[3.30] Singular value decomposition (SVD) of a matrix

Singular value decomposition is a powerful tool for a variety of matrix-related tasks, including working with matrices and sets of linear equations which are nearly singular. You can use SVD to remove the singularities and get numerical results, but they may not be the results you expect. In addition, SVD can be used for linear least-squares regression problems. A future version of this tip may demonstrate some of those examples, but for now I will only present a good implementation of SVD, written by Hernan Rivera. For now, if you would like to see how SVD is used, start with chapter 2.6 of *Numerical Recipes in Fortran*, by Press, Teukolsky, Vetterling, and Flannery (http://www.nr.com).

SVD decomposes a matrix *m* with dimensions *a* x *b* (with *a* < *b*) into three matrices *u*, *d* and *v* such that

$$m = u \cdot d \cdot v^T$$

u is a column-orthogonal matrix with dimensions *a* x *b*. *d* is an *a* x *a* diagonal matrix with positive or zero elements (these are the singular values), and *v* is an *a* x *a* orthogonal matrix. The call for Rivera's *svd()* is

```
svd(m,"u","d","v")
```

where *m* is the matrix to decompose, and "u", "d" and "v" are the names of the variables (as strings) to store the output.

```
svd(aa,á,é,í)
Prgm
© sdv(m,"u","d","v")   m = u*d*vᵀ
© Hernan Rivera

Local st,i,k,n,m,ut,sc

Define sc(st1,n)=Prgm
 Local st2,i
 seq(i,i,1,n)→st2
 SortD st1,st2
 newMat(n,n)→e1
 1→i
 While i≤n
  1→e1[st2[i],i]
  i+1→i
 EndWhile
EndPrgm

eigVl(aa*aaᵀ)→st
eigVc(aa*aaᵀ)→uu
colDim(uu)→n
sc(st,n)
round(uu*e1,11)→uu
eigVl(aaᵀ*aa)→st
eigVc(aaᵀ*aa)→vv
colDim(vv)→m
sc(st,m)
round(vv*e1,11)→vv
min(m,n)→k

1→i
While i≤k
 If (aa*vv)[1,i]*uu[1,i]<Ø Then
  mRow(⁻1,uuᵀ,i)→ut
  utᵀ→uu
 EndIf
 i+1→i
EndWhile
```

```
SortD st
newMat(n,m)→dd

1→i
While i≤k
 If st[i]<1ᴇ⁻Ø11
   Ø→st[i]
 √(st[i])→dd[i,i]
 i+1→i
EndWhile

uu→#á
vv→#í
dd→#é
DelVar e1,uu,vv,dd
EndPrgm
```

Note that *svd()* creates and deletes global variables *e1*, *uu*, *vv* and *dd*; your own variables with those names, if any, will be deleted.

(Credit to Hernan Rivera)