TO: MIT SKYLAB Program Manager
FROM: William M. Robertson
DATE: September 29, 1970
SUBJECT: Detailed Change Evaluation - PCR SLO33 (Fixed Coordinate System for SKYLAB).

I propose that MIT/DL agree to the NASA-originated PCR #SLO33, "Fixed Coordinate System for SKYLAB", and implement it according to the detailed description given in the following sections. This implementation (and a few alternatives) have been discussed over the phone several times with NASA representatives (Bedford Cockrell and Emil Schiesser), and informally agreed upon. As will be pointed out in the evaluation section, this implementation will give NASA a highly flexible program at a relatively small expense.

IMPLEMENTATION DETAILS

1) Modify the formulation of the Planetary-Inertial Orientation Routine to use a full $3 \times 3$ Precession-Nutation Transformation Matrix stored in erasable. Delete (or at least disconnect) the computations in the PIOS routine for the lunar case. See the Appendix for the new PIOS formulation in GSOP format. *STORAGE IMPACT: FIXED; save about six words if PIOS computations are only disconnected, save about 85 more if they are deleted; ERASABLE: Spend 18 unshared erasables. * COMPUTATION time impact: Increase PIOS running time by about 3 milliseconds per call (A call takes the order of 1/4 sec for earth case).

2) Equivalence the AGC variable UNITW (representing the unit vector of the earth's polar axis, expressed in the Basic Reference Coordinate System) to the last row of the full $3 \times 3$ Precession-Nutation Transformation matrix. *STORAGE IMPACT: Fixed; None; ERASABLE: Save 6 words unshared erasable.
3) Delete the unshared erasable quantities AXO and AYO, as their previous function is now accomplished more generally by the full Precession-Nutation matrix in erasable. *STORAGE IMPACT: FIXED: None; ERASABLE: None, as AXO and AYO in UNITW.

4) Move AZO from fixed memory to unshared erasable memory. *STORAGE IMPACT: FIXED: Save 2 words; ERASABLE: Spend 2 words unshared erasable.

5) Modify the Coasting Integration Routine to compute the oblateness acceleration for the earth using the "true of date (of validity of the Precession-Nutation Matrix)" system rather than the current "mean of beginning of the Besselian Year" system. Specifically, since the earth oblateness acceleration contains only zonal terms \((J_2, J_3, J_4)\), use true north (i.e., UNITW or the third row of the Precession-Nutation Matrix) rather than the current approximation \((0, 0, 1)\). In Chapter 5 of the COLOSSUS 2E and LUMINARY 1D GSOP, change Eq. (2.2.13) to read:

\[
\mathbf{u}_Z = \text{unit vector of earth's true polar axis expressed in Basic Reference Coordinate System.}
\]

and use this \(\mathbf{u}_Z\) in Eq. (2.2.11). If it should later be decided to include tesseral terms \((J_{22},\text{ etc.})\) in the earth oblateness acceleration, make calls to the new version of PIOS to obtain \(\mathbf{u}_X, \mathbf{u}_Y, \mathbf{u}_Z\) and the components of the inertial \(\mathbf{r}\) in body-fixed coordinates, similar to those currently made for the lunar oblateness computation. *STORAGE IMPACT (zonals only case): FIXED: Spend about 5 words; ERASABLE: None. Computation Time Impact (zonals only case), Increase Coasting Integration running time by less than 25 milliseconds per step. (A step takes on the order of 1.4 seconds in close earth orbit trajectories without W-matrix extrapolation).

**EVALUATION**

Individual AGC storage and computation time impacts have been given above; the total impact of all five parts is:

**FIXED MEMORY:** Nearly None (+2 or 3 words)

**UNSHARED ERASABLE:** Spend 14 words
COMPUTATION TIME:  
PIO: Negligible (less than 1% increase) 
COASTING INTEGRATION: Negligible (less than 2% increase)

By incorporating all five parts, NASA would gain a great deal of flexibility. Specifically:

A)  
The Coordinate System of any year (e.g. 1900.0, 1950.0, 1975.0, 2000.0) could be chosen as the Basic Reference Coordinate System. The only fixed-memory change* would be to the star unit-vectors, which would have to be expressed in the coordinate system chosen (but properly-motioned to the year of the flight).

