

## AC reactive circuit calculations

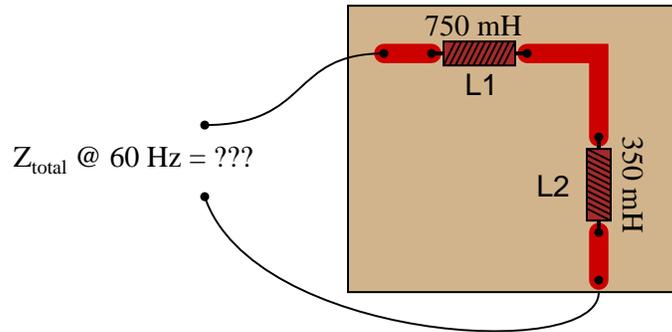
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## Questions

### Question 1

Calculate the total impedance offered by these two inductors to a sinusoidal signal with a frequency of 60 Hz:



Show your work using two different problem-solving strategies:

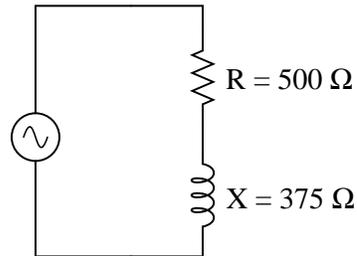
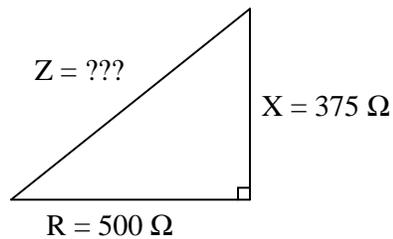
- Calculating total inductance ( $L_{total}$ ) first, then total impedance ( $Z_{total}$ ).
- Calculating individual impedances first ( $Z_{L1}$  and  $Z_{L2}$ ), then total impedance ( $Z_{total}$ ).

Do these two strategies yield the same total impedance value? Why or why not?

[file i01031](#)

### Question 2

Use the “impedance triangle” to calculate the impedance of this series combination of resistance ( $R$ ) and inductive reactance ( $X$ ):



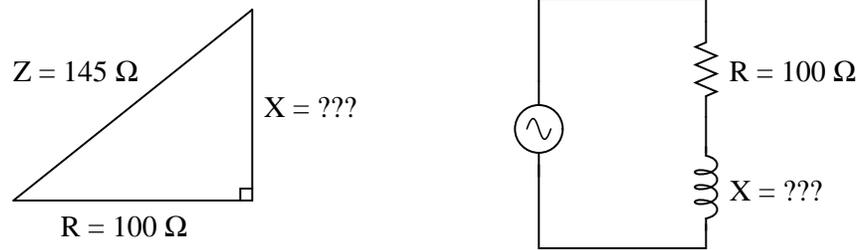
Explain what equation(s) you use to calculate  $Z$ .

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Question 3

Use the “impedance triangle” to calculate the necessary reactance of this series combination of resistance ( $R$ ) and inductive reactance ( $X$ ) to produce the desired total impedance of  $145 \Omega$ :



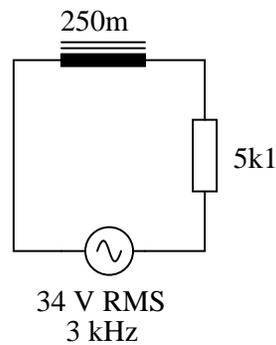
Explain what equation(s) you use to calculate  $X$ , and the algebra necessary to achieve this result from a more common formula.

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Question 4

Calculate all voltages and currents in this circuit, as well as the total impedance:

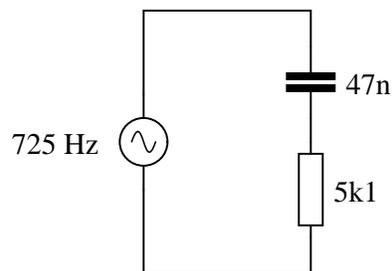


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Question 5

Which component, the resistor or the capacitor, will drop more voltage in this circuit?



