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Space Guidance Analysis Memo #25-65

TO: SGA Distribution
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SUBJECT: Equations for non-optimum estimate of mean square mid-course velocity correction

Introduction

The lunar rendezvous midcourse navigation system is capable of estimating a measurement bias which can be described by three parameters, if the state vector and covariance matrix are to be limited to nine dimensions. In order to analyze the effect on midcourse ΔV of non-estimated biases or CSM ephemeris uncertainties, equations were developed for mean square velocity corrections under these non-optimum estimation conditions.

Error Matrix Equations

Define a covariance matrix E_c (6 x 6) which may represent either the CSM error matrix or the covariance matrix of non-estimated measurement biases. An example of measurement biases might be: 1) a rotation of the radar measurement frame due to structural flexure and 2) constant radar angle pick off biases. The matrix E_c would then be:

$$E_c = \overline{\delta \underline{z} \delta \underline{z}^T} = \begin{pmatrix} * & * \\ E_{TILT} & 0 \\ 0 & E_{CON}^* \end{pmatrix}$$

where: $\delta \underline{z}$ = CSM position and velocity errors, or 6 parameters describing non-estimated measurement biases.

$$E_{TILT}^* = \begin{pmatrix} \overline{\sigma_a^2} & 0 & 0 \\ 0 & \overline{\sigma_b^2} & 0 \\ 0 & 0 & \overline{\sigma_c^2} \end{pmatrix}, \quad E_{CON}^* = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \overline{\sigma_\beta^2} & 0 \\ 0 & 0 & \overline{\sigma_\theta^2} \end{pmatrix}$$

$\overline{\sigma_a^2}, \overline{\sigma_b^2}, \overline{\sigma_c^2}$ = variance of 3 Euler angles describing rotation of radar measurement frame.

$\overline{\sigma_p^2}, \overline{\sigma_\theta^2}$ = variance of constant biases in elevation angle (β) and azimuth angle (θ).

Also define a \underline{b} vector (\underline{b}_c (6 x 1)) which relates the deviation in the measured quantity (Q) to the measurement bias (or CSM uncertainty), according to:

$$\delta Q = \underline{b}_c^T \delta \underline{z}$$

Since the non-estimated biases are not known on board the LEM, the following estimation equations are utilized for navigation i. e.

$$\text{weighting vector} = \underline{W} (9 \times 1) = E' \underline{b} / A$$

$$A = \underline{b}^T E' \underline{b} + \overline{\alpha^2}$$

$$\text{covariance matrix of state estimation errors} = E (9 \times 9) = E' \underline{W} \underline{W}^T A$$

where: \underline{b} satisfies: $\delta Q = \underline{b}^T \delta \underline{x}$, $\delta \underline{x}$ = state deviation vector

$\overline{\alpha^2}$ = variance of random measurement error

The actual estimation error which takes into account the non-estimated biases will be represented by e^* (9 x 1) and its associated covariance matrix is given by:

$$\begin{aligned} E^* &= (I - \underline{W} \underline{b}^T) E^{*'} (I - \underline{W} \underline{b}^T)^T + \underline{W} \underline{W}^T \overline{\alpha^2} - G \underline{b}_c \underline{W}^T \\ &\quad - \underline{W} \underline{b}_c^T G^T - \underline{W} \underline{b}_c^T E_c' \underline{b}_c \underline{W}^T \end{aligned} \quad (1)$$

$$\text{where: } E^{*'} (9 \times 9) = \Phi_L E^* \Phi_L^T \text{ (extrapolation of } E^*) \quad (1a)$$

G (9 x 6) is defined as $\overline{e^* \delta \underline{z}^T}$, $G_0 = 0$

$$G' = \Phi_L G \Phi_c^T \text{ (extrapolation of } G) \quad (1b)$$

Φ_L (9 x 9) = LEM state transition matrix

Φ_c (6 x 6) = CSM state transition matrix, or bias transition matrix

$$E_c' = \Phi_c E_c \Phi_c^T \text{ (extrapolation of } E_c) \quad (1c)$$

$$G = (I - \underline{W} \underline{b}^T) G' - \underline{W} \underline{b}_c^T E_c' \text{ (update of } G \text{ after measurement.)} \quad (1d)$$

