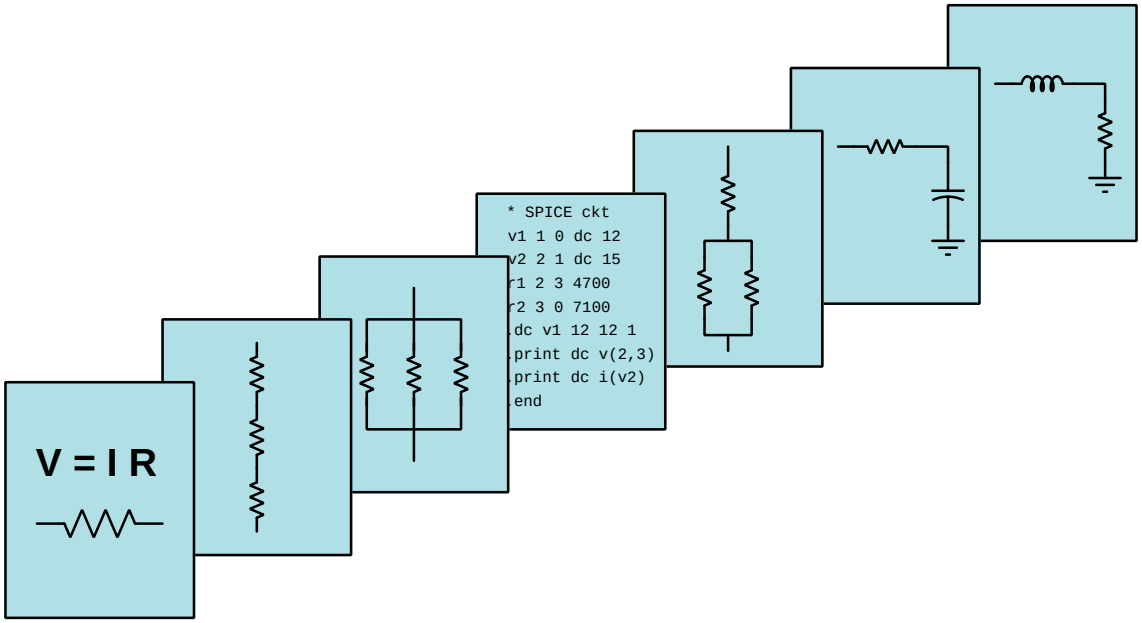
Modular Electronics Learning (ModEL) project



Single-Stage BJT Amplifiers

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Chapter 1

Introduction

1.1 Recommendations for students

Bipolar junction transistors are highly useful electronic amplifying devices, and there is more than one way in which a BJT might be used to construct a practical amplifier circuit. This module explores the different ways in which we commonly find BJTs applied to single-transistor amplifier circuits.

Important concepts related to BJT amplifiers include **gain**, **biasing**, **beta** (β or h_{FE}), **PN junction behavior**, **BJT behavior**, effects of **opens** versus **shorts**, **Ohm's Law**, **Kirchhoff's Voltage Law**, **Kirchhoff's Current Law**, **resistance transformation**, **dynamic emitter resistance**, **voltage dividers**, **swamping**, **quiescent conditions**, **capacitive reactance**, **bypassing**, **series** versus **parallel** networks, **negative feedback**, **Superposition Theorem**, **Maximum Power Transfer Theorem**, **impedance matching**, **transmission lines**, **characteristic impedance**, **Thévenin's Theorem**, and **Norton's Theorem**.

The *Superposition Theorem* is used extensively throughout the tutorial as an analytical technique for understanding the operation of BJT amplifiers. It is based on the concept that for any linear and bilateral system the whole is equal to the sum of its parts; namely, that we may analyze the effects of multiple electrical sources on a linear network by considering the effects caused by each source (at a time) and then summing all those effects together to arrive at a composite result. BJTs are not perfectly linear devices, but they are linear *enough* for the Superposition Theorem to yield approximate results.

Due to the heavy application of math to transistor amplifier analysis, the reader is urged to apply the mathematical principles as soon as possible. The amplifier design example contained in the tutorial serves this purpose well, as it allows you to apply the formulae discussed previously to a real circuit example and then check your work against the completed example to see if your application was correct. Do not simply read the example and assume you understand it just because nothing in the presentation seemed confusing. *Until you can perform the analysis yourself without assistance, you haven't mastered it!*

Here are some good questions to ask of yourself while studying this subject:

- How might an experiment be designed and conducted to measure the beta ratio for a bipolar junction transistor? What hypothesis (i.e. prediction) might you pose for that experiment, and what result(s) would either support or disprove that hypothesis?
- How might an experiment be designed and conducted to plot a family of characteristic curves for a bipolar junction transistor? What hypotheses (i.e. predictions) might you pose for that experiment, and what result(s) would either support or disprove those hypotheses?
- How might an experiment be designed and conducted to measure the gain of an amplifier circuit? What hypotheses (i.e. predictions) might you pose for that experiment, and what result(s) would either support or disprove those hypotheses?
- How is an amplifier circuit able to boost the power of the input signal without violating the Law of Energy Conservation?
- What is the fundamental operating principle of a BJT?
- How do we quantify *gain* for a transistor?
- Why do simple single-transistor amplifier circuits fail to fully reproduce the waveform of an AC signal?
- How does "biasing" work to solve this problem?
- How is the Superposition Theorem useful in analyzing transistor amplifier circuits?
- Why do we replace the transistor with an "open" when applying the Superposition Theorem to an amplifier circuit?
- What does the term *quiescent* mean for an amplifier circuit?
- How may we empirically determine that an amplifier circuit has been properly biased?
- Why is "swamping" a useful engineering strategy, and what kinds of problems does it solve?
- What are some of the advantages and disadvantages of the common-emitter amplifier configuration?
- What are some of the advantages and disadvantages of the common-collector amplifier configuration?
- What are some of the advantages and disadvantages of the common-base amplifier configuration?
- Why does resistance "appear" magnified or diminished when viewed "through" a transistor amplifier circuit?
- Where does dynamic emitter resistance come from in a BJT?
- Why is the common-emitter amplifier configuration so challenging to analyze compared to the common-collector?

- Why does the common-base amplifier configuration have a current gain less than one?
- What is the purpose of a bypass capacitor in a BJT amplifier circuit?
- Which amplifier configuration(s) have/has an inverting characteristic?
- Why might it be important to match the impedance of an amplifier circuit with the signal source and/or load connecting to it?
- How do we determine the input or output impedance of a given amplifier circuit?

1.2 Challenging concepts related to single-stage BJT amplifiers

The following list cites concepts related to this module's topic that are easily misunderstood, along with suggestions for properly understanding them:

- **Superposition of signals** – any linear-behaving circuit with multiple sources may be analyzed one source at a time, the results of those analyses simply added together to determine the composite result. We use this principle extensively in transistor circuit analysis where we have quiescent DC values superimposed on AC signals. Although transistors are not perfectly linear in their behavior, when properly biased for amplifier usage they are linear enough for superposition to yield approximately correct results.

- **BJT beta** – the "beta" (β or h_{FE}) value for a bipolar junction transistor predicts the number of electric charge carriers that are allowed through its collector terminal for every one charge carrier passing through its base terminal. While not particularly stable over different operating conditions, this amplification factor for a BJT is nevertheless useful in making approximate current calculations for BJT-based circuits. One important caveat of beta is that it represents an *upper limit* (i.e. *maximum*) collector-to-base current ratio. If the collector-emitter voltage drop is low enough, it is possible for the transistor to pass less current through its collector terminal than beta would predict, in which case we say the transistor is "saturated".

- **Necessary conditions for transistor operation** – bipolar junction transistors require a base current in the proper direction to inject minority carriers into the base layer (i.e. PNP transistors require hole injection from emitter to base, and so the emitter must be made positive and the base made negative, relative to each other; NPN transistors require electron injection from emitter to base, and so the emitter must be negative and the base positive.), and the collector-to-emitter voltage must be of the correct polarity (i.e. so that collector terminal current goes in the same direction as base terminal current, either *out* or *in*). Both currents join at the emitter terminal, making the emitter current the sum of the base and collector currents.

- **Signal inversion** – some transistor circuits have an "inverting" characteristic in which a rising input signal results in a falling output signal. This is not necessarily obvious upon first inspection, but a good way to make the concept clearer is to run "thought experiments" on the circuit with multiple values of V_{in}, computing the value of V_{out} for each scenario, and seeing for yourself the effect one signal has on the other.

- **Common terminals in amplifier configurations** – BJT amplifier circuits are often categorized as being "common-emitter", "common-collector", or "common-base", and it is usually unclear why this is so. One approach is to simplify these circuits to show them as *grounded-emitter*, *grounded-collector*, and *grounded-base*, respectively.

- **Gain calculations** – gain for any amplifier is most properly calculated as the ratio of output *change* to input *change*, not necessarily the ratio of output to input. For example, voltage gain is calculated as $A_V = \frac{\Delta V_{out}}{\Delta V_{in}}$, the "Delta" ($\Delta$) symbols representing "change in" the designated variable(s). $\frac{\Delta V_{out}}{\Delta V_{in}}$ is equivalent to $\frac{V_{out}}{V_{in}}$ if and only if the amplifier in question outputs zero

when the input is zero (i.e. when the transfer function is *purely linear* rather than merely being *affine*), which for simple one-transistor amplifiers is almost never true.

- **Dynamic emitter resistance** – this concept is confusing to students for two major reasons. First, it emerges only if and when we dispense with the simplifying assumption of base-emitter voltage being exactly 0.7 Volts all the time. Being a PN diode junction, the behavior of the base-emitter junction of a BJT is more accurately described by the Shockley diode equation, which predicts small variations in PN junction voltage drop as junction current varies. Second, dynamic emitter resistance is incremental in nature, which means it is an AC effect rather than a DC effect.

1.3 Recommendations for instructors

This section lists realistic student learning outcomes supported by the content of the module as well as suggested means of assessing (measuring) student learning. The outcomes state what learners should be able to do, and the assessments are specific challenges to prove students have learned.

- **Outcome** – Demonstrate effective technical reading and writing

  Assessment – Students present their outlines of this module's instructional chapters (e.g. Case Tutorial, Tutorial, Historical References, etc.) ideally as an entry to a larger Journal document chronicling their learning. These outlines should exhibit good-faith effort at summarizing major concepts explained in the text.

  Assessment – Students show how quantitative results were obtained by the author in the Tutorial chapter's gain calculation and amplifier design examples.

  Assessment – Students present their "Conceptual Toolbox" Theory Journal entries listing important concepts applied in the Tutorial text(s). A "Conceptual Toolbox" serves as inspiration when solving problems, each concept suggesting a potential problem-solving action. Challenge students to collaboratively generate a Conceptual Toolbox for the day's topic during the class session and then use it when solving problems together, thus fostering the habit of reasoning from principles rather than recalling procedures!

- **Outcome** – Apply the concepts of Ohm's Law, Kirchhoff's Voltage and Current Laws, and principles of bipolar junction transistors to amplifier circuit analysis

  Assessment – Compute DC signal values for given bias voltage conditions at the input of an amplifier; e.g. pose problems in the form of the "Common-emitter calculations" Quantitative Reasoning question.

  Assessment – Calculate voltage gain from the computed signal values at different instantaneous input conditions; e.g. pose problems in the form of the "Common-emitter calculations" Quantitative Reasoning question.

- **Outcome** – Predict amplifier characteristics from schematic diagrams

  Assessment – Calculate amplifier voltage gain as well as input and output impedances from just given component values; e.g. pose problems in the form of the "Common-emitter approximations" Quantitative Reasoning question.

- **Outcome** – Design a transistor amplifier circuit to given specifications

  Assessment – Sketch a schematic diagram of a circuit employing a bipolar junction transistor to amplify a signal with a specified maximum peak-to-peak voltage, with specified input and output impedance values, and with a desired gain.

Chapter 2

Case Tutorial

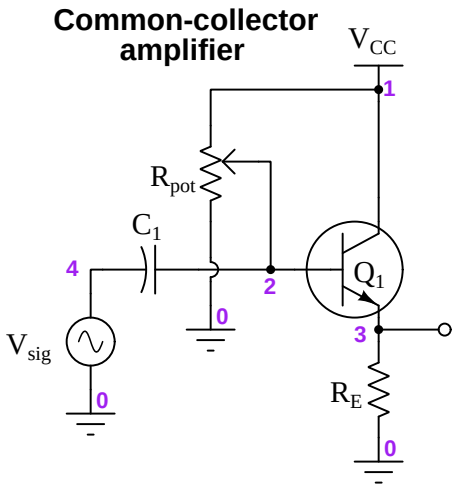
The idea behind a *Case Tutorial* is to explore new concepts by way of example. In this chapter you will read less presentation of theory compared to other Tutorial chapters, but by close observation and comparison of the given examples be able to discern patterns and principles much the same way as a scientific experimenter. Hopefully you will find these cases illuminating, and a good supplement to text-based tutorials.

These examples also serve well as challenges following your reading of the other Tutorial(s) in this module – can you explain *why* the circuits behave as they do?

2.1 Example: effects of bias on common-collector amplifier

In this SPICE simulation we show a common-collector amplifier operating with varying amounts of DC bias:

Circuit schematic diagram (with node numbers listed):



Instead of using two fixed resistors as is customary to form a "voltage divider" biasing network, here we are using a single potentiometer (R_{pot}) and varying the wiper's position to control the amount of DC bias applied to the transistor. In the SPICE netlist we model the single potentiometer as two resistors, R_{pot1} and R_{pot2}, the sum of which always adds to the single potentiometer's total resistance value of 20 kΩ. The input signal (V_{sig}) is a sinusoidal AC voltage with a peak value of 1 Volt and a frequency of 1 kHz. The transistor (Q_1) is a generic NPN type with a beta value of 120, the input coupling capacitor is 2.2 μF, and the emitter resistor has a value of 1 kΩ. The DC power supply (V_{CC} and ground) is 15 Volts.

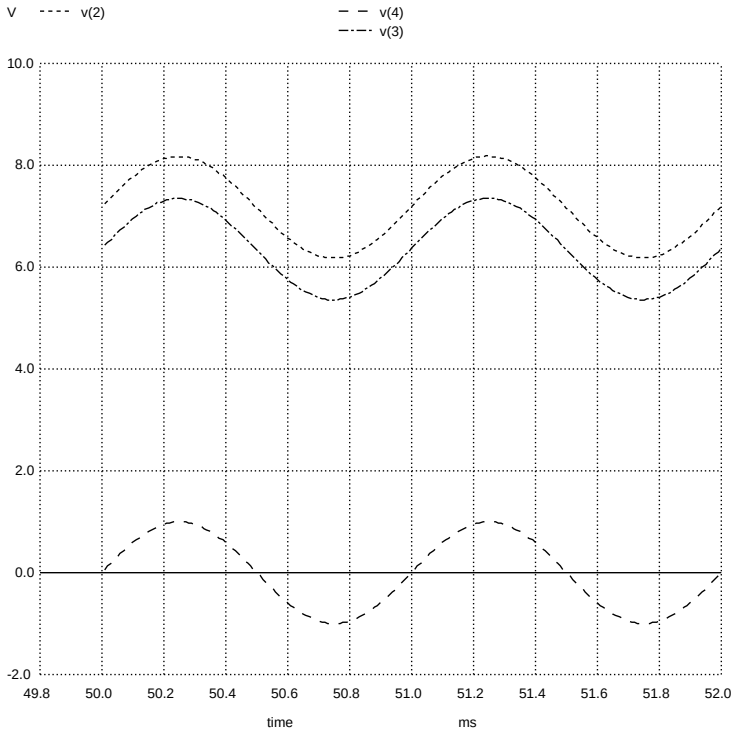
An important detail of this SPICE analysis appears in the `tran` (transient) analysis card of the netlist, where we specify the interval time duration (10 microseconds) followed by the "stop" time (52 milliseconds) followed by the "start" time (50 milliseconds). We intentionally delay the plotting of results until SPICE has performed the first 50 milliseconds of the analysis to give the input capacitor and bias potentiometer enough time for the capacitor's DC voltage to reach a stable value. Then, we plot for 2 milliseconds' worth of time (from 50 ms to 52 ms) in order to show two full cycles of the 1 kHz waveforms.

The following simulations will show the AC input signal (V_{sig}) at node 4, the biased input signal at node 2, and the amplifier's output signal at node 3, all with respect to ground (node 0).

In this first simulation we set the wiper at the 50% position:

SPICE netlist:

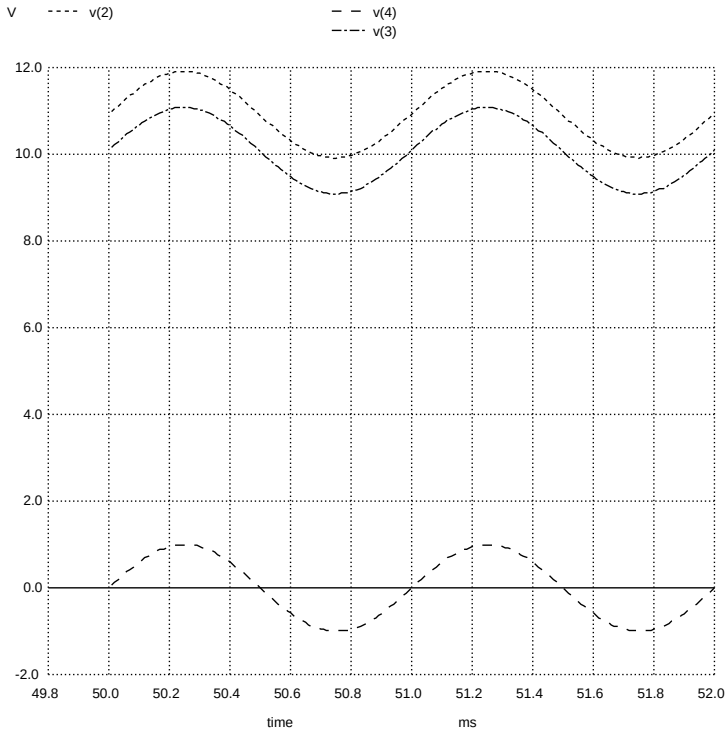
```
* SPICE simulation of C-C amplifier
vcc 1 0 dc 15
vsig 4 0 sin (0 1 1e3 0 0)
rpot1 1 2 10e3
rpot2 2 0 10e3
re 3 0 1e3
c1 2 4 2.2e-6
q1 1 2 3 qmod
.model qmod npn bf=120
.tran 10u 10m 12m uic
.plot tran v(4) v(2) v(3)
.end
```



Next, setting the wiper at the 75% position (i.e. closer to V_{CC} than ground):

SPICE netlist:

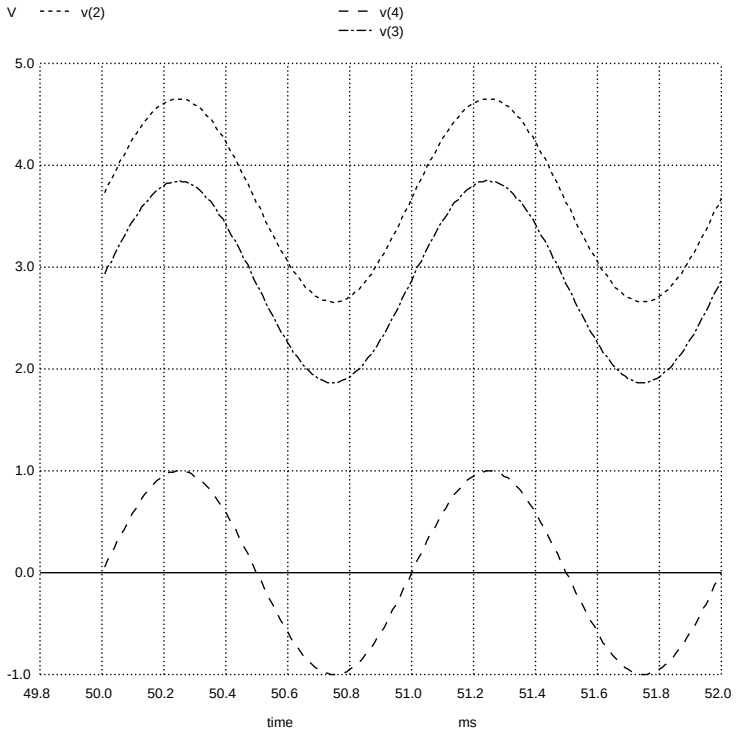
```
* SPICE simulation of C-C amplifier
vcc 1 0 dc 15
vsig 4 0 sin (0 1 1e3 0 0)
rpot1 1 2 5e3
rpot2 2 0 15e3
re 3 0 1e3
c1 2 4 2.2e-6
q1 1 2 3 qmod
.model qmod npn bf=120
.tran 10u 52m 50m uic
.plot tran v(4) v(2) v(3)
.end
```



Next, setting the wiper at the 25% position (i.e. closer to ground than V_{CC}):

SPICE netlist:

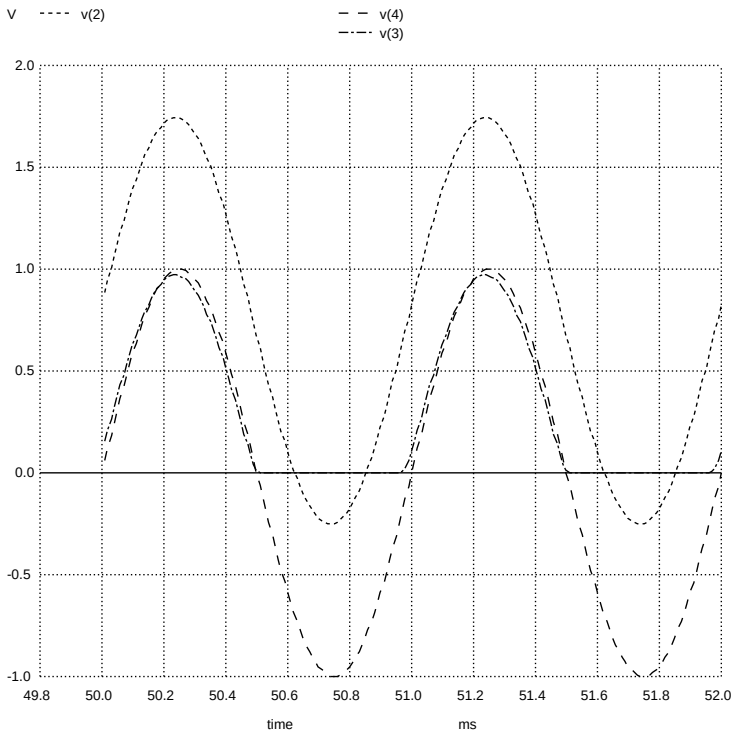
```
* SPICE simulation of C-C amplifier
vcc 1 0 dc 15
vsig 4 0 sin (0 1 1e3 0 0)
rpot1 1 2 15e3
rpot2 2 0 5e3
re 3 0 1e3
c1 2 4 2.2e-6
q1 1 2 3 qmod
.model qmod npn bf=120
.tran 10u 52m 50m uic
.plot tran v(4) v(2) v(3)
.end
```



Lastly, setting the wiper at the 5% position (i.e. almost all the way to ground):

SPICE netlist:

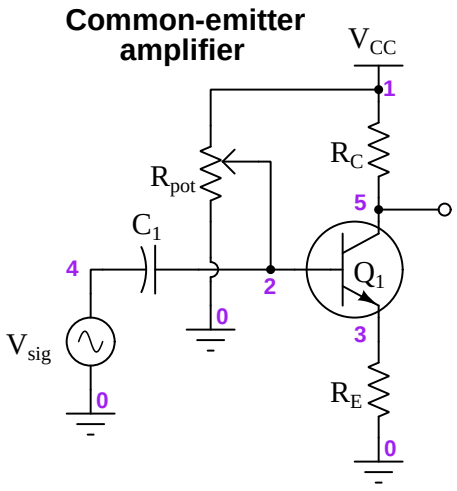
```
* SPICE simulation of C-C amplifier
vcc 1 0 dc 15
vsig 4 0 sin (0 1 1e3 0 0)
rpot1 1 2 19e3
rpot2 2 0 1e3
re 3 0 1e3
c1 2 4 2.2e-6
q1 1 2 3 qmod
.model qmod npn bf=120
.tran 10u 52m 50m uic
.plot tran v(4) v(2) v(3)
.end
```



2.2 Example: effects of bias on common-emitter amplifier

In this SPICE simulation we show a common-emitter amplifier operating with varying amounts of DC bias:

Circuit schematic diagram (with node numbers listed):



Instead of using two fixed resistors as is customary to form a "voltage divider" biasing network, here we are using a single potentiometer (R_{pot}) and varying the wiper's position to control the amount of DC bias applied to the transistor. In the SPICE netlist we model the single potentiometer as two resistors, R_{pot1} and R_{pot2}, the sum of which always adds to the single potentiometer's total resistance value of 20 kΩ. The input signal (V_{sig}) is a sinusoidal AC voltage with a peak value of 1 Volt and a frequency of 1 kHz. The transistor (Q_1) is a generic NPN type with a beta value of 120, the input coupling capacitor is 2.2 μF, and the emitter resistor has a value of 1 kΩ. The collector resistor is five times the value of the emitter resistor. The DC power supply (V_{CC} and ground) is 15 Volts.

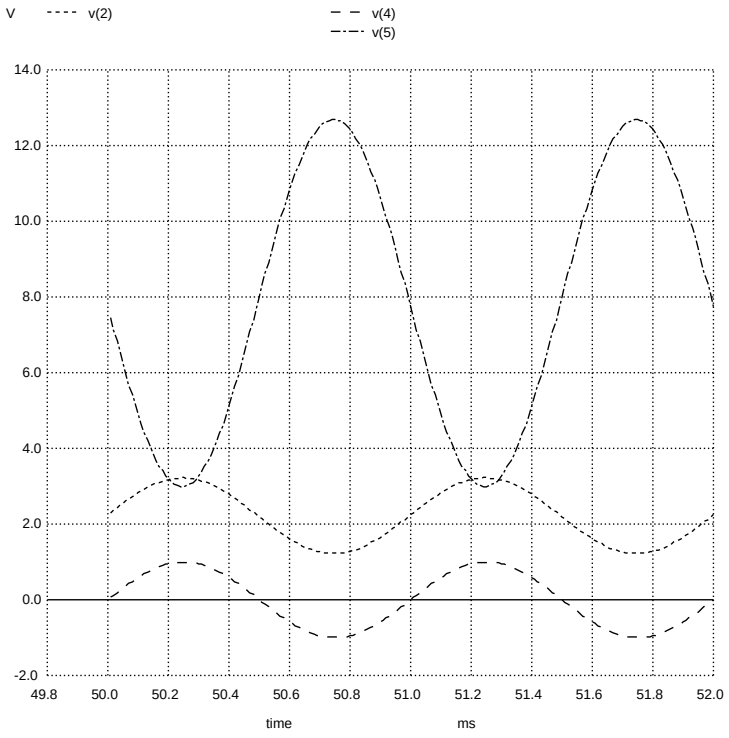
An important detail of this SPICE analysis appears in the `tran` (transient) analysis card of the netlist, where we specify the interval time duration (10 microseconds) followed by the "stop" time (52 milliseconds) followed by the "start" time (50 milliseconds). We intentionally delay the plotting of results until SPICE has performed the first 50 milliseconds of the analysis to give the input capacitor and bias potentiometer enough time for the capacitor's DC voltage to reach a stable value. Then, we plot for 2 milliseconds' worth of time (from 50 ms to 52 ms) in order to show two full cycles of the 1 kHz waveforms.

The following simulations will show the AC input signal (V_{sig}) at node 4, the biased input signal at node 2, and the amplifier's output signal at node 5, all with respect to ground (node 0).

In this first simulation we set the wiper at the 15% position (closer to ground than V_{CC}):

SPICE netlist:

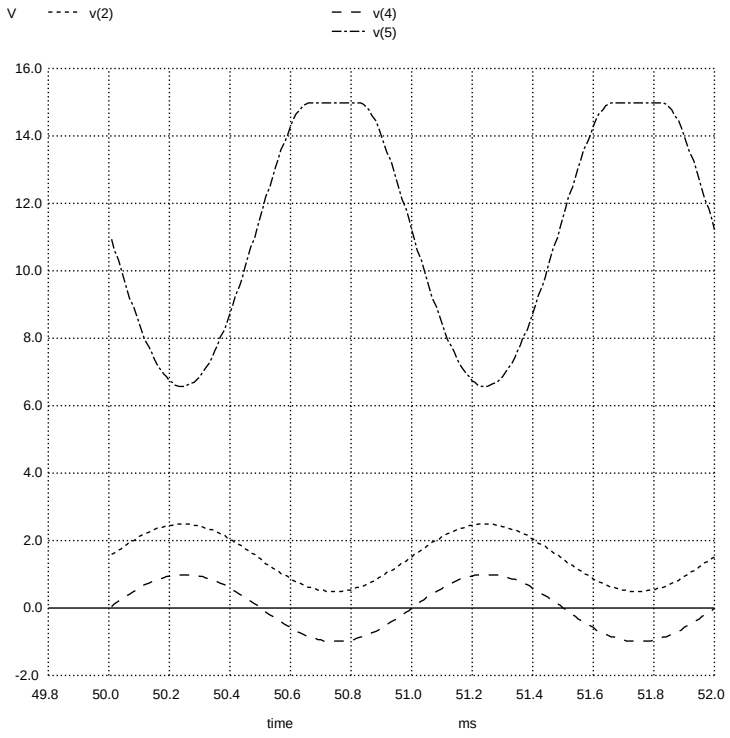
```
* SPICE simulation of C-E amplifier
vcc 1 0 dc 15
vsig 4 0 sin (0 1 1e3 0 0)
rpot1 1 2 17e3
rpot2 2 0 3e3
rc 1 5 5e3
re 3 0 1e3
c1 2 4 2.2e-6
q1 5 2 3 qmod
.model qmod npn bf=120
.tran 10u 52m 50m uic
.plot tran v(4) v(2) v(5)
.end
```



Next we set the wiper at the 10% position (closer to ground than V_{CC}):

SPICE netlist:

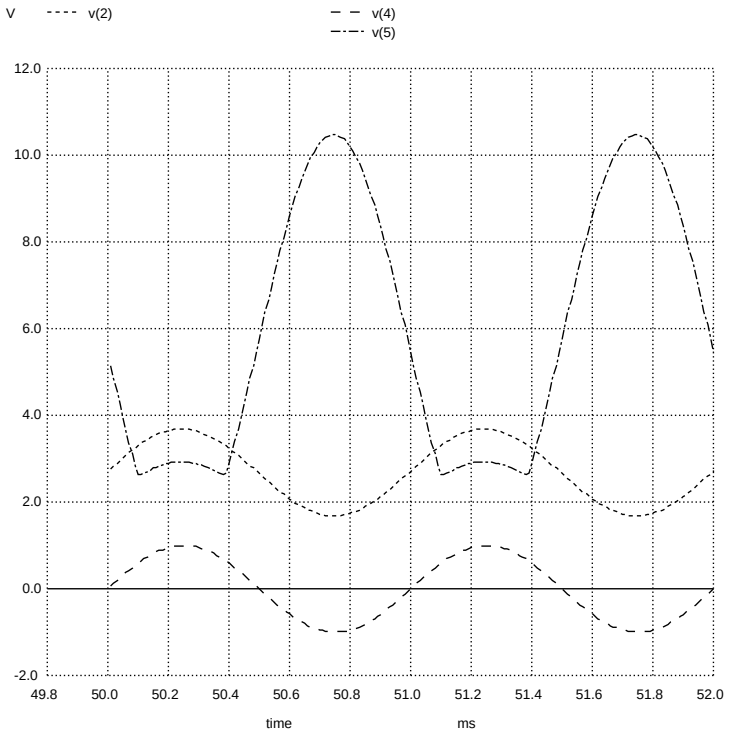
```
* SPICE simulation of C-E amplifier
vcc 1 0 dc 15
vsig 4 0 sin (0 1 1e3 0 0)
rpot1 1 2 18e3
rpot2 2 0 2e3
rc 1 5 5e3
re 3 0 1e3
c1 2 4 2.2e-6
q1 5 2 3 qmod
.model qmod npn bf=120
.tran 10u 52m 50m uic
.plot tran v(4) v(2) v(5)
.end
```



Next we set the wiper at the 20% position (closer to ground than V_{CC}):

SPICE netlist:

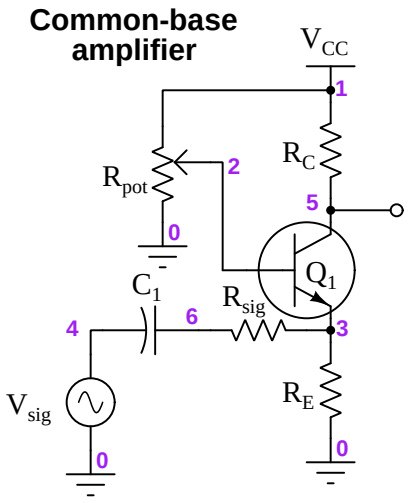
```
* SPICE simulation of C-E amplifier
vcc 1 0 dc 15
vsig 4 0 sin (0 1 1e3 0 0)
rpot1 1 2 16e3
rpot2 2 0 4e3
rc 1 5 5e3
re 3 0 1e3
c1 2 4 2.2e-6
q1 5 2 3 qmod
.model qmod npn bf=120
.tran 10u 52m 50m uic
.plot tran v(4) v(2) v(5)
.end
```



2.3 Example: effects of bias on common-base amplifier

In this SPICE simulation we show a common-emitter amplifier operating with varying amounts of DC bias:

Circuit schematic diagram (with node numbers listed):



Instead of using two fixed resistors as is customary to form a "voltage divider" biasing network, here we are using a single potentiometer (R_{pot}) and varying the wiper's position to control the amount of DC bias applied to the transistor. In the SPICE netlist we model the single potentiometer as two resistors, R_{pot1} and R_{pot2}, the sum of which always adds to the single potentiometer's total resistance value of 20 kΩ. The input signal (V_{sig}) is a sinusoidal AC voltage with a peak value of 1 Volt and a frequency of 1 kHz. The transistor (Q_1) is a generic NPN type with a beta value of 120, the input coupling capacitor is 2.2 μF, and the emitter resistor has a value of 1 kΩ. The collector resistor is five times the value of the emitter resistor. The input resistor (R_{sig}) is 1 kΩ. The DC power supply (V_{CC} and ground) is 15 Volts.