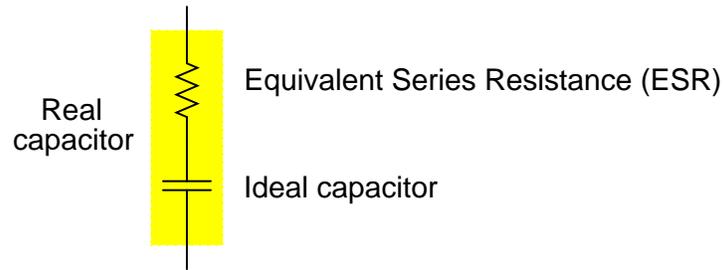
Also, calculate the total impedance ( $Z_{total}$ ) of this circuit, expressing it in both rectangular and polar forms.

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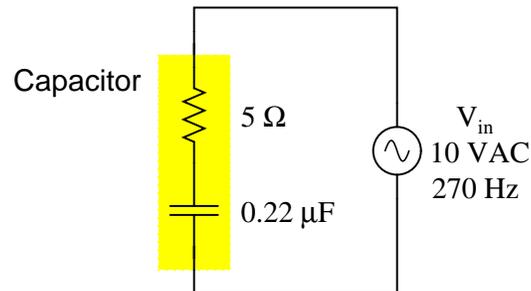
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Question 6

Due to the effects of a changing electric field on the dielectric of a capacitor, some energy is dissipated in capacitors subjected to AC. Generally, this is not very much, but it is there. This dissipative behavior is typically modeled as a series-connected resistance:



Calculate the magnitude and phase shift of the current through this capacitor, taking into consideration its equivalent series resistance (ESR):

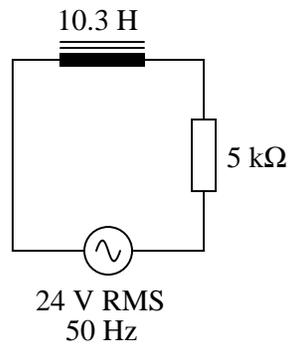


Compare this against the magnitude and phase shift of the current for an ideal  $0.22 \mu\text{F}$  capacitor.  
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Question 7

Solve for all voltages and currents in this series LR circuit, and also calculate the phase angle of the total impedance:

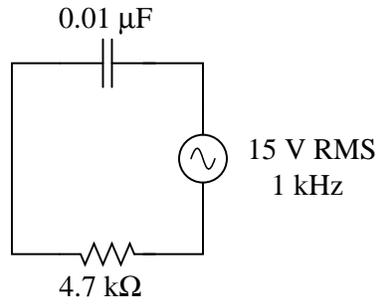


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Question 8

Solve for all voltages and currents in this series RC circuit:

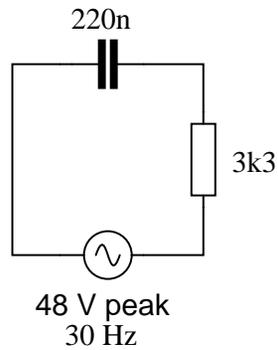


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Question 9

Solve for all voltages and currents in this series RC circuit, and also calculate the phase angle of the total impedance:

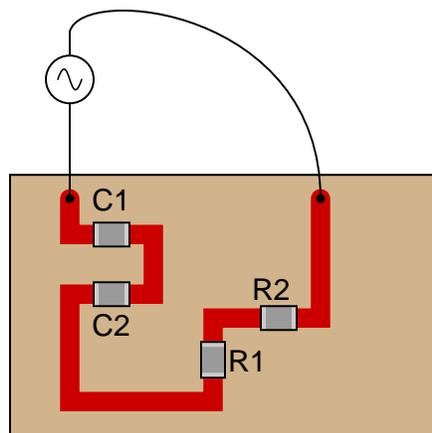


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Question 10

Determine the total current and all voltage drops in this circuit, stating your answers the way a multimeter would register them:



