**APOLLO SPACECRAFT SOFTWARE CONFIGURATION CONTROL BOARD**

**PROGRAM CHANGE REQUEST**

<table>
<thead>
<tr>
<th>No.</th>
<th>941</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Completed by FSB)</td>
<td></td>
</tr>
</tbody>
</table>

**1.0 COMPLETED BY ORIGINATOR**

| 1.1 ORIGINATOR: J. Suddath |
| 1.2 ORGANIZATION: (EG&G) GCD |
| 1.3 EFFECTIVITY: LUMINARY 2 |
| 1.4 TITLE OF CHANGE: Landing Radar Pre-Filter |

**1.5 REASON(S) FOR CHANGE:**
Smooth rough terrain effects out of LR data

**1.6 DESCRIPTION OF CHANGE:**
Technique uses Landing Radar Data and LGC state vectors to construct a least squares straight line fit of the lunar terrain profile. The straight line fit of the terrain will smooth out local terrain anomalies, and provides an estimate of general slope. Details of the technique are given in Lockheed Electronics Co. document LEC/ADM/122.

**2.0 SOFTWARE CONTROL BOARD OR FLIGHT SOFTWARE BRANCH DECISION FOR VISIBILITY IMPACT ESTIMATE BY MIT**

| 2.1 □ APPROVED □ DISAPPROVED |

**2.2 REMARKS:**

**2.3 SOFTWARE CONTROL BOARD OR FLIGHT SOFTWARE BRANCH SIGN OFF:**

**DATE:**

**3.0 MIT VISIBILITY IMPACT EVALUATION:**

**3.1 SCHEDULE IMPACT:**

**3.2 IMPACT OF PROVIDING DETAILED EVALUATION:**

**3.3 STORAGE IMPACT:** 80 WORDS

**3.4 REMARKS:**

**3.5 MIT COORDINATOR: **

| 3.6 MIT COORDINATOR: R.T. Larson |
| DATE: 9-22-69 |

**4.0 SOFTWARE CONTROL BOARD ACTION**

| 4.1 IMPLEMENT AND PROVIDE DETAILED CHANGE EVAL. |
| PROVIDE DETAILED CHANGE EVALUATION □ DISAPPROVED |

**4.2 REMARKS:**

**4.3 SOFTWARE CONTROL BOARD SIGN OFF:**

| 4.4 SOFTWARE CONTROL BOARD SIGN OFF: 10-9-69 |

**5.0 MIT DETAILED PROGRAM CHANGE EVALUATION**

| 5.1 MIT COORDINATOR: |
| DATE: |

**5.2 MIT EVALUATION:**

**6.0 SOFTWARE CONTROL BOARD DECISION ON MIT DETAILED PROGRAM CHANGE EVALUATION**

| 6.1 START OR CONTINUE □ DISAPPROVED OR STOP IMPLEMENTATION |

**6.2 REMARKS:**

**6.3 SOFTWARE CONTROL BOARD SIGN OFF:**

| 6.4 SOFTWARE CONTROL BOARD SIGN OFF: |

**DATE:**
July 23, 1969

National Aeronautics and Space Administration
Manned Spacecraft Center
Houston, Texas 77058

Attention: K. J. Cox, EG23

Subject: LANDING RADAR DATA LEAST SQUARES PREFILTER

Gentlemen:

This memorandum contains the derivation and discussion of a least squares method of preprocessing LM powered descent landing radar data to improve altitude estimation. Powered descent simulations using a three dimensional simulator equivalent to the simulator presented in Ref. 1 have been carried out over the Censorinus and Fra Mauro landing site approach terrains. These simulations have been made to compare the altitude estimating capability of the prefilter with the conventional single measurement method; results will be presented in a future memorandum.

Very truly yours,

H. L. Rozendaal
Guidance, Navigation and Control Section

References:

The success with which landing radar (LR) altimeter data can be used to improve the estimated LM altitude above the landing site (LS) during powered descent is highly dependent upon the characteristics of the approach terrain. While the present method of improving altitude estimates by utilizing single LR estimates has proven adequate for descents over smooth terrain, simulated descents over rough approach paths have shown this technique to be somewhat less than successful, particularly when the approach takes place over terrain which exhibits a general slope. This memorandum describes a method which utilizes a number of LR measurements to obtain a smoothed LM altitude measurement which (1) tends to average out local anomalies in the approach terrain and (2) takes into account the general terrain slope, if it exists. This method has been designated as a LR data prefilter since the data is processed to obtain a smoothed altitude measurement prior to passage through the altitude updating filter. Results of simulated powered descents into the Censorinus and Fra Mauro landing sites with and without the prefilter have been obtained and will follow in a later memorandum.
DERIVATION OF EQUATIONS

Let the mean terrain line be given with respect to an estimated reference altitude, $h_1$, by the equation

$$ S(r) = A + B (r-r_1) $$

$r_1 = \text{neg. range corresp. to } h_1$. 

(1)

Now let $n$ denote the number of LR altitude measurements on which the mean terrain line is based.
From Eqn. 1 and Figure 1, it follows that

\[ S(r_n) = A_n + B_n(r_n - r_1) \] (2)

Let \( \Delta h_n \) be given by

\[ \Delta h_n = - \int_{t_1}^{t_n} h \, dt \] (3)

where \( h \) is obtained from PIPA readouts. If the assumption is made that the straight line given by Eqn. (2) passes through the LS, the altitude of the LM at time \( t_n \) can be estimated from the following expression:

\[ h_n = S(r_n) - B_n r_n - \Delta h_n \] (4)

The constants \( A_n \) and \( B_n \) are now obtained by performing a least squares curve fit of the LR data taken over the terrain segment from \( r_1 \) to \( r_n \).

(See Figure 2, next page)
FIGURE 2 - MEAN TERRAIN LINE COMPUTATION

In Figure 2:

\( r_i \) - estimated downrange distance at time \( t_i \)

\( \Delta h_i \) - integrated vertical distance over time interval \( t_1 \leq t \leq t_i \)

\( q_i \) - altitude component of the LR altimeter beam measurement at time \( t_i \)
For simplicity let

\[ Y_i = \Delta h_i + q_i \]  

(5)

The least squares line of the terrain with respect to the reference altitude \( h_1 \) (and consequently \( A_n \) and \( B_n \)) is now easily obtained as follows:

Let

\[ y_i = A_n + B_n (r_i - r_1) \]  

(6)

Define the function \( F \) to be minimized by

\[ F = \sum_{i=1}^{n} (y_i - Y_i)^2 = \sum_{i=1}^{n} (A_n + B_n (r_i - r_1) - Y_i)^2 \]  

(7)

To minimize \( F \), it is necessary that

\[ \frac{\partial F}{\partial A_n}, \frac{\partial F}{\partial B_n} = 0 \]  

(8)