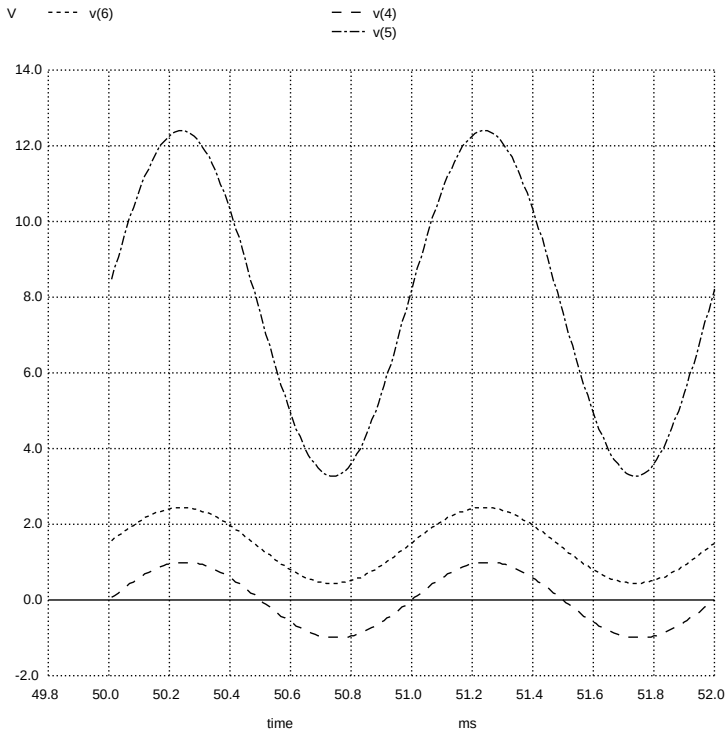
An important detail of this SPICE analysis appears in the `tran` (transient) analysis card of the netlist, where we specify the interval time duration (10 microseconds) followed by the "stop" time (52 milliseconds) followed by the "start" time (50 milliseconds). We intentionally delay the plotting of results until SPICE has performed the first 50 milliseconds of the analysis to give the input capacitor and bias potentiometer enough time for the capacitor's DC voltage to reach a stable value. Then, we plot for 2 milliseconds' worth of time (from 50 ms to 52 ms) in order to show two full cycles of the 1 kHz waveforms.

The following simulations will show the AC input signal (V_{sig}) at node 4, the biased input signal at node 6, and the amplifier's output signal at node 5, all with respect to ground (node 0).

In this first simulation we set the wiper at the 15% position (closer to ground than V_{CC}):

SPICE netlist:

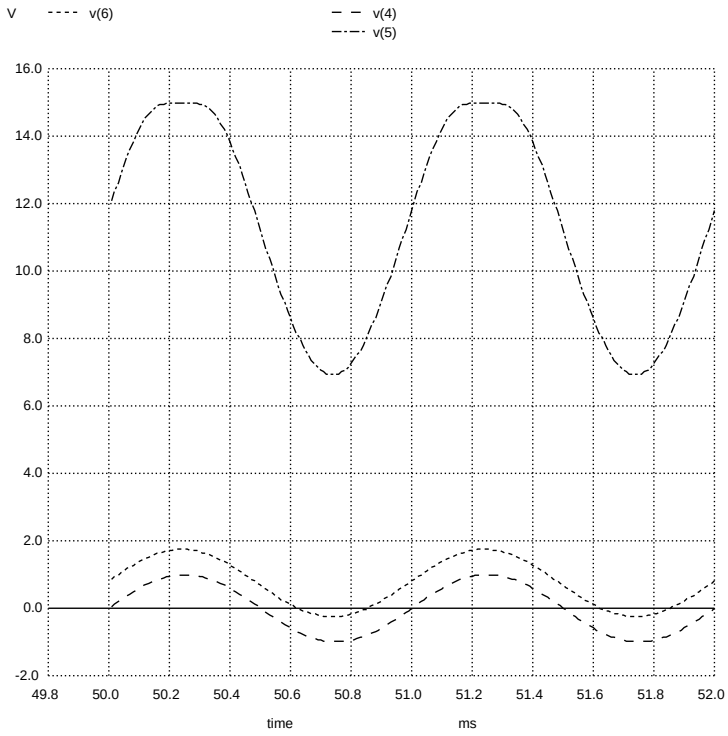
```
* SPICE simulation of C-B amplifier
vcc 1 0 dc 15
vsig 4 0 sin (0 1 1e3 0 0)
rpot1 1 2 17e3
rpot2 2 0 3e3
rc 1 5 5e3
re 3 0 1e3
rsig 6 3 1e3
c1 6 4 2.2e-6
q1 5 2 3 qmod
.model qmod npn bf=120
.tran 10u 52m 50m uic
.plot tran v(4) v(6) v(5)
.end
```



Next we set the wiper at the 10% position (closer to ground than V_{CC}):

SPICE netlist:

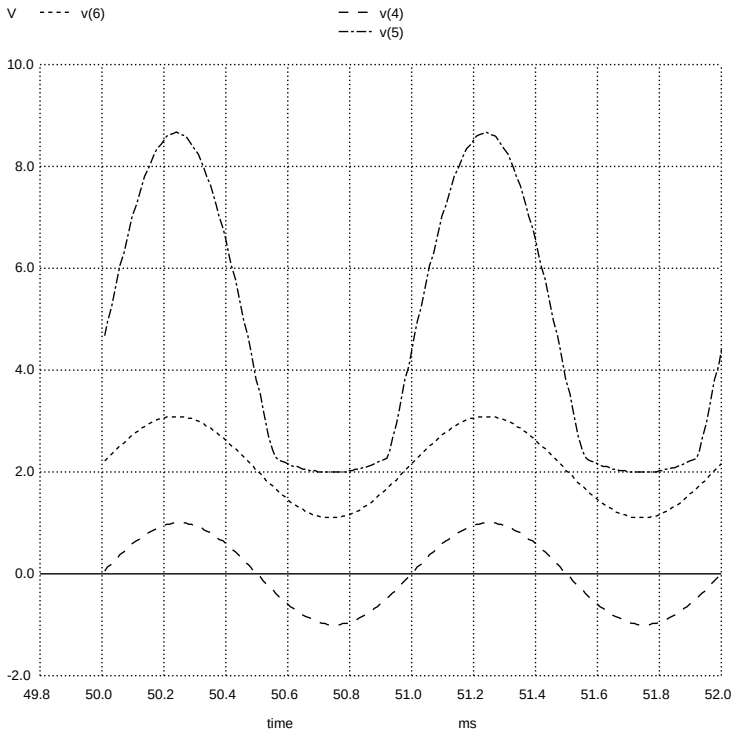
```
* SPICE simulation of C-B amplifier
vcc 1 0 dc 15
vsig 4 0 sin (0 1 1e3 0 0)
rpot1 1 2 18e3
rpot2 2 0 2e3
rc 1 5 5e3
re 3 0 1e3
rsig 6 3 1e3
c1 6 4 2.2e-6
q1 5 2 3 qmod
.model qmod npn bf=120
.tran 10u 52m 50m uic
.plot tran v(4) v(6) v(5)
.end
```



Next we set the wiper at the 20% position (closer to ground than V_{CC}):

SPICE netlist:

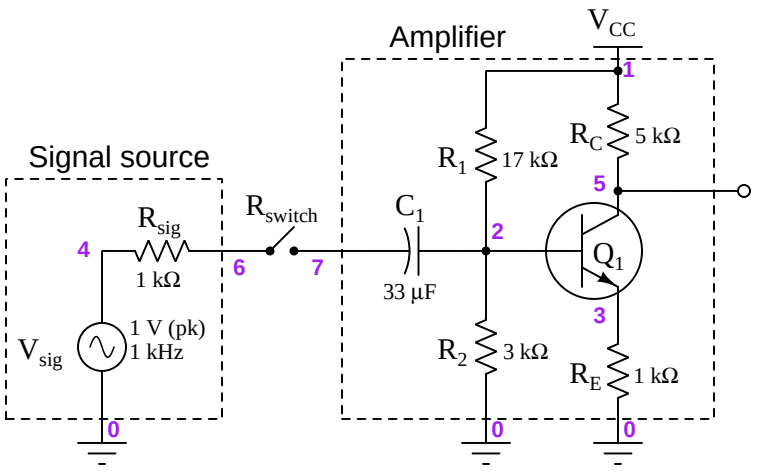
```
* SPICE simulation of C-B amplifier
vcc 1 0 dc 15
vsig 4 0 sin (0 1 1e3 0 0)
rpot1 1 2 16e3
rpot2 2 0 4e3
rc 1 5 5e3
re 3 0 1e3
rsig 6 3 1e3
c1 6 4 2.2e-6
q1 5 2 3 qmod
.model qmod npn bf=120
.tran 10u 52m 50m uic
.plot tran v(4) v(6) v(5)
.end
```



2.4 Example: common-emitter input impedance

In this SPICE simulation we will use a common-emitter amplifier properly biased for Class A operation as a load to a signal source having its own internal resistance:

Circuit schematic diagram (with node numbers listed):



There is nothing noteworthy about the amplifier itself. It uses a two-resistor "voltage divider" biasing network, a coupling capacitor at the input terminal, a 5:1 collector:emitter resistor ratio for a voltage gain of approximately five. The DC power supply (V_{CC} and ground) is 15 Volts. What is a bit unusual is the SPST switch connecting the signal source to the amplifier's input, simulated by a resistance in the SPICE netlist named R_{switch}, set to an extremely high resistance for the "open" condition and a negligible resistance for the "closed" condition. The coupling capacitor was chosen to have a negligible reactance (less than 5 Ohms at 1 kHz).

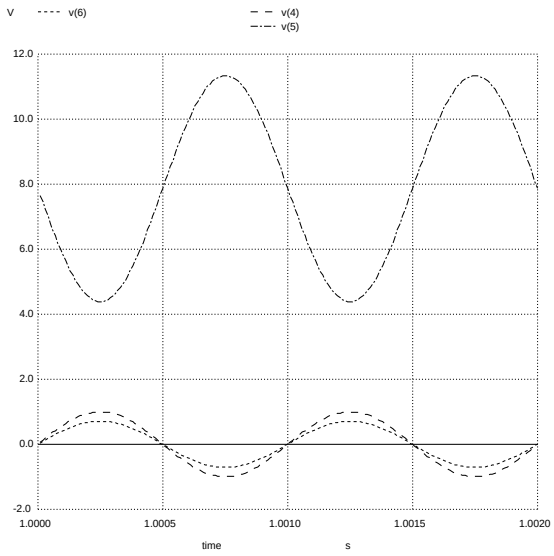
An important detail of this SPICE analysis appears in the `tran` (transient) analysis card of the netlist, where we specify the interval time duration (10 microseconds) followed by the "stop" time (1002 milliseconds) followed by the "start" time (1000 milliseconds). We intentionally delay the plotting of results until SPICE has performed the first 1000 milliseconds of the analysis to give the input capacitor and bias resistors enough time for the capacitor's DC voltage to reach a stable value. Then, we plot for 2 milliseconds' worth of time (from 1000 ms to 1002 ms) in order to show two full cycles of the 1 kHz waveforms.

The following simulations will show the signal's ideal source voltage (node 4) along with the signal voltage at the source's output terminal (node 6), and finally the amplifier's output signal at node 5, all with respect to ground (node 0). Simulations will be shown both for the switch-closed condition and the switch-open condition so you may see the "loading" effect of the amplifier's input impedance on the source signal as measured at node 6.

In this first simulation the switch is closed:

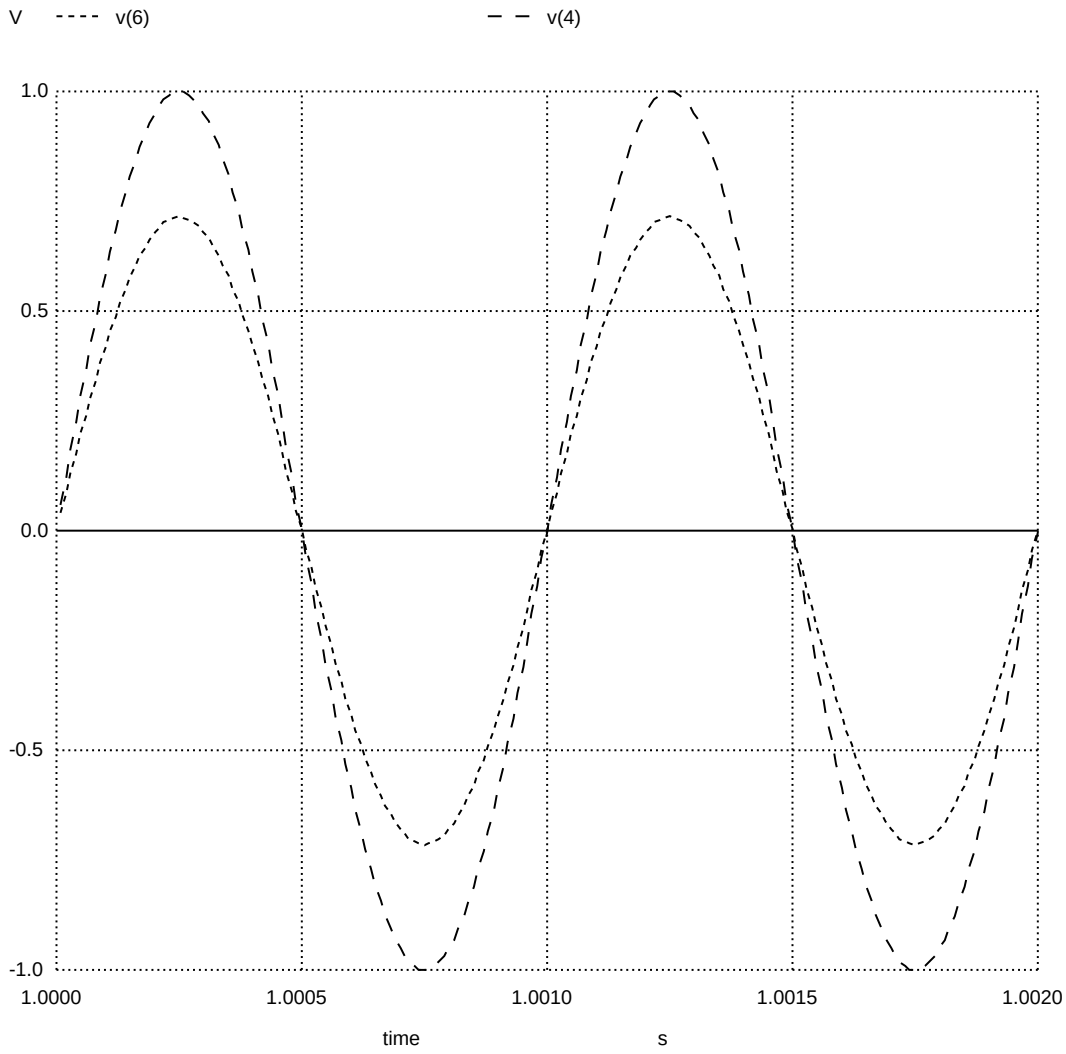
SPICE netlist:

```
* SPICE simulation with switch closed
vcc 1 0 dc 15
vsig 4 0 sin (0 1 1e3 0 0)
rsig 4 6 1e3
rswitch 6 7 1e-9
r1 1 2 17e3
r2 2 0 3e3
rc 1 5 5e3
re 3 0 1e3
c1 7 2 33e-6
q1 5 2 3 qmod
.model qmod npn bf=120
.tran 10u 1002m 1000m uic
.plot tran v(4) v(6)
.end
```



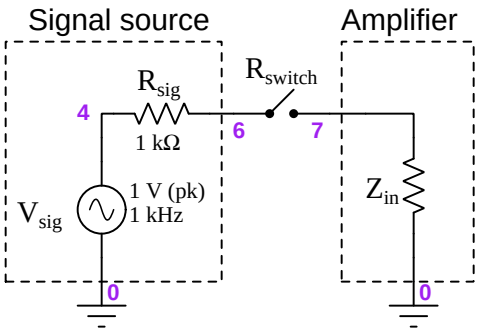
Here we see the signal source's ideal voltage (1 Volt peak at node 4) "loaded" down by the amplifier's input impedance to a lesser value at node 6.

Next we will run the same simulation, only omitting the plot of voltage at node 5 (the amplifier's output terminal) so that we may examine a finer-resolution comparison of the signal source's waveforms at nodes 4 and 6:



Here we see the loading effect in greater detail. The source's ideal (internal) voltage is 1 Volt peak, but the loaded-down signal voltage is slightly less than 0.75 Volts peak.

A simple way to model the amplifier as a load to the signal source is to represent it as a single Thévenin/Norton equivalent resistance, like this:



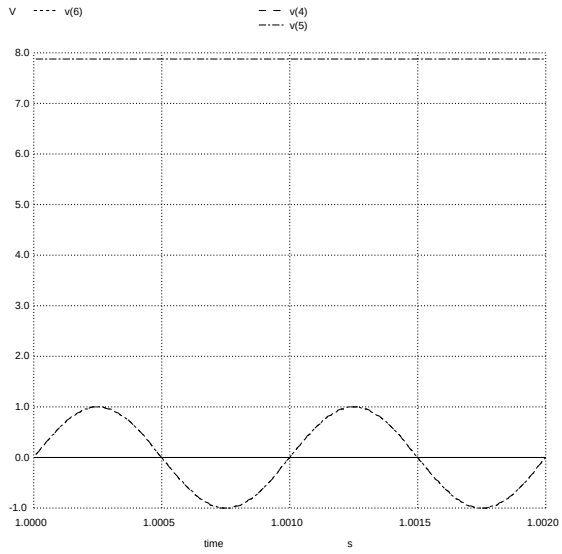
Calculating this impedance using the given resistor values and transistor beta value for the common-emitter amplifier as shown, we predict 2497.4 Ohms of input impedance (using the equation shown below). The voltage-divider effect of this impedance along with the source's internal Thévenin impedance of 1000 Ohms yields a node 6 voltage of 0.7141 Volts peak which agrees well with the simulation results.

$$Z_{in} \approx R_1 \,||\, R_2 \,||\, (\beta + 1)(r'_E + R_E)$$

Next we run a simulation the switch open:

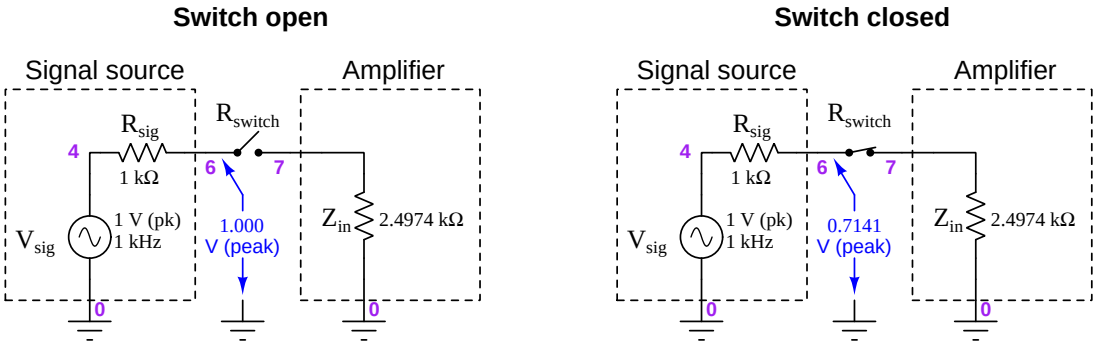
SPICE netlist:

```
* SPICE simulation with switch open
vcc 1 0 dc 15
vsig 4 0 sin (0 1 1e3 0 0)
rsig 4 6 1e3
rswitch 6 7 1e9
r1 1 2 17e3
r2 2 0 3e3
rc 1 5 5e3
re 3 0 1e3
c1 7 2 33e-6
q1 5 2 3 qmod
.model qmod npn bf=120
.tran 10u 1002m 1000m uic
.plot tran v(4) v(6) v(5)
.end
```



With the switch open, the amplifier presents no load to the signal source. Therefore, we see the full (ideal) 1 Volt peak voltage at both nodes 4 and 6. The amplifier's output (node 5) is of course "flat-lined" because the amplifier is not receiving any AC signal to amplify.

Contrasting these two results, using the equivalent circuit model shown earlier:

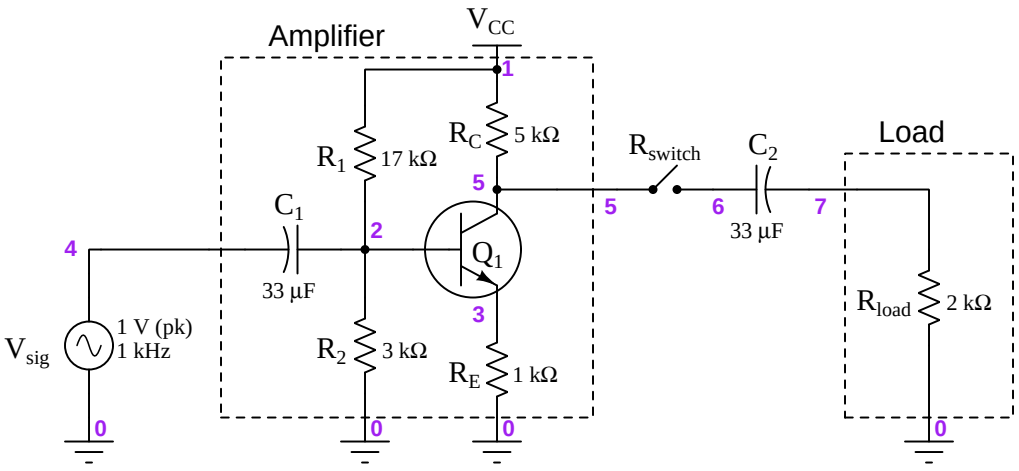


These Thévenin equivalent models of the signal source and amplifier input make it clear how and why the source's signal voltage "sags" when connected to the amplifier. With precise enough measurements of this "sag" we may even compute the amplifier's input impedance without the use of the complex equation employed here to estimate the amplifier's $Z_{in} = 2.4974$ kΩ.

2.5 Example: common-emitter output impedance

In this SPICE simulation we will use a common-emitter amplifier properly biased for Class A operation sourcing a load at its output terminal:

Circuit schematic diagram (with node numbers listed):



There is nothing noteworthy about the amplifier itself. It uses a two-resistor "voltage divider" biasing network, a coupling capacitor at the input terminal, a 5:1 collector:emitter resistor ratio for a voltage gain of approximately five. The DC power supply (V_{CC} and ground) is 15 Volts. What is a bit unusual is the SPST switch connecting a load resistance to the amplifier's output, simulated by a resistance in the SPICE netlist named R_{switch}, set to an extremely high resistance for the "open" condition and a negligible resistance for the "closed" condition. The coupling capacitor was chosen to have a negligible reactance (less than 5 Ohms at 1 kHz).

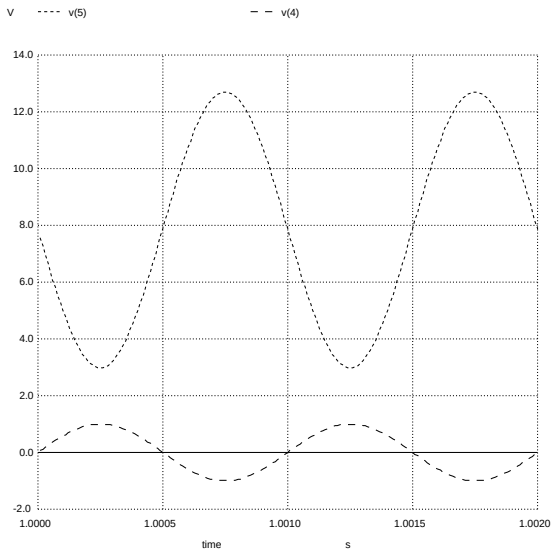
An important detail of this SPICE analysis appears in the `tran` (transient) analysis card of the netlist, where we specify the interval time duration (10 microseconds) followed by the "stop" time (1002 milliseconds) followed by the "start" time (1000 milliseconds). We intentionally delay the plotting of results until SPICE has performed the first 1000 milliseconds of the analysis to give the input capacitor and bias resistors enough time for the capacitor's DC voltage to reach a stable value. Then, we plot for 2 milliseconds' worth of time (from 1000 ms to 1002 ms) in order to show two full cycles of the 1 kHz waveforms.

The following simulations will show the signal's ideal source voltage (node 4) along with the signal voltage at the source's output terminal (node 6), and finally the amplifier's output signal at node 5, all with respect to ground (node 0). Simulations will be shown both for the switch-closed condition and the switch-open condition so you may see the "loading" effect of the amplifier's input impedance on the source signal as measured at node 6.

In this first simulation the switch is open:

SPICE netlist:

```
* SPICE simulation with switch open
vcc 1 0 dc 15
vsig 4 0 sin (0 1 1e3 0 0)
rload 7 0 2e3
rswitch 5 6 1e9
r1 1 2 17e3
r2 2 0 3e3
rc 1 5 5e3
re 3 0 1e3
c1 4 2 33e-6
c2 6 7 33e-6
q1 5 2 3 qmod
.model qmod npn bf=120
.tran 10u 1002m 1000m uic
.plot tran v(4) v(5)
.end
```

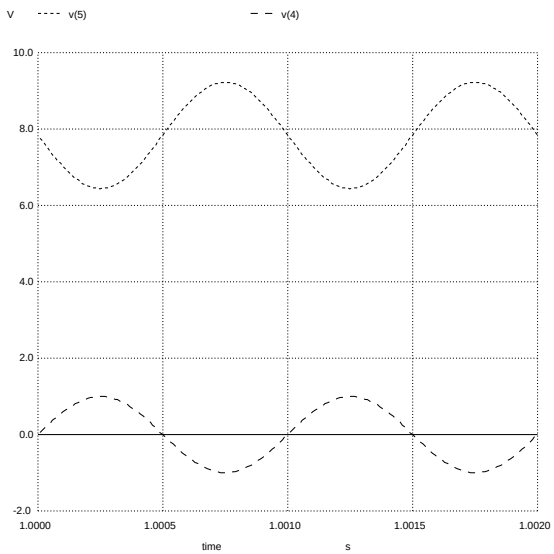


Here we see the amplifier's output voltage exactly where we expect it: approximately five times that of the input, nearly 5 Volts peak.

Next we will close the switch and observe how much the amplifier's output signal "sags" due to the loading effect, compared to the open-switch simulation:

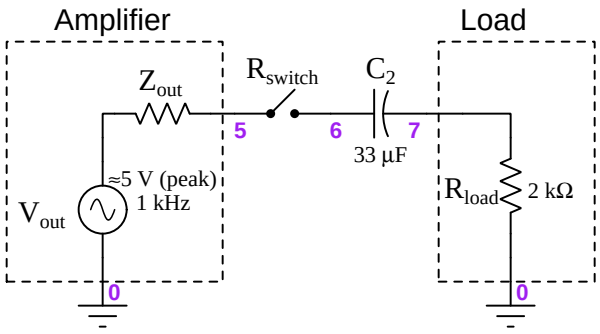
SPICE netlist:

```
* SPICE simulation with switch closed
vcc 1 0 dc 15
vsig 4 0 sin (0 1 1e3 0 0)
rload 7 0 2e3
rswitch 5 6 1e-9
r1 1 2 17e3
r2 2 0 3e3
rc 1 5 5e3
re 3 0 1e3
c1 4 2 33e-6
c2 6 7 33e-6
q1 5 2 3 qmod
.model qmod npn bf=120
.tran 10u 1002m 1000m uic
.plot tran v(4) v(5)
.end
```

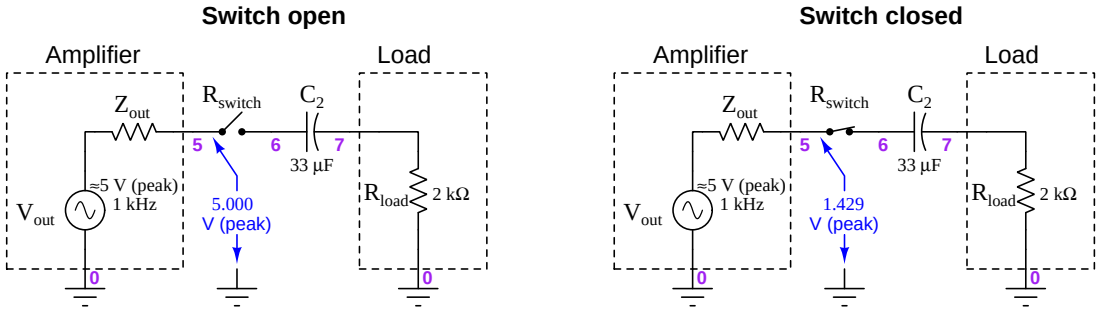


Here we see the output waveform significantly attenuated from before, to a value less than 1.5 Volts peak.

A simple way to model the amplifier as a source to this load is to represent it as a Thévenin equivalent network, like this:



A common-emitter amplifier's output impedance is simply equal to its collector resistor (R_C), which for this amplifier is 5 kΩ. The voltage-divider effect of this impedance along with the load's resistance of 2000 Ω yields a node 5 voltage of 1.429 Volts peak which agrees well with the simulation results:



These Thévenin equivalent models of the signal source and amplifier input make it clear how and why the amplifier's output signal voltage "sags" when connected to the load. With precise enough measurements of this "sag" we may even compute the amplifier's output impedance without knowing any of its internal component values.

Chapter 3

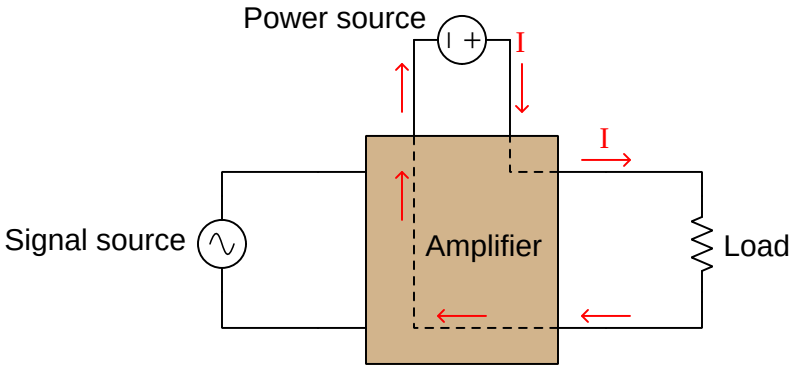
Tutorial

3.1 Amplifier fundamentals

An *amplifier* is any device allowing a small amount of energy to exert control over a larger amount of energy. Every electronic amplifier, regardless of the specific technology used, may be thought of as having three different sets of connection points:

1. Signal input (connects to signal source)
2. Signal output (connects to load)
3. Power (connects to electrical power source)

A simplified diagram of a generic electronic amplifier appears below, without any reference to the active components contained within:



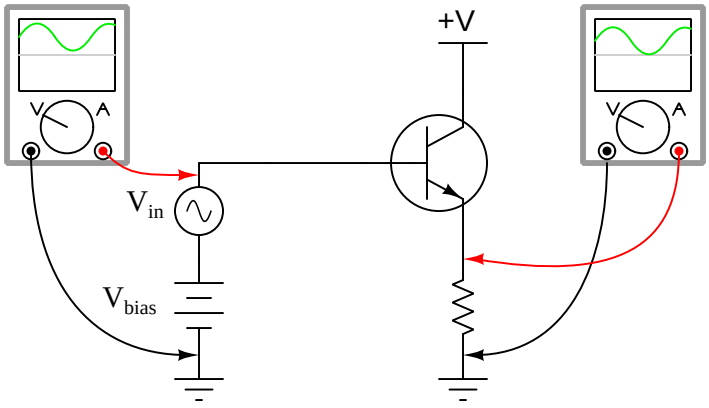
Electronic components such as transistors (or, in previous eras, vacuum tubes) comprise the heart of amplifiers, giving the input signal control over the flow of energy from power source to load.

