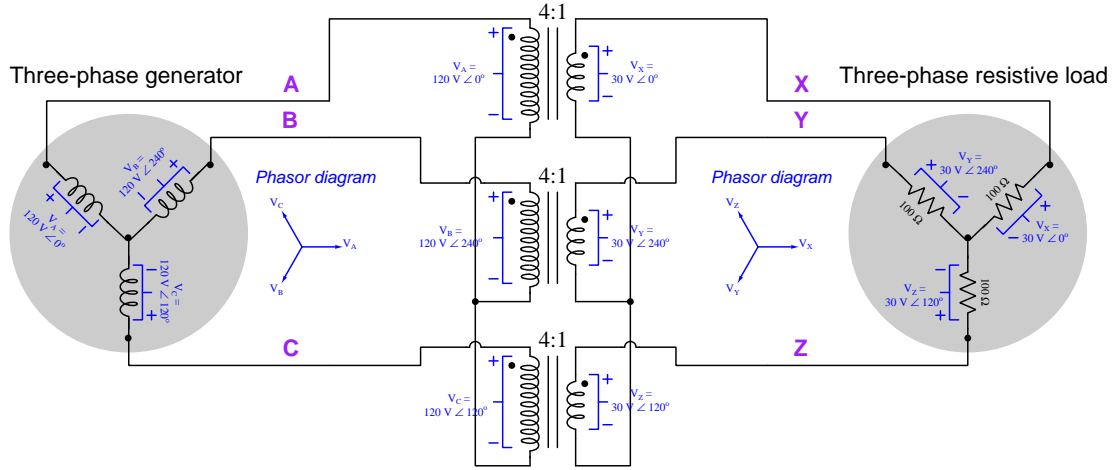
2.2 Wye-Wye transformer bank



Since we have been given the phase voltages of the generator, it makes sense to begin our analysis of this circuit by determining the voltage across each transformer's primary winding. This is rather simple with the transformer primary windings and the generator stator windings both being connected in a Wye configuration: with each generator phase voltage being 120 Volts between the respective line and the center-node of the Wye, we expect each primary transformer winding to also have 120 Volts across it (i.e. between the respective line and the center-node of the transformer Wye):



Following the phase markings on each transformer, we may calculate secondary voltages. Note how the polarity of these voltages follow the “dot” polarity marks on the transformer windings: each secondary winding’s voltage will be “+” on the “dot” terminal because each primary winding is energized with “+” on the “dot” terminal as well. An ideal transformer neither adds to nor subtracts from the phase angle going from primary to secondary, but rather replicates phase according to the phase markings (dots), and so each secondary voltage bears similar polarity and identical phase angle to its primary voltage, only 4 times smaller due to the 4:1 step-down turns ratio of the transformer:

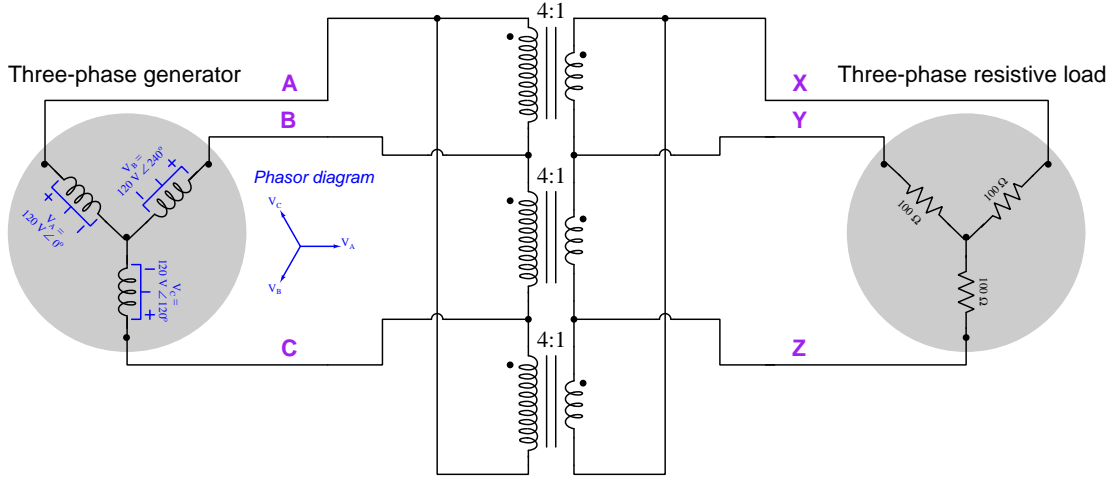


This means each of the 100-Ohm resistors in the “Wye” load receives 30 Volts AC, resulting in a current of 0.3 Amperes (both line and phase current, since those two quantities are equal in “Wye” networks). Each transformer secondary winding therefore carries the same 0.3 Amperes of current, which is stepped down to become a primary winding current of 75 milliAmperes AC. This becomes the phase and line current for the “Wye” source.

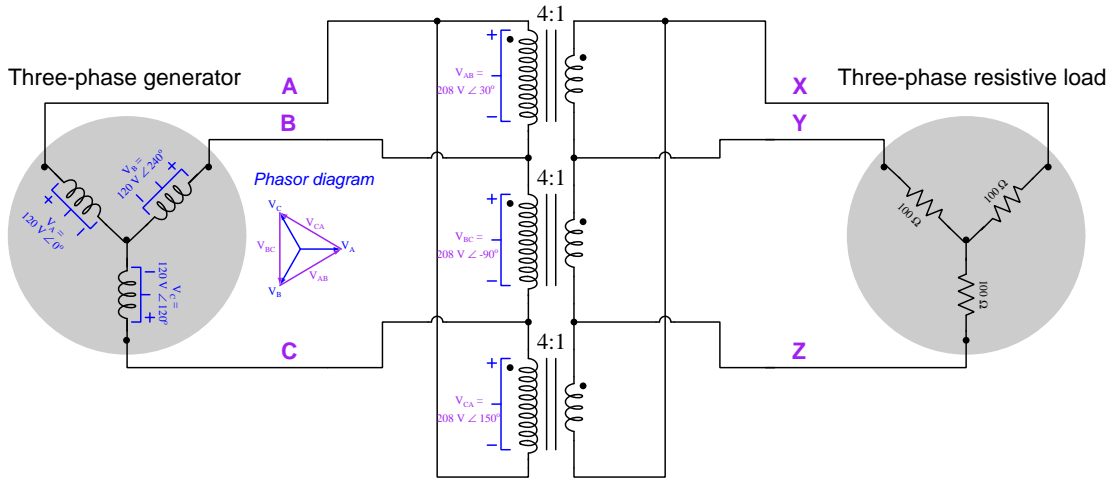
Note how the phasor diagram for the secondary side of this circuit (phasors V_X , V_Y , and V_Z) is geometrically similar to the primary phasor diagram (phasors V_A , V_B , and V_C , respectively). If we were to sketch phasor diagrams for current on the primary and secondary sides of this circuit, they would also be similar because the purely resistive nature of the load causes current to be in-phase with voltage, and therefore all current phasors would have the same angles (i.e. point in the same directions) as their respective voltage phasors.

Note how the *phase rotations* are similar between the primary and secondary circuits as well. On the primary side we see that the phase sequence is A-B-C and on the secondary side X-Y-Z, because convention assumes a counter-clockwise rotation of the phasors over time (i.e. as phase angles increase, they revolve in a counter-clockwise direction around the diagram’s origin point) and any stationary observer watching the phasors spin around would see the lettered phasor tips pass by in these sequences.

2.3 Delta-Delta transformer bank

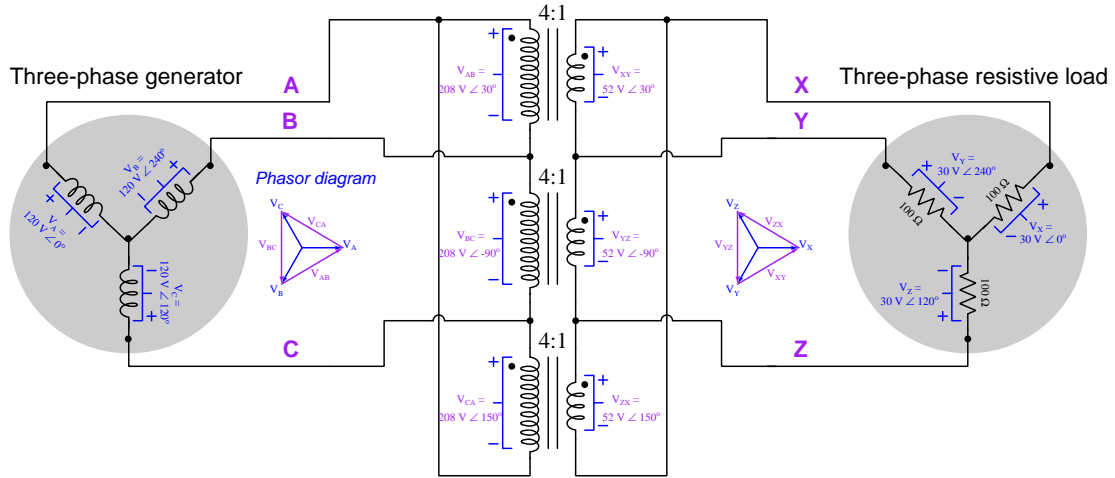


As with the previous example, we will begin analyzing this circuit by determining the voltage across each transformer's primary winding. With each transformer primary winding connected across two of the source's lines, we expect each primary transformer winding to have 208 Volts across it, and we may represent these voltages by drawing phasors spanning the distance between those point-pairs. Each of these new phasors is drawn in such a direction that the head touches the apex corresponding to the source line connected to the “dot” end of the primary winding:



For example, on the upper-most transformer which has the “polarity” (dot) terminal connected to A and the “non-polarity” (non-dot) terminal connected to B, we sketch its phasor with tip touching A and tail touching B, and from that sketch we derive the proper angle for the voltage value written next to that winding (+ representing the phasor tip and – representing the phasor tail).

Following the phase markings on each transformer, we may calculate secondary voltages. Note how the polarity of these voltages follow the “dot” polarity marks on the transformer windings: each secondary winding’s voltage will be “+” on the “dot” terminal because each primary winding is energized with “+” on the “dot” terminal as well. An ideal transformer neither adds to nor subtracts from the phase angle going from primary to secondary, but rather replicates phase according to the phase markings (dots), and so each secondary voltage bears similar polarity and identical phase angle to its primary voltage, only 4 times smaller due to the 4:1 step-down turns ratio of the transformer:

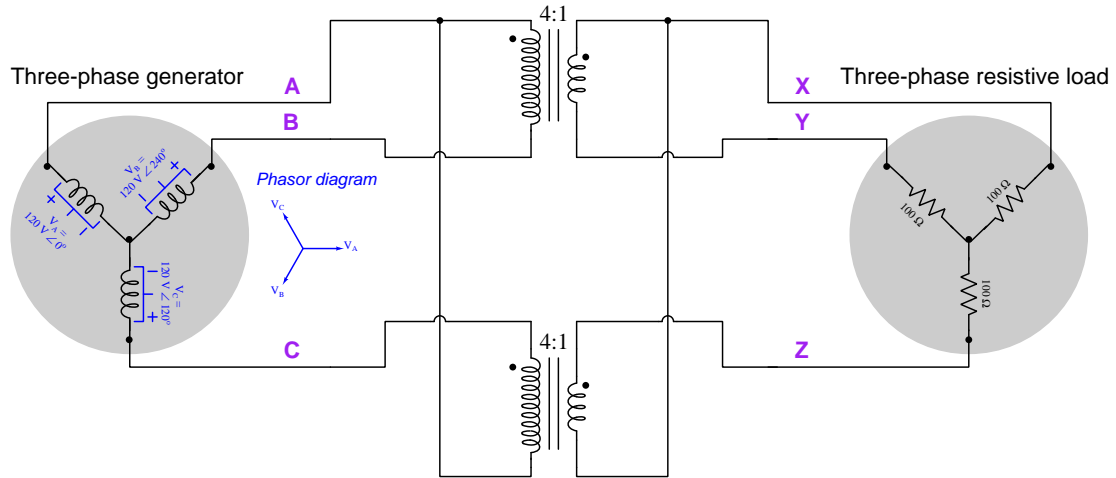


The 52-Volt phasors representing each of the transformer secondary winding voltages circumscribe another set of phasors representing the “Wye”-connected resistors, so that each of the resistors receives $\frac{1}{\sqrt{3}}$ as much voltage, or 30 Volts AC per 100-Ohm resistor. This results in a current of 0.3 Amperes for each resistor (both line and phase current, since those two quantities are equal in “Wye” networks). However, owing to the “Delta” connection of the transformer secondary windings, each of those windings carries $\frac{1}{\sqrt{3}}$ as much current, or 173.21 milliAmperes. This secondary current gets stepped down by a factor of 4:1 to become a primary winding current of 43.30 milliAmperes AC. Due to the “Delta” connection of the primary windings, however, the 43.30 mA primary currents join to become $\sqrt{3}$ larger at the generator’s lines. Thus, each phase/line of the “Wye”-connected generator must source 75 milliAmperes AC.

It is worth comparing the results of this Delta-Delta transformer bank with the previous (Wye-Wye) example. In both cases we end up with load phase voltages of 30 Volts per 100-Ohm resistor, with the same line/phase current values for both Wye-connected load and source, as well as the same phase sequence on each side. The only difference here is that each transformer operates at a higher voltage and lesser current than in the Wye-Wye transformer bank, with those winding voltages and currents having different phase angles than their respective source/load voltages and currents.