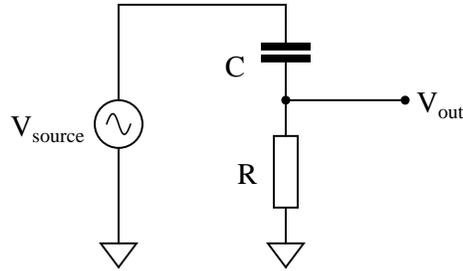
- $C_1 = 125 \text{ pF}$
- $C_2 = 71 \text{ pF}$
- $R_1 = 6.8 \text{ k}\Omega$
- $R_2 = 1.2 \text{ k}\Omega$
- $V_{supply} = 20 \text{ V RMS}$
- $f_{supply} = 950 \text{ kHz}$

Also, calculate the phase angle ( $\theta$ ) between voltage and current in this circuit, and explain where and how you would connect an oscilloscope to measure that phase shift.

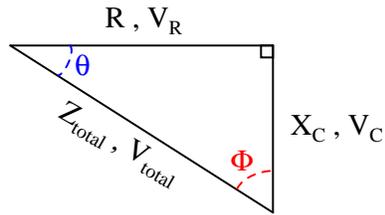
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### Question 11

A student is asked to calculate the phase shift for the following circuit's output voltage, relative to the phase of the source voltage:



He recognizes this as a series circuit, and therefore realizes that a right triangle would be appropriate for representing component impedances and component voltage drops (because both impedance and voltage are quantities that add in series, and the triangle represents phasor addition):



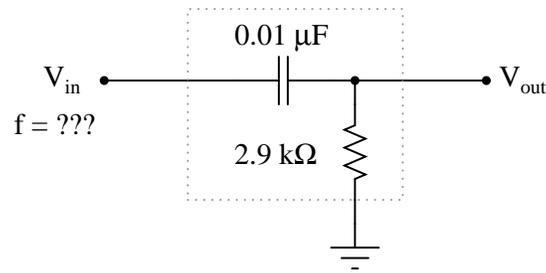
The problem now is, which angle does the student solve for in order to find the phase shift of  $V_{out}$ ? The triangle contains two angles besides the  $90^\circ$  angle,  $\theta$  and  $\Phi$ . Which one represents the output phase shift, and more importantly, *why*?

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Question 12

Determine the input frequency necessary to give the output voltage a phase shift of  $40^\circ$ :

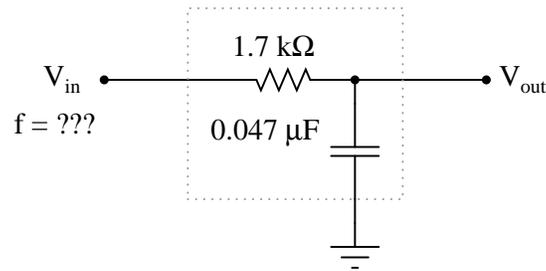


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Question 13

Determine the input frequency necessary to give the output voltage a phase shift of  $-25^\circ$ :

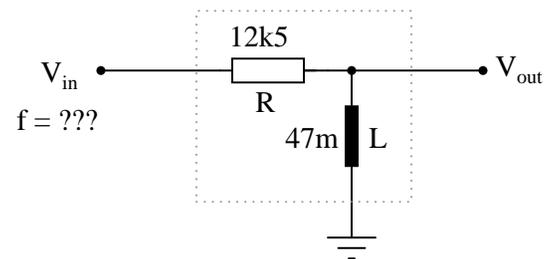


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Question 14

Determine the input frequency necessary to give the output voltage a phase shift of  $75^\circ$ :



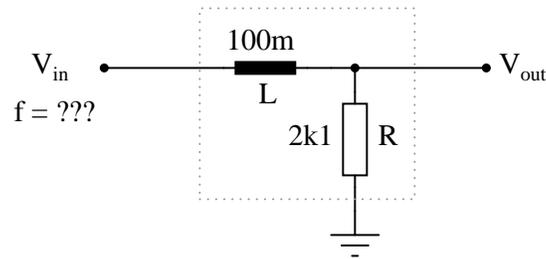
Also, write an equation that solves for frequency ( $f$ ), given all the other variables ( $R$ ,  $L$ , and phase angle  $\theta$ ).