One important parameter for any amplifier is *gain*, defined simply as the ratio of controlled versus controlling quantities, usually symbolized by the variable A. The value of A tells us how many times larger an amplifier's output signal is compared to its input signal. This ratio of output to input may be expressed in multiple ways. For example, we may speak of an amplifier having a *power* gain, a *voltage* gain, and a *current* gain, each gain calculated as the respective ratio of output to input for the specified quantity:

$$A_P = \frac{P_{out}}{P_{in}} \qquad A_V = \frac{V_{out}}{V_{in}} \qquad A_I = \frac{I_{out}}{I_{in}}$$

Gain is always unitless because it is a ratio between quantities having identical units of measurement (e.g. Watts out versus Watts in, Amperes out versus Amperes in).

Amplifiers are often used to boost the amplitude of AC signals, which poses a technical problem because most active components (e.g. transistors, vacuum tubes) are fundamentally DC-only devices. A common solution to this problem is to *bias* the AC signal input to an amplifier with a DC "offset" until it becomes "shifted" so far that its polarity never reverses. This technique is shown in the following schematic diagram, where the AC signal (V_{in}) is connected in series with a DC voltage source (V_{bias}) sufficient to offset the signal waveform entirely above the "zero" line on the oscilloscope display, allowing the transistor to operate on a non-alternating voltage signal:



This amplifier's output signal faithfully represents the wave-shape of the AC input signal, and is also "biased" above zero as a result of the transistor being incapable of controlling a bidirectional current.

When the amplifier circuit in question amplifies an AC signal in this manner – where the input and output signals are both biased, and not necessarily biased by the same amount of DC offset – it becomes useful to define gain in terms of *variations* of power, voltage, or current. This is referred to as the *AC gain* of the circuit, expressed as ratios of *changes* denoted by the Greek capital letter delta (Δ). For example, an AC gain of 2 means the output rises and falls twice as much as the input rises and falls. In order to calculate AC gains from measurements taken of DC circuit quantities at different points in time, we need to compare how far the output and input quantities have *changed* between those different points in time:

$$A_P = \frac{\Delta P_{out}}{\Delta P_{in}} \qquad A_V = \frac{\Delta V_{out}}{\Delta V_{in}} \qquad A_I = \frac{\Delta I_{out}}{\Delta I_{in}}$$

For example, consider the following DC voltage measurements taken at the input and output of a transistor amplifier circuit at different points in time:

V_{in}	V_{out}
1.5 Volts	24.50 Volts
2.0 Volts	22.95 Volts
2.5 Volts	21.40 Volts
3.0 Volts	19.85 Volts
3.5 Volts	18.30 Volts

With each increase of 0.5 Volts at this amplifier's input terminals, the output voltage falls by 1.55 Volts. This represents an AC voltage gain of:

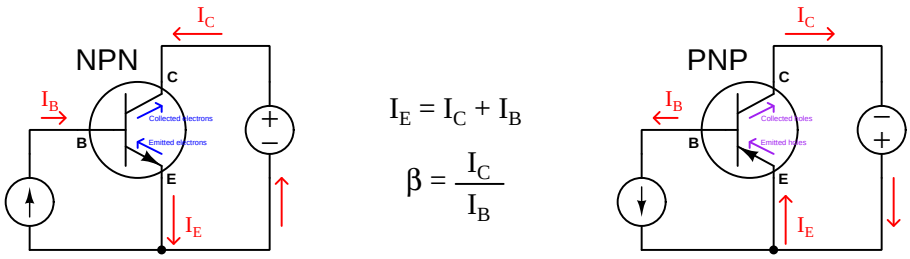
$$A_V = \frac{\Delta V_{out}}{\Delta V_{in}} = \frac{22.95 \text{ V} - 24.50 \text{ V}}{2.0 \text{ V} - 1.5 \text{ V}} = \frac{21.40 \text{ V} - 22.95 \text{ V}}{2.5 \text{ V} - 2.0 \text{ V}} = \frac{19.85 \text{ V} - 21.40 \text{ V}}{3.0 \text{ V} - 2.5 \text{ V}}$$

$$A_V = \frac{\Delta V_{out}}{\Delta V_{in}} = \frac{-\Delta 1.55 \text{ V}}{\Delta 0.5 \text{ V}} = -3.10$$

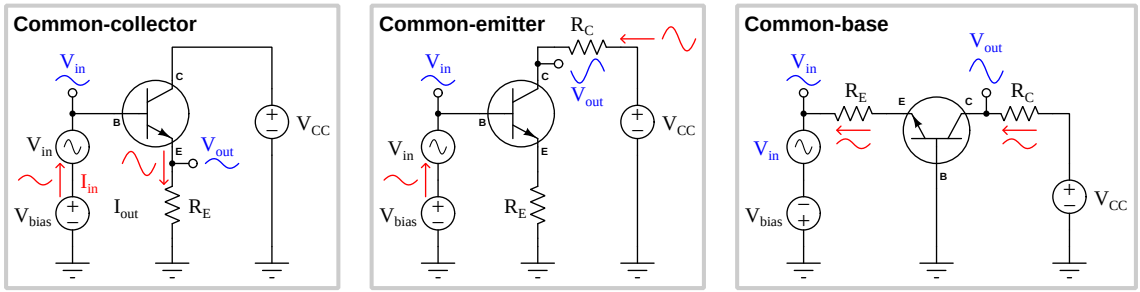
In other words, this particular amplifier has an AC voltage gain of 3.10 with an *inverting* characteristic (i.e. V_{out} falls as V_{in} rises).

3.2 BJT amplifier configurations

Bipolar transistors are three-terminal devices, each having *emitter*, *collector*, and *base* terminals. The controlled current through a BJT flows between the emitter and collector terminals, so named because the emitter "emits" charge carriers which are subsequently injected into the middle (base) region of the transistor, drifting through the base to be "collected" by the collector. BJTs are normally-off devices, and must have their base-emitter PN junctions forward-biased in order to inject minority charge carriers into the base region to permit emitter-collector current. Only a small number of the charge carriers "emitted" by the emitter region of the transistor pass through the base terminal, most of the charge carriers being swept through the thin base region by the external collector-emitter voltage to be "collected" by the collector region. The ratio of collector current to base current is the fundamental amplifying parameter of a BJT, and is symbolized as either β or h_{FE}:



Multiple amplifier circuit configurations exist for bipolar junction transistors, the only difference between the NPN and PNP circuit versions being the necessary DC source polarities to ensure proper transistor action. Three configurations are typical, and are known by the identity of the transistor terminal held in common by both AC input and AC output voltage signals[1]. Thus, we will explore amplifier circuits conventionally called *common-collector*, *common-emitter*, and *common-base*:

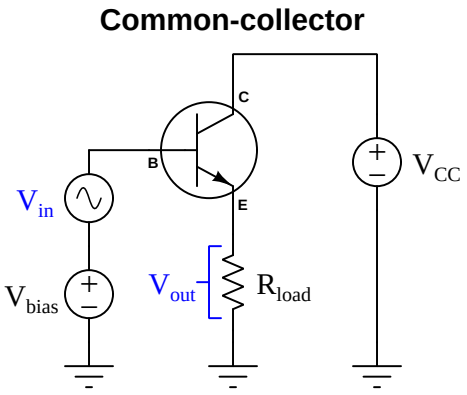


In the interest of simplicity, only NPN transistors will be used, and the necessary DC biasing voltage will be shown in each case as an independent DC voltage source.

[1] This commonality becomes clear if we eliminate all DC sources in the circuit and analyze it from the perspective of AC only, which is done in each of the following sections using standard rules of the Superposition Theorem. With the DC bias voltage source replaced by a short, and the DC power supply also replaced by a short, there will be one transistor terminal shared in common with both the AC signal source and the AC output voltage (i.e. the load).

3.3 Common-collector amplifier

The common-collector amplifier configuration is the simplest of the three, shown here in elementary form with a DC bias source included to permit amplification of the AC voltage signal:

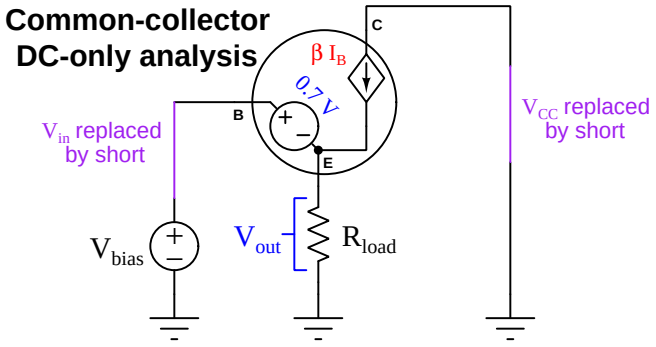


With V_{bias} properly sized, the transistor's base-emitter junction will always be maintained in a state of electrical conduction. This means we can expect V_{BE} to be approximately 0.7 Volts DC at all times: the typical forward voltage drop of a silicon PN junction. Furthermore, from this simple fact we may deduce V_{out} to have the same AC amplitude as V_{in}, the only real difference between V_{out} and V_{in} being a nearly-constant 0.7 Volts DC offset. Thus, the AC voltage gain of a common-collector amplifier should be one, since $\Delta V_{out} = \Delta V_{in}$.

The distinction between DC and AC voltages in this circuit deserves further elaboration, as it will become an important point for understanding more complicated amplifier circuits. When multiple voltage sources are connected in series as we have in this amplifier circuit with V_{in} and V_{bias}, their total voltage is the *superposition* (i.e. algebraic sum) of their individual voltages. This is true regardless of whether the sources in question are all DC, all AC at the same frequency, all AC at different frequencies, or some combination of AC and DC. In accordance with the Superposition Theorem, we may consider the circuit's response to each of those sources – one source at a time – each time ignoring the effects of all other independent sources[2]. This is a very important analytical tool for us, as it allows us to predict where each of those component voltages will end up being dropped across load elements in the circuit.

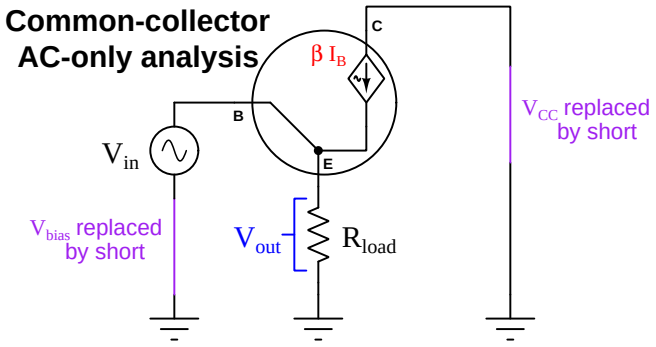
[2]Strictly speaking, the Superposition Theorem only holds true for networks that are both *linear* (obey Ohm's Law) and *bilateral* (permit current to flow in both directions). However, a properly-biased transistor meets these two criteria close enough to permit *approximations* based on the Superposition Theorem: for relatively small AC voltage amplitudes the transistor's amplified response is close enough to being linear for most purposes, and with sufficient V_{bias} the transistor's PN junctions never become reversed-biased and so they never have the opportunity to reveal their unilateral nature.

Let's see how the Superposition Theorem applies to this common-collector circuit. First, we will consider the DC voltage source (V_{bias}) by itself, while negating the effects of the AC source (V_{in}) by replacing that AC voltage source with its internal resistance value of zero Ohms. Likewise, we will replace the power supply (V_{CC}) with a short so that the only active source in the entire circuit is V_{bias}. Note how we model the base-emitter PN junction as a DC voltage source[3] of 0.7 Volts and model the collector-emitter pathway as a current source dependent on base current (I_B):



Applying Kirchhoff's Voltage Law to the loop containing V_{bias} and R_{load}, we see the load resistor must drop 0.7 Volts less than V_{bias}. Therefore, the DC component of V_{out} is $V_{bias} - 0.7$.

Now let's analyze the same circuit with the AC voltage source (V_{in}) by itself, while replacing the DC source (V_{bias}) and the power supply (V_{CC}) with shorts. Once again the transistor's collector-emitter pathway must be modeled as a dependent current source because the amount of collector current is not fixed but rather varies with base current. The major difference this time is how we model the base-emitter junction, as a *short* (dropping zero Volts AC) because we are operating under the assumption that forward-biased PN drops a *constant* 0.7 Volts DC with no variation:



Now the circuit loop we analyzed previously only contains V_{in} and R_{load}, which means the AC component of V_{out} must be equal to V_{in}. Superimposing the DC and AC results across R_{load}, we must conclude V_{out} is a superposition of the full AC input voltage (V_{in}) plus a DC bias equal to 0.7 Volts less than V_{bias}.

[3] We could have regarded the forward voltage drop as a source to be considered independently just like V_{bias}, but this would require another re-drawing of the circuit, and the resulting voltage drop across R_{load} would be the same.

By analyzing this amplifier circuit from two different perspectives – once from a DC-only perspective and another from an AC-only perspective – we are able to determine the composite DC + AC signal appearing as V_{out}. The AC perspective showed us that V_{out} is identical to V_{in}, which means this amplifier configuration has an AC voltage gain (A_V) of one.

While voltage gain for the common-collector amplifier configuration may be one, current gain is significantly greater than one. We know I_C is greater than I_B due to the amplifying nature of the transistor, and that I_E is even greater (being the sum of I_C and I_B in accordance with Kirchhoff's Current Law), therefore the load resistor sees a current *much* greater than the input signal source. Specifically, the current gain (DC or AC) for a common-collector amplifier is $\beta + 1$:

$$\beta = \frac{I_C}{I_B}$$

$$\beta + 1 = \frac{I_C + I_B}{I_B}$$

$$\beta + 1 = \frac{I_E}{I_B} = \frac{I_{out}}{I_{in}} = A_I$$

Another interesting consequence of the transistor's amplifying behavior is the apparent load resistance as experienced by the AC signal source V_{in}. Since most of the load resistor's current is sourced through the collector-emitter pathway of the transistor, the AC signal source does not need to bear the full burden of powering the load: just a small "trickle" of current sourced by V_{in} triggers the transistor to deliver a great deal more current to R_{load}. From the perspective of V_{in}, R_{load} "appears" to be much greater than it actually is[4]. We may calculate this "resistance transformation" by taking the quotient of current gain to voltage gain. The following algebraic expansion proves why $\frac{A_I}{A_V}$ is mathematically equivalent to $\frac{R_{in}}{R_{out}}$:

$$\frac{R_{in}}{R_{out}} = \frac{A_I}{A_V}$$

$$\frac{R_{in}}{R_{out}} = \frac{\frac{I_{out}}{I_{in}}}{\frac{V_{out}}{V_{in}}} = \left(\frac{I_{out}}{I_{in}}\right)\left(\frac{V_{in}}{V_{out}}\right)$$

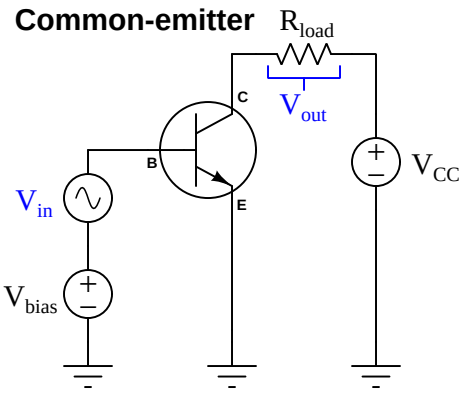
$$\frac{R_{in}}{R_{out}} = \left(\frac{V_{in}}{I_{in}}\right)\left(\frac{I_{out}}{V_{out}}\right) = \frac{\frac{V_{in}}{I_{in}}}{\frac{V_{out}}{I_{out}}} = \frac{R_{in}}{R_{out}}$$

Since we already know A_I is equal to $\beta + 1$ and A_V is equal to one, the resistance transformation ratio of a common-collector amplifier must simply be $\beta + 1$. For example, if this amplifier's load resistance was 3.3 kΩ and the transistor's β ratio was 60, the input signal source would "feel" as though it were powering a 201.3 kΩ load. A practical application is where the signal source is "weak" and cannot deliver much current to the amplifier, and we must determine how much the amplifier's input will "load down" that source.

[4] This resistance-transformation phenomenon is analogous to power-assisted steering on an automobile: the mechanical amplification of force provided by the power-assist mechanism makes it seem from the driver's perspective that the car's steering is exceptionally "light" and effortless, when in fact it is the vehicle's engine providing the strength necessary to angle the turning wheels while the steering wheel's motion merely guides that force. Likewise, the transistor's amplifying nature makes it seem from the input signal's perspective that the load is much "lighter" (i.e. higher resistance, drawing less current for a given amount of voltage) than it actually is.

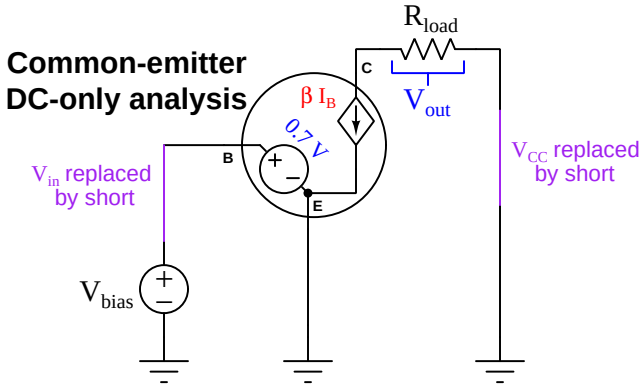
3.4 Common-emitter amplifier

Next in our lineup of BJT amplifier configurations is the *common-emitter* design, shown here in elementary form. It differs from the common-collector configuration only in its placement of the load resistor:



As with the common-collector amplifier examined in the previous section, we will assume V_{bias} is set such that the transistor is always in a state of electrical conduction, never shutting off and never fully turning on. Also as before, we will analyze this amplifier's behavior from both a DC and an AC perspective.

First the DC perspective, disabling all independent sources other than V_{bias} by replacing them with their equivalent internal resistances:



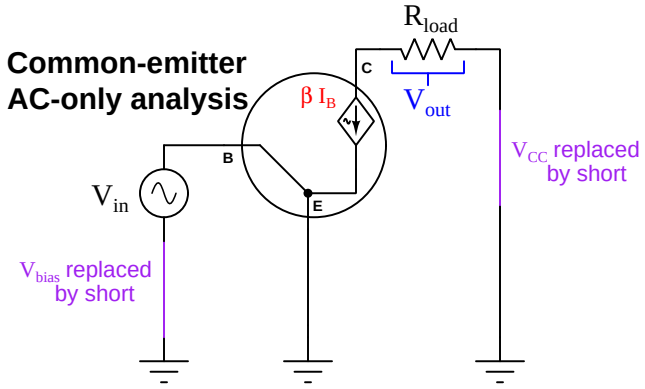
The DC component of V_{out} will be predicted by Ohm's Law, simply being the product of collector current (I_C) and load resistance (R_{load}). Collector current, in turn, is β times greater than the current sourced by V_{bias}. However, our DC-only modeling gives us no insight as to how much collector current we ought to expect. Kirchhoff's Voltage Law simply demands V_{bias} be exactly 0.7 Volts to match the base-emitter voltage drop.

However, we must remember that the 0.7 Volt figure assumed as the forward voltage drop of a silicon PN junction is approximate, and in reality is the variable V in Shockley's diode equation, dependent on both emitter current and base-emitter junction temperature:

$$I = I_S \left(e^{\frac{qV}{nKT}} - 1 \right)$$

If V_{BE} were truly fixed at precisely 0.7 Volts, then V_{bias} would have only one acceptable value, and that would be precisely the same voltage as V_{BE}: 0.7 Volts. In reality, though, V_{BE} (and, correspondingly V_{bias}) can vary slightly above and below 0.7 Volts with the result being a large variation in emitter current. The consequence of all this uncertainty and variability is that it is practically impossible to predict the exact right value of V_{bias} in this common-emitter circuit to keep the transistor between cutoff and saturation. Subtle variations between transistors and even small temperature changes frustrate attempts to accurately predict V_{bias}.

This problem becomes even more evident when we attempt to analyze this amplifier circuit from an AC perspective:



With our assumption that the base-emitter junction drops a constant (DC) voltage, it would appear our AC signal source becomes completely short-circuited! Just as we discovered we had to abandon our assumption of a constant 0.7 Volt drop for the DC analysis, we find here another reason for abandoning the 0.7 VDC V_{BE} assumption: the fact that a *variation* in voltage across the conductive PN junction will indeed cause a variation in current (as predicted by the Shockley equation), and therefore the junction must exhibit something akin to a resistance.

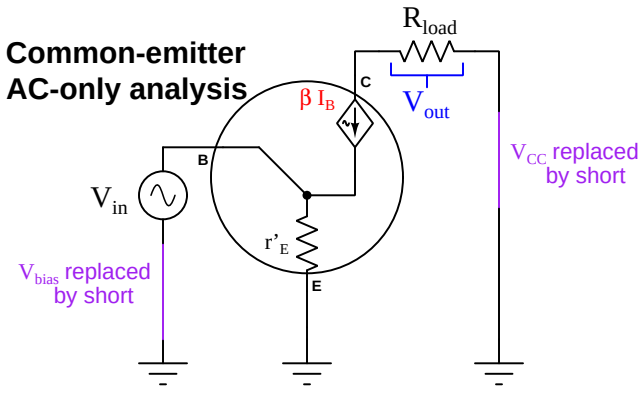
Mathematical analysis of the Shockley equation reveals a quantity called the *dynamic emitter resistance*[5], the value of which depends on the amount of DC bias current (i.e. the *quiescent* emitter current) flowing through the emitter terminal. In other words, for any given amount of DC current through the transistor's emitter terminal, there will be a certain amount *additional* voltage dropped by the base-emitter PN junction in response to any *additional* current passing through it. This

[5]One way to envision this dynamic resistance is to think of it as an *incremental* resistance, or what an electronics-savvy economist might call a *marginal* resistance: it is the amount of resistance the PN junction offered to small *variations* in voltage above and below the DC bias voltage value. This is clearly an application for calculus, because it is an expression of how much one variable changes in response to a related variable changing. For a full derivation of dynamic emitter resistance, refer to page 80.

"dynamic" resistance embedded within the transistor's emitter region is usually denoted as r'_E and its value varies with quiescent current. A commonly accepted range of values for most transistors is 25 mV to 50 mV per Ampere of DC bias current through the emitter terminal:

$$\frac{25 \text{ mV}}{I_{bias}} \leq r'_E \leq \frac{50 \text{ mV}}{I_{bias}}$$

Re-drawing our AC-only schematic diagram to include r'_E, we see this small resistance must drop the entirety of the AC input signal (V_{in}) in accordance with Kirchhoff's Voltage Law:



With r'_E dropping the full AC voltage of V_{in}, the AC emitter current is predictable by Ohm's Law ($I_E = \frac{V_{in}}{r'_E}$). This AC emitter current is the sum of base and collector currents in accordance with Kirchhoff's Current Law, with collector current comprising the majority of I_E. In fact, base current is so small by comparison to collector current that it is reasonable to simplify our analysis by assuming $I_C = I_E$ (i.e. assuming base current to be negligible). Following this assumption, we may conclude that the transistor's AC collector current is equal to V_{in} divided by r'_E.

That AC collector current flows through R_{load}, and in so doing produces the voltage drop V_{out}. Combining the two Ohm's Law equations by substitution of I_C, we may predict the AC component of V_{out} from V_{in} and the two resistance values (r'_E and R_{load}):

$$I_E = \frac{V_{in}}{r'_E} \approx I_C \qquad V_{out} = I_C R_{load}$$

$$V_{out} \approx \left(\frac{V_{in}}{r'_E}\right) R_{load}$$

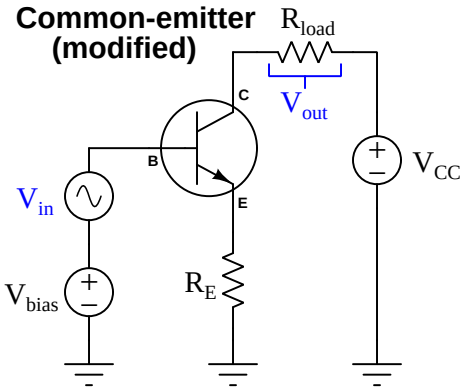
$$V_{out} \approx V_{in} \left(\frac{R_{load}}{r'_E}\right)$$

This equation somewhat resembles the familiar voltage divider formula, but in reverse: it predicts voltage dropped across one resistor (R_{load}) based on an input voltage impressed upon another resistor (r'_E), and if R_{load} exceeds r'_E (which it usually does), the AC output voltage will be greater than the AC input voltage. Moreover, if we divide both sides of that last equation by V_{in}, we arrive at a simple approximation of the amplifier's AC voltage gain: $A_V = \frac{V_{out}}{V_{in}} \approx \frac{R_{load}}{r'_E}$.

At this point it may seem as though we have successfully analyzed the common-emitter transistor amplifier configuration, but we still have an unanswered question: *how do we predict the necessary DC bias voltage?* Knowing the appropriate value for V_{bias} is necessary not just to ensure the transistor remains in its "active" mode to faithfully reproduce the whole AC wave-shape without clipping, but also so we may calculate the quiescent (DC) emitter current, which is necessary to predict r'_E , which is necessary to predict voltage gain. With so many factors depending on V_{bias}, and with so much uncertainty[6] regarding the appropriate value for V_{bias}, we must sadly conclude that this elementary common-emitter amplifier circuit design is unreliable.

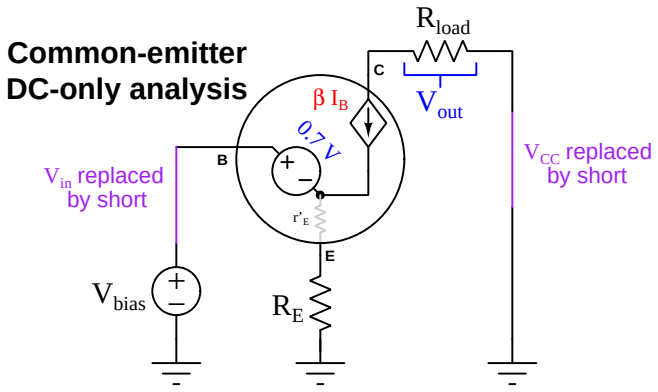
You may recall that we never had to consider dynamic emitter resistance in the common-collector configuration, and the reason for that was the fact that R_{load} was located on the emitter side of the transistor. In that location, R_{load} (which we might reasonably expect to be thousands of Ohms in value) was situated to drop practically all of V_{in}. Whatever minute resistance was posed by the transistor's r'_E in the common-collector amplifier was overshadowed by the much larger series load resistance. In the case of the common-collector amplifier this overshadowing was fortuitous, not intentional. However, we may consider intentionally adding resistance on the emitter terminal of the transistor to achieve the same effect. This design technique – of intentionally overshadowing some small parameter with a much larger parameter – has a colorful name in engineering: *swamping*. We say that the relatively large value of R_{load} "swamps" the relatively small value of r'_E when those two resistance are in the same loop as V_{in}. To "swamp" something means to render it negligible by comparison.

Let's modify our common-emitter amplifier circuit to contain an additional resistor (R_E) intended to "swamp" the transistor's dynamic emitter resistance r'_E:



[6] Remember, the Shockley diode equation ($I = I_S \left(e^{\frac{qV}{nKT}} - 1 \right)$) describes PN junction current being highly dependent on temperature, and of course there is also the ideality factor n which varies depending on certain physical characteristics of the semiconductor PN junction. In other words, two apparently identical common-emitter transistor amplifier circuits could behave very differently simply due to these uncontrolled factors.

Performing a DC-only analysis of this new common-emitter amplifier design:



The transistor's dynamic emitter resistance r'_E is an AC-only phenomenon, and so we will regard it as being zero Ohms for the sake of our DC analysis and show its symbol in grey instead of black on the schematic diagram[7]. Kirchhoff's Voltage Law then tells us R_E must drop the voltage $V_{bias} - 0.7$. Ohm's Law tells us the amount of emitter current resulting from this voltage drop:

$$I_E = \frac{V_{bias} - 0.7}{R_E}$$

This constant (DC) current through the emitter is fairly reliable to predict, as the only uncertainty is the exact magnitude of the forward voltage drop. From this quiescent emitter current we may estimate the quiescent collector current, since I_C is mathematically related to I_B by β and I_E is the sum of I_B and I_C:

$$\text{If } I_C = \beta I_B \text{ and } I_E = I_B + I_C \text{ then } I_C = I_E \left(\frac{\beta}{\beta + 1} \right)$$

$$\frac{I_C}{I_E} = \frac{\beta}{\beta + 1}$$

I_C and I_E will be nearly equal to each other if β is sufficiently large, and so typical amplifier approximations simply assume $I_C = I_E$. V_{out} across the load resistor is then predictable by Ohm's Law ($V_{out} = I_C R_{load} \approx I_E R_{load}$). This value will be the DC component of the amplifier's superimposed AC + DC output voltage signal.

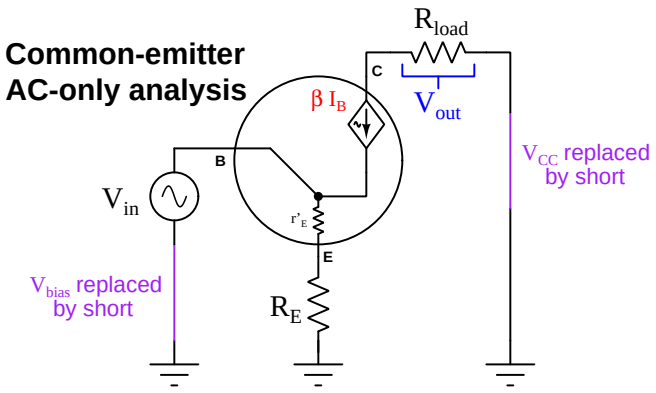
If we wished, we could also use β to calculate base current from the known emitter current:

$$\frac{I_B}{I_E} = \frac{1}{\beta + 1}$$

As you can see, β is the key to determining the value of *any* transistor terminal current given any of the other transistor terminal currents.

[7] "Dynamic" refers to something in a continual state of change. The dynamic resistance of a semiconductor PN junction refers to the amount of *additional* voltage dropped by the junction for a given increase in current. With a steady current (i.e. DC) there is no change taking place over time, and therefore no dynamic resistance.

Now we will perform an AC-only analysis of the "emitter-swamped" common-emitter amplifier:



Kirchhoff's Voltage Law declares the series combination of r'_E and R_E must drop the entirety of V_{in}. From the DC analysis we can anticipate the emitter's quiescent current, and from that compute an approximate value for dynamic emitter resistance ($r'_E \approx \frac{25\text{ mV}}{I_E}$ with I_E being the quiescent emitter current). With r'_E and R_E both known, we may then use Ohm's Law to calculate AC current through the emitter terminal:

$$I_E \approx \frac{V_{in}}{r'_E + R_E}$$

The AC component of this amplifier's output voltage is simply $V_{out} = I_C R_{load}$, and we already know I_C will be very nearly equal to I_E given the typically large value of β. Using algebraic substitution, we may convert the above equation showing V_{out} as a function of V_{in} , as well as AC voltage gain (A_V) as a function of circuit resistances:

$$I_E \approx \frac{V_{in}}{r'_E + R_E} \qquad V_{out} = I_C R_{load} \approx I_E R_{load}$$

$$V_{out} \approx \left(\frac{V_{in}}{r'_E + R_E}\right) R_{load}$$

$$V_{out} \approx V_{in}\left(\frac{R_{load}}{r'_E + R_E}\right)$$

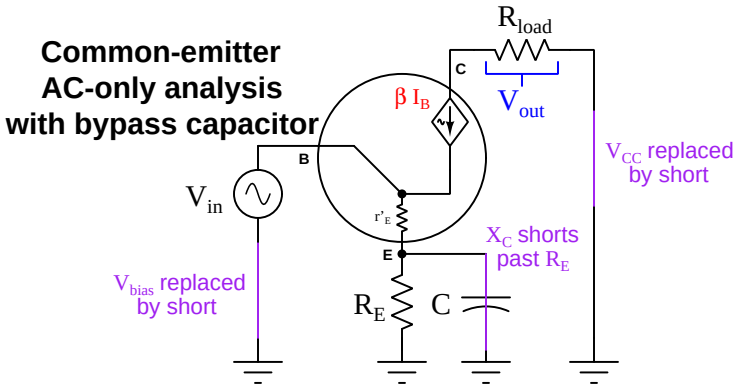
$$A_V \approx \frac{R_{load}}{r'_E + R_E}$$

Now we have a practical common-emitter amplifier design, tolerant of slight variations in transistor construction and operating temperature, having an AC voltage gain nearly equal to $\frac{R_{load}}{R_E}$. AC current gain is simply $\frac{I_C}{I_B}$, the definition of β. The amount of load resistance "perceived" by the AC signal source is a function of β and the emitter resistance just as it was for the common-collector

amplifier, the exact ratio being $\beta + 1$. Interestingly, the actual load resistance is irrelevant to the loading of V_{in}, since the amount of AC current drawn from V_{in} depends solely[8] on β and R_E.