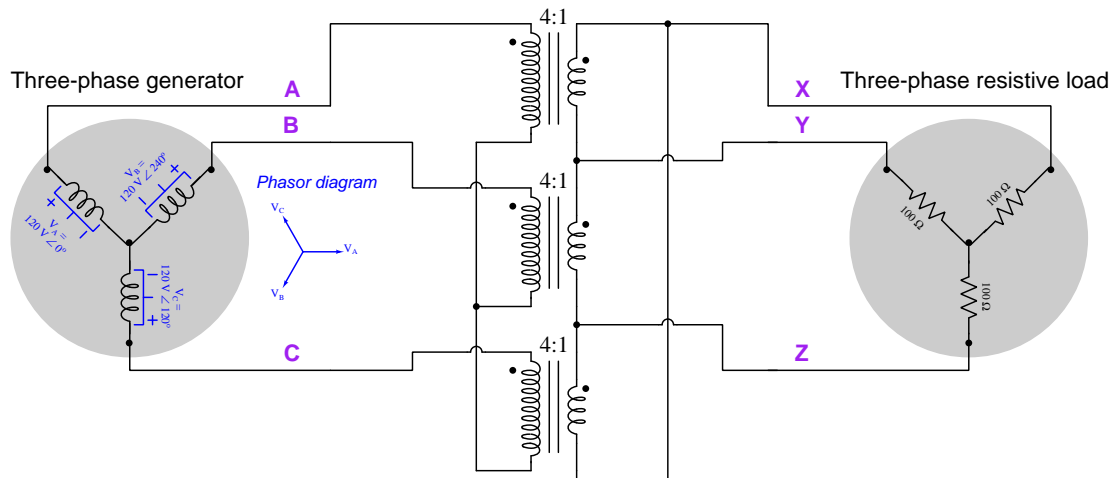
2.4 Open-Delta transformer bank

An option for lower-power applications is the so-called *open-delta* configuration where only two transformers are used:

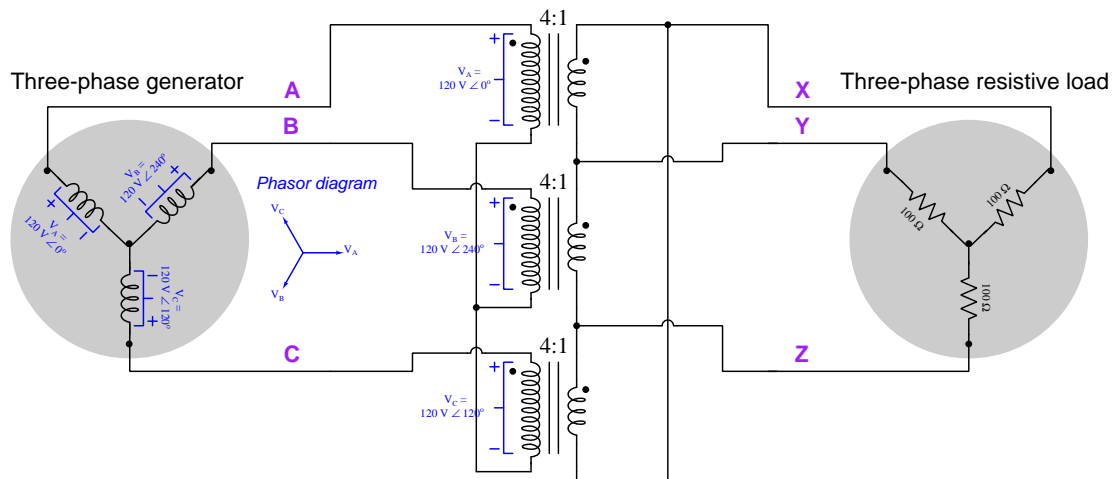


All phase and line voltages remain the same as for a three-transformer Delta-Delta bank. Line and phase currents at the resistive load and at the generator are the same as well, but the two transformers will see greater phase currents through their primary and secondary windings than what would be experienced in a three-transformer bank. With three transformers in a balanced three-phase circuit, each of the three transformers transforms one-third of the total power; in the open-Delta configuration total power is divided equally between the two transformers so that each transformer handles one-half of the total power.

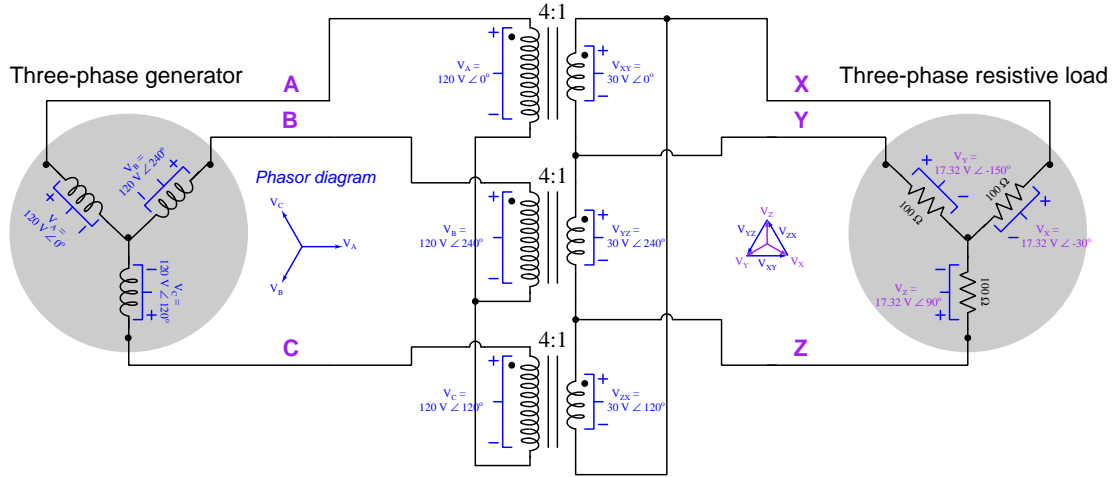
2.5 Wye-Delta transformer bank



Since we have been given the phase voltages of the generator, we will begin our analysis of this circuit by determining the voltage across each transformer's primary winding. This is rather simple with the transformer primary windings and the generator stator windings both being connected in a Wye configuration: with each generator phase voltage being 120 Volts between the respective line and the center-node of the Wye, we expect each primary transformer winding to also have 120 Volts across it (i.e. between the respective line and the center-node of the transformer Wye):



Following the phase markings on each transformer, we may calculate secondary voltages. Note how the polarity of these voltages follow the “dot” polarity marks on the transformer windings: each secondary winding’s voltage will be “+” on the “dot” terminal because each primary winding is energized with “+” on the “dot” terminal as well. An ideal transformer neither adds to nor subtracts from the phase angle going from primary to secondary, but rather replicates phase according to the phase markings (dots), and so each secondary voltage bears similar polarity and identical phase angle to its primary voltage, only 4 times smaller due to the 4:1 step-down turns ratio of the transformer:



These 30-Volt phasors form a “Delta” pattern in the secondary circuit’s phasor diagram, with each phasor maintaining the same angle inherited from its respective primary-winding voltage. Note for example how phasor V_{XY} points horizontally to the right (90°) since that is the direction V_A points. We connect these three 30-Volt phasors together in the phasor diagram according to the electrical connections seen between the secondary windings: the “+” or “polarity” terminal of each secondary winding connecting with the “–” or “non-polarity” terminal of another, which means the tip of one phasor must touch the tail of another.

After sketching the “Delta” phasor diagram comprised of 30-Volt phasors, we see that the “Wye”-connected load experiences a different set of phasor voltages, inscribed within the “Delta” of the secondary windings’ phasors. Each inscribed phasor is $\frac{1}{\sqrt{3}}$ the magnitude of 30 Volts, which means each of the 100-Ohm load resistors experiences 17.32 Volts. Moreover, the phase angle for each of these “Wye” phasors does not match any of the 30-Volt phasors. V_X is 17.32 Volts $\angle -30^\circ$, V_Y is 17.32 Volts $\angle -150^\circ$, and V_Z is 17.32 Volts $\angle 90^\circ$.

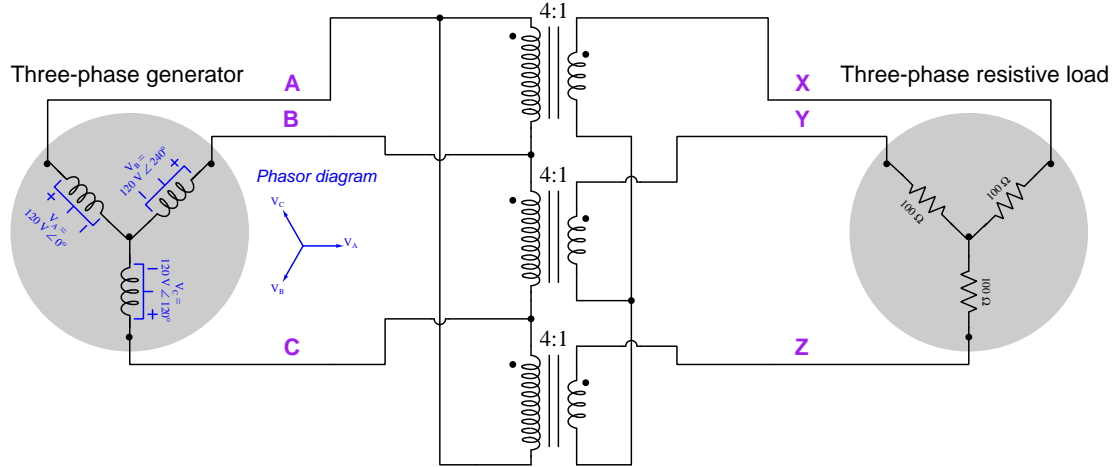
Applying Ohm’s Law to the calculation of load line/phase currents, we get 173.2 milliAmperes through each load resistor at angles of -30° , -150° , and 90° , respectively. These in turn translate to secondary winding currents of lesser magnitude, given the fact that $I_{line} = \sqrt{3}I_{phase}$ for any “Delta” network. Those secondary (phase) currents maintain the same angles as the respective secondary (phase) voltages due to the fact that the load’s characteristic is resistive. Thus, the current through the upper transformer’s secondary winding is 100 mA $\angle 0^\circ$, the middle transformer 100 mA $\angle 240^\circ$, and the lower transformer 100 mA $\angle 120^\circ$. These currents become reduced four-fold by each

transformer's turns ratio, so that the generator line/phase currents are $I_A = 25 \text{ mA} \angle 0^\circ$, $I_B = 25 \text{ mA} \angle 240^\circ$, and $I_C = 25 \text{ mA} \angle 120^\circ$.

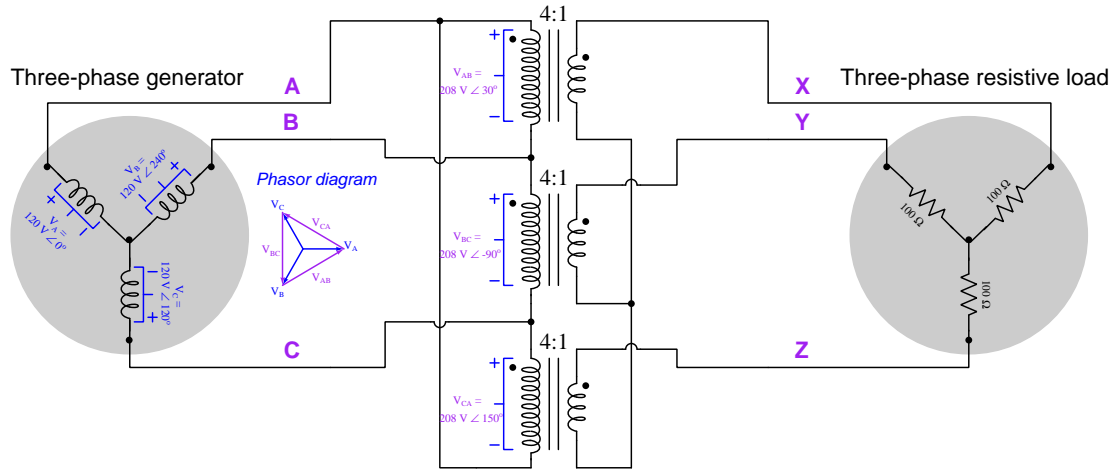
It is worth noting that this is the first three-phase transformer bank example we have seen where the load voltages are out of phase with their respective generator (source) voltages. To be precise, the phase angles for V_X , V_Y , and V_Z lag -30° behind V_A , V_B , and V_C , respectively, even though the primary phase sequence is still A-B-C and the secondary phase sequence still X-Y-Z. Currents for the resistive load, of course, must be in-phase with the resistors' voltages owing to the lack of phase shift between voltage and current for any resistor, and this translates into in-phase currents and voltages for the generator. However, comparing load to source (generator), there is definitely a 30-degree phase shift between the two. It is also important to note that the reason for this 30-degree lagging phase shift from primary to secondary is rooted in the Delta-Wye configuration of the windings. Any transformer bank wired with primary and secondary windings in different configurations will yield a phase shift.

This 30-degree phase shift between primary and secondary windings of the power transformer previously shown is actually a standard specified by the IEEE for standard power system transformers. The IEEE standard C57.12.00-2010 ("IEEE Standard for General Requirements for Liquid-Immersed Distribution, Power, and Regulating Transformers") states that transformers having Wye-Wye or Delta-Delta winding configurations shall exhibit 0° phase shift from input to output, but transformers having Wye-Delta or Delta-Wye winding configurations shall exhibit 30° phase shift between primary and secondary sides with the lower-voltage side of the transformer lagging. That said, there are specialized applications where three-phase transformers are connected to produce phase shifts other than 30 degrees between primary and secondary winding sets. It's just that power transformers used for distribution networks must adhere to the 30-degree standard if mixing Wye and Delta connections.

2.6 Delta-Wye transformer bank

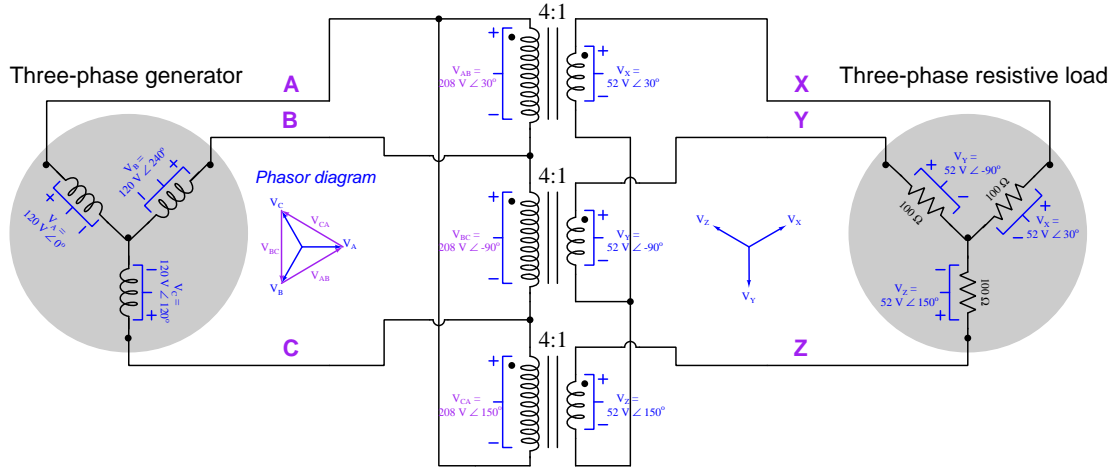


As usual, we will begin analyzing this circuit by determining the voltage across each transformer's primary winding. With each transformer primary winding connected across two of the source's lines, we expect each primary transformer winding to have 208 Volts across it, and we may represent these voltages by drawing phasors spanning the distance between those point-pairs. Each of these new phasors is drawn in such a direction that the head touches the apex corresponding to the source line connected to the "dot" end of the primary winding:



For example, on the upper-most transformer which has the "polarity" (dot) terminal connected to A and the "non-polarity" (non-dot) terminal connected to B, we sketch its phasor with tip touching A and tail touching B, and from that sketch we derive the proper angle for the voltage value written next to that winding (+ representing the phasor tip and - representing the phasor tail).

