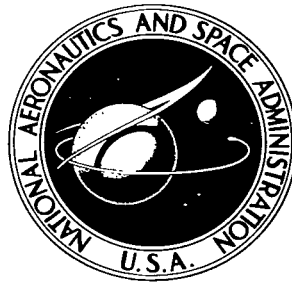


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# A SIMPLIFIED TECHNIQUE FOR DETERMINING DEVIATION IN THE LUNAR TRANSFER ORBIT EPHEMERIS

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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SUMMARY

A linear algebraic equation was derived to determine the altitude deviation of a transfer orbit at selected anomaly angles as a function of errors in the magnitude and direction of the transfer maneuver. The technique is applied to both synchronous orbit and Hohmann transfer maneuvers of the lunar excursion module from the Apollo vehicle in a circular lunar orbit. The technique requires the pilot to make two altitude measurements with known included angles after the orbit transfer maneuver. With precomputed constants, the altitude deviation at any future anomaly angle may then be computed. The results indicate that the technique would be useful for early determination of orbit deviations if accurate altitude measurements can be made.

INTRODUCTION

In the Apollo mission involving manned soft landings on the moon, there are savings in weight and complexity to separate the lunar excursion module (LEM) from the orbiting Apollo vehicle during the orbital phase. The LEM then makes an orbit transfer to an elliptical orbit with a low pericyynthion from which it begins a braking descent to the lunar surface. It is desirable that a technique be developed in which the pilot monitors the transfer and determines the ephemeris independently of the automatic navigation system. The need for such a technique is predicated on the fact that the transfer maneuver of the LEM may not be performed with sufficient accuracy. Although automatic control of the transfer maneuver may be sufficiently accurate to assure a safe transfer, the possibility of a failure which would require manual takeover should be considered.

When deviations in the transfer orbit are determined, there are several procedures for making corrections. The success of any one of them will depend on the accuracy with which the orbit deviation can be determined. A further requirement is that the associated computations be performable in the time available. Rather than to develop methods for all the possible correction procedures, the purpose of this report will be to present and analyze a method of determining the deviation in pericynthion altitude with minimum computation.

The technique requires the measurement of the altitude deviation from nominal at two prescribed angles of travel from the orbit transfer point. Each pair of measurements produces a determination of altitude deviation at any future anomaly angle. The technique is presented and analyzed for both a synchronous orbit and Hohmann transfer maneuver of the LEM from the Apollo vehicle in an established lunar orbit.

#### SYMBOLS

$$A(\varphi) = \frac{\partial r}{\partial v_{0-}}$$

$$B(\varphi) = \frac{\mu r_{0-}}{K^2} (1 - \cos \varphi) + \cos \varphi - \sin \varphi \tan \gamma_{0-}$$

$$C(\varphi) = \frac{\partial r}{\partial \gamma_{0-}}$$

- e            eccentricity
- g            acceleration due to gravity at lunar surface
- G            constant defined in equation (19) for particular angle combination
- H            constant defined in equation (19) for particular angle combination
- h            altitude above lunar surface
- $K = r_{0-} V_{0-} \cos \gamma_{0-}$
- p            semi-latus rectum
- R            remainder term in Taylor series
- r            radial distance to vehicle position
- $r_m$         radius of moon
- $\dot{r}$          derivative of r with respect to time
- V            total velocity of vehicle
- $\Delta V$      magnitude of orbit transfer velocity vector
- $\alpha$        angle of  $\Delta V$
- $\gamma$         flight-path angle, defined with respect to local horizontal

$\delta$  deviation from nominal  
 $\mu$  gravitation constant,  $r_m^2 g$   
 $\phi$  angular travel after orbit transfer  
 $\theta$  anomaly angle  
 $\dot{\theta}$  derivative of  $\theta$  with respect to time

Subscripts:

0+ condition immediately preceding orbit transfer maneuver  
 0- condition immediately following orbit transfer maneuver

For synchronous orbit:

1  $15^\circ$  from injection point  
 2  $30^\circ$  from injection  
 3  $45^\circ$  from injection  
 4  $60^\circ$  from injection  
 5  $75^\circ$  from injection  
 6  $94.04^\circ$  from injection

For Hohmann orbit:

1  $30^\circ$  from injection  
 2  $60^\circ$  from injection  
 3  $80^\circ$  from injection  
 4  $120^\circ$  from injection  
 5  $150^\circ$  from injection  
 6  $180^\circ$  from injection

Subscripts are used to indicate combinations of angles; for example, subscript 126 indicates a combination of  $15^\circ$ ,  $30^\circ$ , and  $94.04^\circ$  for the synchronous orbit.

## METHOD AND ANALYSIS

The effects of errors in magnitude and direction of the transfer  $\Delta V$ , treated separately, are shown in figures 1 and 2 for the synchronous transfer orbit and in figures 3 and 4 for the Hohmann transfer. In each case the nominal transfer originated from an 80-nautical-mile circular orbit and had a pericyynthion of 50,000 feet.

The probable deviation in pericynthion altitude was determined by the use of the Monte Carlo method, simultaneous magnitude and direction errors being taken into account. The results are shown as cumulative probability distributions in figures 5 and 6. Standard deviations in the thrusting maneuver were  $0.5^\circ$  and 5 feet per second. (The fact that the mean deviation in pericynthion is not zero is indicative of the nonlinearity in the relationship between initial errors and deviation in pericynthion altitude. The equations used were those of ref. 1.)

If the aforementioned errors in the thrusting maneuver are accepted as reasonable, it is apparent that determination of the orbit deviation will be necessary. A method is derived in the following section whereby the deviations may be computed manually if the altitudes at two different points along the orbit can be measured.