As useful as the "swamping" technique is for making the behavior of our common-emitter amplifier more predictable, it does come at a price[9]: inserting additional resistance at the transistor's emitter terminal reduces voltage gain. In the unswamped amplifier, AC voltage gain was $\frac{R_{load}}{r'_E}$; in the swamped amplifier it is $\frac{R_{load}}{r'_E + R_E}$.

An interesting solution to the problem of reduced voltage gain is to install a capacitor in parallel with R_E to short AC current past that emitter resistor, so that the AC loop containing V_{in} no longer contains R_E. The goal here is to enjoy the best of both circuit configurations, eliminating R_E for the AC signal but maintaining R_E for DC biasing. All we need to do is select a sufficiently large capacitance that its reactance at the AC signal frequency ($X_C = \frac{1}{2\pi f C}$) is much smaller than the resistance of R_E, essentially acting as a high-pass filter for AC signal current to bypass R_E. This bypassing forces nearly all of V_{in} to be dropped across r'_E, restoring the amplifier's high voltage gain. Yet, R_E is still part of the circuit's DC path so it provides a stable emitter resistance for reliable biasing:

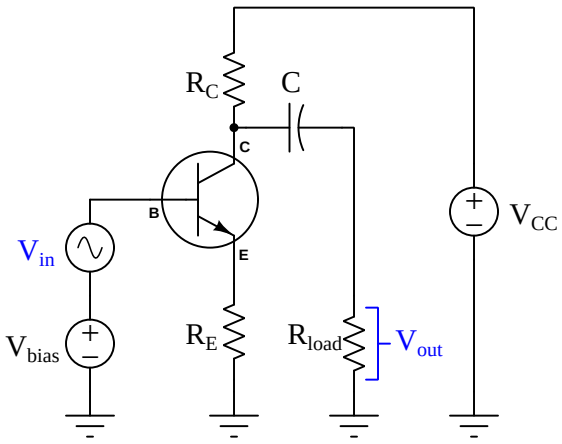


[8] A good "thought experiment" to prove this principle would be to substitute a short for R_{load}, in which case the circuit becomes a common-collector amplifier with R_E as the new load resistance but I_{in} remains unchanged.

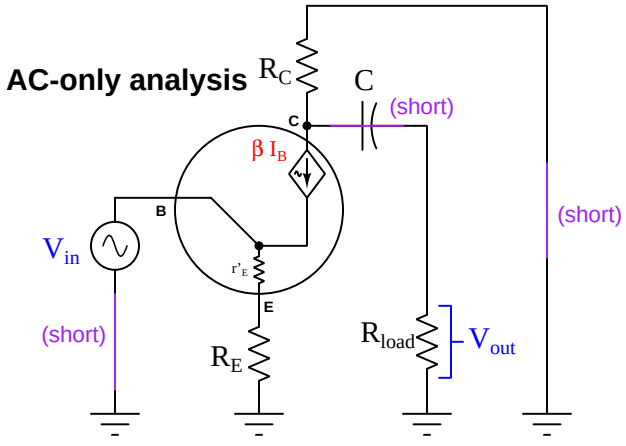
[9] Adding R_E to the common-emitter amplifier circuit is actually an example of an advanced design concept called negative feedback, and negative feedback always optimizes stability at the expense of gain.

Until this point we have shown the load resistance located between the transistor's collector terminal and the positive pole of the DC power supply. However, this location for R_{load} is potentially problematic for two reasons. The first reason is that it requires the load operate at elevated (i.e. non-ground) potential at all times, which may not suit all amplifier applications. The second reason is that it forces the load to operate on a superimposed AC + DC signal, which is unsuitable for loads requiring pure AC.

An alternative location for the load in a common-emitter amplifier is between the collector terminal and ground, usually coupled through a capacitor allowing AC to pass through to the load but blocking DC in the manner of a high-pass filter:



This relocation of the load resistor necessitates the inclusion of a new resistor in the circuit (R_C), and the inclusion of this new resistor affects the amplifier's AC voltage gain:



Examining this diagram closely, we see R_{load} and R_C are actually connected in *parallel* with each other, given the fact that both the coupling capacitor and the DC power supply appear as zero-resistance elements (shorts) to the AC signal.

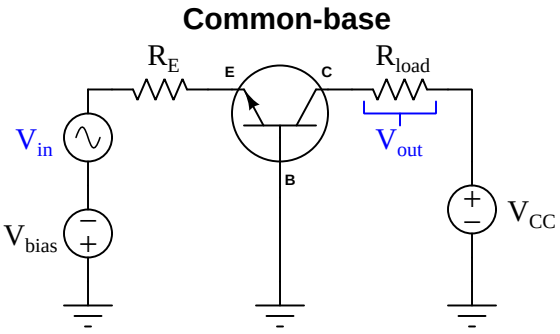
Being in parallel with each other, AC collector current is now split between R_{load} and R_C whereas in the previous version of the circuit all AC collector current passed through R_{load}. This paralleling of resistances decreases their combined (equivalent) resistance, resulting in less voltage gain for the same R_{load} value. Compare the voltage gain formulae[10] for this amplifier circuit versus the amplifier where R_{load} was in series with the collector terminal:

$$A_V \approx \frac{R_{load}}{r'_E + R_E} \qquad A_V \approx \frac{R_{load} || R_C}{r'_E + R_E}$$

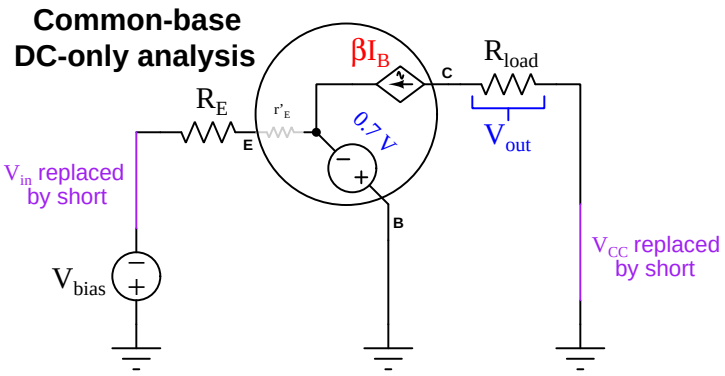
[10] For the sake of brevity I'm using two vertical bars (||) to signify parallel equivalent resistance rather than include a parallel resistance formula within the voltage gain formula. A full, unsimplified version of this formula would be $A_V \approx \frac{\frac{1}{\frac{1}{R_{load}} + \frac{1}{R_C}}}{r'_E + R_E}$.

3.5 Common-base amplifier

Our third and final BJT amplifier circuit configuration is the *common-base*. In this configuration the base terminal of the transistor connects to ground while the emitter is used for the input current and collector for the output current. Having learned our lesson about the necessity of inserting an emitter resistor (to "swamp" the effects of r'_E inside the transistor) for predictable behavior in a common-emitter amplifier circuit, we have included R_E in the input loop of this amplifier too:



Once again we will begin our analysis by considering only the effects of V_{bias} on the circuit, assuming that DC voltage source has been set to a value suitable for maintaining the transistor in a partially-conductive state throughout the entire evolution of the AC V_{in} waveform:

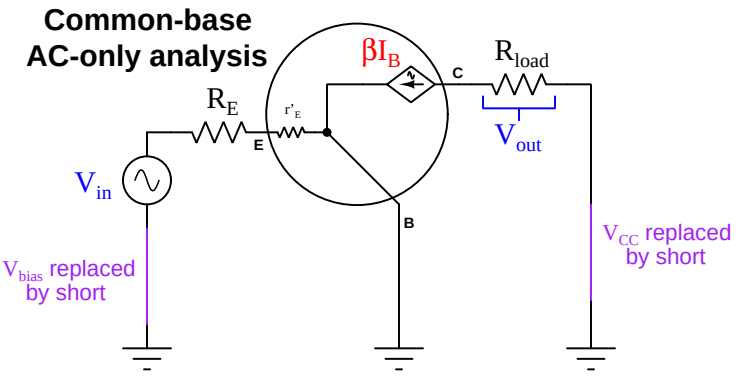


As with the swamped common-emitter amplifier, emitter current is a function of V_{bias}, R_E, and the base-emitter junction's forward voltage drop of 0.7 Volts. r'_E may be ignored because it is strictly an AC effect and does not manifest under constant-current (DC) conditions:

$$I_E = \frac{V_{bias} - 0.7}{R_E}$$

The vast majority of this emitter current passes through the transistor's collector terminal and also through the load resistor, producing a voltage drop approximately equal to $I_E R_{load}$. This will be the DC component of V_{out}.

Next, we will consider only the effects of the AC signal source (V_{in}) on the same circuit:



Once again, we see emitter current calculated the same way it was in the swamped common-emitter configuration, with the entirety of the AC input signal (V_{in}) dropped across the series combination of R_E and r'_E. The value of R_E, of course, is a fixed characteristic of that resistor, while the value of r'_E depends[11] on the magnitude of the DC emitter current determined in the previous analysis:

$$I_E \approx \frac{V_{in}}{r'_E + R_E}$$

Also similar to the common-emitter configuration, this common-base amplifier's AC output voltage is simply $V_{out} = I_C R_{load}$, and with AC emitter current calculated the same as well, the voltage gain for the common-base amplifier will follow the exact same mathematical pattern:

$$I_E \approx \frac{V_{in}}{r'_E + R_E} \qquad V_{out} = I_C R_{load} \approx I_E R_{load}$$

$$V_{out} \approx \left(\frac{V_{in}}{r'_E + R_E} \right) R_{load}$$

$$V_{out} \approx V_{in} \left(\frac{R_{load}}{r'_E + R_E} \right)$$

$$A_V \approx \frac{R_{load}}{r'_E + R_E}$$

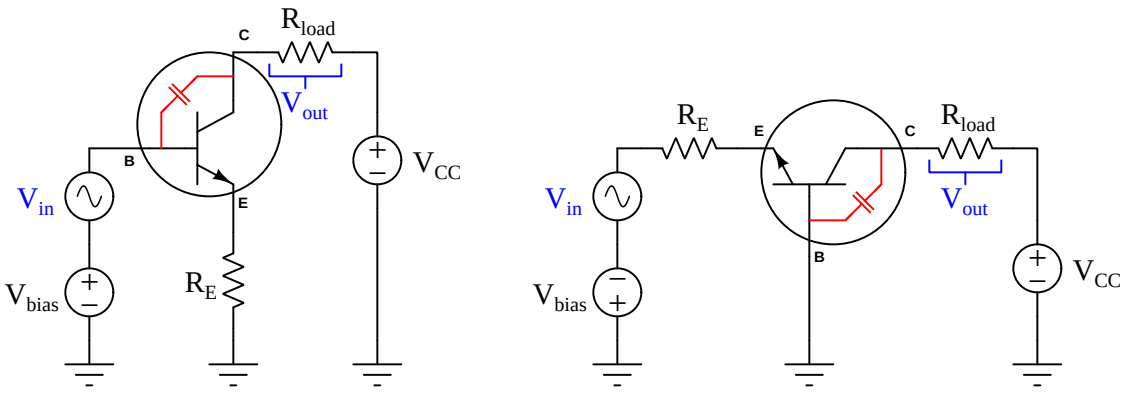
The first difference we see between the common-base and swamped common-emitter amplifiers is current gain. Note how V_{in} must carry the entire emitter current, which in fact is slightly greater than the load (collector) current. This yields a current gain slightly less than one:

$$A_I = \frac{\beta}{\beta + 1}$$

[11] Again, a commonly accepted approximation for r'_E assuming room-temperature silicon is 25 milliVolts per Ampere of DC bias current through the transistor's emitter terminal.

So far it would appear the common-base amplifier offers no advantage over the common-emitter, and in fact gives worse amplifying performance: the voltage gain is the same as with the common-emitter, but the common-base does not boost current at all! Why would anyone wish to use a common-base amplifier given this deficit?

The principal advantage of the common-base amplifier is its superior high-frequency performance. The common-emitter amplifier cannot amplify high-frequency AC signals as well as the common-base amplifier due to the capacitance naturally existing[12] between the base and collector terminals of a BJT in active mode. In the common-emitter configuration where the ground-referenced input signal lies at the transistor's base terminal while the output signal lies at the collector terminal, this natural capacitance provides coupling between output and input signals. No such coupling occurs with the common-base configuration because there is no ground-referenced voltage signal at the transistor's (grounded) base:



Output-input coupling degrades the common-emitter amplifier's voltage gain because the input and output voltage signals happen to be 180^o out of phase with each other in the common-emitter configuration. Every time the input signal's voltage rises, the output signal's voltage falls, and vice-versa. Therefore, any coupling between those two signal points pits the two signals against each other[13], the result being the stronger (output) signal attenuates the weaker (input) signal, the effect worsening as frequency increases because capacitive reactance decreases with frequency and makes the coupling stronger between the base and collector terminals.

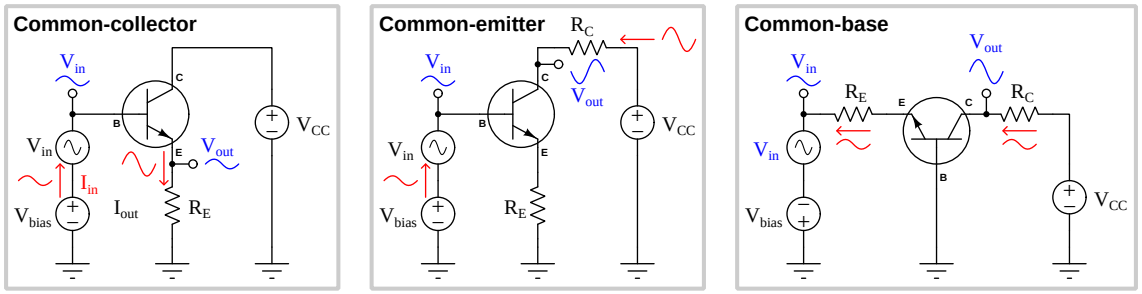
Common-base amplifiers do not suffer from this effect, making this configuration well-suited for high-frequency applications such as RF (radio-frequency) amplification where the base-collector capacitance's reactance (X_C) becomes very small. A popular application for common-base transistor amplifiers is between the antenna and signal-processing circuitry of a radio receiver system, where the common-base amplifier boosts the power level of radio-frequency signals received by the antenna, thereby making it easier for the signal-processing circuitry to interpret those signals.

[12] The base-collector junction of a BJT normally operates in reverse-bias mode, which means a substantial depletion region internally separates those two terminals of the transistor. This depletion region behaves much like a dielectric layer between two capacitor plates (in this case the "plates" are the base and collector regions of the transistor). Thus, a BJT in active mode possesses parasitic capacitance between its base and collector terminals.

[13] This is an unintentional example of *negative feedback*. As with all cases of negative feedback, the result is diminished gain.

3.6 Amplifier gain comparisons

The three different BJT amplifier configurations are shown here side-by-side, with ground-referenced voltage waveforms represented in blue and current waveforms represented in red:



Amplifier configuration	AC voltage gain	AC current gain
Common-collector	≈ 1 maximum	$\beta + 1$ maximum
Common-emitter	$\approx -\frac{R_C}{R_E}$ maximum	β maximum
Common-base	$\approx \frac{R_C}{R_E}$ maximum	≈ 1 maximum

Note how the common-emitter configuration is the only amplifier circuit with a "negative" or "inverting" voltage gain. This means the output voltage signal will be phase-shifted by 180^o from the input voltage signal. For the other two amplifier configurations, there is no phase shift between V_{out} and V_{in} and as such they have "positive" voltage gains.

Gain values are shown as "maximum" because the practical gain depends on the load resistance connected between the output terminal and ground. Values and formulae specified in the table are valid only for cases where the load is one of the circuit resistances (e.g. R_C for the common-emitter and common-base, R_E for the common-collector) and nothing else is connected to the output terminal. Any connected load resistance tends to diminish amplifier gain, a noteworthy exception to this rule being voltage gain for the common-collector amplifier which remains very nearly 1 with or without external load resistance.

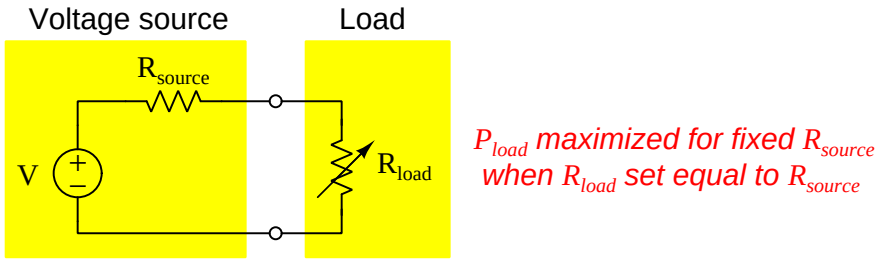
3.7 Input and output impedances

Ideal voltage sources contain zero resistance, while ideal current sources contain infinite (dynamic) resistance. However, real sources fall short of these ideals: real voltage sources possess some degree of internal resistance, while real current sources possess high values of (but not infinite) internal resistance. The existence of finite resistance inside of real sources accounts for why sources "sag" when placed under load[14], which is an important aspect of real-world circuit performance.

When a voltage amplifier powers a load, the burden of that load will cause the amplifier's output voltage to sag below where it would be if operated without a load connected. When a voltage signal source connects to the input of an amplifier, the amplifier acts as a load to that source, causing the source's voltage to also sag below its unloaded amplitude. This is all quite normal, just as it is normal for a DC power supply's output to sag under load.

For this reason, it is important to understand what makes amplifiers non-ideal with respect to their behavior as sources and as loads. With this understanding, we may design amplifiers to have appropriate amounts of internal resistance for their respective applications. In honor of the fact that most amplifiers handle *AC* signals, we refer to these internal resistances by the more general term of *impedance*[15].

The *Maximum Power Transfer Theorem* tells us a load will receive the most power from a fixed-resistance source when the load resistance is equal in value to that of the source, and for the sake of this discussion we will extend the concept to include source and load *impedance*:

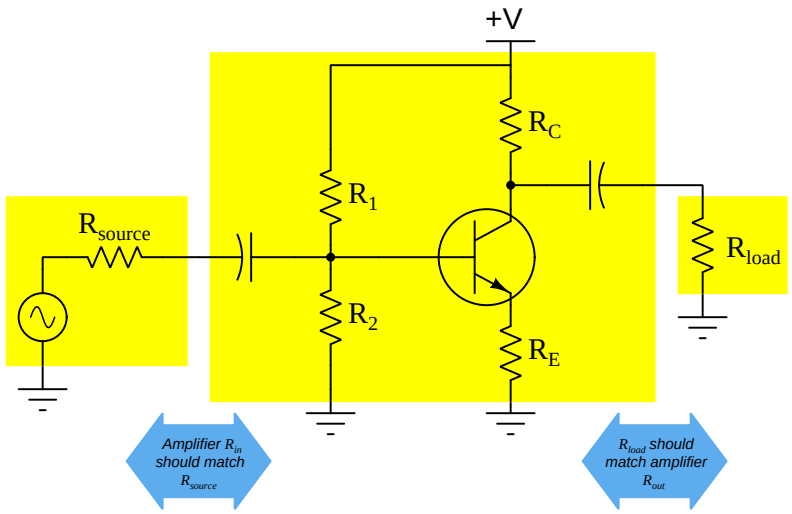


This particular principle of *impedance-matching* loads to sources relates directly to amplifier circuits because every amplifier acts as a load to its signal source, and also acts as a source to the final load. Therefore, any practical amplifier application actually has *two sets of impedances* in need of matching if our aim is to optimize the transfer of energy from signal source to load: the amplifier's input impedance should match the signal source's in order to deliver as much of the signal source's energy as possible to the amplifier, and the final load's impedance should match the amplifier's output impedance in order to deliver as much of the amplifier's output power to the load.

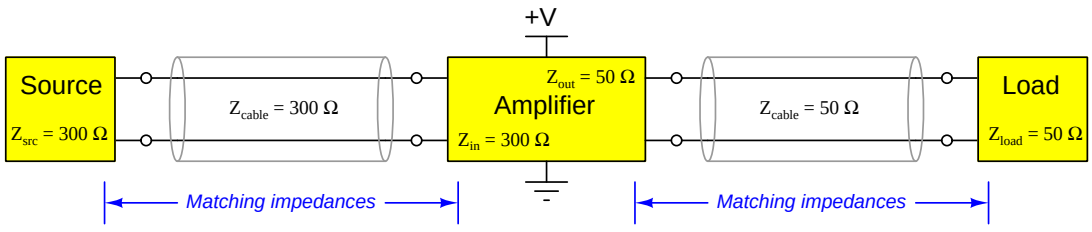
[14] The voltage measured between the terminals of a voltage source will decrease – or *sag* – as that source experiences a "heavier" load, "heavier" defined for a voltage source as a load with low resistance, drawing large amounts of current. Current measured at a terminal of a current source will likewise decrease (sag) as that source becomes more heavily loaded, "heavier" defined for a current source as a load with high resistance, demanding a larger voltage drop.

[15] As you should recall from your study of AC circuits, impedance is a general expression of opposition to electric current that encompasses both resistance and reactance. Resistance is defined as that which dissipates energy from passing electric charge carriers, while reactance is that which alternately absorbs and returns energy from and to passing electric charge carriers. To use a mechanical analogy, resistance is *friction* while reactance is *inertia*. *Impedance* is an aggregate of all such effects.

For example, consider the following common-emitter amplifier connected to a signal source and to a load. If our goal is to extract and deliver as much energy as we can from signal source to load, the following impedances must be matched:

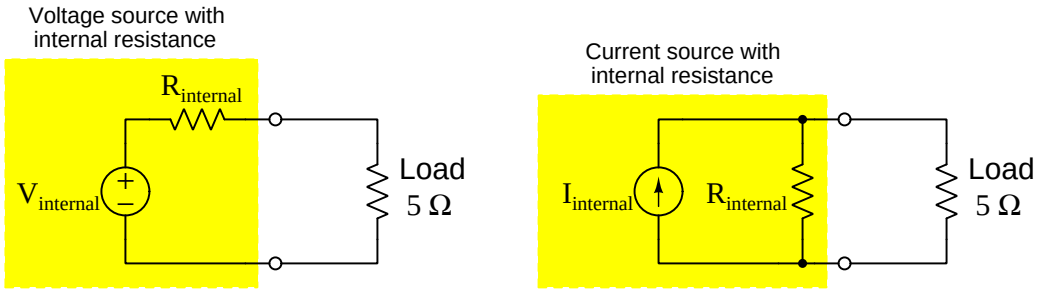


Maximum power transfer is not the only reason we might be concerned with amplifier impedances, either. For example, if the amplifier either receives signals or outputs signals via a *transmission line* (a relatively long set of conductors), it may be important for the amplifier's impedances to match the *characteristic impedances*[16] of those cables. If any circuit located at the end of a transmission line does not match that line's characteristic impedance, the effect will be that some (or all) of the energy contained in a signal pulse traveling along that line will become *reflected* at the interface of that line and the terminating circuit, and return back along the line from where it came rather than be delivered in whole to that circuit. In a well-designed amplified *system*, the impedances of source, source transmission line, and amplifier input all match; likewise, the impedances of amplifier output, load transmission line, and load similarly match:



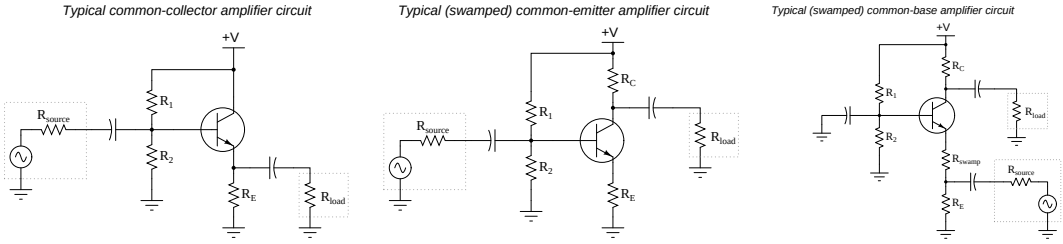
[16] This is a fairly complicated topic, too much to permit adequate explanation in a footnote. Suffice it to say, the characteristic impedance (also known as *surge impedance*) of a cable is the ratio of voltage to current for any *impulse* presented to that cable. This is a function of the cable's distributed inductance and capacitance which both act to store and release electrical energy.

Two theorems help us express complex electrical networks in terms of their equivalent resistance (or impedance): *Thévenin's Theorem* and *Norton's Theorem*. These two theorems tell us that any real source may be modeled as a combination of a single (ideal) source plus a single impedance. In the case of Thévenin's Theorem, any source may be modeled as an ideal voltage source and series impedance; in the case of Norton's Theorem, the equivalent model is an ideal current source and shunt (parallel) impedance. Here we see two such models, each connected to a 5 Ω load:

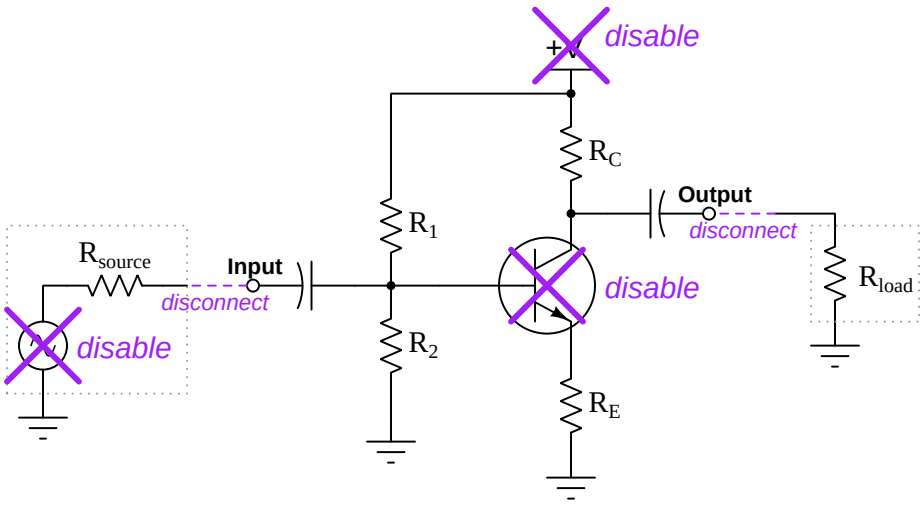


We may determine the equivalent internal resistance (series or parallel) of any DC network by computing the resistance as seen from its connection terminals while all internal sources are disabled: voltage sources replaced by shorts and current sources replaced by opens. This is precisely how we will determine the input and output impedances of transistor amplifier circuits: by disabling all sources and analyzing the series/parallel combinations of the remaining amplifier circuit resistances.

The task set before us, then, is how to analyze the various BJT amplifier circuits for their equivalent internal impedances on the basis of their component values, whether the purpose of doing so is to ensure optimum energy transfer, to prevent unwanted reflection of signals along transmission lines, or both.



When preparing to either Thévenize or Nortonize an amplifier circuit, we must identify all of its sources and as well as its input and output terminals. The schematic shown below is that of a common-emitter amplifier circuit with violet "X" marks drawn over every constant-voltage or constant-current device and both input and output terminals clearly marked. Note that the transistor itself will be regarded as a constant-current device and so must be disabled as well:

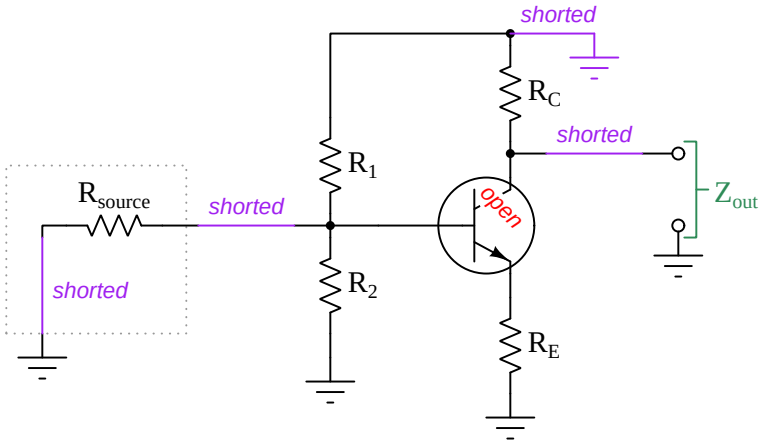


Once the sources have been identified, we must first disable each one by replacing it with its equivalent internal resistance (i.e. replace every constant voltage with a short and every constant current with an open). Then, in the case of an amplifier circuit which has both an input and an output, we will separate one of those terminals from its external component(s) (e.g. separating the AC signal source from the amplifier's input terminal; separating the load from the amplifier's output terminal) while keeping the other terminal connected, and compute the impedance to ground from the perspective of the disconnected terminal.

Let us begin by exploring input and output impedances for the three types of BJT amplifiers. In order to determine the Thévenin/Norton impedance of a transistor amplifier, we must disable all voltage and current sources, replacing each one with its equivalent resistance value. For the DC power supply (V_{CC}) this means *shorting* the positive and negative "rails" together; for the transistor (which behaves as a regulator of collector-terminal current when in its active mode) this means inserting an *open* in its collector terminal (the terminal through which the transistor regulates current).

3.7.1 Common-emitter impedances

The simplest configuration to analyze for output impedance is the common-emitter circuit, shown here with DC supply and AC signal source both shorted, collector terminal open, and load resistor removed to free up the output terminal. Since we are concerned about the amplifier's AC *impedance* and not just its DC *resistance*, we must also consider the reactance of each capacitor. Fortunately, coupling capacitors are typically sized large enough to make their reactance negligible at typical signal frequencies, and so we will just replace them with shorts as well:

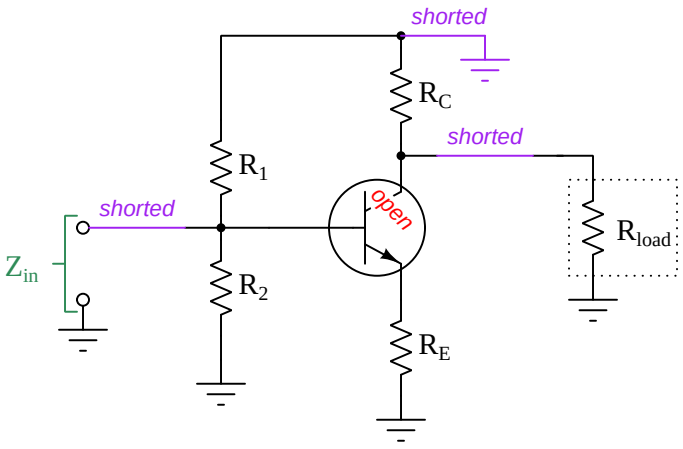


You can see why I said this amplifier configuration is the simplest to analyze for its output impedance: Z_{out} as measured between the output terminal and ground is simply R_C. If we were to model a common-emitter BJT amplifier as either a Thévenin or Norton source, its single internal resistance would approximately[17] equal R_C in the actual amplifier circuit:

$$Z_{out} \approx R_C$$

[17] This is not a *precise* equality largely due to the liberties we took in shorting and opening some of the amplifier's components. For example, we replaced every capacitor with a short based on the assumption that their reactances would be too small to matter, but in fact those reactances will never be *zero*. The same goes for the sources: the actual internal impedance of the DC power supply surely isn't zero, and neither is the current-regulating ability of the transistor so perfect as to justify representing it as a complete and total *open*. The price we pay for a simpler analysis is a reduction in accuracy. We may justify this bargain, though, by appealing both to the variability of transistors themselves as well as the non-critical nature of our goal (determining amplifier impedance for good "matching" to other devices). Transistors are well known for varying in important factors such as β from unit to the next, as well as from one set of operating conditions to the next. Impedance matching need not be perfect to be useful, either.

