

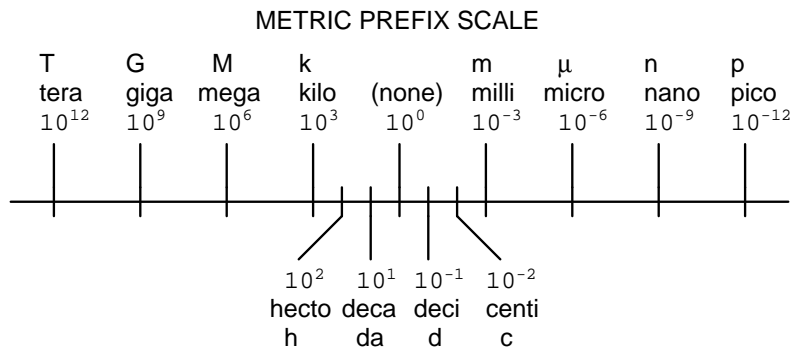
Simple machines

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Metric prefixes and conversion constants

- **Metric prefixes**

- Yotta = 10^{24} Symbol: Y
- Zeta = 10^{21} Symbol: Z
- Exa = 10^{18} Symbol: E
- Peta = 10^{15} Symbol: P
- Tera = 10^{12} Symbol: T
- Giga = 10^9 Symbol: G
- Mega = 10^6 Symbol: M
- Kilo = 10^3 Symbol: k
- Hecto = 10^2 Symbol: h
- Deca = 10^1 Symbol: da
- Deci = 10^{-1} Symbol: d
- Centi = 10^{-2} Symbol: c
- Milli = 10^{-3} Symbol: m
- Micro = 10^{-6} Symbol: μ
- Nano = 10^{-9} Symbol: n
- Pico = 10^{-12} Symbol: p
- Femto = 10^{-15} Symbol: f
- Atto = 10^{-18} Symbol: a
- Zepto = 10^{-21} Symbol: z
- Yocto = 10^{-24} Symbol: y



- **Conversion formulae for temperature**

- $^{\circ}\text{F} = (^{\circ}\text{C})(9/5) + 32$
- $^{\circ}\text{C} = (^{\circ}\text{F} - 32)(5/9)$
- $^{\circ}\text{R} = ^{\circ}\text{F} + 459.67$
- $\text{K} = ^{\circ}\text{C} + 273.15$

Conversion equivalencies for distance

- 1 inch (in) = 2.540000 centimeter (cm)
- 1 foot (ft) = 12 inches (in)
- 1 yard (yd) = 3 feet (ft)
- 1 mile (mi) = 5280 feet (ft)

Conversion equivalencies for volume

1 gallon (gal) = 231.0 cubic inches (in³) = 4 quarts (qt) = 8 pints (pt) = 128 fluid ounces (fl. oz.) = 3.7854 liters (l)

1 milliliter (ml) = 1 cubic centimeter (cm³)

Conversion equivalencies for velocity

1 mile per hour (mi/h) = 88 feet per minute (ft/m) = 1.46667 feet per second (ft/s) = 1.60934 kilometer per hour (km/h) = 0.44704 meter per second (m/s) = 0.868976 knot (knot – international)

Conversion equivalencies for mass

1 pound (lbm) = 0.45359 kilogram (kg) = 0.031081 slugs

Conversion equivalencies for force

1 pound-force (lbf) = 4.44822 newton (N)

Conversion equivalencies for area

1 acre = 43560 square feet (ft²) = 4840 square yards (yd²) = 4046.86 square meters (m²)

Conversion equivalencies for common pressure units (either all gauge or all absolute)

1 pound per square inch (PSI) = 2.03602 inches of mercury (in. Hg) = 27.6799 inches of water (in. W.C.) = 6.894757 kilo-pascals (kPa) = 0.06894757 bar

1 bar = 100 kilo-pascals (kPa) = 14.504 pounds per square inch (PSI)

Conversion equivalencies for absolute pressure units (only)

1 atmosphere (Atm) = 14.7 pounds per square inch absolute (PSIA) = 101.325 kilo-pascals absolute (kPaA) = 1.01325 bar (bar) = 760 millimeters of mercury absolute (mmHgA) = 760 torr (torr)

Conversion equivalencies for energy or work

1 british thermal unit (Btu – “International Table”) = 251.996 calories (cal – “International Table”) = 1055.06 joules (J) = 1055.06 watt-seconds (W-s) = 0.293071 watt-hour (W-hr) = 1.05506 x 10¹⁰ ergs (erg) = 778.169 foot-pound-force (ft-lbf)

Conversion equivalencies for power

1 horsepower (hp – 550 ft-lbf/s) = 745.7 watts (W) = 2544.43 british thermal units per hour (Btu/hr) = 0.0760181 boiler horsepower (hp – boiler)

Acceleration of gravity (free fall), Earth standard

9.806650 meters per second per second (m/s²) = 32.1740 feet per second per second (ft/s²)

Physical constants

Speed of light in a vacuum (c) = 2.9979×10^8 meters per second (m/s) = 186,281 miles per second (mi/s)

Avogadro's number (N_A) = 6.022×10^{23} per mole (mol^{-1})

Electronic charge (e) = 1.602×10^{-19} Coulomb (C)

