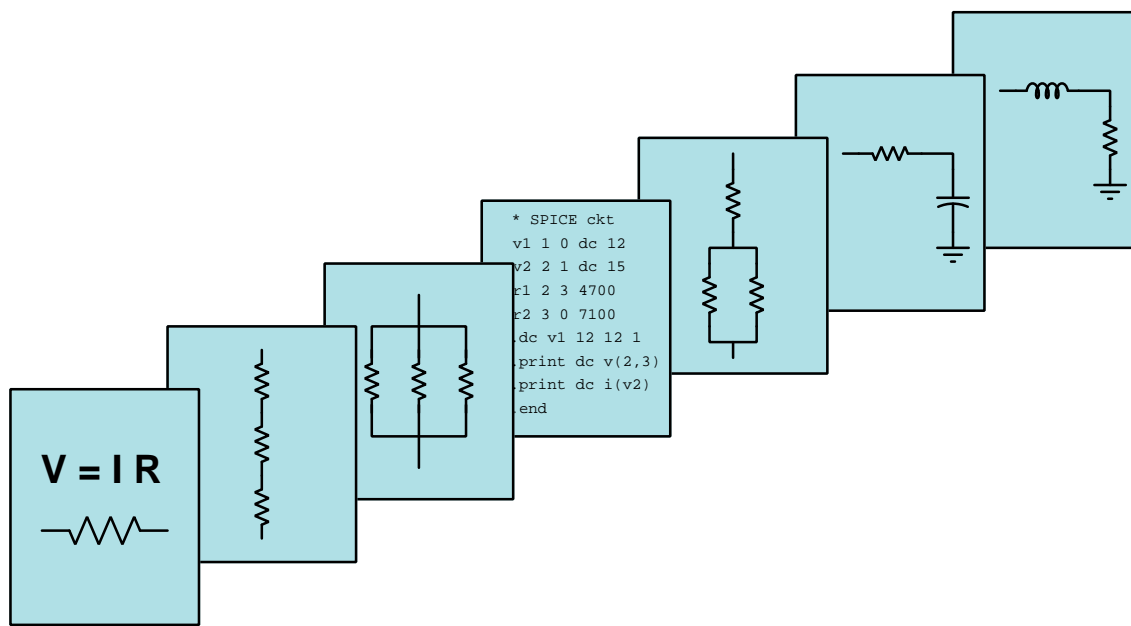


MODULAR ELECTRONICS LEARNING (MODEL) PROJECT



INTRODUCTION TO RF

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Chapter 1

Introduction

1.1 Recommendations for students

Radio-frequency, or RF, signals are those where the wavelength of the signal approaches the physical length of the circuit conductors, and at these frequencies we notice effects quite distinct from those of DC and low-frequency AC circuitry. This module explores some of the basic concepts associated with RF signals as well as common RF signal connectors, cables, and PCB layout practices.

Important concepts related to RF include **parasitic** inductance and capacitance, the **skin effect**, **wavelength**, **resonance**, **crosstalk**, **aggressor** and **victim** conductors, **reactance** and **impedance**, **transmission lines**, **termination**, **matching networks**, **characteristic impedance**, **balanced** versus **unbalanced** signals, and the **Precautionary Principle**.

Here are some good questions to ask of yourself while studying this subject:

- How might an experiment be designed and conducted to explore the phenomenon of skin effect? What hypothesis (i.e. prediction) might you pose for that experiment, and what result(s) would either support or disprove that hypothesis?
- How might an experiment be designed and conducted to map the radiation pattern of an antenna? What hypothesis (i.e. prediction) might you pose for that experiment, and what result(s) would either support or disprove that hypothesis?
- What is the purpose of a *balun*?
- What makes an ordinary signal cable behave as a transmission line?
- What is characteristic impedance, and why are transmission lines typically terminated with resistors having the same value?
- What differentiates the “near-field” region from the “far-field” region for RF fields?
- How may the wavelength of a signal be calculated based on its frequency?

- How are E-field probes constructed so as to preferentially sense electric fields rather than magnetic?
- How are H-field probes constructed so as to preferentially sense magnetic fields rather than electric?
- What is the typical Ohm value for the impedance of free space?
- What differentiates BNC from TNC connectors?
- What differentiates SMA from ST connectors?
- What are some of the known safety hazards of electromagnetic fields?
- What are some of the names and abbreviations used to categorize different frequency ranges of RF?
- What is a *harmonic* frequency?
- Why are a multitude of *vias* typically placed in a PCB design to electrically connect two planes of copper on different layers of the board?
- Why are RF equipment cover plates fastened by so many screws?
- What does it mean if an RF connector is “reverse polarity”?
- What does “coaxial” mean in reference to cables?
- What are some of the different styles of coaxial RF lines?
- How do waveguides differ from coaxial cables in construction and usage?
- What factors dictate the radiation pattern of a transmitting antenna?
- What does the phrase “lumped parameter” mean with reference to electrical models?

1.2 Challenging concepts related to RF

The following list cites concepts related to this module's topic that are easily misunderstood, along with suggestions for properly understanding them:

- **Maxwell's equations** – the existence of electric and magnetic fields within empty space, with no conductors nearby, was a discovery made by James Clerk Maxwell in the late 1800's, and is as nonintuitive today as it was then. RF systems with their extremely high frequencies manifest effects related to detached electric and magnetic fields that are all but undetectable in lower-frequency systems.
- **Decibels** – “decibels” are an attempt to express power ratios (i.e. power gains or attenuation factors) logarithmically rather than linearly, and as such they tend to generate confusion for students less familiar (or unfamiliar) with exponential and logarithmic functions.

1.3 Recommendations for instructors

This section lists realistic student learning outcomes supported by the content of the module as well as suggested means of assessing (measuring) student learning. The outcomes state what learners should be able to do, and the assessments are specific challenges to prove students have learned.

- **Outcome** – Demonstrate effective technical reading and writing

Assessment – Students present their outlines of this module’s instructional chapters (e.g. Case Tutorial, Tutorial, Historical References, etc.) ideally as an entry to a larger Journal document chronicling their learning. These outlines should exhibit good-faith effort at summarizing major concepts explained in the text.

- **Outcome** – Apply the concept of decibels

Assessment – Convert between gain values (as ratios) and decibels; e.g. pose problems in the form of the “Gain ratios into decibels” and “Decibels into gain ratios” Quantitative Reasoning questions.

- **Outcome** – Independent research

Assessment – Locate RF connector datasheets and properly interpret some of the information contained in those documents including connector type, gender, power rating, bandwidth, etc.

Chapter 2

Case Tutorial

The idea behind a *Case Tutorial* is to explore new concepts by way of example. In this chapter you will read less presentation of theory compared to other Tutorial chapters, but by close observation and comparison of the given examples be able to discern patterns and principles much the same way as a scientific experimenter. Hopefully you will find these cases illuminating, and a good supplement to text-based tutorials.

These examples also serve well as challenges following your reading of the other Tutorial(s) in this module – can you explain *why* the circuits behave as they do?

2.1 Example: cable loss decibel calculations

If a radio transmitter sends 15 Watts of power into a coaxial cable having a loss rating of -0.16 dB per meter, and the cable is 20 meters in length, the power available at the far end of the cable must be **7.1795 Watts**.

If an RF signal source sends 10 mW into one end of a 5-meter cable and the other end of the cable outputs 8 mW of signal power, the cable must have a loss rating of **-0.194 dB per meter**.

An RF signal of 2 Watts exits a cable 3 meters in length having a loss rating of -0.14 dB per meter. The amount of power being input to this cable must be **2.203 Watts**.

An RF signal of 15 Watts enters one end of a cable having a loss rating of -0.21 dB per foot, and an RF signal of 10 Watts exits the other end of that same cable. The cable's length must be **8.38530 feet**.

A 50-Watt radio transmitter connected to a 12-foot cable having a loss rating of 0.041 dB/foot terminates at a lightning arrestor with a loss rating of 0.5 dB, and finally to an antenna. The amount of power received by this antenna will be **39.7896 Watts**.

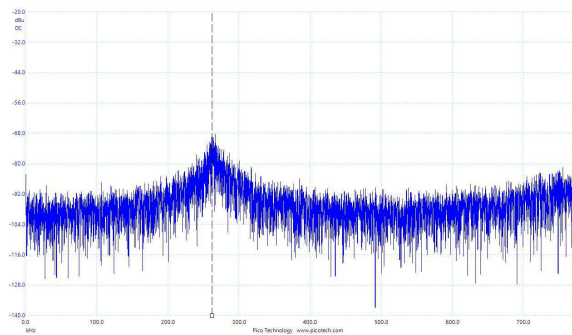
2.2 Example: near-field probing inside a PC

AC electric and magnetic fields may be qualitatively measured on printed circuit boards (PCBs) using special probes called *near-field probes*. These probes usually have BNC- or SMA-style coaxial cable connectors suitable for connection to the input of an oscilloscope or a spectrum analyzer.

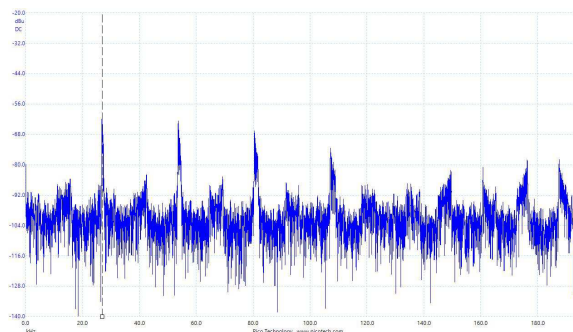
E-field probes sense electric fields with respect to ground, and consist of a metallic electrode coated in plastic (to avoid direct contact with circuit conductors) with a connector appropriate to the input of an oscilloscope or a spectrum analyzer. E-field probes tend to be omnidirectional in their sensing.

H-field probes sense magnetic fields created by current-carrying conductors, and consist of conductive loops (also insulated to avoid direct contact with circuit conductors) generating small AC voltages when exposed to AC magnetic fields. H-field probes tend to be planar in their sensing, which means they are maximally sensitive to magnetic fields caused by current through conductors parallel to the plane of the probe's loop(s) and minimally sensitive to magnetic fields perpendicular to that.

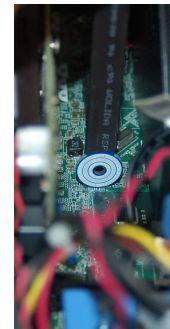
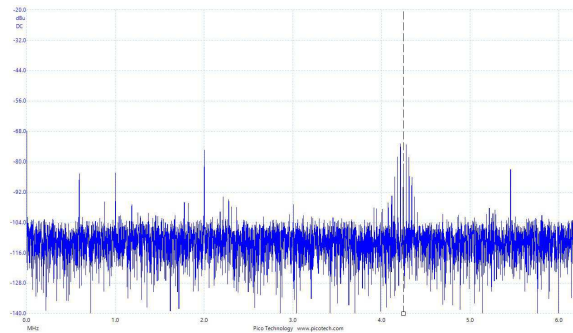
This first set of images shows the signal spectrum (left) and photograph (right) of an E-field probe held near a pair of wires connecting to a video processing PCB inside of a personal computer, picking up a spread of frequencies centered around 260 kHz:



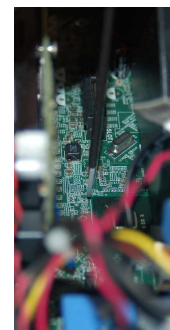
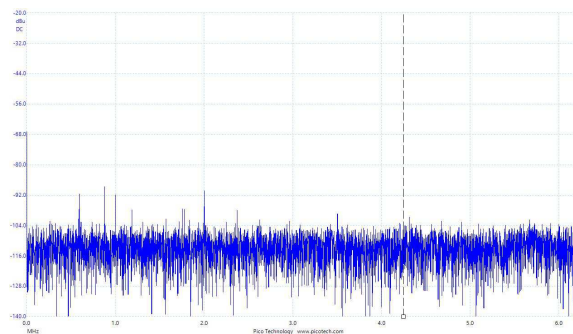
Next we see the same E-field probe held over the hub of a cooling fan motor, intercepting the electric field from a square-wave voltage signal with a fundamental frequency of approximately 27 kHz as well as its associated odd harmonics:



Switching to an H-field probe, the following two examples illustrate the polarization of this probe. In the first set of images we see the spectrum detected by placing the probe with its loop parallel to the current-carrying trace on the PC's motherboard, detecting a cluster of signals centered around 4.25 MHz with maximum sensitivity because parallel conductors ensure mutually perpendicularity with the magnetic field lines:



In the second image set we see the cluster of frequencies all but gone with the probe loop rotated to be perpendicular with the offending PCB trace. The pulsed current still flows through the PCB trace, but because the probe's loop is now perpendicular to that trace it means the magnetic field lines are now parallel to the probe loop, and therefore induction (from that signal) no longer occurs:

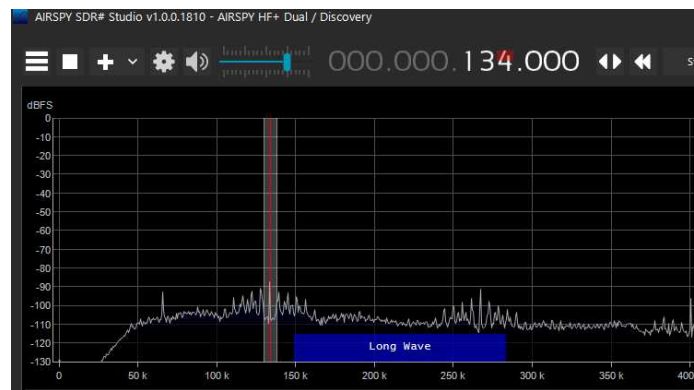


2.3 Example: SDR as a probe analyzer

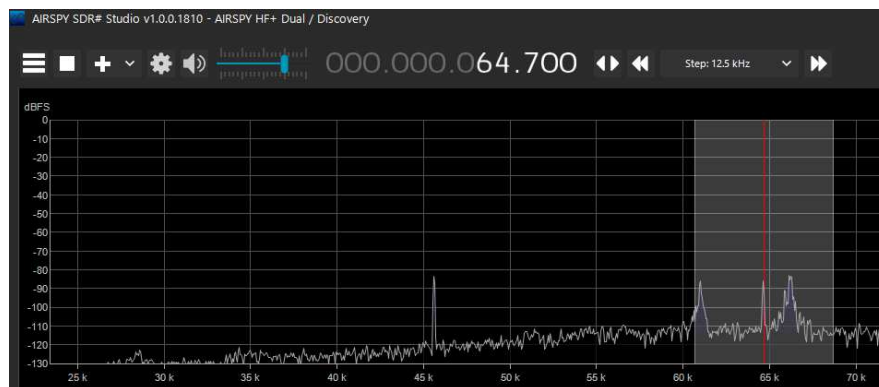
A spectrum analyzer is an ideal instrument to display signals intercepted by E-field and H-field probes, but spectrum analyzers are typically quite expensive. While even inexpensive digital oscilloscopes come equipped with Fast Fourier Transform (FFT) capability to display signal spectra, the bandwidth of an entry-level oscilloscope may be insufficient to reliably show frequencies of interest in near-field probing.

However, a viable alternative exists for affordable near-field probing, and that is the use of a Software-Defined Radio (SDR) receiver unit. An SDR receiver, paired with appropriate software to display the spectrum of any received signals, works quite well as a broadband spectrum analyzer, especially when precise measurement of signal amplitude is unimportant. When probing for electric and/or magnetic fields, we care about the frequencies of the intercepted signals and their *relative* amplitudes, but are generally uninterested in determining precisely how strong each signal is because the signal strength varies so much depending on how we hold the probe.

The first image here is of an AirSpy brand SDR receiving a signal from an E-field probe:



Next, we see the spectrum from an H-field probe:



Both spectra were detected outside the case of an uninterruptible power supply (UPS) unit for a personal computer.

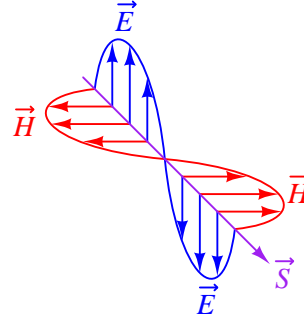
Chapter 3

Tutorial

3.1 Characteristics of RF

The term *RF* refers to *radio frequency*, and simply means any AC power or signal frequency great enough to permit *radiation* of energy in the form of electromagnetic waves, which are oscillating electric (\vec{E}) and magnetic (\vec{H}) fields propagating through space¹. The threshold where electromagnetic radiation becomes significant enough to be of any concern is where the length of the conductors involved approach the AC's wavelength (λ):

Electromagnetic wave



$$\lambda = t_p c = \frac{c}{f}$$

Where,

λ = Wavelength, in meters

t_p = Period of oscillation, in seconds

c = Speed of light in a vacuum, 2.99792458×10^8 meters per second

f = Frequency of oscillation, in Hertz

Typical frequencies associated with “RF” span from hundreds of kiloHertz to hundreds of GigaHertz. Frequencies below this range radiate very ineffectively for circuits of modest physical size. The frequency range of infrared light begins in the hundreds of GigaHertz. It is worth noting that electromagnetic waves comprise much more than just radio waves, but also constitute infrared light, visible light, ultraviolet light, X-rays, gamma rays, and cosmic rays at progressively higher frequencies and shorter wavelengths.

¹The cross-product of perpendicular electric field and magnetic field vectors results in another vector called the *Poynting vector* (\vec{S}) representing the direction of radiated power and measured in Watts per square meter.

The spectrum of frequencies comprising “RF” is quite broad, and commonly subdivided into different categories. Note that none of these categorical definitions is strict, and that the following ranges and lettered designations should be considered approximate only:

Frequency range	Wavelength range	RF subcategory designation
3 GHz to 30 GHz	10 cm to 1 cm	Super High Frequency (SHF)
300 MHz to 3 GHz	1 m to 10 cm	Ultra High Frequency (UHF)
30 MHz to 300 MHz	10 m to 1 m	Very High Frequency (VHF)
3 MHz to 30 MHz	100 m to 10 m	High Frequency (HF)
300 kHz to 3 MHz	1 km to 100 m	Medium Frequency (MF)
30 kHz to 300 kHz	10 km to 1 km	Low Frequency (LF)
3 kHz to 30 kHz	100 km to 10 km	Very Low Frequency (VLF)

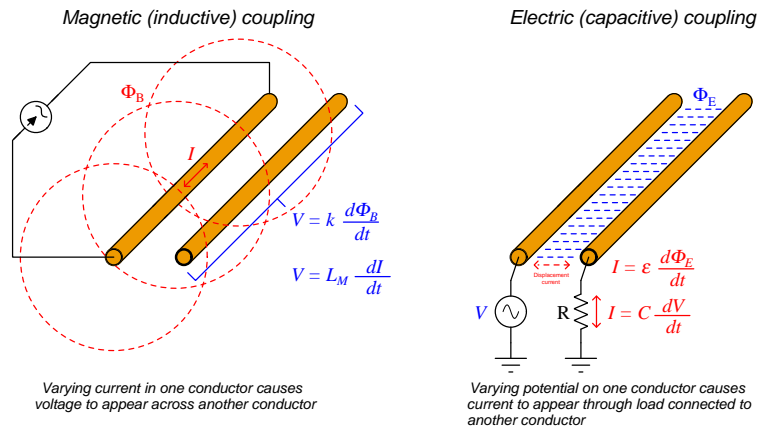
RF is an important topic for any electronics student or professional for several reasons:

- RF is necessary to create radio and radar systems, built for the express purpose of radiating electromagnetic energy and/or receiving electromagnetic energy
- Electromagnetic “emissions” accompany any circuit operating at sufficiently high frequency, thereby posing potential interference for any nearby radio receiver systems
- Parasitic effects such as capacitance and inductance become very significant at these high frequencies – basically, *everything* acts as a capacitor, and an inductor, and a resistor, and an antenna; even traces on a circuit board – and often pose design challenges to mitigate their effects
- Many standard cabling and construction techniques suitable for DC and low-frequency AC circuitry fail miserably if operated at radio frequencies
- RF poses unique safety hazards unknown to DC and to low-frequency AC electric circuits

3.2 Signal coupling and transmission lines

All conductors separated by an insulating medium exhibit capacitance, the ability to store energy in an electric field (related to voltage). All conductors exhibit inductance, the ability to store energy in a magnetic field (related to current). For single wires or collections of wires within a cable, the amount of capacitance and inductance per unit length is very small, negligible for many applications such as DC and line-frequency AC power circuits. However, for signals having high frequency (also, signals with rapid rise/fall times), these parasitic effects can no longer be ignored.

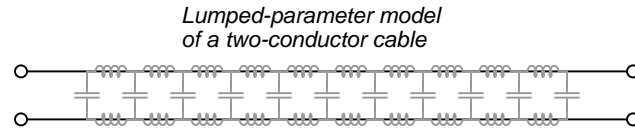
The ability for one conductor to influence another via these effects is referred to as *coupling* or *crosstalk*. Often the conductor exerting influence is deemed the *aggressor* while the conductor receiving this influence is called the *victim*:



Note how both of these effects are proportional to the *rate of change* of either current (inductive coupling) or voltage (capacitive coupling). For a sinusoidal waveshape, maximum rate of change is a direct function of that signal's frequency. For other waveshapes such as square waves, maximum rate-of-change is distinct from the signal's fundamental frequency. However, by Fourier's Theorem we know that any non-sinusoidal waveshape is actually equivalent to a superposition of sinusoids at different frequencies (called *harmonics*), with steeper rise/fall times associated with higher-frequency harmonics. Thus, it is fair to say that rapid rise/fall times and high frequency values are really just two different ways to express the same state of affairs, rise/fall times referencing the *time domain* and harmonic frequencies referencing the *frequency domain*.

All this is to say that RF experiences greater levels of signal coupling ("crosstalk") than low-frequency and DC. This becomes quite apparent if we calculate *reactance* at different frequencies. For example, the amount of capacitive reactance between a pair of conductors exhibiting 0.1 pF of capacitance between them at a frequency of 60 Hz is 26.53 *Giga*Ohms – high enough that the two conductors may be considered un-coupled (i.e. isolated) for all practical purposes. However, at a frequency of 5 GHz this capacitive reactance value falls all the way down to a mere 318.3 Ohms.

Even if crosstalk were of no concern, there would still be the issue of a cable's intrinsic inductance and capacitance bearing on the signal. The combined effect of this L and C within a cable results in the cable being able to store and release energy in its own right, as shown by the following *lumped-parameter model*:



When a pulse signal is applied at one end of this cable, the reactive elements of that cable (i.e. capacitance between the conductors, inductance along the cable length) begin to store energy. This translates to a current drawn by the cable from the source of the pulse, as though the cable were acting as a (momentarily) resistive load. If the cable under test were infinitely long, this charging effect would never end, and the cable would indeed behave exactly like a resistor from the perspective of the signal source. However, real cables (having finite length) do stop “charging” after some time following the application of a voltage signal at one end, the length of that time being a function of the cable's physical length and the speed of light. If we remove the electrical source prior to the pulse reaching the cable's end, the pulse continues to propagate down the cable's length all on its own, because the Law of Energy Conservation means the energy embodied by that pulse cannot simply disappear!

In honor of the cable's capacity to absorb energy and propagate pulses, we typically refer to it as a *transmission line*². All electrical cables act as transmission lines, but the effects just mentioned are so brief in duration that we only notice them when the cable is exceptionally long and/or the pulse durations are exceptionally short (i.e. high-frequency signals). With RF, practically every pair of conductors must be treated as a transmission line, and this includes traces on printed circuit boards (PCBs).

