Table 1: Enhancement factors and most efective configurations

for the truncated Coulomb potential, Eq. (8).

	$\sigma = 0.1$	0.2	0.3	0.4	0.5	0.6	0.7
p + d							
$\log_{10}$ E	6.8	13.9	18.9	22.6	25.4	27.7	29.6
ξ^*	0.44	1.05	1.62	2.17	2.69	3.20	3.69
d + d							
$\log_{10}$ E	8.6	17.1	23.0	27.2	30.5	33.1	35.2
ξ^*	0.49	1.13	1.75	2.33	2.89	3.24	3.94

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of phonons, high hydrogen fractions to increase the multiple vacancy probability, and the flow of a current to induce gross motion of the hydrogen nuclei.

In conclusion, I have shown that fluctuations in the environment of a fusing pair of nuclei within a solid can significantly enhance the rate at which they fuse. This is because of the extreme sensitivity of the fusion rate to the effective potential barrier inhibiting fusion. In a schematic calculation for a shifted coulomb potential, I showed that a fractional *rms* fluctuation of the potential by only 0.1 (about 3 eV) will enhance the fusion rate by some 8 orders of magnitude and that fractional rms fluctuations of 0.5 will lead to enhancements of more than 30 orders of magnitude. While these results are suggestive, a more detailed calculation of the interaction between hydrogen nuclei in a solid (and their tunneling in the presence of non-adiabatic degrees of freedom) would be required to establish the relevance of this mechanism to cold fusion.

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(Θ is the unit step function) and the simple WKB approximation for the zero-energy penetration,

$$S(\xi) = (2\mu)^{1/2} \int_0^{[U(1+\xi)]^{-1}} dr \left[\frac{1}{r} - U(1+\xi)\right]^{1/2} = \pi \left[\frac{\mu}{2U(1+\xi)}\right]^{1/2} . \tag{9}$$

Here, the parameter U describes the average of the constant term $V_0(\xi)$ appearing in Eq. (1), and the fluctuations of this term are given by ξU. A rough estimate of U can be had by considering tunneling in the molecular Hydrogen potential [4], which results in $S = 2.07\mu^{1/2}$, so that $U = 1.15$, or about 31 eV. I adopt this value for the purposes of concreteness in the following discussion.

With the assumptions above, the enhancement is given by

$$E = \int \frac{d\xi}{(2\pi\sigma^2)^{1/2}} e^{-\xi^2/2\sigma^2 - \pi(2\mu/U)^{1/2}[(1+\xi)^{-1/2}-1]} . \tag{10}$$

A saddle-point evaluation of this integral proceeds by finding ξ^*, the most effective value of ξ, as a root of the equation

$$-\frac{\xi^*}{\sigma^2} + \frac{\pi(\mu/2U)^{1/2}}{(1+\xi^*)^{3/2}} = 0 \tag{11}$$

and then

$$E \approx [1 + \frac{3\pi}{4}(2\mu U)^{1/2}(1+\xi^*)^{-5/2}]^{-1/2} e^{-\xi^{*2}/2\sigma^2 - \pi(2\mu/U)^{1/2}[(1+\xi^*)^{-1/2}-1]} . \tag{12}$$

In Table I, I show the enhancement expected for the d + d and p + d fusion rates for various values of σ. Note that even relatively small fractional fluctuations ($\sigma \lesssim 0.5$, or a fluctuation in V_0 of 15 eV) can produce enhancements of some 30 orders of magnitude. The most effective values of ξ generally lie in the extreme wings of $P(\xi)$. Thus, even very improbable configurations can have a significant effect, as the tunneling rate is very sensitive to the potential. Also note that the precise relation between E and σ depends upon what I've assumed for $S(0)$ and the way in which fluctuations perturb the potential.

Are these fluctuations reasonable? A proton at a distance of 1 Bohr radius generates a potential of 1 in atomic units, so that given the possibility of multiple occupation of the interstitial sites or large distortions of the lattice, $\sigma \sim 0.5$ is perhaps not too implausible. Further, it seems likely that the p + d system, with its non-vanishing electric dipole moment operator, will be influenced by fluctuations more than the d + d system, which couples only to the electric quadrupole field. Finally, I note that any conditions in the system that enhance fluctuations will also enhance fusion rates. These might include heating to increase the number

The coordinates ξ can be defined such that $\xi = 0$ is the most probable configuration (e.g., as might be assumed for an interstitial pair in a fixed lattice), so that I can take

$$P(\xi) = \frac{1}{(2\pi\sigma^2)^{1/2}} e^{-\xi^2/2\sigma^2} \ , \tag{3}$$

with σ characterizing the scale of the fluctuations. Further, if we write

$$\Lambda(\xi) = Ae^{-2S(\xi)} \tag{4}$$

with A the nuclear rate constant and $e^{-2S(\xi)} = |\Psi(r = 0;\xi)|^2$ the probability to find the fusing pair at $r = 0$, then the change in the fusion rate due to fluctuations is

$$E = \frac{\Lambda_{\rm eff}}{\Lambda(\xi = 0)} = \int \frac{d\xi}{(2\pi\sigma^2)^{1/2}} e^{-\xi^2/2\sigma^2 - 2[S(\xi) - S(0)]} \ . \tag{5}$$

It is easy to see that most typically $E > 1$, so that fluctuations enhance the effective fusion rate. For example, if a Taylor expansion of $S(\xi)$ about $\xi = 0$ is valid,

$$S(\xi) - S(0) \approx S'\xi + \frac{1}{2} S''\xi^2 + O(\xi^3) \tag{6}$$

where the derivatives are evaluated at $\xi = 0$, then

$$E \approx (1 + 2S''\sigma^2)^{-1/2} e^{2S'^2\sigma^2} \ . \tag{7}$$

Apart from the prefactor, which is unimportant for small σ, $E > 1$ and increases exponentially with σ^2. Moreover, as S is generally large, even small fluctuations in $V(\mathbf{r};\xi)$ (i.e., small σ) can have a significant effect.

