RTCC REQUIREMENTS FOR MISSION G: SELECTING AND VERIFYING USBS DOPPLER DATA SOURCES DURING LM ASCENT AND DESCENT

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PROJECT APOLLO

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RTCC REQUIREMENTS FOR MISSION G: SELECTING AND VERIFYING USBS DOPPLER DATA SOURCES DURING LM ASCENT AND DESCENT

By Alan D. Wylie

SUMMARY AND INTRODUCTION

This note describes a program for computing MSFN high-speed Doppler tracking residuals used in the Real-Time Computer Complex (RTCC) for verifying and selecting USBS Doppler data sources during LM ascent and descent. These same residuals are used to determine whether the primary guidance and navigation control system (PGNCS) or the abort guidance system (AGS) is the more correct LM navigation system if the two significantly disagree. Rendezvous radar observations are also processed to compute rendezvous radar residuals which provide an independent test on the accuracy of PGNCS versus AGS. The program has the capability of simultaneously processing nondestruct Doppler data for up to four USBS ground stations tracking the LM during ascent and descent, one of which may be the transmitter. It is assumed that the Doppler biases for these stations have been computed.

In order to check the validity of the ground tracking data, Doppler residuals are computed for each tracker using common telemetered LM state vectors from either PGNCS or AGS. The "residuals" should agree to within some expected tolerance. Should one station disagree, it can be replaced.

Doppler residuals are actually computed using both telemetry vector sources so that the computer controller may display either set of residuals and so that both sets are available for the flight control vector source selection. For each vector source an average is taken of the valid station residuals in order to indicate which system is providing the better data. The rendezvous radar residuals are computed and displayed for the same purpose.

The information in this note supersedes that in reference 1.

GENERAL PROCEDURE FOR SELECTING AND VERIFYING DOPPLER DATA SOURCES

Residual Computations

The main purpose of this program is to verify the Doppler data of the ground stations tracking the LM during ascent and descent. The most efficient method is to calculate a Doppler residual - that is, to find for each station the difference between observed and computed Doppler values.
From each tracking station, time-tagged raw Doppler count observations arrive at the RTCC at a 10-per-second rate. The observations pass through a MSFN preprocessor routine which makes simple edit checks and stores the valid data for the main program. Telemetered powered-flight LM state vectors arrive at an effective rate of one observation every 2 seconds from each onboard navigation system. Free-flight vectors predicted by the analytic ephemeris generator (AEG) and the lunar stay vectors generated using the selenographic radius ($R$), latitude ($\phi$), and longitude ($\lambda$) of the LM arrive at the same rate during the appropriate stages of the program. The telemetered vector preprocessor and the predicted vector preprocessor make simple edit checks on the corresponding vectors and store the valid values for future processing. Rendezvous radar observations are telemetered down in the form of raw unprocessed data at a rate of one observation every 2 seconds. This data must also pass through a preprocessor routine which includes simple validity tests on the raw data.

The program starts the residual computations process by selecting a recent telemetry vector and iterating to find the transmitter-vehicle (uplink) range and vehicle-receiver (downlink) ranges. A linear interpolation scheme is used for each station to find the "pseudoactual" Doppler count which corresponds in time to the receiver time of the vector.

The program recycles and obtains a second vector from the same source. A position vector is then computed at the time of the second position vector and temporarily replaces the second position vector in the residual computations. In this manner the program avoids a large random error in the computed Doppler due to the large quantization error in telemetered position vectors.

This computed vector is then processed to obtain uplink and downlink ranges and a value of Doppler count for each station. The observed and computed Doppler frequencies are calculated and subtracted to obtain the residual. The residuals are converted from cycles per second to feet per second and plotted continuously in time as in figure 1.

![Figure 1.- Station verification display.](image-url)
Residuals will also be computed using the other vector source. Each set of valid station residuals is then averaged and plotted continuously in time on a flight control display to indicate which telemetry system is in better agreement with the MSFN trackers.

Rendezvous radar range-rate residuals are also plotted on the flight control display. To compute one of the residuals, the program interpolates the current CSM ephemeris to obtain the CSM state vector at the time of the LM vector being processed. The CSM and LM vectors are then used to compute the range rate between the two vehicles. The smoothed value of rendezvous radar range rate evaluated at the time associated with the LM state vector is then used together with the computed range-rate value to construct the residual which is plotted on the display.

Operational Modes and Procedures

The ascent-descent program is designed primarily to function in the "average g" or powered-flight mode. However, it has been extended to cover short periods of free flight and the LM's stay on the moon.

Program use actually begins approximately 5 minutes before ignition for powered descent and is initialized by MED 1. An RTCC predictor, the analytic ephemeris generator (AEG), will use the best LM anchor vector to project LM state vectors which are processed in the high-speed mode in the same manner as the powered-flight PGNCS and AGS vectors. Residuals are computed and displayed for each station for checkout purposes. When the onboard computer switches to the average g mode (approximately 30 seconds before ignition), the LM begins to send updated vectors. A switch is set at the RTCC console to initiate high-speed processing of actual telemetry vectors when they are received.

After touchdown on the lunar surface, the LM continues to send vectors used by this program for approximately 3 minutes. At the end of that time, a MED is entered to terminate the descent phase and thus cease residual generation. About 5 minutes prior to lunar lift-off, the MED to enter high-speed processing and MED 1 are reentered and the program resumes high-speed processing of RTCC-computed LM vectors and MSFN observations in preparation for the ascent phase. A switch will initiate the use of actual telemetry vectors when they are received.

During the ascent stage, high-speed processing of PGNCS and AGS vectors continues until averaged vectors become available following engine cutoff. At that time the controller has the option either to continue using actual telemetry vectors or to switch and use the RTCC-predicted vectors based on the computed average vectors. If the average vectors are not available, the last powered-flight vectors will be considered in the option. The program continues to process the vectors (either actual or predicted) in the high-speed mode until an orbit switch is thrown, signifying the end of the ascent phase.
DETAILED EQUATIONS AND LOGIC

Telemetry Vector Preprocessor

Two telemetry vector preprocessors (appendix A) are used in this program in order that the telemetered powered-flight vectors and the predicted free-flight or lunar-stay vectors may be treated separately. Each preprocessor is a totally independent routine which is started by executive control. Only one of the two may function at any given time.

