

Fundamental principles of algebra

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The following equations are basic algebraic properties: rules that all real numbers adhere to.

Associative property:

$$a + (b + c) = (a + b) + c$$

$$a(bc) = (ab)c$$

Commutative property:

$$a + b = b + a$$

$$ab = ba$$

Distributive property:

$$a(b + c) = ab + bc$$

Properties of exponents:

$$a^x a^y = a^{x+y}$$

$$\frac{a^x}{a^y} = a^{x-y}$$

$$(ab)^x = a^x b^x$$

$$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$$

$$(a^x)^y = a^{xy}$$

Properties of roots:

$$(\sqrt[x]{a})^x = a$$

$$\sqrt[x]{a^x} = a \text{ if } a \geq 0$$

$$\sqrt[x]{ab} = \sqrt[x]{a} \sqrt[x]{b}$$

$$\sqrt[x]{\frac{a}{b}} = \frac{\sqrt[x]{a}}{\sqrt[x]{b}}$$

Questions

Question 1

A very important concept in algebra is the *variable*. What, exactly, is a variable, and why are they so useful to us?

[file i01301](#)

Question 2

What is the difference between these two variable expressions?

$$x^2$$

$$x_2$$

[file i01302](#)

Question 3

Suppose we begin with this mathematical statement:

$$3 = 3$$

Not very stunning, I know, but nevertheless absolutely true in a mathematical sense. If I were to add the quantity “5” to the left-hand side of the equation, though, the quantities on either side of the “equals” sign would no longer be equal to each other. To be proper, I would have to replace the “equals” symbol with a “not equal” symbol:

$$3 + 5 \neq 3$$

Assuming that we keep “3 + 5” on the left-hand side of the statement, what would have to be done to the right-hand side of the statement to turn it into an equality again?

[file i01303](#)

Question 4

Suppose we begin with this mathematical statement:

$$3 = 3$$

If I were to multiply the right-hand side of the equation by the number “7”, the quantities on either side of the “equals” sign would no longer be equal to each other. To be proper, I would have to replace the “equals” symbol with a “not equal” symbol:

$$3 \neq 3 \times 7$$

Assuming that we keep “3 × 7” on the right-hand side of the statement, what would have to be done to the left-hand side of the statement to turn it into an equality again?

[file i01304](#)

Question 5

Suppose we begin with this mathematical statement:

$$3 \times 4 = 10 + 2$$

If I were to add the quantity “1” to the left-hand side of the equation, the quantities on either side of the “equals” sign would no longer be equal to each other. To be proper, I would have to replace the “equals” symbol with a “not equal” symbol:

$$(3 \times 4) + 1 \neq 10 + 2$$

What is the simplest and most direct change I can make to the right-hand side of this expression to turn it into an equality again?

file i01305

Question 6

Suppose we were told that this was a mathematically true statement:

$$x = y$$

In other words, variable x represents the exact same numerical value as variable y . Given this assumption, the following mathematical statements must also be true:

$$x + 8 = y + 8$$

$$-x = -y$$

$$9x = 9y$$

$$\frac{x}{25} = \frac{y}{25}$$

$$\frac{1}{x} = \frac{1}{y}$$

$$x^2 = y^2$$

$$\sqrt{x} = \sqrt{y}$$

$$e^x = e^y$$

$$\log x = \log y$$

$$x! = y!$$

$$\frac{dx}{dt} = \frac{dy}{dt}$$

Explain the general principle at work here. Why are we able to alter the basic equality of $x = y$ in so many different ways, and yet still have the resulting expressions be equalities?

[file i01306](#)

Question 7

Suppose we were given the following equation:

$$\frac{14}{y} = 2$$

What would the equation look like if both sides were multiplied by y ?