The “resistance” exhibited by a transmission line during the “charging” period is called its *characteristic impedance* or *surge impedance* (Z_0), and is a function of the conductors' spacing from each other as well as the permittivity of the insulation separating them. All other factors being equal, greater spacing between conductors results in greater Z_0 . Interestingly, this impedance has absolutely nothing to do with the line's physical length.

If a transmission line is not terminated by a load having the same resistance as the line's characteristic impedance (Z_0), some portion of every pulse's energy will *reflect* back to the source after reaching the cable's end. These reflected pulses are of great concern in RF systems because they represent poor energy transfer to the load, can create interference with subsequent pulses coming from the RF source, and in some cases can even cause cable and/or equipment damage! Therefore, in RF systems we generally take care to ensure cable impedance matches impedances found at both ends of the cable so that these reflections do not occur³. For example, in a radio transmitter system we would be sure to connect a transmission line to the transmitter's output connector having a characteristic impedance equal to the transmitter's output impedance, and at the other end of that

²This use of the term is entirely different from the same term's use in the electric power industry, where a “transmission line” is a set of conductors used to send large amounts of electrical energy over long distances.

³From the traveling signal's perspective, encountering a resistance equal to the cable's own characteristic impedance at the end of that cable is akin to encountering an infinite length of that same cable

line we would be sure to connect an antenna having an impedance also equal to (or nearly equal to) the line's impedance.

Another cause of reflected signals in a transmission line is any *discontinuity* (i.e. sudden change in dimension) in that line. For example, bending a cable around too small of a radius, pinching a cable with a clamp, a sharply-angled corner on a PCB trace, etc. will cause the characteristic impedance of that localized region to differ somewhat from the characteristic impedance of the rest of the transmission line. Since reflections are caused fundamentally by sudden changes in characteristic impedance encountered by the traveling signal, such discontinuities will degrade⁴ transmission line performance.

⁴An anecdotal example of this is a colleague of mine who decided (wisely) to install 75-Ohm RG-59 coaxial cable throughout the new house he was building so that every room could have convenient cable television (or cable internet) service. Being unfamiliar with proper cable installation standards, however, he used electrical staples to anchor the RG-59 cable to the wall stud boards just as he was accustomed to doing for regular type-NM 60 Hz AC electrical cables. After the wall paneling was installed he tried connecting televisions to these new cable receptacles and was sorely disappointed in the “noisy” video quality the units displayed. All of those staples pinching the coaxial cable caused signal reflections, resulting in signal power attenuation at the receivers as well as corrupted signals due to the interference caused between the incident and reflected signal waves. Unwilling as he was at that point to tear down all the paneling and wallboard to replace the now-useless RG-59 cabling, he simply abandoned it and routed other cables to points of use in more conventional fashion (i.e. under the floors, through the crawlspace of the house).

3.3 Skin effect

An interesting AC-only phenomenon called *skin effect* is where most of the electric current oscillating through a conductor travels near the periphery (the “skin”) of that conductor while the center of the conductor carries very little current⁵. This effect becomes more pronounced as frequency increases, and at RF frequencies is severe enough that the surface area of a conductor is a better gauge of its ampacity than its cross-sectional area. The following formula gives an approximation of skin depth for a given material (ρ and μ) and frequency (f):

$$\delta \approx \sqrt{\frac{2\rho}{2\pi f\mu}}$$

Where,

- δ = Skin depth, in meters
- ρ = Resistivity of conductor material, in Ohm-meters
- f = Frequency of AC current
- μ = Absolute permeability of material, in Henrys per meter

“Skin depth” is a measure of how deep into a conductor the current falls off to e^{-1} times (i.e. 36.79% of) the intensity of the current at the conductor’s surface. For example, at 60 Hz the skin depth of a pure copper conductor ($\rho = 1.68 \times 10^{-8}$ Ohm-meters, $\mu = 1.26 \times 10^{-6}$ Henrys per meter) is approximately 8.4 millimeters. For comparison, the radius of a 0000 AWG wire is 5.84 millimeters, which means at 60 Hz the skin effect is of no concern for that large of a copper conductor: the 60 Hz AC current is able to freely occupy the entire cross-sectional area of 0000 AWG copper wire. If we re-calculate for a much higher frequency, however – say, 10 kHz instead of 60 Hz – the skin depth falls to approximately 0.65 millimeters, which means most of a 0000 AWG copper conductor’s cross-sectional area would be effectively unused. Increasing frequency into VHF territory (say, 200 MHz) decreases skin depth to a mere 0.0046 millimeters for copper. At higher frequency, it is as if the conductor were much smaller-gauge than its physical dimensions would suggest, with a correspondingly decreased ampacity rating.

Power conductors used for RF are always hollow rather than solid for this reason: it would be a waste of metal to make a solid conductor when the vast majority of the RF current would not pass through its center anyway.

A special type of wire called *Litz wire* made for high-frequency applications uses individually-insulated wire strands braided together in a pattern designed to equally distribute magnetic flux around the conductor and therefore equally distribute current through all the strands so that none of them are negated due to the skin effect. Each strand in a Litz wire has a cross-sectional radius less than the expected skin depth at the operating frequency, to fully utilize its volume. The weave/twisting pattern places every strand within a Litz wire on the outside of the bundle for some distance as well as inside the bundle for some distance, such that the over-all skin effect for the entire length of each strand is approximately equal.

⁵This is an inductive effect, where the interior regions of a solid conductor experience more mutual induction from current traveling through the exterior region, and as a consequence of this any current in the conductor’s center finds itself opposed by the magnetic fields of the peripheral currents, such that more current travels near the surface of the conductor than through its center.

A good example of hollow RF conductors is the impedance-matching network for a 50 kW AM broadcast radio transmitter seen in the photograph below. This particular radio transmitter operates at 550 kHz, and at that frequency the skin depth for copper is only 0.0878 millimeters. Each of the inductors (coils) clearly visible in the photo are made of hollow copper tubing coated with a thin layer of *silver*⁶ metal to further reduce electrical resistance. The entire network is housed inside of a small shielded room, visible through the open door:



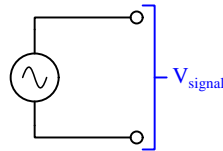
Bare-metal grounding conductors used in RF applications are typically of braided-strand construction or consist of thin strips of sheet metal rather than conventional wire with circular cross-section. The idea here is much the same as with the hollow tubes seen in the above photograph: maximize the conductor's surface area while using the least amount of metal mass.

⁶The electrical resistivity (ρ) for silver is 1.59×10^{-8} Ohm-meters, compared with 1.68×10^{-8} Ohm-meters for copper, making silver the better conductor of electricity.

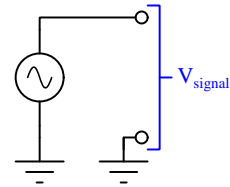
3.4 Balanced versus unbalanced signals

Although not unique to RF applications, the distinction between balanced and unbalanced electrical signals is an important one in RF circuits. Simply defined, a *balanced* signal is where the voltage in question is measured between two ungrounded conductors. This is also known as a *differential* signal. By contrast, an *unbalanced* signal is where the voltage in question is measured between a specific conductor and ground. This is also known as a *ground-referenced* or *single-ended* signal.

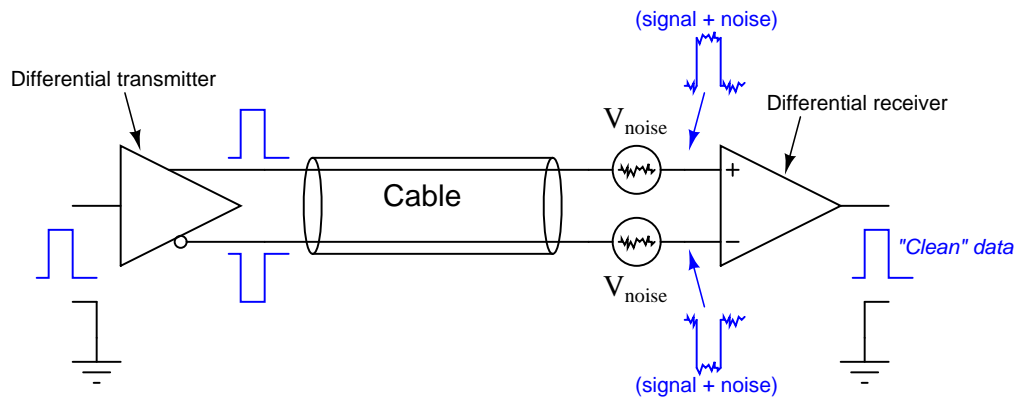
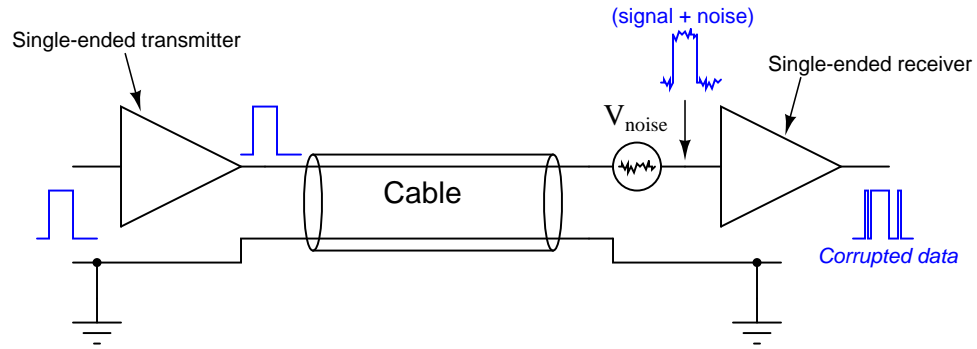
Balanced signal



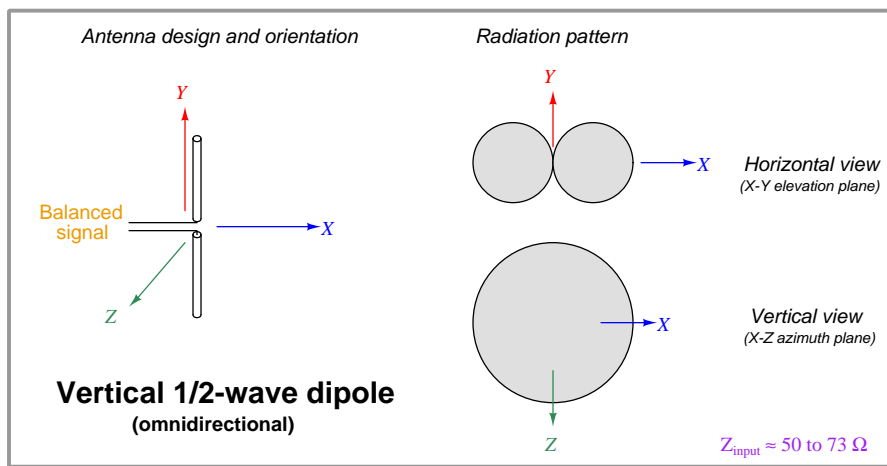
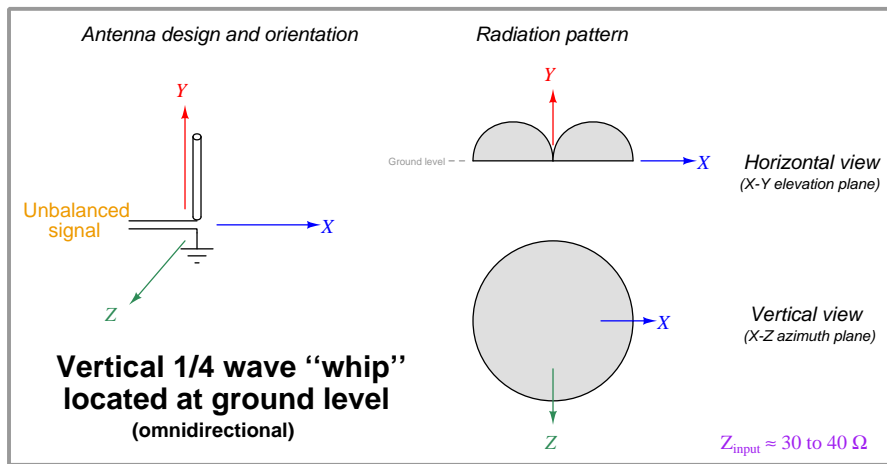
Unbalanced signal



Balanced signals tend to be more resistant to the effects of induced noise than unbalanced signals, as illustrated by the following diagrams showing the transmission of digital pulse signals. Note that the same advantage of noise immunity applies to balanced *analog* signals as well as digital:

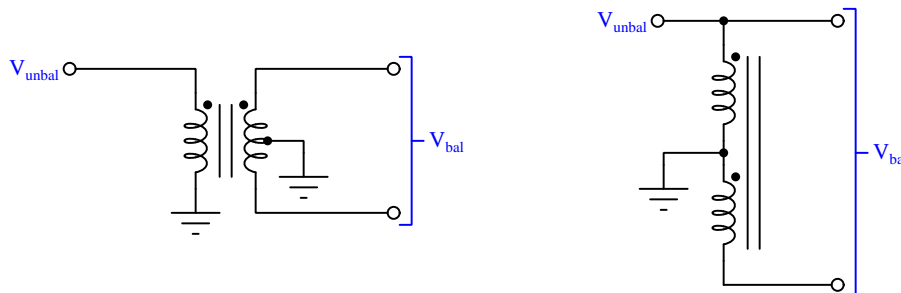


One important application of these two different signal types relates to radio antennas. Single-wire “whip” or *monopole* antennas are inherently unbalanced devices, using Earth ground as the point of reference. Double-element (*dipole*) antennas, however, are inherently balanced devices:

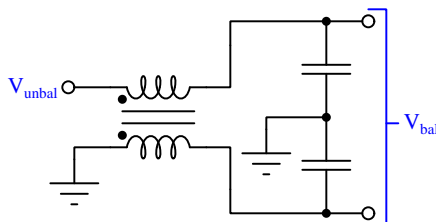


Mis-matching antenna and signal types results in poor performance. For example, if we attempt to drive a dipole antenna from an unbalanced (single-ended) RF source, the result will be an attenuated and mis-shapen radiation pattern from the antenna.

A device useful for translating between balanced and unbalanced signals is a transformer called a *balun*⁷. The following balun transformer examples are known as *voltage baluns* because the energization of each winding is proportional to the applied signal voltage:



A different style of balun called a *current balun* places the windings in series with the load rather than parallel, such that winding energization is proportional to signal current rather than signal voltage:



The high frequency values of RF makes possible transformers having little or no iron in their cores⁸. A balun rated for 3 kW of RF power, for example, may be easily held in one hand. A 3 kVA power transformer rated for 50 Hz or 60 Hz service, by contrast, weighs over 30 kilograms due mostly to the iron core's mass.

⁷As you might guess, the name “balun” is a portmanteau of the words *balanced* and *unbalanced*.

⁸Faraday’s Law of Electromagnetic Induction informs us that induced voltage is proportional to the rate-of-change of magnetic flux. At high frequency, the peak value for magnetic flux does not have to be very great in order to have huge rates-of-change. However, as frequency decreases, the peak value of an oscillating magnetic flux must grow larger and larger to maintain the same maximum rate of rise or fall. Larger quantities of magnetic flux translate to iron cores with greater cross-sectional area, in order to distribute that magnetic flux over a greater area to keep the flux density below the iron’s saturation limit. Therefore, low-frequency transformers have more massive cores than high-frequency transformers, all other factors being equal.

3.5 RF cabling

In RF applications we very rarely find single conductors used to convey signals as we might in DC or low-frequency AC circuits. Instead, a popular alternative is *coaxial cable* where one conductor rests in the center of a hollow conductor sharing the same centerline, or *twinaxial* cable where two conductors rest side-by-side within a hollow conductor. The outer conductor is often a metal-foil wrapping or braided-metal sheath entirely covering the center conductor:



For both types of cables it is common practice to connect the outer (hollow) conductor to Earth ground at one far end of the cable to electrostatically shield the inner conductor(s) from external electric fields, and also to reduce electromagnetic emissions from the cable. This makes coaxial cables well-suited to unbalanced signals, and twinaxial cables to balanced signals.

Industry-standardized RF cables often bear *RG* (“Radio-Grade”) designations, for example RG-6, RG-59, RG-11, etc. Each numbered “RG” type defines a set of basic electrical characteristics including impedance and physical diameter. There seems to be no logical association between the RG- number and any specific property of the cable, as these were arbitrary designators assigned to different cable designs over many years. Some of these characteristics may vary somewhat by manufacturer, which means it’s always prudent to check the detailed specifications before purchasing or using any RG-type cable for a critical application.

The United States military has abandoned the RG designators in favor of “M17” classifications referencing the MIL-DTL-17 standard. Since these designations are defined and enforced by the US military for exacting tolerances and punishing applications, variance between manufacturers tends to be less than with RG-designated cables.

In addition to flexible coaxial cabling, *formable*, *semi-rigid*, and *rigid* coaxial lines also exist. As their names imply, these coaxial transmission line options are stiffer than regular cables. Formable coaxial cables use a “metal-filled” braided outer conductor which allows them to be hand-formed and re-formed as needed. Semi-rigid coaxial line uses solid-metal tube for the outer conductor and often must be formed using special tools. Rigid coaxial line is for high-power applications, using metal tube for the outer conductor and usually smaller-diameter metal tube for the inner conductor. The benefit of using a solid-metal outer conductor rather than foil wrap or metal braid is superior shielding even at extremely high signal frequencies.

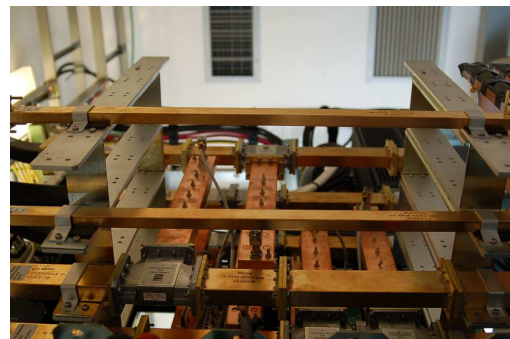
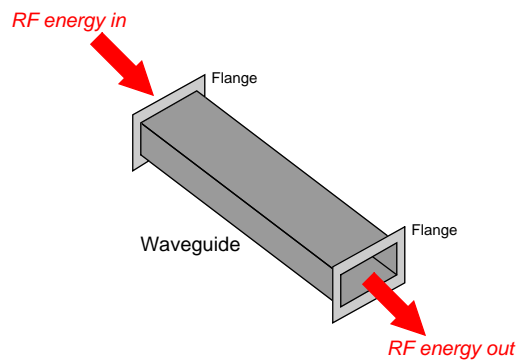
Semi-rigid and rigid coaxial lines commonly utilize plastic or ceramic spacers at set intervals along the length of the line to support the inner conductor and keep it a constant distance from the outer conductor, rather than a solid plastic fill material as is standard in flexible coaxial cable. The use of occasional spacers means most of the insulation between outer and inner conductors is air (or nitrogen fill gas) rather than plastic, which gives these coaxial lines lower losses and higher bandwidth capability.

Rigid coaxial lines resemble copper pipe when viewed from the outside, and typically come with flanges pre-installed at the ends so that they may be easily joined to form longer lengths, or to take corners using special elbow fittings. A photograph of some rigid coaxial lines appear below, as part of the installation for a 50 kW AM radio broadcast transmitter:

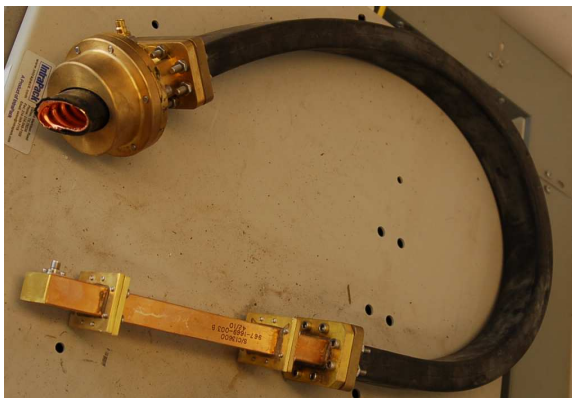


At microwave frequencies it becomes possible to convey RF energy using an entirely different structure called a *waveguide*. A “waveguide” is a completely hollow metal tube lacking any center conductor(s), usually rectangular in cross-section and filled with either air or some other dry gas. Electromagnetic waves propagate through the tube’s interior, with the tube walls functioning as both inductors (length) and capacitor plates (width). At extremely high frequencies, waveguides are far more effective than coaxial lines in terms of shielding and signal attenuation (i.e. power loss).

Below we see an illustration of a waveguide section (left), and a photograph of a waveguide network (right) connecting multiple microwave transceivers to antennas:



Like coaxial lines, waveguides are also available in rigid and semi-rigid forms. The following photograph shows three different types of waveguide joined together, rigid (bottom-left), rectangular semi-rigid (right), and circular semi-rigid (upper-left):



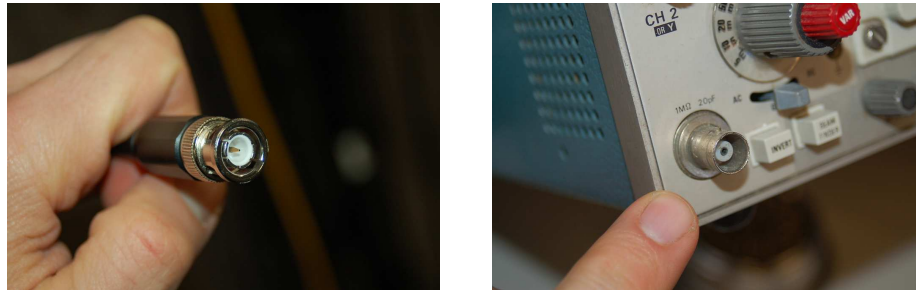
If you closely examine the rigid waveguide section, you will notice an SMA connector at its far end which is how this waveguide connects to the microwave transceiver hardware. The SMA connector leads to a small wire “stub” inside the waveguide which is essentially an antenna for transmitting and receiving RF energy inside the waveguide tube. A short coaxial cable with SMA connectors at either end makes the final connection between the microwave transceiver and the waveguide, the waveguide then leading all the way up to the dish-style antenna at the top of a tall tower.

3.6 RF connectors

A variety of special connectors exist to terminate coaxial cable. The following subsections showcase just a few of these.