The telemetry vector preprocessor receives the LM powered-flight state vectors telemetered from the onboard PGNCS and AGS navigation systems in units of earth radii and earth radii per hour and performs simple tests on the vectors and time tags. Basic validity tests are included, (ref. 2), and all necessary granularities are applied. All vectors not edited out are stored in one table for future processing. The logic is described in flow chart A-1(a).

The predicted-vector preprocessor generates LM state vectors in the same units at the real-time rate while the LM is either in free flight or on the lunar surface. The logic is described in flow chart A-1(b).

MSFN Preprocessor

The MSFN preprocessor (appendix A) also is an autonomous routine started by executive control. It receives from the tracking stations high-speed data consisting of raw Doppler counts accumulated from count initialization and not a count difference over a fixed time interval. Continuous tests are performed to check if the "ascent-descent sites" MED has been entered. The change option of the MED causes one or more new trackers to be entered for which the appropriate storage areas must be reinitialized. The start option causes a new transmitter to be entered which results in all storage areas being reinitialized. Simple validity tests (ref. 2) and a test on the vector time tag are used to detect invalid data. Any necessary granularities are then applied.

The preprocessor logic then proceeds to the edit routine. The initial two values of Doppler count for each station are stored, tagged invalid, and utilized in a linear extrapolation scheme to predict the Doppler count at the time of the third observation. The routine then tests to determine if the observation is greater than the last accepted value and less than some maximum value which is a function of the extrapolate time interval. If the observation passes the test, it is stored but tagged invalid, and the program uses the last two stored values to linearly extrapolate to the time of the next observation. If the observation fails the test, however, it is discarded, and the routine uses the same two data points in repeating
the extrapolation procedure. Once four successive data points pass the edit scheme, the four values and succeeding accepted values are stored and tagged valid. Four successive failures, however, cause the routine to initialize. The program always tests if the linear extrapolation time interval is less than \( \sigma_n \). If not, the time interval is too long to linearly extrapolate, and the program must reinitialize.

The MSFN data passing the edit routine is stored into the arrays \( N_j/i, t_j/i \), where \( i \) represents the station and \( j \) represents the position in the table. Flow chart A-2(a) describes the logic for the preprocessor. Flow chart A-2(b) describes the edit routine logic.

Rendezvous Radar Preprocessor

The rendezvous radar preprocessor (appendix A) is started by executive control. Input consists of raw unprocessed destruct Doppler count data and the associated time tag at the middle of the count interval. The routine performs elementary validity tests (ref. 2) and time tests before applying granularities (ref. 3) which convert the raw data to range rate in earth radii per hour. The routine then tests if the data is within a determined extreme interval. All observations passing the above tests are stored for the main program. The logic for this preprocessor is described in flow chart A-3.

Main Program Control Logic

The main program control logic (appendix B) completes the list of processors which are started by executive control. It is broken down into five parts: the main program logic or driver, the iteration subroutine, the interpolation subroutine, the MSFN residual computation subroutine, and the rendezvous radar residual computation subroutine. Flow charts B-1 through B-5 present the equations and logic for each of the five parts of the program.

All variables must be initialized to zero at the start of the program. The orbit determination constants described in appendix B and the RNP matrix of current time are then read in. In addition, station characteristics and a lunar ephemeris must be available. Tables of time and edited Doppler N-count observations are generated for \( n \) stations.

The program begins each residual computation cycle by picking up a telemetry vector and waiting until the vector is \( T_1 \) (time interval from vector receipt time to current time) seconds old in regard to current time. The first computations involve finding the ranges from the vehicle to the transmitter and receiver for each tracker. Since there is one transmitter and four receivers, one uplink range and four downlink ranges are computed. The iteration routine computes these ranges and the receiver times for each station.
For each station the raw observations are set up in the following arrays: $t_{j/i}$ and $N_{j/i}$ where $i = 1, \ldots, n$ and $j = 1, \ldots, M_l$. The index $M_l$ represents the most current observation at a station and is determined by the time length of the data table. The time test is then performed for station $i$ to determine if the receiver time obtained from the iteration routine lies within the time range of the current MSFN observation table. Failure of the test causes the program logic to check the next station. If all stations fail the test, the program logic transfers to the rendezvous radar residual routine. Otherwise it proceeds to the interpolation routine.

The linear interpolation routine is then called to yield the interpolated value of the Doppler count for each station at each receiver time. The main program then stores all necessary values derived from the onboard telemetry vector and proceeds to the rendezvous radar subroutine.

Output from the rendezvous radar residual subroutine consists of a range-rate residual, It is computed using a PCNCS or AGB telemetry vector, a free-flight GSM ephemeris, and the current table of rendezvous radar observations. The residual is plotted on a flight control display, and program recycle to pick up the second telemetry vector necessary for the Doppler residual computations.

When the second telemetry vector is received from a vector source, the program computes the position vector referenced to the same time. The program uses the position and velocity vectors of the previous telemetered IM state vector and the velocity vector of the current telemetered state vector (both vectors from the same source) in the computation and assumes linear motion of the vehicle. The position vector is computed in order to prevent a large quantization error from appearing in the Doppler MSFN residual computations. The program uses the computed position vector in order to compute the second set of uplink and downlink ranges (iteration routine) and values of Doppler count (interpolation routine) to be used in the residual computations. The logic then transfers to the MSFN residual routine.

(This scheme for predicting a position vector using a previous position vector and the average of two velocity vectors is used only when actual telemetry vectors are being processed during the average Doppler mode. During the non-average Doppler stages, the nominal vectors are processed as they are received.)

The MSFN residual routine is provided with values of uplink and downlink range obtained from one telemetered vector and one computed vector from the same source and the two values of interpolated counts from each of the $n$ stations. The residuals are computed and displayed as shown in figure 1. The average residual $6Y$ computed using vectors from one vector source is then compared on a display with the average residual of the other vector source for the selected stations. The program then returns to process the actual telemetry vector whose time tag is the same as the computed vector just processed.
Iteration Subroutine

Input to the iteration subroutine (flow chart B-2) is the LM position vector, $\hat{R}(t_{LM})$, in a selenocentric coordinate system with the X-Y plane parallel to the earth’s mean equatorial plane, the X-axis through Aries at the beginning of the nearest Besselian year (NEY), and the Z-axis directed along the mean pole. Also input are a vector time tag, $t_{LM}$, referenced to Greenwich midnight prior to launch, station characteristics for station $i$ ($i = 1, \ldots, n$), and a lunar ephemeris. The first computation transforms the mean NEY selenocentric vector to a mean NEY earth-centered inertial (ECI) vector by

$$R'(t_{LM}) = \hat{R}(t_{LM}) + R_M$$

where $R_M$ is the position of the moon in the earth-referenced mean NEY system. The next computation involves finding the range between the transmitting station and lunar orbiting vehicle.

