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THE DESIGN OF SERIES-LOADED
 THERMAL CONDUCTORS FOR
 MINIMUM WEIGHT

by
 Theodore C. Taylor

July 1965



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THE DESIGN OF SERIES-LOADED THERMAL
CONDUCTORS FOR MINIMUM WEIGHT

ABSTRACT

The design of aerospace electrical and electronic equipments frequently makes use of thermal conduction in metal structural members as the primary means of heat removal. This usage of parts can often result in a thermal conductor which is subject to series-loading by a number of concentrated and distributed heat loads. This report presents techniques for designing one-dimensional, series-loaded thermal conductors of minimum weight. The use of the techniques is illustrated by two design examples.

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Introduction

For many of the electrical and electronic equipments in aerospace applications, it is considered desirable to exclude primary or secondary refrigerant fluids from their interior structures, and to rely on thermal conduction of the structures for heat removal. Within structures of modest size, this practice is attractive for two reasons. First, the thermal conduction process can be without rival for its reliability and functional convenience as a heat transport mechanism. Second, the amount of material needed for heat conduction circuits is small where the transport distances involved are short, and in many instances, material which is present for other purposes, such as metal supports or containers, can be configured so as to serve the thermal circuit. The use of multiple-purpose configurations frequently results in the creation of multiply loaded thermal conductors, which transport the heat from several sources to a single sink, or region of external cooling. This report presents a method for the design of such multiply, or series-loaded conductors so as to minimize their weight. The method is therefore of potential importance to aerospace equipments, insofar as thermal considerations may control the design of a part.

Analytical Models

The simple analytical models for a one-dimensional thermal conductor may be taken from Fourier's equation,¹ expressed in the form,

$$W = \left(\frac{kA\theta}{L} \right) \quad (1)$$

As illustrated schematically in Fig. 1, a heat load W is applied to the upper end of a thermal conductor of uniform cross-section A and uniform thermal conductivity k , and is conducted down the length L by reason of the temperature difference $(T_1 - T_2) = \theta$ between the upper end of the conductor and the thermal ground, or sink.

As has been pointed out elsewhere², the volume of material used in the thermal conductor is given by,

$$Q = AL, \quad (2)$$

Hence, a substitution from (1) into (2) gives,

$$Q = W \left(\frac{L^2}{k\theta} \right). \quad (3)$$

Therefore, viewing the conduction removal of a concentrated heat load as a transport problem, the relationship (3) indicates that the amount of material required for this transport over a single conductor with only one concentrated load varies directly with

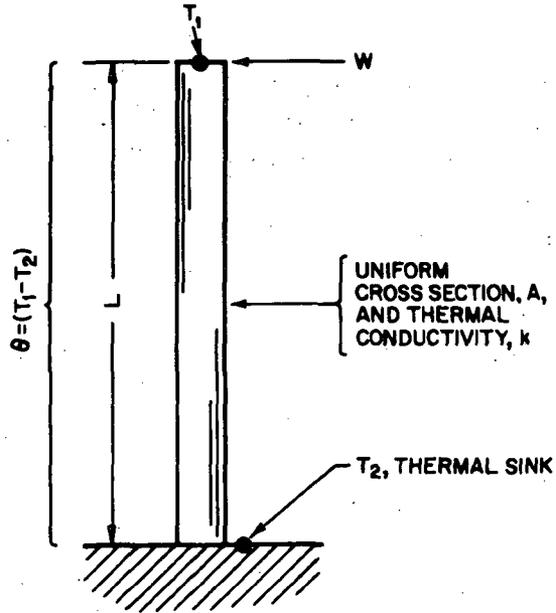


Fig. 1 Schematic illustration of a one-dimensional thermal conductor, having a concentrated heat load W .