[file i01307](#)

Question 8

Solve for the value of a in the following equations:

Equation 1: $a - 4 = 10$

Equation 2: $30 = a + 3$

Equation 3: $-2a = 9$

Equation 4: $\frac{a}{4} = 3.5$

[file i01308](#)

Question 9

Solve for n in the following equations:

Equation 1: $-56 = -14n$

Equation 2: $54 - n = 10$

Equation 3: $\frac{4}{n} = 12$

Equation 4: $28 = 2 - n$

[file i01309](#)

Question 10

What would this equation look like if both sides of it were *reciprocated* (inverted)?

$$\frac{10}{(x + 2)} = -4$$

[file i01311](#)

Question 11

This is the *Ideal Gas Law* equation, expressing the relationship between the pressure (P), volume (V), temperature (T), and molar quantity (n) of an enclosed gas:

$$PV = nRT$$

Manipulate this equation to solve for each of the following variables:

$$P =$$

$$V =$$

$$n =$$

$$R =$$

$$T =$$

[file i01331](#)

Question 12

This formula describes the amount of energy carried by a photon:

$$E = \frac{hc}{\lambda}$$

Where,

E_k = Energy carried by photon (joules)

h = Planck's constant (6.626×10^{-34} joule-seconds)

c = Speed of light in a vacuum ($\approx 3 \times 10^8$ meters per second)

λ = Wavelength of photon (meters)

Manipulate this equation as many times as necessary to express it in terms of all its variables.

[file i02056](#)

Question 13

The velocity of light (normally $c = 3 \times 10^8$ meters per second in a vacuum) slows down proportional to the square-root of the dielectric permittivity of whatever substance it passes through, as described by the following equation:

$$v = \frac{c}{\sqrt{\epsilon}}$$

Manipulate this equation to solve for each of the following variables:

$$c =$$

$$\epsilon =$$

[file i01332](#)

Question 14

A common equation used in physics relates the kinetic energy, velocity, and mass of a moving object:

$$E_k = \frac{1}{2}mv^2$$

Where,

- E_k = Kinetic energy (Joules)
- m = Mass (kilograms)
- v = Velocity (meters per second)

Manipulate this equation as many times as necessary to express it in terms of all its variables.
[file i01312](#)

Question 15

If an object is lifted up against the force of Earth's gravity, it gains *potential energy* (stored energy): the potential to do useful work if released to fall back down to the ground. The exact amount of this potential energy may be calculated using the following formula:

$$E_p = mgh$$

Where,

- E_p = Potential energy (Joules)
- m = Mass (kilograms)
- g = Acceleration of gravity (9.81 meters per second)
- h = Height lifted above the ground (meters)

When that mass is released to free-fall back to ground level, its potential energy becomes converted into *kinetic energy* (energy in motion). The relationship between the object's velocity and its kinetic energy may be calculated using the following formula:

$$E_k = \frac{1}{2}mv^2$$

Where,

- E_k = Kinetic energy (Joules)
- m = Mass (kilograms)
- v = Velocity (meters per second)

Knowing that the amount of potential energy (E_p) an object possesses at its peak height (h) will be the same amount as its kinetic energy (E_k) in the last moment before it hits the ground, combine these two formulae to arrive at one formula predicting the maximum velocity (v) given the object's initial height (h).

[file i02591](#)

Question 16

The electrical resistance of an RTD temperature sensor (R) is given by this equation as a function of its base resistance at a specified reference temperature (R_0), its coefficient of resistance (α), its temperature (T), and the reference temperature (T_{ref}):

$$R = R_0[1 + \alpha(T - T_{ref})]$$

Manipulate this equation to solve for each of the following variables:

$$R_0 =$$

$$\alpha =$$

$$T =$$

$$T_{ref} =$$

[file i01333](#)

Question 17

The velocity necessary for a satellite to maintain a circular orbit around Earth is given by this equation:

$$v_s = \sqrt{g_c + h}$$

Where,

v_s = Satellite velocity (feet per second)

g_c = Acceleration of Earth gravity at sea level (32 feet per second squared)

h = Orbit altitude, (feet)

Manipulate this equation to solve for g_c , and then to solve for h :

$$g_c =$$

$$h = \text{file i01313}$$

Question 18

This equation predicts the volumetric flow rate of liquid through a control valve (Q) given the flow coefficient for the valve (C_v), the upstream and downstream pressures (P_1 and P_2), and the specific gravity of the liquid (G_f):

$$Q = C_v \sqrt{\frac{P_1 - P_2}{G_f}}$$

Manipulate this equation to solve for each of the following variables:

$C_v =$

$P_1 =$

$P_2 =$

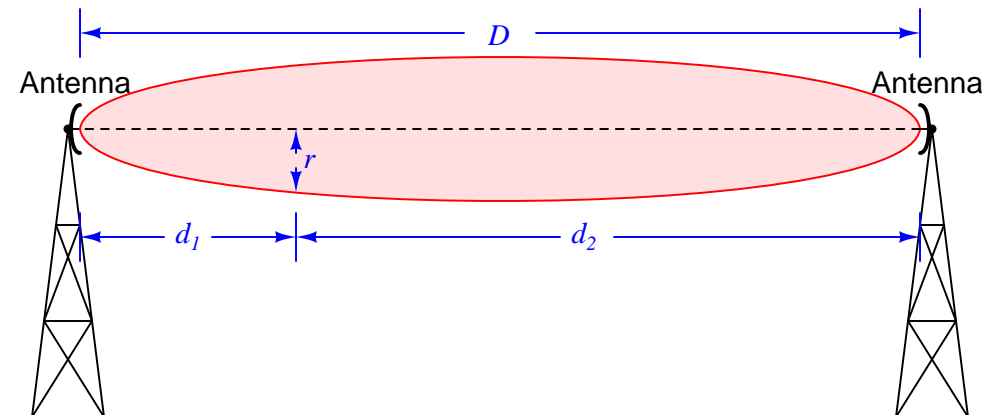
$G_f =$

[file i01336](#)

Question 19

The radius of a *Fresnel zone* between two radio antennas may be predicted by the following equation:

$$r = \sqrt{\frac{n\lambda d_1 d_2}{D}}$$



Manipulate this equation to solve for each of the following variables:

$d_1 =$

$D =$

$\lambda =$

[file i01339](#)

Question 20

The expected operating life of a rolling-contact bearing may be predicted by the following equation:

$$R_L = \left(\frac{C}{L}\right)^{\frac{10}{3}}$$

Where,

R_L = Operating life (millions of shaft revolutions)

C = Dynamic capacity of bearing (pounds)

L = Radial load applied to bearing (pounds)

Manipulate this equation as many times as necessary to express it in terms of all its variables.

[file i01314](#)

Question 21

Mechanical, chemical, and civil engineers must often calculate the size of piping necessary to transport fluids. One equation used to relate the water-carrying capacity of multiple, small pipes to the carrying capacity of one large pipe is as follows:

$$N = \left(\frac{d_2}{d_1}\right)^{2.5}$$

Where,

N = Number of small pipes

d_1 = Diameter of each small pipe

d_2 = Diameter of large pipe

Manipulate this equation as many times as necessary to express it in terms of all its variables.

[file i01315](#)

Question 22

The amount of power required to propel a ship is given by this equation:

$$P = \frac{D^{\frac{2}{3}}V^3}{K}$$

Where,

P = Power required to turn propeller(s) (horsepower)

D = Vessel displacement (long tons)

V = Velocity (nautical miles per hour)

K = Admiralty coefficient (approximately 70 for a 30 foot long ship, load waterline)

Manipulate this equation as many times as necessary to express it in terms of all its variables.

[file i01316](#)

Question 23

This equation solves for the pressure recovery factor (F_L) of a control valve given pressures upstream (P_1), downstream (P_2), and at the vena contracta inside the valve (P_{vc}):

$$F_L = \sqrt{\frac{P_1 - P_2}{P_1 - P_{vc}}}$$

Manipulate this equation to solve for each of the following variables:

$$P_1 =$$

$$P_2 =$$

$$P_{vc} =$$

[file i01337](#)

Question 24

This equation predicts the volumetric flow rate of gas through a control valve (Q) given the flow coefficient for the valve (C_v), the upstream and downstream pressures (P_1 and P_2), the temperature of the gas (T), and the specific gravity of the gas (G_g):

$$Q = 963 C_v \sqrt{\frac{(P_1 - P_2)(P_1 + P_2)}{G_g T}}$$

Manipulate this equation to solve for each of the following variables:

$$P_1 =$$

$$P_2 =$$

$$T =$$

[file i01338](#)