In computing the range, the program must iterate to find the time, $t_{TR}$, the signal was transmitted from the transmitting station in order to have arrived at the vehicle at $t_{LM}$ (ref. 4). Let $c$ be the velocity of light, $R'(t_{LM})$ the geocentric inertial position vector of the vehicle at $t_{LM}$ in the mean NEY coordinate system, and $R_S(t')$ the geocentric inertial position vector of the transmitting station at the computed transmission time, $t'$. Then

$$t' = t_{LM} - \frac{\rho}{c} \quad (1)$$

where

$$\rho = |R'(t_{LM}) - R_S(t')| \quad (2)$$

For an initial guess of $R_S(t')$, we set $t' = t_{LM}$; thus,

$$\rho = |R'(t_{LM}) - R_S(t_{LM})| \quad (3)$$

This $\rho$ from equation (3) is then used in equation (1) for a new estimate of $t'$. The position of the station is then computed for the revised $t'$, a new $\rho$ is calculated in equation (2), and a new $t'$ is calculated in equation (1). This iterative process is repeated until the difference between successive values of $t'$ is less than or equal to $10^{-9}$ seconds or until six iterations have been made.
A refraction correction is then applied to the computed uplink range \( \rho \) to form \( \rho' \) in the following manner. (See ref. 5.) Let

\[
r_s = |R_s(t')|
\]

and

\[
\rho = R'(t') - R_s(t')\]

Then

\[
\rho' = \rho + \frac{c_{nk} r_s \rho}{R_s(t') \cdot \rho}
\]

where \( c_{nk} \) is the product of refraction modulus and radio refractivity \( \times 10^{-6} \), which are available from the station characteristic table. (See appendix B.) The uplink range, \( \rho' \), will then be stored as \( \rho_{TR'} \).

The subroutine then computes downlink ranges and times for the \( n \) stations successively. The only change from the uplink computations is in the time-iteration equation which now becomes

\[
t' = t_{LM} + \frac{\rho}{c}
\]

where \( t' \) is the computed time at which the signal should be received at the station if it left the vehicle at \( t_{LM} \). After the refraction correction has been applied to a downlink range, \( \rho' \) is stored as \( \rho_{R/i} \) and \( t' \) stored as \( t_{R/i} \) (\( i = 1, \ldots, n \)) and the subroutine picks up the proper station characteristics for the downlink computation for the next station.

The position of the station, \( R_s(t') \), is computed for each successive iteration of time, \( t' \), from the station characteristics as follows:

Let \( \theta \) be the angle between the true position of Greenwich at midnight prior to launch and Greenwich at \( t' \).

Then

\[
\theta = \omega t'
\]

where \( \omega \) is the rotational rate of the earth.

Then the angle, \( \lambda' \), of the station is given by

\[
\lambda' = \lambda + \theta
\]
where $\lambda$ is the longitude of the station. Finally, $R_s(t')$ is given by

$$
R_s(t') = (RNP)^T \begin{bmatrix}
\cos \lambda'(r \cos \phi' + h \cos \phi) \\
\sin \lambda'(r \cos \phi' + h \cos \phi) \\
r \sin \phi' + h \sin \phi
\end{bmatrix}
$$

(8)

All the above variables are defined in appendix B.

Interpolation Subroutine

The interpolation subroutine (flow chart B-3) is called to find a pseudoactual Doppler count, $N_{R/i}$, for each station at the receiver time $t_{R/i}$ which is not likely to be a recorded observation time. Inputs to the subroutine are $t_{R/i}$ (obtained from the iteration routine) and $n$ tables of $M_l$ observation times, $t_{j/i}$, versus $M_l$ observed Doppler counts, $N_{j/i}$, $(i = 1, \ldots, n$ and $j = 1, \ldots, M_l)$. All stations for which the flag $MSW(i)$ is zero are processed.

The interpolation scheme tests the $t_{R/i}$ of each station against the corresponding table of recorded observation times to find between which two recorded valid observation times $t_{R/i}$ lies and between which two recorded valid observations $N_{R/i}$ lies. Once these two values of $N_{j/i}$, $t_{j/i}$ are found, the routine tests $t_{j+1/i}$ to see if

$$
t_{j+1/i} - t_{j/i} \leq \sigma_T
$$

If not, the time interval is too long to linearly interpolate. Therefore, do not interpolate, but set $N_{R/i}$ equal to a dummy negative value denoted by $\sigma_N$, and start the time tests for the next station. If, however,

$$
t_{j+1/i} - t_{j/i} \leq \sigma_T
$$

the time interval is sufficiently short to accurately interpolate for $N_{R/i}$. Therefore, the following equation can be solved for $N_{R/i}$.

$$
N_{R/i} = N_{j/i} + \frac{N_{j+1/i} - N_{j/i}}{t_{j+1/i} - t_{j/i}} (t_{R/i} - t_{j/i})
$$

(9)
where \( i \) is the station number. The outputs \( N_{R/1} \) from the interpolation subroutine will then be passed to the main program.

**MSFN Residual Subroutine**

Inputs to the MSFN residual computation subroutine (flow chart B-l) are two receiver times, two uplink ranges, two downlink ranges, two interpolated \( N \)-count observations, and a table of bias values. The subroutine will be called in two different places in the main program depending on whether the vector source is PGNCS or AGS; this selection is controlled by the LSW flag (see appendix B). The inputs to this subroutine can be controlled by the arguments of the calling statements (or equivalent); for example, for a PGNCS vector we might have:

```
CALL MSFN RESIDUAL (Pt_{R/1}, t_{R/1}, Po_{TR}, Po_{R/1}, \rho_{TR}, \rho_{R/1}, FN_{R/1}, N_{R/1}, \\
\Delta Y_{i}, \delta Y)
```

Then for an AGS vector we would have:

```
CALL MSFN RESIDUAL (At_{R/1}, t_{R/1}, Ap_{TR}, Ap_{R/1}, \rho_{TR}, \rho_{R/1}, AN_{R/1}, N_{R/1}, \\
\Delta Y_{i}, \delta Y)
```

Once the input arguments have been established, the program processes the residual subroutine, which is of the following form:

```SUBROUTINE MSFN RESIDUAL (t_{1/1}, t_{2/1}, \rho_{1}, \rho_{2/1}, \rho_{3}, \rho_{4/1}, N_{1/1}, N_{2/1}, \\
\Delta Y_{i}, \delta Y)
```

Before computing a residual for an observation station, the validity of the two Doppler counts \( N_{1/1} \) and \( N_{2/1} \) must be verified. A negative value for either \( N_{1/1} \) or \( N_{2/1} \) causes the residual computation of that station to be omitted, and the routine checks the Doppler counts for the next station.