Following the phase markings on each transformer, we may calculate secondary voltages. Note how the polarity of these voltages follow the “dot” polarity marks on the transformer windings: each secondary winding’s voltage will be “+” on the “dot” terminal because each primary winding is energized with “+” on the “dot” terminal as well. An ideal transformer neither adds to nor subtracts from the phase angle going from primary to secondary, but rather replicates phase according to the phase markings (dots), and so each secondary voltage bears similar polarity and identical phase angle to its primary voltage, only 4 times smaller due to the 4:1 step-down turns ratio of the transformer:

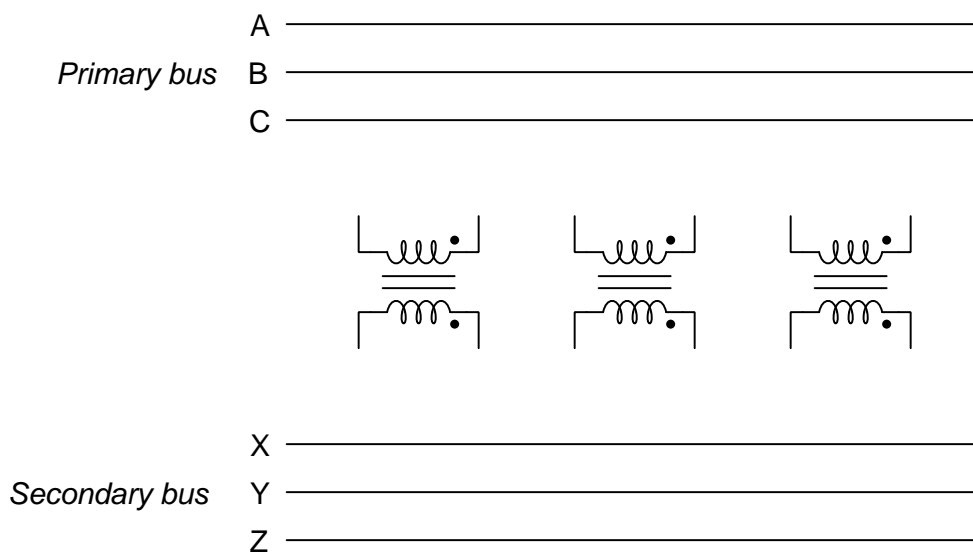


The 52-Volt phasors representing each of the transformer secondary winding voltages form a “Wye” shape in the secondary circuit’s phasor diagram, resulting in each of the “Wye”-connected resistors experiencing 52 Volts. This results in a current of 0.52 Amperes for each resistor (both line and phase current, since those two quantities are equal in “Wye” networks), which will also flow through each secondary winding. This secondary current gets stepped down by a factor of 4:1 to become a primary winding current of 130 milliAmperes AC. Due to the “Delta” connection of the primary windings, however, the 130 mA primary currents join to become $\sqrt{3}$ larger at the generator’s lines. Thus, each phase/line of the “Wye”-connected generator must source 225.2 milliAmperes AC. In each case, current phasors have the same angle as their respective voltage phasors because the load’s characteristic is purely resistive.

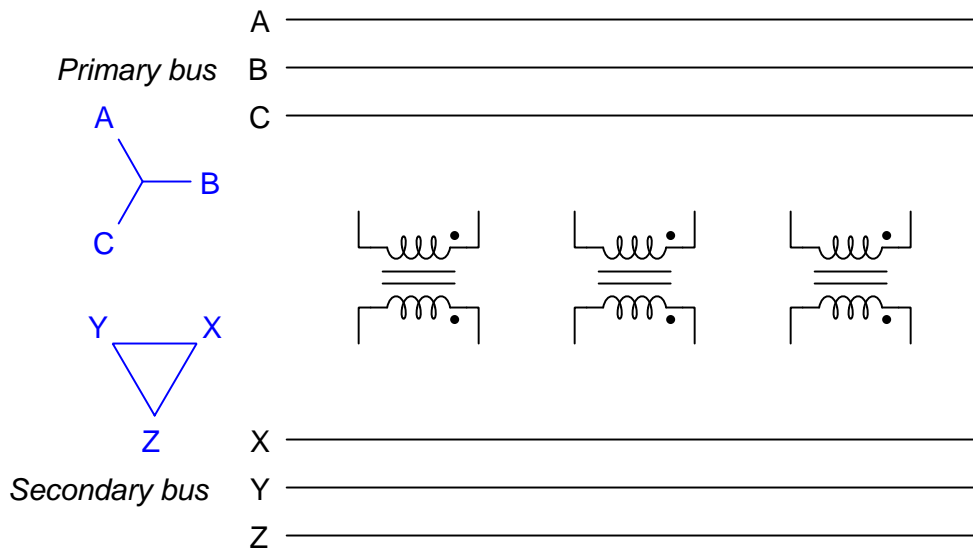
Since this transformer bank’s windings are not wired the same, we should not be surprised to see a phase shift from primary to secondary just as we saw with the Wye-Delta example. In this example, the phase angles for V_X , V_Y , and V_Z lead V_A , V_B , and V_C , respectively. Phase rotation is still A-B-C for primary and X-Y-Z for secondary.

2.7 Wiring from phasor diagrams

When deciding how to interconnect a set of three transformers to form a three-phase transformer bank, it is often the case that a particular phase relationship between primary and secondary circuits be respected. This section will show how to determine proper winding connections to achieve a given phase relationship depicted by primary and secondary phasor diagrams. For simplicity and convenience we will show the three transformers as having primary and secondary windings of equal turns count, and represent the primary and secondary circuit conductors each as three-wire “busses” placed along both sides of the three transformers:

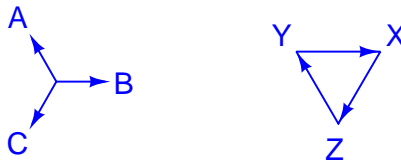


For example, consider this case where we need to interconnect the three transformers to match the given phasor diagrams for primary and secondary circuits:



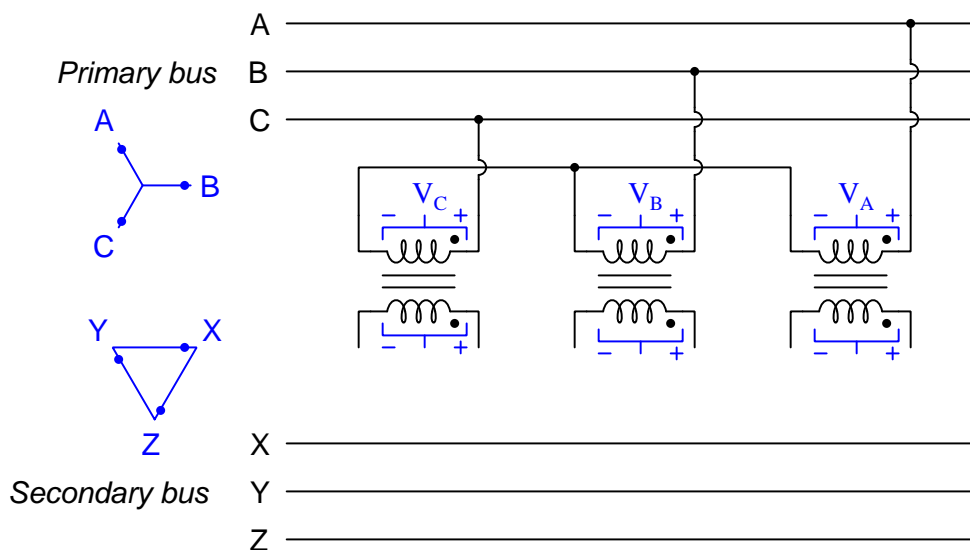
Since we know that each transformer replicates the same phase angle on secondary as it's fed on primary (i.e. the secondary voltage's phase angle will be equal to the primary voltage's phase angle), we know that each vector line segment in the primary circuit phasor diagram must have a matching counterpart in the secondary circuit's phasor diagram. Therefore, we need to identify which vectors in the primary and secondary phasor diagrams have matching angles in order to determine the primary/secondary connections for each transformer. A close inspection reveals matching angles for V_A and V_{YZ} , matching angles for V_B and V_{XY} , and matching angles for V_C and V_{ZX} .

In most phasor diagrams we represent each phasor quantity by a vector having "head" and "tail" ends, but in this phasor diagram (typical of what is found on three-phase transformer nameplates) no arrow-heads are found. This is not as troubling as it might seem to any reader familiar with conventional phasor diagrams. By convention, all "Wye" phasor diagrams are drawn with arrow-heads facing outward, and so we may assume the same is true here. If we edit the "Wye" phasor diagram with arrow-heads (all pointing outward), it becomes clear where the arrow-heads must go in the "Delta" phasor diagram in order to preserve the angles of all the primary phasors:



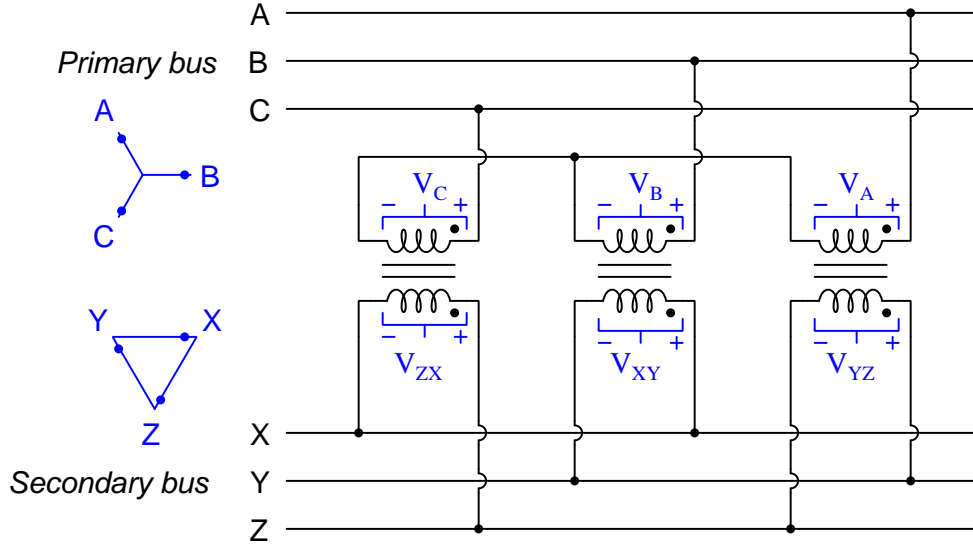
For example, if phasor B (angle of 0° , horizontally to the right) corresponds in phase to phasor XY, then phasor XY must point in the direction of 0° just like phasor B.

The primary phasor diagram’s “Wye” shape tells us all transformer primary windings must connect in a “Wye” configuration. If we are careful to connect the “polarity” (dot) terminal of each winding to the power conductor associated with the label on the arrow-head side of its respective phasor, we will have a proper “Wye”-connected primary winding set. In fact, it is often helpful to add dots to the ends of each phasor line-segment in the phasor diagrams in lieu of arrow-heads to remind us of these polarized connections:



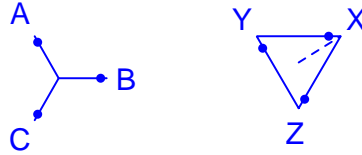
It is completely arbitrary at this stage in the circuit development which transformer connects to which phase in the primary circuit. We could have just as easily and correctly connected the left-most transformer to line A, the middle transformer to line C, and the right-most transformer to line B; any other arrangement is permissible as well. What is *not* arbitrary is what we do next with the transformers’ secondary windings, and indeed our arbitrary choices on the primary side necessarily affect the correct connections on the secondary side.

Next, we identify the matching phasors on the secondary side, and make connections to the XYZ bus accordingly:



The resulting polyphase transformer bank converts the primary power (with “A-B-C” phase rotation) to secondary power with a “Z-Y-X” phase rotation¹. The phase shift between primary voltage V_A (line A to ground) versus secondary voltage V_X (line X to ground) happens to be 90° with X lagging behind A.

A common convention in transformer notation is to show a fourth phasor in the “Delta” diagram, representing the phase voltage of the first (alphabetical order) element of a balanced Wye-connected load if it were powered by the transformers’ “Delta” bus. For the phasor diagram of this example transformer circuit, the phase-voltage line segment is a dashed line extending from X to the geometric center of the “Delta” shape:



With this additional vector shown in the diagram, it is easier to perceive that V_X has a phase angle of 30° , which lags 90° behind V_A ’s angle of 120° .

¹Recall that angle measurements in the phasor plane assume counter-clockwise rotation with increasing angle value. Therefore, as these phasors move over time, they do so in a clockwise direction. Any stationary observer watching the phasor tips rotate past any fixed point will see the sequence ABCABCABC on the primary bus and ZYXZYXZYX on the secondary bus.

Chapter 3

Questions

This learning module, along with all others in the ModEL collection, is designed to be used in an inverted instructional environment where students independently read¹ the tutorials and attempt to answer questions on their own *prior* to the instructor's interaction with them. In place of lecture², the instructor engages with students in Socratic-style dialogue, probing and challenging their understanding of the subject matter through inquiry.

Answers are not provided for questions within this chapter, and this is by design. Solved problems may be found in the Tutorial and Derivation chapters, instead. The goal here is *independence*, and this requires students to be challenged in ways where others cannot think for them. Remember that you always have the tools of *experimentation* and *computer simulation* (e.g. SPICE) to explore concepts!

The following lists contain ideas for Socratic-style questions and challenges. Upon inspection, one will notice a strong theme of *metacognition* within these statements: they are designed to foster a regular habit of examining one's own thoughts as a means toward clearer thinking. As such these sample questions are useful both for instructor-led discussions as well as for self-study.

¹Technical reading is an essential academic skill for any technical practitioner to possess for the simple reason that the most comprehensive, accurate, and useful information to be found for developing technical competence is in textual form. Technical careers in general are characterized by the need for continuous learning to remain current with standards and technology, and therefore any technical practitioner who cannot read well is handicapped in their professional development. An excellent resource for educators on improving students' reading prowess through intentional effort and strategy is the book *Reading For Understanding – How Reading Apprenticeship Improves Disciplinary Learning in Secondary and College Classrooms* by Ruth Schoenbach, Cynthia Greenleaf, and Lynn Murphy.