Question 25

This is the *Nernst* equation, expressing the voltage developed across an ion-permeable membrane as ions migrate through:

$$V = \frac{RT}{nF} \ln \frac{C_1}{C_2}$$

Manipulate this equation to solve for each of the following variables:

$$R =$$

$$F =$$

$$C_1 =$$

$$C_2 =$$

[file i01335](#)

Question 26

This equation predicts the path loss of a radio signal (in units of decibels) as it propagates away from a radiating antenna:

$$L_{path} = -20 \log \left(\frac{4\pi D}{\lambda} \right)$$

Manipulate this equation to solve for each of the following variables:

$$D =$$

$$\lambda =$$

[file i01341](#)

Question 27

In algebra, any equation may be manipulated in any way desired, so long as the same manipulation is applied to *both* sides of the equation equally. In this example, though, only one term on one side of the equation ($\frac{2}{x}$) is manipulated: we multiply it by the fraction $\frac{3x}{3x}$. Is this a “legal” thing to do? Why or why not?

$$y = \frac{2}{x} + \frac{5}{3x^2}$$

$$y = \frac{3x}{3x} \frac{2}{x} + \frac{5}{3x^2}$$

$$y = \frac{6x}{3x^2} + \frac{5}{3x^2}$$

$$y = \frac{6x + 5}{3x^2}$$

[file i01317](#)

Question 28

Manipulate this equation so that it is expressed in terms of x (with all other variables and constants on the other side of the equals sign):

$$y = \frac{3}{2x} - \frac{7}{3y}$$

[file i01318](#)

Question 29

Equations with identical variables represented on *both* sides are often tricky to manipulate. Take this one, for example:

$$\frac{a}{x} = xy + xz$$

Here, the variable x is found on both sides of the equation. How can we manipulate this equation so as to “consolidate” these x variables together so that x is by itself on one side of the equation and everything else is on the other side?

[file i01319](#)

Question 30

The *Brinell hardness* of a metal specimen is measured by pressing a ball into the specimen and measuring the size of the resulting indentation. To calculate Brinell hardness, this equation is used:

$$H = \frac{F}{\left(\frac{\pi d_1}{2}\right)(d_1 - \sqrt{d_1^2 - d_s^2})}$$

Where,

H = Hardness of specimen (Brinell units)

F = Force on ball (kg)

d_1 = Diameter of ball (mm)

d_s = Diameter of indentation (mm)

Manipulate this equation to solve for F and for d_s .

[file i01321](#)

Question 31

In digital electronic systems based on binary numeration, the number of possible *states* representable by the system is given by the following equation:

$$n_s = 2^{n_b}$$

Where,

n_s = Number of possible states

n_b = Number of binary “bits”

How could you manipulate this equation to solve for the number of binary bits necessary to provide a given number of states?

[file i01322](#)

Question 32

Solve for the value of x in this equation:

$$\frac{e^x - e^{-x}}{3} = -1$$

Note: this is a rather challenging problem, requiring you to use (among other things) the quadratic formula in your solution!

[file i01323](#)

Question 33

In this equation there is a **mistake**. Find the mistake, and correct it so that the expressions on both sides of the equals sign are truly equal to each other:

$$5\frac{a}{b} = \frac{5a}{5b}$$

[file i01324](#)

Question 34

In this equation there is a **mistake**. Find the mistake, and correct it so that the expressions on both sides of the equals sign are truly equal to each other:

$$\frac{4y + 8}{4y + 3} = \frac{8}{3}$$

[file i01325](#)

Question 35

In this series of algebraic steps there is a **mistake**. Find the mistake and correct it (as well as all subsequent steps):

$$5a - 6ac = 1 - 3a^2b$$

$$\frac{5a - 6ac}{a} = \frac{1 - 3a^2b}{a}$$

$$5 - 6c = 1 - 3ab$$

$$3ab + 5 - 6c = 1$$

[file i01326](#)

