

respirable coal mine dust or respirable quartz dust does not. Therefore, to ensure that the verification limits have actually been met, it is necessary to provide for a margin of error in each measurement. The critical values provide this margin of error. MSHA can be confident that the verification limits have not been exceeded at the sampled locations. When valid measurements do not exceed the appropriate critical values listed in Table 70-1, corresponding to the number of shifts sampled.

To explain how the verification limits were derived, it is helpful to define some symbolic notation. Let X represent a measurement, and let μ represent the

true value of whatever quantity is being measured *i.e.*, the full-shift average concentration, at a specific sampling location, of either respirable coal mine dust or respirable quartz dust. The difference between X and μ is the measurement error and is denoted by ϵ . $X = \mu + \epsilon$.

In accordance with standard statistical and industrial hygiene practice, ϵ (but not μ) is assumed to be normally distributed. Since the approved sampling and analytical methods for measuring concentrations of respirable coal mine dust and respirable quartz dust are both statistically unbiased, ϵ has a mean value of zero and a degree of variability

represented by its standard deviation, denoted by σ_ϵ . The ratio of σ_ϵ to μ is called the measurement *coefficient of variation* (CV) due to sampling and analytical errors.⁴² The CV relates entirely to variability due to measurement errors and not at all to variability in actual dust concentrations.

For respirable coal mine dust, the value of CV used in calculating critical values was chosen to be consistent with the value proposed at $\mu = 2.0 \text{ mg/m}^3$ in the Notice of Final Policy published in the **Federal Register**: Coal Mine Respirable Dust Standard Noncompliance Determinations (63 FR 5700, February 3, 1998):

$$CV = \sqrt{(7\%)^2 + (5\%)^2 + (5\%)^2} = 10\%$$

The 7-percent term in this formula accounts for uncertainty due to potential weighing error, and the two 5-percent terms account for differences

between individual cyclones and for variability in the exact volume of air pumped through the filter during a 480-minute shift.

For respirable quartz dust, the value of CV used in calculating critical values is:

$$CV = \sqrt{(5.3\%)^2 + (4.2\%)^2 + (5.6\%)^2} = 9\%$$

The 5.3-percent term in this formula accounts for imprecision in the Infrared (Infrared Spectrophotometer or IR) measurement of quartz mass deposited on the filter, the 4.2-percent term represents variability in air volume, and the final 5.6-percent term accounts for uncertainty due to variability between individual cyclones, given the size distribution of quartz dust encountered in mining environments (Bartley, November 1999).

Each critical value (c) was calculated to provide a confidence level of at least 95 percent that the dust control parameters specified in the ventilation plan were effective in preventing dust concentrations from exceeding the verification limits. Using a confidence coefficient of 1.645, based on the standard normal probability distribution, knowledge of the CV makes it possible to calculate a 1-tailed, 95-percent upper confidence limit (UCL) for μ , given a single measurement X . The UCL is $X \cdot (1 + 1.645 \cdot CV)$. When $X \leq c$, the UCL for μ is less than or equal to the verification limit. When $X > c$, the UCL for μ exceeds the verification limit.

For example, suppose $X = 1.71 \text{ mg/m}^3$ respirable dust. Then the UCL for μ would be $1.71 \cdot (1 + (10\% \text{ of } 1.645)) = 1.99 \text{ mg/m}^3$, which is less than the

verification limit for respirable coal mine dust. If, however, $X = 1.72 \text{ mg/m}^3$, then the UCL for μ would be $1.72 \cdot 1.1645 \text{ mg/m}^3$, which slightly exceeds the verification limit. Similarly, for respirable quartz dust, the UCL for μ is $87 \cdot (1 + (9\% \text{ of } 1.645)) = 99.9 \text{ } \mu\text{g/m}^3$ when $X = 87 \text{ } \mu\text{g/m}^3$ and slightly above the verification limit of $100 \text{ } \mu\text{g/m}^3$ when $X = 88 \text{ } \mu\text{g/m}^3$.

If more than one measurement is available, then the confidence coefficient changes to reflect multiplication of the tail probabilities for independent measurement errors. When n measurements are available, the objective is to calculate a critical value (c) such that if each of the n measurements is $\leq c$, then the 1-tailed 95-percent UCL for μ is \leq the verification limit. Since the product of the n individual tail probabilities must equal 0.05, the appropriate 1-tail probability for each measurement individually is the n th root of 0.05.

For example, if $n = 3$, then the appropriate 1-tail probability for each measurement is the cube root of 0.05, or 0.3684. The standard normal confidence coefficient corresponding to this tail probability is 0.336. Therefore, when all three measurements have the same value (X), the UCL is $X \cdot (1 + 0.336 \cdot CV)$.

Substituting the appropriate CV estimate, the UCL is $X \cdot 1.0336$ for respirable coal mine dust or $X \cdot 1.0302$ for respirable quartz dust. Consequently, to obtain the critical value, the verification limit is first divided by 1.0336 (coal mine dust) or 1.0302 (quartz dust) and then truncated to the desired number of decimal digits. This yields 1.93 mg/m^3 for coal mine dust and $97 \text{ } \mu\text{g/m}^3$ for respirable quartz dust.

The confidence coefficients used to establish critical values by this method are as follows:

| n | Confidence coefficient |
|---------|------------------------|
| 1 | 1.645 |
| 2 | 0.760 |
| 3 | 0.336 |
| 4 | 0.068 |

For $n > 4$, the confidence coefficient is less than 0.068. It should be noted that although the critical value calculated for $n \geq 4$ is slightly below the verification limit for both types of respirable dust, for simplicity it was set equal to the verification limit as a close approximation.

⁴² In some publications, this ratio is called the relative standard deviation (RSD). It is sometimes

also denoted by CV_{total} , where "total" refers to all

sources of potential sampling and analytical error but does not cover variability in μ itself.