3.6.1 BNC connectors

This twist-lock style of coaxial connector is extremely common on test instruments capable of high frequency operation, such as signal generators, oscilloscopes, and spectrum analyzers. Photographs below show male (left) and female (right) BNC connectors:



Here we see an oscilloscope probe cable with a male BNC connector on its end, engaged with the female BNC connector on the front panel of an oscilloscope. The quarter-turn twist-lock design of the BNC connector type is evident in this photograph:



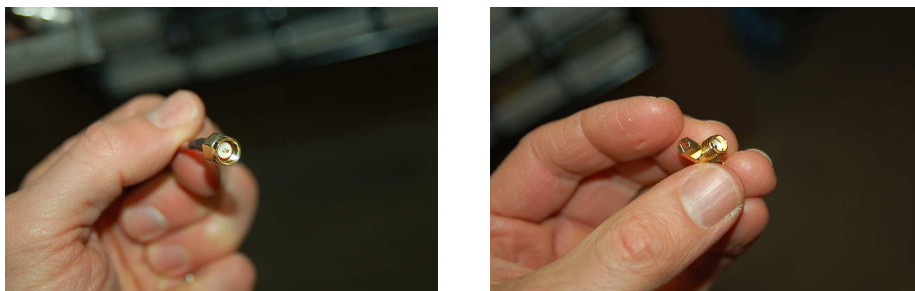
3.6.2 TNC connectors

TNC is nothing more than a *threaded* version of BNC: instead of a quarter-turn locking mechanism, we have a threaded barrel requiring multiple turns to fully engage. Here we see a male TNC connector at the end of a coaxial cable:



3.6.3 SMA and ST connectors

A much smaller connector format than BNC/TNC is SMA (threaded) and its quarter-turn twist-lock counterpart ST. Below we see photographs of a male SMA connector on the end of a cable (left) and a female “bulkhead” SMA connector designed to mount to an instrument panel (right):



Next we see an SMA “cap” with a built-in 50 Ohm termination resistor:

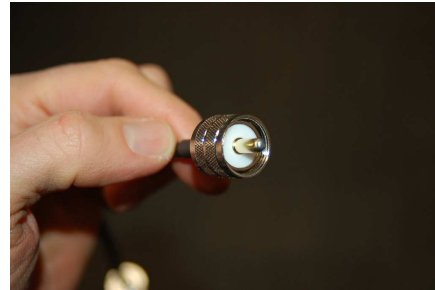


ST connectors are quite popular for fiber-optic cabling. The SMA/ST counterparts are analogous to TNC/BNC, the former being threaded and the latter twist-lock in these cases.

It should be noted that other styles of small-diameter threaded RF connectors exist which are very similar to SMA but have slightly different dimensions. These include the 2.4 mm, 2.92 mm, and 3.5 mm connectors. Just to make matters confusing, while SMA connectors will physically join with 3.5 mm and with 2.92 mm connectors, these are actually different connector designs and should not be intermixed.

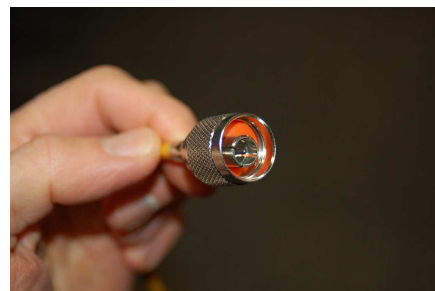
3.6.4 UHF connectors

Another threaded RF connector standard is the so-called UHF, often known by the twin designations of PL-259 (plug) and SO-259 (socket), commonly seen on high-frequency (HF) and ultra high-frequency (UHF) radio equipment for antenna connectors. Below we see female SO-259 (left) and male PL-259 (right) connectors on cable ends:



3.6.5 N connectors

Yet another threaded RF connector standard is the N-type. Below we see a female N connector on a test instrument (left) and a male N connector at the end of a cable:



3.6.6 Reverse-polarity connectors

Threaded RF connectors may have either external or internal threads, and also have either male or female inner-conductor contact points. Typically a internally-threaded connector has a male pin which engages with the female socket of an externally-threaded connector. Below are some photographs of standard SMA-type connectors showing a “male” connector on the left and a “female” connector on the right:



A variation on these connector types is something known as a *reverse-polarity connector*. With these, the thread type remains the same but the center-conductor contact points change. Below we have a “reverse polarity male” connector on the left and a “reverse polarity female” connector on the right: ⁹

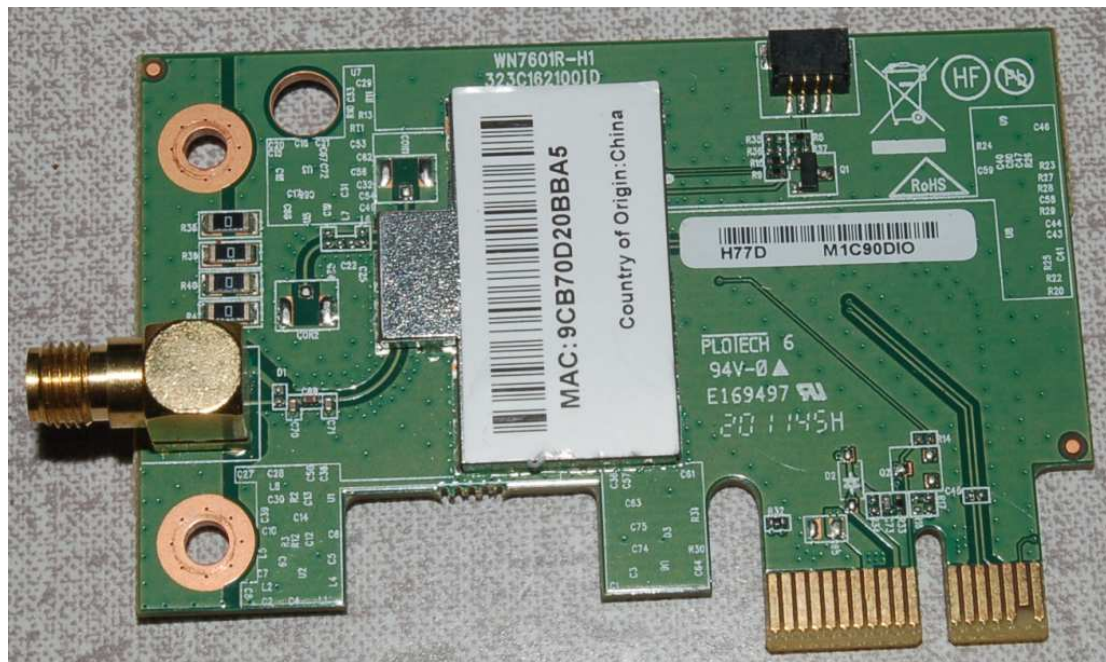


Having standard-polarity and reverse-polarity variations of each connector gender makes sense if the purpose is to prevent an end-user from attaching a cable to the wrong connector.

⁹If you find these conventions illogical, you are not alone. From the standard connectors one might assume that the connector's gender is a function of the center conductor, with “male” having the pin and “female” having the socket. However, with reverse polarity connectors it is this “defining” characteristic that swaps while the *thread* types remain the same! Anyone accustomed to pipe threads will likely find themselves wanting to call a “male” connector “female” instead because the threads are internal to the connector, and vice-versa. However, if you are familiar with industrial compression-style tube fittings the gender designations make a little more sense.

3.7 PCB layout for high frequencies

Traces laid out on printed circuit boards (PCBs) act like transmission lines just like cables and individual wires at radio frequencies. That being the case, careful layout is important for good circuit performance. Below we see a PCB for a personal computer Wi-Fi radio network adapter. Note the female SMA connector on the left-hand edge of this PCB, awaiting connection to an antenna:



Several features of this PCB are worth noting for their high-frequency qualities. First, the curved trace connecting the center-pin of the SMA connector to the RF amplifier (housed under the metal cover) is a good example of careful trace routing intended to maintain a consistent characteristic impedance along its length. Not only does the trace avoid any sharp turns, but the consistent clearance between the trace and the surrounding copper “pours”¹⁰ ensures the traveling signals experience consistent parasitic capacitance and inductance between those conductors. A sharply-angled trace here would have presented a discontinuity to any signals traveling along it, and thereby caused signal reflections; inconsistent clearance between the trace and surrounding ground plans would have also given the line different characteristic impedance values at different locations. Second, we see a differential (balanced) signal pathway in the form of two traces extending from two of the card-edge connector pads, ending in a pair of *vias*¹¹. Third, it is impossible not to notice the

¹⁰A *copper pour* on a PCB is simply an otherwise unused area of a board layer that is filled with copper instead of being left bare. This stands in contrast to a *copper plane* which is an unbroken layer of copper covering the entire area of one board layer.

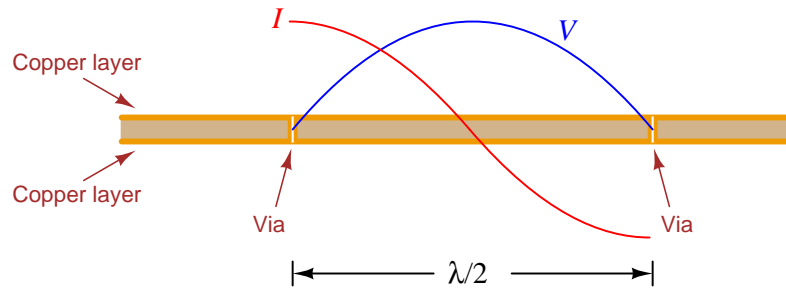
¹¹In PCB design, a *via* is a metal-plated hole passing between two or more copper layers on the board, intended to connect a trace on one layer to a trace on another layer. These differ from through-hole component *pads* in their lack

abundance of vias connecting large copper areas on the top side of the board to corresponding copper areas on the bottom side. This next photograph shows the underside of this same Wi-Fi PCB, where the vias connecting top and bottom copper planes appear as tiny holes:



In DC and low-frequency AC applications, such an abundance of vias would serve no purpose other than for high-current applications. Ordinarily, just a single via would be necessary to make two copper surfaces electrically common to each other, but here we must “stitch together” those opposing copper layers far more frequently. A good practice is to space these “stitching” vias no farther apart from each other than one-twentieth of a wavelength ($\frac{\lambda}{20}$) for the highest anticipated signal frequency, with even closer spacing being better.

Concern over via spacing is rooted in transmission line theory, particularly in the phenomenon of *standing waves*. When two conductive surfaces are electrically joined at two locations, the area between those joints has the ability to resonate like a transmission line shorted at both ends. The short-circuit (via) at either end ensures voltage *nodes* (zero) and current *anti-nodes* (maximum), with the lowest-frequency standing wave being a half-cycle. Therefore, vias placed at $\frac{\lambda}{2}$ intervals invite resonance, which in turn will degrade the performance of adjacent RF circuitry:



A maximum spacing recommendation of $\frac{\lambda}{20}$ comes from the time-honored engineering practice of designing things at least one order of magnitude (10^1) more conservative than the critical limit in order to render that critical effect inconsequential. Placing vias closer together means the resonant frequency for any standing wave must increase (i.e. higher f = shorter λ), and so choosing to

of bare copper areas to solder, since vias are nothing more than inter-layer connectors and not intended as termination points for components.

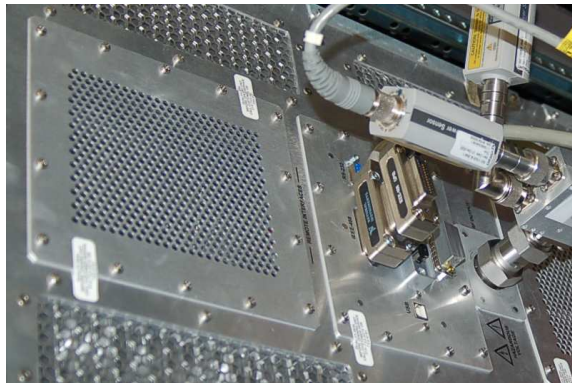
space vias closer together means resonance is possible only at frequencies greater than the circuit is designed to operate.

Below are top- and bottom-layer photographs of another RF board, this one being a low-noise amplifier for boosting the strength of RF signals prior to the input of a radio receiver. The top side of this PCB (left) hosts all the components and is rather simple in layout, having three distinct copper “pour” areas surrounding traces. This board’s bottom side (right) is nothing but a solid copper ground plane, with vias “stitching” the bottom-layer ground plane to all three of the top-layer pours:



Rows of vias “stitching” top-layer ground areas to the bottom-layer ground plane are plentiful on this PCB, and as you can see they are necessary around the circumference of each pour.

The reason for the close spacing of “stitching” vias on a PCB is exactly the same as the reason why metal cover plates on RF devices are fastened by what would appear to be an over-abundance of screws: to ensure the distance between connection points is significantly shorter than a half-wavelength of the RF signal in question. The following photograph is a good example of this, there being far more screws fastening each cover plate to the metal chassis than mechanical considerations alone would require:



Professional-level PCB layout design for custom products is intensely technical, involving detailed analysis of the electric and magnetic fields generated by the circuit, often using specialized electromagnetic modeling software to do so. However, the existence of modern surface-mount integrated circuit (IC) components for radio-frequency designs makes design of simple RF circuits easier than one might suspect.

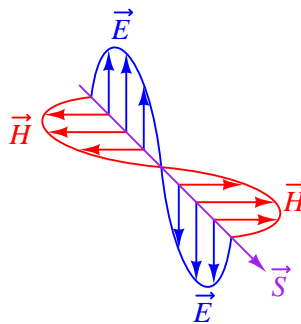
In March of 2016 Michael Ossman gave a technical presentation called “Simple RF Circuit Design” where he offered the following five rules to simplify what might otherwise be an enormously complex task, for the sake of making RF circuit design possible for non-specialists. These are excellent rules for any beginning RF circuit designer to follow!

1. **Always use four-layer PCBs** – Michael’s stack-up recommendation is to restrict all RF traces to the top layer where they directly meet surface-mount device (SMD) terminals, use the middle two layers for DC ground and power respectively, and use the bottom layer for any non-RF signal traces that cannot fit on the top layer. The middle (power rail) layers should be completely uninterrupted aside from any vias that must go through to tie other layers together. No vias should ever be used for any RF signals.
2. **Use ICs wherever possible** – in other words, avoid designing anything RF using discrete resistors, capacitors, inductors, transistors, etc. Modern integrated-circuit offerings for RF include amplifiers (power as well as low-noise), matching networks, baluns, filters, and even entire radio transceivers, and these IC components are fully engineered assemblies guaranteed to work as advertised!
3. **Standardize on 50 Ohms** – using components having (very common) 50-Ohm input and output impedances means being able to make all RF traces have this same characteristic impedance, and this in turn simplifies trace design because the trace dimensions you calculate *once* may be used for all RF signals everywhere in the design. Standardizing on just one impedance value also usually means fewer impedance-matching networks will be needed on your PCB.
4. **Heed manufacturer recommendations** – many datasheets for RF devices provide details on PCB layout best practices, companion devices for RF signal handling, etc. Some manufacturers even sell development PCBs pre-populated with their products, the design principles embodied in these PCBs serving as best-practice models to follow in using those same components.
5. **Route RF traces first** – keep these traces short (compared to the signal wavelength) and as direct as possible between components, also maintaining good separation from other (non-RF) traces residing on the same layer.

3.8 Near-field testing

The abbreviation *RF* of course stands for *Radio Frequency*, because such high frequencies are necessary to produce the electromagnetic waves used for radio communication. Such waves are an amalgam of oscillating electric and magnetic fields at right angles to each other, propagating through open space at the speed of light (along the direction shown by the vector \vec{S}):

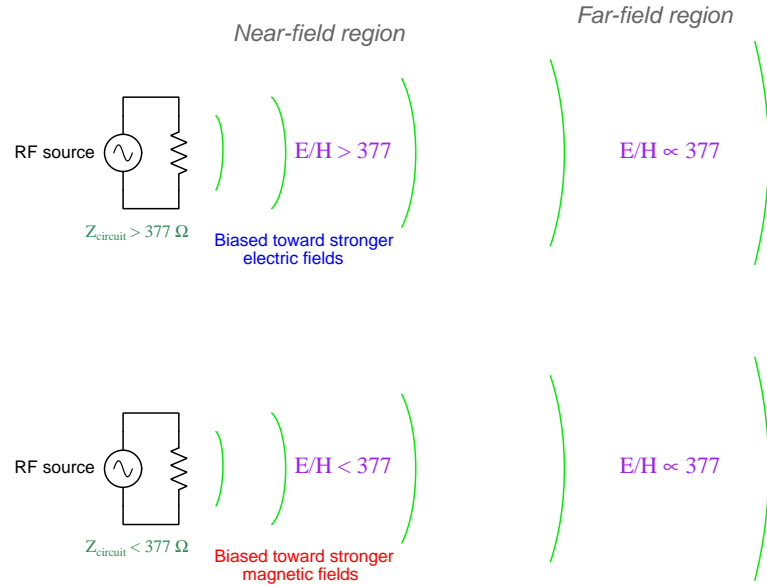
Electromagnetic wave



In order to generate radio waves, all we need to do is create either an oscillating electric field or an oscillating magnetic field of appropriately (high) frequency, and that oscillating field inevitably induces the other type of field as it expands through space (e.g. an expanding electric field will induce a magnetic field, and an expanding magnetic field will induce an electric field). In fact, it is this mutual re-creation of electric and magnetic fields by each other that makes electromagnetic waves possible. At a distance of usually multiple wavelengths from the initial field source called the *far-field region*, the electromagnetic field is “fully formed” and may be received by either its electric properties or its magnetic properties.

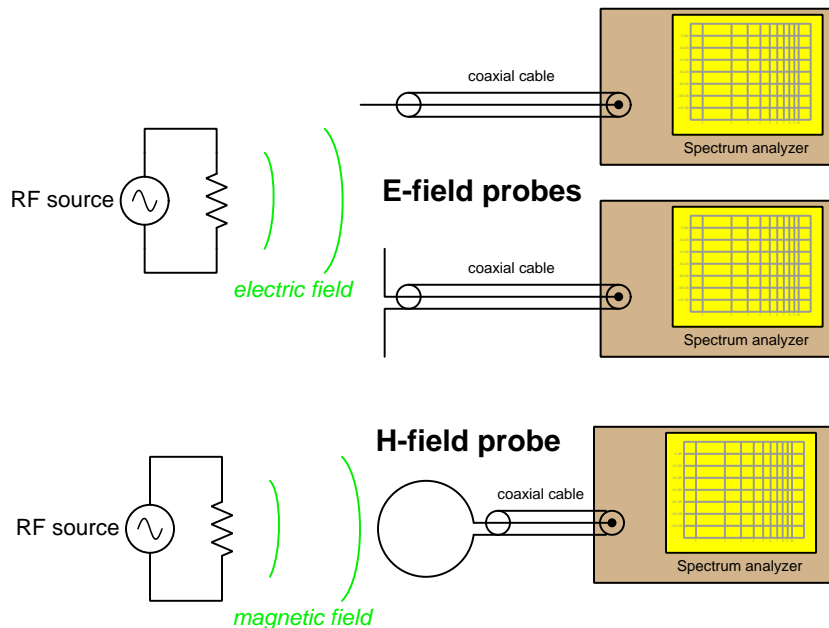
Within this far-field region the oscillating electric and magnetic field strengths of an electromagnetic wave relate to one another by the physical constant called the *characteristic impedance of free space*. Just as impedance is generally defined as the ratio of voltage to current ($Z = \frac{V}{I}$), impedance for an electromagnetic wave is defined as the ratio of electric field to magnetic field magnitudes ($Z = \frac{E}{H}$), where E is expressed in Volts per meter, H in Amperes per meter, and Z (as always) in Ohms. For empty space this impedance ratio is fixed by the magnetic permeability and electric permittivity of a vacuum, $\sqrt{\frac{\mu_0}{\epsilon_0}}$, which happens to be approximately 377 Ohms. This means that for any region of empty space where there is an electromagnetic wave, the electric field value in Volts per meter must be 377 times greater than the magnetic field value in Amperes per meter.

However, when we measure these fields in close proximity to an energized RF circuit, we often find different ratios between E and H , the ratios dominated by circuit impedances more than the impedance of free space. Near high-impedance networks, for example, we find E fields that are more than 377 times stronger than corresponding H fields; near low-impedance networks we find E fields less than 377 times as strong as the accompanying magnetic fields:



This zone of close proximity to the radiating conductor(s) is known as the *near-field region*, where the electric and magnetic field strengths are not necessarily found in the same 377:1 proportion that they will be in the far-field region, and where the fields may not always be perpendicular to each other in orientation. Within this near-field zone we may sense the electric and magnetic fields independently of each other using special probes designed either to preferentially detect electric fields or preferentially detect magnetic fields. The use of such probes is called *near-field testing*, and it is often used to pinpoint locations in a high-frequency circuit most responsible for unwanted electromagnetic emissions. Equipping an RF measuring instrument such as an oscilloscope or spectrum analyzer with a special probe appropriate for detecting either electric or magnetic fields allows us to locate where in a high-frequency circuit these fields are strongest and which component(s) in the circuit are most responsible for unwanted RF emissions.

The following illustrations show E-field probe versus H-field probe designs:



Electric field coupling works on the principle of a voltage rate-of-change inducing a current between two conductors not in direct contact with each other, and so E-field probes consist of open-ended wires ready to receive electric flux lines from energized conductors in the circuit. Magnetic field coupling works on the principle of a current rate-of-change inducing a voltage along a length of conductive material, and so H-field probes consist of closed loops of wire ready to couple with magnetic flux lines from current-carrying conductors in the circuit to produce voltages measurable by the test instrument.

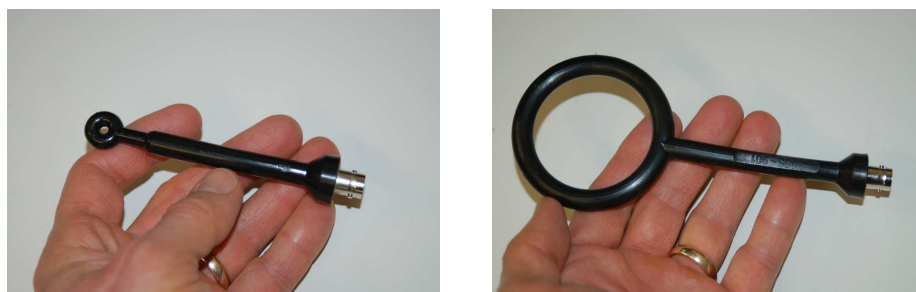
Single-electrode E-field probes work equally well in all physical orientations, but dual-electrode (dipole) E-field probes and H-field probes (which are always *loops*) exhibit polarization in that they are maximally sensitive when oriented *parallel to* the radiating conductors.

Since electric fields and electrical potential (voltage) are associated with one another, and similarly magnetic fields are associated with electrical currents, we find E-field probes most useful near points in a circuit with high potential relative to ground while H-field probes are most useful near conductors carrying high current.

Shown here are two commercial *E-field probes*, both consisting of single electrodes covered in a layer of black plastic to prevent metal-to-metal contact with the circuit under test. The larger-tipped probe is more sensitive to electric fields, but less precise in locating the position of the source:



For contrast, we will now show two commercial *H-field probes*, both consisting of wire loops (with electrostatic shielding to prevent detection of electric fields) covered in black plastic. Again, the larger probe has greater sensitivity to weak fields, but is less useful for pinpointing the exact location of its source:



All of these near-field probes have BNC connectors for attachment to coaxial cable, to route the detected signal to the input of a suitable measuring instrument. Spectrum analyzers work very well as near-field probe indicators because they show a wide range of signal frequencies at once and are able to very intuitively represent the strength of each of those signals by their relative peak heights. Near-field probing is very much a qualitative art, where relative signal strength matters far more than absolute amplitude.

3.9 RF safety

Electromagnetic radiation conveys energy, and as such may pose a physical hazard to living bodies. The following warning sign, found near the base of an FM broadcast transmitting antenna, implies as much:



We will explore some of the hazards of RF exposure, beginning from the most severe and progressing to the least harmful.

At very high levels of RF power – on the order of tens of kiloWatts and more – the electromagnetic energy has a similar effect on the human body to direct electric shock: overriding muscle control. There are cases where RF technicians were accidentally exposed to such levels while working on radio transmitting towers, and actually lost their grip on the tower as a result¹².