Question 36

In this equation there is a **mistake**. Find the mistake, and correct it so that the expressions on both sides of the equals sign are truly equal to each other:

$$(4x + 2)(2x + 3) = 8x^2 + 6$$

[file i01327](#)

Question 37

In this equation there is a **mistake**. Find the mistake, and correct it so that the expressions on both sides of the equals sign are truly equal to each other:

$$\frac{1}{c} + \frac{1}{d} = \frac{1}{c + d}$$

[file i01328](#)

Question 38

In this equation there is a **mistake**. Find the mistake, and correct it so that the expressions on both sides of the equals sign are truly equal to each other:

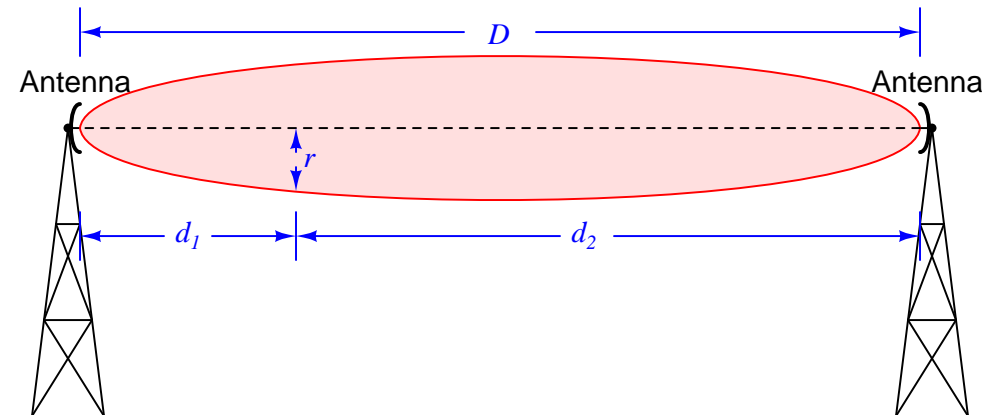
$$\frac{6}{x + 2} - \frac{x - 4}{x + 2} = \frac{6 - x - 4}{x + 2}$$

[file i01329](#)

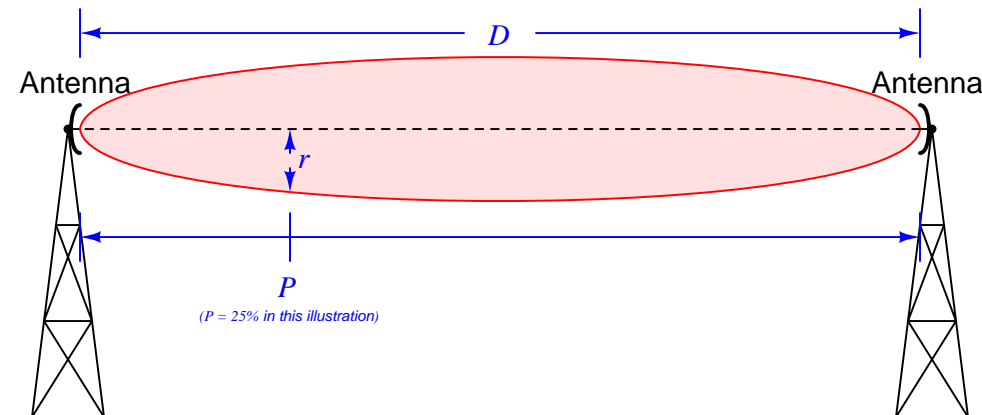
Question 39

The radius of a *Fresnel zone* between two radio antennas may be predicted by the following equation:

$$r = \sqrt{\frac{n\lambda d_1 d_2}{D}}$$



Re-write this equation in a form lacking two distance measurements d_1 and d_2 , replacing them with a single variable P representing the position between the antennas expressed as a percentage. $P = 0\%$ means the point at the left-hand antenna, $P = 100\%$ means the point at the right-hand antenna, and $P = 50\%$ means the point exactly half-way in between the two antennas:



Hint: $d_1 = PD$

[file i01340](#)