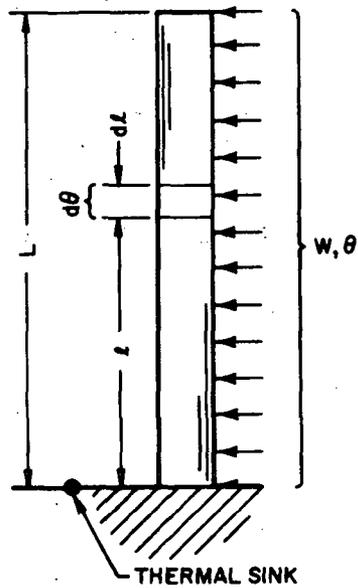


Fig. 2 Schematic illustration of a one-dimensional thermal conductor, having a distributed heat load w per unit length, where the total load is $W = wL$.

the size of the load and the square of the thermal exit length, and inversely with the thermal conductivity and the available temperature difference, or driving potential. For a heat load which is uniformly distributed over the thermal exit length as illustrated in Fig. 2, the value of θ is given by,

$$\theta = \left(\frac{1}{kA} \right) \int_0^L \left[\int_\ell^L w d\ell \right] d\ell, \quad (4)$$

where w is the thermal load per unit length. Therefore, since $W = wL$, the material required to conduct the distributed load to the thermal sink is given by,

$$Q = \frac{W}{2} \left(\frac{L^2}{k\theta} \right). \quad (5)$$

This indicates that the load which is uniformly distributed over its thermal exit length requires only half of the conductor material required to transport the same magnitude total load over the entire length. Having these simple models expressed in equations (3) and (5) for concentrated and uniformly distributed heat loads, we may proceed to the consideration of thermal circuits in systems involving a number of loads, and to formulas for the conductor material requirements of some such circuits.

Series-Loaded Conductors

The design characteristics which determine the minimum amount of material needed in a series-loaded thermal conductor include the positions of the loads and heat sink, the load magnitudes, the thermal conductivity of the conductor, and the temperature distribution requirements. We consider here the class of design problems in which the purpose of a conductor is to remove heat, and thereby limit the temperature rise of the loads on the maximum side only. Thus, if a load temperature is specified, it is assumed that values at or below the specified one are acceptable, while those above it are not. Since a series-loaded conductor must, because of the second law of thermodynamics, increase in temperature as the distance from the sink increases, the load located most distant from the sink will have the highest temperature. As a consequence, there are many situations in which the entire conductor design is dominated by the requirement to meet the temperature limiting requirement of the most remote load. For example, this is the situation when all of the loads consist of components which will tolerate the same maximum temperature. A conductor designed to cool the most remote load in this case will actually "overcool" all of the others. Although there are, for such a conductor having at least two loads, infinitely many designs which will meet its requirements, there is a single design which has the minimum conductor material requirement. This development will present the solution of minimum material for the design case which is controlled by the temperature of the most remote load, and then show the method of solution when the temperature distribution of the loads is more restrictive.

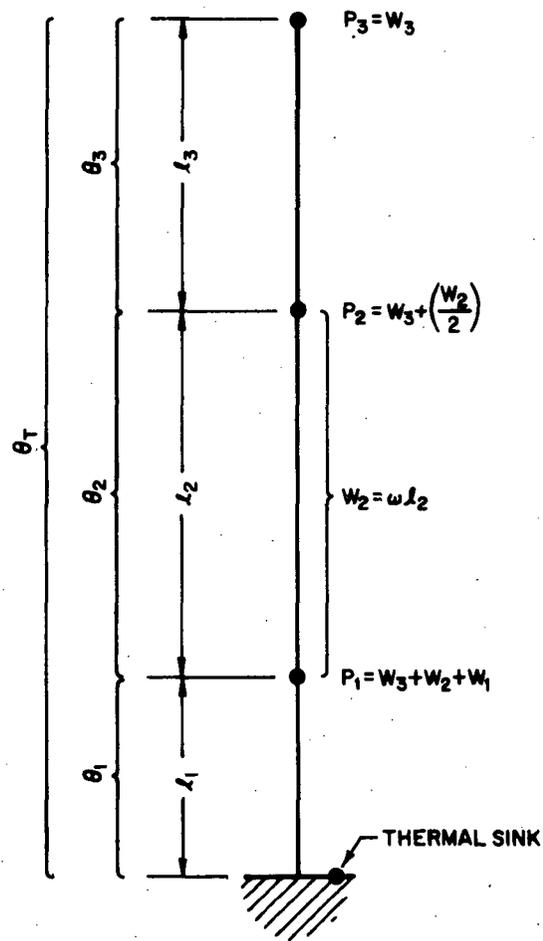


Fig. 3 The thermal circuit of a three leg, series-loaded thermal conductor, illustrating the notation used.