²Lecture is popular as a teaching method because it is easy to implement: any reasonably articulate subject matter expert can talk to students, even with little preparation. However, it is also quite problematic. A good lecture always makes complicated concepts seem easier than they are, which is bad for students because it instills a false sense of confidence in their own understanding; reading and re-articulation requires more cognitive effort and serves to verify comprehension. A culture of teaching-by-lecture fosters a debilitating dependence upon direct personal instruction, whereas the challenges of modern life demand independent and critical thought made possible only by gathering information and perspectives from afar. Information presented in a lecture is ephemeral, easily lost to failures of memory and dictation; text is forever, and may be referenced at any time.

GENERAL CHALLENGES FOLLOWING TUTORIAL READING

- Summarize as much of the text as you can in one paragraph of your own words. A helpful strategy is to explain ideas as you would for an intelligent child: as simple as you can without compromising too much accuracy.
- Simplify a particular section of the text, for example a paragraph or even a single sentence, so as to capture the same fundamental idea in fewer words.
- Where did the text make the most sense to you? What was it about the text's presentation that made it clear?
- Identify where it might be easy for someone to misunderstand the text, and explain why you think it could be confusing.
- Identify any new concept(s) presented in the text, and explain in your own words.
- Identify any familiar concept(s) such as physical laws or principles applied or referenced in the text.
- Devise a proof of concept experiment demonstrating an important principle, physical law, or technical innovation represented in the text.
- Devise an experiment to disprove a plausible misconception.
- Did the text reveal any misconceptions you might have harbored? If so, describe the misconception(s) and the reason(s) why you now know them to be incorrect.
- Describe any useful problem-solving strategies applied in the text.
- Devise a question of your own to challenge a reader's comprehension of the text.

GENERAL FOLLOW-UP CHALLENGES FOR ASSIGNED PROBLEMS

- Identify where any fundamental laws or principles apply to the solution of this problem, especially before applying any mathematical techniques.
- Devise a thought experiment to explore the characteristics of the problem scenario, applying known laws and principles to mentally model its behavior.
- Describe in detail your own strategy for solving this problem. How did you identify and organized the given information? Did you sketch any diagrams to help frame the problem?
- Is there more than one way to solve this problem? Which method seems best to you?
- Show the work you did in solving this problem, even if the solution is incomplete or incorrect.
- What would you say was the most challenging part of this problem, and why was it so?
- Was any important information missing from the problem which you had to research or recall?
- Was there any extraneous information presented within this problem? If so, what was it and why did it not matter?
- Examine someone else's solution to identify where they applied fundamental laws or principles.
- Simplify the problem from its given form and show how to solve this simpler version of it. Examples include eliminating certain variables or conditions, altering values to simpler (usually whole) numbers, applying a limiting case (i.e. altering a variable to some extreme or ultimate value).
- For quantitative problems, identify the real-world meaning of all intermediate calculations: their units of measurement, where they fit into the scenario at hand. Annotate any diagrams or illustrations with these calculated values.
- For quantitative problems, try approaching it qualitatively instead, thinking in terms of “increase” and “decrease” rather than definite values.
- For qualitative problems, try approaching it quantitatively instead, proposing simple numerical values for the variables.
- Were there any assumptions you made while solving this problem? Would your solution change if one of those assumptions were altered?
- Identify where it would be easy for someone to go astray in attempting to solve this problem.
- Formulate your own problem based on what you learned solving this one.

GENERAL FOLLOW-UP CHALLENGES FOR EXPERIMENTS OR PROJECTS

- In what way(s) was this experiment or project easy to complete?
- Identify some of the challenges you faced in completing this experiment or project.

- Show how thorough documentation assisted in the completion of this experiment or project.
- Which fundamental laws or principles are key to this system's function?
- Identify any way(s) in which one might obtain false or otherwise misleading measurements from test equipment in this system.
- What will happen if (component X) fails (open/shorted/etc.)?
- What would have to occur to make this system unsafe?

3.1 Conceptual reasoning

These questions are designed to stimulate your analytic and synthetic thinking³. In a Socratic discussion with your instructor, the goal is for these questions to prompt an extended dialogue where assumptions are revealed, conclusions are tested, and understanding is sharpened. Your instructor may also pose additional questions based on those assigned, in order to further probe and refine your conceptual understanding.

Questions that follow are presented to challenge and probe your understanding of various concepts presented in the tutorial. These questions are intended to serve as a guide for the Socratic dialogue between yourself and the instructor. Your instructor's task is to ensure you have a sound grasp of these concepts, and the questions contained in this document are merely a means to this end. Your instructor may, at his or her discretion, alter or substitute questions for the benefit of tailoring the discussion to each student's needs. The only absolute requirement is that each student is challenged and assessed at a level equal to or greater than that represented by the documented questions.

It is far more important that you convey your *reasoning* than it is to simply convey a correct answer. For this reason, you should refrain from researching other information sources to answer questions. What matters here is that *you* are doing the thinking. If the answer is incorrect, your instructor will work with you to correct it through proper reasoning. A correct answer without an adequate explanation of how you derived that answer is unacceptable, as it does not aid the learning or assessment process.

You will note a conspicuous lack of answers given for these conceptual questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your conceptual answers, where applicable, is to use circuit simulation software to explore the effects of changes made to circuits. For example, if one of these conceptual questions challenges you to predict the effects of altering some component parameter in a circuit, you may check the validity of your work by simulating that same parameter change within software and seeing if the results agree.

³*Analytical* thinking involves the “disassembly” of an idea into its constituent parts, analogous to dissection. *Synthetic* thinking involves the “assembly” of a new idea comprised of multiple concepts, analogous to construction. Both activities are high-level cognitive skills, extremely important for effective problem-solving, necessitating frequent challenge and regular practice to fully develop.

3.1.1 Reading outline and reflections

“Reading maketh a full man; conference a ready man; and writing an exact man” – Francis Bacon

Francis Bacon’s advice is a blueprint for effective education: reading provides the learner with knowledge, writing focuses the learner’s thoughts, and critical dialogue equips the learner to confidently communicate and apply their learning. Independent acquisition and application of knowledge is a powerful skill, well worth the effort to cultivate. To this end, students should read these educational resources closely, journal their own reflections on the reading, and discuss in detail their findings with classmates and instructor(s). You should be able to do all of the following after reading any instructional text:

- ☒ Briefly SUMMARIZE THE TEXT in the form of a journal entry documenting your learning as you progress through the course of study. Share this summary in dialogue with your classmates and instructor. Journaling is an excellent self-test of thorough reading because you cannot clearly express what you have not read or did not comprehend.
- ☒ Demonstrate ACTIVE READING STRATEGIES, including verbalizing your impressions as you read, simplifying long passages to convey the same ideas using fewer words, annotating text and illustrations with your own interpretations, working through mathematical examples shown in the text, cross-referencing passages with relevant illustrations and/or other passages, identifying problem-solving strategies applied by the author, etc. Technical reading is a special case of problem-solving, and so these strategies work precisely because they help solve any problem: paying attention to your own thoughts (metacognition), eliminating unnecessary complexities, identifying what makes sense, paying close attention to details, drawing connections between separated facts, and noting the successful strategies of others.
- ☒ Identify IMPORTANT THEMES, especially GENERAL LAWS and PRINCIPLES, expounded in the text and express them in the simplest of terms as though you were teaching an intelligent child. This emphasizes connections between related topics and develops your ability to communicate complex ideas to anyone.
- ☒ Form YOUR OWN QUESTIONS based on the reading, and then pose them to your instructor and classmates for their consideration. Anticipate both correct and incorrect answers, the incorrect answer(s) assuming one or more plausible misconceptions. This helps you view the subject from different perspectives to grasp it more fully.
- ☒ Devise EXPERIMENTS to test claims presented in the reading, or to disprove misconceptions. Predict possible outcomes of these experiments, and evaluate their meanings: what result(s) would confirm, and what would constitute disproof? Running mental simulations and evaluating results is essential to scientific and diagnostic reasoning.
- ☒ Specifically identify any points you found CONFUSING. The reason for doing this is to help diagnose misconceptions and overcome barriers to learning.

3.1.2 Foundational concepts

Correct analysis and diagnosis of electric circuits begins with a proper understanding of some basic concepts. The following is a list of some important concepts referenced in this module's full tutorial. Define each of them in your own words, and be prepared to illustrate each of these concepts with a description of a practical example and/or a live demonstration.

Polyphase

Phasor diagram

Phase sequence

Wye connection

Delta connection

Phase versus line

Wye voltages and currents

Delta voltages and currents

Advantages of polyphase electrical systems

Power in a three-phase system

Wye grounding

Delta grounding

3.1.3 Identifying windings by inspection

Identify the low-voltage and high-voltage terminals on this three-phase power transformer:



Challenges

- Why do you suppose each wire has a different color of tape on it?

3.1.4 Transformer nameplate inspection

Answer the following questions based on an inspection of this transformer's nameplate:



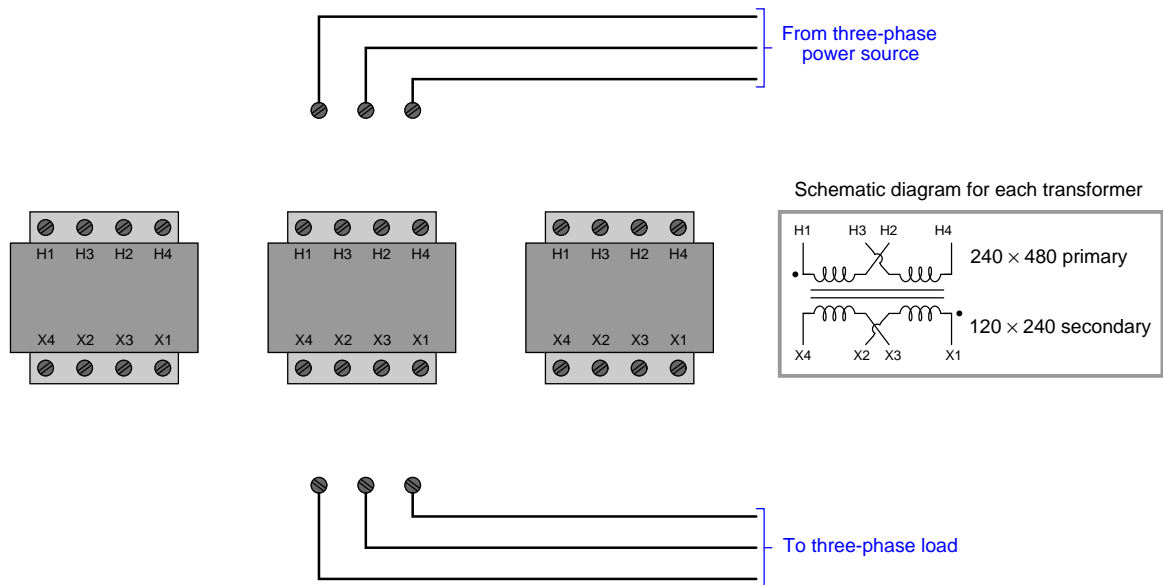
- What is its power rating?
- Does the output lead, lag, or is it in-phase with the input?
- What is the purpose of the “taps” on one set of windings?

Challenges

- Is this transformer *additive* or *subtractive*?

3.1.5 480 Volts to 208 Volts

Suppose you need to connect jumper wires to configure these three transformers to step down 480 VAC three-phase line power to 208 VAC three-phase load power:



Sketch all necessary connections to achieve this goal.

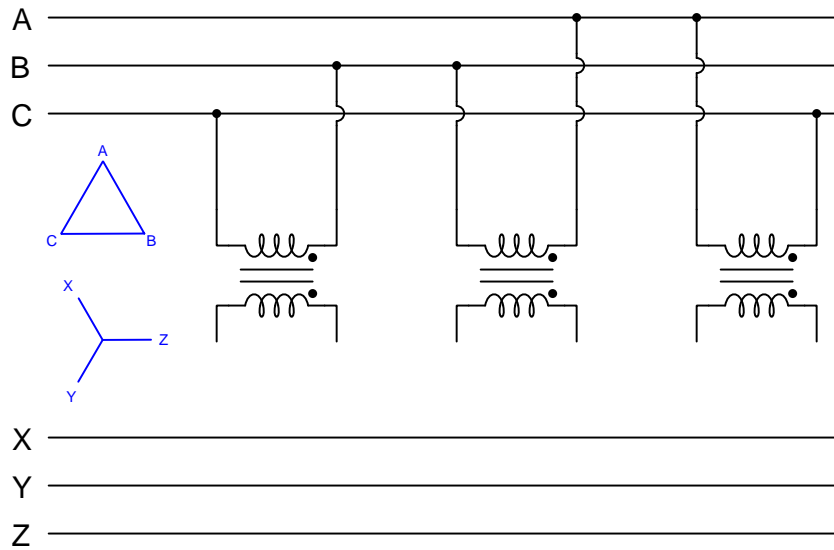
Challenges

- Explain how the “crossed” terminal connections (H2-H3 and X2-X3) are helpful when wiring these transformers using metal “jumpers” the same length as the distance between two adjacent terminal screws.

3.1.6 Sketching winding connections

Sketch the necessary wire connections to create a three-phase transformer bank fulfilling the phasor diagrams shown to the left of the transformers, where ABC is the primary bus and XYZ is the secondary bus. For each of the examples write the phase sequences (phase rotations) for both 3-phase busses, and identify whether the lower bus *leads*, *lags*, or is *in-phase* with the upper bus.