High-power RF exposure also carries direct *thermal* hazards: the electromagnetic energy has the ability heat up living tissue, to the point of inducing deep tissue burns in extreme scenarios. This, in fact, is how microwave cooking ovens work: the RF causes water molecules in the food to rotate, and in so doing translates that energy into heat. This is also the principle at work in *diathermy*, a therapeutic treatment for certain muscular injuries and to induce coagulation of certain tissues where the patient is exposed to localized RF by a qualified medical professional.

At power levels too low to cause muscular paralysis or immediately burn tissue, RF can still be hazardous. Documented cases of RF technicians exposed to high levels of electromagnetic energy from broadcasting antennas reveal symptoms including headaches, blurred vision, paresthesia (strange skin sensations), back pain, diarrhea, physical weakness, insomnia, and general discomfort, in some cases symptoms persisting for *years* following the exposure.

Hazards resulting from much lower levels of exposure to RF are an area of active research and debate. In 2011 the International Agency for Research on Cancer (IARC) classified RF electromagnetic fields as Group 2B carcinogens, which means “possibly carcinogenic to humans”. This designation was based on provisional data showing an increase in *gliomas* (a type of malignant brain tumor) and acoustic neuromas (an often benign tumor affecting the auditory nerve) associated with cellular telephone usage.

The Chairperson of this Working Group, Dr. Jonathan Samet, made the following statement recorded in Press Release 208 from the IARC on 31 March 2011:

“The evidence, while still accumulating, is strong enough to support a conclusion and the 2B classification. The conclusion means that there could be some risk, and therefore we need to keep a close watch for a link between cell phones and cancer risk.”

The Group found limited evidence for cellular telephone use contributing to gliomas and acoustic neuromas, and inadequate evidence for other types of cancer. “Limited evidence” is defined as a positive correlation deemed credible by the Group, but one weak enough that other factors such as bias and chance cannot be eliminated. In other words, the 2B carcinogen classification simply says it’s *possible* low-level RF exposure could cause certain types of cancer. The IARC has other classifications appropriate for definitive causal associations, including Group 1 (*definitely* carcinogenic) and Group 2A (*probably* carcinogenic). Interestingly, the IARC does not specify a quantitative threshold distinguishing Groups 2A and 2B, merely stating that the 2A classification indicates a greater likelihood of carcinogenicity than 2B. This makes these classifications somewhat subjective.

¹²I have personally met people who had this happen to them. One individual testified that his limbs began to flail about uncontrollably as soon as the antenna became energized, causing him to fall off the tower entirely! Fortunately for him, the fall was less injurious than he expected, and in fact his rapid descent removed him from RF exposure faster than if he had been able to climb down, which at the power level he was experiencing would have severely burned him in a very short amount of time.

An example of a recent study exploring correlations between RF exposure and cancer is the so-called *Million Women Study* conducted by the University of Oxford in England, designed to investigate causes of various types of cancers in women. One of the areas of research within this ten-year study was the correlation between cell phone usage and various brain cancers, the most likely form of cancer one would expect as a result of operating a radio transmitter (cell phone) near the head. Risk Ratio (RR) is a measure of how more or less probable it will be that one group of individuals suffers given medical problem than another group, a value of 1.0 meaning equal risk, a value less than 1.0 meaning less risk, and a value greater than 1.0 meaning greater risk. Results from the Million Women Study were mixed, as most studies are comparing RF exposure against cancer incidence tend to be. For example, the Risk Ratio between long-term (10+ years) cell phone users versus women who never or rarely used a cell phone was 0.86 for pituitary gland cancers and 1.32 for acoustic neuromas: this means the risk of developing an acoustic neuroma was 32% greater for women using cell phones for ten years or more versus women who rarely or never used cell phones, but for pituitary gland cancers the long-term cell phone users actually were *less* likely to suffer compared to those who never or rarely used a cell phone.

Risk ratios calculated from the Million Women Study data for all types of head-related cancers, comparing each of the cell phone using groups against those who didn't (or rarely) used cell phones, are shown below:

Ever-use vs. Never-use	Daily-use vs. Never-use	10+ years use vs. Never-use
0.97	1.01	0.95

Such indeterminate results are frustrating for anyone expecting a clear answer as to whether or not RF poses a cancer risk. All of these ratios are quite close to 1.0 which would suggest no statistically significant risk for RF (at least when all types of head cancers are considered together), and the uneven progression of risk ratios from ever-use to daily-use to long-term use (from less risk to more and back to less) – if such ratios are significant at all – defies simple explanation. Possible interpretations include (1) carcinogenicity peaks only with moderate RF exposure but not with infrequent nor with persistent exposure, (2) confounding factors exist which were not controlled in the study, and/or (3) RF exposure is not a significant driver of cancer.

The current lack of a plausible mechanism explaining how electromagnetic waves might possibly cause cancer (i.e. a medical *theory* of RF carcinogenicity) makes research more difficult. It seems prudent, though, to apply what is commonly referred to as the *Precautionary Principle*, where we exercise caution even if we are not sure something causes harm. In other words, it makes no sense to expose oneself to something that *may* be harmful unless there is some clear benefit to doing so (e.g. a definite need for radio communication).

Occupational and general exposure limits established by governmental regulating bodies such as the FCC are largely based on the well-established *thermal* effects of high-power RF emissions. Mitigations when working around RF sources include *lock-out*, *tag-out* (LOTO) and intentionally reducing output power during the exposure period.

Chapter 4

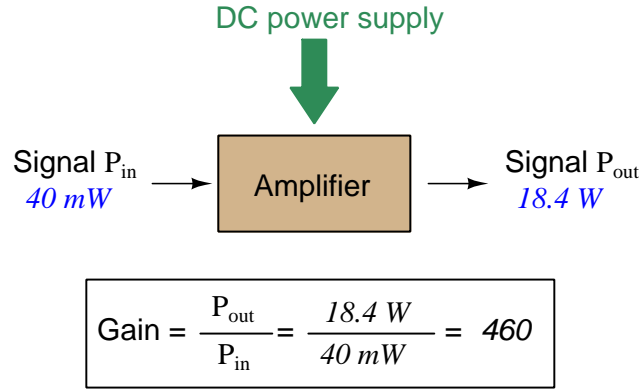
Derivations and Technical References

This chapter is where you will find mathematical derivations too detailed to include in the tutorial, and/or tables and other technical reference material.

4.1 Decibels

One of the mathematical tools popularly used to gauge increases and decreases of electrical power is the *common logarithm*, expressed as a measurement unit called the *decibel*. The basic idea of decibels is to express a ratio of two electrical power quantities in logarithmic terms. Every time you see the unit of “decibel” you can think: *this is an expression of how much greater (or how much smaller) one power is to another*. The only question is which two powers are being compared.

Electronic amplifiers are a type of electrical system where comparisons of power are useful. Students of electronics learn to compare the output power of an amplifier against the input power as a unitless ratio, called a *gain*. Take for example an electronic amplifier with a signal input of 40 milliWatts and a signal output of 18.4 Watts:



An alternative way to express the gain of this amplifier is to do so using the unit of the *Bel*, defined as the common logarithm of the gain ratio:

$$\log \left(\frac{P_{\text{out}}}{P_{\text{in}}} \right) = \log \left(\frac{18.4 \text{ W}}{40 \text{ mW}} \right) = 2.66276 \text{ B}$$

When you see an amplifier gain expressed in the unit of “Bel”, it’s really just a way of saying “The output signal coming from this amplifier is x powers of ten greater than the input signal.” An amplifier exhibiting a gain of 1 Bel outputs 10 times as much power as the input signal. An amplifier with a gain of 2 Bels boosts the input signal by a factor of 100. The amplifier shown above, with a gain of 2.66276 Bels, boosts the input signal 460-fold.

At some point in technological history it was decided that the “Bel” (B) was too large and cumbersome, and so it became common to express powers in fractions of a Bel instead: the *deci*Bel (1 dB = $\frac{1}{10}$ of a Bel). Therefore, this is the form of formula you will commonly see for expressing electrical signal power gains or losses:

$$\text{dB} = 10 \log \left(\frac{P_{\text{out}}}{P_{\text{in}}} \right)$$

The gain of our hypothetical electronic amplifier, therefore, would be more commonly expressed as 26.6276 dB rather than 2.66276 B, although either expression is technically valid¹.

¹It is interesting to note that although the “Bel” is a metric unit, it is seldom if ever used without the metric prefix

An operation students often struggle with is converting a decibel figure back into a ratio, since the concept of logarithms seems to be universally perplexing. Here I will demonstrate how to algebraically manipulate the decibel formula to solve for the power ratio given a dB figure.

First, we will begin with the decibel formula as given, solving for a value in decibels given a power ratio:

$$\text{dB} = 10 \log(\text{Ratio})$$

If we wish to solve for the ratio, we must “undo” all the mathematical operations surrounding that variable. One way to determine how to do this is to reverse the order of operations we would follow if we knew the ratio and were solving for the dB value. After calculating the ratio, we would then take the logarithm of that value, and then multiply that logarithm by 10: start with the ratio, then take the logarithm, then multiply last. To un-do these operations and solve for the ratio, we must un-do each of these operations in reverse order. First, we must un-do the multiplication (by dividing by 10):

$$\frac{\text{dB}}{10} = \frac{10 \log(\text{Ratio})}{10}$$

$$\frac{\text{dB}}{10} = \log(\text{Ratio})$$

Next, we must un-do the logarithm function by applying its mathematical inverse to both sides of the formula – making each expression a power of 10:

$$10^{\frac{\text{dB}}{10}} = 10^{\log(\text{Ratio})}$$

$$10^{\frac{\text{dB}}{10}} = \text{Ratio}$$

To test our algebra, we can take the previous decibel value for our hypothetical amplifier and see if this new formula yields the original gain ratio:

$$\text{Ratio} = 10^{\frac{26.6276 \text{ dB}}{10}}$$

$$\text{Ratio} = 10^{2.66276} \text{ B}$$

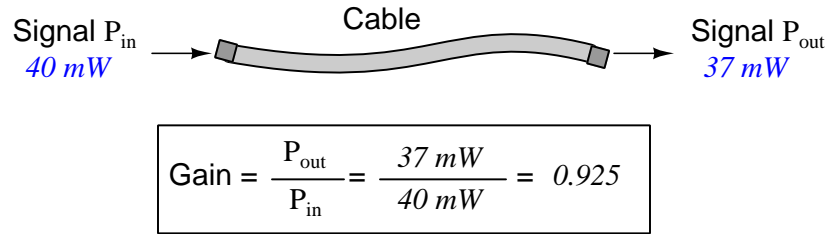
$$\text{Ratio} = 460$$

Sure enough, we arrive at the correct gain ratio of 460, starting with the decibel gain figure of 26.6276 dB.

“deci” ($\frac{1}{10}$). One could express powers in microbels, megabels, or any other metric prefix desired, but it is never done in industry: only the decibel is used.

We may also use decibels to express power *losses* in addition to power *gains*. There are many practical applications of this in signaling systems, both electronic and optical. One such application is *filtering*, where a “filter” circuit screens out certain components of the signal while letting others pass through (e.g. the bass or treble control for an audio system). Another application is *attenuation*, where the entirety of a signal is reduced in magnitude (e.g. the volume control for an audio system).

We will explore yet another application of signal power reduction as a case study for decibels: *cable loss*. Cables designed to convey signals over long distances are not perfect conduits of energy, as some of the signal’s energy is inevitably lost along the way. This is true for different types of signals, electrical and optical being two popular examples. In the following illustration we see a signal cable losing power along its length², such that the power out is less than the power in:



$$10 \log \left(\frac{P_{out}}{P_{in}} \right) = 10 \log \left(\frac{37\text{ mW}}{40\text{ mW}} \right) = -0.3386\text{ dB}$$

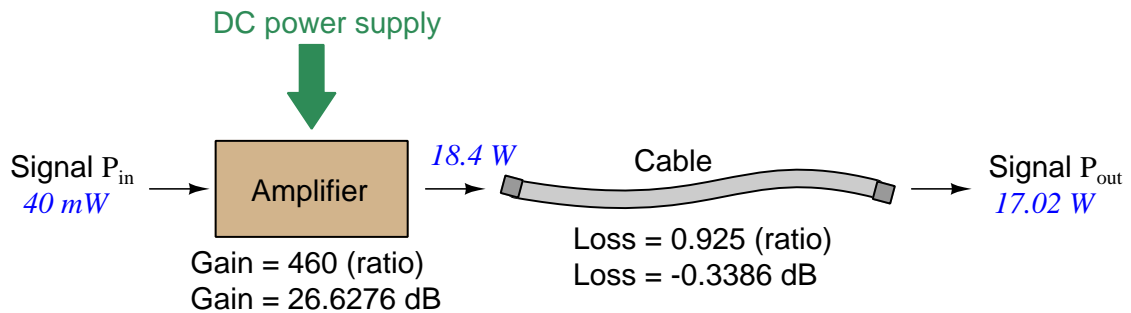
Contrasting this result against the previous result (with the amplifier) we see a very important property of decibel figures: any power *gain* is expressed as a *positive* decibel value, while any power *loss* is expressed as a *negative* decibel value. Any component outputting the exact same power as it takes in will exhibit a “gain” value of 0 dB (equivalent to a gain *ratio* of 1).

Remember that Bels and decibels are nothing more than logarithmic expressions of “greater than” and “less than”. Positive values represent powers that are *greater* while negative values represent powers that are *lesser*. Zero Bel or decibel values represent *no change* (neither gain nor loss) in power.

A couple of simple decibel values are useful to remember for approximations, where you need to quickly estimate decibel values from power ratios (or vice-versa). Each addition or subtraction of 10 dB exactly represents a 10-fold multiplication or division of power ratio: e.g. +20 dB represents a power ratio gain of $10 \times 10 = 100$, whereas −30 dB represents a power ratio reduction of $\frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} = \frac{1}{1000}$. Each addition or subtraction of 3 dB approximately represents a 2-fold multiplication or division of power ratio: e.g. +6 dB is approximately equal to a power ratio gain of $2 \times 2 = 4$, whereas −12 dB is approximately equal to a power ratio reduction of $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16}$. We may combine ± 10 dB and ± 3 dB increments to come up with ratios that are products of 10 and 2: e.g. +26 dB is approximately equal to a power ratio gain of $10 \times 10 \times 2 \times 2 = 400$.

²For high-frequency signals such as those used in radio communications, the dominant mode of energy dissipation is *dielectric heating*, where the AC electric field between the cable conductors excites the molecules of the conductor insulation. This energy loss manifests as heat, which explains why there is less signal energy present at the load end of the cable than is input at the source end of the cable. For DC and low-frequency AC circuits the dominant mode of energy dissipation is cable conductor resistance, which is typically very small.

Observe what happens if we combine a “gain” component with a “loss” component and calculate the overall power out versus power in:



The overall gain of this amplifier and cable system expressed as a ratio is equal to the *product* of the individual component gain/loss ratios. That is, the gain ratio of the amplifier *multiplied* by the loss ratio of the cable yields the overall power ratio for the system:

$$\text{Overall gain} = \frac{17.02 \text{ W}}{40 \text{ mW}} = (460)(0.925) = 425.5$$

The overall gain may be alternatively expressed as a decibel figure, in which case it is equal to the *sum* of the individual component decibel values. That is, the decibel gain of the amplifier *added* to the decibel loss of the cable yields the overall decibel figure for the system:

$$\text{Overall gain} = 10 \log \left(\frac{17.02 \text{ W}}{40 \text{ mW}} \right) = 26.6276 \text{ dB} + (-0.3386 \text{ dB}) = 26.2890 \text{ dB}$$

It is often useful to be able to estimate decibel values from power ratios and vice-versa. If we take the gain ratio of this amplifier and cable system (425.5) and round it down to 400, we may easily express this gain ratio as an expanded product of 10 and 2:

$$425.5 \approx 400 = (10) \times (10) \times (2) \times (2)$$

Knowing that every 10-fold multiplication of power ratio is an addition of +10 dB, and that every 2-fold multiplication of power is an addition of +3 dB, we may express the expanded product as a sum of decibel values:

$$(10) \times (10) \times (2) \times (2) = (10 \text{ dB}) + (10 \text{ dB}) + (3 \text{ dB}) + (3 \text{ dB}) = 26 \text{ dB}$$

Therefore, our power ratio of 425.5 is approximately equal to +26 decibels.

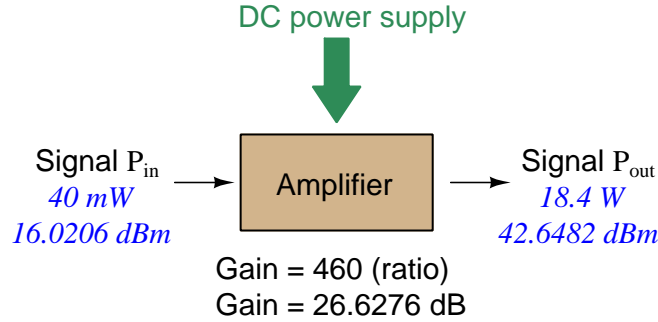
Decibels always represent comparisons of power, but that comparison need not always be P_{out}/P_{in} for a system component. We may also use decibels to express an amount of power compared to some standard reference. If, for example, we wished to express the input power to our hypothetical amplifier (40 milliWatts) using decibels, we could do so by comparing 40 mW against a standard “reference” power of exactly 1 milliWatt. The resulting decibel figure would be written as “dBm” in honor of the 1 milliWatt reference:

$$P_{in} = 10 \log \left(\frac{40 \text{ mW}}{1 \text{ mW}} \right) = 16.0206 \text{ dBm}$$

The unit of “dBm” literally means the amount of dB “greater than” 1 milliWatt. In this case, our input signal of 40 milliWatts is 16.0206 dB greater than a standard reference power of exactly 1 milliWatt. The output power of that amplifier (18.4 Watts) may be expressed in dBm as well:

$$P_{out} = 10 \log \left(\frac{18.4 \text{ W}}{1 \text{ mW}} \right) = 42.6482 \text{ dBm}$$

A signal power of 18.4 Watts is 42.6482 dB greater than a standard reference power of exactly 1 milliWatt, and so it has a decibel value of 42.6482 dBm.



Notice how the output and input powers expressed in dBm relate to the power gain of the amplifier. Taking the input power and simply *adding* the amplifier’s gain factor yields the amplifier’s output power in dBm:

$$P_{in}(\text{dB}) + P_{gain}(\text{dB}) = P_{out}(\text{dB})$$

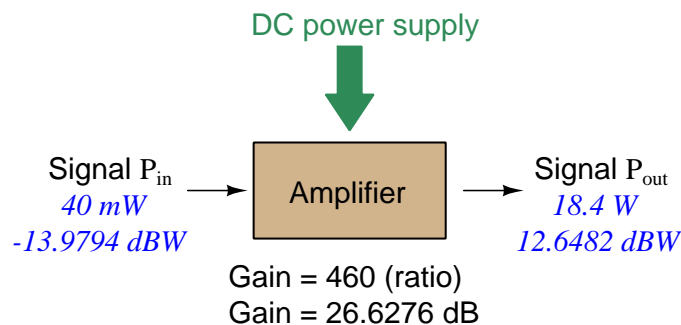
$$16.0206 \text{ dBm} + 26.6276 \text{ dB} = 42.6482 \text{ dBm}$$

An electronic signal that begins 16.0206 dB greater than 1 milliWatt, when boosted by an amplifier gain of 26.6276 dB, will become 42.6482 dB greater than the original reference power of 1 milliWatt.

We may alternatively express all powers in this hypothetical amplifier in reference to a 1-Watt standard power, with the resulting power expressed in units of “dBW” (decibels greater than 1 Watt):

$$P_{in} = 10 \log \left(\frac{40 \text{ mW}}{1 \text{ W}} \right) = -13.9794 \text{ dBW}$$

$$P_{out} = 10 \log \left(\frac{18.4 \text{ W}}{1 \text{ W}} \right) = 12.6482 \text{ dBW}$$



Note how the input power of 40 milliWatts equates to a negative dBW figure because 40 milliWatts is *less* than the 1 Watt reference, and how the output power of 18.4 Watts equates to a positive dBW figure because 18.4 Watts is *more* than the 1 Watt reference. A positive dB figure means “more than” while a negative dB figure means “less than.”

Note also how the output and input powers expressed in dBW still relate to the power gain of the amplifier by simple addition, just as they did when previously expressed in units of dBm. Taking the input power in units of dBW and simply *adding* the amplifier’s gain factor yields the amplifier’s output power in dBW:

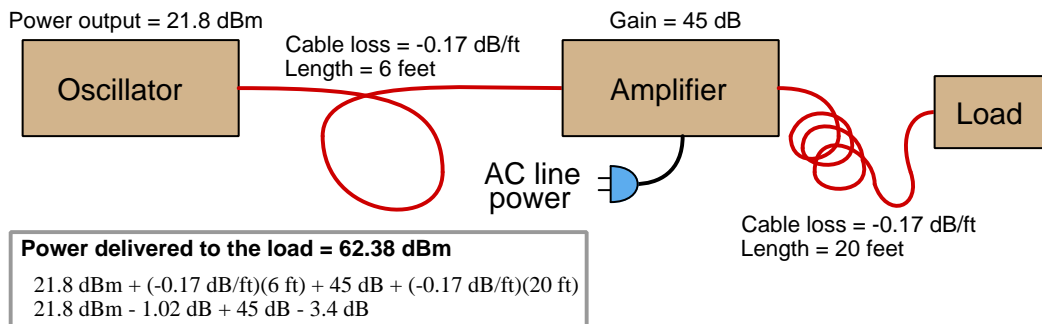
$$P_{in}(\text{dB}) + P_{gain}(\text{dB}) = P_{out}(\text{dB})$$

$$-13.9794 \text{ dBW} + 26.6276 \text{ dB} = 12.6482 \text{ dBW}$$

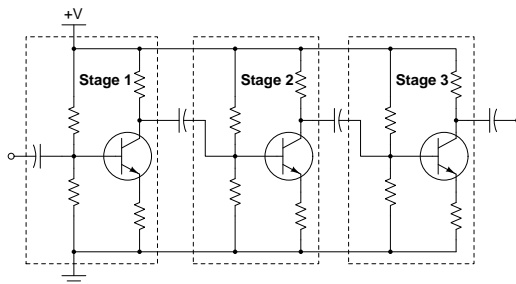
An electronic signal that begins 13.9794 dB less than 1 Watt, when boosted by an amplifier gain of 26.6276 dB, will become 12.6482 dB greater than the original reference power of 1 Watt.

This is one of the major benefits of using decibels to express powers: we may very easily calculate power gains and losses by summing a string of dB figures, each dB figure representing the power gain or power loss of a different system component. Normally, any compounding of *ratios* involves multiplication and/or division of those ratios, but with decibels we may simply add and subtract. One of the interesting mathematical properties of logarithms is that they “transform”³ one type of problem into a simpler type: in this case, a problem of multiplying ratios into a (simpler) problem of adding decibel figures.