Performing the indicated partial differentiation and solving for \( A_n \) and \( B_n \) gives the following expressions:
If Eqns. (9) and (10) are substituted into Eqn. (2), and Eqn. (2) is then substituted into Eqn. (4), simplification of the resulting expression gives the estimated altitude above the LS at time $t_n$.

$$\hat{h}_n = Y - B_n \bar{r} - \Delta h_n$$

(11)

where

$$Y = \frac{1}{n} \sum_{i=1}^{n} Y_i$$

$$\bar{r} = \frac{1}{n} \sum_{i=1}^{n} r_i$$

(12)
The least squares altitude estimate given by Eqn. 11 is used to update the estimated position vector in the platform system in the same manner as the current single measurement updates are performed.

\[ \delta q = h_n - h' \]  

(13)

\[ \bar{r}'_p_N = \bar{r}'_p_o + W \delta q \bar{u}_{hp} \]  

(14)

Where

\[ h' = |\bar{r}'_p_o| - |\bar{r}_{LS}| \]

\[ |\bar{r}_{LS}| = \text{Position vector of the landing site in platform coordinates} \]

\[ \bar{u}_{hp} = \frac{\bar{r}'_p_o}{|\bar{r}'_p_o|} \]

\[ \bar{r}'_p_o = \text{Estimated position vector before the update} \]

\[ \bar{r}'_p_N = \text{Estimated position vector after the update} \]

\[ W = .35 \left(1.0 - \frac{h'}{50,000}\right) \]  

(15)
Discussion of the Method

The major pitfalls inherent in the prefilter presented herein and some possible curative methods are as follows:

1. While it is apparent that the prefilter discussed in the previous section will reduce the dependency of the altitude measurement on local terrain anomalies and tend to account for general terrain slopes, it is equally apparent that the ability of the prefilter to improve altitude estimation is still terrain dependent. This dependency arises because of the assumption that the mean terrain line (the least squares straight line fitted to the last n LR measurements) passes through the LS (LS in platform system). If the general terrain slope changes drastically during the descent then the assumption that the terrain slope based on the last n measurements can be extended to the LS and used to determine the current smoothed altitude measurement is clearly in error. However, drastic changes in the calculated general terrain slope should not occur if n is sufficiently large before the prefiltered altitude measurement is used to update the estimated altitude since LR lock on does not occur until approximately 90 n.m. from the LS. Minor changes in the general slope can be accommodated by periodically reinitializing the sums employed in the prefilter formulas. While a completely recursive prefilter would probably be more effective in accommodating variations in general terrain slope the additional logic and storage requirements make this approach undesirable within the constraints dictated by the present LM computer.
Discussion of the Method (Continued)

(2) Since the prefilter utilizes a least squares smoothing approach, the effect of each LR measurement on the smoothed measurement tends to decrease as the total number of measurements contributing to the relevant sums increases. At best, this gain degradation eventually leads to a steady state error in the altitude estimate. Periodic reinitialization of the measurement sums provides a suitable method for circumventing this problem.

(3) The LR measurement smoothing necessarily introduces a time lag into the estimate updating process. For this reason the prefilter process should be discontinued in favor of single LR measurement altitude updating for the final portion of the descent trajectory. To date, discontinuing use of the prefilter when the estimated altitude falls below 1500 feet has given good results.

The LR data prefiltering method discussed herein has been incorporated into a three dimensional powered descent simulator which is essentially equivalent to the simulator documented in Ref. 1. The only changes necessary to the Ref. 1 simulator are modifications to the estimated state updating routine, STUPDT. Storage requirements for the state update routine utilizing the original single measurement altitude updating approach are approximately 580 decimal words vs. approximately 800 words utilizing the least squares prefilter. Execution time differences have not been calculated but are felt to be insignificant.
Summary

This memo contains a series of viewgraphs prepared for presentation at the design-review meeting held at MSC on February 3, 1970 to consider various "terrain-filtering" schemes for use in the powered landing maneuver. The major results and recommendations are:

1. A simple a-priori terrain model should be stored in the LGC for use during the approach phase. This results in a smoother LPD-angle profile, and reduces LPD pointing errors.

2. The altitude weighting function during the braking phase should be reduced from its present value to limit pitch-angle oscillations of the thrust vector.

3. It is not necessary to include a LR prefilter in the navigation system. Essentially the same effect can be obtained by properly shaping the altitude weighting function.

4. Essentially the same weighting functions as in the present system should be used during the approach phase in order to keep altitude estimation errors small.
On the basis of statistical simulation results it was found that the best choice of weighting functions, in a minimum mean-squared error sense, was dependent on the altitude-variation characteristics of the actual approach terrain to the site and the a-priori models in the LGC. To provide flexibility under these conditions, a two or three segment altitude weighting function should be used.

It is possible to obtain useful estimates of terrain-datum uncertainty with a fairly simple slope estimator, based on the correlation between terrain-datum error and altitude measurement error ($\tilde{h} - h'$). This estimator can, in certain cases, improve the approach-phase trajectory.

The altitude reasonableness test lockout level must be sufficiently high at the start of P64 so that good LR data are not permanently inhibited.

General Information

The problem is concisely defined in Fig. 1. Of particular importance is the fact that it is desired to estimate not local altitude, but rather altitude w.r.t. to the landing site. A major source of difficulty is that the LR measures range to a point on the ground along the range beam rather than range to the landing site.

The key assumptions are given in Fig. 2. The LR acquisition altitude of 35,000 feet for range and 30,000 feet for velocity were considered to be reasonable numbers with the current LR dropout boundaries. The altitude-data reasonableness test was removed for
this study to avoid problems caused by the inhibiting of LR data after High-Gate, if the estimation errors are not sufficiently small at that time.

The basic criteria used to evaluate the navigation system performance during the landing maneuver are given in Fig. 3. In essence, if the altitude estimation errors w.r.t. the site are sufficiently small about one minute before High-Gate and are held down thereafter, then all the various performance indicators will be satisfied. The most important of the performance indicators were found to be the LPD profile, the LPD pointing error, the terminal altitude vs range to go profile, and the thrust-vector elevation-angle profile.

Models for the terrains used in the landing study are given in Fig. 4 as a function of range-to-go, and in Fig. 5 as a function of time. Zero time in Fig. 5 is taken as the time that the full-thrust-position command is nominally issued. The terrain profile seen by the navigation system (Fig. 5) is significantly different from the distance profile (Fig. 4) because of the wide range of vehicle velocities during the landing maneuver. The basic effect is that the number of measurements over a given terrain segment increases as the vehicle slows down en route to the site.

The primary LR data filtering techniques investigated are outlined in Fig. 6. The basic relations for implementing these schemes are given in Figs. 7-10. It should be noted that a terrain-slope estimation option is provided for both Filter #3 (Fig. 9) and Filter #4 (Fig. 10). Also, the values of n and w shown for Filter #3 were preliminary values.