Example #1

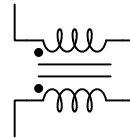
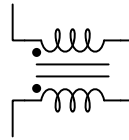
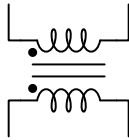
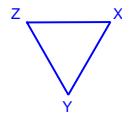
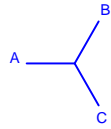


Example #2

A _____

B _____

C _____



X _____

Y _____

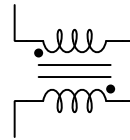
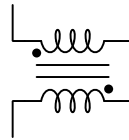
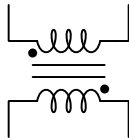
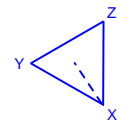
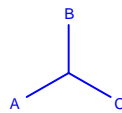
Z _____

Example #3

A _____

B _____

C _____



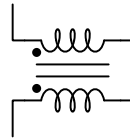
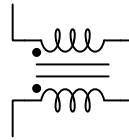
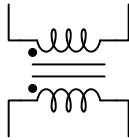
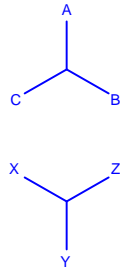
X _____

Y _____

Z _____

Example #4

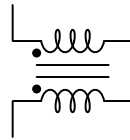
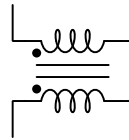
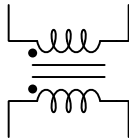
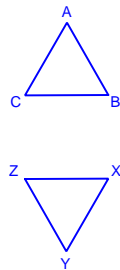
A _____
 B _____
 C _____



X _____
 Y _____
 Z _____

Example #5

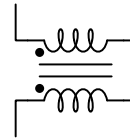
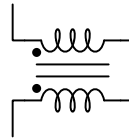
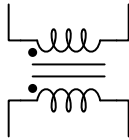
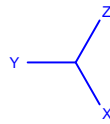
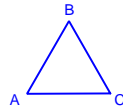
A _____
 B _____
 C _____



X _____
 Y _____
 Z _____

Example #6

A _____
 B _____
 C _____



X _____
 Y _____
 Z _____

Challenges

- Identify two currents in any of these circuits that are guaranteed to be equal in value, even if the source and load happened to be imbalanced.
- Identify two currents in any of these circuits that are unequal in value, and explain why one of them is larger than the other.
- Identify two voltages in any of these circuits that are guaranteed to be equal in value, even if the source and load happened to be imbalanced.
- Identify two voltages in any of these circuits that are unequal in value, and explain why one of them is larger than the other.
- Explain the significance of the dashed-line vector in each phasor diagram. What does it represent, and why is it important?

3.1.7 Pole-mounted transformers

Identify the wiring configurations of the following pole-mounted transformers (i.e. whether the secondary windings are connected in Wye or Delta fashion):



Challenges

- Why is the middle transformer in the right-hand bank larger than the other two?

3.2 Quantitative reasoning

These questions are designed to stimulate your computational thinking. In a Socratic discussion with your instructor, the goal is for these questions to reveal your mathematical approach(es) to problem-solving so that good technique and sound reasoning may be reinforced. Your instructor may also pose additional questions based on those assigned, in order to observe your problem-solving firsthand.

Mental arithmetic and estimations are strongly encouraged for all calculations, because without these abilities you will be unable to readily detect errors caused by calculator misuse (e.g. keystroke errors).

You will note a conspicuous lack of answers given for these quantitative questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. My advice is to use circuit simulation software such as SPICE to check the correctness of quantitative answers. Refer to those learning modules within this collection focusing on SPICE to see worked examples which you may use directly as practice problems for your own study, and/or as templates you may modify to run your own analyses and generate your own practice problems.

Completely worked example problems found in the Tutorial may also serve as “test cases⁴” for gaining proficiency in the use of circuit simulation software, and then once that proficiency is gained you will never need to rely⁵ on an answer key!

⁴In other words, set up the circuit simulation software to analyze the same circuit examples found in the Tutorial. If the simulated results match the answers shown in the Tutorial, it confirms the simulation has properly run. If the simulated results disagree with the Tutorial’s answers, something has been set up incorrectly in the simulation software. Using every Tutorial as practice in this way will quickly develop proficiency in the use of circuit simulation software.

⁵This approach is perfectly in keeping with the instructional philosophy of these learning modules: *teaching students to be self-sufficient thinkers*. Answer keys can be useful, but it is even more useful to your long-term success to have a set of tools on hand for checking your own work, because once you have left school and are on your own, there will no longer be “answer keys” available for the problems you will have to solve.

3.2.1 Miscellaneous physical constants

Note: constants shown in **bold** type are *exact*, not approximations. Values inside of parentheses show one standard deviation (σ) of uncertainty in the final digits: for example, the magnetic permeability of free space value given as $1.25663706212(19) \times 10^{-6}$ H/m represents a center value (i.e. the location parameter) of $1.25663706212 \times 10^{-6}$ Henrys per meter with one standard deviation of uncertainty equal to $0.0000000000019 \times 10^{-6}$ Henrys per meter.

Avogadro's number (N_A) = **$6.02214076 \times 10^{23}$** per mole (mol^{-1})

Boltzmann's constant (k) = **1.380649×10^{-23}** Joules per Kelvin (J/K)

Electronic charge (e) = **$1.602176634 \times 10^{-19}$** Coulomb (C)

Faraday constant (F) = **$96,485.33212...$** $\times 10^4$ Coulombs per mole (C/mol)

Magnetic permeability of free space (μ_0) = $1.25663706212(19) \times 10^{-6}$ Henrys per meter (H/m)

Electric permittivity of free space (ϵ_0) = $8.8541878128(13) \times 10^{-12}$ Farads per meter (F/m)

Characteristic impedance of free space (Z_0) = $376.730313668(57)$ Ohms (Ω)

Gravitational constant (G) = $6.67430(15) \times 10^{-11}$ cubic meters per kilogram-seconds squared ($\text{m}^3/\text{kg}\cdot\text{s}^2$)

Molar gas constant (R) = **$8.314462618...$** Joules per mole-Kelvin (J/mol-K) = $0.08205746(14)$ liters-atmospheres per mole-Kelvin

Planck constant (h) = **$6.62607015 \times 10^{-34}$** joule-seconds (J-s)

Stefan-Boltzmann constant (σ) = **$5.670374419...$** $\times 10^{-8}$ Watts per square meter-Kelvin⁴ ($\text{W}/\text{m}^2\cdot\text{K}^4$)

Speed of light in a vacuum (c) = **$299,792,458$** meters per second (m/s) = 186282.4 miles per second (mi/s)

Note: All constants taken from NIST data "Fundamental Physical Constants – Complete Listing", from <http://physics.nist.gov/constants>, National Institute of Standards and Technology (NIST), 2018 CODATA Adjustment.

3.2.2 Introduction to spreadsheets

A powerful computational tool you are encouraged to use in your work is a *spreadsheet*. Available on most personal computers (e.g. Microsoft Excel), *spreadsheet* software performs numerical calculations based on number values and formulae entered into cells of a grid. This grid is typically arranged as lettered columns and numbered rows, with each cell of the grid identified by its column/row coordinates (e.g. cell B3, cell A8). Each cell may contain a string of text, a number value, or a mathematical formula. The spreadsheet automatically updates the results of all mathematical formulae whenever the entered number values are changed. This means it is possible to set up a spreadsheet to perform a series of calculations on entered data, and those calculations will be re-done by the computer any time the data points are edited in any way.

For example, the following spreadsheet calculates average speed based on entered values of distance traveled and time elapsed:

	A	B	C	D
1	Distance traveled	46.9	Kilometers	
2	Time elapsed	1.18	Hours	
3	Average speed	= B1 / B2	km/h	
4				
5				

Text labels contained in cells A1 through A3 and cells C1 through C3 exist solely for readability and are not involved in any calculations. Cell B1 contains a sample distance value while cell B2 contains a sample time value. The formula for computing speed is contained in cell B3. Note how this formula begins with an “equals” symbol (=), references the values for distance and speed by lettered column and numbered row coordinates (B1 and B2), and uses a forward slash symbol for division (/). The coordinates B1 and B2 function as *variables*⁶ would in an algebraic formula.

When this spreadsheet is executed, the numerical value 39.74576 will appear in cell B3 rather than the formula = B1 / B2, because 39.74576 is the computed speed value given 46.9 kilometers traveled over a period of 1.18 hours. If a different numerical value for distance is entered into cell B1 or a different value for time is entered into cell B2, cell B3’s value will automatically update. All you need to do is set up the given values and any formulae into the spreadsheet, and the computer will do all the calculations for you.

Cell B3 may be referenced by other formulae in the spreadsheet if desired, since it is a variable just like the given values contained in B1 and B2. This means it is possible to set up an entire chain of calculations, one dependent on the result of another, in order to arrive at a final value. The arrangement of the given data and formulae need not follow any pattern on the grid, which means you may place them anywhere.

⁶Spreadsheets may also provide means to attach text labels to cells for use as variable names (Microsoft Excel simply calls these labels “names”), but for simple spreadsheets such as those shown here it’s usually easier just to use the standard coordinate naming for each cell.

Common⁷ arithmetic operations available for your use in a spreadsheet include the following:

- Addition (+)
- Subtraction (-)
- Multiplication (*)
- Division (/)
- Powers (^)
- Square roots (sqrt())
- Logarithms (ln() , log10())

Parentheses may be used to ensure⁸ proper order of operations within a complex formula. Consider this example of a spreadsheet implementing the *quadratic formula*, used to solve for roots of a polynomial expression in the form of $ax^2 + bx + c$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

	A	B
1	x_1	= (-B4 + sqrt((B4^2) - (4*B3*B5))) / (2*B3)
2	x_2	= (-B4 - sqrt((B4^2) - (4*B3*B5))) / (2*B3)
3	a =	9
4	b =	5
5	c =	-2

This example is configured to compute roots⁹ of the polynomial $9x^2 + 5x - 2$ because the values of 9, 5, and -2 have been inserted into cells B3, B4, and B5, respectively. Once this spreadsheet has been built, though, it may be used to calculate the roots of *any* second-degree polynomial expression simply by entering the new a , b , and c coefficients into cells B3 through B5. The numerical values appearing in cells B1 and B2 will be automatically updated by the computer immediately following any changes made to the coefficients.

⁷Modern spreadsheet software offers a bewildering array of mathematical functions you may use in your computations. I recommend you consult the documentation for your particular spreadsheet for information on operations other than those listed here.

⁸Spreadsheet programs, like text-based programming languages, are designed to follow standard order of operations by default. However, my personal preference is to use parentheses even where strictly unnecessary just to make it clear to any other person viewing the formula what the intended order of operations is.

⁹Reviewing some algebra here, a *root* is a value for x that yields an overall value of zero for the polynomial. For this polynomial ($9x^2 + 5x - 2$) the two roots happen to be $x = 0.269381$ and $x = -0.82494$, with these values displayed in cells B1 and B2, respectively upon execution of the spreadsheet.

Alternatively, one could break up the long quadratic formula into smaller pieces like this:

$$y = \sqrt{b^2 - 4ac} \quad z = 2a$$

$$x = \frac{-b \pm y}{z}$$

	A	B	C
1	x_1	= (-B4 + C1) / C2	= sqrt ((B4^2) - (4*B3*B5))
2	x_2	= (-B4 - C1) / C2	= 2*B3
3	a =	9	
4	b =	5	
5	c =	-2	

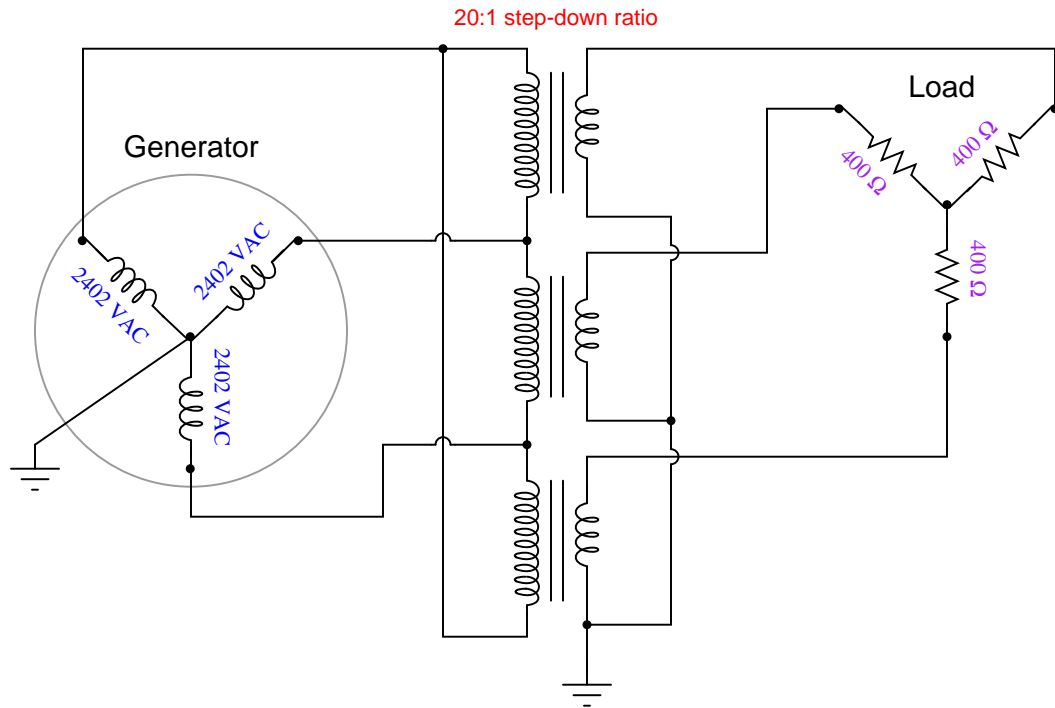
Note how the square-root term (y) is calculated in cell C1, and the denominator term (z) in cell C2. This makes the two final formulae (in cells B1 and B2) simpler to interpret. The positioning of all these cells on the grid is completely arbitrary¹⁰ – all that matters is that they properly reference each other in the formulae.

Spreadsheets are particularly useful for situations where the same set of calculations representing a circuit or other system must be repeated for different initial conditions. The power of a spreadsheet is that it automates what would otherwise be a tedious set of calculations. One specific application of this is to simulate the effects of various components within a circuit failing with abnormal values (e.g. a shorted resistor simulated by making its value nearly zero; an open resistor simulated by making its value extremely large). Another application is analyzing the behavior of a circuit design given new components that are out of specification, and/or aging components experiencing drift over time.

¹⁰My personal preference is to locate all the “given” data in the upper-left cells of the spreadsheet grid (each data point flanked by a sensible name in the cell to the left and units of measurement in the cell to the right as illustrated in the first distance/time spreadsheet example), sometimes coloring them in order to clearly distinguish which cells contain entered data versus which cells contain computed results from formulae. I like to place all formulae in cells below the given data, and try to arrange them in logical order so that anyone examining my spreadsheet will be able to figure out *how* I constructed a solution. This is a general principle I believe all computer programmers should follow: *document and arrange your code to make it easy for other people to learn from it.*

3.2.3 Balanced Delta source and Wye load with transformers

Calculate the following parameters in this three-phase (balanced) AC power system:



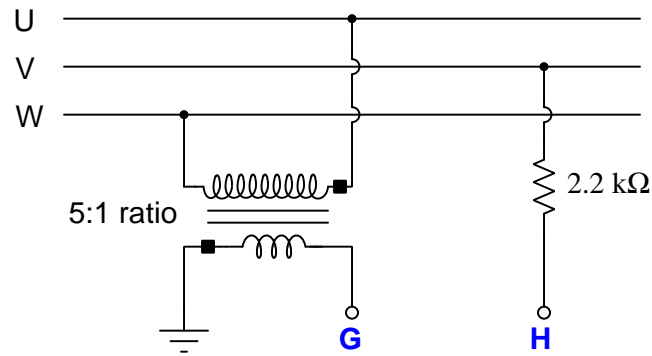
- V_{line} (generator) =
- I_{line} (generator) =
- V_{line} (load) =
- I_{line} (load) =
- P_{total} =

Challenges

- Suppose someone incorrectly calculates a load phase voltage of 120 Volts. Explain the nature of the misconception leading to this wrong result.