For example, we may express the power dissipated along a cable in terms of decibels per foot; the longer the cable, of course, the more power will be lost this way, all other factors being equal. For example, a radio-frequency signal cable having a loss figure of -0.15 decibels per foot at a signal frequency of 2.4 GHz will suffer -15 dB over 100 feet, and -150 dB over 1000 feet. To illustrate how decibels may be used to calculate power delivered to a load in such a system, accounting for various gains and losses along the way using decibel figures:



A similar application of decibels is found in multi-stage amplifier circuits, where one stage amplifies a signal to be fed into a successive stage to be amplified more. The power gains of these stages, each expressed as a ratio, *multiply* to make the over-all amplifier's power gain (ratio). The power gains of those same stages, each expressed as a decibel figure, *add* to make the over-all amplifier's power gain (dB):



³In fact, logarithms are one of the simplest examples of a *transform function*, converting one type of mathematical problem into another type. Other examples of mathematical transform functions used in engineering include the *Fourier transform* (converting a time-domain function into a frequency-domain function) and the *Laplace transform* (converting a differential equation into an algebraic equation).

Another common application of decibels is to express ratios of voltage and/or current rather than power. However, since the unit of the Bel has been defined as an expression of a *power* ratio, we cannot simply substitute V or I for P in any of the formulae we've seen so far.

Suppose an amplifier has a voltage gain of 2 (i.e. V_{out} is twice as large as V_{in}), and we would like to express this gain in decibels. Since decibels are intended to express power gain and not voltage gain, we must figure out how much power gain is equivalent to a voltage gain of two. Obviously, voltage and power are fundamentally different quantities, but if we imagine ourselves connecting a fixed load resistance to the input signal, and then to the output signal, we will realize that load's power dissipation will be more than double when energized by a voltage twice as large. Joule's Law is helpful to determine the exact ratio of power dissipation:

$$P = \frac{V^2}{R}$$

Doubling voltage for any given load resistance results in power *quadrupling* because power is proportional to the square of the voltage applied to a fixed resistance. Using this as the basis for applying decibels to a voltage ratio. Knowing that Joule's Law also declares power is proportional to the square of the current applied to a fixed resistance ($P = I^2 R$) means this same mathematical relationship will apply to current gains and reductions as well as voltage gains and reductions:

$$\text{dB} = 10 \log \left(\frac{P_{out}}{P_{in}} \right) = 10 \log \left(\frac{V_{out}}{V_{in}} \right)^2 = 10 \log \left(\frac{I_{out}}{I_{in}} \right)^2$$

An algebraic identity of logarithms is that the logarithm of any quantity raised to a power is equal to that power multiplied by the logarithm of the quantity. Expressed in general terms:

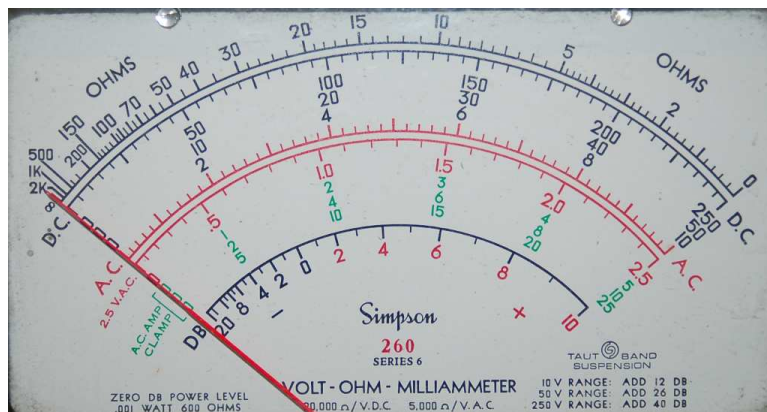
$$\log x^y = y \log x$$

Therefore, we may simplify the decibel formula for voltage gain by removing the "2" power and making it a multiplier:

$$\begin{aligned} 10 \log \left(\frac{V_{out}}{V_{in}} \right)^2 &= (2)(10) \log \left(\frac{V_{out}}{V_{in}} \right) = 20 \log \left(\frac{V_{out}}{V_{in}} \right) \\ 10 \log \left(\frac{I_{out}}{I_{in}} \right)^2 &= (2)(10) \log \left(\frac{I_{out}}{I_{in}} \right) = 20 \log \left(\frac{I_{out}}{I_{in}} \right) \end{aligned}$$

Thus, we may use decibels to express voltage or current ratios if we simply substitute 20 instead of 10 as the multiplier.

We can see the practicality of using decibels to represent something other than electrical *power* by examining this analog meter face, belonging to a Simpson model 260 VOM (Volt-Ohm-Milliammeter). Note the bottom scale on this meter's face, calibrated in decibels (DB):



Pay attention to the note on decibels written in the lower-left corner of the meter face, where 0 dB is defined as 0.001 Watt dissipated by 600 Ohms. The fact that 0 dB is defined as 1 milliWatt means it should (properly) be labeled dBm rather than dB⁴. A load resistance value is necessary as part of this definition for dB because this meter cannot measure power directly but must infer signal power from measurements of AC *voltage*. Without a specific load resistance, there is no clear relation between voltage and power. 600 Ohms is an old telecommunications standard for audio-frequency AC signals, and continues to be used today for voltage-based decibel measurements of audio-frequency AC signals.

The meter as shown is connected to nothing at all, and so registers 0 Volts AC. This, of course, corresponds to zero power, and it has no corresponding decibel value because the logarithm of zero is mathematically *undefined*⁵. Practically, it means $-\infty$ dB, which is why the needle at the 0 Volt position “falls off” the left-hand end of the dB scale.

Close inspection of the dB scale on this meter face reveals another interesting property of decibels, and that is the *nonlinear* nature of the dB scale. This contrasts starkly against all the voltage and current scales on this meter face which are linear. This nonlinearity is a fundamental property of decibels because it is based on the *logarithm* function which is nonlinear.

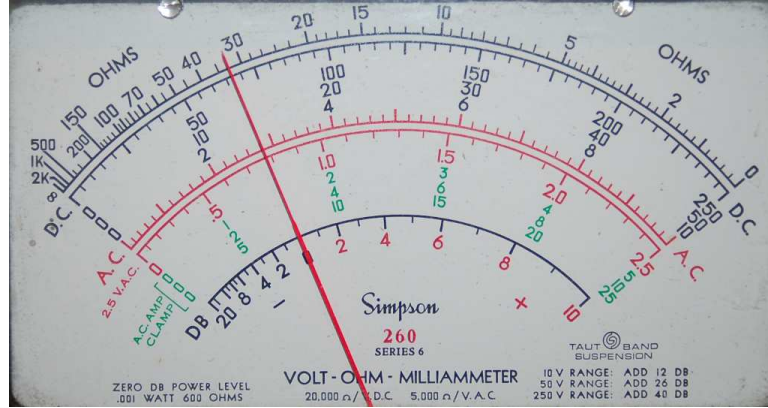
⁴Such mis-labeling is not that uncommon in the profession, the expectation being that the technician or engineer working with the instrument ought to be familiar enough with the concept of decibels to know when dB really means dBm, or dBW, etc.

⁵Your electronic calculator will complain if you attempt to take the logarithm of zero!

Now, we will explore what is necessary to make this meter register 0 dBm (i.e. 1 milliWatt) with an applied AC voltage. 1 milliWatt of power dissipated by 600 Ohms is equivalent to:

$$V = \sqrt{PR} = \sqrt{(0.001)(600)} = 0.7746 \text{ Volts}$$

Setting the VOM to the 2.5 VAC range and applying just enough AC voltage to bring the needle to the 0 dB mark allows us to verify that this is indeed equivalent to just under 0.8 Volts (read on the 2.5 VAC scale):



In the lower-right corner of the meter face we see some notes regarding correction values for decibel measurements when using different AC voltage ranges. The dB scale is read directly when the meter is set on the 2.5 VAC range. When set on the 10 VAC range (i.e. a range *four times* as great), the meter's needle will experience a deflection one-fourth as much as when set to the 2.5 VAC range, and therefore it will point to a lesser (or even negative) value on the dB scale. Converting a voltage ratio of 0.25 into a decibel figure shows us how much less the needle will register on the dB scale when the voltage range is quadrupled:

$$20 \log \left(\frac{2.5}{10} \right) = -12.04 \text{ dB}$$

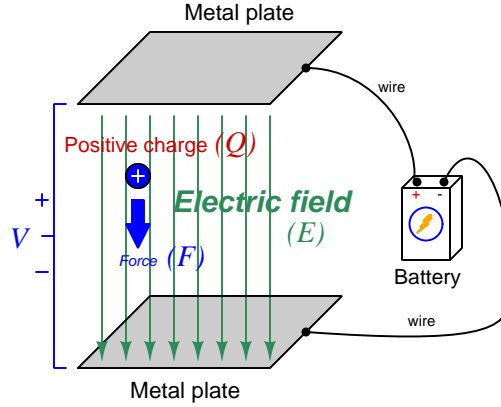
Therefore, when using the 10 VAC range instead of the 2.5 VAC range, one must add 12 dB to the reading. Likewise, we may prove each of the printed correction factors for the alternative voltage-measurement ranges listed (50 Volt AC range and 250 Volt AC range):

$$20 \log \left(\frac{2.5}{50} \right) = -26.02 \text{ dB}$$

$$20 \log \left(\frac{2.5}{250} \right) = -40.0 \text{ dB}$$

4.2 Electric field quantities

A useful definition of electric field (E) is in terms of the force (F) exerted on an electric charge (Q) influenced by that field:



$$\vec{F} = Q\vec{E}$$

Where,

\vec{F} = Force exerted on the charge (Newtons)

Q = Charge quantity (Coulombs⁶)

\vec{E} = Electric field (Newtons per Coulomb)

The small “arrow” symbols above the variables for force and electric field in the equation denote those variables as *vector quantities*, having both magnitude and direction. Charge is a *scalar quantity* having only magnitude but no direction, and as a scalar quantity when multiplied by the electric field vector it simply magnifies the magnitude but does not alter the direction. Therefore, the force and electric field vectors always point in the same direction.

Alternatively electric field may be defined in terms of the voltage between the end-points and the distance separating them, in which case we may express the electric field in units of *Volts per meter* as an alternative to *Newtons per Coulomb*:

$$\vec{E} = \frac{V}{d}$$

This measurement of electric field strength is very important for quantifying the *breakdown* of electrical insulators: the point at which the electric field becomes so powerful that otherwise immobile charges within the insulating substance are torn free to constitute a current and that substance is no longer an insulator. For rating the dielectric strength of insulating materials, we often see electric fields expressed in units of *kiloVolts per millimeter* rather than Volts per meter just to make the numerical quantities easier to manage (1 kV/mm = 1 *million* V/m).

⁶One Coulomb of electric charge is equal to 6.2415×10^{18} electrons.

The vector arrows shown in the previous illustration representing the electric field between two metal plates actually represent electric *flux* (Φ_E). The electric field (E) is related to electric flux by area (A), the field being a measurement of how densely-packed those flux lines are per unit area:

$$\vec{E} = \frac{\Phi_E}{\vec{A}}$$

Where,

\vec{E} = Electric field, or electric flux density (Newtons per Coulomb)

Φ_E = Electric flux (Newton-meter squared per Coulomb)

\vec{A} = Area over which flux is distributed (square meters)

The mere presence of an unbalanced electric charge at any point in space is sufficient to generate lines of electric flux, the total magnitude of that flux predicted by the following equation:

$$\Phi_E = \frac{Q}{\epsilon}$$

Where,

Φ_E = Electric flux (Newton-meter squared per Coulomb)

Q = Charge quantity (Coulombs)

ϵ = Electric permittivity of the surrounding space (Coulombs squared per Newton-meter squared, approximately 8.85×10^{-12} for empty space)

By convention, these flux vectors point *away* from positive charges and point *toward* negative charges, their direction indicating force exerted on any positively-charged particle influenced by that field⁷. As the equation states, the amount of flux depends on how much charge exists at each particle as well as the permittivity of the surrounding space:

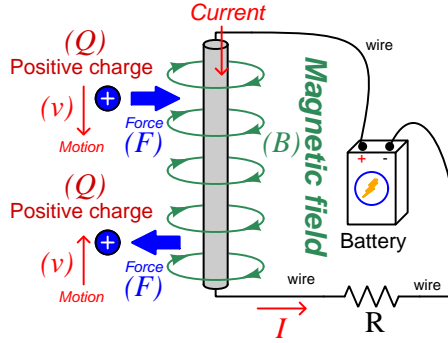


For example, identical charges suspended in a vacuum versus in a substance such as oil will have different amounts of flux associated with them as a result of oil and vacuum having different permittivity values. Perfectly empty space has the least amount of permittivity, which means anything else (gas, liquid, or solid matter) has greater ϵ which acts to diminish the amount of electric flux surrounding any charged particle. Superconducting materials have infinite permittivity, which means they forbid the existence of any electric field inside their bulk.

⁷Conversely, the flux vectors point exactly opposite the direction of force applied to any *negatively*-charged particle within that field. This makes sense of the aphorism that *like charges repel and opposite charges attract*. If you consider the two charges shown in this illustration, the positive charge will be pulled in the direction of the flux vectors pointing toward the negative charge, as the negative charge will also be pulled opposite the direction of the flux vectors pointing away from the positive charge (i.e. the negative charge will be pulled *toward* the positive charge) – thus the positive and negative charges feel mutual attraction.

4.3 Magnetic field quantities

A useful definition of magnetic field (B) is in terms of the force (F , called the *Lorentz force*) exerted on a moving electric charge (Q) influenced by that field:



$$\vec{F} = Q\vec{v} \times \vec{B}$$

Where,

\vec{F} = Force exerted on the charge (Newtons)

Q = Charge quantity (Coulombs⁸)

\vec{v} = Velocity of moving charge (meters per second)

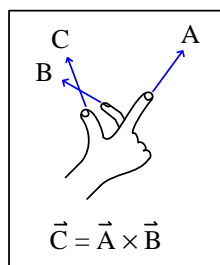
\vec{B} = Magnetic field (Tesla, Webers per square meter, or Newtons per Ampere-meter)

The small “arrow” symbols above the variables for force and velocity and magnetic field in the equation denote those variables as *vector quantities*, having both magnitude and direction. Charge is a *scalar quantity* having only magnitude but no direction, and as a scalar quantity when multiplied by the velocity vector it simply magnifies the magnitude but does not alter the direction. The “cross-product” (\times) is a specific form of vector multiplication, and it results in a product at right angles to the vector directions of both terms. Therefore, the force and velocity and electric field vectors never all point in the same direction.

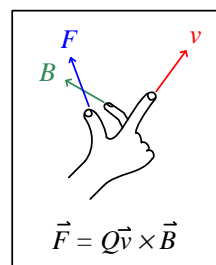
⁸One Coulomb of electric charge is equal to 6.2415×10^{18} electrons.

Vector cross-products conveniently relate to the fingers of the right hand, which is where the “right-hand rule” originates:

General "right-hand rule"
for vector cross-products

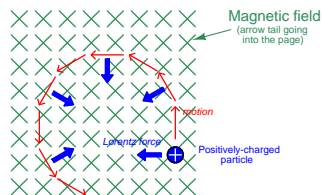


"Right-hand rule" specific
to the Lorentz force



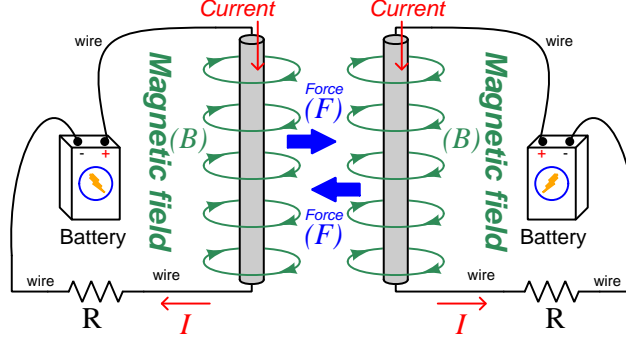
When holding the index finger, middle finger, and thumb of your right hand perpendicular to each other, your index finger points in the direction of the velocity vector (\vec{v}), your middle finger in the direction of the magnetic field vector (\vec{B}), and your thumb in the direction of the force vector (\vec{F}). A simple mnemonic I use to remember these relationships of fingers to vectors is that the **I**ndex finger points in the direction of current⁹ (I), the **M**iddle finger points in the direction of the **m**agnetic field (B), and the **T**humb points in the direction of the **t**hrust (i.e. force) acting upon the moving charge.

The Lorentz force’s effect on electrically-charged particles in motion has many applications, from redirecting the paths of charged-particle beams in particle accelerator machines, to bending the trajectory of electron beams in a cathode-ray tube (CRT), to forcing electrons to travel in spiral or circular trajectories inside of magnetron (microwave oscillator) tubes. An illustration of a positively-charged particle curving perpendicular to a magnetic field appears here:



⁹Some textbooks speak of a “left-hand rule” which is intended to make sense of electric charge motion (current) in terms of *electron flow*. As we know, electrons are the only real mobile charge carriers within metal conductors, and so technically “electron flow” notation is most physically accurate when describing the motion of electric charges in metallic circuits. However, the right-hand rule is a *mathematical definition* for vector cross products, the concept of the cross product arising in the late 18th century when electrical science was still in its infancy. Early explorers of electricity used the established mathematical tools of their time and applied it to their work with electric currents and magnetism. At that time, charge carriers in metal wires were assumed to be “positive” and this is how the motion of positively-charged carriers became associated with the first vector of the cross-product. As a result of this assumption which was later proven false, we have two different conventions for denoting the motion of electricity: electron-flow which is *physically* accurate (for metal wires, at least), and conventional flow which is *mathematically* accurate. This, perhaps more than any other reason, is why educational programs designed for mathematically rigorous fields (e.g. electrical engineering) exclusively use conventional flow notation rather than electron flow notation to denote the direction of current.

If the moving charge in question is not a single charged particle but rather part an electric *current* passing through a conductor parallel to the first, both conductors will experience a mutually-attracting force given by the following equation:



$$\vec{F} = I\vec{l} \times \vec{B}$$

Where,

\vec{F} = Force exerted on both conductors (Newtons)

I = Current (Amperes)

\vec{l} = Length of wire (meters)

\vec{B} = Magnetic field (Tesla, or Webers per square meter, or Newtons per Ampere-meter)

The point-charge Lorentz force equation and the two-conductor Lorentz force equation are not that different from one another. Dimensional analysis validates this: the Lorentz force on a moving charge uses that charge quantity (Coulombs) multiplied by the point-charge's velocity in meters per second to give Coulomb-meters per second for the first term:

$$Q\vec{v} = [\text{C}] \left[\frac{\text{m}}{\text{s}} \right] = \left[\frac{\text{C} \cdot \text{m}}{\text{s}} \right]$$

The Lorentz force on a current-carrying conductor uses the current (Amperes, which is Coulombs per second) multiplied by length in meters, for the same composite units of Coulomb-meters per second:

$$I\vec{l} = \left[\frac{\text{C}}{\text{s}} \right] [\text{m}] = \left[\frac{\text{C} \cdot \text{m}}{\text{s}} \right]$$

This dimensional equivalence makes conceptual sense as well: an electrically-charged particle moving through empty space is an electric current in its own right, and an electric current flowing through a conductor is just a collection of charged particles moving through space (just not *empty* space). In either case, the basis for the Lorentz force remains the same: the moving charge(s) create their own magnetic field, which reacts with the magnetic field of the original current-carrying wire to produce forces acting on both.

If the two currents flow in the same direction, their mutual forces *attract*. If the two currents flow in opposite directions, their mutual forces *repel*. This is the basis of electric motors: causing

mechanical motion by electro-magnetic attraction and repulsion. It also represents an interesting contrast with electric fields:

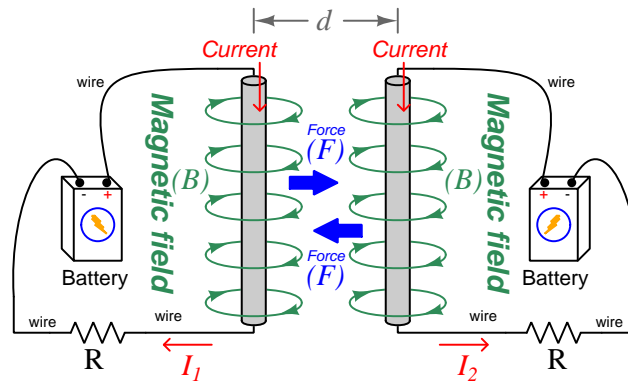
With electric fields, opposite *charges* attract and like *charges* repel.

With magnetic fields, opposite *poles* attract and like *poles* repel.

With parallel currents, opposite *directions* repel and like *directions* attract¹⁰.

Two parallel current-carrying conductors of length l and separated by a distance d will generate a mutual force proportional to both their currents:

$$F = l \frac{\mu I_1 I_2}{2\pi d}$$



¹⁰That is, assuming it's *like* charges moving in these directions! If the charges in question are opposite each other – for example electrons in one circuit and holes in another – then like directions will repel and opposite directions will attract!

The circular loops surrounding the current-carrying conductors in the previous illustrations represent the magnetic lines of *flux* (Φ_B) surrounding each of those conductors. The magnetic field (B) is related to magnetic flux by area (A), the field being a measurement of how densely-packed those flux lines are per unit area. For this reason, magnetic field (B) is more properly known as *magnetic flux density*:

$$\vec{B} = \frac{\Phi_B}{\vec{A}}$$

Where,

\vec{B} = Magnetic field or flux density (Tesla, Webers per square meter, or Newtons per Ampere-meter)

Φ_B = Magnetic flux (Webers)

\vec{A} = Area over which flux is distributed (square meters)

An older unit of measurement for magnetic flux density B is the *Gauss* which is much smaller than a Tesla, with one Tesla equivalent to 10,000 Gauss. To put things into perspective, the Earth's natural magnetic field has a strength of approximately one-half of one Gauss¹¹.

Magnetic field strength is an inverse function of distance from any current-carrying wire, and also depends on the magnetic permeability of the space adjacent to the wire:

$$B = \frac{\mu I}{2\pi d}$$

Where,

B = Magnetic field or flux density (Tesla, Webers per square meter, or Newtons per Ampere-meter)

μ = Magnetic permeability of the surrounding space (Tesla-meters per Ampere, $4\pi \times 10^{-7}$ for empty space)

I = Current (Amperes)

d = Distance from conductor (meters)

¹¹Using the online *Magnetic Field Calculator* application provided by NOAA (the National Oceanic and Atmospheric Administration) at <https://ngdc.noaa.gov/geomag/calculators/magcalc.shtml#igrfwmm>, applying the World Magnetic Model WMM modeling algorithm for years 2019-2024, the total magnetic field strength at my home is 53,584.4 nano-Tesla (53,584.4 nT or 0.535844 Gauss), and presently (May 2020) decaying at a rate of -104.1 nT per year.

The relation of magnetic *flux* to current through a conductor follows a similar equation:

$$\Phi = \frac{\mu AI}{2\pi d}$$

Where,

Φ = Magnetic flux (Webers)

μ = Magnetic permeability of the surrounding space (Tesla-meters per Ampere, $4\pi \times 10^{-7}$ for empty space)

A = Area over which flux is distributed (square meters)

I = Current (Amperes)

d = Distance from conductor (meters)

As this equation makes clear, the amount of magnetic flux surrounding a current-carrying conductor depends not only on the amount of current, but also on the sampled area, the distance from the wire, and also the surrounding material. Most¹² substances (gas, liquid, solid) have permeability values greater than that of empty space, and so this means magnetic flux is usually *enhanced* by the presence of matter around the current-carrying conductor.