Use of A-Priori Terrain Models in LGC

The possibility of storing a-priori terrain-altitude variation models in the LGC was studied in detail for Censorinlus B and C landing sites. The basic models used for the a-priori terrains are shown in Figs. 11 and 12. The major results of the study are summarized in Fig. 13.
An important advantage of using prestored terrains is that the LPD profiles are improved and the pointing errors are reduced. This is demonstrated in Figs. 14 and 15 for Censorinus, using the present navigation system. The advantage of a prestored terrain is shown also in Fig. 16 for a landing at Fra Mauro, using a navigation filter similar to #4-B in Fig. 10.

It should be noted in Fig. 13 that it is suggested that the terrain seen along the range beam should be stored, rather than the local terrain variation. Storing the local terrain can introduce significant errors at long ranges from the site, where the range beam may be 30-40 degrees from local vertical. It should also be noted that it is suggested that the terrain be stored as a function of the down-range distance to the initial site, which will differ from the actual site if redesignations are made.

With the present landing-trajectory targets, it was found that it is better to under-store the terrain rather than to over-store it in the a-priori model, particularly during the approach phase. The important point here is that navigation errors which indicate that the vehicle is higher than it actually is should be avoided.

Performance Comparisons of Candidate Navigation Systems

In order to examine the performance of the various navigation filters described in Fig. 6, a series of simulated landing trajectories were flown over Censorinus B and C with selected combinations of terrain-datum uncertainty, initial velocity estimation errors, and initial altitude estimation errors. Cases with both simple and more complete a-priori terrain models were studied, along with cases where no stored terrains were used. The numerical values used in the test runs were 1 degree for terrain-datum uncertainty, 10 f/s for vertical-velocity estimation error at DDI, and about 2000 ft. for altitude estimation error at the time of LR range acquisition.

A typical set of test data is given in Figs. 17-20 for a Censorinus-C landing with a 1-degree terrain-datum uncertainty, using various candidate navigation filters in combination with a simple
a-priori terrain. Thrust-vector elevation profiles are compared in Fig. 17, LPD characteristics in Fig. 18, and terminal altitude vs. range-to-go profiles are shown in Fig. 19. An evaluation of the relative merits of the different filters based on these data is given in Fig. 20. Also presented in Fig. 20 are comparison data on required ΔV, High-Gate altitude, and LM velocity at an altitude of about 500 feet w.r.t. the site.

The question as to whether a terrain-datum uncertainty should be estimated, is considered in Fig. 21 for Filter #4-B. It has been found that the long baseline uncertainty can be estimated with varying degrees of success. Small local slopes, on the other hand, could not be usefully estimated without introducing other problems. Two extreme cases are shown in Fig. 21: the cases where the terrain-datum estimator most significantly help and hurt the navigation system performance. It should be noted that with a simple a-priori terrain, the major effect is on the terminal altitude or range-to-go profile.

The major results and recommendations are summarized in Fig. 22.
Fig. 1:  **Definition of Problem**

- Estimate altitude of LM w.r.t. landing site
- Primary sensor is LR which measures range to the terrain at which the range beam is pointed
- IMU measures the change in vehicle velocity during powered maneuvers, with alignment & accelerometer bias errors
- At PDI the XP-components of LM position and velocity w.r.t. site are known to about 1000 ft & 1 ft/s (1-sigma)
- Local terrain height variations, terrain-datum uncertainty (slope), and LR measurement noise cause the vertical component of the measurement to differ from altitude w.r.t. the site.
Fig. 2: **Basic Assumptions in Study**

- LR acquisition takes place at $h=35,000$ ft for range measurements and at $h=30,000$ ft for velocity measurements.

- The primary terrain profiles considered were Censorinus B, Censorinus C, and Copernicus.

- Terrain-datum uncertainty taken as 1 degree ($3\sigma$), but with the maximum altitude deviation w.r.t. the site limited to 5000 feet.

- Completely-automatic landing with P63, P64, and P65, using the Apollo-12 reference trajectory.

- No altitude-data reasonableness test or LR dropout boundaries.

- PDI position errors w.r.t. site of 1000 ft down-range and 1500 ft cross-range with N69 (1-sigma); down-range of 4000 ft with no N69 (1-sigma).
Fig. 3: Performance Evaluation Criteria

- Minimize the error in the estimate of LM altitude w.r.t. the landing site: $(r_p - r_m) - (r_t - r_{st})$
- Above error should be reduced to a reasonably small value by HG and kept small thereafter
- If the altitude estimation error is sufficiently small, good values will be obtained for these performance indicators:

<table>
<thead>
<tr>
<th>(1.) ΔV from T1G to TD</th>
<th>(5.) Interval for which LPDang &lt; 57°</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2.) LM velocity at h = 500 ft</td>
<td>(6.) LPD angle profile</td>
</tr>
<tr>
<td>(3.) Thrust-vector elev. angle profile</td>
<td>(7.) LPD pointing error</td>
</tr>
<tr>
<td>(4.) H vs. r_{60}, Last 20,000 ft</td>
<td>(8.) Dead-man's curve cross-over point</td>
</tr>
</tbody>
</table>
Fig. 4: Local Terrain Variation vs Range-to-Go

Censorinus B

Censorinus C

Copernicus
Fig. 5. Local Terrain Variation vs. Time

**Censorinus B**

**Censorinus C**

**Copernicus**
### Fig. 6: LR Data Filtering Techniques Studied

<table>
<thead>
<tr>
<th>Present System</th>
<th>Linear $W_H$ vs $h$, $W_H = 0.35$ at $h=0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Filter #1</td>
<td>Same as present system except $W_H = 0.10$ at $h=0$</td>
</tr>
<tr>
<td>Filter #2</td>
<td>Least-squares smoothing &amp; slope estimation, 30-sample batch processing (LEC Memo ADM/122 by Rozendaal)</td>
</tr>
<tr>
<td>Filter #3 (A)</td>
<td>Simpler &amp; more flexible version of Filter #2 without the slope estimator (T. Moore of MSC, Internal Note MSC-68-EG-05)</td>
</tr>
<tr>
<td>(B)</td>
<td>Slope estimator added to (A)</td>
</tr>
<tr>
<td>Filter #4 (A)</td>
<td>Two segment weighting function: Linear with $h$ from 0 to 0.1 in P63, and linear from 0.1 at start of P64 to 0.25 at $h=0$</td>
</tr>
<tr>
<td>(B)</td>
<td>Simple slope estimator with $W_{sl} = 0.01/N (\text{ft} / \text{mi})$ added to (A),</td>
</tr>
</tbody>
</table>

**Above filters used with:**
1. No prestored terrain model
2. Simple stored terrain model
3. Detailed stored terrain model
Fig. 7: Navigation Equations for Present System

\[ h' = r_p - r_{sp} \]
\[ \delta h = \hat{h} - h' \]
\[ W = LRWH \left(1 - \frac{h'}{LRHMAX}\right) \]
\[ \dot{r}_p = r_p + W \delta h \cdot \mathbf{u}_{hp} \]