Question 40

Bernoulli's equation expresses the relationship between height, velocity, and pressure for a moving fluid. This particular form of Bernoulli's equation uses mass density (ρ) as opposed to weight density (γ) to express how dense the fluid is:

$$z_1 \rho g + \frac{v_1^2 \rho}{2} + P_1 = z_2 \rho g + \frac{v_2^2 \rho}{2} + P_2$$

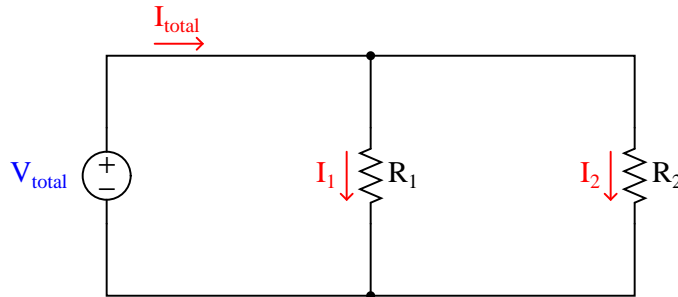
Knowing that mass and weight density are related to each other by the equation $\gamma = \rho g$, re-write Bernoulli's equation so that it does not contain ρ anymore, but instead uses γ :

Re-write Bernoulli's equation one more time so that γ appears only *once* on each side of the equation:

[file i01334](#)

Question 41

Combine the equations $V = IR$, $I_{total} = I_1 + I_2$, and $V_{total} = V_1 = V_2$ for this parallel resistor circuit to solve for I_{total} as a function of V_{total} , R_1 , and R_2 . In other words, write a new equation with just I_{total} on one side of the "equals" symbol and no variables but V_{total} , R_1 , and R_2 on the other side:



$$I_{total} = f(V_{total}, R_1, R_2)$$

[file i01330](#)

Question 42

Solve for values of x and y that will satisfy both of the following equations at the same time:

$$x + 2y = 9$$

$$4x - y = -18$$

[file i03231](#)

Question 43

Solve for values of x and y that will satisfy both of the following equations at the same time:

$$3x - y = -9$$

$$x + 2y = 4$$

[file i03288](#)

Answers

Answer 1

Variables are alphabetical letters used to represent numerical quantities. Instead of writing numerical symbols (0, 1, -5, 2400, etc.), we may write letters (a, b, c, x, y, z), each of which representing a range of possible values.

Answer 2

Superscript numbers represent exponents, so that x^2 means “ x squared”, or x multiplied by itself. Subscript numbers are used to denote separate variables, so that the same alphabetical letter may be used more than once in an equation. Thus, x_2 is a distinct variable, different from x_0, x_1 , or x_3 .

Warning: sometimes subscripts are used to denote specific numerical values of a variable. For instance, x_2 could mean “the variable x when its value is equal to 2”. This is almost always the meaning of subscripts when they are 0 (x_0 is the variable x , set equal to a value of 0). Confusing? Yes!

Answer 3

Without altering the left-hand side of this mathematical expression, the only way to bring both sides into equality again is to add the quantity “5” to the right-hand side as well:

$$3 + 5 = 3 + 5$$

Answer 4

Without altering the right-hand side of this mathematical expression, the only way to bring both sides into equality again is to multiply the left-hand side of the expression by “7” as well:

$$3 \times 7 = 3 \times 7$$

Answer 5

The simplest thing I can do to the right-hand side of the equation to make it equal once again to the left-hand side of the equation is to manipulate it in the same way that I just manipulated the left-hand side (by adding the quantity “1”):

$$(3 \times 4) + 1 = (10 + 2) + 1$$

Answer 6

The principle at work is this: you may perform any mathematical operation you wish to an equation, provided you apply the *same* operation to both sides of the equation in the exact same way.