Let \( \rho_{2/1} \) and \( \rho_{4/1} \) be the downlink ranges computed for the receiver times \( t_{1/1} \) and \( t_{2/1} \) and let \( \rho_{1} \) and \( \rho_{3} \) be the corresponding uplink ranges. The computed Doppler observation is obtained from the following equation (ref. 4).

\[
D_{1} = (w_{3} + b_{1}) + \frac{\omega_{4} f_{TR}}{c(t_{2/1} - t_{1/1})} \left( (\rho_{3} + \rho_{4/1}) - (\rho_{1} + \rho_{2/1}) \right)
\]  

(10)
where the constants $\omega_3$, $\omega_4$, $f_{TR}$, c, $b_i$ are explained in appendix B. A pseudoactual Doppler observation, $S_i$, is obtained by

$$S_i = \frac{N_2/i - N_1/i}{t_2/i - t_1/i}, \quad (11)$$

and the residual is computed from

$$\Delta Y_i = (S_i - D_i) K, \quad (12)$$

where $K$, the conversion factor from Hz to fps, is given by

$$K = \frac{c}{2\omega_4 f_{TR}}$$

where $f_{TR}$ is the frequency transmitted to the LM. These $\Delta Y_i$ are plotted as in figure 1. The average of the residuals from $n'$ stations is then obtained by

$$\delta Y = \frac{\sum_{k=1}^{n'} \Delta Y_k}{n'} \quad (13)$$

where $n'$ is the number of stations selected. This $\delta Y$ will then be plotted against $\delta Y$ computed with the other vector source and displayed. Control is then transferred to the main program in order to process an actual telemetry vector and thus initiate the next residual computations.

Rendezvous Radar Residual Subroutine

The rendezvous radar residual subroutine (flow chart B-5) provides an additional check for testing PGNCS versus AGS. Inputs to this routine are a table of range-rate observations versus time ($r_{RR}$, $t_{RR}^i$ RR = 1, M2), a table of selenocentric CSM ephemeris state vectors versus time ($r_{CSM}$, $v_{CSM}$, $t_{CSM}$), and a LM telemetered state vector ($r_{LM}$, $v_{LM}$, $t_{LM}$). A routine is called to obtain specific rendezvous radar observations in the time range...
from \( t_{LM} - 20 \) seconds to \( t_{LM} + 6 \) seconds (see appendix C). If less than three such observations exist, the rendezvous radar computations are omitted for the particular telemetry vector.

If sufficient rendezvous radar data is available, a sixth-order Lagrange interpolation routine interpolates the CSM ephemeris table to obtain the CSM state vector at the time of the LM state vector being processed. The rendezvous radar range-rate data is smoothed by a second-degree, least-squares filter (see appendix C). This second-degree curve is then evaluated at the time of the telemetry vector to obtain the desired range rate observation.

The position vectors of the LM and CSM allow the program to compute the range between the two vehicles using the equation

\[
\vec{r}_{COMP} = R_{CSM} - R_{LM}.
\]

Then the range rate can be computed by

\[
\dot{\vec{r}}_{COMP} = \frac{\vec{r}_{COMP}}{|\vec{r}_{COMP}|} \cdot (\dot{V}_{CSM} - \dot{V}_{LM}).
\]

This computed range rate is then compared to the observed rendezvous radar range rate, and the residual, \( \Delta \dot{r} \), is displayed on the flight control display in units of feet per second. The logic then returns to the main program to pick up a new telemetry vector.

CONTROLS AND DISPLAYS

The following are general descriptions of the various controls and displays used in this program. Reference 2 presents detailed descriptions.

Manual Entry Devices

1. Ascent-descent sites. This MED specifies up to four sites to be processed in the high-speed mode. This MED also allows the controller to change any of the sites being processed as well as to reinitialize if a change in the identity of the two-way site occurs.
2. Ascent-descent scaling factor.- This MED allows the controller to adjust the ordinate scale for either the MSFN or rendezvous radar residual displays.

**Push Button Indicators**

1. Sources to be considered in the average.- This group of five PBI's determines which sites are to be used in the MSFN average residual computation. The ability exists to edit sites in or out of the average computation.

2. TM selected source.- This group of two PBI's determines whether to display the MSFN residuals computed using the PGNCS vectors or the MSFN residuals computed using the AGS vectors.

3. Processing mode.- This group of two PBI's determines whether actual telemetered vectors or AEG-predicted vectors are to be used in the MSFN residual computation.

**Displays**

1. MSFN source validation 1 and 2.- This display provides an analog trend curve of range rate versus time for the residuals of two MSFN sources computed using PGNCS or AGS telemetered vectors.

2. MSFN source validation 3 and 4.- This display provides the same information as display no. 1 for two other MSFN sources.

3. TM source comparison.- This display provides an analog trend curve of range rate versus time for both telemetry sources. Average MSFN residuals and rendezvous radar range-rate residuals will both be displayed.
APPENDIX A

FLOW CHARTS FOR PREPROCESSOR ROUTINES

TELEMETRY, MSFN, AND

RENDEZVOUS RADAR DATA
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## FLOW CHARTS

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Variables Used in LM State Vector Preprocessors

\( t_a \)  
- time tag (g.e.t.) of a PGNCS or AGS telemetry vector unprocessed by the preprocessor routine.