3.2.4 Ground-referenced voltage calculations

Calculate the V_{GH} in this circuit, expressing your answer in polar form. Assume an UVW phase rotation with $V_U = 277$ Volts $\angle -15^\circ$:



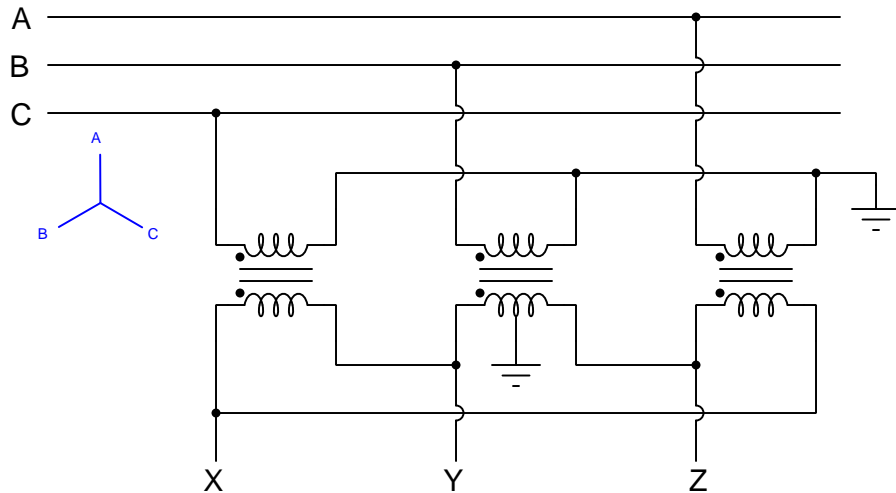
$V_{GH} =$

Challenges

- Is V_{HG} equal to V_{GH} ?

3.2.5 Center-grounded Delta secondary

Calculate the following voltages (in polar form) for this transformer bank, given $V_A = 120 \text{ V} \angle 90^\circ$ and a transformer turns ratio of 1:1.



$$V_X =$$

$$V_{AB} =$$

$$V_{ZY} =$$

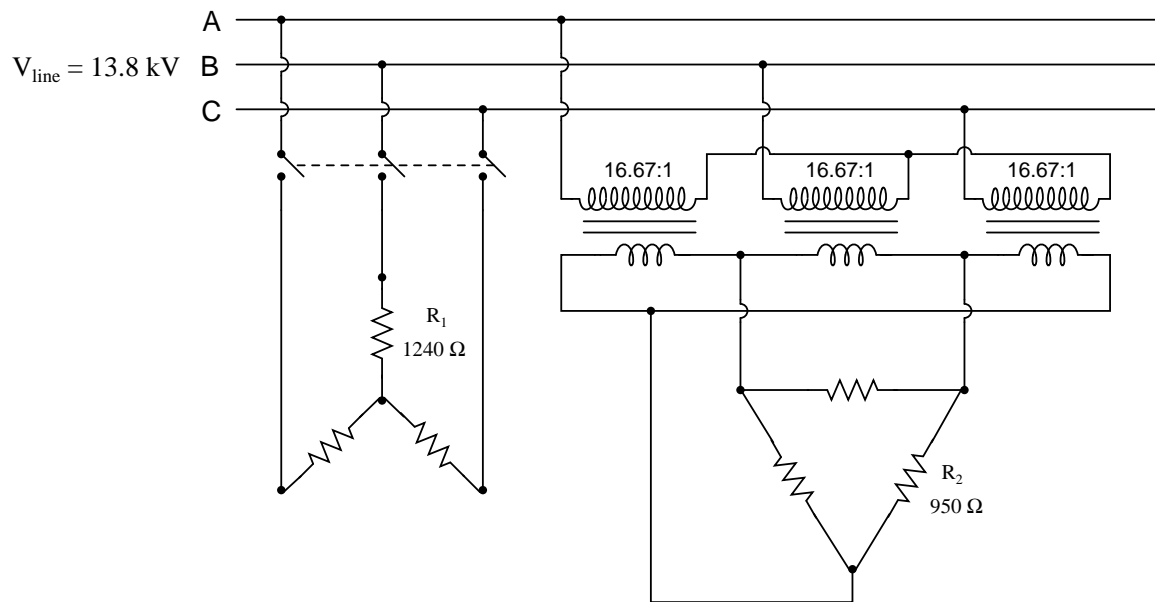
$$V_{XC} =$$

Challenges

- Identify an alternative grounding method for the secondary windings than what is shown here.

3.2.6 Load calculations

Calculate the operating current through each of the load resistances shown in this circuit (assuming each three-phase load is balanced):



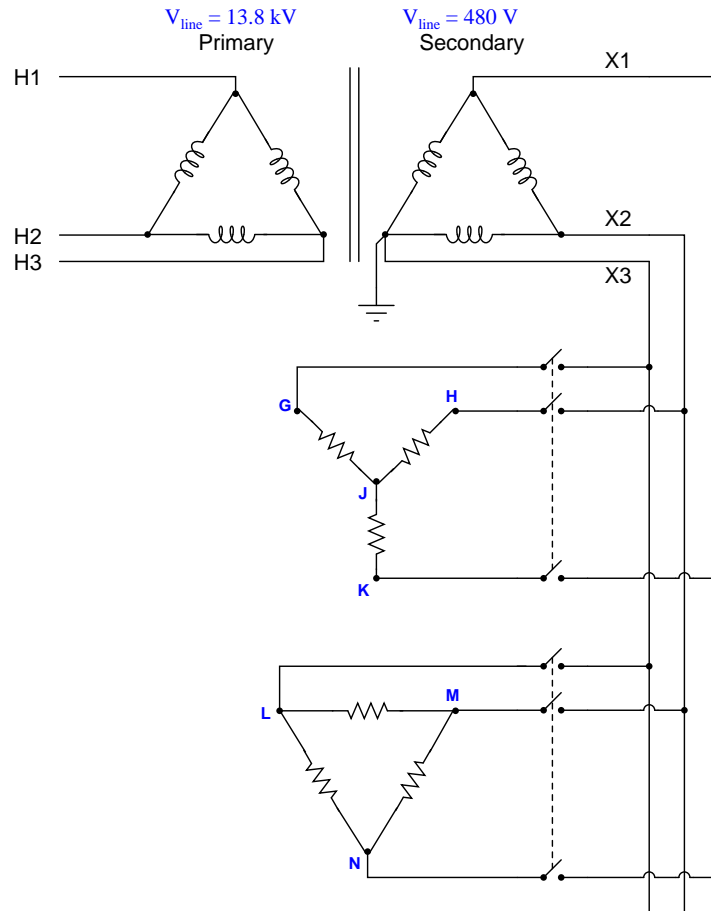
Also, calculate the power dissipated by each load.

Challenges

- Is phase sequence significant for this scenario? Why or why not?

3.2.7 System with multiple loads

A three-phase step-down transformer supplies 480 VAC to a pair of resistive loads. The secondary winding is “corner-grounded” on the X2 leg:



Determine the following phase-to-ground voltages in this system while both loads are energized:

- $V_G =$
- $V_H =$
- $V_J =$
- $V_K =$
- $V_L =$
- $V_M =$

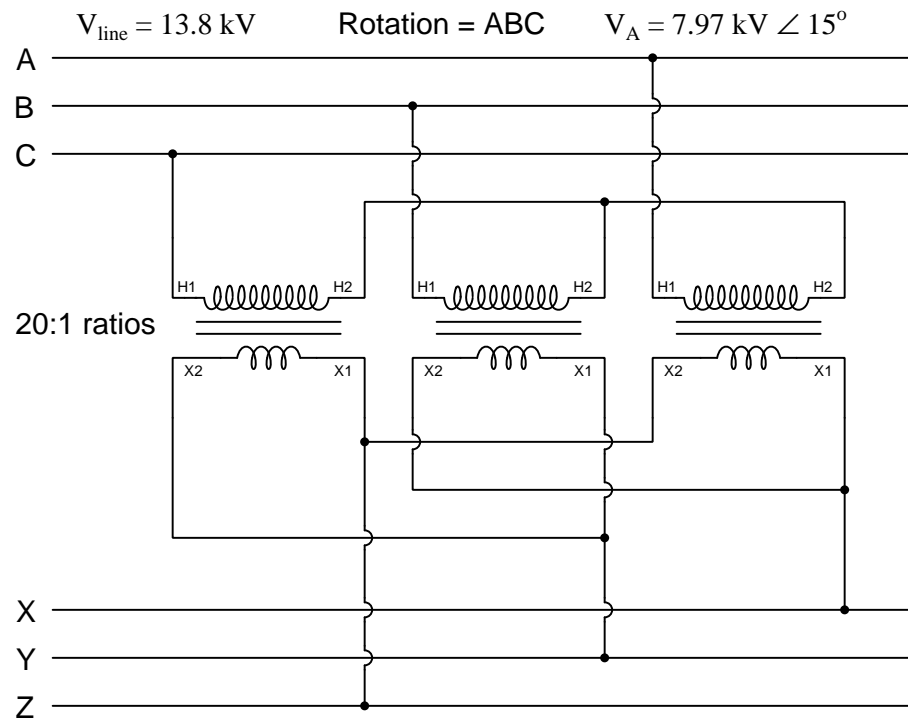
- $V_N =$

Challenges

- Is it safe to ground point J in the Wye-connected load?

3.2.8 Output voltages and phase diagram

Sketch a phasor diagram for this transformer bank's output voltages, and determine the line voltage of the XYZ bus as well as its phase rotation (sequence):

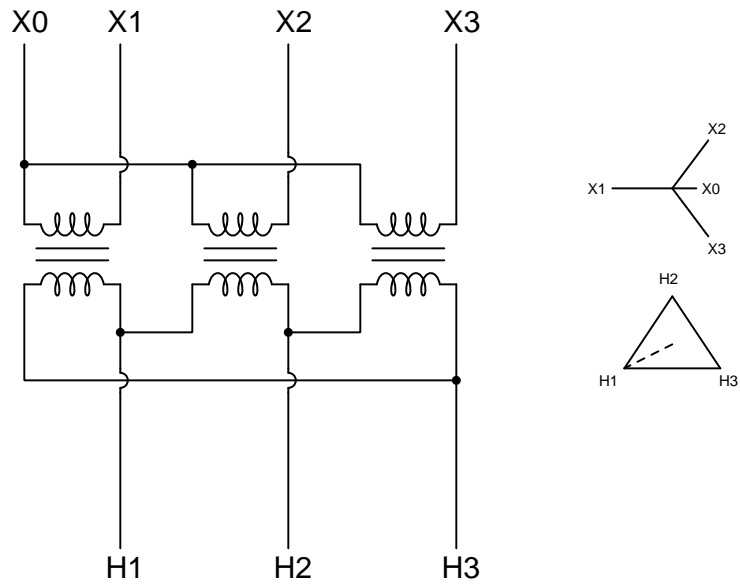


Challenges

- How would the phasor diagram differ if V_A had a phase angle of 0° ?

3.2.9 Nameplate diagram

Examine these schematic and phasor diagrams found on the nameplate of a power transformer, and answer the following questions:



Which is the high-voltage side and which is the low-voltage side of this transformer? How can you tell?

Is the high-voltage side of this transformer *leading*, *lagging*, or *in-phase with* the low-voltage side? How can you tell?

Suppose the turns ratio for each winding pair in this three-phase transformer is 4:1 and the line voltage H1-H2 is 480 Volts. Calculate the line voltage X1-X2.

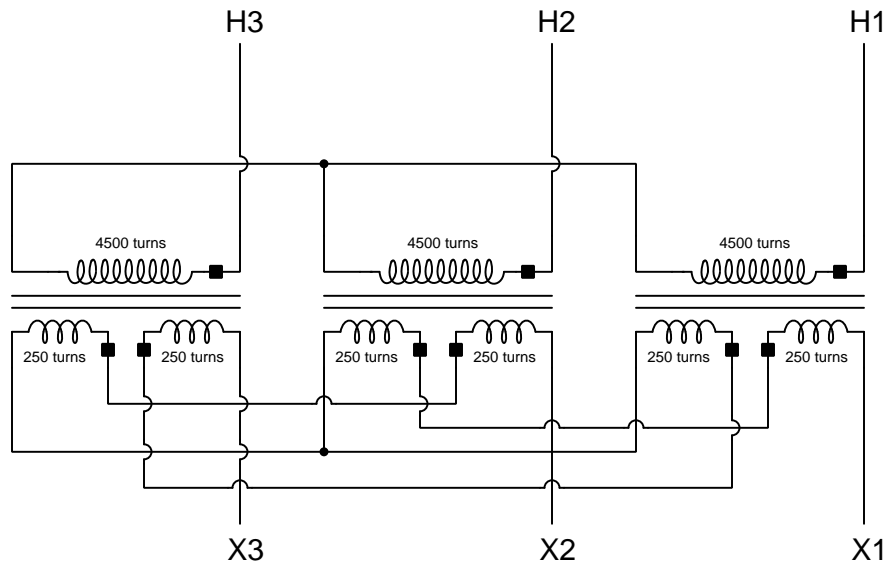
Mark polarity dots for the primary and secondary windings of this transformer in order to produce the phasors shown in the phasor diagram.

Challenges

- Identify the phase rotations of both primary and secondary.

3.2.10 Wye-Zigzag transformer

This three-phase transformer configuration is called a Wye-Zigzag:



Calculate the magnitude and phase angle of V_{X1} , V_{X2} , and V_{X3} assuming V_{H1} is $7.2 \text{ kV} \angle 0^\circ$ and the phase rotation is H1-H2-H3.

$$V_{X1} =$$

$$V_{X2} =$$

$$V_{X3} =$$

Challenges

- Suppose the dual-secondary windings of each transformer had unequal numbers of turns. How would this affect the result?