A series-loaded thermal conductor may be viewed as a series of one-load, one-leg conductors, as illustrated for a simple case in Fig. 3. In the example, loads W_1 and W_3 are concentrated at the ends of their legs most remote from the thermal sink, while W_2 is uniformly distributed over ℓ_2 . As may be justified by a comparative inspection of equations (3) and (5), the distributed load is modeled, within its own distribution length, as a concentrated load of half the distributed load magnitude, situated at the end of the leg remote from the sink. In any legs nearer the sink, it appears at full value. It is most convenient from an algebraic viewpoint to put any thermal conductor into the notation of the example of Fig. 3, replacing any distributed loads by their equivalent concentrated loads.

Consider a thermal conductor, as illustrated in Fig. 4, where θ_T is the design-controlling temperature. Evidently, the quantity of material used in any two adjacent legs of this conductor is given by,

$$Q = \left(\frac{P_j \ell_j^2}{k \theta_j} \right) + \left(\frac{P_{j+1} \ell_{j+1}^2}{k \theta_{j+1}} \right), \quad (6)$$

where θ_j and θ_{j+1} are the temperature rises along the legs involved. If θ_c denotes the total rise for the two leg combination, then,

$$\theta_j + \theta_{j+1} = \theta_c, \quad (7)$$

and (6) can be rearranged to,

$$Q = \left(\frac{1}{k} \right) \left[\frac{P_j \ell_j^2}{\theta_j} \right] - \left[\frac{P_{j+1} \ell_{j+1}^2}{\theta_c - \theta_j} \right]. \quad (8)$$

This equation is found to have a minimum for the point where,

$$\left(\frac{\theta_j}{\theta_c} \right) = \left[\frac{1}{1 + \left(\frac{\ell_{j+1}}{\ell_j} \sqrt{\frac{P_{j+1}}{P_j}} \right)} \right]. \quad (9)$$

So that, if the intermediate temperature rise θ_j is adjusted to conform to (9) while satisfying θ_c , a minimum of material is used in the two leg conductor.

Further algebraic study of the solution will show that other properties of the minimum material solution are given by the relationships,

$$\left(\frac{\theta_{j+1}}{\theta_j} \right) = \left(\frac{\ell_{j+1}}{\ell_j} \right) \sqrt{\frac{P_{j+1}}{P_j}}, \quad (10)$$

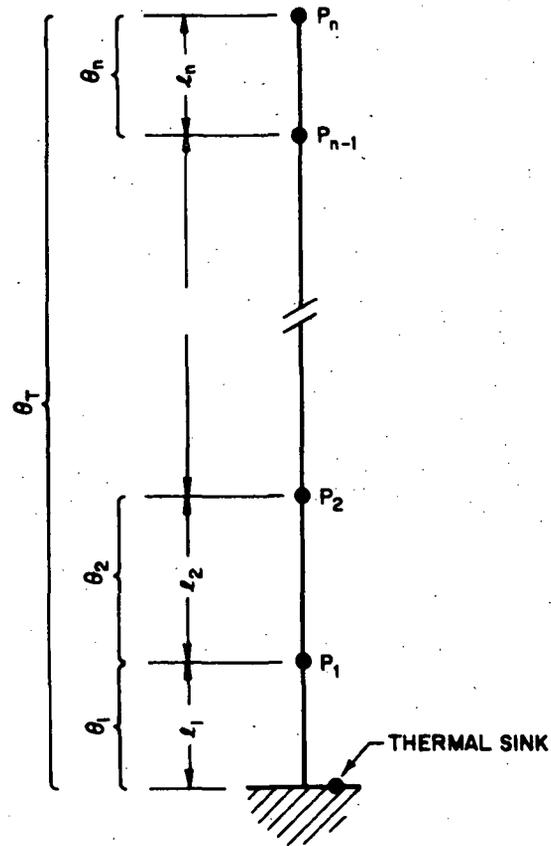


Fig. 4 The thermal circuit of a series-loaded conductor, general case.

$$\left(\frac{A_{j+1}}{A_j}\right) = \sqrt{\frac{P_{j+1}}{P_j}} \quad (11)$$

and,

$$\left(\frac{\theta_{j+1}}{\theta_j}\right) = \left(\frac{Q_{j+1}}{Q_j}\right) \quad (12)$$

Since these results involve only ratios of terms which apply to adjacent legs of the circuit, and do not involve a particular value of θ_c , they may be applied to the problem of designing a minimum material conductor of more than two legs.