Boltzmann's constant (k) = 1.38×10^{-23} Joules per Kelvin (J/K)

Stefan-Boltzmann constant (σ) = 5.67×10^{-8} Watts per square meter-Kelvin⁴ ($\text{W}/\text{m}^2 \cdot \text{K}^4$)

Molar gas constant (R) = 8.314 Joules per mole-Kelvin (J/mol-K)

Properties of Water

Freezing point at sea level = $32^\circ\text{F} = 0^\circ\text{C}$

Boiling point at sea level = $212^\circ\text{F} = 100^\circ\text{C}$

Density of water at $4^\circ\text{C} = 1000 \text{ kg}/\text{m}^3 = 1 \text{ g}/\text{cm}^3 = 1 \text{ kg}/\text{liter} = 62.428 \text{ lb}/\text{ft}^3 = 1.94 \text{ slugs}/\text{ft}^3$

Specific heat of water at $14^\circ\text{C} = 1.00002 \text{ calories}/\text{g} \cdot ^\circ\text{C} = 1 \text{ BTU}/\text{lb} \cdot ^\circ\text{F} = 4.1869 \text{ Joules}/\text{g} \cdot ^\circ\text{C}$

Specific heat of ice $\approx 0.5 \text{ calories}/\text{g} \cdot ^\circ\text{C}$

Specific heat of steam $\approx 0.48 \text{ calories}/\text{g} \cdot ^\circ\text{C}$

Absolute viscosity of water at $20^\circ\text{C} = 1.0019 \text{ centipoise (cp)} = 0.0010019 \text{ Pascal-seconds (Pa}\cdot\text{s)}$

Surface tension of water (in contact with air) at $18^\circ\text{C} = 73.05 \text{ dynes}/\text{cm}$

pH of pure water at $25^\circ\text{C} = 7.0$ (*pH scale = 0 to 14*)

Properties of Dry Air at sea level

Density of dry air at 20°C and 760 torr = $1.204 \text{ mg}/\text{cm}^3 = 1.204 \text{ kg}/\text{m}^3 = 0.075 \text{ lb}/\text{ft}^3 = 0.00235 \text{ slugs}/\text{ft}^3$

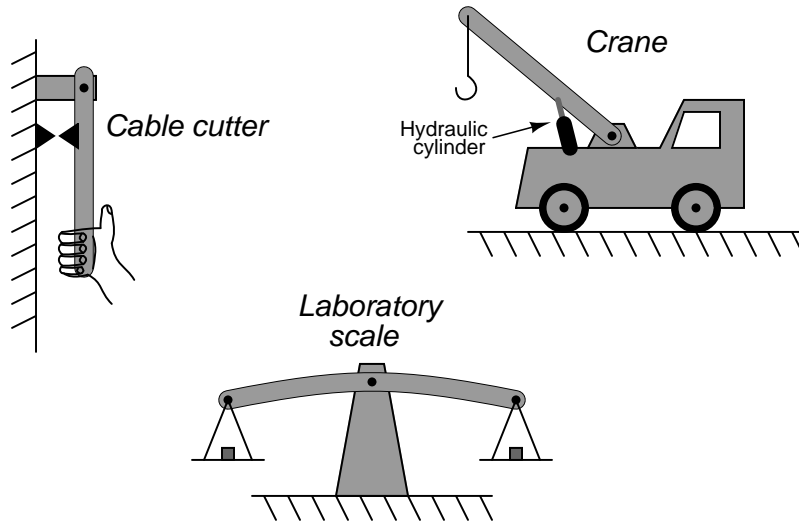
Absolute viscosity of dry air at 20°C and 760 torr = $0.018 \text{ centipoise (cp)} = 1.8 \times 10^{-5} \text{ Pascal-seconds (Pa}\cdot\text{s)}$

file conversion_constants

Questions

Question 1

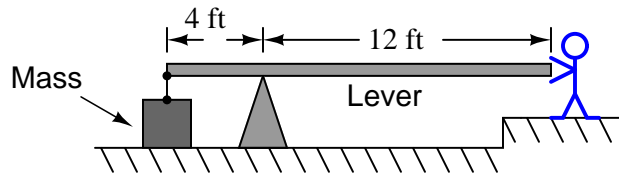
Identify what *class* each of these levers is:



[file i02617](#)

Question 2

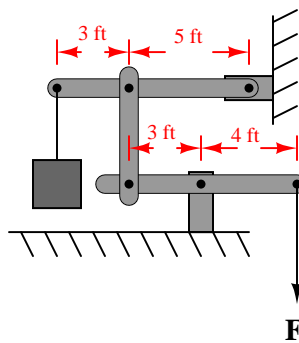
Calculate the *mechanical advantage* of this lever:



[file i02618](#)

Question 3

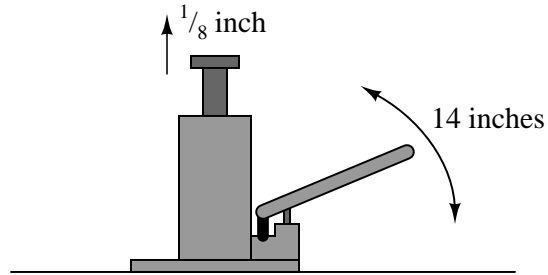
Calculate the mechanical advantage for this lever system:



[file i02619](#)

Question 4

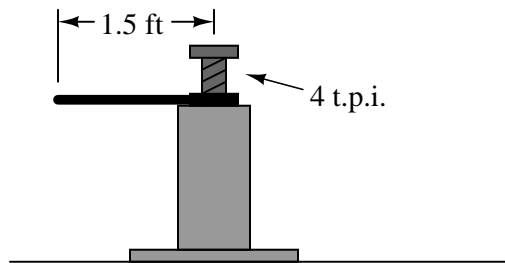
The piston on a hydraulic “bottle” jack lifts up $\frac{1}{8}$ of an inch for every stroke of the lever. In each lever stroke, the handle end moves 14 inches. Calculate the *mechanical advantage* of this hydraulic jack:



file i02625

Question 5

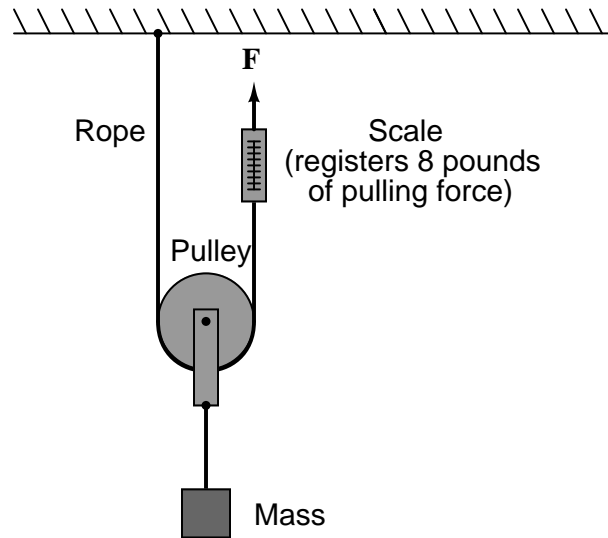
A screw jack has a thread pitch of 4 threads per inch, and is turned by a handle 1.5 feet long (measured from the screw center to the handle's end). Calculate the *mechanical advantage* of this screw jack:



file i02626

Question 6

How much tension is there in the rope where it attaches to the ceiling?



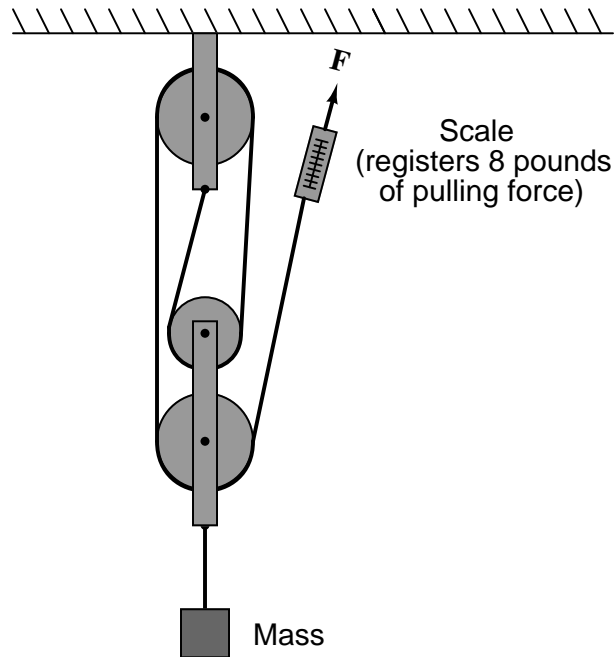
Also, calculate how much this suspended object weighs in units of “pounds”, and how much mass it has in units of “slugs”.

Finally, calculate the mechanical advantage of this pulley system.

file i02627

Question 7

If the pulling force exerted on the end of the rope in this pulley system is 8 pounds, how much upward pulling force is exerted on the mass by the lower pulley?

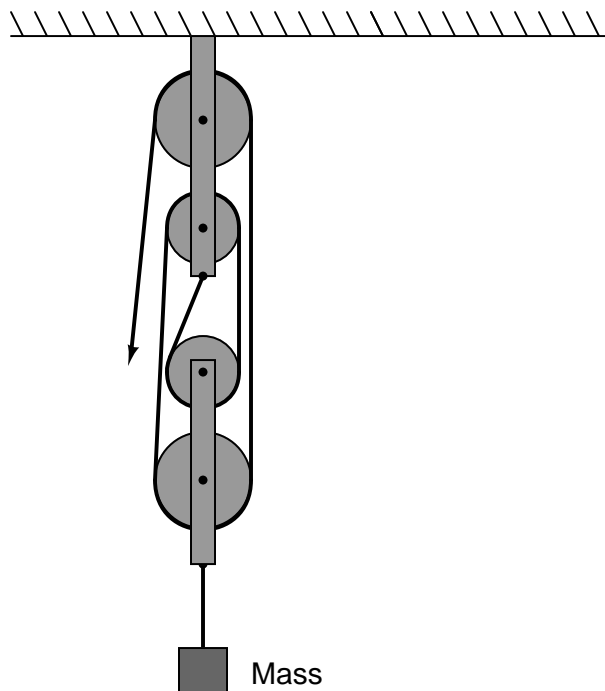


Calculate the amount of work done in lifting the mass 3 feet while pulling on the rope with 8 pounds of force.