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Question 15

Determine the input frequency necessary to give the output voltage a phase shift of  $-40^\circ$ :



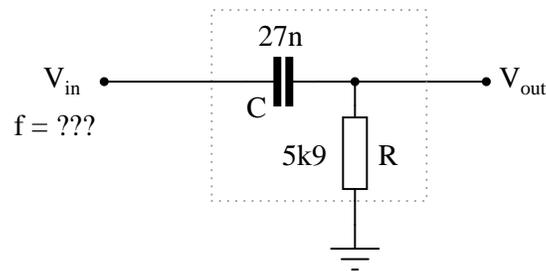
Also, write an equation that solves for frequency ( $f$ ), given all the other variables ( $R$ ,  $L$ , and phase angle  $\theta$ ).

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Question 16

Determine the input frequency necessary to give the output voltage a phase shift of  $25^\circ$ :



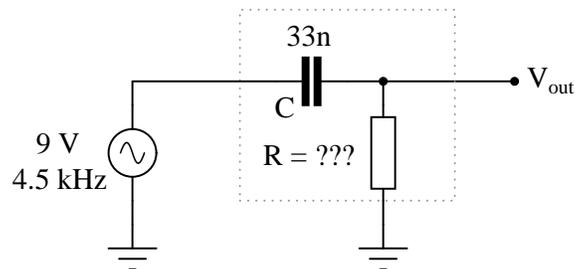
Also, write an equation that solves for frequency ( $f$ ), given all the other variables ( $R$ ,  $C$ , and phase angle  $\theta$ ).

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Question 17

Determine the necessary resistor value to give the output voltage a phase shift of  $58^\circ$ :



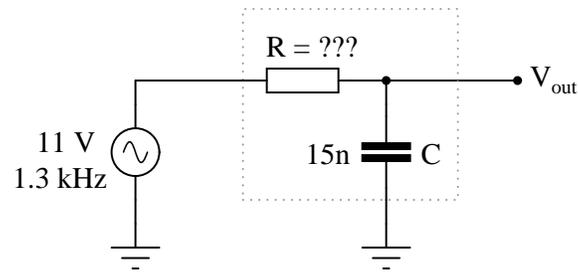
Also, write an equation that solves for this resistance value ( $R$ ), given all the other variables ( $f$ ,  $C$ , and phase angle  $\theta$ ).

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Question 18

Determine the necessary resistor value to give the output voltage a phase shift of  $-64^\circ$ :



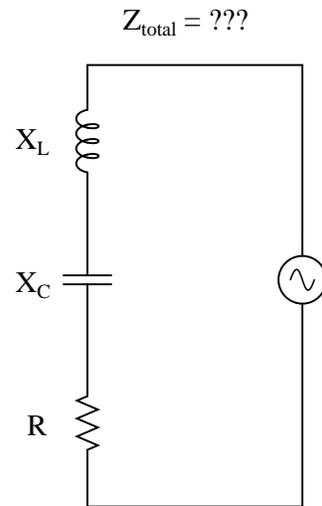
Also, write an equation that solves for this resistance value ( $R$ ), given all the other variables ( $f$ ,  $C$ , and phase angle  $\theta$ ).