The total magnetic flux enclosed by a circular wire loop follows a similar equation:

$$\Phi = \frac{\pi\mu Ir}{2}$$

Where,

Φ = Magnetic flux (Webers)

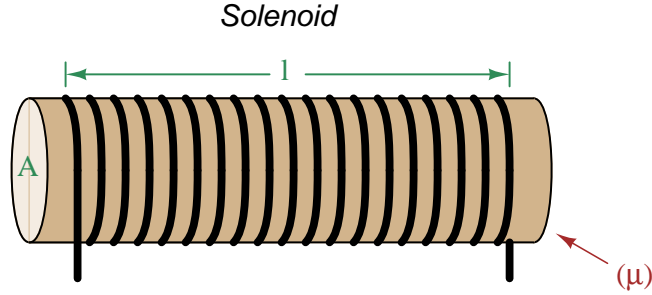
μ = Magnetic permeability of the surrounding space (Tesla-meters per Ampere, $4\pi \times 10^{-7}$ for empty space)

I = Current (Amperes)

r = Radius of circular loop (meters)

¹²Interestingly, superconducting materials *forbid* magnetic fields inside of their bulk, and so the permeability value of any superconductor must be zero!

A common form of electromagnet known as a *solenoid* takes the form of a wire coil wrapped in such a way as to form a long¹³ cylinder, often wrapped around a plastic frame, and often with a ferromagnetic material such as iron in the center:



The amount of magnetic flux, and the flux density, within the interior of a current-carrying solenoid are given by the following formulae:

$$\Phi = \frac{\mu N A I}{l} \qquad B = \frac{\mu N I}{l}$$

Where,

Φ = Magnetic flux (Webers)

B = Magnetic field or flux density (Tesla, Webers per square meter, or Newtons per Ampere-meter)

μ = Magnetic permeability of the surrounding space (Tesla-meters per Ampere, $4\pi \times 10^{-7}$ for empty space)

N = Number of turns of wire in the coil

A = Cross-sectional area of solenoid coil (square meters)

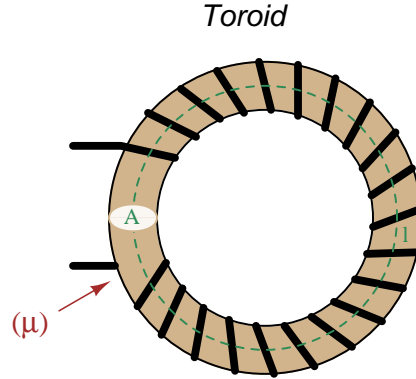
I = Current (Amperes)

l = Length of solenoid coil (meters)

These formulae have interesting implications for solenoid design. Note how a shorter (i.e. smaller length l) solenoid identical in all other respects will generate a stronger magnetic field for a given current. Note also how the flux density (B) remains constant with increasing cross-sectional area (A) if all other factors are equal, and that this necessarily means a greater amount of total magnetic flux (Φ) for a greater area A .

¹³These magnetic field formulae apply perfectly to a solenoid coil that is closely-packed (i.e. each turn adjacent to the next) and infinitely long. Therefore, they only approximate real solenoid behavior. This fact may be understood by performing a thought experiment where we decrease the solenoid coil's length to zero, in which case the formulae predict an *infinite* amount of magnetism for any amount of current at all, which of course cannot be true.

Another common form of electromagnet known as a *toroid* is really just a solenoid bent in a circle so that its two ends meet¹⁴ cylinder, often wrapped around a plastic frame, and often with a ferromagnetic material such as iron in the center. Toroids have the unusual property of *containing* their magnetic flux lines extremely well, unlike solenoids, wires, and simple coils which all radiate magnetic fields. They find application as energy-storage devices, or as electromagnets suitable for applying magnetic fields to specimens placed *inside* the toroid's cross-section:



The amount of magnetic flux, and the flux density, within the interior of a current-carrying toroid are identical to that within an otherwise identical solenoid having a length (l) equal to the toroid's circumference:

$$\Phi = \frac{\mu N A I}{l} \quad B = \frac{\mu N I}{l}$$

Where,

Φ = Magnetic flux (Webers)

B = Magnetic field or flux density (Tesla, Webers per square meter, or Newtons per Ampere-meter)

μ = Magnetic permeability of the surrounding space (Tesla-meters per Ampere, $4\pi \times 10^{-7}$ for empty space)

N = Number of turns of wire in the coil

A = Cross-sectional area of toroid (square meters)

I = Current (Amperes)

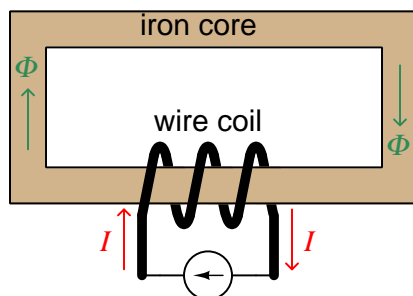
l = Circumference of toroid (meters)

If we wish to substitute toroid radius (r) for circumferential length (l), the formulae become the following:

$$\Phi = \frac{\mu N A I}{2\pi r} \quad B = \frac{\mu N I}{2\pi r}$$

¹⁴Again, the magnetic field formulae are only accurate for a toroidal coil that is closely-packed (i.e. each turn adjacent to the next) and infinitely long, and therefore only approximate real toroid behavior. This fact may be understood by performing an equivalent thought experiment as before where we decrease the toroid's circumference to zero and absurdly end up with *infinite* magnetism for a finite current.

Many applications of electromagnetism involve conductive *coils* wrapped around some form of ferromagnetic core material, the purpose of that core being to provide a higher-permeability pathway for the magnetic flux than would exist otherwise through air, and the purpose of the wire coil being to intensify the amount of magnetism developed by the electric current beyond what would be possible with a straight current-carrying wire. These magnetic cores typically form a closed loop, or *magnetic circuit* for the lines of magnetic flux to naturally form a closed path. A simple example appears here:



The amount of magnetic flux (Φ) present in the magnetic “circuit” formed by the iron core depends on many factors. First and foremost is the amount of electric current (in Amperes) passing through the wire coil and the number of turns that coil makes around the iron core. The product of this current and the number of turns is called the *magnetomotive force* or *mmf* of the magnetic circuit, analogous to “electromotive force” or “emf” often used as a synonym for voltage in an electric circuit. Not surprisingly, the standard metric unit of measurement for magnetomotive force is the *Ampere-turn*.

However, magnetomotive force alone does not fully describe the current’s effect on magnetism within the iron core. The total length of the magnetic circuit is also an important factor, since a longer path distributes that magnetomotive force over a greater distance. The quotient of magnetomotive force and magnetic circuit length is called *magnetic field intensity*, symbolized by the variable H and expressed in units of *Ampere-turns per meter*.

Magnetic permeability (μ) relates magnetic field intensity (H) to the magnetic flux density (B) within the core material, such that a greater permeability will result in higher flux density for any given amount of field intensity. Permeability is a property of the core material and not its geometry, mathematically defined as the ratio of flux density to field intensity: $\mu = \frac{B}{H}$

Magnetic reluctance (\mathfrak{R}) relates magnetomotive force (mmf) to magnetic flux (Φ), and is related not only to the core material’s permeability but also its geometry. It is mathematically defined as the ratio of magnetomotive force to magnetic flux: $\mathfrak{R} = \frac{\text{mmf}}{\Phi}$

If all this seems confusing, you are in good company. Not only are there many magnetic variables, some related to physical geometry and others not, but there are *two different sets of metric units* appropriate for expressing each! The older units were based on the centimeter-gram-second (CGS) version of the metric system, while the newer units are based on the meter-kilogram-second or SI (Système International) version of the metric system.

Quantity	Symbol	SI unit	CGS unit
Magnetomotive force	mmf	Ampere-turn (A-t)	Gilbert (Gb)
Flux	Φ	Weber (Wb)	Maxwell (Mx)
Field intensity	H	Ampere-turns per meter (A-t/m)	Oersted (Oe)
Flux density	B	Tesla (T)	Gauss (G)
Permeability	μ	Tesla-meters per Ampere-turn (T-m/A-t)	Gauss per Oersted (G/Oe)
Reluctance	\mathfrak{R}	Ampere-turns per Weber (A-t/Wb)	Gilberts per Maxwell (G/Mx)

Magnetomotive force (mmf) and magnetic flux (Φ) may be thought of as the “raw” measures of magnetism, with Ampere-turns and Webers being their respective SI metric units. Reluctance (\mathfrak{R}) is the ratio of the two for any given magnetic circuit with known dimensions and core material. Simply put, reluctance tells you how many Ampere-turns of magnetomotive force will be necessary to create one Weber of magnetic flux in a given space.

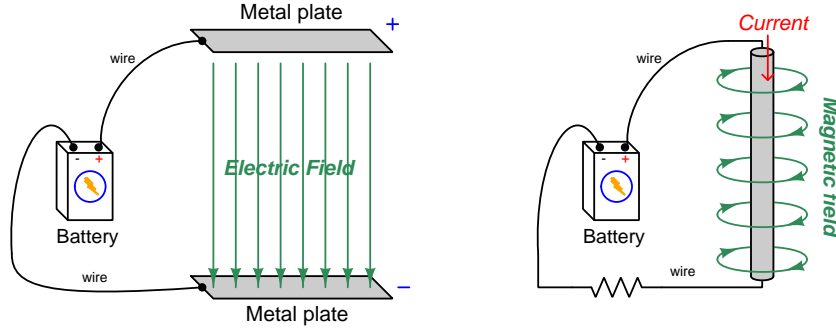
Magnetic field intensity (H) and magnetic flux density (B) may be thought of as the “normalized” measures of magnetism, with Ampere-turns per meter and Tesla being their respective SI metric units. H and B relate to mmf and flux by the physical dimensions of the magnetic circuit (length and cross-sectional area, respectively). Permeability is the ratio of the two for any given magnetic core material. Simply put, permeability tells you how many Tesla of magnetic field (i.e. flux density, or Webers of flux per square meter or cross-sectional core area) you will obtain for one Ampere-turn per meter of magnetic field intensity applied to a given core material.

Conversion between the newer SI and the older CGS metric units are as follows:

Quantity	Conversion equivalence
Magnetomotive force (mmf)	1 Ampere-turn = $\frac{4\pi}{10}$ Gilberts
Magnetic flux (Φ)	1 Weber = 10^8 Maxwells
Magnetic field intensity (H)	1 Ampere-turn/meter = $\frac{4\pi}{1000}$ Oersteds
Magnetic flux density (B)	1 Tesla = 10^4 Gauss
Permeability (μ)	1 Tesla-meter/Ampere-turn = $\frac{10^7}{4\pi}$ Gauss/Oersteds
Reluctance (\mathfrak{R})	1 Ampere-turn/Weber = $\frac{4\pi}{10^9}$ Gilberts/Maxwell

4.4 Near-field versus far-field regions

An electric field exists wherever voltage exists (i.e. wherever an imbalance exists between positive and negative electrical charges), parallel to the axis of that voltage. A magnetic field exists wherever an electric charge moves, perpendicular to the axis of that charge's motion. These phenomena exist for DC (direct) as well as AC (alternating) electricity, and are shown in the two following illustrations:



Electric and magnetic fields, however, are not simply manifestations of voltage and current, respectively. Mathematical discoveries made by the Scottish physicist James Clerk Maxwell (and later simplified by Oliver Heaviside) also relate electric fields and magnetic fields *directly to each other*. Consider two¹⁵ of Maxwell's equations shown below:

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

The first equation describes how an electric field (\vec{E}) is created by a varying magnetic flux ($\frac{d\Phi_B}{dt}$), which is otherwise known as *Faraday's Law of Electromagnetic Induction*. This is the law exploited in the function of electro-mechanical generators whereby coils of wire are subjected to changing magnetic fields, creating voltage between the coils' end-points. The negative sign in this equation is an embodiment of *Lenz's Law*, which states that any current resulting from the induced voltage will produce its own magnetic field opposing the first magnetic field's direction of change.

The second equation shown here describes *two* different ways to produce a magnetic field (\vec{B}). One way is to use a moving stream of electric charges known as a *current* (I), a fact also known as *Ampère's Law*. This is the law exploited in the function of electromagnets, where we produce a magnetic field by connecting a coil of wire to an electrical source. The second way is to vary an electric flux ($\frac{d\Phi_E}{dt}$) in empty space with surrounding magnetic permeability μ_0 and electric permittivity ϵ_0 .

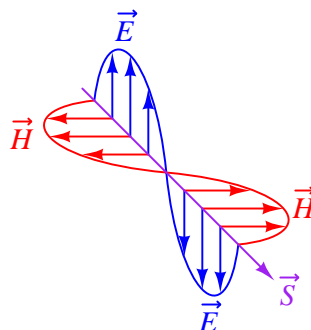
¹⁵Maxwell's equations of electromagnetism are typically presented as a set of *four*. Here we only list two of these, as the other two are not relevant to electromagnetic waves in particular.

Maxwell's equations suggest that in pulsed DC and AC circuits, where the resulting electric and/or magnetic fields vary over time ($\frac{d}{dt}$) rather than holding steady, those time-varying fields will mutually support one another over space and time. That is, a varying electric field will eventually produce a varying magnetic field, and vice-versa, those fields propagating one another as an *electromagnetic wave* radiating away from the circuit through space at the speed of light¹⁶.

At close range, immediately adjacent to the circuit conductors, electric and magnetic fields are strictly functions of those conductors' voltages and currents, respectively. These fields' shapes follow the basic forms shown in the previous illustrations: magnetic field lines circling current-carrying conductors and electric field lines stretching between conductive surfaces. We generally refer to this region of space around an energized circuit as the *near-field region*, where electric and magnetic fields are distinct from one another in form and in relative magnitude.

At farther distances from the energized circuit, the electric and magnetic fields are found in pairs oscillating at right angles to one another in space, forming a single electromagnetic wave radiating away from the source circuit spherically in all directions. This form of wave is a direct consequence of Maxwell's equations describing how the rate-of-change of one field creates the other field, and vice-versa. Below we see a crude representation of an electromagnetic wave, with an electric \vec{E} field, a magnetic \vec{H} field, and the vector of propagation \vec{S} called the *Poynting vector*:

Electromagnetic wave



This region where the electric and magnetic fields exist as coupled pairs comprising a well-formed wave is referred to as the *far-field region*. In this region the electric and magnetic fields are always perpendicular to one another and their relative magnitudes are in a fixed proportion governed by the permeability and permittivity of free space.

The demarcation between near- and far-field regions depends on the physical dimensions of the radiating circuit as well as the *wavelength* of the signal (λ), wavelength being calculated by dividing the speed of light (2.9979×10^8 meters per second in empty space) by the signal frequency in Hertz. There is no simple rule for predicting where the near-field region ends and the far-field region begins, and in fact there is a gradual morphing of one to the other, but one may reliably consider any distance from the circuit in excess of multiple wavelengths to be far-field.

¹⁶In fact, this is precisely what light is: an electromagnetic wave of exceptionally high frequency, well beyond the frequency range of contemporary electric circuits.

The following list compares and contrasts several near-field versus far-field characteristics:

- In the near-field region electric and magnetic field strengths depend greatly on the geometry of the energized circuit's conductors as well as on the specific voltage and current levels existing at each location. In the far-field region, however, the electric and magnetic field strengths are always in fixed proportion to one another.
- Far-field effects may be ignored for low-frequency AC circuits because these circuits' wavelengths are so long. At 60 Hz, for example, the wavelength is nearly 5 *million* meters. If we consider the far-field region of a circuit to begin at least multiple wavelengths from the source, this puts the far-field region of a 60 Hz circuit at least one-quarter of the Earth's circumference away from the circuit in question!
- In the near-field region the proportionality between electric and magnetic field strengths is a function of circuit impedance: high-impedance circuits will have stronger electric fields than magnetic fields, and low-impedance circuits will have stronger magnetic fields than electric fields, all other factors being equal. This is due to Ohm's Law establishing the relationship between voltage and current ($Z = \frac{V}{I}$), and how electric fields originate in a circuit from potential differences while magnetic fields originate in a circuit from currents. In the far-field region, however, where the electric and magnetic fields exist only through mutual support, their strengths are always in the same (fixed) proportion defined as the *characteristic impedance of free space* which is approximately 377 Ohms. This value is equal to the square root of the ratio of magnetic permeability to electric permittivity for free space ($Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$) because these parameters of space dictate just how strongly a varying electric field creates a magnetic field.
- The relationship between field strength and physical distance from the circuit is a very complex one within the near-field region, as this depends greatly on the geometry of the circuit conductors. In the far-field region, however, we find the *Inverse-Square Law* always holds true: as distance from the circuit increases, electromagnetic wave strength simply and reliably diminishes with the *square* of that distance as if the wave were radiating away in a spherical fashion from a point-source¹⁷. For example, if we double the distance from the radiating circuit (from a point in the far-field region to another point in the far-field region twice as far away in empty space), the power conveyed by the electromagnetic radiation will always be *four times* less. Tripling the far-field distance always weakens the wave's power by a factor of *nine*. This is simply because the area over which the radiated energy spreads increases with the square of the distance from any point-source, viewing that point-source as the center of a sphere.
- In the far-field region every radiating circuit has a definite *radiation pattern* where the strength of the radiated electromagnetic wave as a function of the *angle* from the geometric centerline follows a consistent pattern regardless of distance. This is a very important feature of *antennas*, where different antenna designs feature unique far-field radiation patterns. By contrast, in the near-field region of an antenna the radiation pattern varies significantly with distance.

¹⁷A "point source" is a hypothetical point in space having no height nor width nor depth (i.e. it has *zero* physical dimensions) that emits radiation. Point-sources are a theoretical concept only. Real sources have spatial dimensions which makes their near-field strength/distance relationships complex, but the farther away you get from them the more their radiative behavior approaches that of a theoretical point-source.

- The spatial *polarization* of the electromagnetic wave (i.e. the orientation of its perpendicular electric and magnetic fields) is always well-defined in the far-field region but often ill-defined in the near-field region. This means, among other things, that two or more antennas will exchange electromagnetic energy efficiently only if they are appropriately oriented to one another over far-field distances, but may exchange energy fairly well regardless of orientation over near-field distances.
- Any “gain” specifications for an antenna structure apply only to apparent gains in signal power over the far-field range, because they refer to the degree to which an antenna focuses its energy in one direction more than another (i.e. its directionality), implying a distance over which the electromagnetic wave has become well-formed and the antenna’s radiation pattern is reliably established. At near-field distances these “gain” figures are meaningless.

4.5 The Poynting vector

We know that electric fields (E) are always associated with a voltage (V), and magnetic fields (field strength B and field force H) with a current (I). Specifically, electric field strength is the quotient of voltage and distance between two points which is why electric fields are measured in units of *Volts per meter*. Similarly, magnetic field force around a straight current-carrying conductor is the quotient of current and circumference of the circular magnetic path (equal to 2π times the distance from the conductor) which is why magnetic field force is measured in units of *Amperes per meter*.

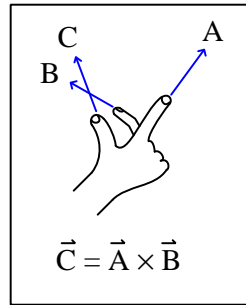
From Joule's Law we know that the rate of energy transfer (called power) whenever there is simultaneous voltage and current ($P = VI$). It stands to reason, then, that just as voltage multiplied by current yields power, an electric field multiplied by a magnetic field must similarly express power transferred by those two fields through space. That is, if $P = VI$, then the product of E and H should yield a power *intensity* which we shall label S :

$$P = VI \qquad [\text{Watts}] = [\text{Volts}][\text{Amperes}]$$

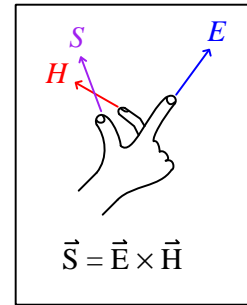
$$S = EH \qquad [\text{Watts/meter}^2] = [\text{Volts/meter}][\text{Amperes/meter}]$$

Notice how the Joule's Law equation $P = VI$ involves scalar quantities, while the $S = EH$ equation necessarily involves spatial dimensions of distance (meters) and area (square meters). This is because both electric and magnetic fields are *vector* quantities having both magnitude and spatial direction. Power intensity (S) is the measure of how much power is flowing per square meter of area in open space. As it so happens, power intensity in Watts per square meter relates to electric and magnetic fields by the right-hand rule of vector cross-products, the vector expression of that power intensity being called the *Poynting vector* (\vec{S}) after its formulator, the English physicist John H. Poynting:

General "right-hand rule"
for vector cross-products



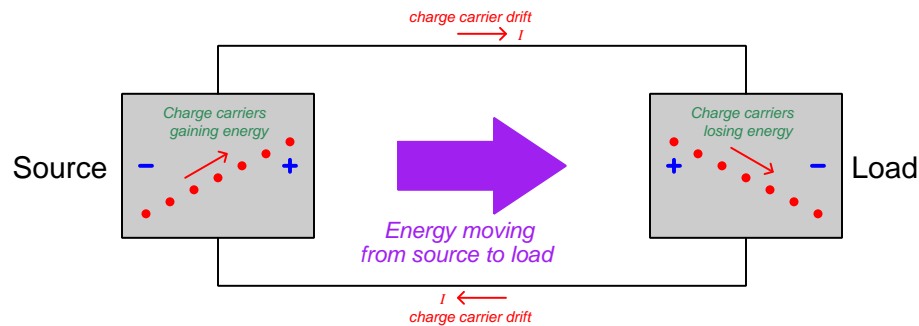
"Right-hand rule" specific
to the Poynting vector



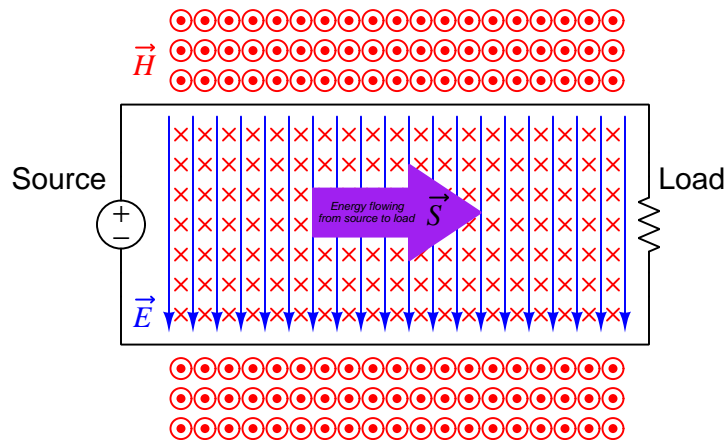
Poynting's contribution to the science of electromagnetism was expressing energy conservation and energy transfer in terms of electric and magnetic fields. That is, the electric and magnetic fields associated with any electrical *source* result in a flow of energy away from that source *through space*, and the electric and magnetic fields associated with any *load* result in a flow of energy toward that load *through space*. The direction of the Poynting vector shows the direction of that energy flow,

and the Poynting vector magnitude shows how many Watts of power flow per square meter of space.