\( R_a \)  
- selenocentric position and velocity of a telemetered vector at \( t_a \) in mean NBY coordinates

\( V_a \)  
- \[ |R_a| \] maximum acceptable value for \( |R_a| \), 80 earth radii

\( |V_a| \)  
- \[ |V_a| \] maximum acceptable value for \( |V_a| \), 50 000 fps

\( t^* \)  
- current time plus 6 seconds

\( t_{\text{MIN}} \)  
- current time minus 20 seconds

\( t_{LM} \)  
- time tag of a current PGNCS or AGS telemetry vector referenced to Greenwich midnight prior to launch

\( R(t_{LM}) \)  
- selenocentric state vector (PGNCS or AGS) of the vehicle at time \( t_{LM} \) in mean NBY coordinates

\( V(t_{LM}) \)  
- selenocentric state vector (PGNCS or AGS) of the vehicle at time \( t_{LM} \) in mean NBY coordinates

\( LSW \)  
- switch set by the identification code of the telemetry vector: \( LSW = 0 \) for a PGNCS vector, \( LSW \neq 0 \) for an AGS vector. \( LSW = 0 \) for all nominal vectors.
"START"

a = 0
LM = 0

1

IS THIS PROCESSOR ON

YES

IS TLM VECTOR AVAILABLE

YES

a = a + 1

OBSERVE LM STATE VECTOR FROM TELEMETRY
--R_a, V_a, t_a, LSW

0 < |R_a| < |R^*| AND 0 < |V_a| < |V^*|

YES

1 < t_a < t^*

YES

FLOW CHART A-1. - Preprocessor for LM vectors.
(a) Logic for the telemetered LM vectors.
(a) Logic for the telemetered vectors – Concluded.
Flow chart A-1.— Preprocessor for LM vectors.
(b) Logic for the predicted LM vectors.
Variables Used in MSFN Preprocessor

$C1(i)$ - station counter of Doppler data which do not pass the extrapolation test; when $C1(i)$ equals four, all counters for station $i$ are reinitialized and the extrapolation edit routine starts over.

$C2(i)$ - station counter of Doppler data which pass the extremum count interval test; $C2(i) = 3$ initializes the use of extrapolating for Doppler.

$C3(i)$ - station counter of Doppler data which pass the extrapolation test; $C3(i) > 4$ causes the data to be tagged valid when stored by the preprocessor routine.

$N_{k/i}$ - an array of raw, non-destruct Doppler counts recorded at station $i$; the element $N_{k/i}$ represents the current value.

$t_{k/i}$ - time array for the $N_{k/i}$.

$t_{\text{MIN}}$ - current time minus 20 seconds.

$t^*$ - current time plus 6 seconds.

$N_{j/i}$ - an array of raw, non-destruct Doppler counts accepted and stored by the MSFN preprocessor routine; the element $N_{j/i}$ represents the last accepted value of $N_{k/i}$.

$t_{j/i}$ - time array for the $N_{j/i}$.

$a_T$ - maximum time interval allowed for linear extrapolation of Doppler.

$N_{\text{MAX}}$ - maximum acceptable count difference between the actual and extrapolated values of $N_{k/i}$; $N_{\text{MAX}}$ is a function of the time interval $t_{j/i} - t_{k/i}$.

$\text{HIGH}$ - maximum acceptable value of $N_{k/i}$ tested when all previous requirements have been met.
"START"

\[ \begin{align*}
C_1 (i) &= 0 \\
C_2 (i) &= 0 \\
C_3 (i) &= 0 \\
i &= 1, n
\end{align*} \]

INITIALIZE ID OF TRANSMITTER FROM 2-WAY SITE TIME LINE AND STORAGE AREA FOR UP TO FOUR TRACKING STATIONS

1

IS THIS PROCESSOR ON

- YES
- NO → "EXIT"

IS HIGH SPEED DATA AVAILABLE

- YES
- NO

HAS ASCENT-DESCENT SITES MED BEEN ENTERED WITH START OPTION

- YES
- NO

HAS ASCENT-DESCENT SITES MED BEEN ENTERED WITH CHANGE OPTION

- YES → REinitialize Appropriate Storage Areas and Counters
- NO

Flow chart A-2. MSFN preprocessor.
(a) Logic for the preprocessor.
Flow chart A-2, MSFN preprocessor.
(a) Logic for the preprocessor - Concluded.
Flow chart A-2. - MSFN preprocessor.
(b) Logic for the edit routine.
Flow chart A-2. - MSFN preprocessor
(b) Logic for the edit routine - Concluded.
Variables Used in Rendezvous Radar Preprocessor

- $t_r$: time tag of a rendezvous radar observation unprocessed by this routine
- $S_r$: raw unprocessed rendezvous radar destruct Doppler count which represents the count of a frequency comprising both the Doppler frequency and a bias frequency over a time interval
- $t_{MIN}$: current time minus 20 seconds
- $t$: current time plus 6 seconds
- $K_r$: scale factor required to obtain the range rate in earth radii per hour. It is of such polarity as to make $\dot{r}_r$ positive for increasing range.
- $f_{Br}$: range-rate bias frequency
- $\tau_r$: counting interval over which $S_r$ is observed
- $t_{RR}$: time tag of a rendezvous radar observation processed by this routine
- $\dot{r}_{MIN}$: lower limit for acceptable rendezvous radar range-rate data
- $\dot{r}_{MAX}$: upper limit for acceptable rendezvous radar range-rate data
- $\dot{r}_{RR}$: processed rendezvous radar range-rate observation at $t_{RR}$
Flow chart A-3, Logic for the rendezvous radar preprocessor.
APPENDIX B

FLOW CHARTS FOR PROGRAM CONTROL LOGIC
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Variables Used In The Main Program

\( \mathbf{R}(t_{LM}) \) selenocentric LM state vector (PGNCS or AGS) of the vehicle at time \( t_{LM} \) in MNBY coordinates

\( \mathbf{V}(t_{LM}) \) selenocentric position vector (PGNCS or AGS) used in order to lessen a quantization error in the Doppler residual computation

\( \mathbf{\hat{R}}(t_{LM}) \) LM selenocentric position vector (PGNCS or AGS) used in order to lessen a quantization error in the Doppler residual computation

PSW switch used to identify the PGNCS vectors; PSW \( \neq 0 \) denotes an actual telemetered PGNCS vector, PSW = 0 denotes a computed PGNCS vector

ASW switch used to identify the AGS vectors; ASW \( \neq 0 \) denotes an actual telemetered AGS vector, ASW = 0 denotes a computed AGS vector

\( T_1 \) elapsed time from vector receipt time to current time

\( T_2 \) time increment used in delaying vector processing

\( C_4 \) counter which counts the number of stations that cannot meet the time tests for a particular telemetry vector

\( \text{MSW}(i) \) switch which denotes MSFN data: \( \text{MSW}(i) \neq 0 \) implies time invalid data for station \( i \); \( \text{MSW}(i) = 0 \) implies valid data which can be processed in interpolation routine

\( J \) position number in an observation table

\( M_1 \) number of MSFN observations copied into a data table

\( K_{SWP} \) switch which denotes the PGNCS vector: \( K_{SWP} = 0 \) for the first PGNCS vector, \( K_{SWP} \neq 0 \) thereafter