Therefore, since, from Fig. 4,

$$\theta_T = \theta_1 + \theta_2 + \dots + \theta_n \quad (13)$$

we may use (10) to obtain,

$$\theta_1 = \frac{\theta_T}{1 + \left(\frac{\ell_2}{\ell_1}\right) \sqrt{\frac{P_2}{P_1}} + \left(\frac{\ell_3}{\ell_1}\right) \sqrt{\frac{P_3}{P_1}} + \dots + \left(\frac{\ell_n}{\ell_1}\right) \sqrt{\frac{P_n}{P_1}}} \quad (14)$$

or, in simpler notation,

$$\theta_1 = \left[\frac{\ell_1 \sqrt{P_1} \theta_T}{\sum_{j=1}^n \ell_j \sqrt{P_j}} \right] \quad (15)$$

After using (15), one may apply (10) to generate the sequence of all θ 's, $\theta_1, \theta_2, \theta_3, \dots, \theta_n$, required for a minimum material use in any and all two adjacent leg portions of the conductor. From this, the cross-sectional area requirement of each leg is found from,

$$A_j = \left(\frac{P_j \ell_j}{k \theta_j} \right) \quad (16)$$

which is simply a rearrangement of (1).

As an example, consider the application of this solution to a conductor as illustrated schematically in upper Fig. 5. The conductor represents a portion of the external casing of an equipment, which is estimated to be subject to distributed ambient heat loadings of 100 watts along its top and upper side wall, and of 50 watts on its middle and