Present Erasable Values:
- LRWH = 0.35
- LRHMAX = 50,000 ft

\[ h' = \text{à-priori LM altitude estimate w.r.t. Landing site} \]
\[ r_p = \text{LM position vector} \]
\[ r_{sp} = \text{Landing-site position vector (} r_{ls} \text{)} \]
\[ \hat{h} = \text{LR altitude measurement (slant range projected along loc. vert.)} \]
\[ W = \text{LR altitude weighting function} \]
\[ \mathbf{u}_{hp} = \text{unit vector along local vertical} \]
Fig. 8: Filter #2: Basic Equations (LEC)

\[
\hat{h}_n = \frac{1}{n} \sum_{i=1}^{n} (\hat{h}_i - \Delta \hat{h}_{1,i}) + \frac{S_n}{n} \sum_{i=1}^{n} r_{G_0,i} + \Delta \hat{h}_{1,n,0}
\]

\[
S_n = \frac{\sum_{i=1}^{n} (\hat{h}_i - \Delta \hat{h}_{1,i}) r_{G_0,i}}{\sum_{i=1}^{n} r_{G_0,i} - \frac{1}{n} \left(\sum_{i=1}^{n} r_{G_0,i}\right)^2}
\]

\[
\Gamma_p = \Gamma_p + W (\hat{h}_n - \hat{h}') U_{HP}, \text{ When } n \geq 30
\]

\(\hat{h}_i\) = LR-derived altitude meas.
\(\Delta \hat{h}_{1,i}\) = meas. change in veh. alt. from \(t_i\) to \(t_{i+1}\)
\(r_{G_0,i}\) = est. distance-to-go from vehicle to landing site
\(S_n\) = est. terrain slope

<table>
<thead>
<tr>
<th>Stored Sums Required</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta \hat{h}<em>{1,n}), (\sum</em>{i=1}^{n} r_{G_0,i}), (\sum_{i=1}^{n} r_{G_0,i}^2), (\sum_{i=1}^{n} (\hat{h}<em>i - \Delta \hat{h}</em>{1,i}) r_{G_0,i})</td>
</tr>
</tbody>
</table>

Note that:
(1) Old data are removed from the stored sums periodically, i.e. in 30-sample batches
(2) Present updating scheme used when \(\hat{h}' < 1500\) ft.
Fig. 9: Filter #3: Navigation Equations (T. Moore)

A. Smoothing Only

\[ \delta h^* = (1-W) \frac{(n-1)}{n} \delta h^* + \frac{1}{n} (\hat{h}' - \hat{h}) \]

\[ m = m + 1 \]

\[ \Gamma_p = \Gamma_p + W \delta h^* \alpha_{HP} \]

<table>
<thead>
<tr>
<th>m</th>
<th>n</th>
<th>W</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-5</td>
<td>m</td>
<td>0.20</td>
</tr>
<tr>
<td>6-29</td>
<td>5</td>
<td>0.20</td>
</tr>
<tr>
<td>30-∞</td>
<td>14</td>
<td>0.05</td>
</tr>
</tbody>
</table>

B. Smoothing and Slope Estimation

\[ \bar{R} = \frac{(nC - 1)}{n} r_{G0} \]

\[ W = 0.05, \quad C = 0.04 \]

\[ \delta s = \bar{R} \left[ (\hat{h}' - h' - s r_{G0}) - (1-W) \delta h^* \right] \]

\[ \delta h^* = (1-W) \frac{(n-1)}{n} \delta h^* + \frac{1}{n} (\hat{h}' - h' - s r_{G0}) - \delta s r_{G0} \]

\[ s = s + \delta s \]

\[ \Gamma_p = \Gamma_p + W \delta h^* \alpha_{HP} \]

\[ m = m + 1 \]

<table>
<thead>
<tr>
<th>m</th>
<th>n</th>
<th>\delta s</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-13</td>
<td>m</td>
<td>0</td>
</tr>
<tr>
<td>14-∞</td>
<td>14</td>
<td>as computed</td>
</tr>
</tbody>
</table>
A.) No slope estimation

\[ W_H = LRWH \left( 1 - \frac{h'}{LRHMAX} \right) \]
\[ h' = r_p - r_{5p} \]
\[ \delta_h = \hat{h} - h' \]
\[ r_p = r_p + W_H \delta h \ U_{HP} \]

B.) With slope estimation

\[ W_{SL} = \frac{0.01}{n} \left( \frac{f_{nm}}{ft} \right) \]
\[ h' = r_p - r_{5p} - s \ M_z \]
\[ r_z = r_{zp} - r_{szp} \]
\[ s = s + W_{SL} \delta h \]
\[ n = n + 1 \]
Fig. 11: Stored-Terrain Models for Censorinus-B

**Simple Model**

**More Detailed Model**
Fig 2: Simple Stored Terrain Model: Censorinus-C

More Complete Stored Terrain Model: Censorinus-C
Fig 13: Use of Stored Terrain Models: Result Summary

- Primary effect is improved LPD-angle profile and reduced LPD pointing error in approach phase.
- Simple terrain models for Censorinus-B and C adequate for above purpose; more important to store accurately in approach phase than to model in more detail in braking phase.
- Store terrain as seen along the range beam on a nominal trajectory; store as a function of the down-range distance to the initial landing site, i.e., vs. \( r_{zp} - r_{szp} \).
- Down-range position estimation errors of up to 4000 ft did not significantly effect the LPD-angle characteristics.
- With the present targets it is desirable to under-store to minimize trajectory drooping.
Fig. 14: LPD Characteristics With & Without Stored Terrain

present system, Censorinus B, simple stored terrain
+1 deg slope, -10 f/s v. vel. error

LPD ANGLE (DEG)

with stored terrain
no stored terrain

ST-SC (FT)

LPD Pointing Error

RUN NO. = 718
Fig 15: Present System with Various Terrain Models

Censorinus-C, -1-deg slope (up-hill)
-10 f/s vert. vel. est error at PDI

LPD Angle

- Detailed Model
- Simple Model
- No Model

LPD Painting Error

- No Model
Fig 16: Use of Stored Terrain for Fra Mauro Landing

Nominal, error-free trajectory for Fra Mauro

LPD Angle

LPD Pointing Error
Fig. 17: Thrust-Vector Profiles with Various Filters

Censorinus-C
Simple terrain model
1-deg slope (down-hill)
+10f/s vert. vel. est err. PDI

- ELEV. ANG. (DEG)

- THRUST (LBS)

TIME (SEC)
Fig. 18: LPD Angle & LPD Pointing Error

for Different LR Filters

Censorinus-C, Simple Stored Terrain Model,
+1 Deg Slope (down-hill), +10 f/s vert. vel. est. error PDI
Fig. 19: Altitude vs. Range-to-Go for Various Filters

Censorusus-C, Simple Terrain models

+1 degree slope, +10 ft/s vert vel est. err at PDI
Fig. 20: Censorinus-C, Simple Stored Terrains