Answer 7

$$14 = 2y$$

Answer 8

Equation 1: $a = 14$

Equation 2: $a = 27$

Equation 3: $a = -4.5$

Equation 4: $a = 14$

Answer 9

Equation 1: $n = 4$

Equation 2: $n = 44$

Equation 3: $n = 0.\overline{333}$

Equation 4: $n = -26$

Answer 10

$$\frac{(x+2)}{10} = -\frac{1}{4}$$

Answer 11

$$P = \frac{nRT}{V}$$

$$V = \frac{nRT}{P}$$

$$n = \frac{PV}{RT}$$

$$R = \frac{PV}{nT}$$

$$T = \frac{PV}{nR}$$

Answer 12

$$\lambda = \frac{hc}{E}$$

$$h = \frac{E\lambda}{c}$$

$$c = \frac{E\lambda}{h}$$

Answer 13

$$c = v\sqrt{\epsilon}$$

$$\epsilon = \left(\frac{c}{v}\right)^2$$

Answer 14

$$m = 2\frac{E_k}{v^2}$$

$$v = \sqrt{2\frac{E_k}{m}}$$

Answer 15

If 100% of the potential energy at the peak height gets converted into kinetic energy just before contact with the ground, we may set $E_p = E_K$:

$$E_p = mgh = E_k = \frac{1}{2}mv^2$$

$$mgh = \frac{1}{2}mv^2$$

$$gh = \frac{1}{2}v^2$$

$$v^2 = 2gh$$

$$\sqrt{v^2} = \sqrt{2gh}$$

$$v = \sqrt{2gh}$$

The paradoxical conclusion we reach from this combination and manipulation of formulae is that the amount of mass (m) doesn't matter: any object dropped from a certain height will hit the ground with the same velocity. Of course, this assumes 100% conversion of energy from potential to kinetic with no losses, which is never completely practical, and explains why lighter objects do in fact fall slower than heavy objects in air. However, in a vacuum (no air resistance), the fall velocities are precisely equal!

Answer 16

$$R_0 = \frac{R}{1 + \alpha(T - T_{ref})}$$

$$\alpha = \frac{\frac{R}{R_0} - 1}{T - T_{ref}}$$

$$T = \frac{\frac{R}{R_0} - 1}{\alpha} + T_{ref}$$

$$T_{ref} = T - \frac{\frac{R}{R_0} - 1}{\alpha}$$

Answer 17

$$g_c = v_s^2 - h$$

$$h = v_s^2 - g_c$$

Answer 18

$$C_v = \frac{Q}{\sqrt{\frac{P_1 - P_2}{G_f}}}$$

$$P_1 = \frac{Q^2 G_f}{C_v^2} + P_2$$

$$P_2 = P_1 - \frac{Q^2 G_f}{C_v^2}$$

$$G_f = \frac{(P_1 - P_2) C^2}{Q^2}$$

Answer 19

$$d_1 = \frac{Dr^2}{n\lambda d_2}$$

$$D = \frac{n\lambda d_1 d_2}{r^2}$$

$$\lambda = \frac{Dr^2}{nd_1 d_2}$$

Answer 20

$$C = L(R_L)^{\frac{3}{10}}$$

$$L = \frac{C}{(R_L)^{\frac{3}{10}}}$$

Answer 21

$$d_2 = d_1 N^{0.4}$$

$$d_1 = \frac{d_2}{N^{0.4}}$$

Answer 22

$$D = \left(\frac{PK}{V^3} \right)^{\frac{3}{2}}$$

$$V = \left(\frac{PK}{D^{\frac{2}{3}}} \right)^{\frac{1}{3}}$$

$$K = \frac{D^{\frac{2}{3}} V^3}{P}$$

Answer 23

$$P_1 = \frac{P_2 - P_{vc}F_L^2}{1 - F_L^2}$$

$$P_2 = P_1 - F_L^2(P_1 - P_{vc})$$

$$P_{vc} = P_1 - \frac{P_1 - P_2}{F_L^2}$$

Answer 24

$$P_1 = \sqrt{\frac{Q^2 G_g T}{927369 C_v^2} + P_2^2}$$

$$P_2 = \sqrt{P_1^2 - \frac{Q^2 G_g T}{927369 C_v^2}}$$

$$T = 927369 C_v^2 \frac{P_1^2 - P_2^2}{G_g Q^2}$$

Answer 25

$$R = \frac{VnF}{T \ln \frac{C_1}{C_2}}$$

$$F = \frac{RT}{nV} \ln \frac{C_1}{C_2}$$

$$C_1 = C_2 e^{\frac{VnF}{RT}}$$

$$C_2 = \frac{C_1}{e^{\frac{VnF}{RT}}} \quad \text{or} \quad C_2 = C_1 e^{-\frac{VnF}{RT}}$$