This should be a rather startling conclusion to anyone familiar with voltage, current, and power in simple DC circuits. Beginning students of electricity regard energy transfer in simple circuits as the gain and loss of potential energy in mobile electric charges, those charges gaining energy as they pass through a source and losing energy as they pass through a load:

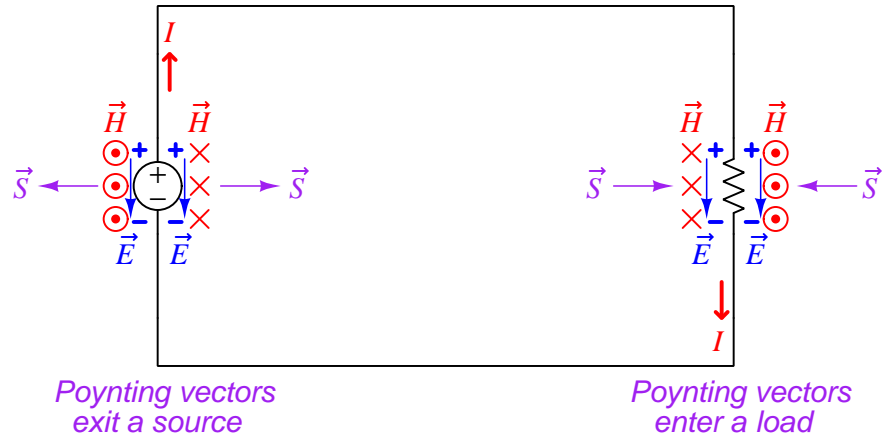


Though this model is conceptually correct, *so is Poynting's model of energy flowing through space*. In fact, if we re-draw this simple circuit illustration showing the electric and magnetic field vectors between the two wires, we can see the Poynting vector expressing that flow of energy from source to load¹⁸:



¹⁸Holding your right hand such that your index finger points downward (\vec{E}) and your middle finger into the page (\vec{H}), your thumb will point to the right (\vec{S}) from source to load.

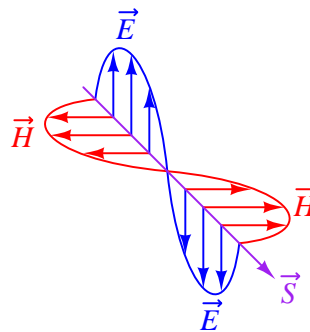
If we examine the source and load separately using the right-hand rule, we see how the Poynting vector in each case shows the proper direction of energy transfer:



It is well-worth your time to apply the right-hand rule to each of the four Poynting vectors shown above, to familiarize yourself with the use of this tool as a means to determine vector cross-product directions. For example, to the left of the source we see the electric field vector pointing down which means your index finger should point downward as well. The magnetic field vector on that side is coming out of the page (middle finger pointing toward you), and holding your right hand in that manner points your thumb to the left.

For simple DC circuits and circuit elements, Poynting's theory of power transfer may seem like a mere curiosity. It fails to explain anything we could not already explain using the model of potential energy gained and lost by mobile charge carriers, and it requires contortions of your right hand to properly align all the vectors. However, in some applications such as *radio* where energy radiates away from structures called *antennas* in the form of electromagnetic waves (oscillating electric and magnetic fields), Poynting's theory is the *only* suitable explanation for power transfer because there are no wires to convey mobile electric charges from source (transmitting antenna) to load (receiving antenna). Below we see a simplified representation of an electromagnetic wave consisting of two oscillating fields (shown here as an electric field oscillating vertically and a magnetic field oscillating horizontally), and if you use the right-hand rule in each half-cycle you will find that the Poynting vector consistently points in the same direction at the speed of light:

Electromagnetic wave



Based on the direction of the Poynting vector we see in this illustration, the source (transmitting antenna) would be located somewhere above and to the left of the drawn wave. There is no definite location for a load, as electromagnetic waves spread out in *all directions* away from the source. Our simple illustration merely shows a “cross-section” of a wave radiating *cylindrically* away from the transmitting antenna. The real electromagnetic wave (of which this illustration shows just a portion) would have an electric field resembling ripples on a pond oscillating vertically while radiating away in all horizontal directions from a single point, and have a magnetic field resembling a set of concentric circles oscillating rotationally in the horizontal plane while also radiating in all directions from that same point.

4.6 Maxwell's equations of electromagnetism

The Scottish physicist James Clerk Maxwell mathematically proved the interrelationships of electric and magnetic fields, publishing his results in the book *A Treatise on Electricity and Magnetism* in 1873. His equations were later simplified and consolidated by the work of Oliver Heaviside, commonly expressed in the following forms describing electric fields (\mathbf{E}) and magnetic fields (\mathbf{B}) within a vacuum:

$$\begin{aligned}\oint \mathbf{E} \cdot d\mathbf{A} &= \frac{Q}{\epsilon_0} \\ \oint \mathbf{B} \cdot d\mathbf{A} &= 0 \\ \oint \mathbf{E} \cdot d\mathbf{s} &= -\frac{d\Phi_B}{dt} \\ \oint \mathbf{B} \cdot d\mathbf{s} &= \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}\end{aligned}$$

We will now explore each of these four equations, relating them to laws codified by other early electrical researchers, as well as describe how they relate to the phenomenon of *electromagnetic waves*.

4.6.1 Gauss's Law of electric fields

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\epsilon_0}$$

The first of these equations simply describes electric fields as spreading out from a quantity of electric charge (Q). If we imagine a three-dimensional surface completely enclosing an electric charge (e.g. a soap bubble surrounding a charge), the total quantity of electric flux passing through that surface will be the product of the electric field (also known as the electric flux density, \mathbf{E}) and the enclosing area (\mathbf{A}). This is also known as *Gauss's Law of electric fields*.

4.6.2 Gauss's Law of magnetic fields

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0$$

The second of these equations describes magnetic fields passing through a three-dimensional enclosing surface. If we imagine a magnet completely enclosed within a soap bubble, the sum total of magnetic flux passing out through that soap bubble's surface must equal the sum total of magnetic flux re-entering it somewhere else. Literally, this equation says the product of magnetic field (also known as magnetic flux density \mathbf{B}) and the enclosing area (\mathbf{A}) must be algebraically equal to zero. This is to say, magnetic fields must always exist as *closed loops*, or alternatively we may say there is no such thing as a magnetic monopole (i.e. a solitary North or South pole without its complement, or a *magnetic charge*). In this way magnetic fields are quite different from electric fields which radiate away from discrete electric charges without ever returning. This is also known as *Gauss's Law of magnetic fields*.

4.6.3 Faraday's Law of electromagnetic induction, extended

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

The third equation states that an electric field (\mathbf{E}) will be produced in open space by a varying magnetic flux ($\frac{d\Phi_B}{dt}$). Michael Faraday expressed this phenomenon in terms of voltage induced along the length of a conductor by a time-varying magnetic field, but Maxwell demonstrated mathematically that this same thing happens even in empty space!

4.6.4 Ampère's Circuital Law, extended

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

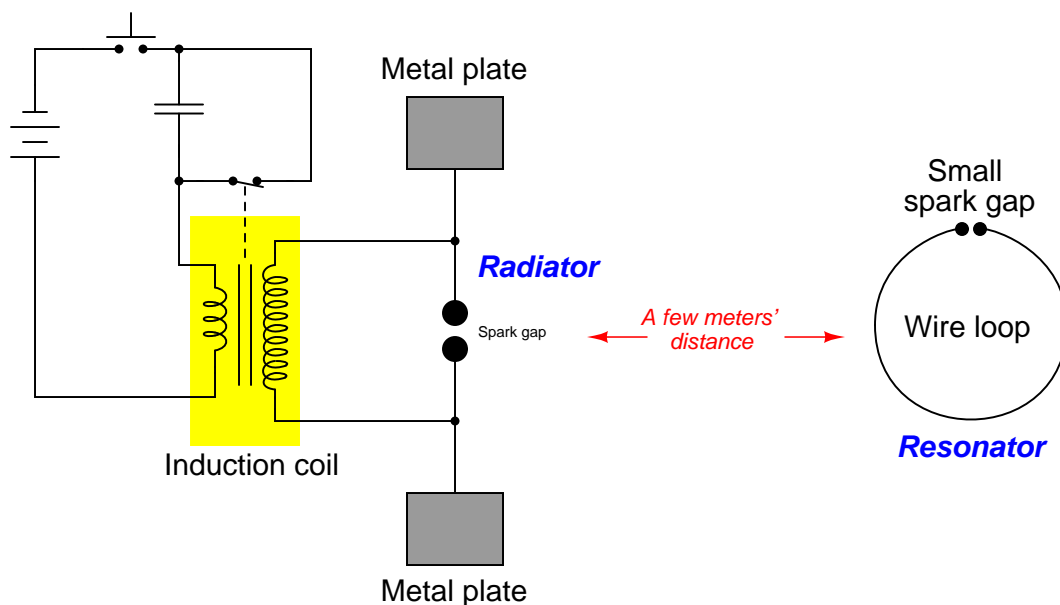
The fourth equation states that a magnetic field (\mathbf{B}) will be produced in open space either by an electric current (I) and/or by a varying electric flux ($\frac{d\Phi_E}{dt}$). The French scientist André-Marie Ampère described the relationship between current passing through a conductor and the magnetic field created around that conductor by the current, but once again Maxwell extended this concept to include magnetic fields generated in empty space. In empty space, where there are no moving electric charges (i.e. current, I) to generate a magnetic field, only the second term of the equation applies with a time-varying electric field substituting for actual electric current.

4.6.5 Proof of electromagnetic waves

Given the complementary relationship between time-varying electric and magnetic fields, Maxwell reasoned, it was possible for a varying electric field to create a varying magnetic field which would then create another varying electric field, and so on. This cause-and-effect cycle could continue, *ad infinitum*, with fast-changing electric and magnetic fields radiating off into open space without needing wires to carry or guide them, or a tank circuit to sustain the oscillations. In other words, *these complementary fields self-sustain as they travel*, although they do become weaker due to their spreading out of a finite amount of energy over larger and larger volumes of space.

The Prussian Academy of Science offered a reward to anyone who could experimentally validate Maxwell’s prediction, and this challenge was answered by Professor Heinrich Hertz at the Engineering College in Karlsruhe, Germany in 1887, eight years after Maxwell’s death. Hertz constructed and tested a pair of devices: a “radiator” to produce the electromagnetic waves, and a “resonator” to receive them.

A simplified diagram showing Hertz’s experimental device is shown here:



An “induction coil” (a buzzing device constructed of a self-interrupting relay and step-up transformer winding to generate a continuous pulsing waveform of high voltage) provided an extremely noisy (i.e. harmonically rich)¹⁹ AC signal to the radiator, while a spark gap at the

¹⁹A fundamental property of waves is that any wave-shape, no matter how complex, is mathematically equivalent to a series of perfectly sinusoidal waves of differing frequencies added together. Waveforms with fast rise/fall times, such as the pulse waveforms produced by switched induction coils, contain many such high-frequency “harmonic” signals, and it is these high-frequency signals that created the radiated electromagnetic waves in Hertz’s experiments. The induction coil’s secondary winding inductance combined with the parasitic capacitance of the radiator’s wires and plates provided the means to momentarily sustain some of these high-frequency oscillations long enough to sustain visible sparks at the resonator’s gap.

resonator provided indication that the electromagnetic waves were captured and converted into voltage by the resonator wire.

Both the radiator and the resonator are antennas. The purpose of the transmitting antenna (radiator) is to take high-frequency AC power and radiate that power in the form of electromagnetic waves: self-sustaining electric and magnetic fields propagating outward into open space. The purpose of the receiving antenna is to capture those electromagnetic waves and convert them into an electrical signal (i.e. AC voltage and current). All antennas – from historical to the most modern – behave in fundamentally the same way: energize them with high-frequency AC electrical power, and they will radiate electromagnetic waves at that frequency; expose them to electromagnetic waves, and they will produce a very small AC electrical signal at the same frequency as the incident radiation.

Chapter 5

Questions

5.1 Quantitative reasoning

These questions are designed to stimulate your computational thinking. In a Socratic discussion with your instructor, the goal is for these questions to reveal your mathematical approach(es) to problem-solving so that good technique and sound reasoning may be reinforced. Your instructor may also pose additional questions based on those assigned, in order to observe your problem-solving firsthand.

Mental arithmetic and estimations are strongly encouraged for all calculations, because without these abilities you will be unable to readily detect errors caused by calculator misuse (e.g. keystroke errors).

You will note a conspicuous lack of answers given for these quantitative questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. My advice is to use circuit simulation software such as SPICE to check the correctness of quantitative answers. Refer to those learning modules within this collection focusing on SPICE to see worked examples which you may use directly as practice problems for your own study, and/or as templates you may modify to run your own analyses and generate your own practice problems.

Completely worked example problems found in the Tutorial may also serve as “test cases¹” for gaining proficiency in the use of circuit simulation software, and then once that proficiency is gained you will never need to rely² on an answer key!

¹In other words, set up the circuit simulation software to analyze the same circuit examples found in the Tutorial. If the simulated results match the answers shown in the Tutorial, it confirms the simulation has properly run. If the simulated results disagree with the Tutorial’s answers, something has been set up incorrectly in the simulation software. Using every Tutorial as practice in this way will quickly develop proficiency in the use of circuit simulation software.

²This approach is perfectly in keeping with the instructional philosophy of these learning modules: *teaching students to be self-sufficient thinkers*. Answer keys can be useful, but it is even more useful to your long-term success to have a set of tools on hand for checking your own work, because once you have left school and are on your own, there will no longer be “answer keys” available for the problems you will have to solve.

5.1.1 Miscellaneous physical constants

Note: constants shown in **bold** type are *exact*, not approximations. Values inside of parentheses show one standard deviation (σ) of uncertainty in the final digits: for example, the magnetic permeability of free space value given as $1.25663706212(19) \times 10^{-6}$ H/m represents a center value (i.e. the location parameter) of $1.25663706212 \times 10^{-6}$ Henrys per meter with one standard deviation of uncertainty equal to $0.0000000000019 \times 10^{-6}$ Henrys per meter.

Avogadro's number (N_A) = **$6.02214076 \times 10^{23}$** per mole (mol^{-1})

Boltzmann's constant (k) = **1.380649×10^{-23}** Joules per Kelvin (J/K)

Electronic charge (e) = **$1.602176634 \times 10^{-19}$** Coulomb (C)

Faraday constant (F) = **$96,485.33212...$** $\times 10^4$ Coulombs per mole (C/mol)

Magnetic permeability of free space (μ_0) = $1.25663706212(19) \times 10^{-6}$ Henrys per meter (H/m)

Electric permittivity of free space (ϵ_0) = $8.8541878128(13) \times 10^{-12}$ Farads per meter (F/m)

Characteristic impedance of free space (Z_0) = $376.730313668(57)$ Ohms (Ω)

Gravitational constant (G) = $6.67430(15) \times 10^{-11}$ cubic meters per kilogram-seconds squared ($\text{m}^3/\text{kg}\cdot\text{s}^2$)

Molar gas constant (R) = **$8.314462618...$** Joules per mole-Kelvin (J/mol-K) = $0.08205746(14)$ liters-atmospheres per mole-Kelvin

Planck constant (h) = **$6.62607015 \times 10^{-34}$** joule-seconds (J-s)

Stefan-Boltzmann constant (σ) = **$5.670374419...$** $\times 10^{-8}$ Watts per square meter-Kelvin⁴ ($\text{W}/\text{m}^2\cdot\text{K}^4$)

Speed of light in a vacuum (c) = **$299,792,458$** meters per second (m/s) = 186282.4 miles per second (mi/s)

Note: All constants taken from NIST data "Fundamental Physical Constants – Complete Listing", from <http://physics.nist.gov/constants>, National Institute of Standards and Technology (NIST), 2018 CODATA Adjustment.

5.1.2 Introduction to spreadsheets

A powerful computational tool you are encouraged to use in your work is a *spreadsheet*. Available on most personal computers (e.g. Microsoft Excel), *spreadsheet* software performs numerical calculations based on number values and formulae entered into cells of a grid. This grid is typically arranged as lettered columns and numbered rows, with each cell of the grid identified by its column/row coordinates (e.g. cell B3, cell A8). Each cell may contain a string of text, a number value, or a mathematical formula. The spreadsheet automatically updates the results of all mathematical formulae whenever the entered number values are changed. This means it is possible to set up a spreadsheet to perform a series of calculations on entered data, and those calculations will be re-done by the computer any time the data points are edited in any way.

For example, the following spreadsheet calculates average speed based on entered values of distance traveled and time elapsed:

	A	B	C	D
1	Distance traveled	46.9	Kilometers	
2	Time elapsed	1.18	Hours	
3	Average speed	= B1 / B2	km/h	
4				
5				

Text labels contained in cells A1 through A3 and cells C1 through C3 exist solely for readability and are not involved in any calculations. Cell B1 contains a sample distance value while cell B2 contains a sample time value. The formula for computing speed is contained in cell B3. Note how this formula begins with an “equals” symbol (=), references the values for distance and speed by lettered column and numbered row coordinates (B1 and B2), and uses a forward slash symbol for division (/). The coordinates B1 and B2 function as *variables*³ would in an algebraic formula.

When this spreadsheet is executed, the numerical value 39.74576 will appear in cell B3 rather than the formula = B1 / B2, because 39.74576 is the computed speed value given 46.9 kilometers traveled over a period of 1.18 hours. If a different numerical value for distance is entered into cell B1 or a different value for time is entered into cell B2, cell B3’s value will automatically update. All you need to do is set up the given values and any formulae into the spreadsheet, and the computer will do all the calculations for you.

Cell B3 may be referenced by other formulae in the spreadsheet if desired, since it is a variable just like the given values contained in B1 and B2. This means it is possible to set up an entire chain of calculations, one dependent on the result of another, in order to arrive at a final value. The arrangement of the given data and formulae need not follow any pattern on the grid, which means you may place them anywhere.

³Spreadsheets may also provide means to attach text labels to cells for use as variable names (Microsoft Excel simply calls these labels “names”), but for simple spreadsheets such as those shown here it’s usually easier just to use the standard coordinate naming for each cell.

Common⁴ arithmetic operations available for your use in a spreadsheet include the following:

- Addition (+)
- Subtraction (-)
- Multiplication (*)
- Division (/)
- Powers (^)
- Square roots (sqrt())
- Logarithms (ln() , log10())

Parentheses may be used to ensure⁵ proper order of operations within a complex formula. Consider this example of a spreadsheet implementing the *quadratic formula*, used to solve for roots of a polynomial expression in the form of $ax^2 + bx + c$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

	A	B
1	x_1	= (-B4 + sqrt((B4^2) - (4*B3*B5))) / (2*B3)
2	x_2	= (-B4 - sqrt((B4^2) - (4*B3*B5))) / (2*B3)
3	a =	9
4	b =	5
5	c =	-2

This example is configured to compute roots⁶ of the polynomial $9x^2 + 5x - 2$ because the values of 9, 5, and -2 have been inserted into cells B3, B4, and B5, respectively. Once this spreadsheet has been built, though, it may be used to calculate the roots of *any* second-degree polynomial expression simply by entering the new a , b , and c coefficients into cells B3 through B5. The numerical values appearing in cells B1 and B2 will be automatically updated by the computer immediately following any changes made to the coefficients.

⁴Modern spreadsheet software offers a bewildering array of mathematical functions you may use in your computations. I recommend you consult the documentation for your particular spreadsheet for information on operations other than those listed here.

⁵Spreadsheet programs, like text-based programming languages, are designed to follow standard order of operations by default. However, my personal preference is to use parentheses even where strictly unnecessary just to make it clear to any other person viewing the formula what the intended order of operations is.

⁶Reviewing some algebra here, a *root* is a value for x that yields an overall value of zero for the polynomial. For this polynomial ($9x^2 + 5x - 2$) the two roots happen to be $x = 0.269381$ and $x = -0.82494$, with these values displayed in cells B1 and B2, respectively upon execution of the spreadsheet.

Alternatively, one could break up the long quadratic formula into smaller pieces like this:

$$y = \sqrt{b^2 - 4ac} \quad z = 2a$$

$$x = \frac{-b \pm y}{z}$$

	A	B	C
1	x_1	= (-B4 + C1) / C2	= sqrt ((B4^2) - (4*B3*B5))
2	x_2	= (-B4 - C1) / C2	= 2*B3
3	a =	9	
4	b =	5	
5	c =	-2	

Note how the square-root term (y) is calculated in cell C1, and the denominator term (z) in cell C2. This makes the two final formulae (in cells B1 and B2) simpler to interpret. The positioning of all these cells on the grid is completely arbitrary⁷ – all that matters is that they properly reference each other in the formulae.

Spreadsheets are particularly useful for situations where the same set of calculations representing a circuit or other system must be repeated for different initial conditions. The power of a spreadsheet is that it automates what would otherwise be a tedious set of calculations. One specific application of this is to simulate the effects of various components within a circuit failing with abnormal values (e.g. a shorted resistor simulated by making its value nearly zero; an open resistor simulated by making its value extremely large). Another application is analyzing the behavior of a circuit design given new components that are out of specification, and/or aging components experiencing drift over time.

⁷My personal preference is to locate all the “given” data in the upper-left cells of the spreadsheet grid (each data point flanked by a sensible name in the cell to the left and units of measurement in the cell to the right as illustrated in the first distance/time spreadsheet example), sometimes coloring them in order to clearly distinguish which cells contain entered data versus which cells contain computed results from formulae. I like to place all formulae in cells below the given data, and try to arrange them in logical order so that anyone examining my spreadsheet will be able to figure out *how* I constructed a solution. This is a general principle I believe all computer programmers should follow: *document and arrange your code to make it easy for other people to learn from it.*

5.1.3 Gain ratios into decibels

Convert the following amplifier gains (either power, voltage, or current gain ratios) into gains expressed in the unit of decibels (dB):

- $A_P = 25$; $A_{P(dB)} =$
- $A_V = 10$; $A_{V(dB)} =$
- $A_I = 37$; $A_{I(dB)} =$
- $A_P = 150$; $A_{P(dB)} =$
- $A_I = 41$; $A_{I(dB)} =$
- $A_V = 3.4$; $A_{V(dB)} =$
- $A_P = 18$; $A_{P(dB)} =$
- $A_V = 100$; $A_{V(dB)} =$

Challenges

- Suppose a student mistakenly computes 10 dB for $A_V = 10$ (ratio). Explain what went wrong, and why the correct answer is different.

5.1.4 Decibels into gain ratios

Convert the following amplifier gains expressed in the unit of decibels (dB), to gain figures expressed as unitless ratios:

- $A_P = 5 \text{ dB}$; $A_{P(ratio)} =$
- $A_V = 23 \text{ dB}$; $A_{V(ratio)} =$
- $A_I = 20 \text{ dB}$; $A_{I(ratio)} =$
- $A_P = 2.5 \text{ dB}$; $A_{P(ratio)} =$
- $A_I = 7.4 \text{ dB}$; $A_{I(ratio)} =$
- $A_V = 45 \text{ dB}$; $A_{V(ratio)} =$
- $A_P = 12.8 \text{ dB}$; $A_{P(ratio)} =$
- $A_V = 30 \text{ dB}$; $A_{V(ratio)} =$

Challenges

- Suppose a student mistakenly computes 100 (ratio) for $A_I = 20 \text{ dB}$. Explain what went wrong, and why the correct answer is different.

5.1.5 Amplifier gains

An amplifier has a voltage gain of 5 and a current gain of 75, both figures being ratios. Calculate the following gains:

- Power gain (as a ratio) =
- Power gain (dB) =
- Voltage gain (dB) =
- Current gain (dB) =

Challenges

- Suppose a student calculates 6.9897 dB for the voltage gain of this amplifier. Explain where they made a mistake in this calculation.

Appendix A

Problem-Solving Strategies

The ability to solve complex problems is arguably one of the most valuable skills one can possess, and this skill is particularly important in any science-based discipline.