[file i02628](#)

Question 8

If the free end of the rope is pulled a distance of 10 cm, how far will the mass be lifted?



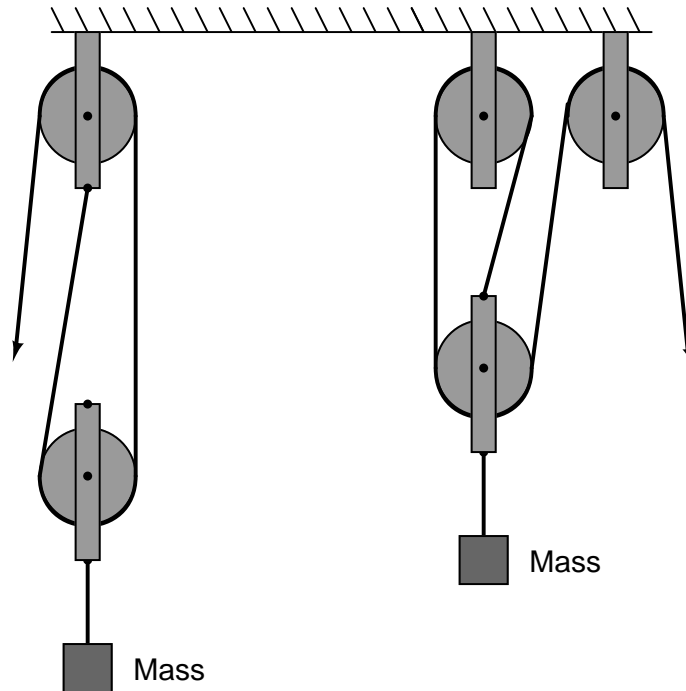
Assuming the mass is 55 kg, how much work is done while pulling the rope 10 cm?
[file i02629](#)

Question 9

Determine a way to arrange pulleys to provide a 3:1 mechanical advantage.
[file i02630](#)

Question 10

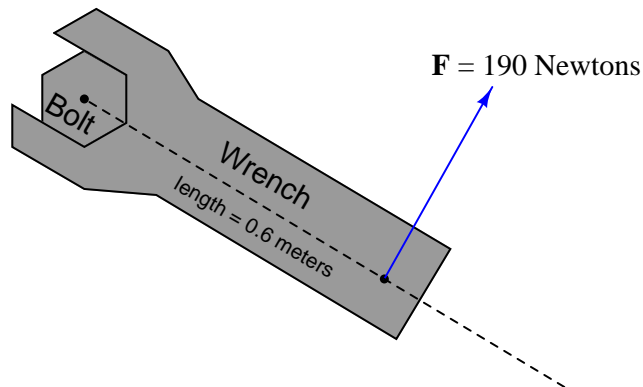
Calculate the mechanical advantage (M_A) for each of these pulley systems:



[file i02824](#)

Question 11

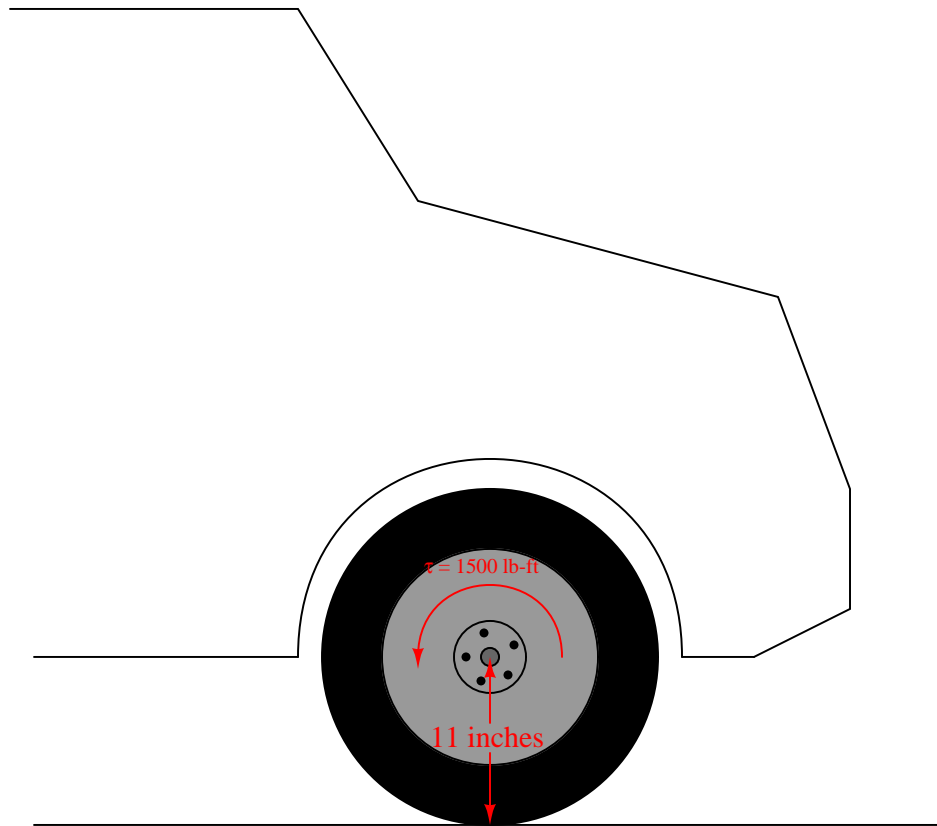
Suppose a mechanic pulls perpendicularly at the end of a wrench 0.6 meters in length with a steady force of 190 newtons for two complete revolutions. Calculate the amount of work done by the mechanic (in newton-meters or joules), and also calculate his power output in watts if those two turns were completed in 6 seconds.