### Derivation of Equations

The equation of an ellipse in terms of its semi-latus rectum  $p$ , eccentricity  $e$ , and true anomaly  $\theta$  is (from ref. 2):

$$r = \frac{p}{1 + e \cos \theta} \quad (1)$$

or, since  $p = \frac{K^2}{\mu}$ ,

$$r = \frac{K^2/\mu}{1 + e \cos \theta} \quad (2)$$

In terms of the anomaly at injection  $\theta_{0-}$ , the radius at any later time is

$$r = \frac{K^2/\mu}{1 + e \cos(\theta_{0-} + \varphi)} \quad (3)$$

where  $\varphi$  is the angular travel from injection to the point of observation. Expansion of the cosine term gives

$$r = \frac{K^2/\mu}{1 + e \cos \theta_{0-} \cos \varphi - e \sin \theta_{0-} \sin \varphi} \quad (4)$$

An expression for  $e \sin \theta_{0-}$  may be derived by differentiating equation (2) to give

$$e \sin \theta_{0-} = \frac{K^2}{\mu r_{0-}^2} \left( \frac{dr}{d\theta} \right)_{0-} \quad (5)$$

The flight-path angle at injection is

$$\gamma_{0-} = \tan^{-1} \frac{1}{r_{0-}} \left( \frac{dr}{d\theta} \right)_{0-} \quad (6)$$

Therefore equation (5) may be written

$$e \sin \theta_{0-} = \frac{K^2 \tan \gamma_{0-}}{\mu r_{0-}} \quad (7)$$

The substitution of equations (2) and (7) into equation (4) gives the following:

$$r = \frac{K^2/\mu}{1 + \left( \frac{K^2}{\mu r_{0-}} - 1 \right) \cos \varphi - \frac{K^2}{\mu r_{0-}} \sin \varphi \tan \gamma_{0-}} \quad (8)$$

Multiplication of numerator and denominator of equation (8) by  $\mu r_{0-}/K^2$  gives

$$r = \frac{r_{0-}}{\frac{\mu r_{0-}}{K^2} (1 - \cos \varphi) + \cos \varphi - \sin \varphi \tan \gamma_{0-}} \quad (9)$$

The deviation from nominal of  $r$  at selected angles is obtained with a Taylor's series expansion of equation (9), second and higher order terms being neglected.

$$\delta r(\varphi) = \left( \frac{\partial r}{\partial r_{0-}} \right) \delta r_{0-} + \left( \frac{\partial r}{\partial V_{0-}} \right) \delta V_{0-} + \left( \frac{\partial r}{\partial \gamma_{0-}} \right) \delta \gamma_{0-} \quad (10)$$

The partial derivatives in equation (10) are determined by taking the partial derivative of equation (9) first with respect to  $r_{0-}$ ,  $V_{0-}$ , and then  $\gamma_{0-}$

$$\frac{\partial r}{\partial r_{0-}} = \frac{1}{B(\varphi)} + \frac{r_{0-} \mu (1 - \cos \varphi)}{K^2 B^2(\varphi)}$$

$$\frac{\partial r}{\partial V_{0-}} = \frac{2r_{0-}^2 \mu (1 - \cos \varphi)}{K^2 B^2(\varphi) V_{0-}}$$

$$\frac{\partial r}{\partial \gamma_{0-}} = \frac{r_{0-} \sin \varphi \sec^2 \gamma_{0-}}{B^2(\varphi)} - \frac{2\mu r_{0-}^2 (1 - \cos \varphi) \tan \gamma_{0-}}{K^2 B^2(\varphi)}$$

where  $B(\varphi)$  is the denominator of equation (9).

In the application of the technique it is assumed that the orbiting Apollo vehicle has a well-established circular orbit ( $\delta r_{0-} = 0$ ). The only transfer maneuver errors considered are in  $\Delta V$  and  $\alpha$ ; therefore, the resultant components of velocity after the transfer maneuver are:

$$\dot{r} = -\Delta V \cos \alpha \quad (11)$$

$$(\dot{r}\hat{\theta})_{0-} = (\dot{r}\hat{\theta})_{0+} - \Delta V \sin \alpha \quad (12)$$

The resultant flight-path angle is

$$\gamma_{0-} = \tan^{-1} \frac{-\Delta V \cos \alpha}{(\dot{r}\hat{\theta})_{0+} - \Delta V \sin \alpha} \quad (13)$$

and the resultant total velocity magnitude is

$$V_{0-} = \left\{ (\Delta V \cos \alpha)^2 + [(\dot{r}\hat{\theta})_{0+} - \Delta V \sin \alpha]^2 \right\}^{1/2} \quad (14)$$

An error in either magnitude  $\Delta V$  or direction  $\alpha$  of the orbit transfer thrusting maneuver will give an error in resultant flight-path angle  $\gamma_{0-}$  and velocity magnitude  $V_{0-}$ . Therefore,  $\delta V_{0-}$  and  $\delta \gamma_{0-}$  of equation (10) are determined with a Taylor's series expansion of equations (13) and (14) about nominal  $\Delta V$  and  $\alpha$  as

$$\delta V_{0-} = \left( \frac{\partial V_{0-}}{\partial \Delta V} \right) \delta \Delta V + \left( \frac{\partial V_{0-}}{\partial \alpha} \right) \delta \alpha + R(\delta \Delta V, \delta \alpha) \quad (15)$$

$$\delta \gamma_{0-} = \left( \frac{\partial \gamma_{0-}}{\partial \Delta V} \right) \delta \Delta V + \left( \frac{\partial \gamma_{0-}}{\partial \alpha} \right) \delta \alpha + R(\delta \Delta V, \delta \alpha) \quad (16)$$



Equations (15) and (16) are substituted into equation (10) to obtain

$$\begin{aligned} \delta r(\varphi) = & \frac{\partial r}{\partial v_{0-}} \left[ \left( \frac{\partial v_{0-}}{\partial \Delta V} \right) \delta \Delta V + \left( \frac{\partial v_{0-}}{\partial \alpha} \right) \delta \alpha + R(\delta \Delta V, \delta \alpha) \right] \\ & + \frac{\partial r}{\partial \gamma_{0-}} \left[ \left( \frac{\partial \gamma_{0-}}{\partial \Delta V} \right) \delta \Delta V + \left( \frac{\partial \gamma_{0-}}{\partial \alpha} \right) \delta \alpha + R(\delta \Delta V, \delta \alpha) \right] \end{aligned} \quad (17)$$

which gives the altitude deviation at any angle as a function of the magnitude and direction of the orbit-transfer thrusting maneuver.

