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Space Guidance Analysis Memo #8-65

TO: SGA Distribution
FROM: E. M. Copps, Jr.
DATE: May 3, 1965
SUBJECT: An Analysis of Control of Track Deviation During Lunar Deboost

1. This memo answers two questions:

- 1) How much track error results from no track position control?
- 2) How much extra delta-v does it cost to have track position control?

For lunar orbit insertion, the diagram of velocities is;

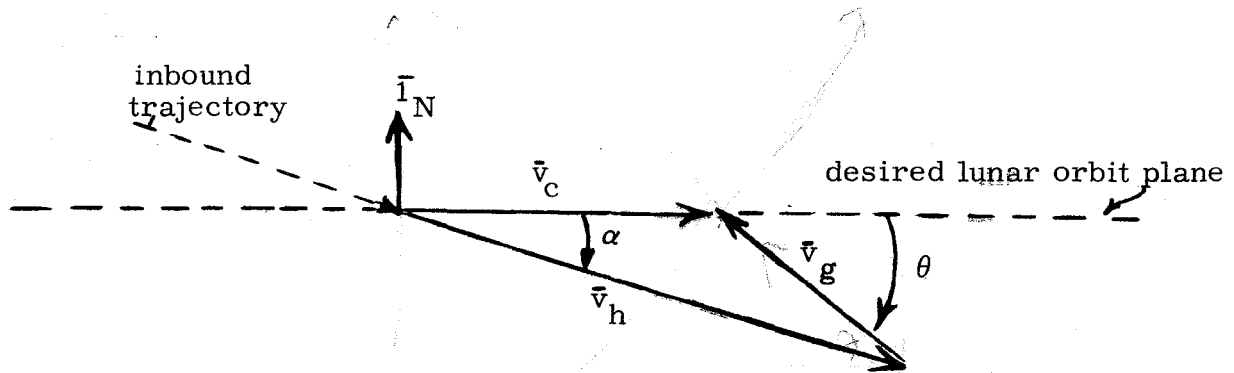


Figure 1 Vector Diagram

where

- \bar{v}_c = circular velocity ~ 5335 ft./sec.
- \bar{v}_h = inbound hyperbolic velocity ~ 8300 ft./sec.
- \bar{v}_g = velocity to be gained during insertion
- \bar{i}_N = unit vector perpendicular to desired plane

The thrust acceleration vector \bar{a}_T is essentially along \bar{v}_g . We work first in a coordinate along $\bar{v}_g \sim$ the x coordinate. We assume constant $|\bar{a}_T|$ for ease of operation. In these coordinates, the change in position in the x direction due to thrust acceleration is;

$$\Delta x = \frac{a_T T^2}{2} \quad 1)$$

where T is the burning time ~ (320 secs.)

By referring to the vector diagram of Fig. 1, the final y position where y is along \bar{i}_N is;

$$y = \frac{a_T T^2}{2} \sin \theta - v_H T \sin \alpha + y_0 \text{ (ignition)} \quad 2)$$

This equation permits the calculation of three important partial derivatives:

1. Change in y due to late ignition. In this case the burn time T is invariant, but the absolute time of cut off increases. This influences the third term in Eq. 2, and the partial is approximately,

$$\frac{\partial y}{\partial t_{\text{ignition}}} = -v_H \sin \alpha \quad 3)$$

2. Change in y due to unpredicted changes in thrust to mass ratio:

Here make the substitution

$$T = |\bar{v}_g| / a_T$$

where

$$|\bar{v}_g| \cong 3200 \text{ ft. /sec.}$$

$$a_T \cong 10 \text{ ft. /sec.}^2$$

then

$$\frac{\partial y}{\partial a_T} = - \frac{|\bar{v}_g|^2}{2a_T^2} \sin \theta + \frac{v_H \sin \alpha |\bar{v}_g|}{a_T^2} \quad 4)$$