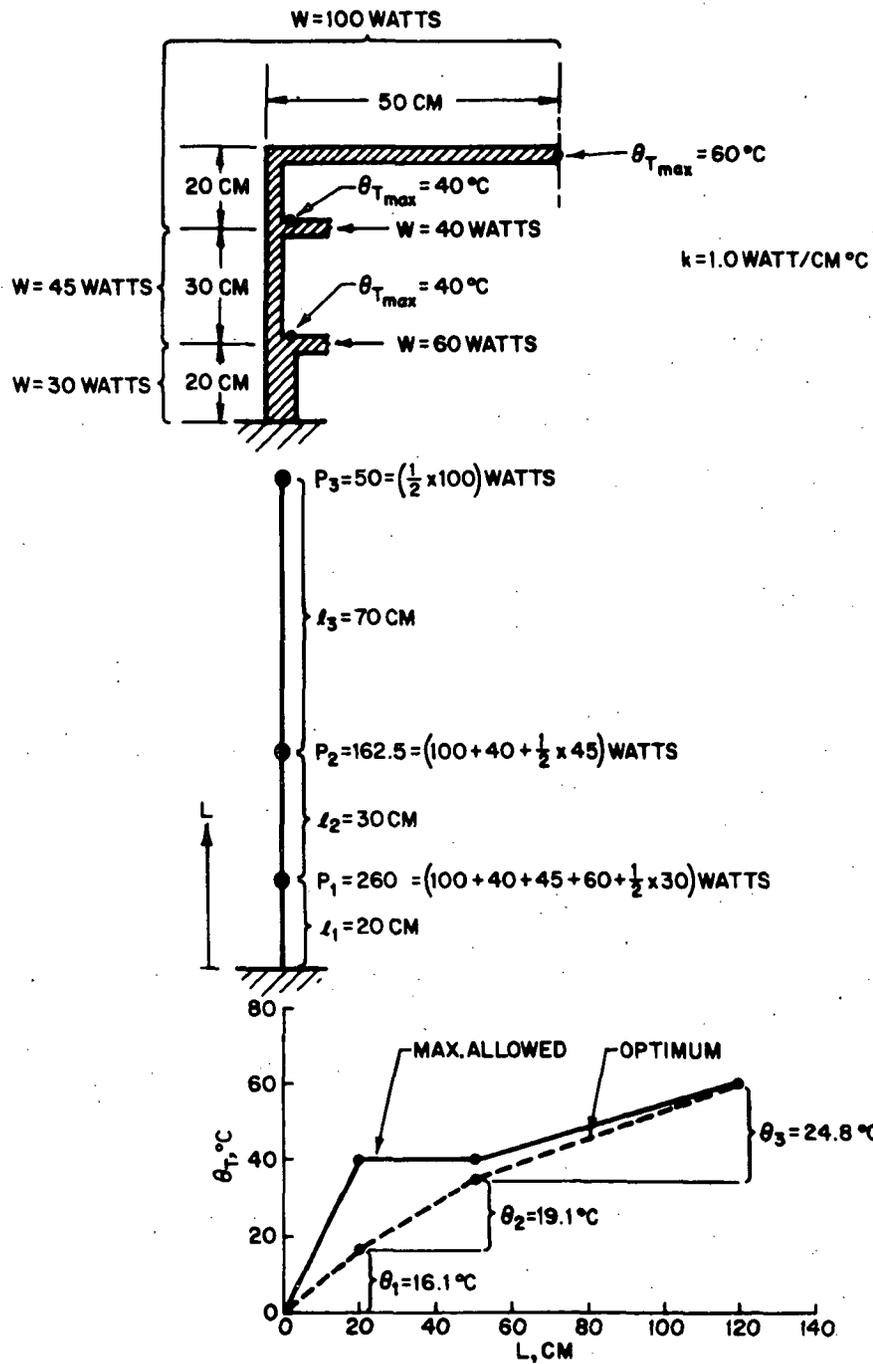


Fig. 5 Example of thermal design of minimum weight, where temperatures below the maximum distribution are allowed.

lower side walls. It also is loaded with two concentrated loads, of 40 and 60 watts, coming from component decks within the casing. It is desired to limit the maximum temperature rise of the casing to 60°C , and those of the component deck edges to 40°C . The center of Fig. 5 shows a thermal equivalent circuit for the situation where all of the heat loads have been appropriately represented as concentrated loads. The lower portion of the Figure shows, as a solid line, the allowed maximum temperature distribution. The dashed line represents the distribution which results from applying equations (15) and (10) to produce a design of minimum material. Arriving at this point, a designer might use this temperature distribution as a basis for refining the estimate of ambient heat loadings, and proceed to recompute. Assuming that recomputation was not necessary in this case, one may proceed to the determination of the thermal cross-sections, or areas of the conductor legs. Using (16), the area is found to be 323 cm^2 , and using (11), the other areas are found to be $A_2 = 255\text{ cm}^2$ and $A_3 = 141.4\text{ cm}^2$, based on a material (such as a magnesium alloy) having a thermal conductivity of $k = 1.0\text{ watt/cm}^{\circ}\text{C}$. Thus, if the structure of upper Fig. 5 were 100 cm long, the wall thicknesses required of the casing would be 3.23 cm in the lower side wall, 2.55 cm in the middle side wall, and 1.41 cm in the upper side wall and top.

The example just given is one wherein the allowed temperature distribution lies above that corresponding to an optimum, or minimum material design based on the maximum allowed temperature for the point in the system most remote from the thermal sink. It is clear that a design based on the most remote point cannot always be used. A situation where it cannot is illustrated schematically in Fig. 6. The allowed maximum temperature distribution of the loads is indicated by the solid line, and that of a minimum material conductor design based on the temperature of load 3 is shown by the upper dashed line. This design would not suitably cool load number 2, and so could not be used. However, a minimum conductor material design could be prepared for the first and second legs of the circuit, as indicated by the lower dashed line. Thereafter, the third leg is simply designed using equation (16) based on P_3 , l_3 , and θ_3 .

After obtaining a little experience with the characteristic shape of optimum design temperature distributions, there is no difficulty in estimating which portions of a conductor are amenable to achieving a minimum weight design through optimizing the temperature distribution. Those portions which are not are then point-by-point designed, using equation (16) applied for the conditions of the required load and temperature distributions. In general, one may note that the optimum distribution is always convex upward so that portions of a required distribution which plot concave upward may require step-by-step analysis using (16) to obtain their minimum sizes. If one prefers a more analytic approach, it is easily shown that for any two-leg two-load segment of a distribution, the material required can be reduced by manipulating θ_j only if the required temperature distribution satisfies,

$$\left(\frac{\theta_{j+1}}{\theta_j}\right) < \left(\frac{l_{j+1}}{l_j}\right) \sqrt{\frac{P_{j+1}}{P_j}} \quad (17)$$