The magnitude of the fluctuations required to produce a given enhancement is then the crucial question. A full evaluation of E requires a realistic specification of $V(\mathbf{r};\xi)$ and a numerical integration of the Schroedinger equation to find $S(\xi)$. However, for the present illustrative discussion, it is sufficient to adopt a shifted Coulomb potential characterized by a single non-adiabatic parameter

$$V(\mathbf{r};\xi) = \left[\frac{1}{r} - U(1+\xi)\right] \Theta\big(r^{-1} - U(1+\xi)\big) \ , \tag{8}$$

For two hydrogen nuclei in a solid, I must consider the effect of all other degrees of freedom beyond $\mathbf{r}$. The electronic coordinates are still safely adiabatic, but the coordinates of the other nuclear degrees of freedom are likely not. For a solid composed of nuclei of mass number A_L, the lattice (phonon) time scale is $\tau_L \approx A_L^{1/2} M_n$, the time scale for the center-of-mass motion of the fusing pair is $\tau_{CM} \approx (A_1 + A_2)^{1/2} M_n$, and the time scale for motion

of the other hydrogen nuclei is $\tau_H \approx A_{1,2}^{1/2} M_n$. Typically,

I expect $\tau_L \gg \tau_f$ ($A_L = 106$ for Pd), and $(\tau_{CM}, \tau_H) \gtrsim \tau_f$.

Although methods for treating tunneling in general multi-dimensional situations have been developed using functional-integral methods [7], the sudden limit is one plausible and tractable approach to the present problem. The tunneling of the fusing nuclei must be calculated in an instantaneous potential $V(\mathbf{r};\xi)$ that depends parametrically upon the non-adiabatic coordinates, which we have denoted collectively by ξ. Thus, $V(\mathbf{r};\xi)$ as a function of ξ embodies variations of the interaction potential between the fusing pair associated with their location within the lattice, the shape of the interstitial cavity in which they might be contained, the possible presence of a third hydrogen nucleus nearby, etc. For $r \ll 1$, I expect the inter-nuclear coulomb repulsion to dominate, so that

$$V(\mathbf{r};\xi) \approx \frac{1}{r} + V_0(\xi) \ , \tag{1}$$

while for distances smaller than the electronic screening length, $V(\mathbf{r};\xi)$ can be expanded in terms of the instantaneous local electric field, quadrupole field, etc.

The coordinates ξ are not fixed, but rather fluctuate on time scales longer than τ_f due to zero-point and thermal motion, as well as non-equilibrium conditions. If the instantaneous fusion rate of the pair is $\Lambda(\xi)$, and if $P(\xi)$ denotes the normalized probability distribution of the non-adiabatic coordinates, then the effective fusion rate is

$$\Lambda_{\rm eff} = \int d\xi P(\xi) \Lambda(\xi) \ . \tag{2}$$

This expression is analogous to the usual one for thermonuclear rates in which the fusion cross section is averaged over a Maxwellian distribution of relative velocities [6]. Here, however, the average is over the potential through which the nuclei tunnel, rather than their asymptotic kinetic energy.

Enhancement of cold fusion rates by fluctuations

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The rate at which two nuclei tunnel through the coulomb barrier inhibiting their fusion can be enhanced significantly by modest fluctuations in their environment. Such enhancements might play a role in recent experiments claiming to observe cold fusion.

Recent reports have suggested the observation of fusion of deuterium nuclei introduced by electrolysis into solid Palladium [1] and Titanium [2]. The fusion rates implied by these results are 40–50 orders of magnitude larger than might be expected naively [3,4], although the temperatures involved are far too low to achieve such rates by the conventional thermonuclear mechanism of surmounting the coulomb barrier [5]. Thus, if these experiments are indeed revealing nuclear processes, alternative explanations must be sought. In this Letter, I suggest that modest fluctuations in the environment of the fusing nuclei can lead to the required enhancements. The mechanism is closely related to an explanation of large sub-barrier enhancements of heavy-ion fusion cross sections [6].

The rate at which two nuclei will fuse depends upon the extent to which they tunnel through the potential $V(\mathbf{r})$ describing their interaction as a function of their separation $\mathbf{r}$. In an isolated diatomic molecule, the electronic time scale τ_e is far shorter than the fusion time scale τ_f (i.e., the "time" the system is in a classically forbidden region), and $V(\mathbf{r})$ can be taken to be the usual Born-Oppenheimer potential. Indeed, a simple estimate is $\tau_f/\tau_e \approx \mu^{1/2}$, where μ is the reduced mass of the fusing nuclei. [Note that I am using atomic units where $\hbar = m_e = e = 1$, so that $\mu = M_n A_1 A_2/(A_1 + A_2)$, where $A_{1,2}$ are the mass numbers of the fusing nuclei and $M_n = 1833$ is the ratio of the nucleon and electron masses. Lengths are therefore measured in Bohr radii (0.53×10^{-8} cm), energies are measured in Hartrees (27.2 eV), and times are measured in $\tau_e = \hbar/(27.2\text{eV}) \approx 2.5 \times 10^{-17}$ s.]