\( T_{MAX} \) the maximum time interval between two successive telemetry vectors from the same source for which a Doppler residual may be computed

\( K_{SWA} \) switch which denotes the AGS vector: \( K_{SWA} = 0 \) for the first AGS vector, \( K_{SWA} \neq 0 \) thereafter
\[ PR(t_{LM}) \]
\[ PV(t_{LM}) \]
\[ PtLM \]
\[ PtR/i \]
\[ PrR/i \]
\[ PrTR \]
\[ PR(i) \]
\[ PR(i') \]
\[
\begin{align*}
\text{temporary storage of previous values of } & R(t_{LM}), V(t_{LM}), t_{LM}, \\
& t_{R/i}, \rho_{R/i}, \rho_{TR}, \text{ and } N_{R/i}, \text{ respectively, resulting from a PGNCS telemetry vector}
\end{align*}
\]
\[ AtLM \]
\[ AV(t_{LM}) \]
\[ AtR/i \]
\[ APR/i \]
\[ APTR \]
\[ APR(i) \]
\[ ANR/i \]
\[
\begin{align*}
\text{temporary storage of previous values of } & R(t_{LM}), V(t_{LM}), t_{LM}, \\
& t_{R/i}, \rho_{R/i}, \rho_{TR}, \text{ and } N_{R/i}, \text{ respectively, resulting from an AGS telemetry vector}
\end{align*}
\]
\[ GtLM \]
\[ GR(t_{LM}) \]
\[ GV(t_{LM}) \]
\[
\begin{align*}
\text{generalized temporary storage locations for } & PtLM, PR(t_{LM}), \\
& PV(t_{LM}), \text{ or } AtLM, AR(t_{LM}), AV(t_{LM}) \text{ in the computation of } R(t_{LM})
\end{align*}
\]
INPUT: ORBIT DETERMINATION CONSTANTS; JPL EPHEMERIS TAPES; STATION CHARACTERISTICS TABLES AND EDITED OBSERVATION TABLES FOR n STATIONS.

FLOW CHART

"START"

PICK UP RNP MATRIX

SET ALL VARIABLES AND SWITCHES TO ZERO

1

IS THIS PROCESSOR ON

YES

EXIT

NO

IS NEXT VECTOR IN TABLE AVAILABLE

YES

PICK UP NEXT VECTOR IN TABLE -- \( \mathbf{R}(t_{LM}) \) AND VECTOR RECEIPT TIME; SET LSW = 0

NO

NOMINAL VECTOR ?

YES

PICK UP NEXT TLM VECTOR IN TABLE -- \( \mathbf{R}(t_{LM}), \mathbf{V}(t_{LM}) \), LSW -- AND VECTOR RECEIPT TIME

NO

2A

Flow chart B-1.- Logic for the main program.
Flow chart B-1 - Logic for the main program - Continued.
Flow chart B-1. - Logic for the main program - Continued.
Flow chart B-1.- Logic for the main program - Continued.
Flow chart B-1: Logic for the main program - Concluded.
Variables Used in the Iteration Subroutine

\( R'(t_{LM}) \) ECI position vector of the vehicle at \( t_{LM} \) in the Aries mean NBY coordinate system

\( R_M \) position vector of moon, ECI coordinates

\( i \) station number, where \( i = 1, n \)

\( ISW \) switch which denotes uplink or downlink range computation: \( ISW = 0 \) for uplink range computation, \( ISW \neq 0 \) for downlink computations

\( LOOP \) counter which limits the number of iterations

\( \rho, \rho', t', t'' \) temporary storage locations used in the iteration cycle

\( \theta \) angle between true-of-date position of Greenwich and the position of Greenwich at midnight prior to launch

\( \omega \) rotational rate of the earth

\( DT \) difference between UT1 and UTC, \( DT = UT1 - UTC \)

\( \lambda' \) \( \lambda + \theta \)

\( \lambda \) longitude of station

\( R_s(t') \) ECI position vector of a station at the computed time \( t' \) in the Aries mean NBY coordinate system

\( R_s(t_{LM}) \) initial guess of \( R_s(t') \)

\( (RNP)^T \) matrix which converts from the Greenwich position at midnight prior to launch true-of-date coordinate system to the Aries mean NBY system

The following constants are found on the station characteristics tape:

\( r \cos \phi' + h \cos \phi \) projection of position vector of the station into the x-y plane in the ECI coordinate system

\( r \sin \phi' + h \sin \phi \) z coordinate of station in ECI system
where $r$ radius to ellipsoid below station

$\phi'$ geocentric latitude of station

$\phi$ geodetic latitude of station.

$h$ height of station above ellipsoid

$\bar{\rho}$ vector difference between $R'(t_{LM})$ and $R(t')$

$r_s$ magnitude of the vector $R(t')$

$c_{nk}$ product of the radio $r^\nu$-activity $\times 10^{-6}$ and the decay constant calculated from radio refractivity with an exponential reference atmosphere; both variables are found on the station characteristics tab.

$t_{TR}$ time the signal must be transmitted from the transmitting station to arrive at the vehicle at $t_{LM}$

$\rho_{TR}$ uplink range from the transmitting station at $t'$ to the vehicle at $t_{LM}$

$t_{R/i}$ time at which the signal would arrive at station $i$ if it had left the vehicle at $t_{LM}$

$\rho_{R/i}$ downlink range from the vehicle at $t_{LM}$ to station $i$ at $t_{R/i}$ ($i = 1, n$)

$n$ number of stations
INPUT: SELENOCENTRIC LM POSITION VECTOR \( \vec{r}_{LM} \); STATION CHARACTERISTICS FOR TRANSMITTER AND STATION \( i = 1, n \); LUNAR EPHEMERIS; ORBIT DETERMINATION CONSTANTS

"START"

TRANSFORM MEAN NBY SELENOCENTRIC TLM VECTOR TO MEAN NBY ECI VECTOR

\[ \vec{r}'_{LM} = \vec{r}_{LM} + \vec{r}_M \]

\( i = 1 \)

ISW = 0

1

YES

ISW = 0

NO

SET STATION CHARACTERISTICS EQUAL TO CHARACTERISTICS OF TRANSMITTER

SET STATION CHARACTERISTICS EQUAL TO CHARACTERISTICS OF STATION \( i \)

LOOP = 1

\( t' = t_{LM} \)

2

Page 1 of 3

Flow chart B-2. - Logic for the iteration subroutine.
CALCULATE INERTIAL POSITION VECTOR OF STATION I AT TIME $t'$

\[ \omega t' \quad \lambda' = \lambda + \theta \]

\[ R_s(t') = (RNP)^T \]

\[ R_s(t') = \left( \begin{array}{c} \cos \lambda' \cos \phi' + h \cos \lambda' \\ \sin \lambda' \cos \phi' + h \cos \lambda' \\ r \sin \phi' + h \sin \phi \end{array} \right) \]

\[ \rho = |R'(t_{LM}' - R_s(t')| \]

**Flowchart B-2. Logic for the iteration subroutine - Continued.**
\[ P = R(t_{LM}) - R(t) \]

\[ r_s = |R(t)| \]

**APPLY REFRACTION**

\[ P' = P + c_n k r_s P \]

Flow chart B-2. - Logic for the iteration subroutine - Concluded.

