

[6.8] Convert trigonometric expressions to exponential format

Trigonometric expressions can be expressed as powers of the natural logarithm base e . Hyperbolic expressions can be easily converted to the equivalent exponential format with `expand()`. For example:

`expand(cosh(t))`

returns $\frac{e^t}{2} + \frac{1}{2e^t}$

which is equivalent to the more common form $\frac{e^t + e^{-t}}{2}$

Other trigonometric functions can also be expressed in exponential form, but the results are more complicated. The principle is to use complex numbers for the function arguments, then eliminate the imaginary parts of the arguments. The procedure is:

1. Set the Complex Format mode to Polar.
2. Enter the trigonometric expression with the argument variables, using the "with" operator `|` to assign complex numbers to the arguments in rectangular coordinates.
3. Replace the imaginary components with zero to get the final result, using the "with" operator.

As an example, convert $\cos(x+y)$ to exponential format. Enter

`cos(x+y) | x=a+bi and y=c+di`

Note that x and y are replaced with complex numbers. The result is a sizable function of a , b , c and d . Next, enter

`ans(1) | b=0 and d=0`

This eliminates the imaginary components of x and y . This also results in a big function, but it is only a function of a and c . The result is in the general form of $re^{i\theta}$, where r is the magnitude and θ is the angle of the result. a corresponds to the original x , and c corresponds to the original y . To make this substitution automatically, use this:

`ans(1) | b=0 and d=0 and a=x and c=y`

Setting constraints on a and b may result in a more simple expression.

(Credit to Glenn Fisher)