3.3 Diagnostic reasoning

These questions are designed to stimulate your deductive and inductive thinking, where you must apply general principles to specific scenarios (deductive) and also derive conclusions about the failed circuit from specific details (inductive). In a Socratic discussion with your instructor, the goal is for these questions to reinforce your recall and use of general circuit principles and also challenge your ability to integrate multiple symptoms into a sensible explanation of what's wrong in a circuit. Your instructor may also pose additional questions based on those assigned, in order to further challenge and sharpen your diagnostic abilities.

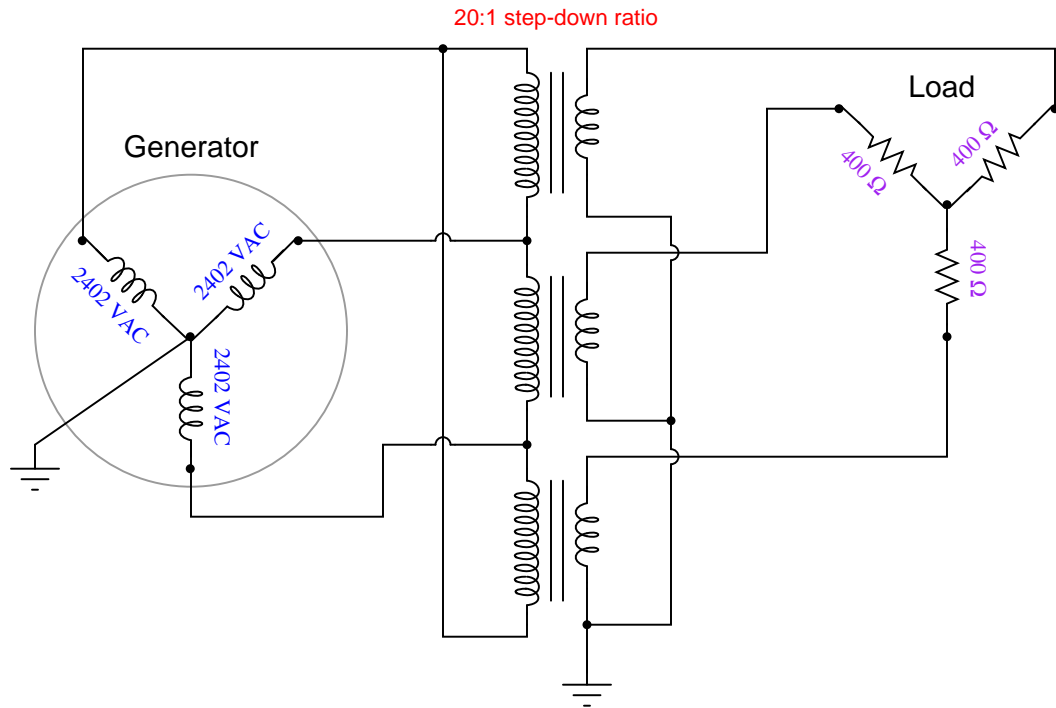
As always, your goal is to fully *explain* your analysis of each problem. Simply obtaining a correct answer is not good enough – you must also demonstrate sound reasoning in order to successfully complete the assignment. Your instructor's responsibility is to probe and challenge your understanding of the relevant principles and analytical processes in order to ensure you have a strong foundation upon which to build further understanding.

You will note a conspicuous lack of answers given for these diagnostic questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your diagnostic answers, where applicable, is to use circuit simulation software to explore the effects of faults placed in circuits. For example, if one of these diagnostic questions requires that you predict the effect of an open or a short in a circuit, you may check the validity of your work by simulating that same fault (substituting a very high resistance in place of that component for an open, and substituting a very low resistance for a short) within software and seeing if the results agree.

3.3.1 Failed transformer winding

Suppose the secondary winding of the middle transformer fails open:



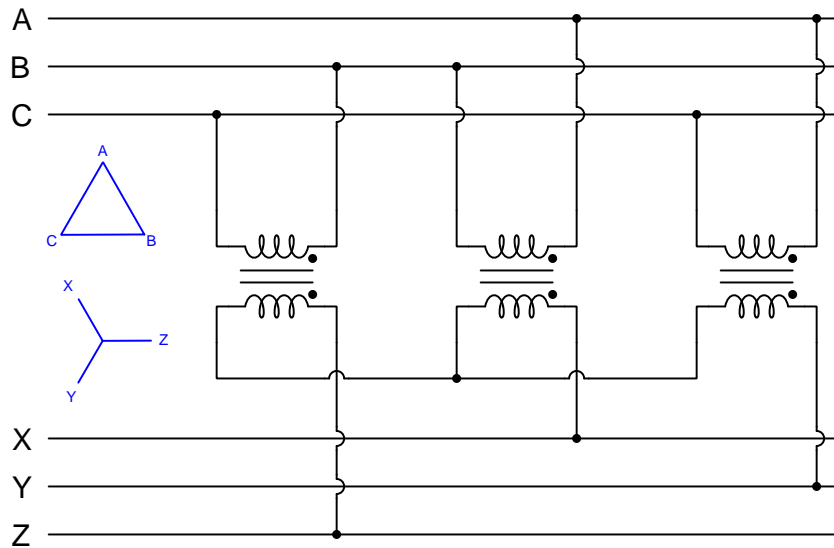
Identify all the consequences of this fault.

Challenges

- Identify a different fault that would result in the same consequences.

3.3.2 Find the mistake(s)

Identify all connection errors in this transformer circuit:



Challenges

- How would the phasor diagram look with the error in place?

Appendix A

Problem-Solving Strategies

The ability to solve complex problems is arguably one of the most valuable skills one can possess, and this skill is particularly important in any science-based discipline.

- Study principles, not procedures. Don't be satisfied with merely knowing how to compute solutions – learn *why* those solutions work.
- Identify what it is you need to solve, identify all relevant data, identify all units of measurement, identify any general principles or formulae linking the given information to the solution, and then identify any “missing pieces” to a solution. Annotate all diagrams with this data.
- Sketch a diagram to help visualize the problem. When building a real system, always devise a plan for that system and analyze its function *before* constructing it.
- Follow the units of measurement and meaning of every calculation. If you are ever performing mathematical calculations as part of a problem-solving procedure, and you find yourself unable to apply each and every intermediate result to some aspect of the problem, it means you don't understand what you are doing. Properly done, every mathematical result should have practical meaning for the problem, and not just be an abstract number. You should be able to identify the proper units of measurement for each and every calculated result, and show where that result fits into the problem.
- Perform “thought experiments” to explore the effects of different conditions for theoretical problems. When troubleshooting real systems, perform *diagnostic tests* rather than visually inspecting for faults, the best diagnostic test being the one giving you the most information about the nature and/or location of the fault with the fewest steps.
- Simplify the problem until the solution becomes obvious, and then use that obvious case as a model to follow in solving the more complex version of the problem.
- Check for exceptions to see if your solution is incorrect or incomplete. A good solution will work for *all* known conditions and criteria. A good example of this is the process of testing scientific hypotheses: the task of a scientist is not to find support for a new idea, but rather to *challenge* that new idea to see if it holds up under a battery of tests. The philosophical

principle of *reductio ad absurdum* (i.e. disproving a general idea by finding a specific case where it fails) is useful here.

- Work “backward” from a hypothetical solution to a new set of given conditions.
- Add quantities to problems that are qualitative in nature, because sometimes a little math helps illuminate the scenario.
- Sketch graphs illustrating how variables relate to each other. These may be quantitative (i.e. with realistic number values) or qualitative (i.e. simply showing increases and decreases).
- Treat quantitative problems as qualitative in order to discern the relative magnitudes and/or directions of change of the relevant variables. For example, try determining what happens if a certain variable were to increase or decrease before attempting to precisely calculate quantities: how will each of the dependent variables respond, by increasing, decreasing, or remaining the same as before?
- Consider limiting cases. This works especially well for qualitative problems where you need to determine which direction a variable will change. Take the given condition and magnify that condition to an extreme degree as a way of simplifying the direction of the system’s response.
- Check your work. This means regularly testing your conclusions to see if they make sense. This does *not* mean repeating the same steps originally used to obtain the conclusion(s), but rather to use some other means to check validity. Simply repeating procedures often leads to *repeating the same errors* if any were made, which is why alternative paths are better.

Appendix B

Instructional philosophy

“The unexamined circuit is not worth energizing” – Socrates (if he had taught electricity)

These learning modules, although useful for self-study, were designed to be used in a formal learning environment where a subject-matter expert challenges students to digest the content and exercise their critical thinking abilities in the answering of questions and in the construction and testing of working circuits.

The following principles inform the instructional and assessment philosophies embodied in these learning modules:

- The first goal of education is to enhance clear and independent thought, in order that every student reach their fullest potential in a highly complex and inter-dependent world. Robust reasoning is *always* more important than particulars of any subject matter, because its application is universal.
- Literacy is fundamental to independent learning and thought because text continues to be the most efficient way to communicate complex ideas over space and time. Those who cannot read with ease are limited in their ability to acquire knowledge and perspective.
- Articulate communication is fundamental to work that is complex and interdisciplinary.
- Faulty assumptions and poor reasoning are best corrected through challenge, not presentation. The rhetorical technique of *reductio ad absurdum* (disproving an assertion by exposing an absurdity) works well to discipline student’s minds, not only to correct the problem at hand but also to learn how to detect and correct future errors.
- Important principles should be repeatedly explored and widely applied throughout a course of study, not only to reinforce their importance and help ensure their mastery, but also to showcase the interconnectedness and utility of knowledge.

These learning modules were expressly designed to be used in an “inverted” teaching environment¹ where students first read the introductory and tutorial chapters on their own, then individually attempt to answer the questions and construct working circuits according to the experiment and project guidelines. The instructor never lectures, but instead meets regularly with each individual student to review their progress, answer questions, identify misconceptions, and challenge the student to new depths of understanding through further questioning. Regular meetings between instructor and student should resemble a Socratic² dialogue, where questions serve as scalpels to dissect topics and expose assumptions. The student passes each module only after consistently demonstrating their ability to logically analyze and correctly apply all major concepts in each question or project/experiment. The instructor must be vigilant in probing each student’s understanding to ensure they are truly *reasoning* and not just *memorizing*. This is why “Challenge” points appear throughout, as prompts for students to think deeper about topics and as starting points for instructor queries. Sometimes these challenge points require additional knowledge that hasn’t been covered in the series to answer in full. This is okay, as the major purpose of the Challenges is to stimulate analysis and synthesis on the part of each student.

The instructor must possess enough mastery of the subject matter and awareness of students’ reasoning to generate their own follow-up questions to practically any student response. Even completely correct answers given by the student should be challenged by the instructor for the purpose of having students practice articulating their thoughts and defending their reasoning. Conceptual errors committed by the student should be exposed and corrected not by direct instruction, but rather by reducing the errors to an absurdity³ through well-chosen questions and thought experiments posed by the instructor. Becoming proficient at this style of instruction requires time and dedication, but the positive effects on critical thinking for both student and instructor are spectacular.

An inspection of these learning modules reveals certain unique characteristics. One of these is a bias toward thorough explanations in the tutorial chapters. Without a live instructor to explain concepts and applications to students, the text itself must fulfill this role. This philosophy results in lengthier explanations than what you might typically find in a textbook, each step of the reasoning process fully explained, including footnotes addressing common questions and concerns students raise while learning these concepts. Each tutorial seeks to not only explain each major concept in sufficient detail, but also to explain the logic of each concept and how each may be developed

¹In a traditional teaching environment, students first encounter new information via *lecture* from an expert, and then independently apply that information via *homework*. In an “inverted” course of study, students first encounter new information via *homework*, and then independently apply that information under the scrutiny of an expert. The expert’s role in lecture is to simply *explain*, but the expert’s role in an inverted session is to *challenge*, *critique*, and if necessary *explain* where gaps in understanding still exist.

²Socrates is a figure in ancient Greek philosophy famous for his unflinching style of questioning. Although he authored no texts, he appears as a character in Plato’s many writings. The essence of Socratic philosophy is to leave no question unexamined and no point of view unchallenged. While purists may argue a topic such as electric circuits is too narrow for a true Socratic-style dialogue, I would argue that the essential thought processes involved with scientific reasoning on *any* topic are not far removed from the Socratic ideal, and that students of electricity and electronics would do very well to challenge assumptions, pose thought experiments, identify fallacies, and otherwise employ the arsenal of critical thinking skills modeled by Socrates.

³This rhetorical technique is known by the Latin phrase *reductio ad absurdum*. The concept is to expose errors by counter-example, since only one solid counter-example is necessary to disprove a universal claim. As an example of this, consider the common misconception among beginning students of electricity that voltage cannot exist without current. One way to apply *reductio ad absurdum* to this statement is to ask how much current passes through a fully-charged battery connected to nothing (i.e. a clear example of voltage existing without current).

from “first principles”. Again, this reflects the goal of developing clear and independent thought in students’ minds, by showing how clear and logical thought was used to forge each concept. Students benefit from witnessing a model of clear thinking in action, and these tutorials strive to be just that.

Another characteristic of these learning modules is a lack of step-by-step instructions in the Project and Experiment chapters. Unlike many modern workbooks and laboratory guides where step-by-step instructions are prescribed for each experiment, these modules take the approach that students must learn to closely read the tutorials and apply their own reasoning to identify the appropriate experimental steps. Sometimes these steps are plainly declared in the text, just not as a set of enumerated points. At other times certain steps are implied, an example being assumed competence in test equipment use where the student should not need to be told *again* how to use their multimeter because that was thoroughly explained in previous lessons. In some circumstances no steps are given at all, leaving the entire procedure up to the student.

This lack of prescription is not a flaw, but rather a feature. Close reading and clear thinking are foundational principles of this learning series, and in keeping with this philosophy all activities are designed to *require* those behaviors. Some students may find the lack of prescription frustrating, because it demands more from them than what their previous educational experiences required. This frustration should be interpreted as an unfamiliarity with autonomous thinking, a problem which must be corrected if the student is ever to become a self-directed learner and effective problem-solver. Ultimately, the need for students to read closely and think clearly is more important both in the near-term and far-term than any specific facet of the subject matter at hand. If a student takes longer than expected to complete a module because they are forced to outline, digest, and reason on their own, so be it. The future gains enjoyed by developing this mental discipline will be well worth the additional effort and delay.

Another feature of these learning modules is that they do not treat topics in isolation. Rather, important concepts are introduced early in the series, and appear repeatedly as stepping-stones toward other concepts in subsequent modules. This helps to avoid the “compartmentalization” of knowledge, demonstrating the inter-connectedness of concepts and simultaneously reinforcing them. Each module is fairly complete in itself, reserving the beginning of its tutorial to a review of foundational concepts.