Page 3 of 3
Variables Used in the Interpolation Subroutine

\[ \sigma_N \]
dummy negative value for \( N_{R/i} \) which signifies that a residual will not be computed for station \( i \) for the current telemetry vector being processed

\[ \sigma_T \]
the maximum time interval over which the Doppler data may be linearly interpolated

\[ N_{R/i} \]
an interpolated N-count situated between \( N_{j/i} \) and \( N_{j+1/i} \)
Flow chart B-3. - Logic for the interpolation subroutine.
Variables Used in the MSFN Residual Subroutine

\( N_{1i} \)
- interpolated values of N-count at \( t_{1i} \) and \( t_{2i} \)

\( N_{2i} \)
- computed Doppler frequency at station \( i \)

\( D_i \)
- a bias introduced by the tracking equipment that guarantees that the Doppler shift will never become negative

\( \omega_3 \)
- a multiplying factor which adjusts the frequency of the signal at the spacecraft

\( b_i \)
- bias for station \( i \) on the actual Doppler observables (previously solved for in a preascent routine)

\( f_{TR} \)
- frequency transmitted to the LM, as specified on the station characteristics tape

\( c \)
- velocity of light

\( t_{1i} \)
- successive computed observation times for station \( i \)

\( t_{2i} \)
- uplink ranges computed for \( t_{1i} \) and \( t_{2i} \) respectively

\( \rho_2 \)
- dowlink ranges computed for \( t_{1i} \) and \( t_{2i} \) respectively

\( s_i \)
- observed Doppler frequency shift over \( t_{2i} - t_{1i} \)

\( \Delta Y_i \)
- residual computed for station \( i \) (actual minus computed)

\( K \)
- constant which converts Hz to fps

\( STA(i) \)
- flag subject to external control via the station PBI

\( \delta Y \)
- average value of the \( \Delta Y_i \)

\( n' \)
- number of stations used in computing \( \delta Y \); the value of \( n' \) is subject to external control via the station PBI
INPUT: ORBIT DETERMINATION CONSTANTS; TABLE OF DOPPLER BIAS VALUES; THE FOLLOWING VARIABLES OBTAINED FROM THE ITERATION AND INTERPOLATION SUBROUTINES: $t_{1/1}$, $t_{2/1}$, $p_{1}$, $p_{2/1}$, $p_{3}$, $p_{4/1}$, $N_{1/1}$, $N_{2/1}$ ($i = 1, n$).

ARE BOTH DOPPLER COUNTS VALID FOR FINDING DOPPLER?

**COMPUTED "DOPPLER FREQUENCY"**

$$d_{i} = \omega_{3} + b_{i} + \frac{\omega_{4}^{f \text{ TR}}}{c_{1/2/1} - t_{1/1}} \left[ (p_{3} + p_{4/1}) - (p_{1} + p_{2/1}) \right]$$

**ACTUAL "DOPPLER FREQUENCY"**

$$s_{i} = \frac{N_{2/1} - N_{1/1}}{t_{2/1} - t_{1/1}}$$

COMPUTE RESIDUALS

$$\Delta y_{i} = \left( s_{i} - d_{i} \right) \times k$$

DISPLAY

$$\Delta y_{i}$$

DO NOT USE DATA FROM STATION $i$ IN COMPUTATION OF $\Delta y$

DO NOT USE DATA FROM STATION $i$ IN COMPUTATION OF $\Delta y$

Flow chart B-4. Logic for the MSFN residual subroutine.
Variables Used in Rendezvous Radar Residual Subroutine

\( t_s \)  
- Time tag of first rendezvous radar observation after \( t_{LM} \)

\( \rho_s \)  
- Rendezvous radar range-rate observation used in smoothing routine

\( t_s' \)  
- Time tag of \( \rho_s' \)

\( R_{CSM} \)  
- CSM ephemeris state vector whose time tag is the time of the IM telemetry vector being processed

\( V_{CSM} \)  
- CSM ephemeris state vector whose time tag is the time of the IM telemetry vector being processed

\( t_{LM} \)  
- Time tag of last rendezvous radar observation

\( m \)  
- Number of rendezvous radar observations used in the data smoothing subroutine

\( h \)  
- Counter of rendezvous radar observations

\( At_h \)  
- Time interval between the time tag of a rendezvous radar value in the table and the time tag of the telemetry vector being processed

\( \rho_h \)  
- Raw rendezvous radar range-rate value which corresponds to \( At_h \)

\( S_1 \)
\( S_2 \)
\( S_3 \)
\( S_4 \)
\( S_5 \)
\( S_6 \)
\( S_7 \)

- Rendezvous radar data summations for \( m \) points

\( \text{MAT 1} \)
\( \text{MAT 2} \)
\( \text{MAT 3} \)

- Matrices which define the equations used in the routine
coefficients of the second-degree curve which best fits the rendezvous radar values being processed

\[
a_0, a_1, a_2
\]

\( \rho_{OBS} \) rendezvous radar range-rate value observed at \( t_{LM} \)

\( \rho_{COMP} \) computed range from LM to CSM

\( \rho_{COMP} \) magnitude of \( \rho_{COMP} \)

\( \dot{\rho}_{COMP} \) computed range rate from LM to CSM

\( \Delta \rho \) range-rate residual (actual minus computed)
Flow chart B-5 - Rendezvous radar residual subroutine.

