r_0 (Å)	R_{DD} ($P_n = 1$)	R_{pD} ($P_n = 10^{-6}$)	R_{pp} ($P_n = 10^{-23}$)
0.5	10^{-64}	10^{-55}	10^{-63}
0.25	10^{-40}	10^{-36}	10^{-46}
0.125	10^{-23}	10^{-22}	10^{-35}
0.1	10^{-19}	10^{-19}	10^{-32}

TABLE I

Fusion rates (per second) versus the width of a "generic" fusion barrier r_0 calculated from eq (11) for D+D, p+D and p+p reactions. Here P_n is the nuclear reaction probability once the nuclei have reached a separation of 5 Fm. [These are estimated from the S values in table II.]

			Kinematic Enhancement		Relative Rate	
Reaction	Mass	S	Cold	Hot	Cold	Hot
$D + D \rightarrow {}^3\mathrm{He} + n$	1	5.2×10^{-2}	1	1	1	1
$p + D \rightarrow {}^3\mathrm{He} + \gamma$	2/3	2.5 x 10^{-7}	2 x 10^7	6	100	10^{-5}
$D + T \rightarrow {}^4\mathrm{He} + n$	6/5	11	10^{-4}	.4	10^{-2}	100
$p + p \rightarrow D + e^+ + \nu$	1/2	4 x 10^{-25}	6 x 10^{11}	18	10^{-12}	10^{-22}

Table II

Relative fusion rates compared to the $D + D \rightarrow {}^3\mathrm{He} + n$ reaction. The S factors are in Mev-b while the kinematic factors describe the increase in the tunneling exponential in eq (11) for systems with a lighter reduced mass. The hot fusion numbers assume an energy of 10 keV which corresponds to a barrier width of r_0=144 Fm.

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example, hydrogen is expected to have a larger zero point motion then deuterium because of its smaller mass and Fermi statistics.

Can cold fusion be qualitatively different from hot fusion? One experiment claimed to see heat corresponding to a fusion rate of ten to the thirteen per second [2]. Furthermore, the observed neutron flux was some nine orders of magnitude too small for the heat to be from the $D + D \rightarrow {}^3\mathrm{He} + n$ reaction. The authors claim that the heat is from unknown nuclear reactions which are very different from those in hot fusion.

Clearly, one possibility is that the heat is from the p+D reaction which can not produce neutrons. Thus one should control even trace amounts of hydrogen in the apparatus and perform experiments with different H to D ratios. It is possible that the $p + D$ reaction rate is even higher then that estimated in table II because of a previously unmeasured pair production branch,

$$p + D \rightarrow {}^3\mathrm{He} + e^+ + e^-. \tag{14}$$

Pair production can compete with the ^{3}He+γ reaction because the gamma emission is highly suppressed.

The cross section for $D + D \rightarrow {}^4\mathrm{He} + \gamma$ is very small because of the 1^+ spin and parity of the deuteron. Given the 0^+ ^{4}He and two D in a relative s state one needs an electric quadrupole (2^+) photon. Therefore, the rate is lower then in an electric dipole (1^-) transition. Furthermore, since the photon couples most strongly to the nuclear orbital motions the reaction probably proceeds through a small d state admixture in either a D or the ^{4}He [7]. The pair production reaction could proceed through a virtual 0^+ Coulomb monopole photon and will not suffer either suppression.

The situation for the $p + D \rightarrow {}^3\mathrm{He} + \gamma$ reaction is very similar because of the $1/2^+$ spins and parities of the p and ^{3}He. Thus, pair production may be important for this reaction also. Indeed muon capture (a similar process) is significant in muon induced p+D fusion. About seven percent of the time a 5.4 MeV muon is ejected [8].

In conclusion, we have examined cold fusion in metallic hydrogen and other materials. Fusion is not expected to be important in the metallic hydrogen core of Jupiter. The fusion rate is very sensitive to the width of the fusion barrier. With a conventional mechanism, there should not be a substantial fusion rate until the barrier width has been reduced to about 0.125 Å. Because cold fusion involves a large tunneling exponential it is very sensitive to the nuclear reduced mass. This will favor the p+D and hinder the D+T reaction. Finally, we speculate that reactions involving pair production could be important.