Analyzing the input impedance of a common-emitter amplifier is more challenging. First, we set up the schematic diagram with +V shorted to ground and all capacitors shorted and the transistor's collector terminal open as before, but this time we keep the load resistance connected and eliminate the AC signal source to leave the amplifier's input terminal disconnected:



At first it would appear Z_{in} is simply the two biasing resistors R_1 and R_2 in parallel with each other, but also included in the input impedance is R_E in parallel as "perceived" through the base terminal of the transistor. Recall in earlier sections of this tutorial how the input signal source for a common-collector amplifier and for a common-emitter amplifier experiences an "apparent" load resistance value many times greater than the actual emitter resistance ($R_{in} >> R_E$) due to the "transforming" effects of the transistor's beta ratio. This is due to the transistor's amplifying action bearing most of the energy burden of that emitter resistance, leaving the input signal (at the base terminal) merely to direct the power supply's energy to that resistance rather than the AC signal source having to bear it all alone. We see that same effect here, with R_E and r'_E together appearing to be $\beta+1$ times greater[18] than they actually are when "viewed" from the transistor's base terminal. Therefore, the actual input impedance of this amplifier is the parallel combination[19] of R_1, R_2, and $(\beta + 1)(r'_E + R_E)$:

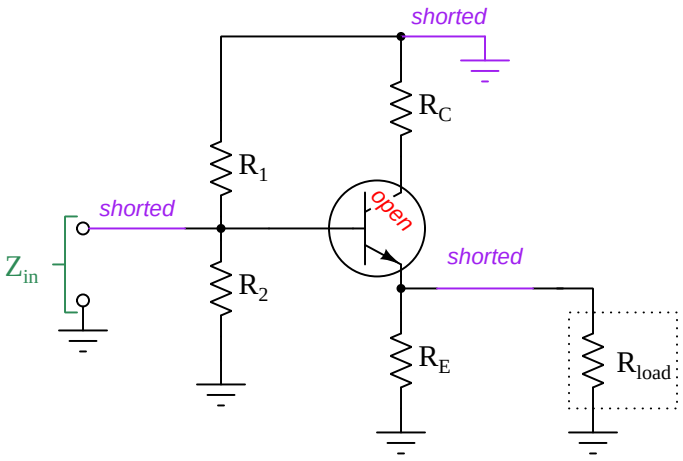
$$Z_{in} \approx R_1 \,||\, R_2 \,||\, (\beta + 1)(r'_E + R_E)$$

[18] Recall the reason for this: emitter current is $\beta + 1$ times greater than base current, and so for any given voltage the impedance must be that much less in accordance with Ohm's Law ($Z = \frac{V}{I}$). If we happened to be analyzing impedance from the perspective of the emitter terminal, "perceiving" the input impedance through the transistor, that impedance would appear to be $\beta+1$ times *less* than it actually was for the same reason. This, in fact, is precisely what we do when analyzing the output impedance of a common-collector amplifier (where the amplifier's output terminal is the transmitter's emitter terminal).

[19] The formula shown here uses "parallel" symbols (|| as a short-hand notation for the lengthier parallel impedance formula $Z_{parallel} = \frac{1}{\frac{1}{Z_1}+\frac{1}{Z_2}+\cdots\frac{1}{Z_n}}$.)

3.7.2 Common-collector impedances

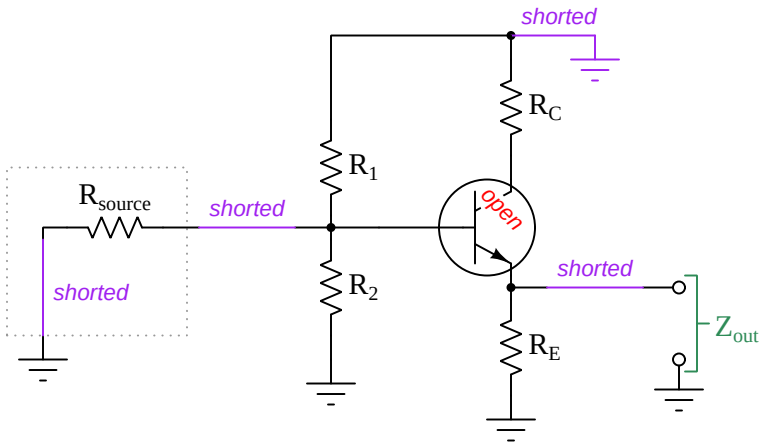
Now let us consider the input impedance of a common-collector amplifier:



As viewed from the input terminals, the common-collector amplifier's input impedance is remarkably similar to that of the swamped common-emitter amplifier. The only real difference with the common-collector amplifier is that the load resistance is in parallel with the emitter resistor:

$$Z_{in} \approx R_1 \,||\, R_2 \,||\, (\beta + 1)[r'_E + (R_E \,||\, R_{load})]$$

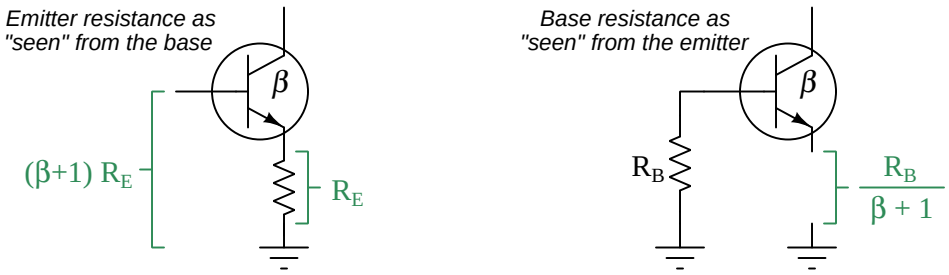
Output impedance is another matter entirely for the common-collector circuit, though:



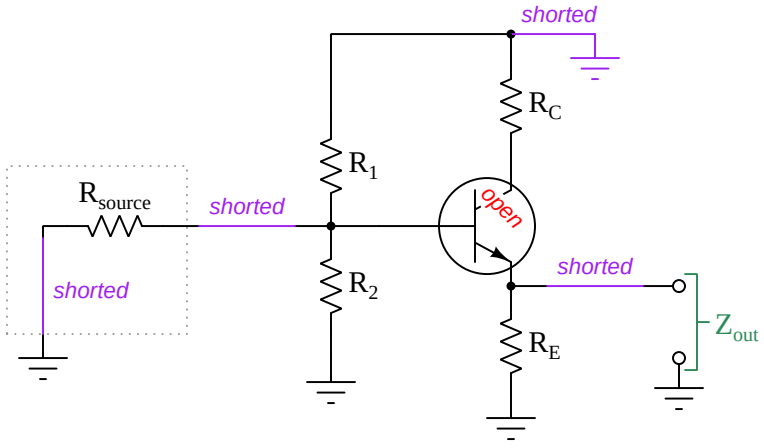
At first it appears as though Z_{out} might simply be equal to R_E, as it was equal to R_C in the common-emitter circuit. However, the common-collector amplifier is a bit more complicated than this. On the next page we will discover why!

Recall from the common-emitter amplifier example that any time we "view" the impedance from one side of the transistor to the other, the apparent impedance becomes "transformed" by the amplifying action of the transistor. We know that as voltage between base and ground (V_B) varies, voltage between emitter and ground (V_E) will vary by approximately the same amount because the base-emitter voltage drop is a nearly-constant 0.7 Volts, yet base current and emitter current differ from each other by a factor of $\beta + 1$.

If we look at the impedance of the amplifier as seen from the signal source (i.e. Z_{in}), there will be $\beta + 1$ times *less* current drawn from that signal source as its voltage changes than there will be current passing through the transistor's emitter. Thus, the resistance between emitter and ground becomes "multiplied" or "transformed" by the transistor's amplifying action to appear as a much larger amount of resistance as "seen" from the perspective of the signal source. *The converse of this is also true*: the resistance between base and ground becomes "divided" or "transformed" by the transistor's amplifying action to appear as a much smaller amount of resistance as "seen" from the perspective of the emitter, which in the case of a common-collector amplifier is the output terminal:



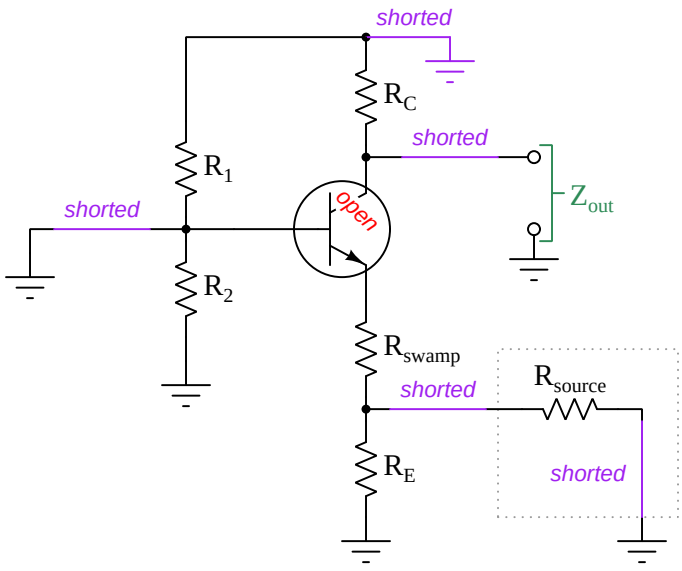
Examining the common-collector amplifier circuit again from the perspective of the emitter (output) terminal, we see Z_{out} to be R_E in parallel with the sum of the dynamic emitter resistance (r'_E) and the "transformed" equivalent of all the resistances connected to the base terminal:



$$Z_{out} \approx R_E \,||\, \left(r'_E + \frac{R_1 \,||\, R_2 \,||\, R_{source}}{\beta + 1} \right)$$

3.7.3 Common-base impedances

Now let us consider the output impedance of a common-base amplifier:

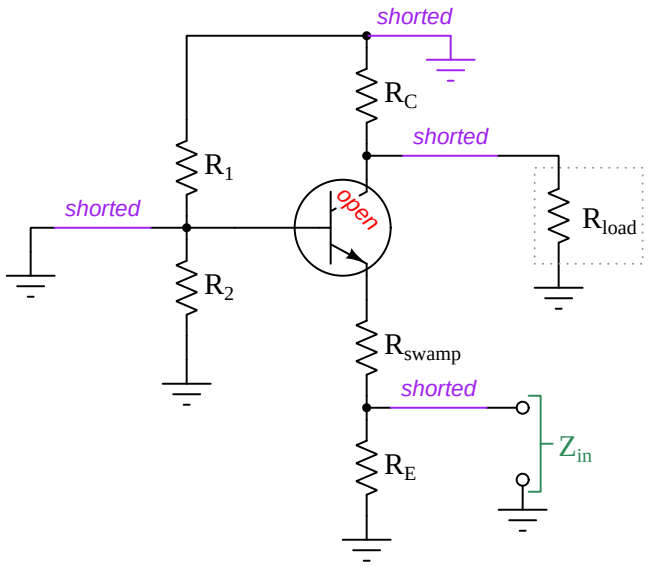


With the transistor considered as a disabled current source (i.e. open-circuited at the collector terminal which is the terminal through which current is regulated by the transistor's action), there is only one component left to define the amplifier's output impedance, and that is R_C. Therefore the output impedance of a common-base amplifier is the same as for the common-emitter amplifier, namely:

$$Z_{out} \approx R_C$$

This conclusion should not be too surprising, as we already know the swamped versions of the common-base and common-emitter amplifiers share the same voltage gain characteristic ($\approx \frac{R_C}{R_E}$).

When viewed from the perspective of the input terminal, though, the common-base amplifier's impedance is unique. With the base terminal essentially being grounded by the capacitor (for AC signals of sufficient frequency), the only resistances "seen" from the perspective of the input are R_E, R_{swamp}, and r'_E:



$$Z_{in} \approx R_E \,||\, (r'_E + R_{swamp})$$

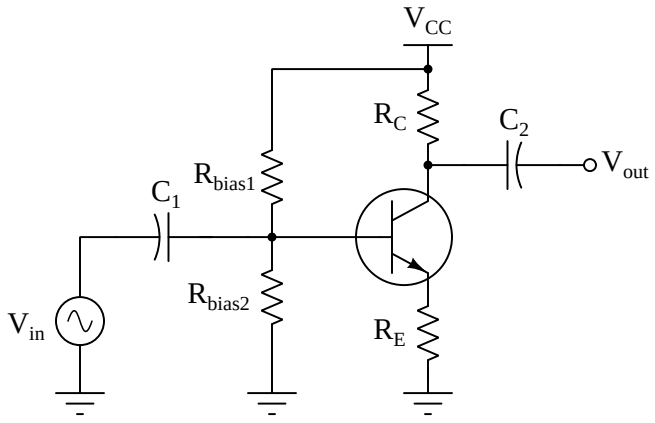
The purpose of the "swamping" resistor is to overshadow the effect of r'_E, which otherwise would dominate this amplifier's input impedance with its very small value. Even with R_{swamp} in place, though, common-base amplifiers typically exhibit low input impedance which means they "load down" whatever source provides the driving signal for V_{in}. This agrees with what we already know of the common-base amplifier's unusually low current gain: it demands much current from the signal source, and so that source needs to have a relatively low impedance itself in order to successfully drive this type of amplifier.

With each of these amplifier types (common-collector, -emitter, and -base) having different impedances and different gain characteristics, it may seem impossible to find one amplifier type able to fulfill all the necessary specifications for a particular circuit design. This indeed is often the case, which is a reason why we find amplifier circuits built using *multiple stages*, each stage having some of the necessary characteristics but not all of them. An amplifier circuit made up of many transistors, with the input stage designed to meet Z_{in} requirements, the output stage designed to meet Z_{out} requirements, and possibly additional stages in between meeting over-all gain requirements, is generally the only practical solution.

3.8 Amplifier design example: swamped common-emitter

Selecting appropriate components to design a transistor amplifier for a specific application is an exercise in compromise, and it is a task aided by the generous use of approximations and idealized assumptions. Here we will explore an example of a swamped common-emitter amplifier. To begin, here are some desired performance parameters:

- Must operate on a supply voltage (V_{CC}) of 15 Volts DC
- Output impedance should be approximately 3 kΩ
- Voltage gain should be approximately 2.75
- Input impedance should be at least 10 kΩ



Voltage gain for a swamped common-emitter amplifier is set by the ratio of collector resistance to emitter resistance ($\frac{R_C}{R_E}$). Output impedance for a common-emitter amplifier is simply equal to collector resistance ($Z_{out} = R_C$). Input impedance is the most complicated calculation of them all, involving the two bias resistors as well as β and R_E: $Z_{in} \approx R_{bias1} \,||\, R_{bias2} \,||\, (\beta + 1)(r'_E + R_E)$. We can see that output impedance requirement of 3 kΩ constrains our choice of R_C to that same value, and so with that we have our first component value.

From this known value of R_C we may take our desired voltage gain and use it to calculate R_E:

$$A_V = \frac{R_C}{R_E}$$

$$R_E = \frac{R_C}{A_V}$$

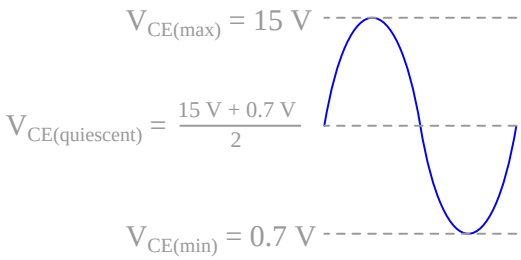
$$R_E = \frac{3000\ \Omega}{2.75} = 1090.9\ \Omega$$

So, now we know R_E should be 1090.9 Ω. It is important to bear in mind that many of the values we compute in this design exercise will be *approximate*, and so it is okay to make small adjustments

in order to select common-size components. When we make such adjustments it is helpful to know which way the real performance parameter differs from ideal. In the case of voltage gain, real amplifier A_V is always less than the predicted $A_V = \frac{R_C}{R_E}$ due to the imperfection of our simplifying assumption, namely that $I_C = I_E$. Given a high β value these two currents will be *nearly* equal, but not exactly equal. I_C is always slightly less than I_E, and so the voltage drop developed across R_C will also be slightly less, and that leads to less voltage gain. Knowing this, when we choose a value for R_E it is safer to round *down* to the nearest common resistor value because this will inflate voltage gain and thus help offset that ideal/real gain error.

The next resistances to compute are the two bias resistors. Their primary function is to set the bias voltage at the transistor's base terminal (V_B, between base and ground) such that the transistor is exactly *half-way-on* so that it has plenty of "headspace" to turn off (to raise V_{out}) and turn on (to lower V_{out}) for class-A amplification. V_B must be approximately 0.7 Volts greater than V_E (voltage dropped across R_E) due to the base-emitter PN junction's forward-voltage drop. So, if we can determine V_E we will be able to determine V_B, and from there choose a voltage-division ratio for R_{bias1} and R_{bias2} to fulfill.

Calculations related to bias are exclusively DC, not AC. Here we ignore the AC signal source (V_{in}) and treat the amplifier circuit as though its only source was V_{CC}. If our goal is to have the transistor be 50% "on" in its purely DC state, we must define what 100% "on" and 0% "on" (i.e. *off*) look like for the transistor. In the full-on state we want the transistor to be as conductive as it can without entering a state of saturation, because we are using the transistor here as an analog amplification device and not an on/off switch. This means the minimum collector-emitter voltage drop will be the same as V_{BE}, approximately 0.7 Volts. The maximum collector-emitter voltage drop will be the full supply voltage (V_{CC}) which we know is 15 Volts for this design exercise. These two limits establish the peak values of V_{CE}:



V_{CE} during a condition of quiescence (when there is only DC and no AC in the amplifier circuit) must be the mid-point of these two limits, if our goal is to give the amplifier circuit as much "space" to swing its output in the positive direction as in the negative direction. Finding the midpoint between any two numbers is equivalent to finding the average of those two numbers, in this case adding 15 Volts to 0.7 Volts and dividing by two:

$$V_{CE(quiescent)} = \frac{15 \text{ V} + 0.7 \text{ V}}{2} = 7.85 \text{ V}$$

If the transistor drops 7.85 Volts between its collector and emitter terminals, then by Kirchhoff's Voltage Law the remainder of the 15 Volt supply must be dropped across the series combination of R_C and R_E. Using Ohm's Law we can then calculate quiescent current (assuming $I_C = I_E$):

$$I_C = I_E = \frac{15 \text{ V} - 7.85 \text{ V}}{3000\,\Omega + 1000\,\Omega} = 1.7875 \text{ mA}$$

With emitter current at 1.7875 mA and the emitter resistor being 1 kΩ, voltage from the emitter terminal to ground must be 1.7875 Volts, also by Ohm's Law:

$$V_E = I_E R_E = (1.7875 \text{ mA})(1000\,\Omega) = 1.7875 \text{ V}$$

Voltage from the transistor's base to ground (V_B) must be 0.7 Volts greater than this, or 2.4845 Volts. This is the amount of DC bias voltage the two bias resistors must develop from the 15 Volt source in order to properly bias the amplifier for class-A operation.

R_{bias1} and R_{bias2} form a resistive voltage divider, and as such may produce this 2.4845 Volt bias from the 15 Volt source using any two resistance values that happen to make the necessary *ratio*. The voltage divider formula makes this clear:

$$\frac{V_B}{V_{CC}} = \frac{R_{bias2}}{R_{bias1} + R_{bias2}}$$

Yet, our choices for these two resistors are not unconstrained. If we make them too large the division ratio will become "loaded" by the very small bias current entering the transistor's base terminal (I_B), and this will result in the base bias voltage being less than it should be, in turn making the transistor less than half-way-on under quiescent conditions. If we make these resistors too small they will pose an unreasonable burden on the input signal source (i.e. Z_{in} will be too low). Fortunately, we've been given a minimum value for Z_{in} which will help us choose R_{bias1} and R_{bias2}.

Input impedance for a common-emitter amplifier with voltage-divider biasing is predicted by this long formula:

$$Z_{in} \approx R_{bias1} \,||\, R_{bias2} \,||\, (\beta + 1)(r'_E + R_E)$$

With β typically being around 100 for small-signal transistors and R_E being 1 kΩ, the last term of this formula will be greater than 100 kΩ. If we only need Z_{in} to be greater than 10 kΩ, it means the parallel combination of R_{bias1} and R_{bias2} can be slightly greater than 10 kΩ, since 100+ kΩ will not make much of a difference in parallel with an $R_{bias1}||R_{bias2}$ combination nearly ten times smaller in size. In other words, $R_{bias1}||R_{bias2}$ *swamps* $(\beta + 1)(r'_E + R_E)$. Therefore, we will ignore that last term and approximate input impedance as the parallel combination of the two bias resistors:

$$Z_{in} \approx R_{bias1} \,||\, R_{bias2}$$

We are still left with a difficult problem: namely, how to calculate reasonable values for *two* resistances given a known voltage division ratio and a minimum specification on their combined parallel resistance. Techniques exist for solving multiple variables given multiple equations[20], but these are two fairly ugly equations to manipulate:

$$\frac{2.4845}{15} = \frac{R_{bias2}}{R_{bias1} + R_{bias2}} \qquad \frac{1}{\frac{1}{R_{bias1}} + \frac{1}{R_{bias2}}} \geq 10000$$

A perfectly valid problem-solving technique for this case is to use a method called *cut and try*. This is where we make an educated guess about one of these resistor values and then plug in that value into the equations to see if this satisfies our design criteria. Since we do not have an exact specification for Z_{in}, there should still be a range of appropriate values for R_{bias1} and R_{bias2}, and so we ought to be able to find a perfectly acceptable solution using this method[21].

Parallel resistances diminish, and so if we want $R_{bias1} \,||\, R_{bias2}$ to be no less than 10 kΩ it means both R_{bias1} and R_{bias2} must be greater than 10 kΩ. How much greater? Let's just "cut and try" to find out! With our desired V_B bias voltage being a less than half of 15 Volts, we know R_{bias2} will be smaller than R_{bias1}.

[20] Using such techniques, we arrive at $R_{bias1} = 60.374$ kΩ and $R_{bias2} = 11.985$ kΩ. Perhaps the easiest method is to use algebraic substitution, manipulating the voltage divider equation to solve for R_{bias1} in terms of R_{bias2}, which yields $R_{bias1} = \left(\frac{15}{2.4845} - 1\right) R_{bias2}$, after which we may substitute this definition of R_{bias1} into the parallel resistance formula and finally solve for R_{bias2}.

[21] The "cut and try" method is less attractive in cases where we need an *exact* solution, because it invariably requires many attempts at guessing values until you arrive at a solution that is close enough.

Here, we will try $R_{bias2} = 12\ \text{k}\Omega$ (this being common resistor size greater than 10 kΩ) and use this value to solve for R_{bias1} based on the exact voltage division ratio we require for proper biasing. Manipulating the voltage divider formula[22] to solve for R_{bias1}:

$$\frac{V_B}{V_{CC}} = \frac{R_{bias2}}{R_{bias1} + R_{bias2}}$$

$$\frac{V_{CC}}{V_B} = \frac{R_{bias1} + R_{bias2}}{R_{bias2}}$$

$$\frac{V_{CC} R_{bias2}}{V_B} = R_{bias1} + R_{bias2}$$

$$\frac{V_{CC} R_{bias2}}{V_B} - R_{bias2} = R_{bias1}$$

$$\frac{V_{CC} R_{bias2}}{V_B} - \frac{V_B R_{bias2}}{V_B} = R_{bias1}$$

$$\frac{V_{CC} R_{bias2} - V_B R_{bias2}}{V_B} = R_{bias1}$$

$$R_{bias2} \left(\frac{V_{CC} - V_B}{V_B} \right) = R_{bias1}$$

$$12000\ \Omega \left(\frac{15\ \text{V} - 2.4845\ \text{V}}{2.4845\ \text{V}} \right) = R_{bias1}$$

$$R_{bias1} = 60449.2\ \Omega$$

Now let's check to see if these two resistance values in parallel are greater than 10 kΩ:

$$\frac{1}{\frac{1}{R_{bias1}} + \frac{1}{R_{bias2}}}$$

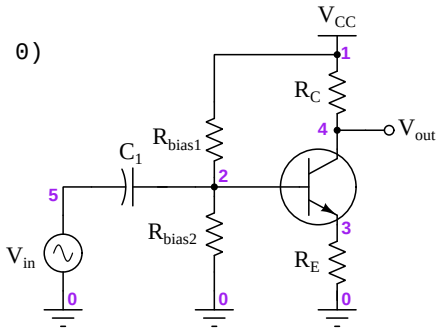
$$\frac{1}{\frac{1}{60449.2\ \Omega} + \frac{1}{12000\ \Omega}} = 10012.4\ \Omega$$

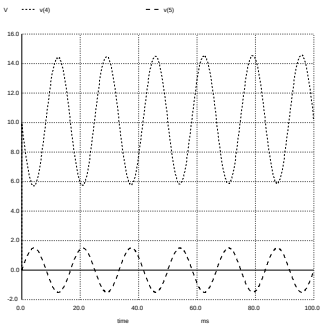
Looks like our first cut-and-try solution worked rather well!

[22] Alternatively, we could have solved for DC current through R_{bias2} given its assumed cut-and-try value and its intended voltage drop of 2.4845 Volts, using Ohm's Law. Then we could take that divider network current and use it to calculate R_{bias1} knowing that the upper resistor must drop the remainder of the 15 Volts, using KVL and Ohm's Law.

Let's try out our design using SPICE to simulate the amplifier. We will set it up using a 1.5 Volt peak AC source, and use a 10 μF coupling capacitor[23] between the AC signal source and the divider network. The output capacitor may be eliminated since we are not actually driving a load, and as such may monitor V_C as the amplifier's output voltage:

```
* SPICE circuit
vin 5 0 sin (0 1.5 60 0 0)
vcc 1 0 dc 15
rbias1 1 2 60.4492k
rbias2 2 0 12k
c1 2 5 10u ic=2.4845
rc 1 4 3000
re 3 0 1000
q1 4 2 3 qmod
.model qmod npn
.tran 0.1m 100m uic
.plot tran v(1) v(4)
.end
```





The output waveform appears to be about 8.67 Volts peak-to-peak while the input waveform is 3 Volts peak-to-peak. This gives an empirical voltage gain of approximately 2.89 which is not far from our intended voltage gain ratio of 2.75, being higher due to the fact we rounded the value of R_E down from 1090.9 Ω to 1000 Ω. Generally speaking, it is better to design an amplifier with more gain than necessary rather than less gain, because that way we still have the option of attenuating the input signal to get the desired output amplitude. Attenuation won't help an amplifier with insufficient gain.

More accurate methods exist for designing transistor amplifier circuits, but this works fairly well. In practice, with imperfect components and temperature-dependent transistor behaviors unaccounted for in the simulation, you end up having to make slight adjustments to the design while in the prototyping stage. What you need is a method that gets you *close enough*. Summarizing our steps:

- Select values of R_C and R_E for desired gain and Z_{out}
- Calculate V_{CE} at the half-way-on point for class-A operation based on the limits of V_{CC} and the transistor's minimum V_{CE} (without saturating)
- Determine I_E at that quiescent point
- Determine V_E based on that current and R_E
- Determine V_B based on V_E and the V_{BE} forward voltage drop
- Select biasing resistors for desired V_B and Z_{in}
- Select coupling capacitor sizes yielding negligible X_C at the lowest signal frequency

[23] Coupling capacitors should be sized such that their AC reactance is negligible compared to the amplifier's impedance. Such is the case for C_1 at our test frequency of 60 Hz, with $X_C = 265.3\ \Omega$ in comparison to $Z_{in} = 10\ \text{k}\Omega$. Note also the "initial condition" (`ic=`) of 2.4845 Volts specified in the simulation to account for the DC bias voltage, so we won't have to wait for this capacitor to accumulate that charge.

Chapter 4

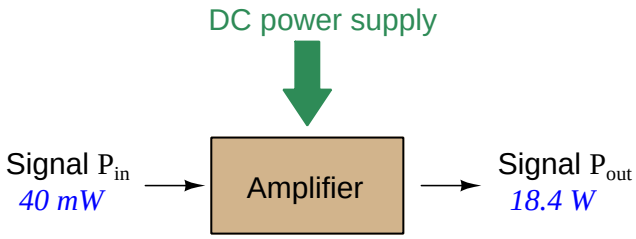
Derivations and Technical References

This chapter is where you will find mathematical derivations too detailed to include in the tutorial, and/or tables and other technical reference material.

4.1 Decibels

One of the mathematical tools popularly used to gauge increases and decreases of electrical power is the *common logarithm*, expressed as a measurement unit called the *decibel*. The basic idea of decibels is to express a ratio of two electrical power quantities in logarithmic terms. Every time you see the unit of "decibel" you can think: *this is an expression of how much greater (or how much smaller) one power is to another*. The only question is which two powers are being compared.

Electronic amplifiers are a type of electrical system where comparisons of power are useful. Students of electronics learn to compare the output power of an amplifier against the input power as a unitless ratio, called a *gain*. Take for example an electronic amplifier with a signal input of 40 milliWatts and a signal output of 18.4 Watts:



$$\text{Gain} = \frac{P_{out}}{P_{in}} = \frac{18.4\ W}{40\ mW} = 460$$

An alternative way to express the gain of this amplifier is to do so using the unit of the *Bel*, defined as the common logarithm of the gain ratio:

$$\log\left(\frac{P_{out}}{P_{in}}\right) = \log\left(\frac{18.4\ \text{W}}{40\ \text{mW}}\right) = 2.66276\ \text{B}$$

When you see an amplifier gain expressed in the unit of "Bel", it's really just a way of saying "The output signal coming from this amplifier is x powers of ten greater than the input signal." An amplifier exhibiting a gain of 1 Bel outputs 10 times as much power as the input signal. An amplifier with a gain of 2 Bels boosts the input signal by a factor of 100. The amplifier shown above, with a gain of 2.66276 Bels, boosts the input signal 460-fold.

At some point in technological history it was decided that the "Bel" (B) was too large and cumbersome, and so it became common to express powers in fractions of a Bel instead: the *deci*Bel (1 dB = $\frac{1}{10}$ of a Bel). Therefore, this is the form of formula you will commonly see for expressing electrical signal power gains or losses:

$$\text{dB} = 10 \log\left(\frac{P_{out}}{P_{in}}\right)$$

The gain of our hypothetical electronic amplifier, therefore, would be more commonly expressed as 26.6276 dB rather than 2.66276 B, although either expression is technically valid[1].

[1] It is interesting to note that although the "Bel" is a metric unit, it is seldom if ever used without the metric prefix

An operation students often struggle with is converting a decibel figure back into a ratio, since the concept of logarithms seems to be universally perplexing. Here I will demonstrate how to algebraically manipulate the decibel formula to solve for the power ratio given a dB figure.

First, we will begin with the decibel formula as given, solving for a value in decibels given a power ratio:

$$\text{dB} = 10 \log(\text{Ratio})$$

If we wish to solve for the ratio, we must "undo" all the mathematical operations surrounding that variable. One way to determine how to do this is to reverse the order of operations we would follow if we knew the ratio and were solving for the dB value. After calculating the ratio, we would then take the logarithm of that value, and then multiply that logarithm by 10: start with the ratio, then take the logarithm, then multiply last. To un-do these operations and solve for the ratio, we must un-do each of these operations in reverse order. First, we must un-do the multiplication (by dividing by 10):

$$\frac{\text{dB}}{10} = \frac{10 \log(\text{Ratio})}{10}$$

$$\frac{\text{dB}}{10} = \log(\text{Ratio})$$

Next, we must un-do the logarithm function by applying its mathematical inverse to both sides of the formula – making each expression a power of 10:

$$10^{\frac{\text{dB}}{10}} = 10^{\log(\text{Ratio})}$$

$$10^{\frac{\text{dB}}{10}} = \text{Ratio}$$

To test our algebra, we can take the previous decibel value for our hypothetical amplifier and see if this new formula yields the original gain ratio:

$$\text{Ratio} = 10^{\frac{26.6276 \text{ dB}}{10}}$$

$$\text{Ratio} = 10^{2.66276 \text{ B}}$$

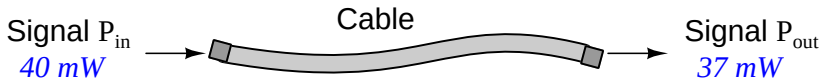
$$\text{Ratio} = 460$$

Sure enough, we arrive at the correct gain ratio of 460, starting with the decibel gain figure of 26.6276 dB.

"deci" ($\frac{1}{10}$). One could express powers in microbels, megabels, or any other metric prefix desired, but it is never done in industry: only the decibel is used.

We may also use decibels to express power *losses* in addition to power *gains*. There are many practical applications of this in signaling systems, both electronic and optical. One such application is *filtering*, where a "filter" circuit screens out certain components of the signal while letting others pass through (e.g. the bass or treble control for an audio system). Another application is *attenuation*, where the entirety of a signal is reduced in magnitude (e.g. the volume control for an audio system).