B)  
The full Precession-Nutation matrix would be at the disposal of NASA (it could be updated during a flight if desired). Moreover it is in a convenient and more understandable form as a 3 x 3 transformation matrix as opposed to the five parameters now used (AXO, AYO and the vector UNITW). Also, more or less "everybody" has a computer routine which will generate the two separate matrices whose product is this Precession-Nutation matrix.

C)  
By adjusting the erasable AZO, NASA would be able to correct for small errors in the earth rotational orientation which are caused by roundoff, and which have been estimated by TRW** to be about 30 ft/year at the surface of the earth.

D)***  
By adjusting the erasable AZO, and eleven of the 22 erasable coefficients**** in the Lunar-Solar Ephemerides Routine, NASA would have at their disposal the possibility of arbitrarily selecting the origin away from which TEPHEM (the time interval from this origin to the time the computer clock is zeroed) is measured. The origin

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*It is assumed that the 22 constants (19 if PCR SLO34 is disapproved) of the Lunar-Solar Ephemerides Routine are in erasable.

**TRW Inter-Office Correspondence #5522.7-70-43 by J. B. Clifford.

***This capability has not been fully verified, but it is believed to be substantially correct (99.44/100% pure).

****If PCR SLO34 is disapproved, nine of the 19 erasable coefficients. They are specifically LOS, LON, LOM, A, PHASEA, B, PHASEB, [C'MOON', PHASEC, C'SUN, PHASECSUN].
for TEPHEM has always been the Greenwich midnight between June 30 and July 1 which begins the Nearest Besselian Year during which the mission takes place, but this is not necessary.

To obtain the high flexibility outlined above at the relatively low impact stated, it is recommended that PCR SLO33 be implemented as described.
APPENDIX

5.5.2 PLANETARY INERTIAL ORIENTATION SUBROUTINE

This subroutine is used to transform vectors between the Basic Reference Coordinate System and a Planetary (Earth-fixed) Coordinate System at a specified time. These three coordinate systems are defined in Section 5.1.4.

Let \( \mathbf{r} \) be a vector in the Basic Reference Coordinate System, \( \mathbf{r}_P \) the same vector expressed in the Planetary Coordinate System, and \( t \) the specified ground elapsed time (GET). Then,

\[
\mathbf{r}_P = M(t) L \mathbf{r}
\]  \hspace{1cm} (5.2.1)

and

\[
\mathbf{r} = L^T M^T(t) \mathbf{r}_P
\]  \hspace{1cm} (5.2.2)

where \( M(t) \) is a time dependent orthogonal transformation matrix, and \( L \) is a \( 3 \times 3 \) matrix included in the pre-launch erasable data load. The matrix \( L \) is considered constant in one coordinate system for the duration of the mission, although for long missions \( L \) may be updated if desired.

For the earth, the matrix \( M(t) \) describes a rotation about the polar axis of the earth (the \( Z \)-axis of the Earth-fixed Coordinate System), and the matrix \( L \) expresses the precession and nutation of the polar axis from the \( Z \)-axis of the Basic Reference Coordinate System. Let

\[
A_Z = A_{Z0} + \omega_E (t + t_0)
\]  \hspace{1cm} (5.2.3)

Then

\[
M(t) = \begin{pmatrix}
\cos A_Z & \sin A_Z & 0 \\
-sin A_Z & \cos A_Z & 0 \\
0 & 0 & 1
\end{pmatrix}
\]
where \( \alpha_{Z0} \) is the angle between the X-axis of the Basic Reference Coordinate System and the X-axis of the Earth-fixed Coordinate System (the intersection of the Greenwich meridian and the equatorial plane of the earth) at some epoch* universal time, \( t_0 \) is the elapsed time between that epoch and the time that the computer clock was zeroed, and \( \omega_E \) is the angular velocity of the earth.

The lunar orientation capability previously existing in mainline Apollo, has been deleted for SKYLAB.

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*The epoch is generally taken to be that midnight at Greenwich between June 30 and July 1 which precedes the launch.