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Supported in part by Department of Energy Contract DF-FG02- 87ER40365.

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Here, r_0 is the width of the fusion barrier to the classical turning point. The vibrational frequency is estimated to be about that of an isolated H_2 molecule, $h\nu \approx 0.5$ eV. Rates from eq (11) are collected in table I for different isotopes. The tunneling greatly prefers a smaller reduced mass. Indeed the weak interaction process $p+p \rightarrow D+e^+ +\nu$ has a larger rate (at large r_0) then $D+D$ fusion despite its very small reaction probability $P_n \approx 10^{-23}$ (ten to the minus twenty three).

If hydrogen is dissolved in a normal metal such as Palladium the width of the fusion barrier r_0 can be reduced both by forcing the equilibrium position of the atoms closer together and through electron screening of the repulsive coulomb interaction. Note, the fusion rate is very sensitive to r_0. Using eq (11) we estimate that the scale of the fusion barrier must change by about a factor of five (compared to $r_0 \approx$0.7 Å in H_2) until,

$$r_0 \approx 0.125\text{Å}, \tag{12}$$

in order for the fusion rate to be near the claimed ten to the minus twenty three (10^{-23}) per second [1]. This factor of five is much smaller then the factor of 200 in muonic fusion. Nevertheless, it is a major change. It is not at all clear how to obtain an electronic configuration with such a small length scale. Thus the fusion observations are surprising and must be carefully confirmed.

If the experiments are confirmed and this length scale can be achieved (perhaps by a combination of effects including a large effective electron mass [6]) then the relative rates of various reactions will be quite different for cold compared to thermonuclear fusion. This is because the much larger tunneling exponential in cold fusion is extremely sensitive to the reduced mass. In table II we evaluate the ratio of the exponential factors, eq (5), for various reactions compared to the D+D reaction. This enhancement factor depends only on the reduced mass. [We assume the same fusion barrier for all reactions.] As an example we consider cold fusion with $r_0 = 0.125$ Å and thermonuclear (hot) fusion with $r_0 = 144$ Fm (which corresponds to an energy of 10 keV). The relative reaction rate (per isotope pair) is the product of this enhancement factor and the ratio of the cross section factors S for the basic nuclear reactions. The cross section at an energy E, $\sigma(E)$, is commonly expressed in terms of an S factor.

$$\sigma = \frac{S}{E}\exp(-e^2\pi(2M)^{1/2}/E^{1/2}) \tag{13}$$

We collect the experimental S factors in table II. For hot fusion the larger S factor suggests that the $D+T$ reaction will dominate. However, for cold fusion the smaller reduced mass will favor the $p+D$ reaction even though this has a small S factor. Clearly, more attention should be focused on the $p+D \rightarrow {}^3\text{He}+\gamma$ reaction.

It is interesting to note that the small mass favors the weak $p+p$ reaction by twelve orders of magnitude. If a cold fusion "reactor" could be set up with a large fusion rate then even this weak interaction might be observable at a rate only some ten to the minus twelve of the $D+D$ reaction.

It is important to emphasize that table II assumes the same solubility, chemical environment, etc. for the different hydrogen isotopes. However, these could be different. For

a nuclear reaction P_n (once the nuclei have made it to r_n). The probability P_n is about 0.1 to 1 for a strong interaction process such as $D + D \rightarrow {}^3\mathrm{He} + n$. However, P_n is about ten to the minus six (10^{-6}) for the electromagnetic reaction $D + p \rightarrow {}^3\mathrm{He} + \gamma$. (See the discussion of S factors below).

$$R = \nu P_n \Big| \frac{k(r_n)}{k(r_e)} \Big| \exp(-\alpha) \tag{7}$$

This equation will serve as our "generic" estimate of a fusion rate in the remainder of this paper.