(a) Logic for the subroutine.
Flow chart B-5. - Rendezvous radar residual subroutine.
(a) Logic for the subroutine - Concluded.
Flow chart B-5.- Rendezvous radar residual subroutine.
(b) Logic for obtaining data for data smoothing routine.
Flow chart B-5, - Rendezvous radar residual subroutine.
(b) Logic for obtaining data for data smoothing routine - Concluded.
Flow chart B-5. - Rendezvous radar residual subroutine.
(c) Logic for the data smoothing subroutine.
Flow chart B-5. - Rendezvous radar residual subroutine.
(c) Logic for the data smoothing subroutine - Concludeu,
APPENDIX 3

PROCEDURE FOR SMOOTHING RENDEZVOUS RADAR DATA
PROCEDURE FOR SMOOTHING RENDEZVOUS RADAR DATA

Data Acquisition Routine

Rendezvous radar data is received via the telemetry downlink in the raw unprocessed form at a rate of one observation every 2 seconds. It is initially smoothed by a series of validity tests as described in reference 2. Further smoothing is necessary, however, as is an interpolation scheme for the data. A second-degree, least-squares curve fit has been selected to accomplish both purposes.

The objective of this routine is to find six rendezvous radar range-rate values in the data table which are in the time vicinity of the time tag of the telemetry vector being processed \( t_{LM} \). This time interval is from \( t_{LM} - 20 \) seconds to \( t_{LM} + 6 \) seconds. The rendezvous radar residual routine can function if only three, four, or five such points are available; otherwise computations are omitted for the particular telemetry vector.

The logic for this routine begins by locating the two data values between which the time tag of the telemetry vector lies. The program then goes forward in time from \( t_{LM} \) to \( t_{LM} + 6 \) seconds or to the end of the table and records all data values (not more than 3) in the interval. It then goes back in time from \( t_{LM} \) and records all data values until a total of six values has been recorded or until it reaches the time \( t_{LM} - 20 \) seconds or the end of the table.

For each data value the time interval between its time tag and \( t_{LM} \) is also recorded. The logic then returns to the rendezvous radar residual routine.

Data Smoothing Routine

Any second-degree curve of range rate versus time can be represented by the equation

\[
\dot{\rho} = a_0 + a_1 \Delta t + a_2 \Delta t^2.
\]  

(1)

Let the x's below represent the raw data points \( \rho_1, \rho_2, \ldots, \rho_m \) acquired from the data acquisition routine to which it is desired to fit the second-degree least squares curve.
For each data point the difference between the raw observed value and the corresponding value on the curve is its residual. The objective of this routine is to solve for the coefficients $a_0$, $a_1$, $a_2$ which determine a unique second-degree curve such that the sum of the squares of the residuals of the data points is a minimum.

The zero time to which the $\Delta t$ of equation (1) is referenced is the time tag of the telemetry vector being processed. Let $t_{LM}$ be this reference time and let $h$ be the counter of the data points recorded in the data acquisition routine. Then equation (1) can be evaluated at different points ($t_{s'}$) on the curve as follows:

$$\rho_{1c} = a_0 + a_1 \Delta t_1 + a_2 \Delta t_1^2$$

$$\rho_{2c} = a_0 + a_1 \Delta t_2 + a_2 \Delta t_2^2$$

$$\vdots$$

$$\rho_{mc} = a_0 + a_1 \Delta t_m + a_2 \Delta t_m^2$$

(2)
where

\[ \Delta t_h = t_{LM} - t_s, \]

\[ \rho_h = \rho_s, \quad \text{for } h = 1, \ldots, m. \]

The sum of the squares of the residuals is given by

\[
3 = \left[ (a_0 + a_1 \Delta t_1 + a_2 \Delta t_1^2) - \rho_1 \right]^2 + \left[ (a_0 + a_1 \Delta t_2 + a_2 \Delta t_2^2) - \rho_2 \right]^2 + \ldots + \left[ (a_0 + a_1 \Delta t_m + a_2 \Delta t_m^2) - \rho_m \right]^2.
\]

In order for \( S \) to be a minimum,

\[
\frac{\partial S}{\partial a_0} = 0, \quad \frac{\partial S}{\partial a_1} = 0, \quad \frac{\partial S}{\partial a_2} = 0.
\]

Solving the partials, we get three equations with three unknowns, which in matrix form can be expressed as

\[
\begin{bmatrix}
\sum_{h=1}^{m} \Delta t_h^2 & \sum_{h=1}^{m} \Delta t_h & m \\
\sum_{h=1}^{m} \Delta t_h & \sum_{h=1}^{m} \Delta t_h^2 & \sum_{h=1}^{m} \Delta t_h \\
\sum_{h=1}^{m} \Delta t_h^3 & \sum_{h=1}^{m} \Delta t_h^2 & \sum_{h=1}^{m} \Delta t_h \\
\end{bmatrix}
\begin{bmatrix}
a_0 \\
a_1 \\
a_2
\end{bmatrix}
=
\begin{bmatrix}
\sum_{h=1}^{m} \Delta t_h^2 \rho_h \\
\sum_{h=1}^{m} \Delta t_h \rho_h \\
\sum_{h=1}^{m} \rho_h
\end{bmatrix}.
By applying a matrix inversion scheme to the $3 \times 3$ matrix, the coefficients $a_0, a_1, a_2$ can be readily computed. These coefficients determine a unique second-degree, least-squares curve which can then be evaluated for any $\Delta t$ in the defined interval. In particular, at the time tag of the vector,

$$\Delta t = 0$$

and the desired smoothed rendezvous radar range rate, $\rho_{OBS}$, is given by

$$\dot{\rho}_{OBS} = a_0.$$
Memorandum

TO: See List Below

FROM: FM/Mission Planning and Analysis Division

DATE: 6 Jun 1968

68-FM47-140

SUBJECT: RTCC Requirements for Mission G: Selecting and Verifying USBS Doppler Data Sources During IM Ascent and Descent

1. References:


   b. TRW Memorandum, "Residual Random Error Due to Quantization Error," Dr. W. M. Lear, April 8, 1968.

2. This internal note is a revision of the internal note referenced above (reference a).

3. The revision was made to delete large quantization errors in the MSFPN Doppler residual computations which were due to quantization errors in the telemetered position vectors (reference b).

   James C. McPherson, Chief
   Mathematical Physics Branch
   The Flight Software Branch concurs with the above recommendation and requests IBM to proceed accordingly.

   James C. Stokes, Jr., Chief
   Flight Software Branch

   APPROVED BY:

   John P. Mayer
   Chief, Mission Planning
   and Analysis Division

Enclosure

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