3. Change in y due to changes in magnitude of inbound velocity. (This is easily accounted for by choice of ignition time since it known beforehand).

$$\frac{\partial y}{\partial |\bar{v}_H|} = -T \sin \alpha \quad 5)$$

We now turn to Δv lost in maneuvering to make up track displacements. Using coordinates along $\bar{v}_g \sim x$ and perpendicular to $v_g \sim y$, we proceed:

A near optimum track steering law, for track position control is;

$$\psi = A + Bt \quad 6)$$

$$\ddot{y} = a_T (A + Bt) \quad 7)$$

yielding
$$\dot{y}_F = a_T (AT + BT^2/2) + \dot{y}_0 \quad 8)$$

$$y_F = a_T (AT^2/2 + BT^3/6) + \dot{y}_0 T + y_0 \quad 9)$$

The appropriate boundary values to correct for a displacement yield.

$$\left[\frac{a_T T^2}{2} \right] A + \left[\frac{a_T T^3}{6} \right] B = y_F \quad 10)$$

$$\left[a_T T \right] A + \left[\frac{a_T T^2}{2} \right] B = 0$$

for lunar deboost, we can use

$$5.11 \times 10^5 A + 5.45 \times 10^7 B = y_F$$

$$3.2 \times 10^3 A + 5.11 \times 10^5 B = 0$$

Inverting these equations yields A and B in terms of y_F .

$$A = 5.89 \times 10^{-6} y_F$$

$$B = -3.68 \times 10^{-8} y_F \quad 11)$$

Delta-v lost can be expressed by the approximate formula.

$$\Delta v_L = \int_{\text{ignition}}^{\text{cut-off}} \frac{a_T \psi^2}{2} dt \quad 12)$$

yielding

$$\Delta v_L = \frac{a_T}{2} (A^2 T + \frac{B^2 T^3}{3} + ABT^2) \quad 13)$$

Returning to Fig. 1, we note that displacements of interest are along the $\bar{1}_n$ vector, since displacements in the desired plane are not of interest. We must therefore modify our calculations by the sine and cosine of the angle θ .

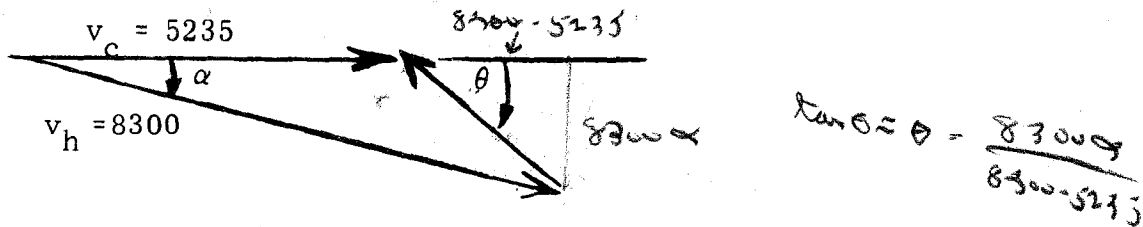


Figure 2 Velocity Diagram

An approximate relation between θ and α is;

$$\theta = \frac{8300 \alpha}{8300 - 5235} = 2.7 \alpha \quad (14)$$

We now use A, B, (y_F) to relate Δv_L to y_F , using $T = 320$ secs.

$$\Delta v_L = 1.85 \times 10^{-6} y_F^2 \quad (15)$$

3. Some typical numerical applications.

For a 6 degree plane change at lunar orbit insertion,

Table 1

Thrust Perturbation	Max distance out of plane if no track position control	Δv lost if track position control is used
3%	3900 ft.	.306 ft./sec.
6%	7800	1.22
9%	11,700	2.75

For a 9 degree plane change at lunar orbit insertion,

Table 2

Thrust Perturbation	Max distance out of plane if no track position constraint	Δv lost if track position control is used
3%	5900 ft.	.795 ft./sec.
6%	11,800	3.12
9%	17,700	6.35