The Born Oppenheimer potential energy surface V_{eff} includes the interaction of the nuclei with all of their neighbors. However, because the screening length is short, the total potential energy surface for two nuclei as the move together in the crystal is essentially just the interaction, eq (1), between the fusing nuclei, $V_{eff}(r) \approx V(r)$. It is now a simple matter to evaluate eqs (1, 6 and 7). For the p + D$\rightarrow$ ${}^3\mathrm{He} + \gamma$ reaction using $P_n = 10^{-6}$ (ten to the minus six) we get,

$$R_{pD} \approx 10^{-54}\,(\text{ten to the minus fifty four})\,\mathrm{sec}^{-1}, \tag{8}$$

per H, deuterium pair. *This fusion rate is much too small to contribute to the planets heating.* The rate for the D+D$\rightarrow$ ${}^3\mathrm{He} + n$ reaction (using $P_n = 1$) is,

$$R_{DD} \approx 10^{-63}\,(\text{ten to the minus sixty three})\,\mathrm{sec}^{-1}, \tag{9}$$

even smaller because of the larger reduced mass in eq (6).

Increasing the density beyond n=3.15 $\mathrm{\AA}^{-3}$ will increase the rate. However, we note the small 1/6 exponent in eq (2). For example at n=25 $\mathrm{\AA}^{-3}$ we estimate $R_{pD} \approx 10^{-35}$ and $R_{DD} \approx 10^{-39}$ per second. At this density the pressure is about ten to the third (10^3) Mbar.

We have used a simple Fermi gas model for metallic hydrogen. However, in the limit of very high density this model is expected to become increasingly valid. We conclude that fusion is unlikely to be important in Jupiter. In a latter paper we will present fusion rates at higher densities for other astrophysical objects such as white dwarfs.

We turn now to fusion of deuterium dissolved in Palladium. The interaction V_{eff} is assumed to be the Born Oppenheimer potential energy function for this complicated system. However, we will be interested in V_{eff} primarily at short distances where it can be approximated,

$$V_{eff}(r) \approx \frac{e^2}{r} - Const. \tag{10}$$

For example in H_2 at small distances, the electronic energy is very close to that of an isolated He atom (E=-79.0eV). Hence, the constant would be just the difference between this and the 27.2 eV binding energy of two H atoms, (Const.=-51.8 eV). Replacing eq (10) with the full potential energy surface will not change our results very much at short distances. Using eq. (10) the integral in eq (6) can be easily evaluated. This gives for the fusion rate,

$$R \approx \nu P_n \Big| \frac{k(r_n)}{k(r_e)} \Big| \exp\Big\{ -\pi \Big[\frac{2M}{m_e} \frac{r_0}{a_0} \Big]^{1/2} \Big\}. \tag{11}$$

The density dependent Thomas Fermi screening length λ is,

$$\lambda(n) = (\frac{\pi a_0}{4k_F})^{1/2} = \Big[(\frac{3}{\pi})^{1/3} 4e^2 m_e\Big]^{-1/2} n^{-1/6}. \tag{2}$$

Here a_0 is the Bohr radius, m_e the electron mass and k_F is the electron Fermi wave-number which is related to the electron density $n = k_F^3/(3\pi^2)$. We note that λ decreases only as the one sixth power of the density. More sophisticated electron screening calculations may modify eq (1) somewhat at large distances. However, eqs (1-2) are expected to be qualitatively correct at short distances and it is the short distance behavior of V(r) that will be important for fusion rate calculations.

As an example, we consider a density of $n = 3.15$ Å^{-3} which corresponds to a density parameter r_s ($n^{-1} = \frac{4}{3}\pi a_0^3 r_s^3$) of 0.8. At this density, the pressure is estimated to be 73 Mbar [4]. This compares to the roughly 60 Mbar pressure expected at the center of Jupiter [5]. The screening length is

$$\lambda = 0.30\text{Å}, \tag{3}$$

which is shorter then the inter-particle spacing of ≈0.68 Å. Therefore electron screening reduces the width of the Coulomb barrier substantially and this should increase the fusion rate.

