APOLLO Memo No. 218

To: John Miller  
From: J. H. Flanders  
Date: 5 October 1952  
Subject: Proposed Tolerances for Misalignments of IRIGs and PIPAs With Respect to the Stable Member Axis System

1. General. Misalignments of the IRIGs and PIPAs with respect to the ideal Stable Member Triad are the subject of alignment tests now being generated in the IMU System Group. In the course of this work, criteria for individual component misalignment have been reviewed. The following paragraphs contain proposals for specifying the misalignments in detail.

2. IRIG Misalignment. It is proposed that IRIG misalignment should be specified by its effect on fine alignment in space. The misalignment angle $\gamma_{n,m}$ is defined as the rotation of the "n" gyro about the "m" stable member axis. (See Figure 1.)

Rotation of IRIGs about their own input axes do not do damage. Therefore, $\gamma_x^x, \gamma_y^y, \gamma_z^z$ will not be discussed.

Next, consider a case where a one-degree error, $A_1$, left over from Coarse Alignment is to be eliminated by Fine Alignment torquing of a given gyro about its input. (See Figure 2.) Consider a rotation of one degree about the $Z_{SM}$ axis, for example, when the Z IRIG is misaligned by $\gamma_z^z$.

The correct number of pulses (approximately 1440) are fed to the Z IRIG. However, rotation must occur about the Middle Gimbal Axis in order to null the gyro. All of the gyro error goes to the middle gimbal servo because $A_{IG}^1 = 0$. Because of the misalignment, more rotation $A_2$ must occur about MGA to satisfy the Z IRIG null. The error, $E_1$, in fine alignment due to this condition is given by:
\[
E_1 = A_2 - A_1 = \frac{A_1}{\cos \gamma_{zy}} - A_1 \\
= A_1 \left[ \frac{1}{\cos \gamma_{zy}} - 1 \right]
\]

Assume gross IRIG misalignment; \( \gamma_{zy} = 1^\circ \)

\[
A_1 = 1^\circ = 3600 \text{ seconds}
\]

\[
E_1 = 3600 [1.0001 - 1.0000] = 0.36 \text{ seconds (Negligible)}
\]

Next consider the effects of \( \gamma_{zy} \); when a one-degree error, \( A_1 \), left over from Coarse Alignment is to be corrected by torquing the X IRIG about \( X_{SM} \). When this occurs the misaligned Z IRIG will sense the motion and introduce a misalignment \( E_2 \) about \( Z_{SM} \). (See Figure 3.) The error in fine alignment due to this condition is given by

\[
E_2 = A_1 \sin \gamma_{zy}.
\]

To place a figure of merit on allowable \( \gamma_{zy} \), assume that \( E_2 \) should be no larger than other errors which you have listed as contributing to Fine Alignment error. Then:

\[
E_2 \leq 20 \text{ arc seconds}
\]

\[
A_1 = 3600 \text{ arc seconds}
\]

\[
\gamma_{zy} \leq \frac{20}{3600} = 0.00555 \text{ radians}
\]

\[
= 6 \text{ milliradians (approx.)}
\]

To allow for a factor of safety, I recommend that:

a. We specify that \( \gamma_{zy} \leq 10 \text{ arc minutes} \)

b. Our IRIG Alignment Tests should measure to at least \( 11 \text{ arc minute} \).
3. PIPA Misalignments. It is proposed that PIPA misalignment should be specified by its effect on powered flight trajectories. The misalignment angle $\alpha_n$ is defined as the rotation of the "n" PIPA about the "m" stable member axis. (See Figure 4.)

Rotation of PIPAs about their own input axes do not do damage. Therefore, $\alpha_x$, $\alpha_y$, and $\alpha_z$ will not be discussed.

Next, consider the typical situation planned for powered flight. Assume that $X_{SM}$ has been perfectly aligned along the boost direction in Inertial Space. Any rotation of the velocity desired during the maneuver is assumed to be small and in the $X_{SM} - Z_{SM}$ plane. Thus, $Y_{SM}$ is normal to the maneuver plane. (See Figure 5.)