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Question 19

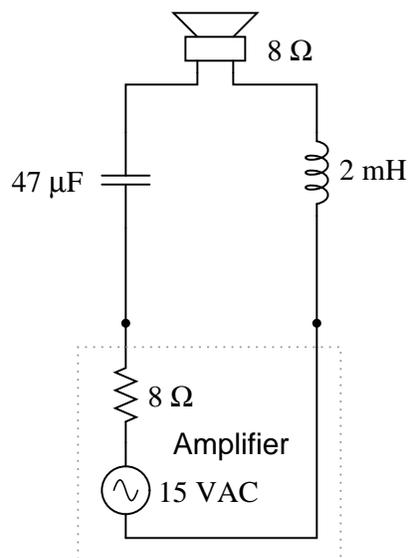
Write an equation that solves for the impedance of this series circuit. The equation need not solve for the phase angle between voltage and current, but merely provide a scalar figure for impedance (in ohms):



[file i01074](#)

Question 20

Calculate the voltage dropped across the inductor, the capacitor, and the 8-ohm speaker in this sound system at the following frequencies, given a constant source voltage of 15 volts:



- $f = 200$  Hz
- $f = 550$  Hz
- $f = 900$  Hz

Regard the speaker as nothing more than an 8-ohm resistor.

**Suggestions for Socratic discussion**

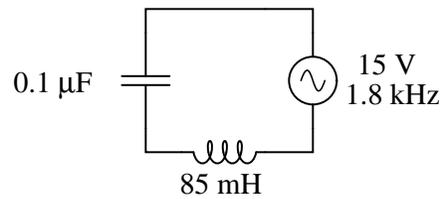
- As part of an audio system, would this LC network tend to emphasize the *bass*, *treble*, or *mid-range* tones?

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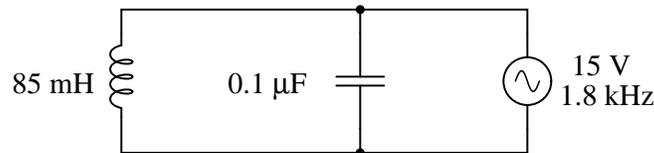
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Question 21

Is this circuit's overall behavior capacitive or inductive? In other words, from the perspective of the AC voltage source, does it "appear" as though a capacitor is being powered, or an inductor?



Now, suppose we take these same components and re-connect them in parallel rather than series. Does this change the circuit's overall "appearance" to the source? Does the source now "see" an equivalent capacitor or an equivalent inductor? Explain your answer.



**Suggestions for Socratic discussion**

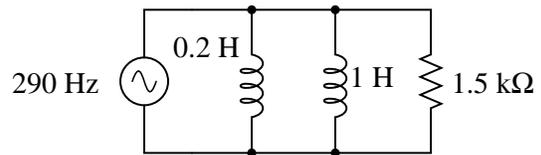
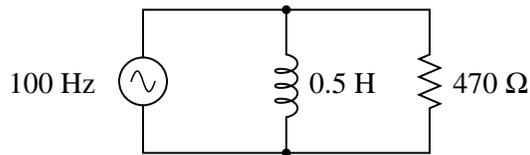
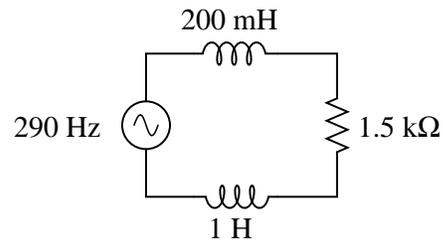
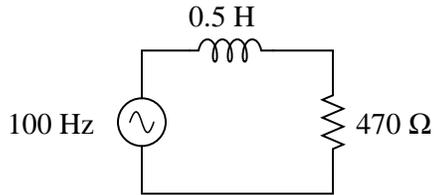
- Which component "dominates" the behavior of a series LC circuit, the one with the least reactance or the one with the greatest reactance?
- Which component "dominates" the behavior of a parallel LC circuit, the one with the least reactance or the one with the greatest reactance?