[file i03777](#)

Question 12

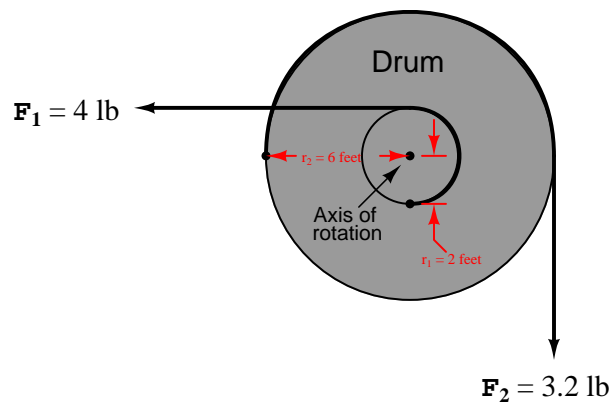
How much linear force will the car's tire exert on the ground if the axle exerts a torque of 1500 lb-ft on the wheel, and the tire's radius is 11 inches?



file i01402

Question 13

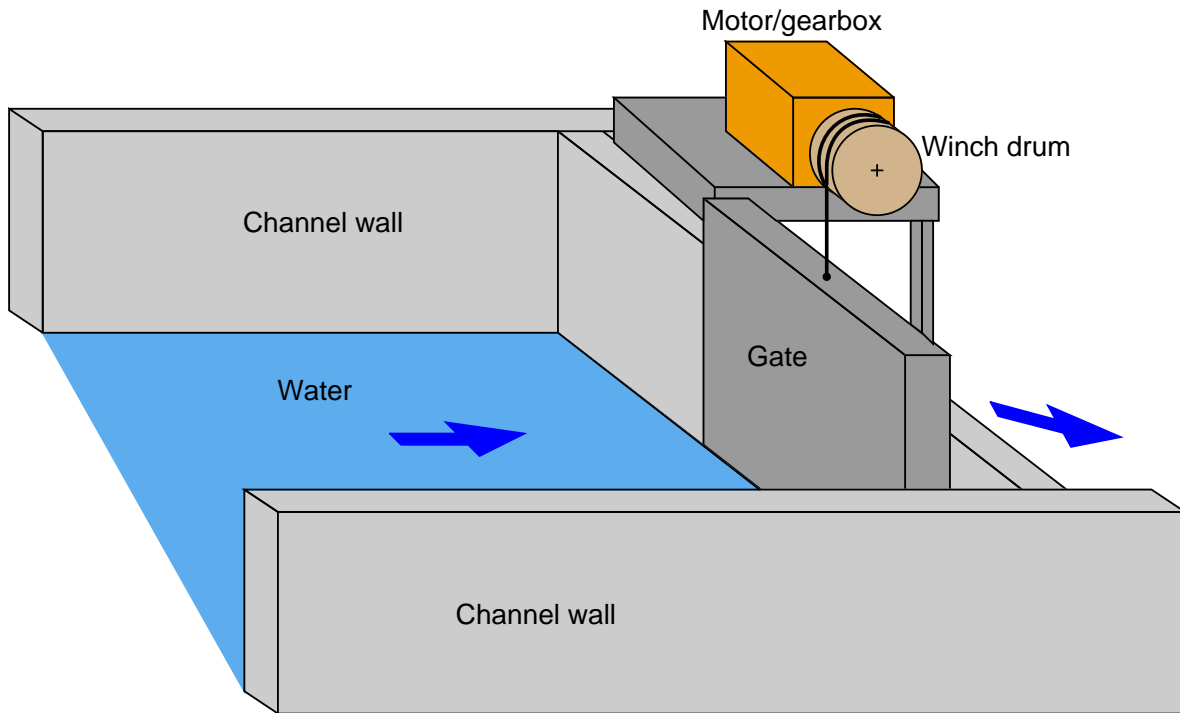
Calculate the net torque applied to the drum from the two forces shown. The drum's outside radius is 6 feet, and the radius of the smaller pulley (attached to the drum) is 2 feet:



Also, calculate the mechanical advantage of this system, if F_1 is considered the *input* force.
[file i01428](#)

Question 14

Suppose an electric actuator is used to lift a large concrete gate in an irrigation water flow control facility. The gate effectively acts as a control valve for water flowing through an open irrigation channel, and a powerful winch is necessary to control its position:



The winch drum measures 20 inches in diameter, and the concrete gate weighs 12,740 pounds. Calculate the torque required at the drum to lift the gate, and also the torque required by the electric motor given a gearbox speed-reduction ratio of 1200:1.

Assuming the electric motor powering this speed-reducing gearbox spins at 1720 RPM (at full load), calculate the vertical lifting speed of the gate in feet per minute. Finally, calculate the horsepower output of the electric motor lifting this much weight (12,740 pounds) at this vertical speed.