The $X$ PIPA is used to determine the propulsion cut-off command. The $Y$ PIPA and $Z$ PIPA are used to direct the velocity buildup in yaw and pitch respectively.

Observing the $Y_{SM} - Z_{SM}$ plane in Figure 5, it is apparent that $\alpha_y$ and $\alpha_z$ introduce errors which amount to gain changes in the pitch and yaw velocity steering loops. These changes are expressed by: Error in Gain = 1 - $\cos \alpha$

It would take a value of $\alpha$ = 3 degrees to make a 1% change in steering loop gain. As long as the boost occurs along $X_{SM}$, $\alpha_y$ and $\alpha_z$ are not critical.

Misalignments of the $X$ PIPA, $\alpha_x$, and $\alpha_x$, introduce secant errors into the cutoff velocity. These errors are always in the direction of late cutoff or too much velocity. Using 100 parts per million as a rough figure of merit, $\alpha_x$ and $\alpha_x$ must be maintained at less than 37 arc minutes of misalignment. (Secant of $00^0 38' 00" = 1.0001$).

Finally, the steering bias misalignments are considered. $\alpha_y$ error means that the $Y$ PIPA picks up boost velocity erroneously and steers the vehicle out of the maneuver plane by that angle. Similarly, $\alpha_z$ produces pitch steering errors directly. These two then are the most critical PIPA alignments.

However, $\alpha_x$ is almost zero by definition. Actually it is only off by the four arc seconds within which we hope to zero the Inner Gimbal Resolver. This is an electrical adjustment. On the other hand, $\alpha_y$ emerges as the most critical mechanical PIPA alignment. A tolerance of 10 arc seconds on these two items would relate $\alpha_y$ and $\alpha_z$ to the other secondary items in your IMU/SXT error budget.
In the discussion above, it was assumed that the boost accelerations remained closely associated with the $X_{SM}$ axis. If the IMU was involved in a $90^\circ$ vehicle turn during boost, the boost accelerations would rotate from an $X_{SM}$ orientation to a $Z_{SM}$ orientation. Then the cutoff would be affected by $\alpha_{xz}$ and $\alpha_{yz}$. $\alpha_{xy}$ would now become significant as a source of yaw steering error. $\alpha_{xz}$ would play a similar role in pitch steering. $\alpha_{xz}$ and $\alpha_{yz}$ then would become gain term errors.

This information is summarized in Table I as a recommendation for $\alpha$ tolerances and a description of their relationship to physical dimensions on the Stable Member.

**TABLE I. Proposed PIPA Alignment Tolerances**

<table>
<thead>
<tr>
<th>Angle</th>
<th>Determining Element</th>
<th>Tolerance $(X \text{ Boost})$</th>
<th>Tolerance $(Z \text{ Boost})$</th>
<th>Tolerance (Overall Min.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_{xy}$</td>
<td>Machine normal to prime surface</td>
<td>37'</td>
<td>10'</td>
<td>10'</td>
</tr>
<tr>
<td>$\alpha_{xz}$</td>
<td>Pins</td>
<td>37'</td>
<td>Not critical</td>
<td>37'</td>
</tr>
<tr>
<td>$\alpha_{yz}$</td>
<td>Pins</td>
<td>10'</td>
<td>Not critical</td>
<td>10'</td>
</tr>
<tr>
<td>$\alpha_{yx}$</td>
<td>Machine parallel to IGA bore</td>
<td>not critical</td>
<td>10'</td>
<td>10'</td>
</tr>
<tr>
<td>$\alpha_{zx}$</td>
<td>Pins</td>
<td>not critical</td>
<td>37'</td>
<td>37'</td>
</tr>
<tr>
<td>$\alpha_{zy}$</td>
<td>Prime surface</td>
<td>10'</td>
<td>37'</td>
<td>37'</td>
</tr>
</tbody>
</table>

Distribution

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FIGURE 1. IRIG MISALIGNMENTS.
Figure 2: Misalignment due to $\cos \phi_{Z_Y}$
Figure 3: Misalignment due to $\sin \theta_{xy}$
Figure 5: Individual PIPA Misalignments and Their Effects.