This methodology of assigning text-based modules to students for digestion and then using Socratic dialogue to assess progress and hone students’ thinking was developed over a period of several years by the author with his Electronics and Instrumentation students at the two-year college level. While decidedly unconventional and sometimes even unsettling for students accustomed to a more passive lecture environment, this instructional philosophy has proven its ability to convey conceptual mastery, foster careful analysis, and enhance employability so much better than lecture that the author refuses to ever teach by lecture again.

Problems which often go undiagnosed in a lecture environment are laid bare in this “inverted” format where students must articulate and logically defend their reasoning. This, too, may be unsettling for students accustomed to lecture sessions where the instructor cannot tell for sure who comprehends and who does not, and this vulnerability necessitates sensitivity on the part of the “inverted” session instructor in order that students never feel discouraged by having their errors exposed. *Everyone* makes mistakes from time to time, and learning is a lifelong process! Part of the instructor’s job is to build a culture of learning among the students where errors are not seen as shameful, but rather as opportunities for progress.

To this end, instructors managing courses based on these modules should adhere to the following principles:

- Student questions are always welcome and demand thorough, honest answers. The only type of question an instructor should refuse to answer is one the student should be able to easily answer on their own. Remember, *the fundamental goal of education is for each student to learn to think clearly and independently*. This requires hard work on the part of the student, which no instructor should ever circumvent. Anything done to bypass the student's responsibility to do that hard work ultimately limits that student's potential and thereby does real harm.
- It is not only permissible, but encouraged, to answer a student's question by asking questions in return, these follow-up questions designed to guide the student to reach a correct answer through their own reasoning.
- All student answers demand to be challenged by the instructor and/or by other students. This includes both correct and incorrect answers – the goal is to practice the articulation and defense of one's own reasoning.
- No reading assignment is deemed complete unless and until the student demonstrates their ability to accurately summarize the major points in their own terms. Recitation of the original text is unacceptable. This is why every module contains an "Outline and reflections" question as well as a "Foundational concepts" question in the Conceptual reasoning section, to prompt reflective reading.
- No assigned question is deemed answered unless and until the student demonstrates their ability to consistently and correctly apply the concepts to *variations* of that question. This is why module questions typically contain multiple "Challenges" suggesting different applications of the concept(s) as well as variations on the same theme(s). Instructors are encouraged to devise as many of their own "Challenges" as they are able, in order to have a multitude of ways ready to probe students' understanding.
- No assigned experiment or project is deemed complete unless and until the student demonstrates the task in action. If this cannot be done "live" before the instructor, video-recordings showing the demonstration are acceptable. All relevant safety precautions must be followed, all test equipment must be used correctly, and the student must be able to properly explain all results. The student must also successfully answer all Challenges presented by the instructor for that experiment or project.

Students learning from these modules would do well to abide by the following principles:

- No text should be considered fully and adequately read unless and until you can express every idea *in your own words, using your own examples*.
- You should always articulate your thoughts as you read the text, noting points of agreement, confusion, and epiphanies. Feel free to print the text on paper and then write your notes in the margins. Alternatively, keep a journal for your own reflections as you read. This is truly a helpful tool when digesting complicated concepts.
- Never take the easy path of highlighting or underlining important text. Instead, *summarize* and/or *comment* on the text using your own words. This actively engages your mind, allowing you to more clearly perceive points of confusion or misunderstanding on your own.
- A very helpful strategy when learning new concepts is to place yourself in the role of a teacher, if only as a mental exercise. Either explain what you have recently learned to someone else, or at least *imagine* yourself explaining what you have learned to someone else. The simple act of having to articulate new knowledge and skill forces you to take on a different perspective, and will help reveal weaknesses in your understanding.
- Perform each and every mathematical calculation and thought experiment shown in the text on your own, referring back to the text to see that your results agree. This may seem trivial and unnecessary, but it is critically important to ensuring you actually understand what is presented, especially when the concepts at hand are complicated and easy to misunderstand. Apply this same strategy to become proficient in the use of *circuit simulation software*, checking to see if your simulated results agree with the results shown in the text.
- Above all, recognize that learning is hard work, and that a certain level of frustration is unavoidable. There are times when you will struggle to grasp some of these concepts, and that struggle is a natural thing. Take heart that it will yield with persistent and varied⁴ effort, and never give up!

Students interested in using these modules for self-study will also find them beneficial, although the onus of responsibility for thoroughly reading and answering questions will of course lie with that individual alone. If a qualified instructor is not available to challenge students, a workable alternative is for students to form study groups where they challenge⁵ one another.

To high standards of education,

Tony R. Kuphaldt

⁴As the old saying goes, “Insanity is trying the same thing over and over again, expecting different results.” If you find yourself stumped by something in the text, you should attempt a different approach. Alter the thought experiment, change the mathematical parameters, do whatever you can to see the problem in a slightly different light, and then the solution will often present itself more readily.

⁵Avoid the temptation to simply share answers with study partners, as this is really counter-productive to learning. Always bear in mind that the answer to any question is far less important in the long run than the method(s) used to obtain that answer. The goal of education is to empower one’s life through the improvement of clear and independent thought, literacy, expression, and various practical skills.

Appendix C

Tools used

I am indebted to the developers of many open-source software applications in the creation of these learning modules. The following is a list of these applications with some commentary on each.

You will notice a theme common to many of these applications: a bias toward *code*. Although I am by no means an expert programmer in any computer language, I understand and appreciate the flexibility offered by code-based applications where the user (you) enters commands into a plain ASCII text file, which the software then reads and processes to create the final output. Code-based computer applications are by their very nature *extensible*, while WYSIWYG (What You See Is What You Get) applications are generally limited to whatever user interface the developer makes for you.

The GNU/Linux computer operating system

There is so much to be said about Linus Torvalds' **Linux** and Richard Stallman's **GNU** project. First, to credit just these two individuals is to fail to do justice to the *mob* of passionate volunteers who contributed to make this amazing software a reality. I first learned of **Linux** back in 1996, and have been using this operating system on my personal computers almost exclusively since then. It is *free*, it is completely *configurable*, and it permits the continued use of highly efficient **Unix** applications and scripting languages (e.g. shell scripts, Makefiles, **sed**, **awk**) developed over many decades. **Linux** not only provided me with a powerful computing platform, but its open design served to inspire my life's work of creating open-source educational resources.

Bram Moolenaar's **Vim** text editor

Writing code for any code-based computer application requires a *text editor*, which may be thought of as a word processor strictly limited to outputting plain-ASCII text files. Many good text editors exist, and one's choice of text editor seems to be a deeply personal matter within the programming world. I prefer **Vim** because it operates very similarly to **vi** which is ubiquitous on **Unix/Linux** operating systems, and because it may be entirely operated via keyboard (i.e. no mouse required) which makes it fast to use.

Donald Knuth's \TeX typesetting system

Developed in the late 1970's and early 1980's by computer scientist extraordinaire Donald Knuth to typeset his multi-volume magnum opus *The Art of Computer Programming*, this software allows the production of formatted text for screen-viewing or paper printing, all by writing plain-text code to describe how the formatted text is supposed to appear. \TeX is not just a markup language for documents, but it is also a Turing-complete programming language in and of itself, allowing useful algorithms to be created to control the production of documents. Simply put, *\TeX is a programmer's approach to word processing*. Since \TeX is controlled by code written in a plain-text file, this means anyone may read that plain-text file to see exactly how the document was created. This openness afforded by the code-based nature of \TeX makes it relatively easy to learn how other people have created their own \TeX documents. By contrast, examining a beautiful document created in a conventional WYSIWYG word processor such as Microsoft **Word** suggests nothing to the reader about *how* that document was created, or what the user might do to create something similar. As Mr. Knuth himself once quipped, conventional word processing applications should be called WYSIAYG (What You See Is *All* You Get).

Leslie Lamport's \LaTeX extensions to \TeX

Like all true programming languages, \TeX is inherently extensible. So, years after the release of \TeX to the public, Leslie Lamport decided to create a massive extension allowing easier compilation of book-length documents. The result was \LaTeX , which is the markup language used to create all ModEL module documents. You could say that \TeX is to \LaTeX as **C** is to **C++**. This means it is permissible to use any and all \TeX commands within \LaTeX source code, and it all still works. Some of the features offered by \LaTeX that would be challenging to implement in \TeX include automatic index and table-of-content creation.

Tim Edwards' **Xcircuit** drafting program

This wonderful program is what I use to create all the schematic diagrams and illustrations (but not photographic images or mathematical plots) throughout the ModEL project. It natively outputs PostScript format which is a true vector graphic format (this is why the images do not pixellate when you zoom in for a closer view), and it is so simple to use that I have never had to read the manual! Object libraries are easy to create for **Xcircuit**, being plain-text files using PostScript programming conventions. Over the years I have collected a large set of object libraries useful for drawing electrical and electronic schematics, pictorial diagrams, and other technical illustrations.

Gimp graphic image manipulation program

Essentially an open-source clone of Adobe's **PhotoShop**, I use **Gimp** to resize, crop, and convert file formats for all of the photographic images appearing in the **ModEL** modules. Although **Gimp** does offer its own scripting language (called **Script-Fu**), I have never had occasion to use it. Thus, my utilization of **Gimp** to merely crop, resize, and convert graphic images is akin to using a sword to slice bread.

SPICE circuit simulation program

SPICE is to circuit analysis as **T_EX** is to document creation: it is a form of markup language designed to describe a certain object to be processed in plain-ASCII text. When the plain-text "source file" is compiled by the software, it outputs the final result. More modern circuit analysis tools certainly exist, but I prefer **SPICE** for the following reasons: it is *free*, it is *fast*, it is *reliable*, and it is a fantastic tool for *teaching* students of electricity and electronics how to write simple code. I happen to use rather old versions of **SPICE**, version 2g6 being my "go to" application when I only require text-based output. **NGSPICE** (version 26), which is based on Berkeley **SPICE** version 3f5, is used when I require graphical output for such things as time-domain waveforms and Bode plots. In all **SPICE** example netlists I strive to use coding conventions compatible with all **SPICE** versions.

Andrew D. Hwang's ePiX mathematical visualization programming library

This amazing project is a **C++** library you may link to any **C/C++** code for the purpose of generating PostScript graphic images of mathematical functions. As a completely free and open-source project, it does all the plotting I would otherwise use a Computer Algebra System (CAS) such as **Mathematica** or **Maple** to do. It should be said that **ePiX** is *not* a Computer Algebra System like **Mathematica** or **Maple**, but merely a mathematical *visualization* tool. In other words, it won't determine integrals for you (you'll have to implement that in your own **C/C++** code!), but it can graph the results, and it does so beautifully. What I really admire about **ePiX** is that it is a **C++** programming library, which means it builds on the existing power and toolset available with that programming language. Mr. Hwang could have probably developed his own stand-alone application for mathematical plotting, but by creating a **C++** library to do the same thing he accomplished something much greater.

`gnuplot` mathematical visualization software

Another open-source tool for mathematical visualization is `gnuplot`. Interestingly, this tool is *not* part of Richard Stallman’s GNU project, its name being a coincidence. For this reason the authors prefer “gnu” *not* be capitalized at all to avoid confusion. This is a much “lighter-weight” alternative to a spreadsheet for plotting tabular data, and the fact that it easily outputs directly to an X11 console or a file in a number of different graphical formats (including PostScript) is very helpful. I typically set my `gnuplot` output format to default (X11 on my Linux PC) for quick viewing while I’m developing a visualization, then switch to PostScript file export once the visual is ready to include in the document(s) I’m writing. As with my use of `Gimp` to do rudimentary image editing, my use of `gnuplot` only scratches the surface of its capabilities, but the important points are that it’s *free* and that it *works well*.

Python programming language

Both Python and C++ find extensive use in these modules as instructional aids and exercises, but I’m listing Python here as a *tool* for myself because I use it almost daily as a *calculator*. If you open a Python interpreter console and type `from math import *` you can type mathematical expressions and have it return results just as you would on a hand calculator. Complex-number (i.e. *phasor*) arithmetic is similarly supported if you include the complex-math library (`from cmath import *`). Examples of this are shown in the Programming References chapter (if included) in each module. Of course, being a fully-featured programming language, Python also supports conditionals, loops, and other structures useful for calculation of quantities. Also, running in a console environment where all entries and returned values show as text in a chronologically-ordered list makes it easy to copy-and-paste those calculations to document exactly how they were performed.

Appendix D

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Appendix E

References

Blackburn, J. Lewis and Domin, Thomas J., *Protective Relaying Principles and Applications*, Third Edition, CRC Press, Taylor & Francis Group, Boca Raton, FL, 2007.

Boylestad, Robert L., *Introductory Circuit Analysis*, 9th Edition, Prentice Hall, Upper Saddle River, NJ, 2000.

IEEE C57.12.00-2010, IEEE Standard for General Requirements for Liquid-Immersed Distribution, Power, and Regulating Transformers, IEEE Power Engineering Society, New York, NY, 2010.

Mason, C. Russell, *The Art and Science of Protective Relaying*, First Edition, John Wiley & Sons, 1956.

Appendix F

Version history

This is a list showing all significant additions, corrections, and other edits made to this learning module. Each entry is referenced by calendar date in reverse chronological order (newest version first), which appears on the front cover of every learning module for easy reference. Any contributors to this open-source document are listed here as well.

27 September 2024 – divided the Introduction chapter into sections, one with recommendations for students, one with a listing of challenging concepts, and one with recommendations for instructors.

10 March 2024 – edited some of the instructor notes.

12 December 2023 – added a Tutorial section on open-Delta transformer banks.

17-18 October 2023 – added a “Review of polyphase networks” section title to the Tutorial, and also added more questions to the Introduction chapter.

15 March 2023 – changed the given fault in the “Failed transformer winding” Diagnostic Reasoning question to have the secondary winding fail open rather than the primary. This makes the consequences less ambiguous.

28 November 2022 – placed questions at the top of the itemized list in the Introduction chapter prompting students to devise experiments related to the tutorial content.

8 May 2021 – commented out or deleted empty chapters.

17 April 2021 – added Conceptual questions based on photographs of three-phase pole-mounted transformer banks.

18 March 2021 – corrected multiple instances of “volts” that should have been capitalized “Volts”.

15 March 2021 – significantly edited the Introduction chapter to make it more suitable as a pre-study guide and to provide cues useful to instructors leading “inverted” teaching sessions. Also corrected two typographical errors pointed out to me by Jacob Stormes.

12 March 2021 – added Conceptual questions based on photographs of three-phase power transformers.

15-28 February 2021 – added content to the Tutorial, still very incomplete.

9 January 2021 – changed title of “Full Tutorial” to simply “Tutorial”.

8 July 2018 – document first created.

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