Equation (17) is used to develop a technique for determining altitude deviation at any angle after injection by measuring the altitude deviation at two prior angles and using precomputed constants. Equation (17) is written for three angles of travel  $\varphi$ . By solving the system of equations to eliminate the terms in brackets, an equation is derived for the altitude deviation at the third angle as a function of the deviations at the first two angles:

$$\begin{aligned} \delta r(\varphi_3) = & \left[ \frac{A(\varphi_1)A(\varphi_2)C(\varphi_3) - A(\varphi_1)A(\varphi_3)C(\varphi_2)}{A(\varphi_1)A(\varphi_2)C(\varphi_2) - A^2(\varphi_2)C(\varphi_1)} + \frac{A(\varphi_3)}{A(\varphi_2)} \right] \delta r(\varphi_2) \\ & - \frac{A^2(\varphi_2)C(\varphi_3) - A(\varphi_2)A(\varphi_3)C(\varphi_2)}{A(\varphi_1)A(\varphi_2)C(\varphi_2) - A^2(\varphi_2)C(\varphi_1)} \delta r(\varphi_1) \end{aligned} \quad (18)$$

where  $A( )$  and  $C( )$  are the partial derivatives  $\frac{\partial r}{\partial v_{0-}}$  and  $\frac{\partial r}{\partial \gamma_{0-}}$  for the particular angles.

The use of the technique would require knowing the nominal orbit in advance and deciding the angles after injection at which measurements of altitude would be taken. The terms in brackets in equation (18) then could be precomputed and equation (18) would reduce to

$$\delta r(\varphi_3) = G\delta r(\varphi_2) - H\delta r(\varphi_1) \quad (19)$$

with a different  $G$  and  $H$  for each angle combination used.

#### Evaluation of Method

To illustrate the technique, an example using a synchronous orbit transfer from the parent vehicle in an 80-nautical-mile circular lunar orbit is presented. Table I lists the constants necessary to establish the ephemeris of the orbit after making two altitude measurements. In this case the pilot would measure

his altitude after  $15^\circ$  of travel from the orbit transfer point to obtain  $\delta r(\varphi_1)$ . A similar measurement at  $30^\circ$  gives  $\delta r(\varphi_2)$ . By using  $G_{126}$  and  $H_{126}$ , the altitude deviation at pericyynthion is determined from:

$$\delta r(\varphi_3) = G_{126} \delta r(\varphi_2) - H_{126} \delta r(\varphi_1)$$

By using appropriate constants, the altitude deviation at other points in the orbit can be determined with these two measurements. At  $45^\circ$  the process is repeated with  $G_{136}$ ,  $H_{136}$ , and so forth. The agreement of the measured and predicted value of altitude deviation at orbit points provides a check on the prior measurements. The process is again repeated at  $60^\circ$  and  $75^\circ$ . When the pilot has a correlation between measured and analytically determined deviations he can decide whether corrections or abort procedures are necessary. The necessary constants for applying the technique to a Hohmann transfer are also listed in table I. The constants are computed in  $30^\circ$  increments and an example can easily be computed.

It should be pointed out that angular travel can be closely approximated by time rather than by a direct measurement. By using a nominal time for  $150^\circ$  of travel as the point of making the altitude measurement, errors in the transfer maneuver of 5 feet per second in  $\Delta V$  and  $0.5^\circ$  in  $\alpha$  result in a  $0.3^\circ$  error in the point of measurement. Since the angular rate of the LEM never exceeds  $0.1^\circ$  per second and the rate of descent is small, use of time as the reference would have little effect on the accuracy of the technique.

To determine the effect of systematic errors which result from dropping second and higher order terms in equation (10), the equations of motion in reference 1 were solved to give altitude deviations along trajectories that had discrete injection errors. The method of this report was then applied to errors at various angle combinations to determine the deviation in altitude at the pericyynthion. The actual altitude deviation and the various predicted deviations for both synchronous and Hohmann transfers are given in tables II and III.

An estimate of the random error in the predicted pericyynthion may be obtained by applying the theory of the propagation of error in linear systems. Thus,

$$\sigma_{\delta r(\varphi_3)} = \left[ G^2 (\sigma_{\delta r(\varphi_2)})^2 + H^2 (\sigma_{\delta r(\varphi_1)})^2 \right]^{1/2} \quad (20)$$

where  $\sigma_{\delta r(\varphi_2)}$  and  $\sigma_{\delta r(\varphi_1)}$  are the standard deviations in the measured values of altitude.

For practical purposes, it may be assumed that  $(\sigma_{\delta r(\varphi_2)})^2$  equals  $(\sigma_{\delta r(\varphi_1)})^2$  and equation (20) may be written as

$$\sigma_{\delta r}(\varphi_3) = \sigma_{\delta r}(\varphi_1) (G^2 + H^2)^{1/2} \quad (21)$$

The values of  $(G^2 + H^2)^{1/2}$  for angle combination where the constants were computed is given in table I. For any given value of  $\sigma_{\delta r}(\varphi_1)$ , the standard deviation of altitude determination can be computed.