- 1 degree slope
- 10 f/s vert. vel. est. error at PDI
- Slope = down-hill to site
- Vert. vel. error: vehicle low

<table>
<thead>
<tr>
<th>Filter</th>
<th>Run No.</th>
<th>ΔV (f/s)</th>
<th>HG Alt at Site (ft)</th>
<th>Vel at 500 ft (deg)</th>
<th>LPD Ang. Chars</th>
<th>Terminal H vs. RGO</th>
<th>Thrust Angle Profile</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present System</td>
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<td>Good</td>
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<td>-13.9</td>
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<td>Good</td>
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</table>

Comments:
1. Runs 709 and 737 had the same ΔV at h=500 ft, Run 737 had an alt. est. error of 65 ft at this point whereas Run 709 had 21 feet.
2. Present system oscillatory in thrust angle in P63, otherwise fairly good.
3. Filter #3-A corrects altitude est. error too slowly, too high in P64.
4. Filter #3-B too oscillatory in thrust-vector & LPD angles.
5. Most favorable situation for #4-B slope estimator.
Fig. 21: Performance of Simple Slope Estimator
Censorinus-B, Simple Stored Terrain, Filters #4-A/#4-B

Best Case: +1 deg slope
+10 ft/s vel. error

Worst Case: -1 deg slope
+10 ft/s vel. error

H vs. RGO

#4-A

#4-B (slope est.)
Summary of Results and Recommendations
(Based primarily on Censorinus B and C data)

- Store simple terrain model for approach phase to improve LPD profile and reduce LPD pointing errors.

- Reduce altitude weighting function in braking phase to decrease pitch-angle oscillation, e.g. linear to 0:1. Trade-off terminal-phase trajectory constraints vs. pitch oscillations.

- Use weighting functions similar to present in approach phase to keep altitude estimation errors small.

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<th>Filter Type</th>
<th>Pitch Profile</th>
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<td>Fair</td>
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<td>#3-A</td>
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<td>Poor</td>
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<td>#3-B</td>
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<td>#4-B</td>
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<td>Fair-Good</td>
<td>Good</td>
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Mission Simulation Memo # 4-70

TO: Distribution
FROM: Bernard Kriegsman and Donald Gustafson
DATE: February 23, 1970

SUMMARY

An extensive study has been made of navigation systems for use during the powered landing maneuver when the terrain en-route to the site is rough. A technical presentation on this subject was made at MSC on February 3, 1970. The data used in the formal presentation are given and discussed in Ref. 1. The present memo gives additional data used in obtaining the results and conclusions of Ref. 1.

For convenience, the data are presented in a viewgraph format. The general background material and assumptions are given in Ref. 1, and, therefore, are not repeated here. It should be emphasized that this memo is intended as a supplement to Ref. 1.

REFERENCE

GENERAL INFORMATION
Fig. 1: Simulation Data

The following figures contain the key results of the LR-filter simulation study. The curves presented here are of five general types:

1. Thrust-vector elevation angle vs. time
   Thrust magnitude vs. time

2. Error in estimate of vehicle altitude w.r.t site vs. time

3. Computed LPD angle vs. time
   Displacement between actual landing point and the one indicated by the LPD vs. time

4. Altitude vs. range-to-go for the last 16,000 feet down range from the site

5. Local terrain altitude variation including slope vs. time
   Prestored terrain, and estimated terrain deviation by filter vs. time
Fig2: Simulation-Result Notes

(1.) Positive slopes are down-hill to the landing site
(2.) Slope contributions to terrain deviation are limited to 5000 feet
(3.) Zero time is at the time that the full-thrust-position command is first issued, i.e. T1G+26 seconds.
(4.) LR acquisition at 35,000 ft altitude for range data and 30,000 feet for velocity-component data
(5.) Altitude-data reasonableness test not included
(6.) Present guidance used unless specifically noted that delta-guidance is used
(7.) Initial velocity errors are along local vertical at PDI
(8.) Estimation errors are defined as the estimated value minus true value, i.e. $\text{err.} = \text{est.} - \text{true}$
(9.) Thrust-vector elevation is w.r.t. local horizontal
(10.) The site altitude error is the LGC altitude estimate minus the true altitude w.r.t. the site
Fig 3: Simulation-Result Notes -- continued

(11.) Flight-path angle (θ) is w.r.t. local horizontal, positive value means vehicle is dropping

(12.) LPD angles shown positive for convenience, values outside lower window boundary (65 deg) not shown

(13.) The altitude-vs-R6O curves have the 12 and 20 deg terminal-constraint boundaries superposed on the frame

(14.) The algebraic sum of the prestored terrain and the filter's estimated terrain deviation should ideally equal the local terrain deviation w.r.t. the site
**Fig. 4: Navigation-Sensor and Initial-Condition Errors**

**IMU (3-sigma values)**
- SM alignment = 2 mrad/axis
- Drift-rate bias = .09 deg/hr/axis
- Accelerom. bias = .0067 ft/s²
- Accel. scale factor = 450 ppm.

**LR (3-sigma values)**
- Range: 1.5% random, 5 ft threshold
- Speed: 1.5% random (VXA), 2.0% random (VYA), 3.0% random (VZA), 0.8 ft/s threshold/axis
- Alignment: 10 mrad/axis

**Initial Errors at PDI⁺ (TIG)**
- RMS 1-sigma values

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<th>LM Pos. (ft)</th>
<th>LM Vel. (ft/sec)</th>
<th>LM Pos. w.r.t. Site (ft)</th>
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<td>XP YP ZP</td>
<td>XP YP ZP</td>
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<td>1567 1011 4115</td>
<td>4.37 1.37 1.42</td>
<td>991 714 3150 1000*</td>
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</tbody>
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* Assumed value with N69 landing site update
† Est. error cov. matrix for H-1 from MSC Memo 69-FM46-334
Fig 5: Clarification of Certain Performance Evaluation Criteria

**LPD Angle Chars.:** Variation during visibility interval, oscillations, minimum value, peak error between true & indicated site.

**Terminal H vs RGO:** Remaining within 12-20 degree tube, drooping of trajectory, particular emphasis on last 500-1000 ft alt.