Suggestions for Socratic discussion

- Like all “story problems” involving mathematical calculation, the most important aspect of your answer is *how* you arrived at it, not the numerical value(s) of your answer. Explain how you were able to set up the proper equations to solve for drum torque, motor torque, lifting speed, and motor output power.
- A useful problem-solving technique is to sketch a simple diagram of the system you are asked to analyze. This is useful even when you already have some graphical representation of the problem given to you, as a simple sketch often reduces the complexity of the problem so that you can solve it more easily. Draw your own sketch showing how the given information in this problem inter-relates, and use this sketch to explain your solution.

[file i00584](#)

Question 15

The General Electric “Frame 6” gas turbine engine is a popular choice for natural gas powered electrical generators in the United States, with an output power of approximately 42 megawatts. The turbine itself spins at 5163 RPM, but the 2-pole AC generator must spin at a different speed in order to generate 60 Hz electrical power.

Calculate the gear ratio necessary to couple the turbine’s shaft to the generator’s shaft, and identify whether this gear set needs to *reduce* speed or *increase* speed.

[file i04790](#)

Question 16

The Siemens model SGT-800 gas turbine engine has an output power of approximately 47 megawatts. The turbine itself spins at 6608 RPM, but the 2-pole AC generator must spin at a different speed in order to generate 50 Hz electrical power for European power markets.

Calculate the gear ratio necessary to couple the turbine’s shaft to the generator’s shaft, and identify whether this gear set needs to *reduce* speed or *increase* speed.

[file i04789](#)

Question 17

Roy has the meanest pulling tractor in his county: its engine outputs a maximum torque of 1200 lb-ft, and the total geartrain (transmission combined with rear axle differential gearing) has a 12:1 reduction ratio in the lowest gear. With 5.5 foot tall tires, how much horizontal pulling force can this tractor (theoretically) exert?

If Roy’s tractor drags a weight 300 feet along the ground while pulling at maximum engine torque, how much work was done by the tractor?

Rate the horsepower of Roy’s tractor if it took exactly 1 minute to drag that weight 300 feet along the ground.

When Roy goes to the county fair to compete in the tractor-pull contest, he notices that the front end of the tractor tends to raise up off the ground when pulling a heavy load. Explain to Roy why this happens.

[file i01429](#)

Question 18

Calculate the RPM of the engine in a truck as it drives down the highway at 70 MPH with 30-inch tall wheels and an axle gear speed reduction ratio of 3.55:1, in top gear (a transmission ratio of 1:1).

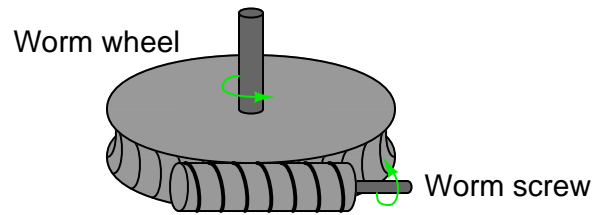
Next, calculate the engine’s RPM at the same highway speed if the truck is equipped with an *overdrive* gear in the transmission (turning the axle driveshaft faster than the engine is turning), having an effective ratio of 0.7:1.

[file i04791](#)

Question 19

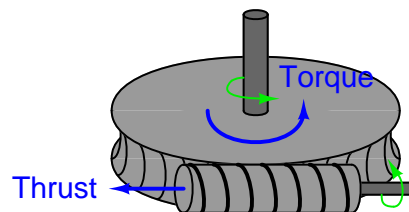
Electric motors usually rotate at too high of speed to be used directly as valve actuators. Nearly all electric valve actuators use gear mechanisms to reduce the speed of the electric motor (and multiply its torque). One of the more popular gear mechanisms for achieving great speed reduction (and torque multiplication) is called the *worm gear*:

Worm gear mechanism



The worm wheel's teeth match the pitch of the threads on the worm screw, allowing the two pieces to mesh like gears. It should be evident from inspection that it takes many, many turns of the worm screw to obtain one revolution of the worm wheel. In electric valve actuators, the motor couples to the worm screw and the wheel turns the valve mechanism.

What might not be so evident is how torque on the worm wheel directly translates to linear thrust on the worm screw. In other words, the more twisting force output by the worm wheel, the greater the straight-line force experienced by the screw:



If we can find a way to measure this linear thrust on the worm screw, we may infer the torque output by the wheel. Explain how this could be done in an electric valve actuator mechanism.

[file i01390](#)

Answers

Answer 1

- 1st class = *laboratory scale*
- 2nd class = *cable cutter*
- 3rd class = *crane*

Answer 2

For any lever – regardless of class – the mechanical advantage (M_A) is always calculated as the ratio of radii between the input and output points on the lever (i.e. the ratio of lengths – also called “moment arms” – between each point and the fulcrum).

In this case, since the ratio of moment arms lengths is 12:4 (3:1), the mechanical advantage will be 3:1 as well. The person only needs to exert 1/3 the weight of the mass on the right-hand end of the lever in order to lift the mass, but the *motion* at that end of the lever will be three times as much as the motion of the mass.