## RESULTS AND DISCUSSION

The results shown in tables II and III indicated that the linearized equation provides a very good determination of the pericyynthion altitude deviation. It is shown that the accuracy of the determination is increased by using the first and most recent measurements. The equation is sufficiently accurate to determine a large deviation early in the transfer orbit and small deviations can be predicted accurately prior to reaching the pericynthion.

Table I indicates the effect of measurement errors. Some improvement over the values in table I can be effected by making the first measurement earlier. The second measurement should be made as late as possible but while sufficient time remains to calculate and perform the corrective procedures. The time required for the calculations and corrections depends on the final mission strategy; therefore no optimization has been attempted.

In applying the technique to the lunar mission, there are several suggested methods of obtaining altitude measurements. When the LEM and Apollo spacecrafts are in close proximity, visual sightings and rendezvous radar may be used to obtain altitude information. At other points in the orbit the landing radar, celestial navigation, or visual sightings on the lunar surface may be used to obtain altitude measurements.

## CONCLUDING REMARKS

A linear algebraic equation was derived and studied to determine the altitude deviation of a transfer orbit at selected anomaly angles as a function of errors in the magnitude and direction of the transfer maneuver. The technique requires two altitude measurements at known angles of travel and simple calculations that could be performed onboard the lunar excursion module are required. Systematic errors in the technique are small and the random errors depend primarily on the accuracy of the altitude measurements.

Langley Research Center,  
National Aeronautics and Space Administration,  
Langley Station, Hampton, Va., July 21, 1964.

## REFERENCES

1. Queijo, Manuel J.; and Miller, G. Kimball, Jr.: Analysis of Two Thrusting Techniques for Soft Lunar Landings Starting From a 50-Mile Altitude Circular Orbit. NASA TN D-1230, 1962.
2. Stephenson, Reginald J.: Mechanics and Properties of Matter. 2nd ed., John Wiley & Sons, Inc., c.1952.

TABLE I.- CONSTANTS FOR ORBIT DETERMINATION

Angle combination	G	H	$(G^2 + H^2)^{1/2}$
Synchronous orbit transfer			
126	12.798	20.438	24.114
136	4.495	8.068	9.236
146	2.381	3.848	4.525
156	1.519	1.664	2.253
Hohmann transfer orbit			
126	7.482	12.968	14.972
136	2.737	5.481	6.126
146	1.579	2.740	3.162
156	1.155	1.158	1.636

TABLE II.- SYSTEMATIC ERRORS IN APPLYING THE TECHNIQUE  
TO SYNCHRONOUS TRANSFER ORBIT

Injection errors for -		Pericyynthion altitude deviation, ft	Predicted pericyynthion altitude deviation for measured deviations	
$\Delta V$ , fps	$\alpha$ , deg		Angle combination*	Predicted error, ft
5.0	-0.5	2,425	126	-1,274
			136	358
			146	1,277
			156	2,087
5.0	0.5	-12,180	126	-12,836
			136	-12,821
			146	-12,694
			156	-12,119
-5.0	-0.5	13,160	126	13,918
			136	13,823
			146	13,209
			156	13,319
-5.0	0.5	-1,095	126	-420
			136	-1,906
			146	-1,808
			156	-1,141

\*See symbol list for angles indicated by the numbers.

TABLE III.- SYSTEMATIC ERRORS IN APPLYING THE TECHNIQUE  
TO HOHMANN TRANSFER ORBIT

Injection errors for -		Pericyynthion altitude deviation, ft	Predicted pericyynthion altitude deviation for measured deviations	
$\Delta V$ , fps	$\alpha$ , deg		Angle combinations	Predicted error, ft
5.0	-0.5	-21,360	126	-22,146
			136	-21,469
			146	-21,294
			156	-21,154
5.0	0.5	-21,366	126	-24,417
			136	-22,613
			146	-21,584
			156	-21,400
-5.0	-0.5	21,486	126	23,574
			136	22,686
			146	21,963
			156	21,516
-5.0	0.5	21,496	126	24,780
			136	23,202
			146	22,223
			156	21,630