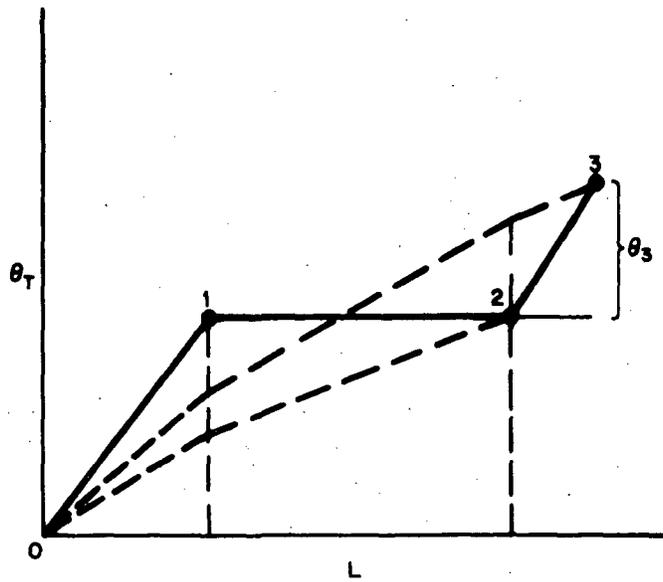


Fig. 6 Example of an allowed maximum temperature distribution (solid line) for which an optimum design based on θ_{T3} (upper dashed line) is unsatisfactory.

Otherwise, it is necessary to use a step-by-step design of the two legs to satisfy the required temperature distribution.

Another instance wherein the principles of designing for minimum weight can be useful is that of comparing the weight or material requirements of alternate structural configurations. As an example of this, consider the two configurations shown in Fig. 7. On the left is shown a portion of a structure which serves both as a casing and chassis-deck support for an equipment. The components of the equipment load the deck with 200 watts, while the casing surfaces receive an ambient heat loading which totals 200 watts. On the right is shown an alternative, wherein the casing and the chassis are not integral, and have a thermal insulating gap between them. The thermal equivalent circuits and the temperature distributions are shown in the lower portion of the illustration. Calculation for a thermal conductor of $k = 1.0 \text{ watt/cm}^{\circ}\text{C}$ indicates that the integral configuration would require a total of $14,530 \text{ cm}^3$ of thermal conductor material if an optimum design were used, while the configuration on the right would use about $13,000 \text{ cm}^3$ of the same material by using an optimum design for the casing, and a separate one for the chassis. The saving of over ten per cent in the latter instance results from the separation of the loads into two thermal circuits, where one of them (the chassis) can be operated at a higher, and therefore more efficient, value of thermal gradient. The example serves to illustrate the fact that quite similar looking configurations in equipment structures can be somewhat different in their thermal characteristics. The second configuration, in particular, reveals that over $11,000 \text{ cm}^3$ of the total material is used in the casing to handle the ambient heat loading, suggesting that an insulated casing using an external conductor skin at a maximum external temperature substantially in excess of $\theta_T = 60^{\circ}\text{C}$ could produce a much lighter design. Such design freedom applies only where a heavy metal casing is not needed for reasons other than thermal design, such as structural rigidity or strength.

Conclusion

The preceding analysis and examples have shown how to achieve a minimum weight design in a series-loaded thermal conductor, based on the manipulation, where such is possible, of the temperature distribution with length along the conductor. The technique is applicable to situations which conform to equation (17), and which permit "overcooling", or lower temperatures than the maximum prescribed for portions of the conductor. Where the situation in a two-leg conductor does not conform to (17), the design of the legs to provide the prescribed maximum temperature distribution is in fact the minimum weight design.