Answer 3

The first lever (where force \mathbf{F} works on the right-hand end) is a first-class, with a mechanical advantage of 4:3. It connects to a second lever (third-class), with an mechanical advantage ratio (actually, a *disadvantage* ratio) of 5:8. The overall mechanical advantage is the product of these two advantage ratios:

$$M_A = \left(\frac{4}{3}\right) \left(\frac{5}{8}\right) = \frac{5}{6} = 0.83333$$

In other words, for every pound of force applied at \mathbf{F} , there will be 0.8333 pounds of force available to move the mass.

Answer 4

The mechanical advantage of any machine may be empirically determined by dividing input displacement by output displacement:

$$M_A = \frac{s_{in}}{s_{out}}$$

In this case, an input displacement of 14 inches yields an output displacement of 0.125 inches, so:

$$M_A = \frac{14 \text{ in}}{0.125 \text{ in}} = 112$$

This means the handle tip moves 112 times farther than the jack’s lifting piston, but the lifting piston exerts 112 times more force than it takes to move the handle.

Answer 5

For each turn of the screw (or nut), the jack will lift $1/4$ of an inch. We know this because there are four threads per inch, which means four complete turns are required to lift one inch. Now all we need to do is calculate how far the tip of the handle travels in one turn, and we have the necessary data to calculate mechanical advantage ($M_A = \frac{s_{in}}{s_{out}}$).

Given a radius of 1.5 feet (18 inches), the circumference of the circle described by one full rotation of the handle will be 113.1 inches according to the formula $C = \pi D = 2\pi r$. Thus, with an input displacement of 113.1 inches and an output displacement of 0.25 inch, the mechanical advantage must be:

$$M_A = \frac{113.1 \text{ in}}{0.25 \text{ in}} = 452.4$$

This means the handle tip moves 452.4 times farther than the jack's lifting screw, but the lifting screw exerts 452.4 times more force than it takes to move the handle.

Answer 6

Since the scale on the right-hand end of the rope registers a tension of 8 pounds, the tension at the other end of the rope must be 8 pounds as well, not counting any friction in the pulley. All the pulley does is *redirect* the force pulling on the rope.

Since the object is being supported by the tension in *two* rope lengths, its weight must be twice the tension:

$$W = (2)(8 \text{ lb}) = 16 \text{ lb}$$

The relationship between mass and weight is the basic $F = ma$ formula where force F is the weight of the object, m is its mass, and a is the acceleration of gravity:

$$m = \frac{F}{a} = \frac{16 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.4969 \text{ slugs}$$

The mechanical advantage of any machine is (ideally) the ratio between output force and input force. Since in this case the output force is the 16 lb weight and the input force is the 8 lb rope tension, the calculation looks like this:

$$M_A = \frac{F_{out}}{F_{in}}$$
$$M_A = \frac{16 \text{ lb}}{8 \text{ lb}} = 2$$

In other words, for every pound of force applied on the rope, there will be 2 pounds of force available to move the mass.

Answer 7

With 8 pounds of tension in the cable, and *four* cables pulling upward on the lower pulley assembly, the total upward force exerted on the mass will be four times the tension, or 32 pounds.

With a mechanical advantage of 4:1, the mass moves $\frac{1}{4}$ the distance that the rope is pulled. So, if the mass moves 3 feet, the rope must be pulled 12 feet. This gives the following values for work done (calculated either at the mass or at the rope's end, yields the same result):

$$W = Fx = (8 \text{ lb})(12 \text{ ft}) = 96 \text{ ft}\cdot\text{lb} \quad (\text{calculated at rope's end})$$

$$W = Fx = (32 \text{ lb})(3 \text{ ft}) = 96 \text{ ft}\cdot\text{lb} \quad (\text{calculated at mass})$$

Answer 8

Seeing that the lower pulley assembly is supported by *four* lengths of rope, the mechanical advantage in this system must be 4:1. Thus, the output displacement will be four times less than the input displacement:

$$M_A = \frac{x_{in}}{x_{out}}$$

$$x_{out} = \frac{x_{in}}{M_A}$$

$$x_{out} = \frac{10 \text{ cm}}{4} = 2.5 \text{ cm}$$

Expressing the same result in meters instead of centimeters:

$$x_{out} = \frac{0.1 \text{ m}}{4} = 0.025 \text{ m}$$

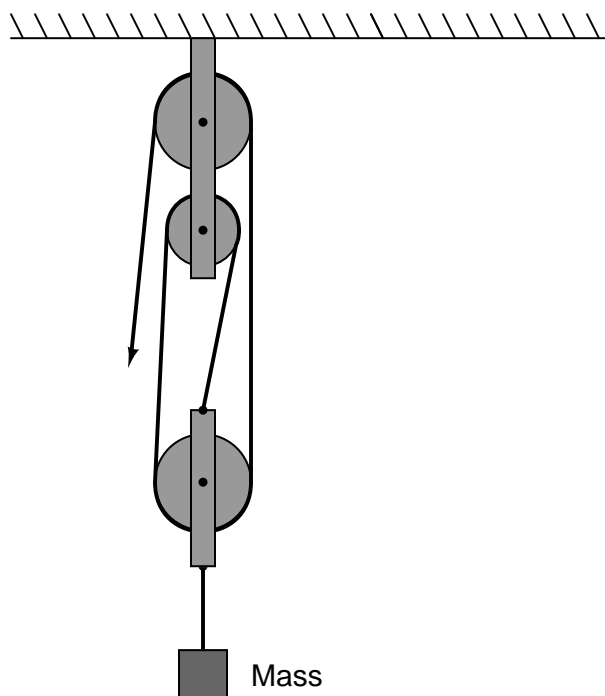
A mass of 55 kg weighs 539.99 newtons in Earth's gravity ($F = ma$, where F is the force that gravity exerts on the mass, m is the amount of mass, and a is the acceleration of Earth gravity: 9.81 meters per second squared). This is the amount of force exerted upward on the mass by the pulley system. Given our mechanical advantage of 4:1, it means the rope's tension at the pulled end must be $\frac{1}{4}$ this value, or 134.89 newtons.