[file i01076](#)

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Question 22

Calculate the total impedances (complete with phase angles) for each of the following inductor-resistor circuits:

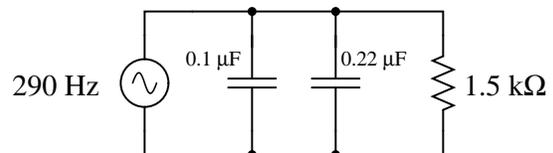
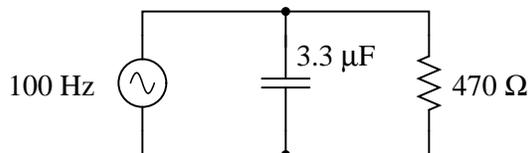
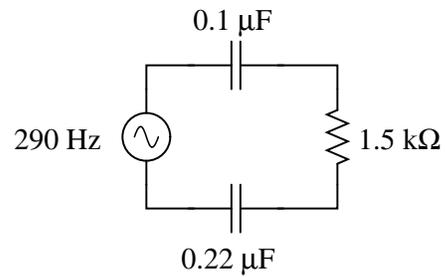
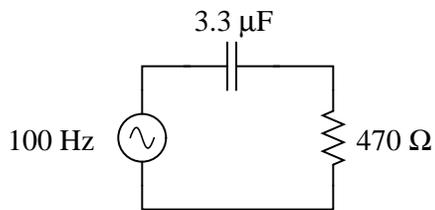


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Question 23

Calculate the total impedances (complete with phase angles) for each of the following capacitor-resistor circuits:

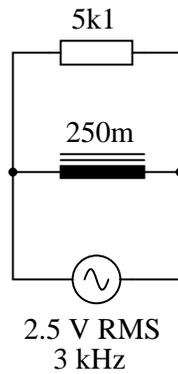


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Question 24

Calculate the individual currents through the inductor and through the resistor, the total current, and the total circuit impedance:

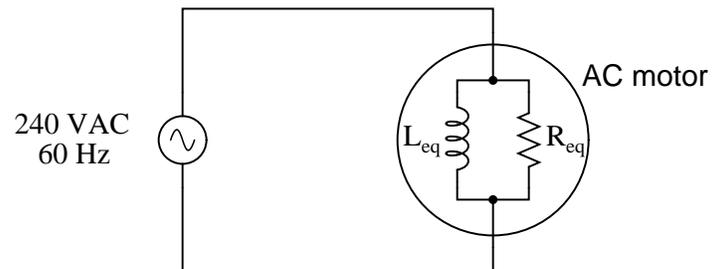


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Question 25

An AC electric motor under load can be considered as a parallel combination of resistance and inductance:



Calculate the current necessary to power this motor if the equivalent resistance and inductance is  $20 \Omega$  and  $238 \text{ mH}$ , respectively.

[file i01058](#)

Answer 1

**First strategy:**

$$L_{total} = 1.1 \text{ H}$$

$$X_{total} = 414.7 \Omega$$

$$\mathbf{Z}_{total} = 414.7 \Omega \angle 90^\circ \text{ or } \mathbf{Z}_{total} = 0 + j414.7 \Omega$$

**Second strategy:**

$$X_{L1} = 282.7 \Omega \quad \mathbf{Z}_{L1} = 282.7 \Omega \angle 90^\circ$$

$$X_{L2} = 131.9 \Omega \quad \mathbf{Z}_{L2} = 131.9 \Omega \angle 90^\circ$$

$$\mathbf{Z}_{total} = 414.7 \Omega \angle 90^\circ \text{ or } \mathbf{Z}_{total} = 0 + j414.7 \Omega$$

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Answer 2

$Z = 625 \Omega$ , as calculated by the Pythagorean Theorem.

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Answer 3

$X = 105 \Omega$ , as calculated by an algebraically manipulated version of the Pythagorean Theorem.

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Answer 4

$$Z_{total} = 6.944 \text{ k}\Omega$$

$$I = 4.896 \text{ mA RMS}$$

$$V_L = 23.07 \text{ V RMS}$$

$$V_R = 24.97 \text{ V RMS}$$

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Answer 5

The resistor will drop more voltage.

$$Z_{total} \text{ (rectangular form)} = 5100 \Omega - j4671 \Omega$$

$$Z_{total} \text{ (polar form)} = 6916 \Omega \angle -42.5^\circ$$

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Answer 6

$\mathbf{I} = 3.732206 \text{ mA} \angle 89.89^\circ$  for the real capacitor with ESR.