It can be quite important to explore for the minimum weight design in a series-loaded conductor for aerospace applications, where the weight of a structure is critical. Otherwise, the application of one leg at a time design procedures to conductors having

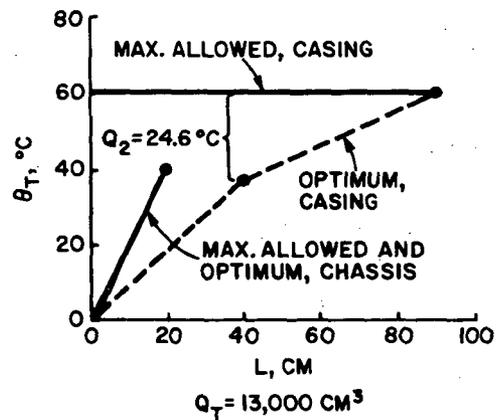
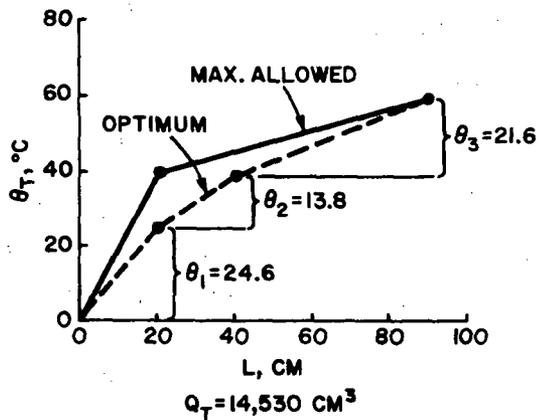
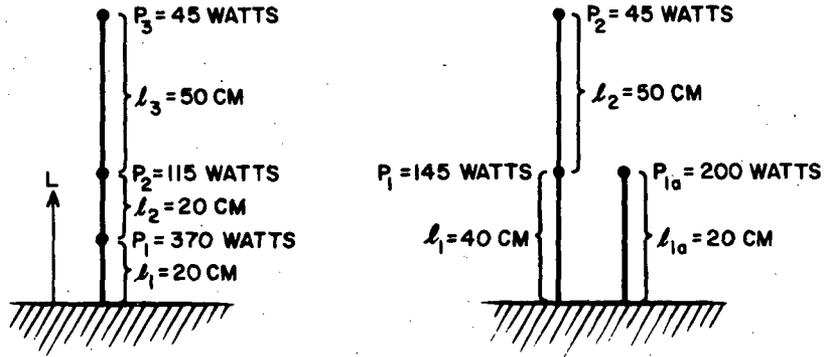
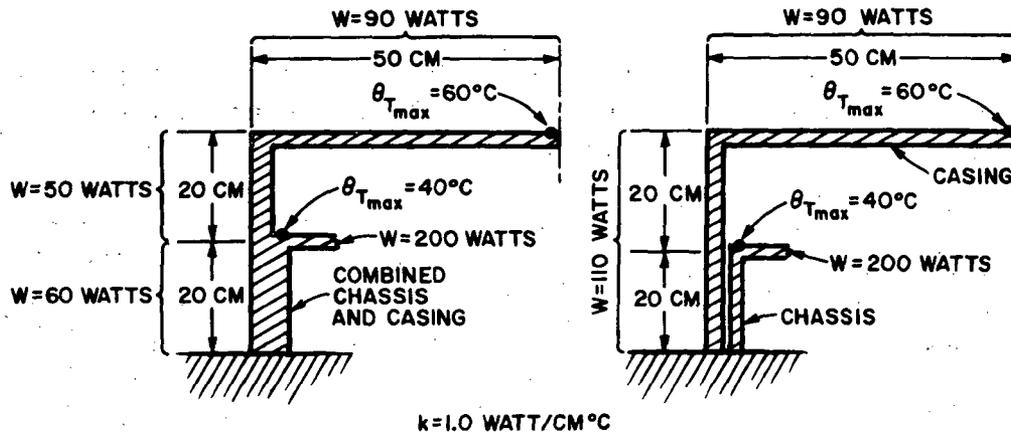


Fig. 7. Examples showing the optimum thermal design requirements of two functionally similar structures.

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small prescribed maximum temperature increments can lead to excessively large conductors. As an extreme example, the conduction of heat between two loads at the same temperature would require infinite material in the conductor. The formulas and their application are sufficiently straightforward to permit the easy establishment of thermal design requirements, which may be compared to those arising from other design considerations. The basic requirement for applicability is that a conductor be representable as one-dimensional, and that the thermal resistances in the system other than those involved with heat transport in the conductor may be accounted for, so as to permit the assignment of meaningful values of θ to the transport problem.

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