We will explore yet another application of signal power reduction as a case study for decibels: *cable loss*. Cables designed to convey signals over long distances are not perfect conduits of energy, as some of the signal's energy is inevitably lost along the way. This is true for different types of signals, electrical and optical being two popular examples. In the following illustration we see a signal cable losing power along its length[2], such that the power out is less than the power in:



$$\text{Gain} = \frac{P_{out}}{P_{in}} = \frac{37\ mW}{40\ mW} = 0.925$$

$$10 \log \left(\frac{P_{out}}{P_{in}} \right) = 10 \log \left(\frac{37 \text{ mW}}{40 \text{ mW}} \right) = -0.3386 \text{ dB}$$

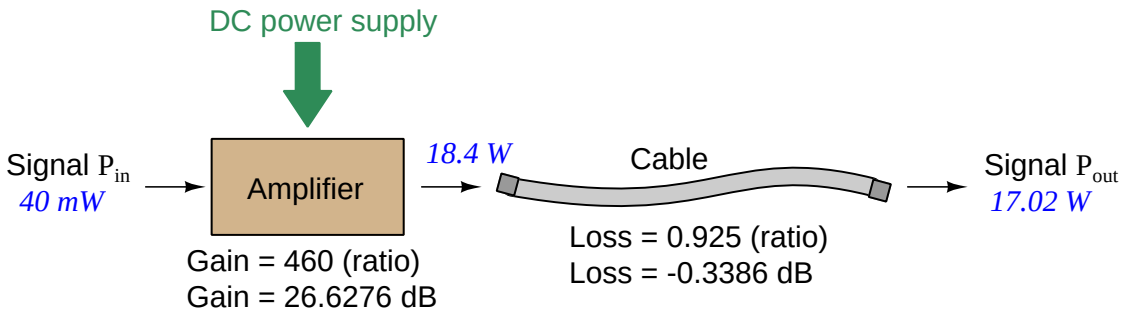
Contrasting this result against the previous result (with the amplifier) we see a very important property of decibel figures: any power *gain* is expressed as a *positive* decibel value, while any power *loss* is expressed as a *negative* decibel value. Any component outputting the exact same power as it takes in will exhibit a "gain" value of 0 dB (equivalent to a gain *ratio* of 1).

Remember that Bels and decibels are nothing more than logarithmic expressions of "greater than" and "less than". Positive values represent powers that are *greater* while negative values represent powers that are *lesser*. Zero Bel or decibel values represent *no change* (neither gain nor loss) in power.

A couple of simple decibel values are useful to remember for approximations, where you need to quickly estimate decibel values from power ratios (or vice-versa). Each addition or subtraction of 10 dB exactly represents a 10-fold multiplication or division of power ratio: e.g. +20 dB represents a power ratio gain of $10 \times 10 = 100$, whereas −30 dB represents a power ratio reduction of $\frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} = \frac{1}{1000}$. Each addition or subtraction of 3 dB approximately represents a 2-fold multiplication or division or power ratio: e.g. +6 dB is approximately equal to a power ratio gain of $2 \times 2 = 4$, whereas −12 dB is approximately equal to a power ratio reduction of $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16}$. We may combine ± 10 dB and ± 3 dB increments to come up with ratios that are products of 10 and 2: e.g. +26 dB is approximately equal to a power ratio gain of $10 \times 10 \times 2 \times 2 = 400$.

[2]For high-frequency signals such as those used in radio communications, the dominant mode of energy dissipation is *dielectric heating*, where the AC electric field between the cable conductors excites the molecules of the conductor insulation. This energy loss manifests as heat, which explains why there is less signal energy present at the load end of the cable than is input at the source end of the cable. For DC and low-frequency AC circuits the dominant mode of energy dissipation is cable conductor resistance, which is typically very small.

Observe what happens if we combine a "gain" component with a "loss" component and calculate the overall power out versus power in:



The overall gain of this amplifier and cable system expressed as a ratio is equal to the *product* of the individual component gain/loss ratios. That is, the gain ratio of the amplifier *multiplied* by the loss ratio of the cable yields the overall power ratio for the system:

$$\text{Overall gain} = \frac{17.02 \text{ W}}{40 \text{ mW}} = (460)(0.925) = 425.5$$

The overall gain may be alternatively expressed as a decibel figure, in which case it is equal to the *sum* of the individual component decibel values. That is, the decibel gain of the amplifier *added* to the decibel loss of the cable yields the overall decibel figure for the system:

$$\text{Overall gain} = 10 \log \left(\frac{17.02 \text{ W}}{40 \text{ mW}} \right) = 26.6276 \text{ dB} + (-0.3386 \text{ dB}) = 26.2890 \text{ dB}$$

It is often useful to be able to estimate decibel values from power ratios and vice-versa. If we take the gain ratio of this amplifier and cable system (425.5) and round it down to 400, we may easily express this gain ratio as an expanded product of 10 and 2:

$$425.5 \approx 400 = (10) \times (10) \times (2) \times (2)$$

Knowing that every 10-fold multiplication of power ratio is an addition of +10 dB, and that every 2-fold multiplication of power is an addition of +3 dB, we may express the expanded product as a sum of decibel values:

$$(10) \times (10) \times (2) \times (2) = (10 \text{ dB}) + (10 \text{ dB}) + (3 \text{ dB}) + (3 \text{ dB}) = 26 \text{ dB}$$

Therefore, our power ratio of 425.5 is approximately equal to +26 decibels.

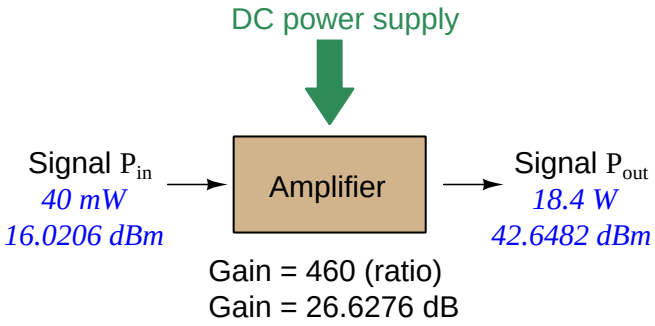
Decibels always represent comparisons of power, but that comparison need not always be P_{out}/P_{in} for a system component. We may also use decibels to express an amount of power compared to some standard reference. If, for example, we wished to express the input power to our hypothetical amplifier (40 milliWatts) using decibels, we could do so by comparing 40 mW against a standard "reference" power of exactly 1 milliWatt. The resulting decibel figure would be written as "dBm" in honor of the 1 *m*illiWatt reference:

$$P_{in} = 10 \log \left(\frac{40 \text{ mW}}{1 \text{ mW}} \right) = 16.0206 \text{ dBm}$$

The unit of "dBm" literally means the amount of dB "greater than" 1 milliWatt. In this case, our input signal of 40 milliWatts is 16.0206 dB greater than a standard reference power of exactly 1 milliWatt. The output power of that amplifier (18.4 Watts) may be expressed in dBm as well:

$$P_{out} = 10 \log \left(\frac{18.4 \text{ W}}{1 \text{ mW}} \right) = 42.6482 \text{ dBm}$$

A signal power of 18.4 Watts is 42.6482 dB greater than a standard reference power of exactly 1 milliWatt, and so it has a decibel value of 42.6482 dBm.



Notice how the output and input powers expressed in dBm relate to the power gain of the amplifier. Taking the input power and simply *adding* the amplifier's gain factor yields the amplifier's output power in dBm:

$$P_{in}(\text{dB}) + P_{gain}(\text{dB}) = P_{out}(\text{dB})$$

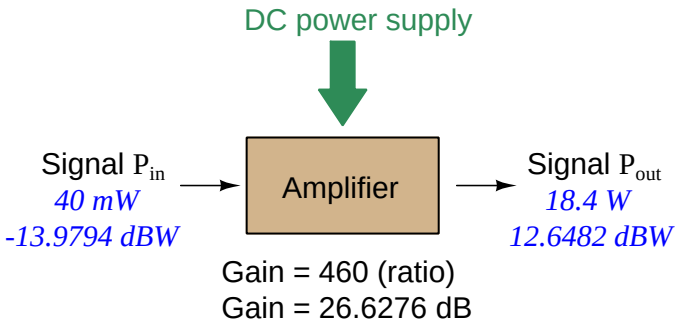
$$16.0206 \text{ dBm} + 26.6276 \text{ dB} = 42.6482 \text{ dBm}$$

An electronic signal that begins 16.0206 dB greater than 1 milliWatt, when boosted by an amplifier gain of 26.6276 dB, will become 42.6482 dB greater than the original reference power of 1 milliWatt.

We may alternatively express all powers in this hypothetical amplifier in reference to a 1-Watt standard power, with the resulting power expressed in units of "dBW" (decibels greater than 1 Watt):

$$P_{in} = 10 \log \left(\frac{40 \text{ mW}}{1 \text{ W}} \right) = -13.9794 \text{ dBW}$$

$$P_{out} = 10 \log \left(\frac{18.4 \text{ W}}{1 \text{ W}} \right) = 12.6482 \text{ dBW}$$



Note how the input power of 40 milliWatts equates to a negative dBW figure because 40 milliWatts is *less* than the 1 Watt reference, and how the output power of 18.4 Watts equates to a positive dBW figure because 18.4 Watts is *more* than the 1 Watt reference. A positive dB figure means "more than" while a negative dB figure means "less than."

Note also how the output and input powers expressed in dBW still relate to the power gain of the amplifier by simple addition, just as they did when previously expressed in units of dBm. Taking the input power in units of dBW and simply *adding* the amplifier's gain factor yields the amplifier's output power in dBW:

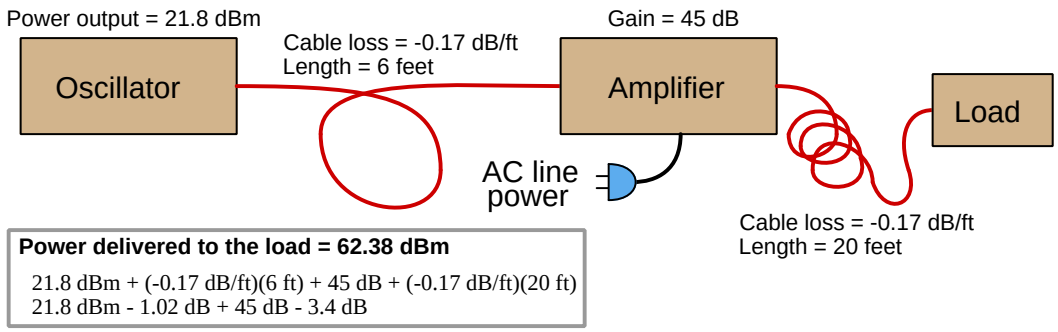
$$P_{in}(\text{dB}) + P_{gain}(\text{dB}) = P_{out}(\text{dB})$$

$$-13.9794 \text{ dBW} + 26.6276 \text{ dB} = 12.6482 \text{ dBW}$$

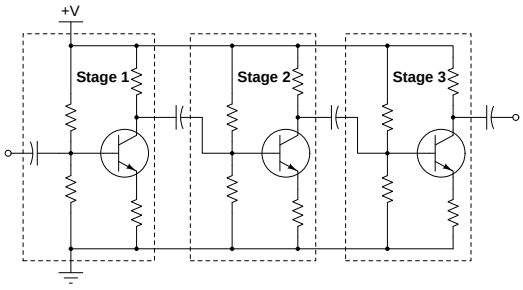
An electronic signal that begins 13.9794 dB less than 1 Watt, when boosted by an amplifier gain of 26.6276 dB, will become 12.6482 dB greater than the original reference power of 1 Watt.

This is one of the major benefits of using decibels to express powers: we may very easily calculate power gains and losses by summing a string of dB figures, each dB figure representing the power gain or power loss of a different system component. Normally, any compounding of *ratios* involves multiplication and/or division of those ratios, but with decibels we may simply add and subtract. One of the interesting mathematical properties of logarithms is that they "transform[3]" one type of problem into a simpler type: in this case, a problem of multiplying ratios into a (simpler) problem of adding decibel figures.

For example, we may express the power dissipated along a cable in terms of decibels per foot; the longer the cable, of course, the more power will be lost this way, all other factors being equal. For example, a radio-frequency signal cable having a loss figure of −0.15 decibels per foot at a signal frequency of 2.4 GHz will suffer −15 dB over 100 feet, and −150 dB over 1000 feet. To illustrate how decibels may be used to calculate power delivered to a load in such a system, accounting for various gains and losses along the way using decibel figures:



A similar application of decibels is found in multi-stage amplifier circuits, where one stage amplifies a signal to be fed into a successive stage to be amplified more. The power gains of these stages, each expressed as a ratio, *multiply* to make the over-all amplifier's power gain (ratio). The power gains of those same stages, each expressed as a decibel figure, *add* to make the over-all amplifier's power gain (dB):



[3]In fact, logarithms are one of the simplest examples of a *transform function*, converting one type of mathematical problem into another type. Other examples of mathematical transform functions used in engineering include the *Fourier transform* (converting a time-domain function into a frequency-domain function) and the *Laplace transform* (converting a differential equation into an algebraic equation).

Another common application of decibels is to express ratios of voltage and/or current rather than power. However, since the unit of the Bel has been defined as an expression of a *power* ratio, we cannot simply substitute V or I for P in any of the formulae we've seen so far.

Suppose an amplifier has a voltage gain of 2 (i.e. V_{out} is twice as large as V_{in}), and we would like to express this gain in decibels. Since decibels are intended to express power gain and not voltage gain, we must figure out how much power gain is equivalent to a voltage gain of two. Obviously, voltage and power are fundamentally different quantities, but if we imagine ourselves connecting a fixed load resistance to the input signal, and then to the output signal, we will realize that load's power dissipation will be more than double when energized by a voltage twice as large. Joule's Law is helpful to determine the exact ratio of power dissipation:

$$P = \frac{V^2}{R}$$

Doubling voltage for any given load resistance results in power *quadrupling* because power is proportional to the square of the voltage applied to a fixed resistance. Using this as the basis for applying decibels to a voltage ratio. Knowing that Joule's Law also declares power is proportional to the square of the current applied to a fixed resistance ($P = I^2R$) means this same mathematical relationship will apply to current gains and reductions as well as voltage gains and reductions:

$$\text{dB} = 10 \log \left(\frac{P_{out}}{P_{in}} \right) = 10 \log \left(\frac{V_{out}}{V_{in}} \right)^2 = 10 \log \left(\frac{I_{out}}{I_{in}} \right)^2$$

An algebraic identity of logarithms is that the logarithm of any quantity raised to a power is equal to that power multiplied by the logarithm of the quantity. Expressed in general terms:

$$\log x^y = y \log x$$

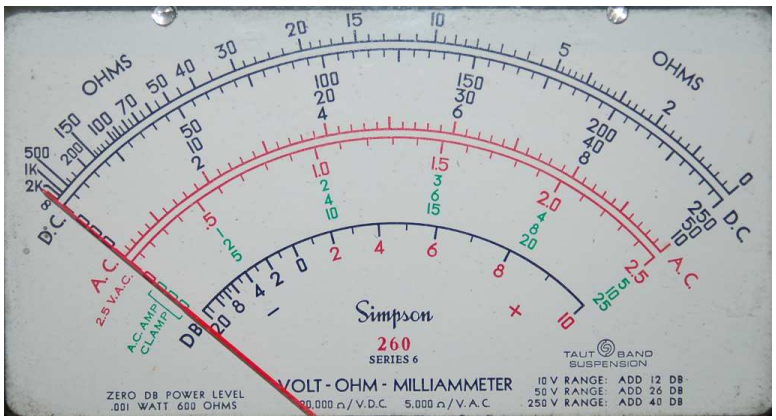
Therefore, we may simplify the decibel formula for voltage gain by removing the "2" power and making it a multiplier:

$$10 \log \left(\frac{V_{out}}{V_{in}} \right)^2 = (2)(10) \log \left(\frac{V_{out}}{V_{in}} \right) = 20 \log \left(\frac{V_{out}}{V_{in}} \right)$$

$$10 \log \left(\frac{I_{out}}{I_{in}} \right)^2 = (2)(10) \log \left(\frac{I_{out}}{I_{in}} \right) = 20 \log \left(\frac{I_{out}}{I_{in}} \right)$$

Thus, we may use decibels to express voltage or current ratios if we simply substitute 20 instead of 10 as the multiplier.

We can see the practicality of using decibels to represent something other than electrical *power* by examining this analog meter face, belonging to a Simpson model 260 VOM (Volt-Ohm-Milliammeter). Note the bottom scale on this meter's face, calibrated in decibels (DB):



Pay attention to the note on decibels written in the lower-left corner of the meter face, where 0 dB is defined as 0.001 Watt dissipated by 600 Ohms. The fact that 0 dB is defined as 1 milliWatt means it should (properly) be labeled dBm rather than dB[4]. A load resistance value is necessary as part of this definition for dB because this meter cannot measure power directly but must infer signal power from measurements of AC *voltage*. Without a specific load resistance, there is no clear relation between voltage and power. 600 Ohms is an old telecommunications standard for audio-frequency AC signals, and continues to be used today for voltage-based decibel measurements of audio-frequency AC signals.

The meter as shown is connected to nothing at all, and so registers 0 Volts AC. This, of course, corresponds to zero power, and it has no corresponding decibel value because the logarithm of zero is mathematically *undefined*[5]. Practically, it means $-\infty$ dB, which is why the needle at the 0 Volt position "falls off" the left-hand end of the dB scale.

Close inspection of the dB scale on this meter face reveals another interesting property of decibels, and that is the *nonlinear* nature of the dB scale. This contrasts starkly against all the voltage and current scales on this meter face which are linear. This nonlinearity is a fundamental property of decibels because it is based on the *logarithm* function which is nonlinear.

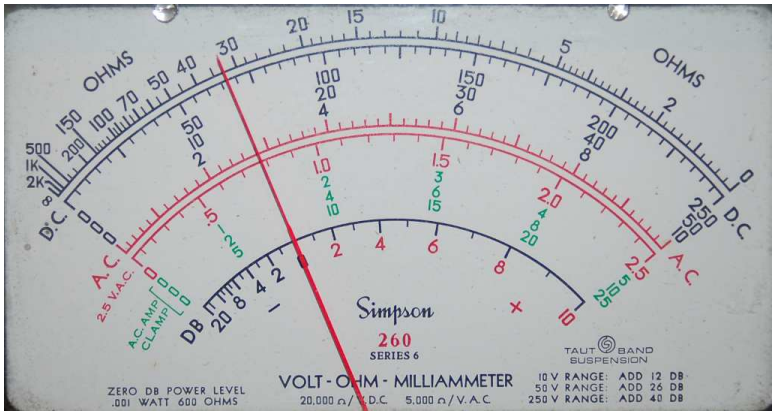
[4] Such mis-labeling is not that uncommon in the profession, the expectation being that the technician or engineer working with the instrument ought to be familiar enough with the concept of decibels to know when dB really means dBm, or dBW, etc.

[5] Your electronic calculator will complain if you attempt to take the logarithm of zero!

Now, we will explore what is necessary to make this meter register 0 dBm (i.e. 1 milliWatt) with an applied AC voltage. 1 milliWatt of power dissipated by 600 Ohms is equivalent to:

$$V = \sqrt{PR} = \sqrt{(0.001)(600)} = 0.7746 \text{ Volts}$$

Setting the VOM to the 2.5 VAC range and applying just enough AC voltage to bring the needle to the 0 dB mark allows us to verify that this is indeed equivalent to just under 0.8 Volts (read on the 2.5 VAC scale):



In the lower-right corner of the meter face we see some notes regarding correction values for decibel measurements when using different AC voltage ranges. The dB scale is read directly when the meter is set on the 2.5 VAC range. When set on the 10 VAC range (i.e. a range *four times* as great), the meter's needle will experience a deflection one-fourth as much as when set to the 2.5 VAC range, and therefore it will point to a lesser (or even negative) value on the dB scale. Converting a voltage ratio of 0.25 into a decibel figure shows us how much less the needle will register on the dB scale when the voltage range is quadrupled:

$$20 \log \left(\frac{2.5}{10} \right) = -12.04 \text{ dB}$$

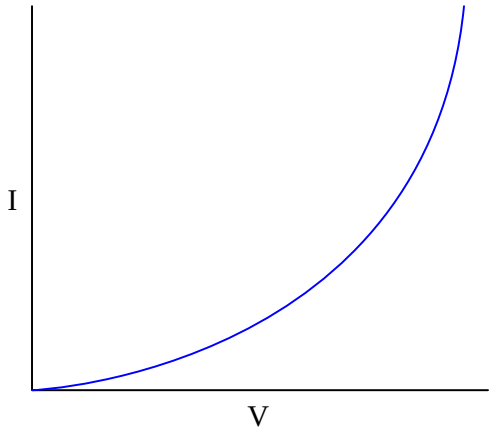
Therefore, when using the 10 VAC range instead of the 2.5 VAC range, one must add 12 dB to the reading. Likewise, we may prove each of the printed correction factors for the alternative voltage-measurement ranges listed (50 Volt AC range and 250 Volt AC range):

$$20 \log \left(\frac{2.5}{50} \right) = -26.02 \text{ dB}$$

$$20 \log \left(\frac{2.5}{250} \right) = -40.0 \text{ dB}$$

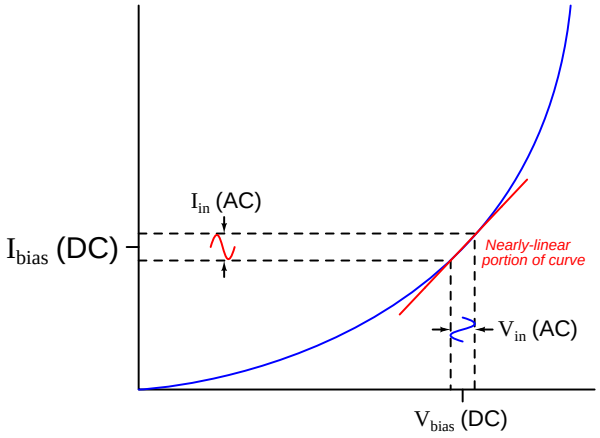
4.2 Derivation of dynamic emitter resistance

The Shockley diode equation gives the relationship of PN junction current to PN junction voltage drop. When graphed, it yields a curve resembling this:



This function is clearly non-linear, and so we know Ohm's Law in its familiar form ($V = IR$) does not apply to a semiconductor diode junction. Another way of saying this is that a PN diode junction does not possess a fixed amount of resistance R.

However, if we know that the amount of AC voltage applied to the junction is small compared to the amount of DC (bias) voltage applied to the junction, we may regard the short section of curve encompassing the biased AC voltage range as being essentially linear, and thus having an (approximately) fixed resistance value within that narrow range of operation:



The linearized slope of any curved mathematical function is known as the *derivative* of that function, and this is one of the fundamental principles of calculus. If we apply the calculus process of "differentiation" to the Shockley diode equation, we will generate a new equation describing the slope of the curve at any given value or DC bias. If we arrange this "derivative" equation to express

change in voltage divided by change in current ($\frac{dV}{dI}$) we will have our equation for computing the *dynamic resistance* (r') of a PN junction.

First, we will review the Shockley diode equation:

$$I = I_S \left(e^{\frac{qV}{nKT}} - 1 \right)$$

Where,
I = Forward-bias current through the diode, Amperes
I_S = Reverse-bias saturation current[6] through the diode, Amperes
e = Euler's constant (≈ 2.71828)
V = Voltage applied to the PN junction externally, Volts
q = Elementary charge of an electron (1.602×10^{-19} Coulombs)
n = Ideality factor (1 for a perfect junction)
k = Boltzmann's constant ($1.3806504 \times 10^{-23}$ J / K)
T = Absolute temperature (Kelvin), 273.15 more than degrees Celsius

For most practical values of applied voltage, the term $e^{\frac{qV}{nKT}}$ is very much larger than one, making $e^{\frac{qV}{nKT}} - 1$ very nearly equal to $e^{\frac{qV}{nKT}}$ and so we may simplify the diode equation to the following form:

$$I \approx I_S \left(e^{\frac{qV}{nKT}} \right)$$

This equation form just happens to be very simple to differentiate, as the derivative of an exponential function of Euler's number (e) is that same function. In other words:

$$\frac{d}{dx}(e^x) = e^x$$

The elementary exponential function is the only function for which its derivative is identical. Variations of exponential functions are similarly easy to differentiate. For example, a constant multiplied by the exponent becomes a multiplying coefficient in the derivative:

$$\frac{d}{dx}(e^{xy}) = ye^{xy}$$

[6] A very small amount of current will still flow in the reverse-biased condition, due to so-called *minority carriers* in the P and N halves of the diode. This tiny current, usually in the range of *nano-Amperes* is referred to as the *reverse saturation current* because its value does not increase appreciably with greater reverse-bias voltage but rather "saturates" or "plateaus" at a constant value. This saturation current, while fairly independent of applied voltage, varies greatly with changes in device *temperature*.

Armed with these calculus tools, let's differentiate our simplified version of the diode equation with respect to voltage:

$$\frac{d}{dV}I \approx \frac{d}{dV}\left(I_S e^{\frac{qV}{nKT}}\right)$$

The complicated exponent $\frac{qV}{nKT}$ may be thought of as our variable of interest (V) multiplied by a collection of constants ($\frac{q}{nKT}$). Applying our rule of the exponent's multiplying coefficient becoming a multiplying coefficient for the derivative function:

$$\frac{dI}{dV} \approx \frac{q}{nKT} I_S e^{\frac{qV}{nKT}}$$

Recall our simplified version of the diode equation ($I \approx I_S \left(e^{\frac{qV}{nKT}}\right)$), where junction current is approximately equal to the product of saturation current and the exponential function. Knowing this (near) equality, we may simplify our derivative by substituting I for $I_S e^{\frac{qV}{nKT}}$:

$$\frac{dI}{dV} \approx \frac{qI}{nKT}$$

What we're trying to determine here is dynamic resistance (r'), which is the ratio $\frac{dV}{dI}$. So, we need to invert both sides of this equation:

$$r' = \frac{dV}{dI} \approx \frac{nKT}{qI}$$

n, K, and q are all constants, and T of course is the temperature of the PN junction which is usually fairly stable. If we assume room temperature for the junction (usually considered to be 20 ^{o}C or 293.15 K), we may simplify the equation even further, down to a function of one independent variable (I):

$$r' = \frac{dV}{dI} \approx \frac{(1)(1.380 \times 10^{-23})(293.15)}{(1.602 \times 10^{-19})I}$$

$$r' = \frac{dV}{dI} \approx \frac{25.2397 \times 10^{-3}}{I}$$

And there we have it: the approximate dynamic resistance for an ideal PN junction as a function of its DC (quiescent) current I. At a forward DC current of 1 Ampere DC, this dynamic resistance will be approximately 25 milliOhms; at a forward DC current of 1 milliAmpere DC, dynamic resistance will be about 25 Ohms. This trend makes sense when we examine the diode equation's exponential curve: less current follows the curve down and to the left, in a region where smaller variations in current occur for a given variation in voltage (i.e. more dynamic resistance).

To put this into practical context, assume our diode junction is forward-biased with 10 mA of DC current. The equation tells us the dynamic resistance of that junction carrying 10 mA at room temperature will be approximately 2.5 Ω. If we superimpose an AC current of 0.3 mA onto the 10 mA DC bias, the PN junction will drop an AC voltage of 0.75 mV:

$$V = Ir' = (0.3 \text{ mA})(2.5\,\Omega) = 0.75 \text{ mV}$$

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Chapter 5

Questions

This learning module, along with all others in the ModEL collection, is designed to be used in an inverted instructional environment where students independently read[1] the tutorials and attempt to answer questions on their own *prior* to the instructor's interaction with them. In place of lecture[2], the instructor engages with students in Socratic-style dialogue, probing and challenging their understanding of the subject matter through inquiry.

Answers are not provided for questions within this chapter, and this is by design. Solved problems may be found in the Tutorial and Derivation chapters, instead. The goal here is *independence*, and this requires students to be challenged in ways where others cannot think for them. Remember that you always have the tools of *experimentation* and *computer simulation* (e.g. SPICE) to explore concepts!

The following lists contain ideas for Socratic-style questions and challenges. Upon inspection, one will notice a strong theme of *metacognition* within these statements: they are designed to foster a regular habit of examining one's own thoughts as a means toward clearer thinking. As such these sample questions are useful both for instructor-led discussions as well as for self-study.

[1] Technical reading is an essential academic skill for any technical practitioner to possess for the simple reason that the most comprehensive, accurate, and useful information to be found for developing technical competence is in textual form. Technical careers in general are characterized by the need for continuous learning to remain current with standards and technology, and therefore any technical practitioner who cannot read well is handicapped in their professional development. An excellent resource for educators on improving students' reading prowess through intentional effort and strategy is the book textitReading For Understanding – How Reading Apprenticeship Improves Disciplinary Learning in Secondary and College Classrooms by Ruth Schoenbach, Cynthia Greenleaf, and Lynn Murphy.

[2] Lecture is popular as a teaching method because it is easy to implement: any reasonably articulate subject matter expert can talk to students, even with little preparation. However, it is also quite problematic. A good lecture always makes complicated concepts seem easier than they are, which is bad for students because it instills a false sense of confidence in their own understanding; reading and re-articulation requires more cognitive effort and serves to verify comprehension. A culture of teaching-by-lecture fosters a debilitating dependence upon direct personal instruction, whereas the challenges of modern life demand independent and critical thought made possible only by gathering information and perspectives from afar. Information presented in a lecture is ephemeral, easily lost to failures of memory and dictation; text is forever, and may be referenced at any time.

General challenges following tutorial reading

- Summarize as much of the text as you can in one paragraph of your own words. A helpful strategy is to explain ideas as you would for an intelligent child: as simple as you can without compromising too much accuracy.
- Simplify a particular section of the text, for example a paragraph or even a single sentence, so as to capture the same fundamental idea in fewer words.
- Where did the text make the most sense to you? What was it about the text's presentation that made it clear?
- Identify where it might be easy for someone to misunderstand the text, and explain why you think it could be confusing.
- Identify any new concept(s) presented in the text, and explain in your own words.
- Identify any familiar concept(s) such as physical laws or principles applied or referenced in the text.
- Devise a proof of concept experiment demonstrating an important principle, physical law, or technical innovation represented in the text.
- Devise an experiment to disprove a plausible misconception.
- Did the text reveal any misconceptions you might have harbored? If so, describe the misconception(s) and the reason(s) why you now know them to be incorrect.
- Describe any useful problem-solving strategies applied in the text.
- Devise a question of your own to challenge a reader's comprehension of the text.

General follow-up challenges for assigned problems

- Identify where any fundamental laws or principles apply to the solution of this problem, especially before applying any mathematical techniques.
- Devise a thought experiment to explore the characteristics of the problem scenario, applying known laws and principles to mentally model its behavior.
- Describe in detail your own strategy for solving this problem. How did you identify and organized the given information? Did you sketch any diagrams to help frame the problem?
- Is there more than one way to solve this problem? Which method seems best to you?
- Show the work you did in solving this problem, even if the solution is incomplete or incorrect.
- What would you say was the most challenging part of this problem, and why was it so?
- Was any important information missing from the problem which you had to research or recall?
- Was there any extraneous information presented within this problem? If so, what was it and why did it not matter?
- Examine someone else's solution to identify where they applied fundamental laws or principles.
- Simplify the problem from its given form and show how to solve this simpler version of it. Examples include eliminating certain variables or conditions, altering values to simpler (usually whole) numbers, applying a limiting case (i.e. altering a variable to some extreme or ultimate value).
- For quantitative problems, identify the real-world meaning of all intermediate calculations: their units of measurement, where they fit into the scenario at hand. Annotate any diagrams or illustrations with these calculated values.
- For quantitative problems, try approaching it qualitatively instead, thinking in terms of "increase" and "decrease" rather than definite values.
- For qualitative problems, try approaching it quantitatively instead, proposing simple numerical values for the variables.
- Were there any assumptions you made while solving this problem? Would your solution change if one of those assumptions were altered?
- Identify where it would be easy for someone to go astray in attempting to solve this problem.
- Formulate your own problem based on what you learned solving this one.

General follow-up challenges for experiments or projects

- In what way(s) was this experiment or project easy to complete?
- Identify some of the challenges you faced in completing this experiment or project.

- Show how thorough documentation assisted in the completion of this experiment or project.
- Which fundamental laws or principles are key to this system's function?
- Identify any way(s) in which one might obtain false or otherwise misleading measurements from test equipment in this system.
- What will happen if (component X) fails (open/shorted/etc.)?
- What would have to occur to make this system unsafe?

5.1 Conceptual reasoning

These questions are designed to stimulate your analytic and synthetic thinking[3]. In a Socratic discussion with your instructor, the goal is for these questions to prompt an extended dialogue where assumptions are revealed, conclusions are tested, and understanding is sharpened. Your instructor may also pose additional questions based on those assigned, in order to further probe and refine your conceptual understanding.

Questions that follow are presented to challenge and probe your understanding of various concepts presented in the tutorial. These questions are intended to serve as a guide for the Socratic dialogue between yourself and the instructor. Your instructor's task is to ensure you have a sound grasp of these concepts, and the questions contained in this document are merely a means to this end. Your instructor may, at his or her discretion, alter or substitute questions for the benefit of tailoring the discussion to each student's needs. The only absolute requirement is that each student is challenged and assessed at a level equal to or greater than that represented by the documented questions.