Mean Square Velocity Correction (DEL V)

The estimate of the midcourse velocity correction is given as usual by:

$$\Delta \hat{V} = c^* \delta \hat{r} - \delta \hat{v} = B \delta \hat{x} = B(\underline{e}^* + \delta \underline{x})$$

where:

$$B(3 \times 9) = [c^* \quad -I \quad 0]$$

Then:
$$DELV = \overline{\Delta \hat{V} \Delta \hat{V}^T} = B(\underline{E}^* + \underline{x} + \underline{e}^* \delta \underline{x}^T + \delta \underline{x} \underline{e}^{*T}) B^T \quad (2)$$

and:

$$DELV = TRACE(DELV) \quad (3)$$

where:
$$\underline{x} = \overline{\delta \underline{x} \delta \underline{x}^T} = \text{covariance matrix of actual state deviations} \quad (4)$$

and:
$$\underline{x}' = \Phi_L \underline{x} \Phi_L^T \quad (\text{extrapolation}) \quad (4)$$

For an optimum estimate, it is recalled that $\underline{e} \delta \underline{x}^T = -E$ so that $DELV^*$ reduces simply to $B(\underline{x} - E) B^T$. However, this is not true for the non-optimum estimate and $\underline{e}^* \delta \underline{x}^T$ must be computed in order to compute $DELV$.

Thus, define $F(9 \times 9) = \underline{e}^* \delta \underline{x}^T$

$$F_0 = \underline{e}_0 \delta \underline{x}_0^T = \underline{e}_0 (\delta \hat{x}_0 - \underline{e}_0)^T = -\underline{e}_0 \underline{e}_0^T = -E_0$$

$$F' = \Phi_L F \Phi_L^T \quad (\text{extrapolation}) \quad (5a)$$

$$F = (I - \underline{W} \underline{b}^T) F' - \underline{W} \underline{b}_c^T L' \quad (\text{measurement update}) \quad (5b)$$

$$L' = \Phi_c L \Phi_L^T \quad (\text{extrapolation}) \quad (5c)$$

where:
$$L(6 \times 9) = \overline{\delta \underline{z} \delta \underline{x}^T}$$

The L matrix is usually zero before any velocity corrections since the actual LEM state is not correlated with measurement errors. (It may have an initial value, however, when $\delta \underline{z}$ represents the CSM state vector. L will probably be quite small even in this case when the rendezvous phase is initiated).

After a velocity correction, $\delta \underline{x}$ is correlated with $\delta \underline{z}$ since $\Delta \hat{V}$ is a function of $\delta \underline{z}$. Thus, the update of L after a velocity correction is given by:

$$L = \overline{\delta \underline{z} \delta \underline{x}_+^T} = L'(I + MB)^T + G^T B^T M^T \quad (\text{velocity correction update}) \quad (6)$$

$$(\text{noting } \delta \underline{x}_+^T = \delta \underline{x}^T (I + MB)^T + \underline{e}^{*T} B^T M^T)$$

where:

$$M(9 \times 3) = \begin{pmatrix} * \\ 0 \\ * \\ I \\ * \\ 0 \end{pmatrix}$$

The x matrix and F matrix must also be updated after a velocity

correction:

$$x = M(\text{DELV}^*) M^T + x' + MB (x' + F') + (x' + F')^T B^T M^T \quad (7)$$

(velocity correction update)

$$F = F' + (F' + E^{*'}) B^T M^T \quad (8)$$

(velocity correction update)

Thus the procedure followed to compute DELV is:

- 1) Between measurement points compute $E^{*'} , G' , E_c'$ $x' , F' ,$ and L' using equations (1a), (1b), (1c), (4), (5a), and (5c) respectively.
- 2) Compute $E^* , G , F ,$ after each measurement using equations (1), (1d), and (5b) respectively.
- 3) Compute DELV^* and DELV when desired using equations (2) and (3) respectively.
- 4) Compute $L , x ,$ and F after each velocity correction using equations (6), (7), and (8) respectively.