- Study principles, not procedures. Don't be satisfied with merely knowing how to compute solutions – learn *why* those solutions work.
- Identify what it is you need to solve, identify all relevant data, identify all units of measurement, identify any general principles or formulae linking the given information to the solution, and then identify any “missing pieces” to a solution. Annotate all diagrams with this data.
- Sketch a diagram to help visualize the problem. When building a real system, always devise a plan for that system and analyze its function *before* constructing it.
- Follow the units of measurement and meaning of every calculation. If you are ever performing mathematical calculations as part of a problem-solving procedure, and you find yourself unable to apply each and every intermediate result to some aspect of the problem, it means you don't understand what you are doing. Properly done, every mathematical result should have practical meaning for the problem, and not just be an abstract number. You should be able to identify the proper units of measurement for each and every calculated result, and show where that result fits into the problem.
- Perform “thought experiments” to explore the effects of different conditions for theoretical problems. When troubleshooting real systems, perform *diagnostic tests* rather than visually inspecting for faults, the best diagnostic test being the one giving you the most information about the nature and/or location of the fault with the fewest steps.
- Simplify the problem until the solution becomes obvious, and then use that obvious case as a model to follow in solving the more complex version of the problem.
- Check for exceptions to see if your solution is incorrect or incomplete. A good solution will work for *all* known conditions and criteria. A good example of this is the process of testing scientific hypotheses: the task of a scientist is not to find support for a new idea, but rather to *challenge* that new idea to see if it holds up under a battery of tests. The philosophical

principle of *reductio ad absurdum* (i.e. disproving a general idea by finding a specific case where it fails) is useful here.

- Work “backward” from a hypothetical solution to a new set of given conditions.
- Add quantities to problems that are qualitative in nature, because sometimes a little math helps illuminate the scenario.
- Sketch graphs illustrating how variables relate to each other. These may be quantitative (i.e. with realistic number values) or qualitative (i.e. simply showing increases and decreases).
- Treat quantitative problems as qualitative in order to discern the relative magnitudes and/or directions of change of the relevant variables. For example, try determining what happens if a certain variable were to increase or decrease before attempting to precisely calculate quantities: how will each of the dependent variables respond, by increasing, decreasing, or remaining the same as before?
- Consider limiting cases. This works especially well for qualitative problems where you need to determine which direction a variable will change. Take the given condition and magnify that condition to an extreme degree as a way of simplifying the direction of the system’s response.
- Check your work. This means regularly testing your conclusions to see if they make sense. This does *not* mean repeating the same steps originally used to obtain the conclusion(s), but rather to use some other means to check validity. Simply repeating procedures often leads to *repeating the same errors* if any were made, which is why alternative paths are better.

Appendix B

Instructional philosophy

“The unexamined circuit is not worth energizing” – Socrates (if he had taught electricity)

These learning modules, although useful for self-study, were designed to be used in a formal learning environment where a subject-matter expert challenges students to digest the content and exercise their critical thinking abilities in the answering of questions and in the construction and testing of working circuits.

The following principles inform the instructional and assessment philosophies embodied in these learning modules:

- The first goal of education is to enhance clear and independent thought, in order that every student reach their fullest potential in a highly complex and inter-dependent world. Robust reasoning is *always* more important than particulars of any subject matter, because its application is universal.
- Literacy is fundamental to independent learning and thought because text continues to be the most efficient way to communicate complex ideas over space and time. Those who cannot read with ease are limited in their ability to acquire knowledge and perspective.
- Articulate communication is fundamental to work that is complex and interdisciplinary.
- Faulty assumptions and poor reasoning are best corrected through challenge, not presentation. The rhetorical technique of *reductio ad absurdum* (disproving an assertion by exposing an absurdity) works well to discipline student’s minds, not only to correct the problem at hand but also to learn how to detect and correct future errors.
- Important principles should be repeatedly explored and widely applied throughout a course of study, not only to reinforce their importance and help ensure their mastery, but also to showcase the interconnectedness and utility of knowledge.

These learning modules were expressly designed to be used in an “inverted” teaching environment¹ where students first read the introductory and tutorial chapters on their own, then individually attempt to answer the questions and construct working circuits according to the experiment and project guidelines. The instructor never lectures, but instead meets regularly with each individual student to review their progress, answer questions, identify misconceptions, and challenge the student to new depths of understanding through further questioning. Regular meetings between instructor and student should resemble a Socratic² dialogue, where questions serve as scalpels to dissect topics and expose assumptions. The student passes each module only after consistently demonstrating their ability to logically analyze and correctly apply all major concepts in each question or project/experiment. The instructor must be vigilant in probing each student’s understanding to ensure they are truly *reasoning* and not just *memorizing*. This is why “Challenge” points appear throughout, as prompts for students to think deeper about topics and as starting points for instructor queries. Sometimes these challenge points require additional knowledge that hasn’t been covered in the series to answer in full. This is okay, as the major purpose of the Challenges is to stimulate analysis and synthesis on the part of each student.

The instructor must possess enough mastery of the subject matter and awareness of students’ reasoning to generate their own follow-up questions to practically any student response. Even completely correct answers given by the student should be challenged by the instructor for the purpose of having students practice articulating their thoughts and defending their reasoning. Conceptual errors committed by the student should be exposed and corrected not by direct instruction, but rather by reducing the errors to an absurdity³ through well-chosen questions and thought experiments posed by the instructor. Becoming proficient at this style of instruction requires time and dedication, but the positive effects on critical thinking for both student and instructor are spectacular.

An inspection of these learning modules reveals certain unique characteristics. One of these is a bias toward thorough explanations in the tutorial chapters. Without a live instructor to explain concepts and applications to students, the text itself must fulfill this role. This philosophy results in lengthier explanations than what you might typically find in a textbook, each step of the reasoning process fully explained, including footnotes addressing common questions and concerns students raise while learning these concepts. Each tutorial seeks to not only explain each major concept in sufficient detail, but also to explain the logic of each concept and how each may be developed

¹In a traditional teaching environment, students first encounter new information via *lecture* from an expert, and then independently apply that information via *homework*. In an “inverted” course of study, students first encounter new information via *homework*, and then independently apply that information under the scrutiny of an expert. The expert’s role in lecture is to simply *explain*, but the expert’s role in an inverted session is to *challenge*, *critique*, and if necessary *explain* where gaps in understanding still exist.

²Socrates is a figure in ancient Greek philosophy famous for his unflinching style of questioning. Although he authored no texts, he appears as a character in Plato’s many writings. The essence of Socratic philosophy is to leave no question unexamined and no point of view unchallenged. While purists may argue a topic such as electric circuits is too narrow for a true Socratic-style dialogue, I would argue that the essential thought processes involved with scientific reasoning on *any* topic are not far removed from the Socratic ideal, and that students of electricity and electronics would do very well to challenge assumptions, pose thought experiments, identify fallacies, and otherwise employ the arsenal of critical thinking skills modeled by Socrates.

³This rhetorical technique is known by the Latin phrase *reductio ad absurdum*. The concept is to expose errors by counter-example, since only one solid counter-example is necessary to disprove a universal claim. As an example of this, consider the common misconception among beginning students of electricity that voltage cannot exist without current. One way to apply *reductio ad absurdum* to this statement is to ask how much current passes through a fully-charged battery connected to nothing (i.e. a clear example of voltage existing without current).

from “first principles”. Again, this reflects the goal of developing clear and independent thought in students’ minds, by showing how clear and logical thought was used to forge each concept. Students benefit from witnessing a model of clear thinking in action, and these tutorials strive to be just that.

Another characteristic of these learning modules is a lack of step-by-step instructions in the Project and Experiment chapters. Unlike many modern workbooks and laboratory guides where step-by-step instructions are prescribed for each experiment, these modules take the approach that students must learn to closely read the tutorials and apply their own reasoning to identify the appropriate experimental steps. Sometimes these steps are plainly declared in the text, just not as a set of enumerated points. At other times certain steps are implied, an example being assumed competence in test equipment use where the student should not need to be told *again* how to use their multimeter because that was thoroughly explained in previous lessons. In some circumstances no steps are given at all, leaving the entire procedure up to the student.

This lack of prescription is not a flaw, but rather a feature. Close reading and clear thinking are foundational principles of this learning series, and in keeping with this philosophy all activities are designed to *require* those behaviors. Some students may find the lack of prescription frustrating, because it demands more from them than what their previous educational experiences required. This frustration should be interpreted as an unfamiliarity with autonomous thinking, a problem which must be corrected if the student is ever to become a self-directed learner and effective problem-solver. Ultimately, the need for students to read closely and think clearly is more important both in the near-term and far-term than any specific facet of the subject matter at hand. If a student takes longer than expected to complete a module because they are forced to outline, digest, and reason on their own, so be it. The future gains enjoyed by developing this mental discipline will be well worth the additional effort and delay.

Another feature of these learning modules is that they do not treat topics in isolation. Rather, important concepts are introduced early in the series, and appear repeatedly as stepping-stones toward other concepts in subsequent modules. This helps to avoid the “compartmentalization” of knowledge, demonstrating the inter-connectedness of concepts and simultaneously reinforcing them. Each module is fairly complete in itself, reserving the beginning of its tutorial to a review of foundational concepts.

This methodology of assigning text-based modules to students for digestion and then using Socratic dialogue to assess progress and hone students’ thinking was developed over a period of several years by the author with his Electronics and Instrumentation students at the two-year college level. While decidedly unconventional and sometimes even unsettling for students accustomed to a more passive lecture environment, this instructional philosophy has proven its ability to convey conceptual mastery, foster careful analysis, and enhance employability so much better than lecture that the author refuses to ever teach by lecture again.

Problems which often go undiagnosed in a lecture environment are laid bare in this “inverted” format where students must articulate and logically defend their reasoning. This, too, may be unsettling for students accustomed to lecture sessions where the instructor cannot tell for sure who comprehends and who does not, and this vulnerability necessitates sensitivity on the part of the “inverted” session instructor in order that students never feel discouraged by having their errors exposed. *Everyone* makes mistakes from time to time, and learning is a lifelong process! Part of the instructor’s job is to build a culture of learning among the students where errors are not seen as shameful, but rather as opportunities for progress.

To this end, instructors managing courses based on these modules should adhere to the following principles:

- Student questions are always welcome and demand thorough, honest answers. The only type of question an instructor should refuse to answer is one the student should be able to easily answer on their own. Remember, *the fundamental goal of education is for each student to learn to think clearly and independently*. This requires hard work on the part of the student, which no instructor should ever circumvent. Anything done to bypass the student's responsibility to do that hard work ultimately limits that student's potential and thereby does real harm.
- It is not only permissible, but encouraged, to answer a student's question by asking questions in return, these follow-up questions designed to guide the student to reach a correct answer through their own reasoning.
- All student answers demand to be challenged by the instructor and/or by other students. This includes both correct and incorrect answers – the goal is to practice the articulation and defense of one's own reasoning.
- No reading assignment is deemed complete unless and until the student demonstrates their ability to accurately summarize the major points in their own terms. Recitation of the original text is unacceptable. This is why every module contains an "Outline and reflections" question as well as a "Foundational concepts" question in the Conceptual reasoning section, to prompt reflective reading.
- No assigned question is deemed answered unless and until the student demonstrates their ability to consistently and correctly apply the concepts to *variations* of that question. This is why module questions typically contain multiple "Challenges" suggesting different applications of the concept(s) as well as variations on the same theme(s). Instructors are encouraged to devise as many of their own "Challenges" as they are able, in order to have a multitude of ways ready to probe students' understanding.
- No assigned experiment or project is deemed complete unless and until the student demonstrates the task in action. If this cannot be done "live" before the instructor, video-recordings showing the demonstration are acceptable. All relevant safety precautions must be followed, all test equipment must be used correctly, and the student must be able to properly explain all results. The student must also successfully answer all Challenges presented by the instructor for that experiment or project.

Students learning from these modules would do well to abide by the following principles:

- No text should be considered fully and adequately read unless and until you can express every idea *in your own words, using your own examples*.
- You should always articulate your thoughts as you read the text, noting points of agreement, confusion, and epiphanies. Feel free to print the text on paper and then write your notes in the margins. Alternatively, keep a journal for your own reflections as you read. This is truly a helpful tool when digesting complicated concepts.
- Never take the easy path of highlighting or underlining important text. Instead, *summarize* and/or *comment* on the text using your own words. This actively engages your mind, allowing you to more clearly perceive points of confusion or misunderstanding on your own.
- A very helpful strategy when learning new concepts is to place yourself in the role of a teacher, if only as a mental exercise. Either explain what you have recently learned to someone else, or at least *imagine* yourself explaining what you have learned to someone else. The simple act of having to articulate new knowledge and skill forces you to take on a different perspective, and will help reveal weaknesses in your understanding.
- Perform each and every mathematical calculation and thought experiment shown in the text on your own, referring back to the text to see that your results agree. This may seem trivial and unnecessary, but it is critically important to ensuring you actually understand what is presented, especially when the concepts at hand are complicated and easy to misunderstand. Apply this same strategy to become proficient in the use of *circuit simulation software*, checking to see if your simulated results agree with the results shown in the text.
- Above all, recognize that learning is hard work, and that a certain level of frustration is unavoidable. There are times when you will struggle to grasp some of these concepts, and that struggle is a natural thing. Take heart that it will yield with persistent and varied⁴ effort, and never give up!

Students interested in using these modules for self-study will also find them beneficial, although the onus of responsibility for thoroughly reading and answering questions will of course lie with that individual alone. If a qualified instructor is not available to challenge students, a workable alternative is for students to form study groups where they challenge⁵ one another.

To high standards of education,

Tony R. Kuphaldt

⁴As the old saying goes, “Insanity is trying the same thing over and over again, expecting different results.” If you find yourself stumped by something in the text, you should attempt a different approach. Alter the thought experiment, change the mathematical parameters, do whatever you can to see the problem in a slightly different light, and then the solution will often present itself more readily.

⁵Avoid the temptation to simply share answers with study partners, as this is really counter-productive to learning. Always bear in mind that the answer to any question is far less important in the long run than the method(s) used to obtain that answer. The goal of education is to empower one’s life through the improvement of clear and independent thought, literacy, expression, and various practical skills.

Appendix C

Tools used

I am indebted to the developers of many open-source software applications in the creation of these learning modules. The following is a list of these applications with some commentary on each.

You will notice a theme common to many of these applications: a bias toward *code*. Although I am by no means an expert programmer in any computer language, I understand and appreciate the flexibility offered by code-based applications where the user (you) enters commands into a plain ASCII text file, which the software then reads and processes to create the final output. Code-based computer applications are by their very nature *extensible*, while WYSIWYG (What You See Is What You Get) applications are generally limited to whatever user interface the developer makes for you.

The GNU/Linux computer operating system

There is so much to be said about Linus Torvalds' **Linux** and Richard Stallman's **GNU** project. First, to credit just these two individuals is to fail to do justice to the *mob* of passionate volunteers who contributed to make this amazing software a reality. I first learned of **Linux** back in 1996, and have been using this operating system on my personal computers almost exclusively since then. It is *free*, it is completely *configurable*, and it permits the continued use of highly efficient **Unix** applications and scripting languages (e.g. shell scripts, Makefiles, **sed**, **awk**) developed over many decades. **Linux** not only provided me with a powerful computing platform, but its open design served to inspire my life's work of creating open-source educational resources.

Bram Moolenaar's **Vim** text editor

Writing code for any code-based computer application requires a *text editor*, which may be thought of as a word processor strictly limited to outputting plain-ASCII text files. Many good text editors exist, and one's choice of text editor seems to be a deeply personal matter within the programming world. I prefer **Vim** because it operates very similarly to **vi** which is ubiquitous on **Unix/Linux** operating systems, and because it may be entirely operated via keyboard (i.e. no mouse required) which makes it fast to use.

Donald Knuth's \TeX typesetting system

Developed in the late 1970's and early 1980's by computer scientist extraordinaire Donald Knuth to typeset his multi-volume magnum opus *The Art of Computer Programming*, this software allows the production of formatted text for screen-viewing or paper printing, all by writing plain-text code to describe how the formatted text is supposed to appear. \TeX is not just a markup language for documents, but it is also a Turing-complete programming language in and of itself, allowing useful algorithms to be created to control the production of documents. Simply put, *\TeX is a programmer's approach to word processing*. Since \TeX is controlled by code written in a plain-text file, this means anyone may read that plain-text file to see exactly how the document was created. This openness afforded by the code-based nature of \TeX makes it relatively easy to learn how other people have created their own \TeX documents. By contrast, examining a beautiful document created in a conventional WYSIWYG word processor such as Microsoft **Word** suggests nothing to the reader about *how* that document was created, or what the user might do to create something similar. As Mr. Knuth himself once quipped, conventional word processing applications should be called WYSIAYG (What You See Is *All* You Get).

Leslie Lamport's \LaTeX extensions to \TeX

Like all true programming languages, \TeX is inherently extensible. So, years after the release of \TeX to the public, Leslie Lamport decided to create a massive extension allowing easier compilation of book-length documents. The result was \LaTeX , which is the markup language used to create all ModEL module documents. You could say that \TeX is to \LaTeX as **C** is to **C++**. This means it is permissible to use any and all \TeX commands within \LaTeX source code, and it all still works. Some of the features offered by \LaTeX that would be challenging to implement in \TeX include automatic index and table-of-content creation.

Tim Edwards' **Xcircuit** drafting program

This wonderful program is what I use to create all the schematic diagrams and illustrations (but not photographic images or mathematical plots) throughout the ModEL project. It natively outputs PostScript format which is a true vector graphic format (this is why the images do not pixellate when you zoom in for a closer view), and it is so simple to use that I have never had to read the manual! Object libraries are easy to create for **Xcircuit**, being plain-text files using PostScript programming conventions. Over the years I have collected a large set of object libraries useful for drawing electrical and electronic schematics, pictorial diagrams, and other technical illustrations.

Gimp graphic image manipulation program

Essentially an open-source clone of Adobe's **PhotoShop**, I use **Gimp** to resize, crop, and convert file formats for all of the photographic images appearing in the **ModEL** modules. Although **Gimp** does offer its own scripting language (called **Script-Fu**), I have never had occasion to use it. Thus, my utilization of **Gimp** to merely crop, resize, and convert graphic images is akin to using a sword to slice bread.

SPICE circuit simulation program

SPICE is to circuit analysis as **T_EX** is to document creation: it is a form of markup language designed to describe a certain object to be processed in plain-ASCII text. When the plain-text "source file" is compiled by the software, it outputs the final result. More modern circuit analysis tools certainly exist, but I prefer **SPICE** for the following reasons: it is *free*, it is *fast*, it is *reliable*, and it is a fantastic tool for *teaching* students of electricity and electronics how to write simple code. I happen to use rather old versions of **SPICE**, version 2g6 being my "go to" application when I only require text-based output. **NGSPICE** (version 26), which is based on Berkeley **SPICE** version 3f5, is used when I require graphical output for such things as time-domain waveforms and Bode plots. In all **SPICE** example netlists I strive to use coding conventions compatible with all **SPICE** versions.

Andrew D. Hwang's ePiX mathematical visualization programming library

This amazing project is a **C++** library you may link to any **C/C++** code for the purpose of generating PostScript graphic images of mathematical functions. As a completely free and open-source project, it does all the plotting I would otherwise use a Computer Algebra System (CAS) such as **Mathematica** or **Maple** to do. It should be said that **ePiX** is *not* a Computer Algebra System like **Mathematica** or **Maple**, but merely a mathematical *visualization* tool. In other words, it won't determine integrals for you (you'll have to implement that in your own **C/C++** code!), but it can graph the results, and it does so beautifully. What I really admire about **ePiX** is that it is a **C++** programming library, which means it builds on the existing power and toolset available with that programming language. Mr. Hwang could have probably developed his own stand-alone application for mathematical plotting, but by creating a **C++** library to do the same thing he accomplished something much greater.

`gnuplot` mathematical visualization software

Another open-source tool for mathematical visualization is `gnuplot`. Interestingly, this tool is *not* part of Richard Stallman’s GNU project, its name being a coincidence. For this reason the authors prefer “gnu” *not* be capitalized at all to avoid confusion. This is a much “lighter-weight” alternative to a spreadsheet for plotting tabular data, and the fact that it easily outputs directly to an X11 console or a file in a number of different graphical formats (including PostScript) is very helpful. I typically set my `gnuplot` output format to default (X11 on my Linux PC) for quick viewing while I’m developing a visualization, then switch to PostScript file export once the visual is ready to include in the document(s) I’m writing. As with my use of `Gimp` to do rudimentary image editing, my use of `gnuplot` only scratches the surface of its capabilities, but the important points are that it’s *free* and that it *works well*.

Python programming language

Both Python and C++ find extensive use in these modules as instructional aids and exercises, but I’m listing Python here as a *tool* for myself because I use it almost daily as a *calculator*. If you open a Python interpreter console and type `from math import *` you can type mathematical expressions and have it return results just as you would on a hand calculator. Complex-number (i.e. *phasor*) arithmetic is similarly supported if you include the complex-math library (`from cmath import *`). Examples of this are shown in the Programming References chapter (if included) in each module. Of course, being a fully-featured programming language, Python also supports conditionals, loops, and other structures useful for calculation of quantities. Also, running in a console environment where all entries and returned values show as text in a chronologically-ordered list makes it easy to copy-and-paste those calculations to document exactly how they were performed.

Appendix D

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Appendix E

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Appendix F

Version history

This is a list showing all significant additions, corrections, and other edits made to this learning module. Each entry is referenced by calendar date in reverse chronological order (newest version first), which appears on the front cover of every learning module for easy reference. Any contributors to this open-source document are listed here as well.

28 March 2025 – added recommendations from Michael Ossman on simple PCB layout for RF circuits. Also made some minor edits to the “RF safety” section of the Tutorial.

26-27 February 2025 – added text about the UK’s Million Women Study to the “RF Safety” section of the Tutorial chapter, and also corrected some minor typographical errors throughout.

22 October 2024 – added a new Case Tutorial section with example decibel calculations for RF cable losses.

1 October 2024 – minor edits to the Tutorial.

26 September 2024 – divided the Introduction chapter into sections, one with recommendations for students, one with a listing of challenging concepts, and one with recommendations for instructors.

24 September 2024 – added a new Technical Reference section on Maxwell’s Laws of electromagnetism.

25-27 February 2024 – made minor edits to the Tutorial chapter, including a rather embarrassing omission in the “Signal coupling and transmission lines” section where I neglected to mention a frequency of 60 Hz for a capacitive reactance example. Also made significant edits to the section on Near-field testing for technical accuracy and readability. Lastly, populated the Introduction chapter with concepts and questions.

20 January 2024 – added a new section to the Technical References chapter on near-field versus far-field effects. Also made minor edits to the Tutorial chapter.

7 January 2024 – expanded on the description of the PL-259 style RF connector, adding that

SO-259 refers to the socket, and that these are also generically known as “UHF” connectors.

1 June 2023 – minor wording change in the “RF safety” section of the Tutorial.

29 November 2022 – placed questions at the top of the itemized list in the Introduction chapter prompting students to devise experiments related to the tutorial content.

26 April 2022 – added subsection on “reverse-polarity” RF connectors.

3 January 2022 – added comments regarding 2.4 mm, 2.92 mm, and 3.5 mm connectors. Also added information on different rigidities of coaxial cable as well as waveguides.

26-31 December 2021 – document first created.

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