$\mathbf{I} = 3.732212 \text{ mA} \angle 90.00^\circ$  for the ideal capacitor.

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Answer 7

$$V_L = 13.04 \text{ volts RMS}$$

$$V_R = 20.15 \text{ volts RMS}$$

$$I = 4.030 \text{ milliamps RMS}$$

$$\theta_Z = 32.91^\circ$$

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Answer 8

$$V_C = 14.39 \text{ volts RMS}$$

$$V_R = 4.248 \text{ volts RMS}$$

$$I = 903.9 \mu\text{A RMS}$$

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Answer 9

$$V_C = 47.56 \text{ volts peak}$$

$$V_R = 6.508 \text{ volts peak}$$

$$I = 1.972 \text{ milliamps peak}$$

$$\theta_Z = -82.21^\circ$$

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Answer 10

- $I_{total} = 2.269 \text{ mA}$
- $V_{C1} = 3.041 \text{ V}$
- $V_{C2} = 5.354 \text{ V}$
- $V_{R1} = 15.43 \text{ V}$
- $V_{R2} = 2.723 \text{ V}$
- $\theta = -24.82^\circ$  (voltage lagging current)

I suggest using a dual-trace oscilloscope to measure total voltage (across the supply terminals) and voltage drop across resistor  $R_2$ . Theoretically, measuring the voltage dropped by either resistor would be fine, but  $R_2$  works better for practical reasons (oscilloscope input lead grounding). Phase shift then could be measured either in the time domain or by a Lissajous figure analysis.

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Answer 11

The proper angle in this circuit is  $\theta$ , and it will be a positive (leading) quantity.

The reason we must focus on  $\theta$  in this problem is because this is the angle separating the resistor's voltage drop from the source's voltage. Note how in the circuit  $V_{out}$  is measured across the resistor, not across the capacitor.

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Answer 12

$$f = 6.54 \text{ kHz}$$

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Answer 13

$$f = 929 \text{ Hz}$$

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Answer 14

$$f = 11.342 \text{ kHz}$$

$$f = \frac{R}{2\pi L \tan \theta}$$

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Answer 15

$$f = 2.804 \text{ kHz}$$

$$f = -\frac{R \tan \theta}{2\pi L}$$

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Answer 16

$$f = 2.143 \text{ kHz}$$

$$f = \frac{1}{2\pi RC \tan \theta}$$

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Answer 17

$$R = 669.7 \ \Omega$$

$$R = \frac{1}{2\pi fC \tan \theta}$$

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Answer 18

$$R = 16.734 \text{ k}\Omega$$

$$R = -\frac{\tan \theta}{2\pi fC}$$

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Answer 19

$$Z_{total} = \sqrt{R^2 + (X_L - X_C)^2}$$

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Answer 20

- $f = 200 \text{ Hz}$  ;  $V_L = 1.750 \text{ V}$  ;  $V_C = 11.79 \text{ V}$  ;  $V_{speaker} = 5.572 \text{ V}$
- $f = 550 \text{ Hz}$  ;  $V_L = 6.472 \text{ V}$  ;  $V_C = 5.766 \text{ V}$  ;  $V_{speaker} = 7.492 \text{ V}$
- $f = 900 \text{ Hz}$  ;  $V_L = 9.590 \text{ V}$  ;  $V_C = 3.763 \text{ V}$  ;  $V_{speaker} = 6.783 \text{ V}$