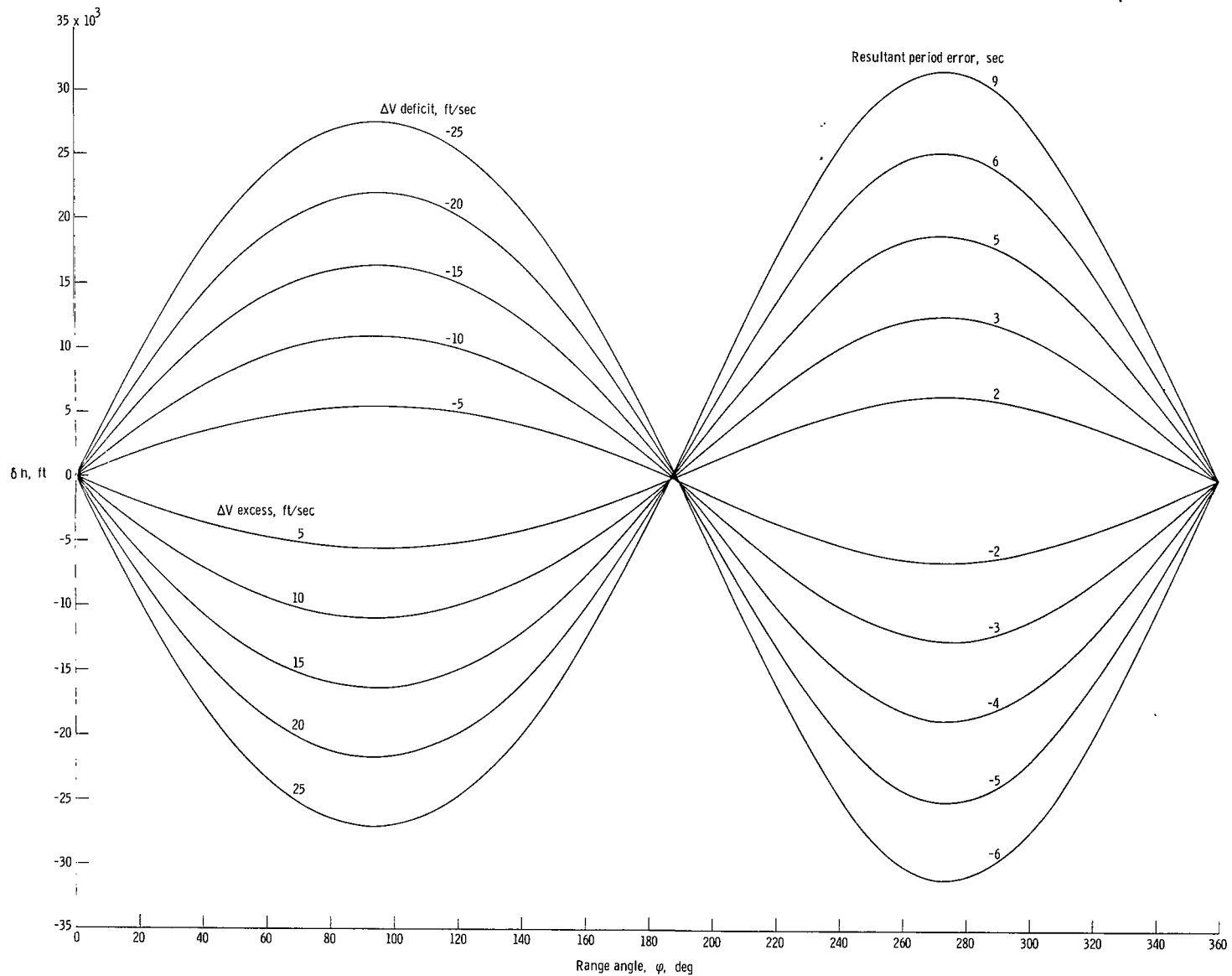


Figure 1.- Variation of altitude deviation with range angle for errors in magnitude of the synchronous orbit transfer  $\Delta V$ .



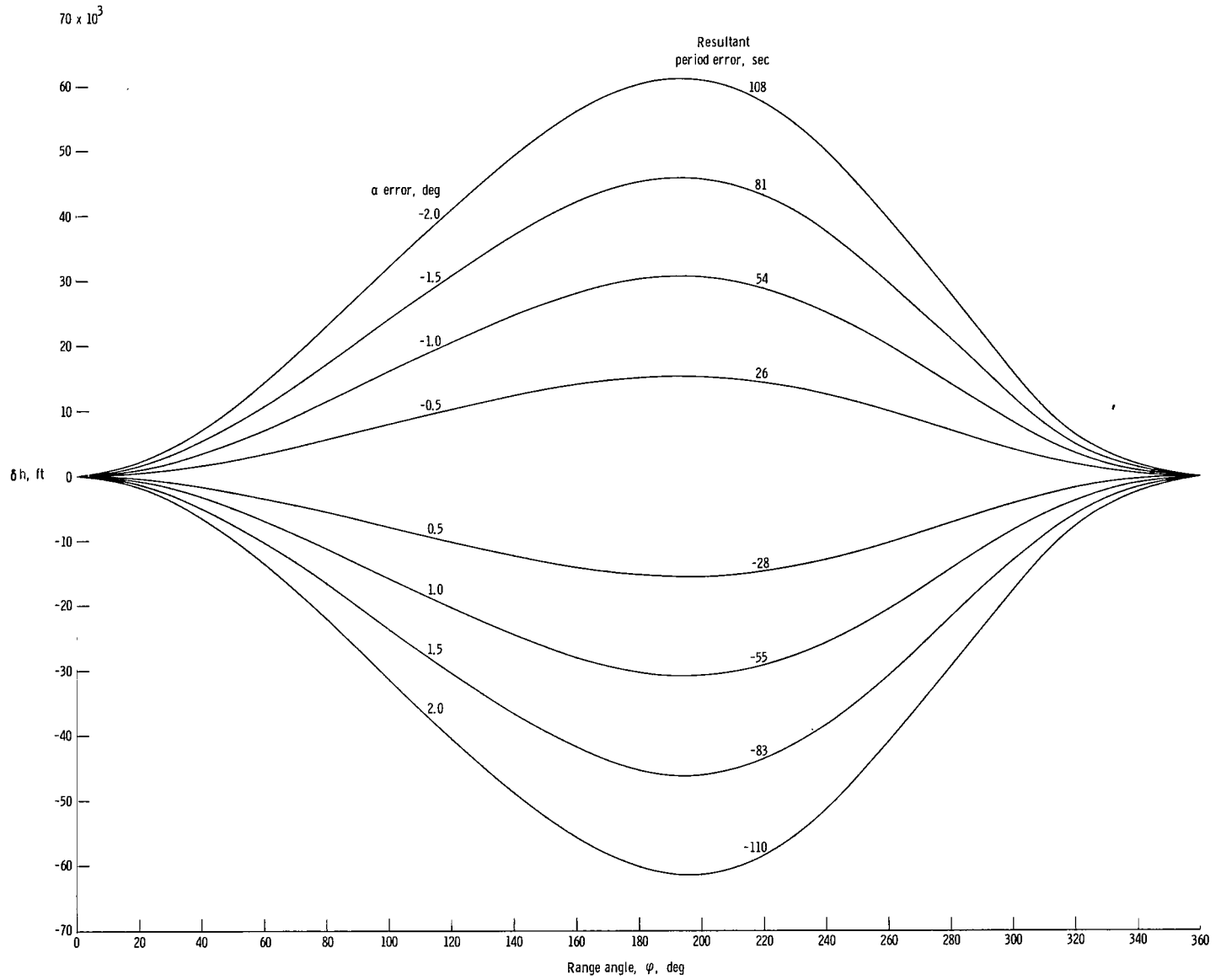


Figure 2.- Variation of altitude deviation with range angle for errors in direction of the synchronous orbit transfer  $\Delta V$ .

