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Space Guidance Analysis Memo #5-68

TO: SGA Distribution
FROM: Wayne H. Tempelman
DATE: March 4, 1968
SUBJECT: A Modified Acceleration Expression for Bodies in
Gravitation Fields

The processing of the Lunar Orbiter tracking data has consistently led to large doppler residuals near pericyynthion.¹ These residuals are from 2 to 10 Hz (one Hz equals 65 mm/s) on a least-squares fit, which are approximately three orders of magnitude larger than the residuals on the translunar trajectories. The corresponding position errors are on the order of kilometers after a four hour lapse from the tracking period.² Attempts to explain the large residuals have not been successful.

The pericyynthion location of the large residuals (plus the fact that the residuals are largest for low altitude orbits) suggests that a perturbing acceleration term which is proportional to the velocity might eliminate the residuals. The simplest approach to deriving such a term is by analogy with the expression for the acceleration of a charged body in a static electric field, which is

$$\bar{a} = \frac{q}{m} \left[\frac{\bar{E}_{\parallel}}{(1 - u^2/c^2)^{-3/2}} + \frac{\bar{E}_{\perp}}{(1 - u^2/c^2)^{-1/2}} \right] \quad (1)$$

where $\bar{E} = Q \bar{r}/r^3$, $\bar{E}_{\parallel} = (\bar{E} \cdot \bar{u})\bar{u}/u^2$, $\bar{E}_{\perp} = \bar{E} - \bar{E}_{\parallel}$ and c is the speed of light.

The acceleration of a body in a gravitation field can be assumed to be

$$\bar{a} = -\frac{\mu}{r^3} \left[\frac{\bar{r}_{\parallel}}{(1 - u^2/c^2)^{\alpha}} + \frac{\bar{r}_{\perp}}{(1 - u^2/c^2)^{\beta}} \right] \quad (2)$$

where

$$\bar{r}_{\parallel} = (\bar{r} \cdot \bar{u})\bar{u}/u^2, \quad \bar{r}_{\perp} = \bar{r} - \bar{r}_{\parallel}$$

α and β are real numbers to be determined from either theoretical or experimental considerations. Maintaining only terms of order u^2/c^2 , a new perturbing acceleration term \bar{a}_d is obtained

$$\bar{a} = \bar{a}_d - \mu \bar{r}/r^3 \quad (3)$$

where

$$\bar{a}_d = \frac{\mu u^2}{r^3 c^2} (\alpha \bar{r}_{\parallel} + \beta \bar{r}_{\perp})$$

A reduction from two arbitrary parameters to one can be obtained using the "observed" value for the advance in the perihelion of Mercury, which is given by

$$\eta_0 = 6\pi\mu/pc^2 = 43'' \text{ arc/century} \quad (4)$$

where p is the semilatus rectum. The differential equation for this advance is³

$$\frac{d\eta}{df} = - (e r p \mu)^{-1} \left\{ (\mu p^3)^{1/2} \cos f (\bar{r} \cdot \bar{a}_d) - (p + r) \sin f \left[r^2 (\bar{u} \cdot \bar{a}_d) - (\bar{r} \cdot \bar{u}) (\bar{r} \cdot \bar{a}_d) \right] \right\} \quad (5)$$

where f is the true anomaly and e is the eccentricity.

Integration of this equation shows that the solution is multivalued and represented by

$$\alpha = 3 K + 2 \beta \quad (6)$$

where

$$\eta = K \eta_0$$

Setting K equal to one results in the standard value given in Eq. (4); a K equal to 11/12 results in advance of 39.4'' arc/century which compares favorably with Dicke's value of 39.6 arc/century based on the solar oblateness.⁴

In order for the perturbing acceleration term to have an effect on the order of the lunar residuals α and/or β must be very large. As an illustration, α and β equal to -2×10^7 and -10^7 , respectively, were used in preparing Fig. 1 and 2. The results are valid over at least a range of K 's from -10 to 10. The residual in the semimajor axis was 223 meters at perigee and close to zero at apogee. With the possible exception of the longitude curve, the curves seem to indicate that the new perturbing acceleration term might be effective in reducing the lunar residuals.

The parameter β (and perhaps also α or K) can be chosen to minimize the sums of the square of the residuals using the Lunar Orbiter data.

References

1. J. Lorell and W. Sjogren, "Lunar Orbiter Data Analysis," Technical Report 32-1220 (Jet Propulsion Laboratory, Pasadena, Calif., Nov. 15, 1967).
2. Personal communication with W. Sjogren.
3. R. Battin, "Astronautical Guidance" (McGraw Hill, New York, 1964), p. 195.
4. R. Dicke and H. Holdenberg, "Solar Oblateness and General Relativity," Physical Review Letters, Vol. 18, No. 9, (1967), pp. 313-316.