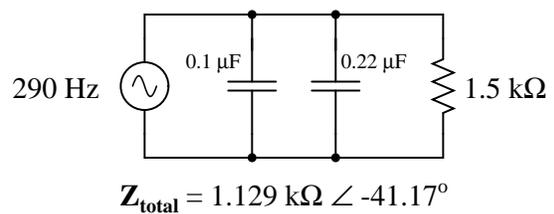
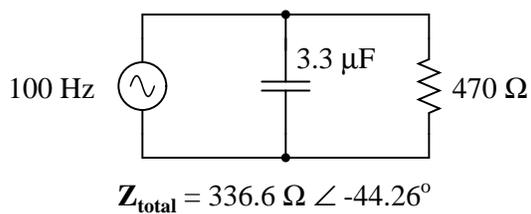
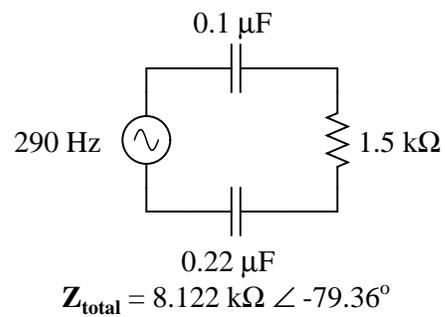
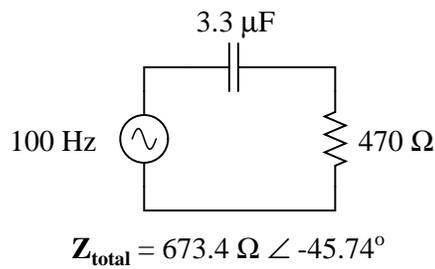
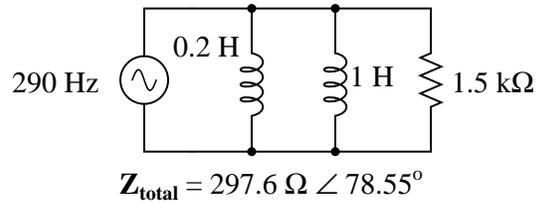
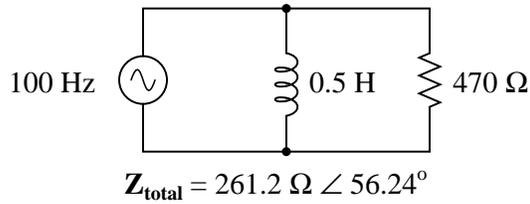
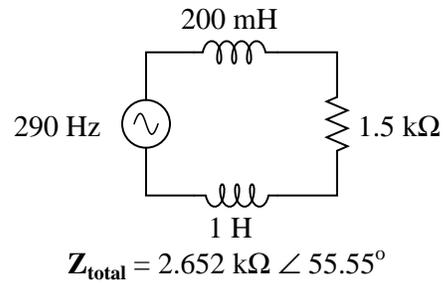
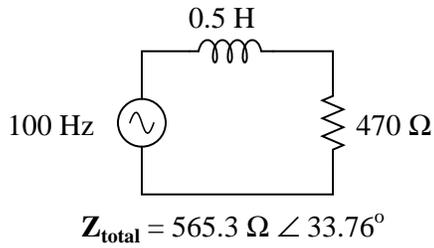
This circuit is known as a *midrange crossover* in stereo system design.

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Answer 21

The first (series) circuit's behavior is predominantly *inductive*, with 961.3 ohms of inductive reactance overshadowing 884.2 ohms of capacitive reactance, for a total circuit impedance of 77.13 ohms  $\angle +90^\circ$  (polar form) or  $0 + j77.13$  ohms (rectangular form).

The second (parallel) circuit's behavior is *capacitive*, with 1.131 millisiemens of capacitive susceptance overshadowing 1.040 millisiemens of inductive susceptance, for a total circuit admittance of 90.74 microsiemens  $\angle +90^\circ$ . This translates into a total circuit impedance of 11.02 k $\Omega$   $\angle -90^\circ$ , the negative phase angle clearly indicating this to be a predominantly capacitive circuit.



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Answer 24

$$I_L = 530.5 \mu\text{A RMS} \angle -90^\circ$$

$$I_R = 490.2 \mu\text{A RMS} \angle 0^\circ$$

$$I_{total} = 722.3 \mu\text{A RMS} \angle -47.26^\circ$$

$$Z_{total} = 3.461 \text{ k}\Omega \angle 47.26^\circ$$

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Answer 25

$$I_{supply} = 12.29 \text{ A}$$