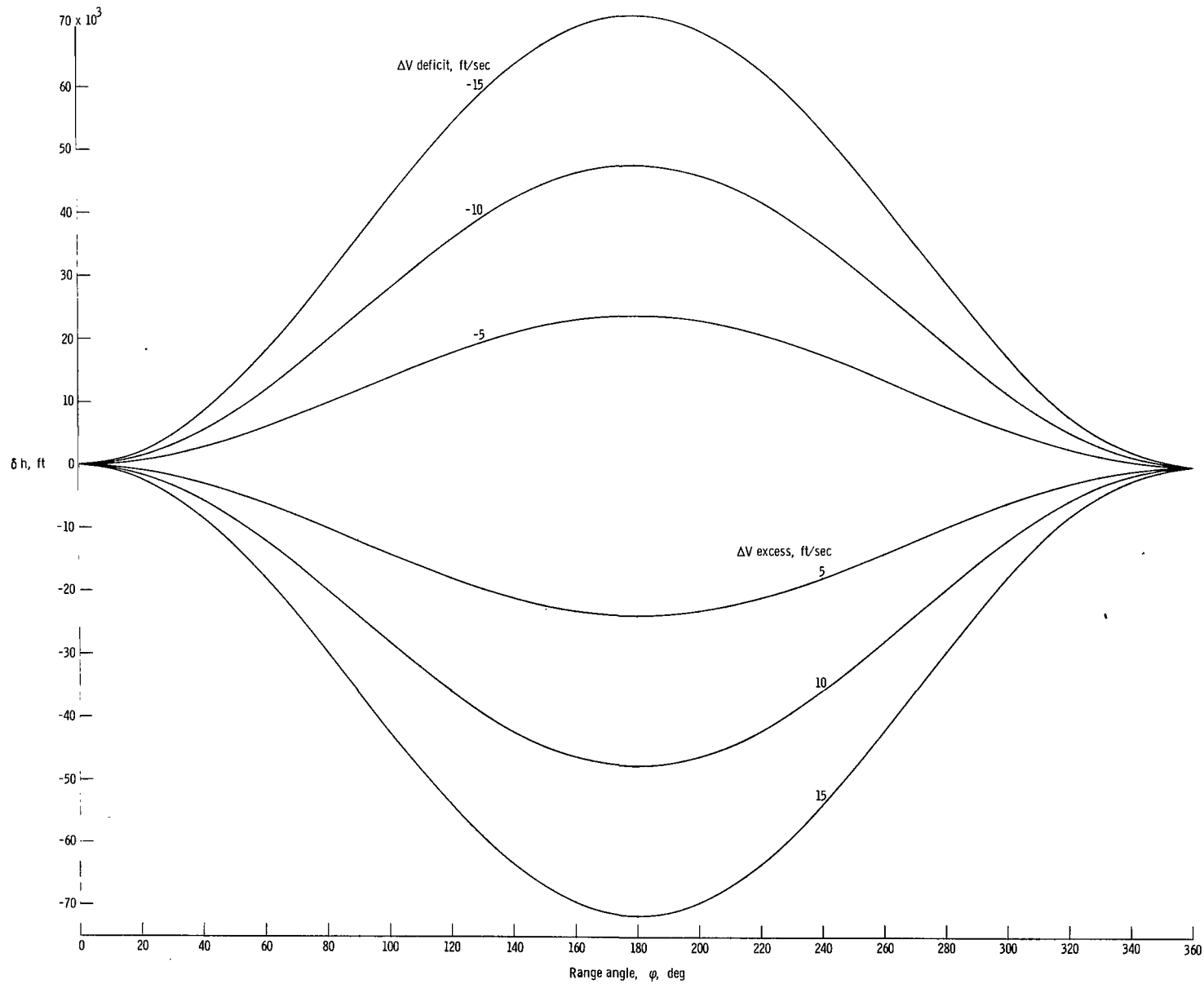


Figure 3.- Variation of altitude deviation with range angle for errors in magnitude of the Hohmann orbit transfer  $\Delta V$ .

