

arbitrarily large number of sub-interval errors. By the Central Limit Theorem, such a summation tends to be normally distributed, regardless of the distribution of sub-interval errors. This does not depend on the distribution of  $\mu_i$ , which is generally represented as being lognormal.

Furthermore, each measurement made by MSHA personnel is based on the difference between pre- and post-exposure weights of a dust sample, as determined in the same laboratory, and adjusted by the weight gain or loss of the control filter capsule. Any systematic error or bias in the weighing process attributable to the laboratory is mathematically canceled out by subtraction. Furthermore, any bias that may be associated with day-to-day changes in laboratory conditions or introduced during storage and handling of the filter capsules is also mathematically canceled out. Elimination of the sources of systematic errors identified above, together with the fact that the concentration of respirable dust is defined by section 202(e) of the Mine Act to mean the average concentration of respirable dust measured by an approved sampler unit, implies that the measurements are unbiased. This means that  $\varepsilon_i$  is equally likely to be positive or negative and, on average, equal to zero.

Therefore, each  $\varepsilon_i$  is assumed to be normally distributed, with a mean value of zero and a degree of variability represented by its standard deviation

$$\sigma_i = \mu_i \cdot CV_{\text{total}}.$$

Since  $X_i = \mu_i + \varepsilon_i$ , it follows that for a given value of  $\mu_i$ ,  $X_i$  is normally distributed with expected value equal to  $\mu_i$  and standard deviation equal to  $\sigma_i$ .  $CV_{\text{total}}$ , is the coefficient of variation in measurements corresponding to a given value of  $\mu_i$ .  $CV_{\text{total}}$  relates entirely to variability due to measurement errors and not at all to variability in actual dust concentrations.

MSHA's procedure for citing noncompliance based on the CTV table consists of formally testing a presumption of compliance at every location sampled. Compliance with the applicable dust standard at the  $i$ th sampling location is expressed by the relation  $\mu_i \leq S$ .  $\text{Max}\{\mu_i\}$  denotes the maximum dust concentration, among all of the sampling locations within a MMU. Therefore, if  $\text{Max}\{\mu_i\} \leq S$ , none of the sampler units in the MMU were exposed to excessive dust concentration. Since the burden of proof is on MSHA to demonstrate noncompliance, the hypothesis being tested (called the null hypothesis, or  $H_0$ ) is that the concentration at every location sampled is *in* compliance with the applicable dust standard.

Equivalently, for a MMU the null hypothesis ( $H_0$ ) is that  $\text{max}\{\mu_i\} \leq S$ . In other areas, where only one, full-shift measurement is made, the null hypothesis is simply that  $\mu_i \leq S$ .

The test consists of evaluating the likelihood of measurements obtained during an MSHA sample, under the assumption that  $H_0$  is true. Since  $X_i = \mu_i + \varepsilon_i$ ,  $X_i$  (or  $\text{max}\{X_i\}$  in the case of a MMU) can exceed  $S$  even under that assumption. However, based on the normal distribution of measurement errors, it is possible to calculate the probability that a measurement error would be large enough to fully account for the measurement's exceeding the standard. The greater the amount by which  $X_i$  exceeds  $S$ , the less likely it is that this would be due to measurement error alone. If, under  $H_0$ , this probability is less than five percent, then  $H_0$  can be rejected at a 95-percent confidence level and a citation is warranted. For a MMU, rejecting  $H_0$  (and therefore issuing a citation) is equivalent to determining that  $\mu_i > S$  for at least one value of  $i$ .

Each CTV listed was calculated to ensure that citations will be issued at a confidence level of at least 95 percent. As described in MSHA's February 1994 notice and explained further by Kogut [2], the tabled CTV corresponding to each  $S$  was calculated on the assumption that, at each sampling location:

$$CV_{\text{total}} \leq CV_{\text{CTV}} = \sqrt{\left(\frac{0.14 \text{ mg/m}^3}{\mu_i \text{ mg/m}^3} \cdot 100\%\right)^2 + (5\%)^2 + (5\%)^2}$$

The MSHA and NIOSH joint finding establishes that for valid measurements made with an approved sampler unit,  $CV_{\text{total}}$  is in fact less than  $CV_{\text{CTV}}$  at all dust concentrations ( $\mu_i$ ).

The situation in which measurement error is most likely to cause an erroneous noncompliance determination is the hypothetical case of  $\mu_i = S$  for either a single-shift sample measurement or for all of the measurements made in the same MMU. In that borderline situation—*i.e.*, the worst case consistent with  $H_0$ —the standard deviation is identical for all measurement errors. Therefore, the value of  $\sigma$  used in constructing the CTV table is the product of  $S$  and  $CV_{\text{CTV}}$  evaluated for a dust concentration equal to  $S$ :

$$\sigma = S \cdot \sqrt{\left(\frac{0.14}{S}\right)^2 + (.05)^2 + (.05)^2}$$

Assuming a normal distribution of measurement errors as explained above, it follows that the probability a single measurement would equal or exceed the critical value

$$c = S + 1.64 \cdot \sigma$$

is five percent under  $H_0$  when  $CV_{\text{total}} = CV_{\text{CTV}}$ . The tabled CTV corresponding to  $S$  is derived by simply raising the critical value  $c$  up to the next exact multiple of 0.01 mg/m<sup>3</sup>.

For example, at a dust concentration ( $\mu_i$ ) just meeting the applicable dust standard of  $S = 2 \text{ mg/m}^3$ ,  $CV_{\text{CTV}}$  is 9.95 percent. Therefore, the calculated value of  $c$  is 2.326 and the CTV is 2.33 mg/m<sup>3</sup>. Any valid single-shift measurement at or above this CTV is unlikely to be this large simply because of measurement error. Therefore, any such measurement warrants a noncompliance citation.

The probability that a measurement exceeds the CTV is even smaller if

$\mu_i < S$  for any  $i$ . Furthermore, to the extent that  $CV_{\text{total}}$  is actually less than  $CV_{\text{CTV}}$ ,  $\sigma$  is actually less than  $S \cdot CV_{\text{CTV}}$ . This results in an even lower probability that the critical value would be exceeded under the null hypothesis. Consequently, if any single-shift measurement equals or exceeds  $c$ , then  $H_0$  can be rejected at confidence level of at least 95-percent. Since rejection of  $H_0$  implies that  $\mu_i > S$  for at least one value of  $i$ , this warrants a noncompliance citation.

It should be noted that when each of several measurements is separately compared to the CTV table, the probability that at least one  $\varepsilon_i$  will be large enough to force  $X_i \geq \text{CTV}$  when  $\mu_i \leq S$  is greater than the probability when only a single comparison is made. For example (still assuming  $S = 2 \text{ mg/m}^3$ ), if  $CV_{\text{total}}$  is actually 6.6%, then the standard deviation of  $\varepsilon_i$  is 6.6% of 2.0 mg/m<sup>3</sup>, or 0.132 mg/m<sup>3</sup>, when  $\mu_i = S$ . Using properties of the normal