Figure 1 Effect of \bar{a}_d ($\alpha = -2 \times 10^7$, $\beta = 10^7$) on Position in Lunar Orbit ($e = .05$, $a = 1970$ km, $I = 21^\circ$)

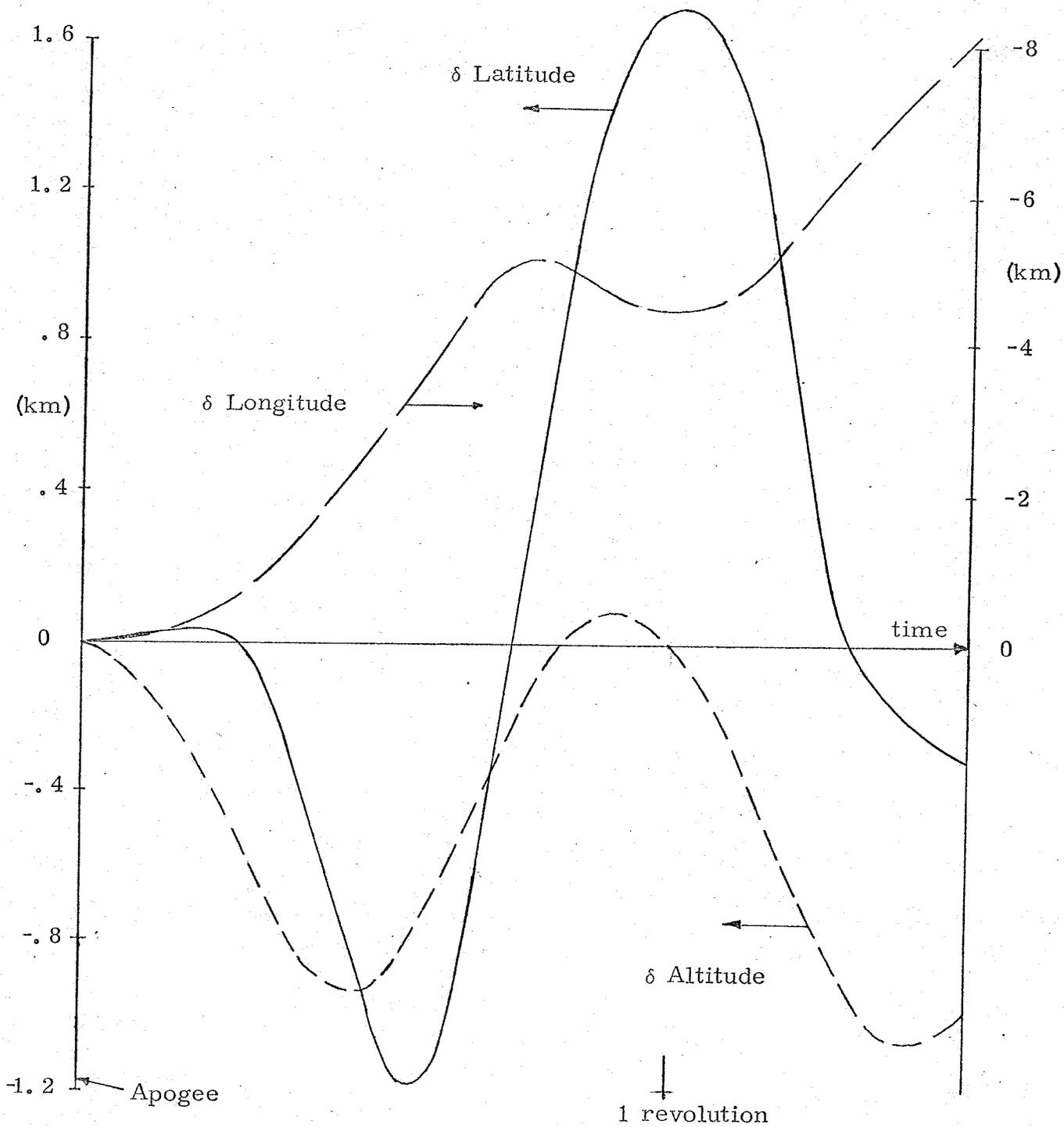


Figure 2 Effect of \bar{a}_d ($\alpha = -2 \times 10^7$, $\beta = 10^7$) on Velocity in Lunar Orbit ($e = .05$, $a = 1970$ km, $I = 21^\circ$)