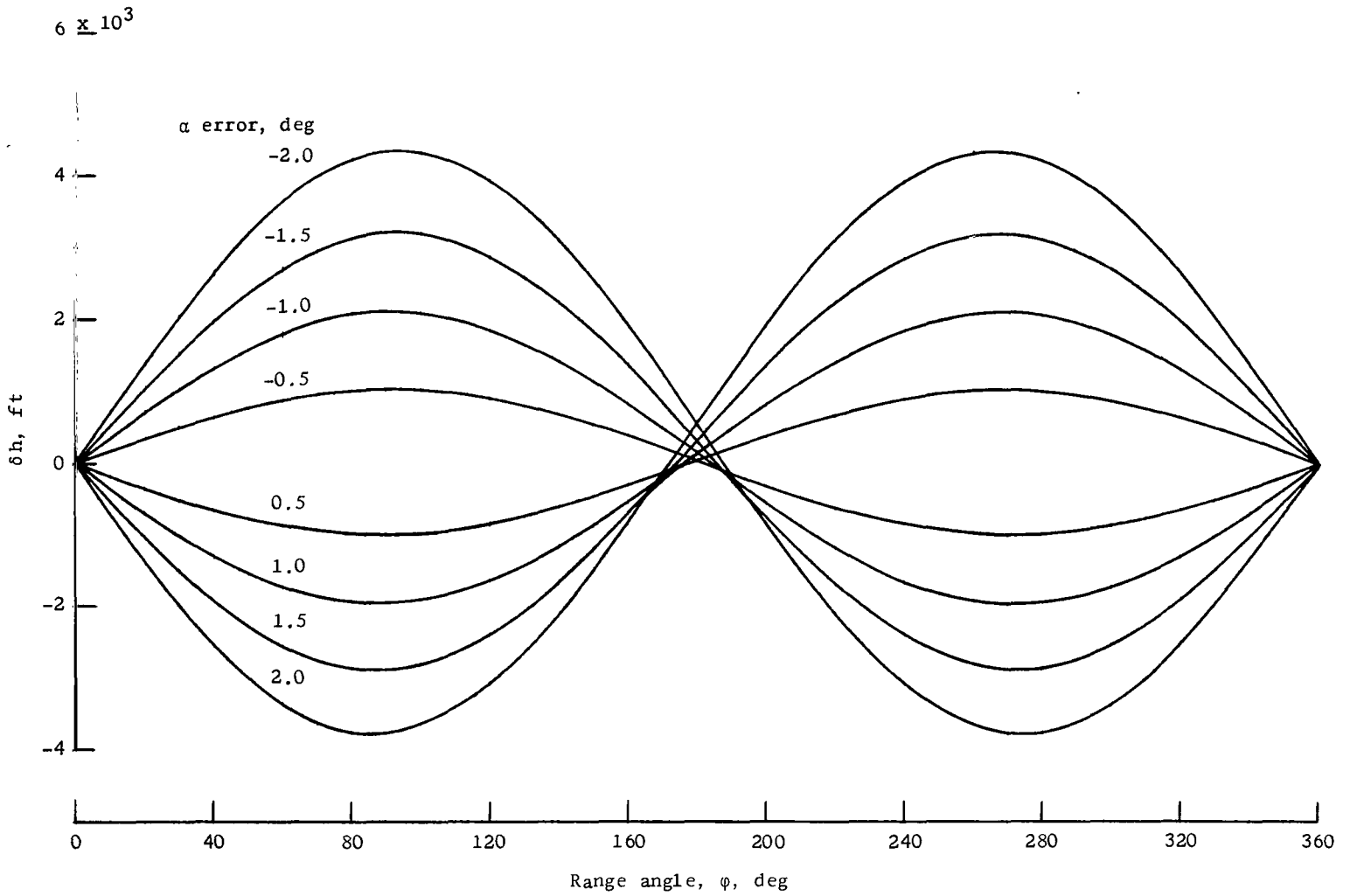


Figure 4.- Variation of altitude deviation with range angle for errors in direction of the Hohmann orbit transfer  $\Delta V$ .

