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Space Guidance Analysis Memo # 3-67

TO: Distribution  
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SUBJECT: Constraints on Selecting b-Vectors to Represent  
Measurements

For computational ease it may be desirable to use an alternate set of measurement b-vectors rather than the set of b-vectors which actually represent the given measurements. If it is desired to process individual measurements sequentially, there are some constraints which must be satisfied in order to assure that the random measurement errors remain uncorrelated. In addition, the measurement variances associated with the new b-vectors are not necessarily the same as the variances associated with the actual measurements.

The correlation matrix of the measurement variances which is to be used with the new set of b-vectors is given by the following equation:

$$\underline{\alpha'} \underline{\alpha'}^T = (A B^{-1}) \underline{\alpha} \underline{\alpha}^T (A B^{-1})^T \quad (1)$$

where: B is a matrix, the rows of which are the actual measurement b-vectors associated with the actual measurement covariance matrix  $\underline{\alpha} \underline{\alpha}^T$ ,

and: A is a matrix, the rows of which are the new measurement b-vectors associated with the measurement covariance matrix  $\underline{\alpha'} \underline{\alpha'}^T$ .

For sequential processing of the measurement data, it is required that  $\underline{\alpha} \underline{\alpha}^T$  be a diagonal matrix\*. It is then necessary that  $\underline{\alpha}' \underline{\alpha}'^T$  also be a diagonal matrix to allow the sequential processing of the measurement data as represented by the new set of b-vectors.

Specializing Eq. 1 to the case of 2 dimensions, and using measurements with orthogonal b-vectors (e. g. optics measurements), and also assuming that the new b-vectors are chosen to be orthogonal to each other, the cross-correlation terms in the matrix  $\underline{\alpha}' \underline{\alpha}'^T$  are zero, only if one of the following statements is true.

1. The new b-vectors are in the same direction as the original b-vectors.
2. The new set of b-vectors are orthogonal to the original b-vectors

$$3. \frac{\overline{\alpha_1^2}}{\overline{\alpha_2^2}} = \frac{|\underline{b}_1|^2}{|\underline{b}_2|^2}$$

$\overline{\alpha_1^2}$  is the variance corresponding to one measurement,  
 $|\underline{b}_1|$  is the magnitude of the first measurement b-vector

and  $\overline{\alpha_2^2}$ ,  $|\underline{b}_2|$  are similar quantities pertaining to the other measurement.

Assuming that one of the above statements is true, then  $\underline{\alpha}' \underline{\alpha}'^T$  has the following form,

$$\underline{\alpha}' \underline{\alpha}'^T = \begin{bmatrix} \alpha_1'^2 & 0 \\ 0 & \alpha_2'^2 \end{bmatrix}$$

\* In order to sequentially process measurements whose random errors are correlated a new coordinate frame must be used in which the measurement correlation matrix is diagonalized.

thus permitting the sequential processing of the measurement data as represented by the new set of b-vectors. It should be noted that it is still necessary to use the proper values of  $\alpha_1^2$  and  $\alpha_2^2$  (as given by Eq. 1) so that the updates of the state vector and covariance matrix using the new b-vectors are the same as those obtained with the actual measurement b-vectors.

For example, if there is no cross-correlation in the  $\overline{\alpha' \alpha'^T}$  matrix due to reason 3 above, that is, if

$$\frac{\overline{\alpha_1^2}}{\alpha_2^2} = \frac{|\underline{b}_1|^2}{|\underline{b}_2|^2}$$

then the  $\overline{\alpha' \alpha'^T}$  matrix, in terms of the original measurement variances,  $\alpha_1^2$  and  $\alpha_2^2$ , becomes

$$\overline{\alpha' \alpha'^T} = \begin{bmatrix} \frac{|\underline{b}'_1|^2}{|\underline{b}_1|^2} & \overline{\alpha_1^2} & & 0 \\ \frac{|\underline{b}'_1|^2}{|\underline{b}_1|^2} & \overline{\alpha_1^2} & & 0 \\ 0 & & \frac{|\underline{b}'_1|^2}{|\underline{b}_2|^2} & \overline{\alpha_2^2} \\ 0 & & \frac{|\underline{b}'_2|^2}{|\underline{b}_2|^2} & \overline{\alpha_2^2} \end{bmatrix}$$

where  $\underline{b}_1, \underline{b}_2$  are the original set of b-vectors

and  $\underline{b}'_1, \underline{b}'_2$  are the new set of b-vectors.