We may calculate the amount of work done at the mass or at the rope's end. Either way, we will get the exact same result:

$$W = Fx = (134.89 \text{ N})(0.1 \text{ m}) = 13.489 \text{ N}\cdot\text{m} \quad (\text{calculated at rope's end})$$

$$W = Fx = (539.55 \text{ N})(0.025 \text{ m}) = 13.489 \text{ N}\cdot\text{m} \quad (\text{calculated at mass})$$

Answer 9



Answer 10

$$M_A = 2:1 \text{ (on left)} \quad M_A = 3:1 \text{ (on right)}$$

Answer 11

The mechanic's work is 1432 Newton-meters, or 1432 Joules. The mechanic's average power output during the 6 seconds is 238.7 watts.

Answer 12

$$F = 1636.36 \text{ lb}$$

To solve for force, we simply need to manipulate the torque equation so that force (F) is by itself on one side of the equality sign:

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\vec{F} = \frac{\vec{\tau}}{\vec{r}}$$

Since we happen to know in this problem that all three vectors are orthogonal (perpendicular) to each other, we may re-write the equation in simpler terms of scalar quantities instead of vector quantities:

$$F = \frac{\tau}{r}$$

Before we may insert the given values for torque and moment arm length, we need to convert units of length for the moment arm:

$$(11 \text{ inches})(1 \text{ foot} / 12 \text{ inches}) = 0.916667 \text{ feet}$$

Now, solving for force:

$$F = \frac{1500 \text{ lb-ft}}{0.916667 \text{ ft}}$$

$$F = 1636.36 \text{ lb}$$

Answer 13

$$\tau_{net} = 11.2 \text{ lb-ft, clockwise}$$

$$M_A = \frac{F_{out}}{F_{in}} = \frac{3.2 \text{ lb}}{4 \text{ lb}} = 0.8$$

Answer 14

Partial answer:

$$\tau_{motor} = 8.8472 \text{ lb-ft}$$

$$\text{Motor output} = 2.897 \text{ horsepower}$$

Answer 15

A 60 Hz, 2-pole generator must spin at 3600 RPM exactly (60 revolutions per second). We therefore require a *reduction* gear set between the turbine and the generator with a gear ratio of 1.434:1 ($\frac{5163}{3600}$).

Answer 16

A 50 Hz, 2-pole generator must spin at 3000 RPM exactly (50 revolutions per second). We therefore require a *reduction* gear set between the turbine and the generator with a gear ratio of 2.203:1 ($\frac{6608}{3000}$).

Answer 17

Maximum pulling force = 5236.36 pounds

$$W = Fx = (5236.36 \text{ lb})(300 \text{ ft}) = 1570909.1 \text{ ft-lb}$$

1 horsepower is 550 ft-lb of work done per second. If Roy's tractor did 1,570,909.1 ft-lb of work in 60 seconds, it is equivalent to 26,181.8 ft-lb/s of power, which is 47.6 horsepower.

As the tractor mechanism exerts torque on the wheels, and the weight of the load opposes the wheels' turning, the tractor experiences this torque about the axis of rotation: the axles. As the wheels *try* to rotate in a forward direction, but are impeded by the resistance of the load, the reaction torque *tries to rotate the tractor backward about the same axis*. This manifests itself in the form of the front tires of the tractor lifting off the ground.

Answer 18

Each revolution of the truck's 30-inch diameter wheels is equivalent to 30π inches of linear motion along the highway. Converting 70 miles per hour into inches per minute, and then from linear inches per minute into revolutions per minute, gives us the rotational speed of the wheels:

$$\left(\frac{70 \text{ miles}}{\text{hour}}\right) \left(\frac{5280 \text{ feet}}{1 \text{ mile}}\right) \left(\frac{12 \text{ inches}}{1 \text{ foot}}\right) \left(\frac{1 \text{ rev}}{30\pi \text{ inches}}\right) \left(\frac{1 \text{ hour}}{60 \text{ minutes}}\right) = 784.3 \text{ RPM}$$

Since the axle's gear ratio is a speed *reducer* from driveshaft to wheels, it acts as a speed *increaser* from wheels to driveshaft. Thus, with a ratio of 3.55:1, the driveshaft will spin at a speed of 2784.3 RPM.

With a transmission ratio of 1:1, this means the engine will also spin at 2784.3 RPM.

With an overdrive transmission ratio of 0.7:1, this means the engine will also spin at 1949.0 RPM.

Answer 19

One way to measure worm screw thrust force is with a *load cell*. Another way is to spring-load the screw shaft and use an LVDT or other motion-sensing device to measure displacement.