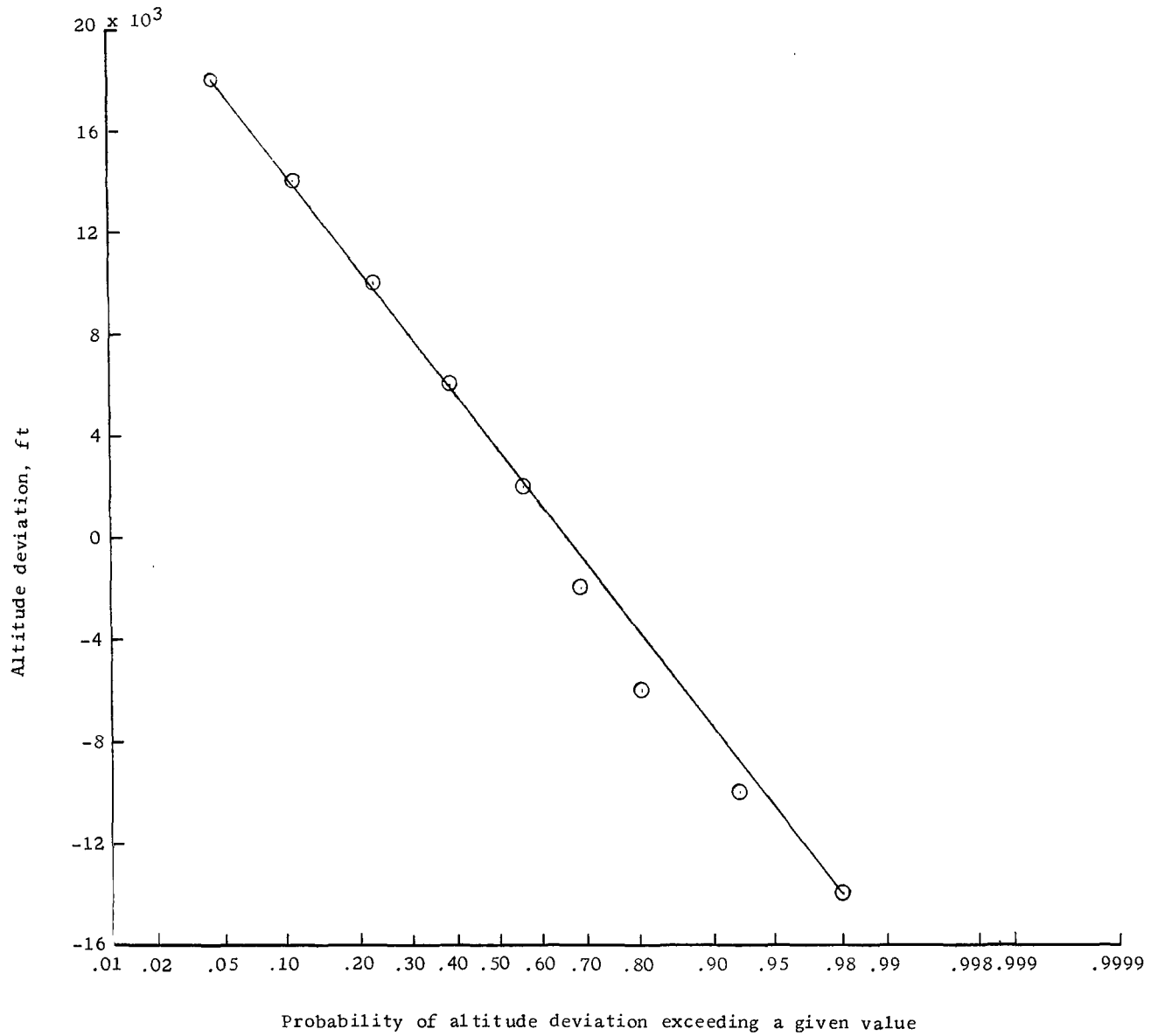


Figure 5.- Probability distribution of altitude deviation at normal pericynthion of synchronous orbit for injection errors with zero mean and  $\sigma$  of  $0.5^\circ$  in angle and 5.0 feet per second in  $\Delta V$ .

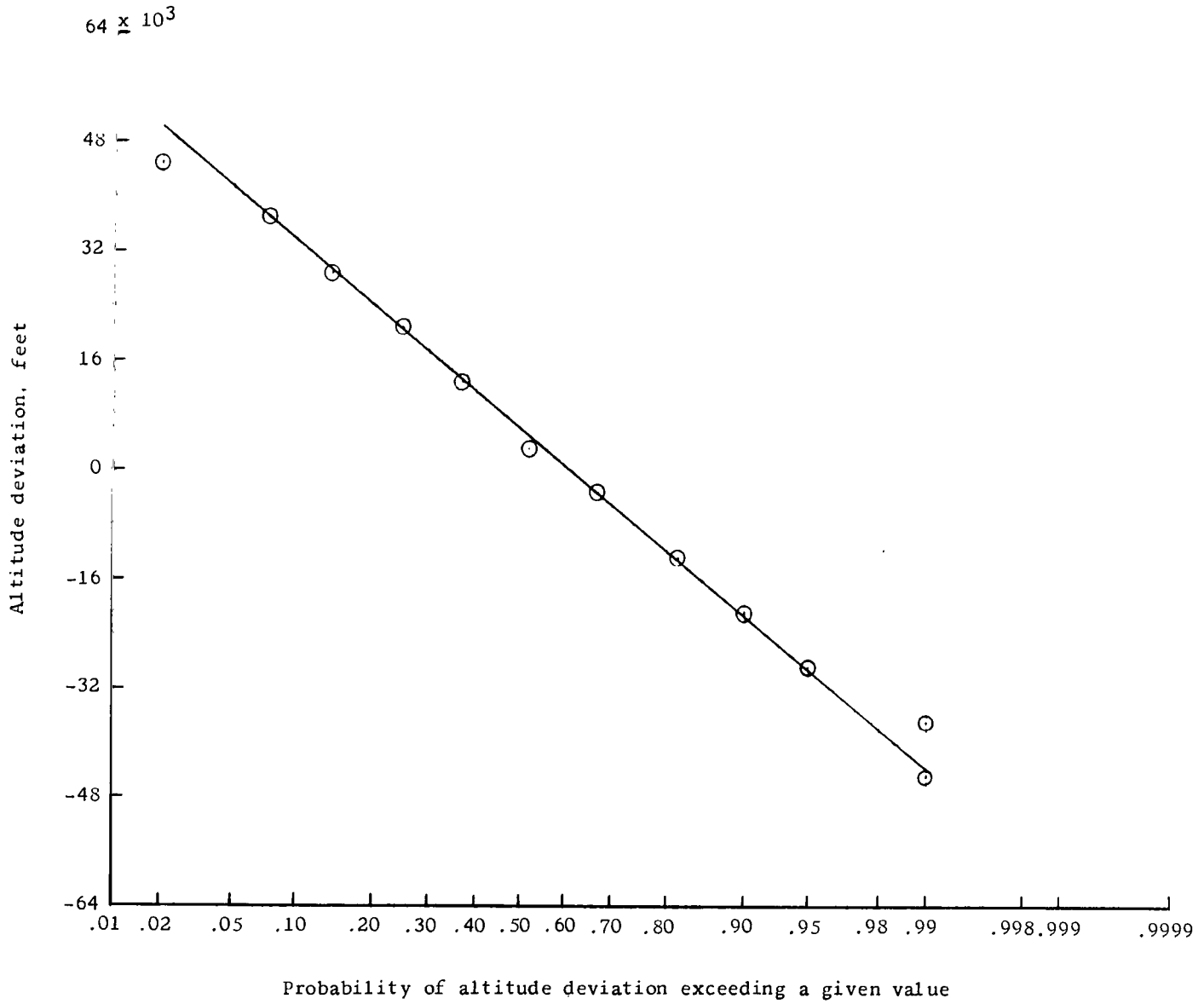


Figure 6.- Probability distribution of altitude deviation at normal pericynthion of Hohmann orbit for injection errors with zero mean and  $\sigma$  of  $0.5^\circ$  in angle and 5.0 feet per second in  $\Delta V$ .

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