**Thrust Angle Profile:** Oscillations
Simulation Data for Censorinus B & C with Various Filters

Simple Terrain Models
1-degree slopes
10 ft/s vert vel est errors at PDI
Fig. 6 Censorinus-C, Simple Stored Terrains  
(see Ref. 1 for models)

+1 degree slope
-10 ft/s vert vel. error at PDI

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<th>Filter</th>
<th>Run No.</th>
<th>ΔV (f/s)</th>
<th>HG Alt WRT Site (ft)</th>
<th>Vel at 500 ft Y(deg.)</th>
<th>Speed (f/s)</th>
<th>LPD Ang. Chars.</th>
<th>Terminal H vs. RGO</th>
<th>Thrust Angle Profile</th>
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<td>Poor</td>
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<td>46</td>
<td>Good</td>
<td>Fair-Good</td>
<td>Good</td>
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</table>

Comments: (1.) Relative performance essentially same as for case with 10 ft/s vert vel. error at PDI
(2.) Slope estimator in #4-B estimated about 0.5 deg for slope
Fig 7: Thrust-Vector Profiles for Various Filters
Censorinus-C, Simple terrain models
+ 1 deg. slope (down-hill)
- 10 f/s vert. vel. est. err. at PDI
Fig. 8: LPD Angle & Pointing Errors for Various Filters

Censorinus-C, Simple terrain models
+1 Degree Slope (down-hill)
-10 f/s vert. vel. est. err. at PDI
Fig. 9: Altitude vs. Range-to-Go for Various Filters

Censorinus-C, Simple terrain models
+1 degree slope (down-hill)
-10 f/s vert. vel. est. err. at PDI
**Fig 10: Censorinus-C, Simple Stored Terrains**

-1 degree slope

+ 10 f/s vert vel est error at PDI

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<th>Filter</th>
<th>Run No.</th>
<th>ΔV (f/s)</th>
<th>H.G Alt. W/1st Site (ft)</th>
<th>Vel at 500ft (deg)</th>
<th>Speed (f/s)</th>
<th>LPD Ang. Char.</th>
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<td>Fair</td>
<td>Fair</td>
<td>Good</td>
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</table>

Comments:
1. Run #760 resulted in LM hitting surface at 74 f/s.
2. Altitude est errors corrected too slowly in Run #760.
3. Slope estimator did very little in Run #708.
4. No significant difference between Filters #1, 4-A, & 4-B.
5. Present system slightly better H vs RGO than #1 & 4-A or #4-B.
Fig. 11: Thrust-Vector Profiles for Various Filters

Censorinus-C, Simple terrain models
-1 Degree Slope (up-hill)
+10 f/s Vert. Vel. est. errors at PDI
Fig. 12: Altitude Est. Errors for Various Filters

Censorinus-C
Simple terrain models
-1 deg slope (up-hill)
+10 fps vert. vel. error at PDI
Fig 13  LPD Angle & Pointing Errors for Various Filters

Censorinus-C
Simple terrain models
-1 deg. slope (up-hill)
+1 ft/s Vert. vel. est.error at PDI
Fig 14: Altitude vs. Range-to-Go for Various Filters

Censorinus-C
Simple terrain models
-1 deg slope (up-hill)
+1 ft/s vert vel est error at PDI
**Fig. 15: Censorinus-C, Simple Stored Terrains**

- 1 degree slope
- 10 f/s vert. vel. est. error at PDI

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<th>Filter</th>
<th>Run No.</th>
<th>ΔV (f/s)</th>
<th>HG Alt. W/ft Site (ft)</th>
<th>Vel. at 500 ft. Y (deg.)</th>
<th>Speed (f/s)</th>
<th>LPD Ang. Chars.</th>
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**Comments:**

(1) Less severe case than with 110 f/s vert. vel. error.
(2) Slope estimator in #4-B estimated about 1.2 deg and smoothed out LPD profile.
(3) Filter #3-B very oscillatory thrust elev. profile.
(4) Present system good except for thrust elev. osc.
Fig. 16: Thrust-Vector Profiles for Various Filters

Censorinus-C, Simple terrain model

-1 deg slope (up-hill), -10 ft/s vert. vel. est. err. PDI
Fig. 17: LPD Angle & Pointing Errors for Various Filters

Censorinus-C
Simple stored terrain models
-1 deg slope (up-hill to site)
-1 ft/s vert vel. est. err. at PDI
Fig 18: Altitude vs. Range-to-Go for Various Filters
Censorinus-C, Simple terrain model
- 1 deg slope (up-hill to site)
- 10 ft/s vert vel est. error at PDI
Fig. 19: Performance of Slope Estimator of Filter #4-B.

Terrain: Censorinus C
Slope: -1 deg.
-10 ft/s vert. vel. est err. PDI
Simple terrain model
Fig 19A: Operation of Slope Estimator

Censorinus-C, Simple terrain models
+1-degree slope, +10% vert. vel est. error at PDI

TERRAIN=CENSORINUS C
SLOPE= 1 DEG.

HBI LOCAL = ---
EST. SL. BIAS = ---
STORED TERR. = O--O

ALT. VAR. (FEET)

RANGE-TO-GO (NM)

RUN NO. = 705
Fig. 20: *Censorinus-B* - Simple Stored Terrain

+1 degree slope
+10 ft/s vert. vel. est. error at PDI

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<th>Filter</th>
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<th>HG Alt. w/r.t. Site (ft)</th>
<th>Vel. at 300 ft. (ft/s)</th>
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<td>#3-B (slope ext)</td>
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<td>good</td>
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</table>
Fig 21: Censorinus-B, Simple Stored Terrain

+1 degree slope
-10 f/s vert. vel. est. error at PDI

+ slope = down-hill to site
+ vert. vel. error = vehicle low

<table>
<thead>
<tr>
<th>Filter</th>
<th>Run No.</th>
<th>AV (f/s)</th>
<th>HG Alt w.r.t. Site (ft)</th>
<th>Vel. at 500 ft. Y (deg)</th>
<th>Speed (f/s)</th>
<th>LPD Ang Chars</th>
<th>Terminal H vs. RGO</th>
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Notes
Fig 22: **Censorinus-B**, Simple Stored Terrain

- 1 degree slope
+ 10 f/s vert. vel. est. error at PDR
+ Slope = down-hill to site
+ Vert. vel. error: vehicle low

<table>
<thead>
<tr>
<th>Filter</th>
<th>Run No.</th>
<th>ΔV (f/s)</th>
<th>HG Alt w.r.t. Site (ft)</th>
<th>Vel at 500 ft. (deg)</th>
<th>Speed (f/s)</th>
<th>LPD Ang Chars</th>
<th>Terminal H vs. RGO</th>
<th>Thrust Angle Profile</th>
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<td></td>
<td></td>
</tr>
<tr>
<td>#4-A</td>
<td>803</td>
<td>6579</td>
<td>6301</td>
<td>-13</td>
<td>62</td>
<td>poor</td>
<td>fair</td>
<td>good</td>
</tr>
<tr>
<td>#4-B (slope ext)</td>
<td>807</td>
<td>6578</td>
<td>5920</td>
<td>-9</td>
<td>70</td>
<td>poor</td>
<td>fair</td>
<td>good</td>
</tr>
</tbody>
</table>