It is far more important that you convey your *reasoning* than it is to simply convey a correct answer. For this reason, you should refrain from researching other information sources to answer questions. What matters here is that *you* are doing the thinking. If the answer is incorrect, your instructor will work with you to correct it through proper reasoning. A correct answer without an adequate explanation of how you derived that answer is unacceptable, as it does not aid the learning or assessment process.

You will note a conspicuous lack of answers given for these conceptual questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your conceptual answers, where applicable, is to use circuit simulation software to explore the effects of changes made to circuits. For example, if one of these conceptual questions challenges you to predict the effects of altering some component parameter in a circuit, you may check the validity of your work by simulating that same parameter change within software and seeing if the results agree.

[3] *Analytical* thinking involves the "disassembly" of an idea into its constituent parts, analogous to dissection. *Synthetic* thinking involves the "assembly" of a new idea comprised of multiple concepts, analogous to construction. Both activities are high-level cognitive skills, extremely important for effective problem-solving, necessitating frequent challenge and regular practice to fully develop.

5.1.1 Reading outline and reflections

"Reading maketh a full man; conference a ready man; and writing an exact man" – Francis Bacon

Francis Bacon's advice is a blueprint for effective education: reading provides the learner with knowledge, writing focuses the learner's thoughts, and critical dialogue equips the learner to confidently communicate and apply their learning. Independent acquisition and application of knowledge is a powerful skill, well worth the effort to cultivate. To this end, students should read these educational resources closely, journal their own reflections on the reading, and discuss in detail their findings with classmates and instructor(s). You should be able to do all of the following after reading any instructional text:

☑ Briefly SUMMARIZE THE TEXT in the form of a journal entry documenting your learning as you progress through the course of study. Share this summary in dialogue with your classmates and instructor. Journaling is an excellent self-test of thorough reading because you cannot clearly express what you have not read or did not comprehend.

☑ Demonstrate ACTIVE READING STRATEGIES, including verbalizing your impressions as you read, simplifying long passages to convey the same ideas using fewer words, annotating text and illustrations with your own interpretations, working through mathematical examples shown in the text, cross-referencing passages with relevant illustrations and/or other passages, identifying problem-solving strategies applied by the author, etc. Technical reading is a special case of problem-solving, and so these strategies work precisely because they help solve any problem: paying attention to your own thoughts (metacognition), eliminating unnecessary complexities, identifying what makes sense, paying close attention to details, drawing connections between separated facts, and noting the successful strategies of others.

☑ Identify IMPORTANT THEMES, especially GENERAL LAWS and PRINCIPLES, expounded in the text and express them in the simplest of terms as though you were teaching an intelligent child. This emphasizes connections between related topics and develops your ability to communicate complex ideas to anyone.

☑ Form YOUR OWN QUESTIONS based on the reading, and then pose them to your instructor and classmates for their consideration. Anticipate both correct and incorrect answers, the incorrect answer(s) assuming one or more plausible misconceptions. This helps you view the subject from different perspectives to grasp it more fully.

☑ Devise EXPERIMENTS to test claims presented in the reading, or to disprove misconceptions. Predict possible outcomes of these experiments, and evaluate their meanings: what result(s) would confirm, and what would constitute disproof? Running mental simulations and evaluating results is essential to scientific and diagnostic reasoning.

☑ Specifically identify any points you found CONFUSING. The reason for doing this is to help diagnose misconceptions and overcome barriers to learning.

5.1.2 Foundational concepts

Correct analysis and diagnosis of electric circuits begins with a proper understanding of some basic concepts. The following is a list of some important concepts referenced in this module's full tutorial. Define each of them in your own words, and be prepared to illustrate each of these concepts with a description of a practical example and/or a live demonstration.

Energy

Conservation of Energy

Cut-and-try problem-solving strategy

Amplification

Electrical source

Electrical load

Gain

Biasing

BJT principles

Beta

Superposition

Kirchhoff's Voltage Law

Kirchhoff's Current Law

Quiescent

Shockley diode equation

Swamping

Reactance

Filter

Phase shift

Thévenin's theorem

Norton's theorem

Voltage source

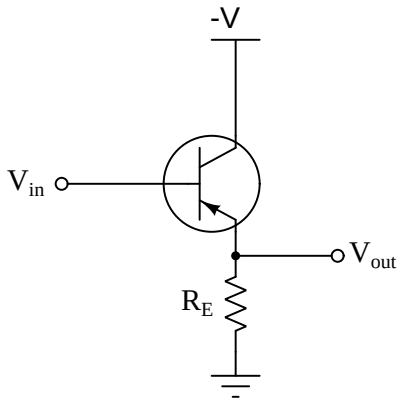
Current source

Impedance

5.1.3 Common-collector voltage gain

The following schematic diagram shows a simple *common-collector* transistor amplifier circuit:

Common-collector amplifier



Explain why the AC voltage gain ($A_{V(AC)}$) of such an amplifier is approximately 1, using any or all of the following principles:

- $I_E = I_C + I_B$
- $I_E \approx I_C$
- $V_{BE} \approx 0.7$ Volts
- $\beta = \frac{I_C}{I_B}$
- Kirchhoff's Voltage Law
- Kirchhoff's Current Law
- Superposition Theorem

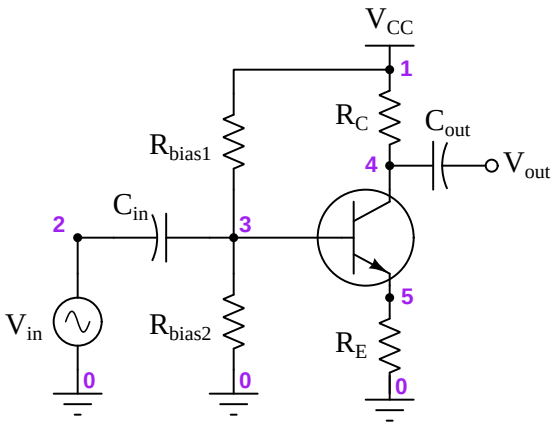
Remember that AC voltage gain is defined as $\frac{\Delta V_{out}}{\Delta V_{in}}$.

Challenges

- Identify the type of transistor being used in this circuit.

5.1.4 Altering common-emitter resistor values

Determine what will happen to the voltage gain of a common-emitter transistor amplifier circuit if the following resistance values change (consider one change at a time):



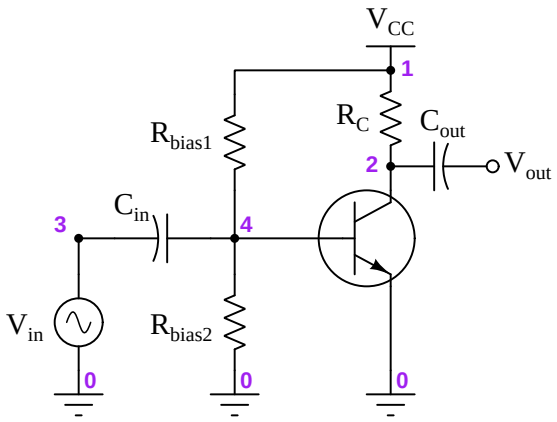
- Resistance R_C increase; A_V . . .
- Resistance R_E increase; A_V . . .
- Resistance R_{bias1} increase; A_V . . .
- Resistance R_{bias2} increase; A_V . . .

Challenges

- Is the orientation of the two polarized capacitors proper? How may we tell?
- Modify this amplifier circuit to work with a PNP transistor instead of the NPN (shown).
- Is r'_E affected by any of these changes?

5.1.5 Impossible gain value

A student attempts to calculate the voltage gain of the following common-emitter amplifier circuit, and arrives at an incalculable value (divide-by-zero error):



Identify the student's error.

Challenges

- Explain what one would have to know in order to calculate this amplifier's voltage gain.

5.1.6 Empirical voltage gain determination

Explain how you could *measure* the AC voltage gain of a functioning transistor amplifier circuit, as opposed to *predicting* its gain from known component values.

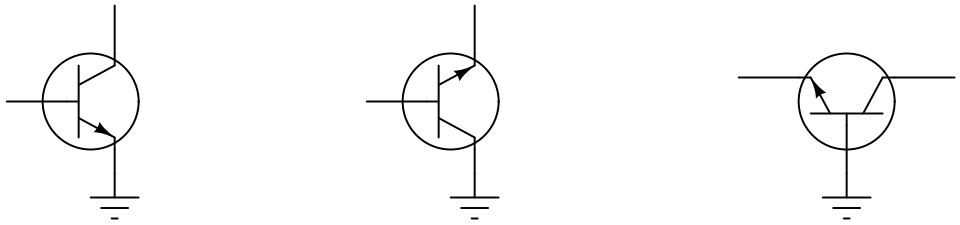
Challenges

- Identify at least one error that could be made in such a test to yield an incorrect gain value.

5.1.7 Sketching amplifier configurations

Common-emitter, common-collector, and common-base amplifier circuits are sometimes referred to as *grounded-emitter*, *grounded-collector*, and *grounded-base*, respectively, because these configurations may actually be built with those respective terminals connected straight to ground.

Although this may not be very practical for ease of biasing, it can be done. Draw the rest of the circuit necessary to provide class-A operation for each of these (partial) transistor circuits. Be sure to show where the DC power source, signal input, and signal output connect:

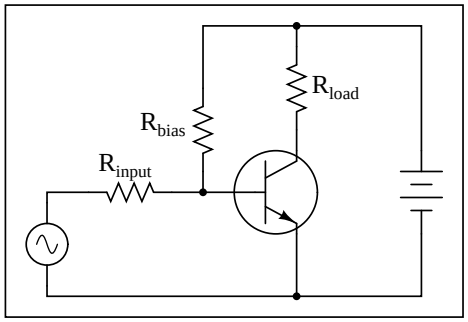


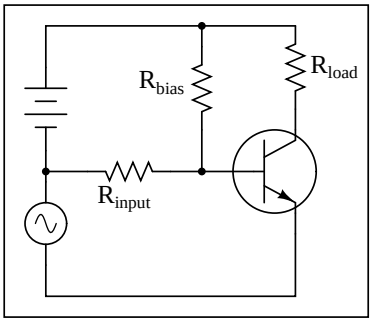
Challenges

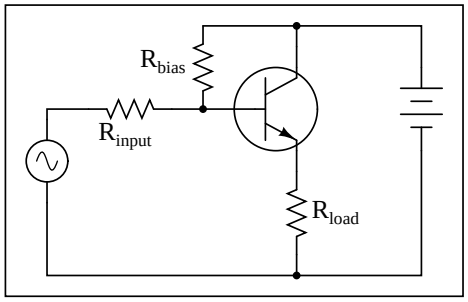
- Identify which of these configurations is able to yield a voltage gain greater than 0 dB.
- Identify what else must change in the circuit if these NPN transistors are exchanged for PNP.

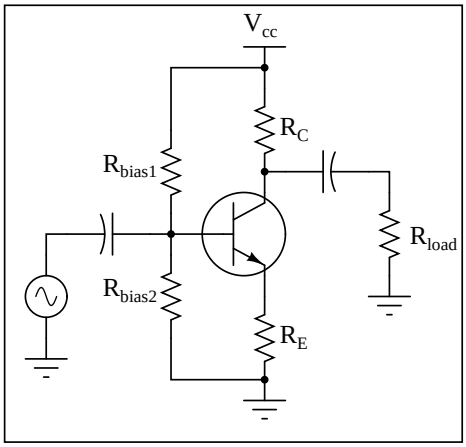
5.1.8 Identifying amplifier configurations

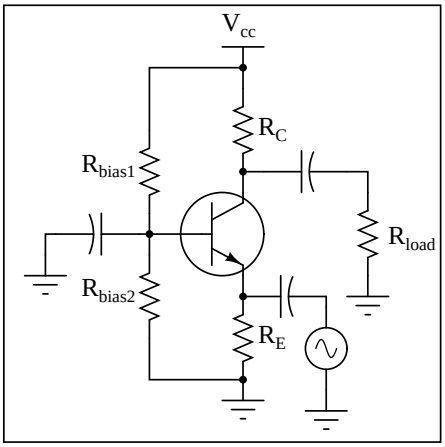
Identify the type of transistor amplifier configuration in these schematic diagrams as either *common-emitter*, *common-collector*, or *common-base*.

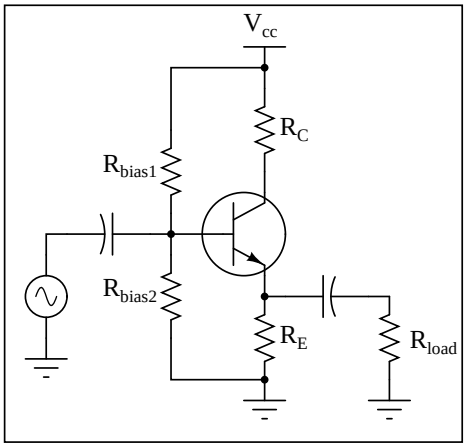










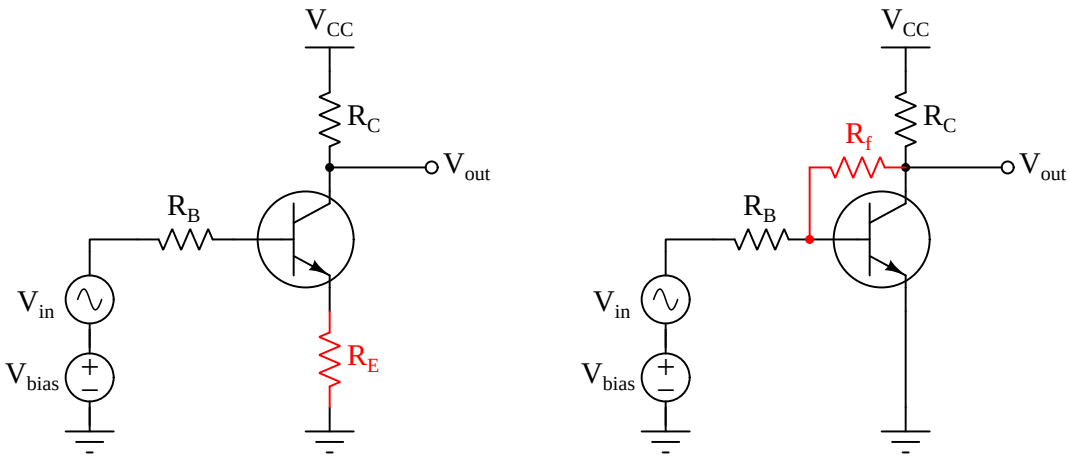


Challenges

- Identify rules by which one may determine the type of "common-" amplifier configuration from a schematic diagram.

5.1.9 Negative feedback for common-emitter amplifiers

Two simple methods of incorporating negative (i.e. "degenerative") feedback into common-emitter amplifier circuits are shown below:



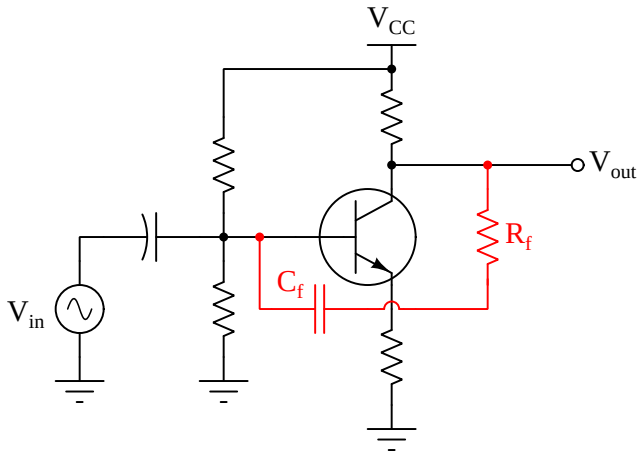
Explain what the term *negative* (or *degenerative*) means with reference to feedback, and explain how each of these techniques works to produce this type of feedback. Also, explain one *disadvantage* of applying negative feedback to a common-emitter amplifier circuit.

Challenges

- Suppose each of the colored resistors were exchanged for *inductors* – how would each amplifier's performance be altered?
- Identify where a "bypass" capacitor would be connected in the left-hand amplifier circuit, and explain how the addition of that capacitor would affect the circuit's voltage gain.

5.1.10 RC feedback network

Sometimes a *feedback network* is purposely placed in an amplifier circuit, like the R_f-C_f combination shown in the following schematic:



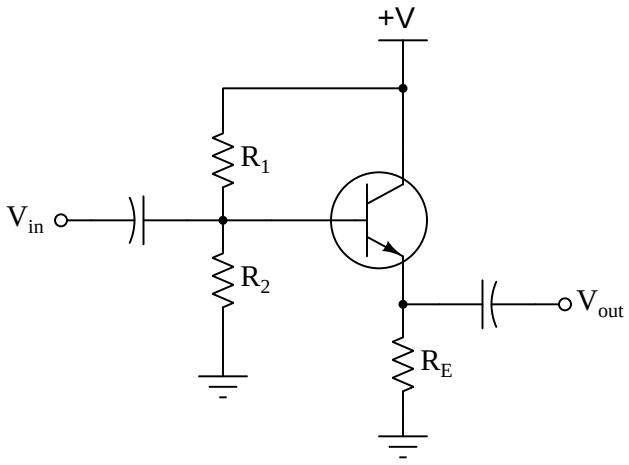
Contrast the performance of this feedback network against that of a plain resistor-only feedback network in the same type of amplifier circuit.

Challenges

- Which way should the capacitor be installed if it were polarized (e.g. electrolytic)?

5.1.11 Bootstrapping

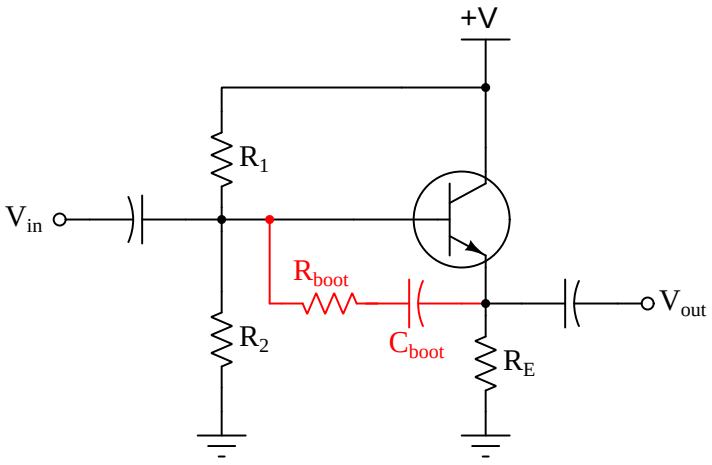
In a common-collector transistor amplifier circuit with voltage divider biasing, the input impedance (Z_{in}) is a function of load impedance, emitter resistance (R_E), and the two biasing resistances (R_1 and R_2). Often, the biasing resistances are of sufficiently low value to swamp the input impedance of the transistor, so that R_1 and R_2 constitute the heaviest load for any input signals driving the amplifier.



$$Z_{in} \approx R_1 \,||\, R_2 \,||\, (\beta + 1)[r'_E + (R_E \,||\, R_{load})]$$

This is a shame, because the only practical purpose served by R_1 and R_2 is to provide a stable bias voltage so the transistor always functions in class A mode. In order to provide a stable bias, these resistors have to be relatively low in value compared to the impedance seen at the base of the transistor (resulting from the load). Otherwise, changes in dynamic emitter resistance (r'_E) could result in significant bias shifts. So, the naturally high input impedance of the common-collector transistor configuration is spoiled by the necessary presence of R_1 and R_2.

A clever way to recover some of that naturally large input impedance is to add a bit of *regenerative* (positive) feedback to the circuit in the form of a capacitor and another resistor. This technique is given an equally clever name: *bootstrapping*.



Explain how bootstrapping works, and why that particular name is given to the technique.

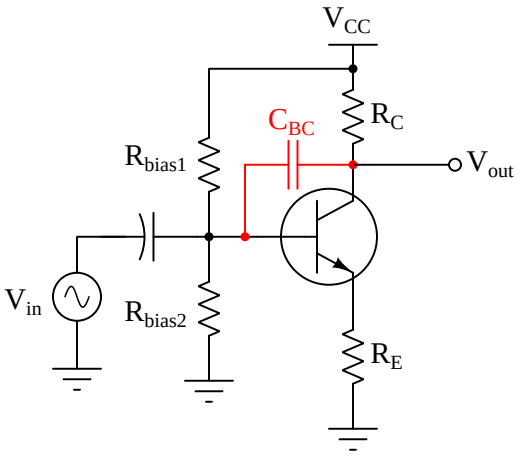
Challenges

- Bootstrapping works well for common-collector amplifiers because the natural voltage gain of such amplifiers is slightly less than 1 (i.e. 0 dB). Explain why this is important, and what undesirable effects might occur if bootstrapping were applied to amplifiers having larger gain values.

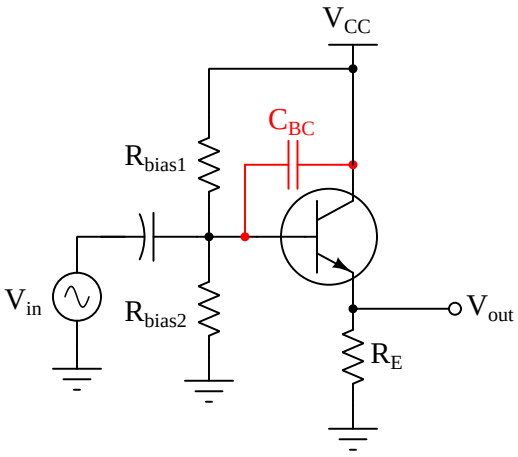
5.1.12 Base-collector capacitance

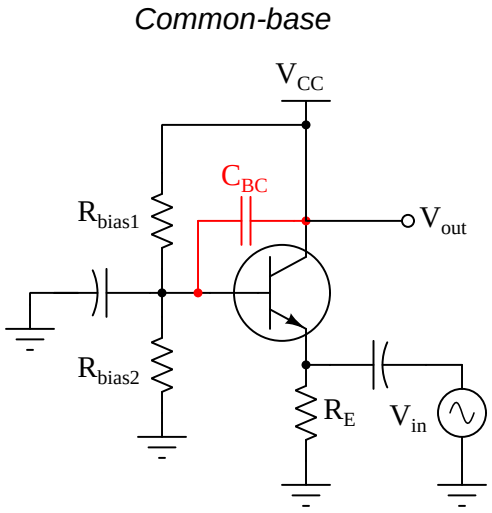
The base-collector PN junction of a bipolar transistor operates in reverse-bias while the transistor functions as an analog amplifying device, and there exists a certain amount of capacitance (C_{BC}) across the resulting base-collector depletion region. The BJT amplifier configuration most affected by this parasitic capacitance at high frequencies is the *common-emitter*. Common-collector and common-base amplifier configurations are not affected as much. After examining the following amplifier circuits (with this parasitic capacitance shown external to the transistors), explain why:

Common-emitter



Common-collector





Challenges

- What kind of signal feedback occurs through C_{BC}, *regenerative* or *degenerative* feedback?

5.1.13 Miller effect

The parasitic capacitance across a BJT's base-collector PN junction (C_{BC}) is often multiplied by $\beta + 1$ and called the *Miller capacitance* of that transistor:

$$C_{Miller} = C_{BC} \left(\beta + 1\right)$$

Why is this? What purpose does it serve to include the transistor's gain into the calculation, rather than just expressing the junction capacitance as it is?

Challenges

- Is the $\beta + 1$ factor an advantage, or a disadvantage, to the operation of the amplifier circuit?

5.2 Quantitative reasoning

These questions are designed to stimulate your computational thinking. In a Socratic discussion with your instructor, the goal is for these questions to reveal your mathematical approach(es) to problem-solving so that good technique and sound reasoning may be reinforced. Your instructor may also pose additional questions based on those assigned, in order to observe your problem-solving firsthand.

Mental arithmetic and estimations are strongly encouraged for all calculations, because without these abilities you will be unable to readily detect errors caused by calculator misuse (e.g. keystroke errors).

You will note a conspicuous lack of answers given for these quantitative questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. My advice is to use circuit simulation software such as SPICE to check the correctness of quantitative answers. Refer to those learning modules within this collection focusing on SPICE to see worked examples which you may use directly as practice problems for your own study, and/or as templates you may modify to run your own analyses and generate your own practice problems.

Completely worked example problems found in the Tutorial may also serve as "test cases[4]" for gaining proficiency in the use of circuit simulation software, and then once that proficiency is gained you will never need to rely[5] on an answer key!

[4] In other words, set up the circuit simulation software to analyze the same circuit examples found in the Tutorial. If the simulated results match the answers shown in the Tutorial, it confirms the simulation has properly run. If the simulated results disagree with the Tutorial's answers, something has been set up incorrectly in the simulation software. Using every Tutorial as practice in this way will quickly develop proficiency in the use of circuit simulation software.

[5] This approach is perfectly in keeping with the instructional philosophy of these learning modules: *teaching students to be self-sufficient thinkers*. Answer keys can be useful, but it is even more useful to your long-term success to have a set of tools on hand for checking your own work, because once you have left school and are on your own, there will no longer be "answer keys" available for the problems you will have to solve.

5.2.1 Miscellaneous physical constants

Note: constants shown in **bold** type are *exact*, not approximations. Values inside of parentheses show one standard deviation (σ) of uncertainty in the final digits: for example, the magnetic permeability of free space value given as $1.25663706212(19) \times 10^{-6}$ H/m represents a center value (i.e. the location parameter) of $1.25663706212 \times 10^{-6}$ Henrys per meter with one standard deviation of uncertainty equal to $0.0000000000019 \times 10^{-6}$ Henrys per meter.

Avogadro's number (N_A) = **6.02214076** $\times 10^{23}$ **per mole** (mol^{-1})

Boltzmann's constant (k) = **1.380649** $\times 10^{-23}$ **Joules per Kelvin** (J/K)

Electronic charge (e) = **1.602176634** $\times 10^{-19}$ **Coulomb** (C)

Faraday constant (F) = **96,485.33212...** $\times 10^{4}$ **Coulombs per mole** (C/mol)

Magnetic permeability of free space (μ_0) = $1.25663706212(19) \times 10^{-6}$ Henrys per meter (H/m)

Electric permittivity of free space (ϵ_0) = $8.8541878128(13) \times 10^{-12}$ Farads per meter (F/m)

Characteristic impedance of free space (Z_0) = 376.730313668(57) Ohms (Ω)

Gravitational constant (G) = $6.67430(15) \times 10^{-11}$ cubic meters per kilogram-seconds squared ($m^3/kg\text{-}s^2$)

Molar gas constant (R) = **8.314462618... Joules per mole-Kelvin** (J/mol-K) = 0.08205746(14) liters-atmospheres per mole-Kelvin

Planck constant (h) = **6.62607015** $\times 10^{-34}$ **joule-seconds** (J-s)

Stefan-Boltzmann constant (σ) = **5.670374419...** $\times 10^{-8}$ **Watts per square meter-Kelvin**^{4} ($W/m^2 \cdot K^4$)

Speed of light in a vacuum (c) = **299,792,458 meters per second** (m/s) = 186282.4 miles per second (mi/s)

Note: All constants taken from NIST data "Fundamental Physical Constants – Complete Listing", from `http://physics.nist.gov/constants`, National Institute of Standards and Technology (NIST), 2018 CODATA Adjustment.

5.2.2 Introduction to spreadsheets

A powerful computational tool you are encouraged to use in your work is a *spreadsheet*. Available on most personal computers (e.g. Microsoft Excel), *spreadsheet* software performs numerical calculations based on number values and formulae entered into cells of a grid. This grid is typically arranged as lettered columns and numbered rows, with each cell of the grid identified by its column/row coordinates (e.g. cell `B3`, cell `A8`). Each cell may contain a string of text, a number value, or a mathematical formula. The spreadsheet automatically updates the results of all mathematical formulae whenever the entered number values are changed. This means it is possible to set up a spreadsheet to perform a series of calculations on entered data, and those calculations will be re-done by the computer any time the data points are edited in any way.

For example, the following spreadsheet calculates average speed based on entered values of distance traveled and time elapsed:

	A	B	C	D
1	Distance traveled	46.9	Kilometers	
2	Time elapsed	1.18	Hours	
3	Average speed	= B1 / B2	km/h	
4				
5				

Text labels contained in cells `A1` through `A3` and cells `C1` through `C3` exist solely for readability and are not involved in any calculations. Cell `B1` contains a sample distance value while cell `B2` contains a sample time value. The formula for computing speed is contained in cell `B3`. Note how this formula begins with an "equals" symbol (`=`), references the values for distance and speed by lettered column and numbered row coordinates (`B1` and `B2`), and uses a forward slash symbol for division (`/`). The coordinates `B1` and `B2` function as *variables*[6] would in an algebraic formula.

When this spreadsheet is executed, the numerical value `39.74576` will appear in cell `B3` rather than the formula `= B1 / B2`, because 39.74576 is the computed speed value given 46.9 kilometers traveled over a period of 1.18 hours. If a different numerical value for distance is entered into cell `B1` or a different value for time is entered into cell `B2`, cell `B3`'s value will automatically update. All you need to do is set up the given values and any formulae into the spreadsheet, and the computer will do all the calculations for you.

Cell `B3` may be referenced by other formulae in the spreadsheet if desired, since it is a variable just like the given values contained in `B1` and `B2`. This means it is possible to set up an entire chain of calculations, one dependent on the result of another, in order to arrive at a final value. The arrangement of the given data and formulae need not follow any pattern on the grid, which means you may place them anywhere.

[6] Spreadsheets may also provide means to attach text labels to cells for use as variable names (Microsoft Excel simply calls these labels "names"), but for simple spreadsheets such as those shown here it's usually easier just to use the standard coordinate naming for each cell.

Common[7] arithmetic operations available for your use in a spreadsheet include the following:

- Addition (`+`)
- Subtraction (`-`)
- Multiplication (`*`)
- Division (`/`)
- Powers (`^`)
- Square roots (`sqrt()`)
- Logarithms (`ln()` , `log10()`)

Parentheses may be used to ensure[8] proper order of operations within a complex formula. Consider this example of a spreadsheet implementing the *quadratic formula*, used to solve for roots of a polynomial expression in the form of $ax^2 + bx + c$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

	A	B
1	x_1	= (-B4 + sqrt((B4^2) - (4*B3*B5))) / (2*B3)
2	x_2	= (-B4 - sqrt((B4^2) - (4*B3*B5))) / (2*B3)
3	a =	9
4	b =	5
5	c =	-2

This example is configured to compute roots[9] of the polynomial $9x^2 + 5x - 2$ because the values of 9, 5, and -2 have been inserted into cells `B3`, `B4`, and `B5`, respectively. Once this spreadsheet has been built, though, it may be used to calculate the roots of *any* second-degree polynomial expression simply by entering the new a, b, and c coefficients into cells `B3` through `B5`. The numerical values appearing in cells `B1` and `B2` will be automatically updated by the computer immediately following any changes made to the coefficients.

[7] Modern spreadsheet software offers a bewildering array of mathematical functions you may use in your computations. I recommend you consult the documentation for your particular spreadsheet for information on operations other than those listed here.

[8] Spreadsheet programs, like text-based programming languages, are designed to follow standard order of operations by default. However, my personal preference is to use parentheses even where strictly unnecessary just to make it clear to any other person viewing the formula what the intended order of operations is.

[9] Reviewing some algebra here, a *root* is a value for x that yields an overall value of zero for the polynomial. For this polynomial ($9x^2 + 5x - 2$) the two roots happen to be $x = 0.269381$ and $x = -0.82494$, with these values displayed in cells `B1` and `B2`, respectively upon execution of the spreadsheet.

Alternatively, one could break up the long quadratic formula into smaller pieces like this:

$$y = \sqrt{b^2 - 4ac} \qquad z = 2a$$

$$x = \frac{-b \pm y}{z}$$

	A	B	C
1	x_1	= (-B4 + C1) / C2	= sqrt((B4^2) - (4*B3*B5))
2	x_2	= (-B4 - C1) / C2	= 2*B3
3	a =	9	
4	b =	5	
5	c =	-2	

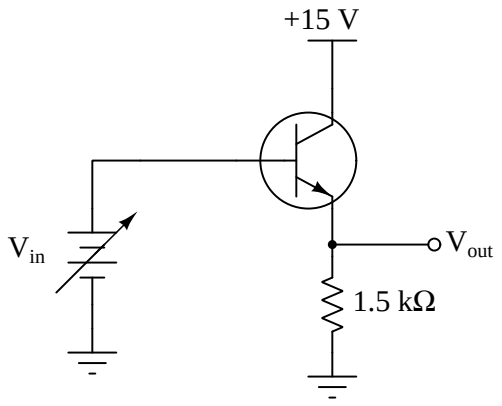
Note how the square-root term (y) is calculated in cell C1, and the denominator term (z) in cell C2. This makes the two final formulae (in cells B1 and B2) simpler to interpret. The positioning of all these cells on the grid is completely arbitrary[10] – all that matters is that they properly reference each other in the formulae.