Answer 26

$$D = \frac{\lambda 10^{-\frac{L}{20}}}{4\pi} \quad \dots \text{ or } \dots \quad D = \frac{\lambda}{4\pi 10^{\frac{L}{20}}}$$

$$\lambda = \frac{4\pi D}{10^{-\frac{L}{20}}} \quad \dots \text{ or } \dots \quad \lambda = 4\pi D 10^{\frac{L}{20}}$$

Answer 27

This type of manipulation is perfectly “legal” to do, following the algebraic identity:

$$1a = a$$

Answer 28

$$x = \frac{9y}{6y^2 + 14}$$

Answer 29

$$x = \sqrt{\frac{a}{y+z}}$$

Answer 30

$$F = H \left(\frac{\pi d_1}{2} \right) \left(d_1 - \sqrt{d_1^2 - d_s^2} \right)$$

$$d_s = \sqrt{d_1^2 - \left(d_1 - \frac{2F}{H\pi d_1} \right)^2}$$

Answer 31

$$n_b = \frac{\log n_s}{\log 2}$$

Answer 32

$$x = \frac{\sqrt{13} - 3}{2}$$

Answer 33

Correction:

$$5\frac{a}{b} = \frac{5a}{1b} = \frac{5a}{1b} = \frac{5a}{b}$$

Answer 34

Correction:

$$\frac{(4y)(8)}{(4y)(3)} = \frac{8}{3}$$

Answer 35

A mistake was made between the second and third equations. Here is the correction:

$$5a - 6ac = 1 - 3a^2b$$

$$\frac{5a - 6ac}{a} = \frac{1 - 3a^2b}{a}$$

$$5 - 6c = \frac{1}{a} - 3ab$$

$$3ab + 5 - 6c = \frac{1}{a}$$

Answer 36

Correction:

$$(4x + 2)(2x + 3) = 8x^2 + 16x + 6$$

Answer 37

Correction:

$$\frac{1}{c} + \frac{1}{d} = \frac{d}{cd} + \frac{c}{cd} = \frac{d+c}{cd}$$

Answer 38

Correction:

$$\frac{6}{x+2} - \frac{x-4}{x+2} = \frac{6-x+4}{x+2}$$

Answer 39

$$r = \sqrt{\frac{n\lambda PD(D - PD)}{D}}$$

... or ...

$$r = \sqrt{n\lambda(PD - P)}$$

Answer 40

$$z_1\gamma + \frac{v_1^2\gamma}{2g} + P_1 = z_2\gamma + \frac{v_2^2\gamma}{2g} + P_2$$

$$z_1 + \frac{v_1^2}{2g} + \frac{P_1}{\gamma} = z_2 + \frac{v_2^2}{2g} + \frac{P_2}{\gamma}$$

Answer 41

$$I_{total} = \frac{V_{total}}{R_1} + \frac{V_{total}}{R_2}$$

Here is how we get this answer:

- First, we identify the variable we are trying to solve for; in this case it is I_{total}
- Next we identify which of the given equations contains this variable; in this case, it is $I_{total} = I_1 + I_2$
- Next, we look for ways to substitute V and the R variables for I_1 and I_2 in the first equation
- We know from Ohm's Law that $V = IR$. Therefore, $V_1 = I_1R_1$ and $V_2 = I_2R_2$
- We then substitute $\frac{V_1}{R_1}$ for I_1 , and $\frac{V_2}{R_2}$ for I_2 ; the result is $I_{total} = \frac{V_1}{R_1} + \frac{V_2}{R_2}$
- Using the equality $V_{total} = V_1 = V_2$ for parallel circuits, we now replace V_1 and V_2 with V_{total} to get $I_{total} = \frac{V_{total}}{R_1} + \frac{V_{total}}{R_2}$

Answer 42

$$x = -3 \quad y = 6$$

Answer 43

$$x = -2 \quad y = 3$$