**Notes**
**Fig. 23: Censorinus-B > Simple Stored Terrain**

- 1 degree slope
- 10 f/s vert. vel. error at PDI

<table>
<thead>
<tr>
<th>Filter</th>
<th>Run No.</th>
<th>ΔV (f/s)</th>
<th>HG Alt w.r.t. Site (ft)</th>
<th>Vel. at 500 ft. (deg)</th>
<th>Speed (f/s)</th>
<th>LPD Ang Chars</th>
<th>Terminal H vs. RGO</th>
<th>Thrust Angle Profile</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present System</td>
<td>812</td>
<td>6594</td>
<td>7668</td>
<td>-9</td>
<td>106</td>
<td>good</td>
<td>fair-mid</td>
<td>fair</td>
</tr>
<tr>
<td>#1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>#3-A</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>#3-B (slope est)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>#4-A</td>
<td>804</td>
<td>6593</td>
<td>7900</td>
<td>-10</td>
<td>101</td>
<td>good</td>
<td>fair-good</td>
<td>good</td>
</tr>
<tr>
<td>#4-B (slope est)</td>
<td>808</td>
<td>6596</td>
<td>8900</td>
<td>-15</td>
<td>60</td>
<td>good</td>
<td>good</td>
<td>good</td>
</tr>
</tbody>
</table>

**Notes**
Fig. 24 Pitch and Thrust Profiles for Present System
+10 f/s v. vel. error, 1 deg slope

TERRAIN = CENSORINUS B

Pitch ang. vs. time (wrt local hor.)

Terrain slope:
--- + 1 deg
-- - 1 deg

Thrust vs. time

ELEV. ANG. (DEG)

TIME (SEC)

THRUST (LBS)

TIME (SEC)
Fig 25: Approach Trajectory Profile - Present System

Censorinus B, +10 f/s v. vel. error
1 deg slopes

ALTITUDE VS. RANGE-TO-GO

ALTITUDE (FEET)

+1 deg slope

-1 deg slope

H vs. RGO

RANGE-TO-GO (FT)

RUN NO. = 717
Fig. 26: LPD Characteristics - Present System

Censorinus B, -10 f/s v. vel. error, 1 deg slopes
simple stored terrain

LPD ANGLE (DEG)

terrain slope:
- + 1 deg
- - 1 deg

LPD Painting Error

RUN NO. = 717
Fig. 27: Pitch & Thrust Profiles for Pres. System & Filters 4A, 4B

Censorinus B, +1 deg slope, +10 f/s v.rel. error
Simple stored terrain

TERRAIN = CENSORINUS B

ELEV. ANG. (DEG)

TIME (SEC)

THRUST (LBS)

TIME (SEC) BAK DEG RUN NO. = 809

# 4-A

# 4-B

Pres.

thrust vs. time

# 4-A

# 4-B

Pres.
Fig 28: Approach Trajectory Profile with Pres. System, Filters 4A & 4B

Censorinus B, +1 deg. slope, +10 f/s v. vel. error
simple stored terrain

ALTITUDE VS. RANGE-TO-GO

ALTIMETER (FEET)

-8000 -4000 0 4000 8000 12000 16000
RANGE-TO-GO (FT)

RUN NO. = 809
Fig. 29: Pitch & Thrust Profiles for Prec System & Filter 4A, 4B.

Censorinus B, -1 deg slope, ±10 f/s v.rel. error
Simple stored terrains

TERRAIN = CENSORINUS B
SLOPE = -1 DEG.

ELEV. ANG. (DEG.)

TIME (SEC)

THRUST (LBS)

TIME (SEC)
Fig. 30: Approach Trajectory Profile for Pres. System Filters 4A & 4B

Censorinus B, -1 deg. slope, +10 f/s v.vel. error
Simple stored terrain

ALTITUDE VS. RANGE-TO-GO

ALTIMETER (FEET)

-8000 -4000 0 4000 8000 12000 16000
RANGE-TO-GO (FT)

RUN NO. = 811
Fig. 31  Performance of Simple Slope Estimator
Censorinus-C, Simple Stored Terrain, Filters #4-A & 4-B

Best Case: +1 deg. slope
+10 f/s. v.vel.err

H vs. RGO

Worst Case: -1 deg. slope
+10 f/s. v.vel.err

H vs. RGO

#4-A

#4-B
(slope est.)

#4-A & B

ALTITUDE (FEET)

0

1000

2000

3000

4000

5000

6000

RANGE-TO-GO (FT)

0

4000

8000

12000

0

1000

2000

3000

4000

ALTITUDE (FEET)

RANGE-TO-GO (FT)
Stored Terrain Models
Fig 32: Considerations in Use of Stored Terrain Models

- Over what distance span should a terrain model be stored.
- Accuracy of available 2-dimensional terrain models.
- Effect of down-range & cross-range position estimation errors.
- Difference between local terrain and terrain along range beam.
- How should the terrain model be stored and how accurately.
- Effect of site redesignations on stored models.
- Capability of removing stored model in flight.
- Zeroing the weighting function in certain high-slope regions.
Fig 33: Stored-Terrain Models for Censorinus-B

Simple Model

More Detailed Model
Simple Stored Terrain Model: Censorinus-C

More Complete Stored Terrain Model: Censorinus-C
Fig. 35: Present System with Various Terrain Models

Censorinus-C, 1-deg. slope (down-hill)
10 f/s vert. vel. est. error at PDI

Thrust-Vector Elevation

<table>
<thead>
<tr>
<th>ELEV. ANG. (DEG)</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
</tr>
<tr>
<td>60</td>
</tr>
<tr>
<td>40</td>
</tr>
<tr>
<td>20</td>
</tr>
<tr>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TIME (SEC)</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
</tr>
<tr>
<td>300</td>
</tr>
<tr>
<td>400</td>
</tr>
<tr>
<td>500</td>
</tr>
<tr>
<td>600</td>
</tr>
<tr>
<td>700</td>
</tr>
</tbody>
</table>

No Model
Simple Model
Detailed Model
Fig 36: Present System with Various Terrain Models
Censorinus-C, 1-deg slope (down-hill)
10 f/s vert. vel. est. error at PDI

LPD ANGLE (DEG)

LPD POINTING ERROR

No Model
Simple Model
Detailed Model
Fig. 37: Comparison of Simple vs. No Stored Terrain Models

Filter #4-A
Censorinus-C
1-deg. slope (down-hill)
10 f/s vert vel est err. PDI

LPD Angle

LPD Pointing Error

* Similar results with reversed sign of vert vel error; similar results with Filter #4-B
Fig. 38: Comparison of Simple vs. No Stored Terrain Models

Filter #4-A
Censorinus-C

1-deg. slope (down-hill)
10-f/s vert. vel. est.err. PDI

ALTITUDE VS. RANGE-TO-GO

ALTITUDE (FEET)

RANGE-TO-GO (FT)
Fig. 39 Present System with Various Terrain Models

Censorinus-C, -1-deg slope (up-hill)
-10 f/s vert. vel. est. error at PDI

ELEV. ANG. (DEG)

No Model
Detailed Model

THRUST (LBS)

No Model
Detailed Model
Fig 4.0: Comparison of Simple vs. No Stored Terrain Models

