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Space Guidance Analysis Memo #37-64

TO: SGA Distribution
FROM: Donald Fraser
DATE: September 25, 1964
SUBJECT: Computer Program Description - Root Locus Technique

This memo describes a MH-1800 computer program developed to calculate the points for a root locus diagram with gain as a variable. Up to 19 open loop poles may be processed.

The method of calculation used is to form the polynomial $1 + KG$ (assuming negative feedback) and then factoring it using a binary version of the ROOTFINDER program* written by R. W. Baker.

Gain Control:

The polynomial KG is expressed in the form:

$$KG = K \frac{(s^m + a_{m-1} s^{m-1} + a_{m-2} s^{m-2} + \dots + a_1 s + a_0)}{(s^n + b_{n-1} s^{n-1} + b_{n-2} s^{n-2} + \dots + b_1 s + b_0)}$$

Note that the highest order coefficient in both numerator and denominator is unity.

The gain K is incremented logarithmically according to the rule

$$K = (\Delta K) K$$

from a specified minimum gain to a specified maximum gain and may be either positive or negative. If negative gain is used (positive feedback) the minimum gain should be a negative number with a larger magnitude than the maximum gain.

Output Control:

Output from the program includes a printed list of the closed loop poles for each value of gain plus, if desired, a completely plotted root locus diagram.

* See SGA Memo #32-64 for description of factoring technique

Provision is included to mark certain points on each branch to facilitate matching points on the plot with corresponding gains on the printout. The frequency of these points is controlled by the input variable "Space". If it is desired to mark every fifth point on each branch, space = 5.

Plotting of all points outside of a given radius can be suppressed. This radius is equal to the input variable "Limit".

Provision is included to plot either all the points or only those lying in the top half plane. This is controlled by the input variable "Plane" as follows:

Plane = 0 Both top and bottom half plane points are plotted

Plane = 1 Only top half plane points are plotted

Input Cards Required

- * Job Card
- * Userfile = Apollo
- * Run Rootlocus
- F As many sets of data cards as desired
- * Endjob

Data Cards

Card 1	Word 1	Gain Increment (ΔK)
	Word 2	Minimum Gain
	Word 3	Maximum Gain
	Word 4	Root Flag If = 0 Read poles and zeros in factored form If = 1 Read poles and zeros in polynomial form
	Word 5	Space
	Word 6	Plot Flag If = 0 No plotted output If = 1 Input control of plot format as described below is possible If = 2 Plot is done in Z transform format described below

If Plot Flag = 1

Control of the plot format with the following additional input cards is possible. For a description of the variables see the FILEPLOT memo (PR - 0007) published by the digital computation group. This group of cards should be included only on the first run of any job.

Card A1

Word 1 Point (for marked points only)

Card A2

Word 1 XMIN

Word 2 XMAX

Word 3 XINCH

Word 4 XTICK

Card A3

Word 1 YMIN

Word 2 YMAX

Word 3 YINCH

Word 4 YTICK

Plot Format

With Plot Flag non-zero, a point plot is made with open loop poles marked with X's, open loop zeros marked with small circles and locus points marked with small dots. If Plot Flag = 1.0, control of special point marking is possible, otherwise these points are marked with small squares.

The Z transform plot format is set up to plot only top half plane points in a frame small enough to fit in an 8 1/2 x 11 ring binder. The radius of the unit semicircle is four inches and tick spacing on both axes is 0.1. Limit is automatically reset to 1.25 if this plotting mode is selected; however it still must be read in.