We will need the vibrational frequency ν of the crystal's zero point motion. This is easily estimated from the classical energy of a crystal lattice using the two body interaction in eq (1). This gives,

$$h\nu \approx 1eV, \tag{4}$$

which agrees well with the frequency estimated from the Lindemann ratio (of the amplitude of zero point motion to the inter-particle spacing) calculated in ref [4]. The frequency is relatively low because the nuclei are weakly interacting given that the screening length is smaller then the average separation.

It is now a simple matter to make a WKB estimate of the fusion rate. The ratio of the square of the wave function $\psi^*\psi$ at some small distance $r_n \approx 5$ Fm compared to $\psi^*\psi$ at the classical turning point $r_0 \approx 0.68$ Å is,

$$P = \Big|\frac{k(r_n)}{k(r_e)}\Big| \exp[-\alpha(r_n, r_0)]. \tag{5}$$

Here the local wave vector is $k(r) = [2M(V_{eff}(r) - E)]^{1/2}$, M is the reduced mass of the two nuclei and the tunneling exponential is

$$\alpha(r_n, r_0) = 2\int_{r_n}^{r_0} dr' [2M(V_{eff}(r) - E)]^{1/2}. \tag{6}$$

In eq (5) we have approximated the WKB connection of the wave function across the classical turning point by simply evaluating $k(r_e)$ at the equilibrium distance r_e. Thus $k(r_e)$ is the wave number of the zero point oscillation ($\hbar^2 k(r_e)^2/2m_e = h\nu$).

The fusion rate R is calculated by multiplying P in eq (5) by the frequency of attacks on the Coulomb barrier (which is just the vibrational frequency ν) and the probability of

Cold Nuclear Fusion in Metallic Hydrogen and Normal Metals

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Submitted to Physical Review C

ABSTRACT

The rate of nuclear fusion from tunnelling in very dense metallic hydrogen in the core of Jupiter is calculated to be ten to the minus fifty four (10^{-54}) per hydrogen-deuterium pair per second. It is estimated that the width of the fusion barrier for deuterium in Palladium or a similar metal must be reduced to, of order, 0.125 Angstroms for the fusion rate to be ten to the minus twenty three (10^{-23}) per deuterium per second. If this scale is achieved, the ratios of various nuclear reaction rates will be very different for cold versus thermonuclear fusion.

There is great interest in cold nuclear fusion induced by quantum mechanical tunnelling of the zero point motion of Deuterium in a solid. Recently, two groups have claimed to observe neutrons [1] and possibly heat [2] from such reactions. Furthermore, there could be geophysical and astrophysical implications of cold fusion. For example, cold fusion in the dense metallic hydrogen core of Jupiter could contribute to the planets heating [1].

We first consider fusion in metallic hydrogen because of its very simple structure. In addition to direct applications to Jupiter, our results may provide insight for fusion in more complicated materials. Next, we estimate the change in scale of the fusion barrier required in a normal metal such as Palladium to produce the claimed fusion rate of ten to the minus twenty three (10^{-23}) fusions per deuterium per second [1].

Cold fusion, if it can be achieved, will involve a much larger tunneling exponential then in conventional thermonuclear fusion. This will strongly favor reactions with light reduced masses such as deuterium plus proton going to ^{3}He plus γ. We will discuss the very different ratios of reaction rates expected for cold versus thermonuclear fusion.

In the limit of very high density, the electrons in metallic hydrogen should become a Fermi gas. Therefore, we model metallic hydrogen as a Fermi gas of electrons and a crystal of nuclei interacting via screened coulomb potentials. [Our results are not expected to be qualitatively changed if the protons are in a liquid phase.] The effective potential between two nuclei $V(r)$ which includes the effects of electron screening is given, in a simple Thomas Fermi model [3], by

$$V(r) = \frac{e^2}{r}\exp[-\frac{r}{\lambda(n)}]. \quad (1)$$