**Filter #4-A**  
- 1 deg slope (up-hill)

**Censorinus-C**  
- 10 fps vert. vel. est. error PDI

**LPD Angle**

**LPD Pointing Error**
Fig 41: Comparison of Simple vs. No Stored Terrain Models

Filter #4-A
-1 deg. slope (up-hill)

Censorinus-C
-10 ft/s vert. vel. est. error PDI

ALTITUDE VS. RANGE-TO-GO

[Graph showing altitude vs. range-to-go for different models]
Fig 48: LPD Characteristics With Simple Stored Terrain

Censorium B, +1 deg. slope, +10 f/s v. vel. error
Pres. system, filters 4A & 4B

LPD ANGLE (DEG)

TIME (SEC)

LPD ANGLE (DEG)

ST-SC (FT)

LPD Pointing Error

RUN NO. = 809
Fig 4B: LPD Characteristics With Simple Stored Terrains

Censorinus B, -1 deg. slope, +10 ft/s v.rel. error
Pres. system, filters 4A & 4B

LPD ANGLE (DEG)

ST-SC (FT)

TIME (SEC)

RUN NO. = 803
Effect of Down-Range Position Errors with Simple Terrain Models
Fig 4-4: Effect of Down-Range Pos. Est. Errors

Filter #4-A, Censorinus-C

LPD ANGLE (DEG)

LPD POINTING ERROR

Pos. lst. Errors

no error
4000 ft (short)

no error
4000 ft (short)
Fig 45: Effect of Down-Range Pos. Est. Errors

Filter #4-A, Censorinus-C

-1 deg. slope (up-hill), -10 ft/s vert. vel. est. err.

LPD Angle

LPD Pointing Error
Statistical Results
Fig46: Navigation-Filter Design Approach

(1.) Develop **statistical** models for all important error sources

- IMU modeled with alignment, drift, accel-bias, & scale-factor errors
- LR modeled with random altitude meas. errors, random speed meas. errors, and alignment errors (consistent with LR spec. values)
- Terrain-datum uncertainty modeled as a constant slope, which is treated as a measurement error proportional to range-to-go
- Terrain altitude variations modeled as a random meas. error (white noise) or as correlated noise (white noise through first-order filter)
- Initial-condition errors modeled by 9-dimensional covariance matrix for LM position, LM velocity, and site position errors
(2) Using the statistical models, a simulated landing trajectory is run during which the estimation-error covariance matrix is continuously computed & updated after LR data are incorporated.

(3) The correlations between the sensor-bias and state-vector estimation errors are computed at the same time.

(4) LR weighting functions are computed to minimize the mean-squared errors in the estimation of LM pos. and vel. w.r.t. the site, based on the est. error cov. matrix and the statistics of the measurement errors.
Fig48: **Navigation-Filter Design Approach—concluded**

(5.) **Uncoupled** weighting functions are assumed, i.e. altitude data are used to update altitude and velocity data to update velocity. The previously described procedure is repeated to generate weighting functions that minimize mean-squared estimation errors w.r.t. the site.

(6.) The uncoupled weighting functions computed for the simulated trajectory are stored in the guidance computer as simple functions of altitude, speed, or as constants for the landing maneuver.

(7.) Simulated landing trajectories over realistic terrains are now run to check out the navigation system performance, including sensor bias and random errors, initial-condition errors, and terrain-datum uncertainty. Of particular importance here is interaction with the guidance system.
**Fig. 49: Basic Terrain Uncertainty Models**

1. Terrain uncertainty treated as unknown error added to landing radar altitude measurement.
2. Terrain datum uncertainty modeled as constant slope with known variance emanating from landing site.

**Local Terrain Uncertainty Models**

<table>
<thead>
<tr>
<th>Model #</th>
<th>Description</th>
<th>How Used</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>White gaussian noise. RMS value constant far from site and decreasing exponentially to zero at site as function of correlation dist.</td>
<td>Used to model alt. meas. error when employing prestored terrains on-board.</td>
</tr>
<tr>
<td>2</td>
<td>Correlated noise. Terrain variation is output of first-order lag filter driven by white gaussian noise.</td>
<td>Model alt. meas. error with no prestored terrains.</td>
</tr>
<tr>
<td>3</td>
<td>White gaussian noise. RMS value constant at high forward vel. and decreasing exponentially to zero as forward vel. goes to zero.</td>
<td>Model point-to-point terrain deviations independent of range-to-go to landing site. (Prestored terrains)</td>
</tr>
</tbody>
</table>
Fig. 50: Navigation System Performance With Various Altitude Weighting Functions

terrain model #1 (1000 ft rms, 5 n.m. corr. dist)
rms datum slope uncertainty = 33 ft/n.m.

RMS Altitude Est. Errors

with optimum $W_H$
with present $W_H$
with recommended $W_H$

Altitude Weighting Functions

- optimum +++
- present system --
- recommended system •••
**Fig. 51:** Navigation System Performance With Various Velocity Weighting Functions

**Terrain Model #1** (1000 ft rms, 5 n.m. corr. dist)

- **RMS Velocity Est. Errors**
- **Optimum Coupled W's**
  - Vert. at Site
  - Downrange
  - Crossrange

- Recommended system indicated by broken curves

**Velocity Weighting Functions**

- (LR antenna-fixed frame)
  - Local vert
  - Downrange
  - Crossrange

- Recommended system (all axes)
Fig. 52 Slope Estimation Performance With Various Statistical Terrain Models

RMS Slope Est. Error

TIME (SEC)

Optimum Slope Weighting Function

W_{SL}(ft/nm)

RANGE-TO-GO (NM)
Filter Responses to Step Changes in Terrain & Slopes
**Fig 53: Down-Hill Slope of 100 f/minute to Site (-1 Deg)**

Maximum alt. dev. Limited to 5000 ft  
Initial est. errors = 0, No IMU or LR errors  
LR acquisition at 45,000 ft. altitude

**Filter #3 (Moore)**  
Alt. Est. Error w.r.t. Site

- **TIME (SEC)**
  - (A) slope est. out
  - (B) slope est. in

**Filter #3-B**  
for above case

**True** --- **Est.***
**Fig 54: Constant Slope of 0.25 Deg.: Alt. Est. Error**

Error is w.r.t. site -- \((r_p - r_{LS}) - (r_T - r_{ST})\)

Present System

Filter #2 (LEC)

Filter #3 (Moore)

Filter #4-A
Fig 55: Step Change in Terrain of 3000 Ft at 20 Miles Range-to-Go

Position Est. Error ($x_p$) vs. Time

Filter #2 (LEC)

Filter #3-B (Moore): Slope Est In

Filter #4-A

Filter #3-A: No Slope Est.
Fig 5: Step Change in Terrain at 7.5 Miles Range-to-Go, \( (V_0 \approx 750 \text{ f/s}) \)

Altitude Est. Errors w.r.t. Site vs. Time

Filter #3-B (Moore): Slope Est. In

Filter #3-A (Moore): No Slope Est.

Filter #4-A