Spreadsheets are particularly useful for situations where the same set of calculations representing a circuit or other system must be repeated for different initial conditions. The power of a spreadsheet is that it automates what would otherwise be a tedious set of calculations. One specific application of this is to simulate the effects of various components within a circuit failing with abnormal values (e.g. a shorted resistor simulated by making its value nearly zero; an open resistor simulated by making its value extremely large). Another application is analyzing the behavior of a circuit design given new components that are out of specification, and/or aging components experiencing drift over time.

[10] My personal preference is to locate all the "given" data in the upper-left cells of the spreadsheet grid (each data point flanked by a sensible name in the cell to the left and units of measurement in the cell to the right as illustrated in the first distance/time spreadsheet example), sometimes coloring them in order to clearly distinguish which cells contain entered data versus which cells contain computed results from formulae. I like to place all formulae in cells below the given data, and try to arrange them in logical order so that anyone examining my spreadsheet will be able to figure out *how* I constructed a solution. This is a general principle I believe all computer programmers should follow: *document and arrange your code to make it easy for other people to learn from it.*

5.2.3 Emitter-follower calculations

Complete the table of output voltages for several given values of input voltage in this common-collector amplifier circuit. Assume that the transistor is a standard silicon NPN unit, with a nominal base-emitter junction forward voltage of 0.7 Volts:



- V_{in} = 0.0 V ; V_{out} =
- V_{in} = 0.5 V ; V_{out} =
- V_{in} = 1.0 V ; V_{out} =
- V_{in} = 2.5 V ; V_{out} =
- V_{in} = 5.8 V ; V_{out} =
- V_{in} = 10.3 V ; V_{out} =

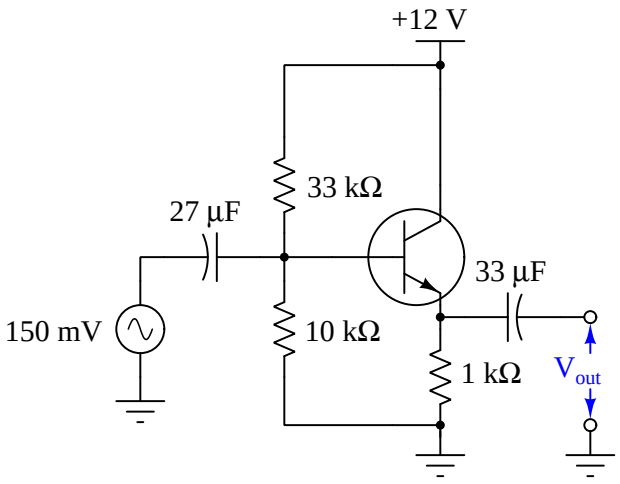
Based on the values you calculate, explain why the common-collector circuit configuration is often referred to as an *emitter follower*.

Challenges

- Suppose the 1.5 kΩ resistor is exchanged for a 2.2 kΩ resistor. What effect, if any, will this have on the operation of this emitter follower circuit?
- As V_B increases, does V_{out} increase or decrease? As AC signals, will V_{out} be in-phase or out-of-phase with V_{in}?
- Modify this emitter follower to work with a PNP transistor instead of the NPN (shown).

5.2.4 Common-collector output voltage

Calculate the approximate amount of AC voltage output by this common-collector amplifier circuit:



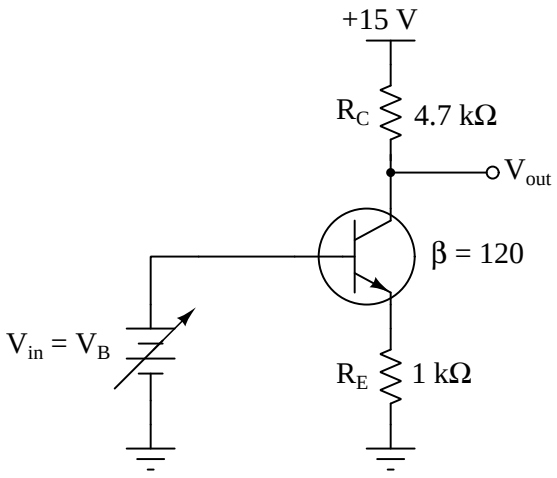
Also, explain why the reactance of each capacitor is a negligible factor in the operation of this amplifier circuit, assuming a signal frequency of 5 kHz.

Challenges

- Calculate the voltage gain for this circuit.
- What would happen if the 33 kΩ resistor failed open?

5.2.5 Common-emitter calculations

Complete the table of voltages and currents for several given values of input voltage in this common-emitter amplifier circuit. Assume that the transistor is a standard silicon NPN unit, with a nominal base-emitter junction forward voltage of 0.7 Volts:



V_B	V_E	I_E	I_C	V_{R_C}	V_{CE}	V_{out}
0.0 V						
0.5 V						
1.0 V						
1.5 V						
2.0 V						
2.5 V						
3.0 V						

Calculate the voltage gain of this circuit from the numerical values in the table using the following formula:

$$A_V = \frac{\Delta V_{out}}{\Delta V_{in}}$$

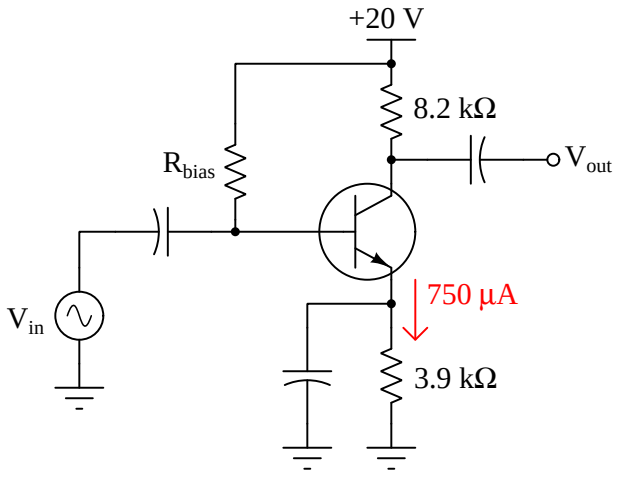
Finally, identify how some of the foundational concepts you've studied apply to this circuit: **Ohm's Law**, **Joule's Law**, **Kirchhoff's Voltage Law**, **Kirchhoff's Current Law**, properties of **series** and **parallel** networks, behavior of **PN junctions**, behavior of **BJTs**, the **Shockley diode equation**, etc. Feel free to include any other relevant foundational concepts not listed here.

Challenges

- As V_B increases, does V_{out} increase or decrease? As AC signals, will V_{out} be in-phase or out-of-phase with V_{in}?
- What relationship do you see between the AC voltage gain value and the resistor values?
- What would happen if the 4.7 kΩ resistor failed open?

5.2.6 Voltage gain of a bypassed common-emitter amplifier

Calculate the approximate voltage gain (A_V) for the following bypassed common-emitter amplifier circuit, assuming a quiescent (DC) emitter current value of 750 μA. Also calculate the quiescent DC voltage measured at the transistor's collector terminal with respect to ground (V_C). Assume a silicon transistor:



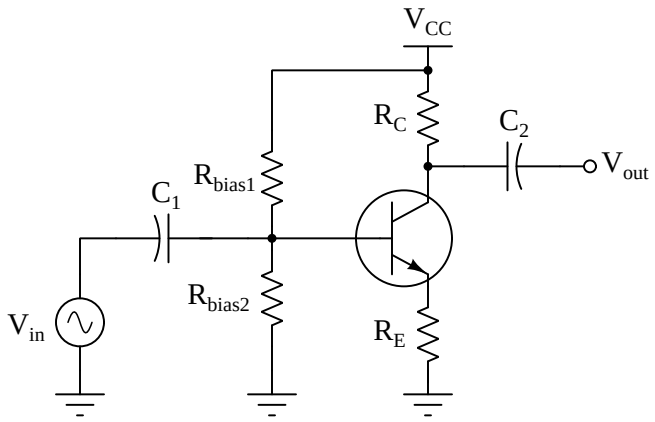
- $A_V \approx$
- $V_C \approx$

Challenges

- What would happen if R_{bias} failed open?
- Explain the purpose of the bypass capacitor in this circuit.

5.2.7 Achieving a specified voltage gain

Choose values for the collector and emitter resistors that will yield a voltage gain of approximately 5 for the following common-emitter amplifier circuit:



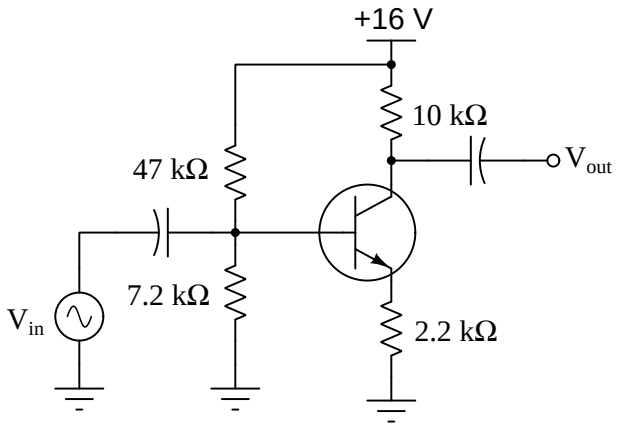
Also, identify design constraints other than voltage gain that limit our choices for R_C and/or R_E.

Challenges

- What would happen if R_{bias2} failed open?
- Technically, there are many correct answers to this question, each of which will result in a voltage gain of 5. Among those various solutions, which might be preferable, and for what reason(s)?

5.2.8 Gain and quiescent values for common-emitter amplifier

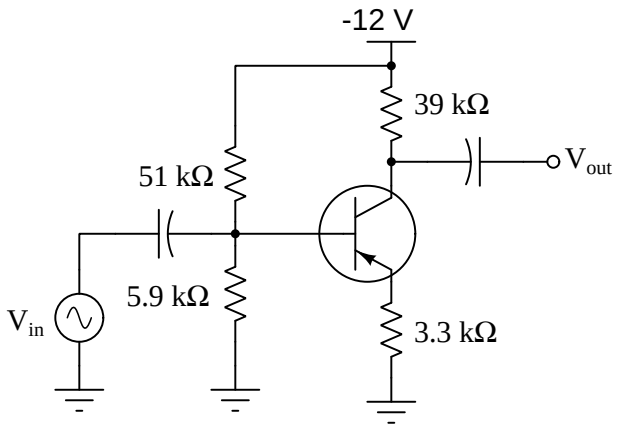
Calculate the approximate voltage gain (A_V) for the following common-emitter amplifier circuit, and also calculate the quiescent DC voltages measured at the three terminals of the transistor with respect to ground (V_B, V_E, and V_C):



- $A_V \approx$
- $V_B \approx$
- $V_E \approx$
- $V_C \approx$

Identify any simplifying assumptions made in your calculations.

Next, do the same for this amplifier circuit:



- $A_V \approx$
- $V_B \approx$
- $V_E \approx$
- $V_C \approx$

Again, identify any simplifying assumptions made in your calculations.

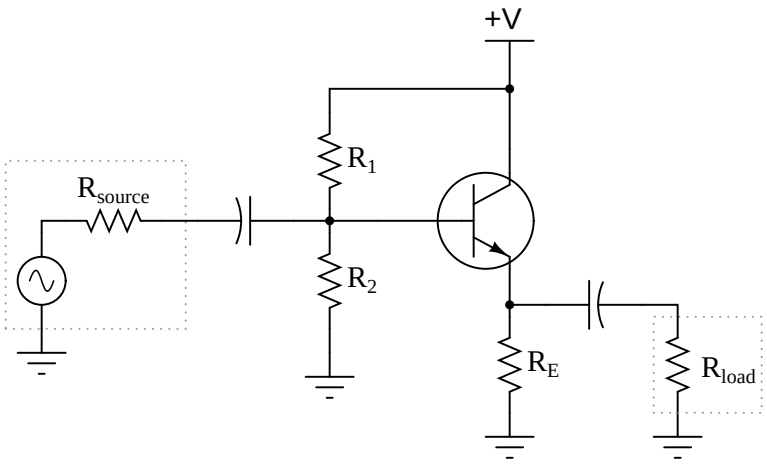
Challenges

- Your calculated values are bound to be approximate, given uncertainties of the transistor itself along with simplifying assumptions often made to ease the burden of performing these calculations. For each of your (estimated) values, identify whether you think they are higher or lower than the true values.

5.2.9 Common-collector impedance formulae

Explain each of the mathematical approximations for this typical common-collector amplifier circuit:

Typical common-collector amplifier circuit



$$A_V \approx 1$$

$$Z_{in} \approx R_1 \,||\, R_2 \,||\, (\beta + 1)[r'_E + (R_E \,||\, R_{load})]$$

$$Z_{out} \approx R_E \,||\, \left(r'_E + \frac{R_1 \,||\, R_2 \,||\, R_{source}}{\beta + 1}\right)$$

What does each term in each expression represent, and why do they relate to one another as shown?

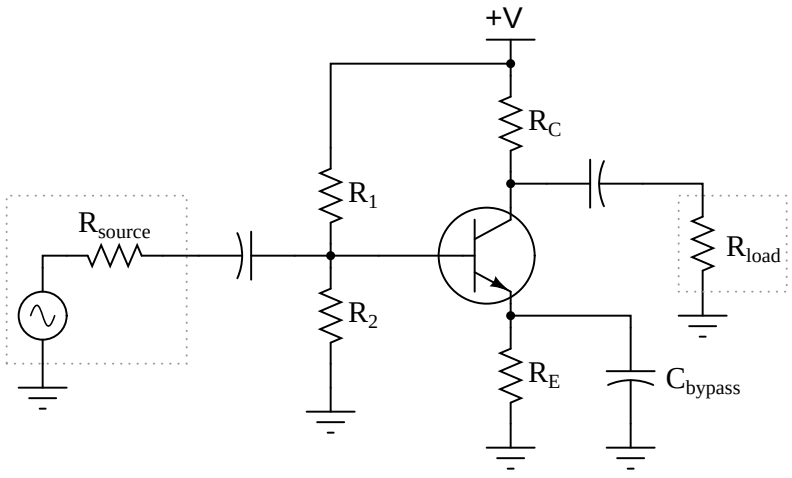
Challenges

- Why do you suppose the || symbols were used instead of more conventional mathematical symbols?

5.2.10 Common-emitter impedance formulae

Explain each of the mathematical approximations for this typical common-emitter amplifier circuit (with a bypass capacitor):

Typical (bypassed) common-emitter amplifier circuit



$$A_V \approx \frac{R_C \,||\, R_{load}}{r'_E}$$

$$Z_{in} \approx R_1 \,||\, R_2 \,||\, (\beta + 1)r'_E$$

$$Z_{out} \approx R_C$$

What does each term in each expression represent, and why do they relate to one another as shown?

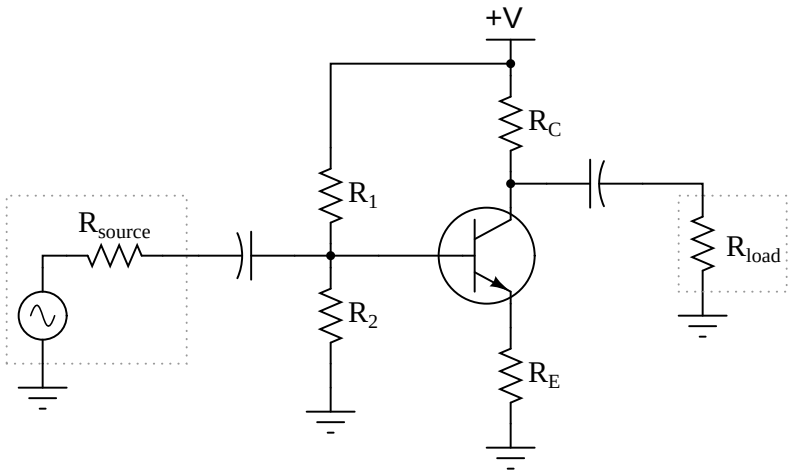
Challenges

- Why do you suppose the || symbols were used instead of more conventional mathematical symbols?

5.2.11 Swamped common-emitter impedance formulae

Explain each of the mathematical approximations for this typical common-emitter amplifier circuit (with the dynamic emitter resistance "swamped" by R_E):

Typical (swamped) common-emitter amplifier circuit



$$A_V \approx \frac{R_C \,||\, R_{load}}{r'_E + R_E}$$

$$Z_{in} \approx R_1 \,||\, R_2 \,||\, (\beta + 1)(r'_E + R_E)$$

$$Z_{out} \approx R_C$$

What does each term in each expression represent, and why do they relate to one another as shown?

Challenges

- Why do you suppose the || symbols were used instead of more conventional mathematical symbols?

5.2.12 Simplifying common-collector impedance formulae

A common set of formulae for calculating input and output impedances of common-collector amplifier circuits is as follows:

$$Z_{in} \approx R_1 \,||\, R_2 \,||\, (\beta + 1)[r'_E + (R_E \,||\, R_{load})]$$

$$Z_{out} \approx R_E \,||\, \left(r'_E + \frac{R_1 \,||\, R_2 \,||\, R_{source}}{\beta + 1}\right)$$

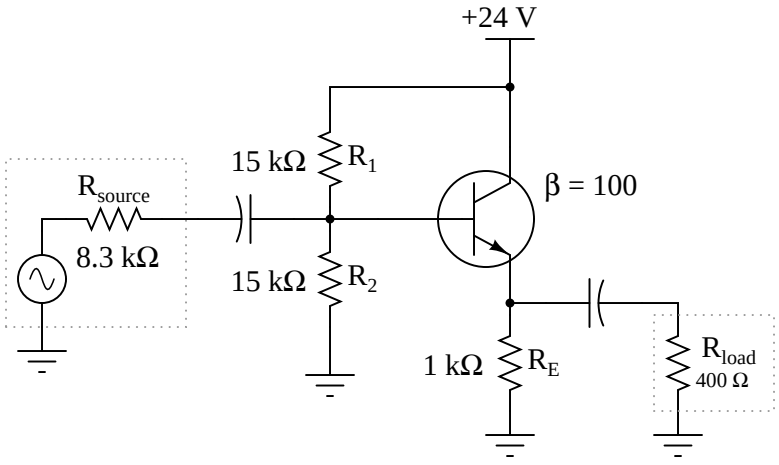
If precision is not required, we may greatly simplify these formulae by assuming the transistor to be ideal; i.e. having an infinite current gain ($\beta = \infty$). Re-write these formulae accordingly, and explain how you simplified each one.

Challenges

- Devise a way to compute the amount of error incurred by using the simplified formulae.

5.2.13 Common-collector approximations

Approximate the following values for this common-collector amplifier circuit, assuming the use of a silicon transistor:



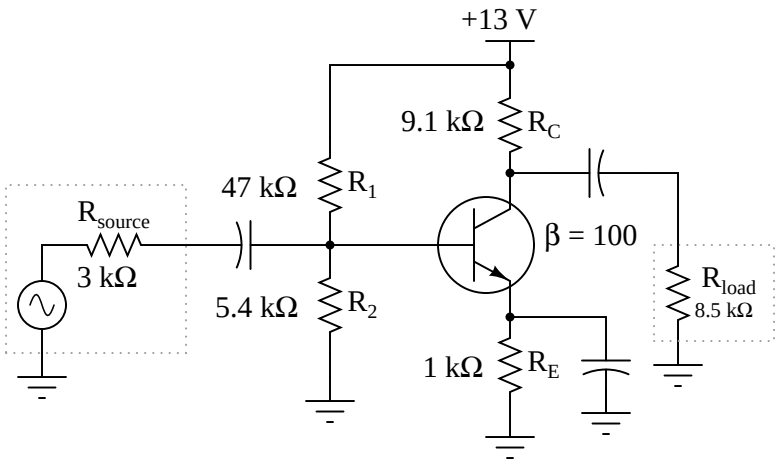
- A_V (as a ratio) $\approx$
- A_V (in decibels) $\approx$
- $Z_{in} \approx$
- $Z_{out} \approx$

Challenges

- How would these figures change, if at all, supposing the transistor had an infinite current gain ($\beta = \infty$)?

5.2.14 Common-emitter approximations

Approximate the following values for this common-emitter amplifier circuit, assuming the use of a silicon transistor:



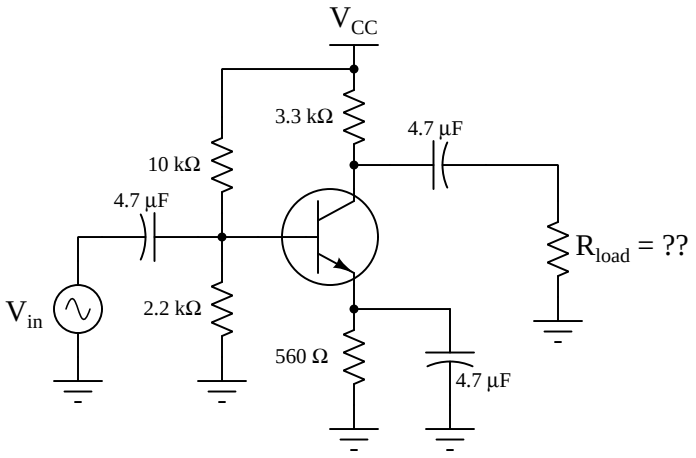
- A_V (as a ratio) $\approx$
- A_V (in decibels) $\approx$
- $Z_{in} \approx$
- $Z_{out} \approx$

Challenges

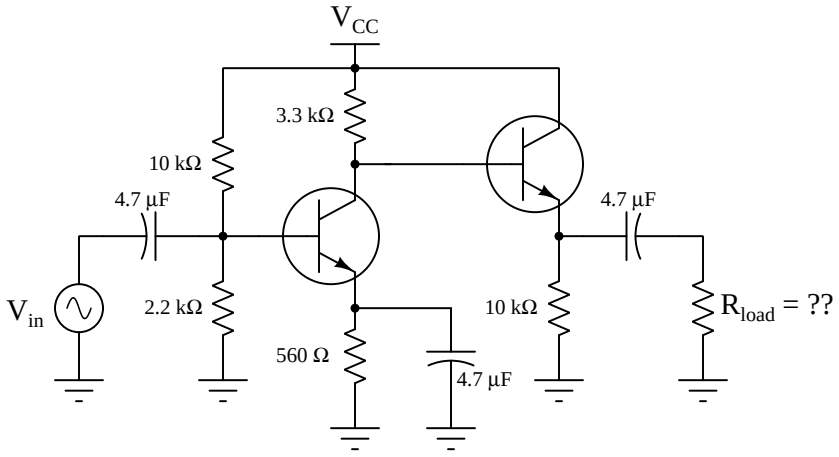
- How would these figures change, if at all, supposing the transistor had an infinite current gain ($\beta = \infty$)?

5.2.15 Ideal load resistance value

What is the ideal amount of load impedance for this amplifier circuit, so that maximum power will be delivered to it? Assume a β value for the transistor of 100:



If we augment this amplifier with a common-collector output stage, what will the ideal load resistance be now? The new transistor has a β value of 35:

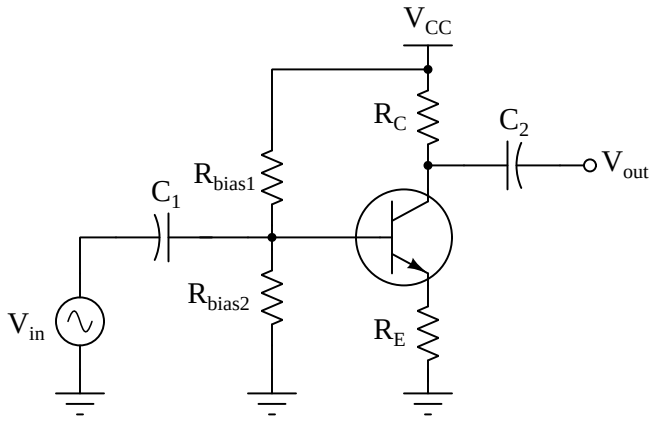


Challenges

- Suppose we wished to drive an 8 Ohm audio speaker with this amplifier. How could we achieve an optimum impedance match for the 8 Ohm speaker?
- Does the 4.7 μF capacitor influence amplifier impedance at all?

5.2.16 Common-emitter quiescent simulation program

Write a text-based computer program (e.g. C, C++, Python) to approximate input impedance, output impedance, voltage gain, quiescent (DC) base voltage, DC collector voltage, and DC emitter voltage for the following amplifier circuit given user-input component values shown in this schematic:



Here is some code (written in C) to get you started:

```
#include <stdio.h>

int main (void)
{
  float rb1, rb2, rc, re, vcc;

  printf("Enter resistance of Rbias1: ");  scanf("%f", &rb1);
  printf("Enter resistance of Rbias2: ");  scanf("%f", &rb2);
  printf("Enter resistance of Rc: ");      scanf("%f", &rc);
  printf("Enter resistance of Re: ");      scanf("%f", &re);
  printf("Enter supply voltage: ");        scanf("%f", &vcc);

  // Your code goes here . . .


  return 0;
}
```

Identify any simplifying assumptions made in your calculations.

Then, use the program you wrote to experiment with resistor values and source voltage value until you find a combination where the quiescent DC collector voltage is approximately one-half of

V_{CC}.

Challenges

- Why don't the capacitor values matter in the requested calculations?

5.3 Diagnostic reasoning

These questions are designed to stimulate your deductive and inductive thinking, where you must apply general principles to specific scenarios (deductive) and also derive conclusions about the failed circuit from specific details (inductive). In a Socratic discussion with your instructor, the goal is for these questions to reinforce your recall and use of general circuit principles and also challenge your ability to integrate multiple symptoms into a sensible explanation of what's wrong in a circuit. Your instructor may also pose additional questions based on those assigned, in order to further challenge and sharpen your diagnostic abilities.

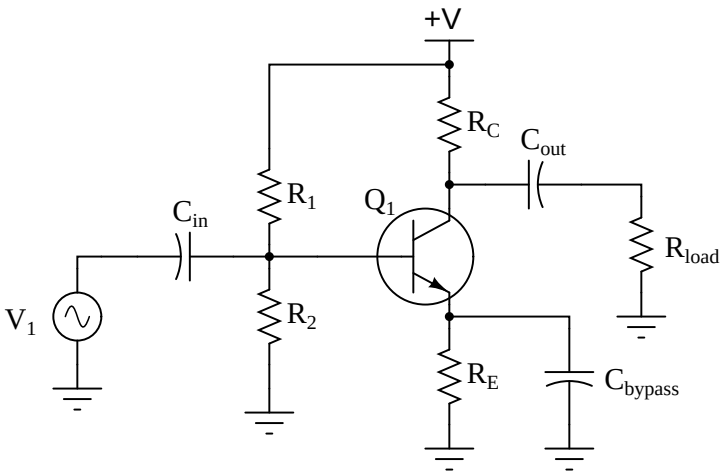
As always, your goal is to fully *explain* your analysis of each problem. Simply obtaining a correct answer is not good enough – you must also demonstrate sound reasoning in order to successfully complete the assignment. Your instructor's responsibility is to probe and challenge your understanding of the relevant principles and analytical processes in order to ensure you have a strong foundation upon which to build further understanding.

You will note a conspicuous lack of answers given for these diagnostic questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your diagnostic answers, where applicable, is to use circuit simulation software to explore the effects of faults placed in circuits. For example, if one of these diagnostic questions requires that you predict the effect of an open or a short in a circuit, you may check the validity of your work by simulating that same fault (substituting a very high resistance in place of that component for an open, and substituting a very low resistance for a short) within software and seeing if the results agree.

5.3.1 Effects of faults in a bypassed common-emitter amplifier

Predict how all transistor currents (I_B, I_C, and I_E) and the output voltage signal will be affected as a result of the following faults. Consider each fault independently (i.e. one at a time, no coincidental faults):



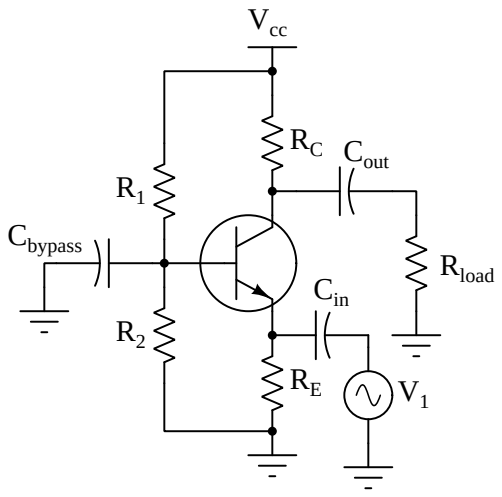
- Capacitor C_{in} fails open:
- Solder bridge (short) past resistor R_1:
- Resistor R_1 fails open:
- Resistor R_C fails open:
- Resistor R_E fails open:
- Capacitor C_{bypass} fails shorted:

Challenges

- Suppose this amplifier had a known-good input signal (V_1) but no measured output signal across the load resistor. What diagnostic steps would you take to isolate the location of the fault?

5.3.2 Effects of faults in a common-base amplifier

Predict how all transistor currents (I_B, I_C, and I_E) and the output voltage signal will be affected as a result of the following faults. Consider each fault independently (i.e. one at a time, no coincidental faults):



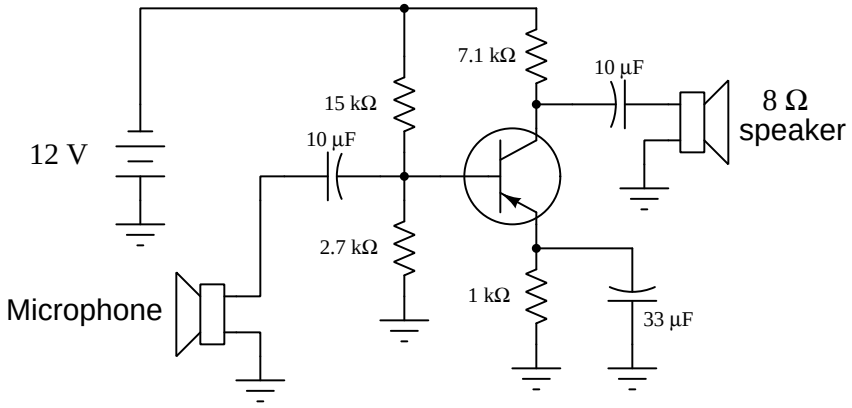
- Capacitor C_{out} fails open:
- Solder bridge (short) past resistor R_1:
- Resistor R_1 fails open:
- Resistor R_C fails open:
- Resistor R_E fails open:
- Capacitor C_{bypass} fails shorted:

Challenges

- Suppose this amplifier had a known-good input signal (V_1) but no measured output signal across the load resistor. What diagnostic steps would you take to isolate the location of the fault?

5.3.3 Faulty audio amplifier design

A student builds this common-emitter amplifier so they may amplify the audio signals from a microphone to power a speaker:



Unfortunately, the results are considerably less than expected: although some sound does come out of the speaker, it is not enough to be considered a success. Another student inspects the design and cryptically mumbles something about "poor impedance matching", leaving the first student somewhat confused.

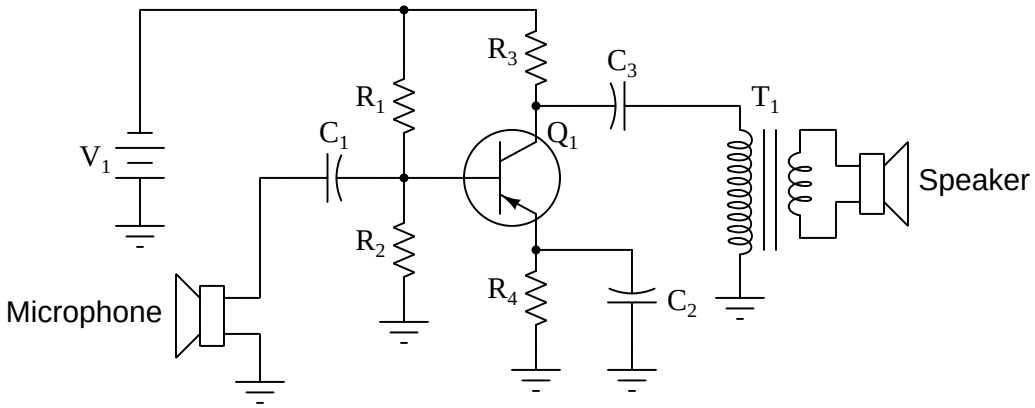
Explain what *impedance matching* means in this context, where the mis-match might be in this circuit, and what might be done to correct it.

Challenges

- There is more than one location in this amplifier circuit where an impedance mis-match might exist. Identify them all!
- Explain how the Maximum Power Transfer Theorem applies to this circuit.

5.3.4 Effects of faults in an audio amplifier

Each of the following faults will cause this audio amplifier circuit to stop working. Determine what diagnostic voltage measurement(s) would positively identify each one of the faults.



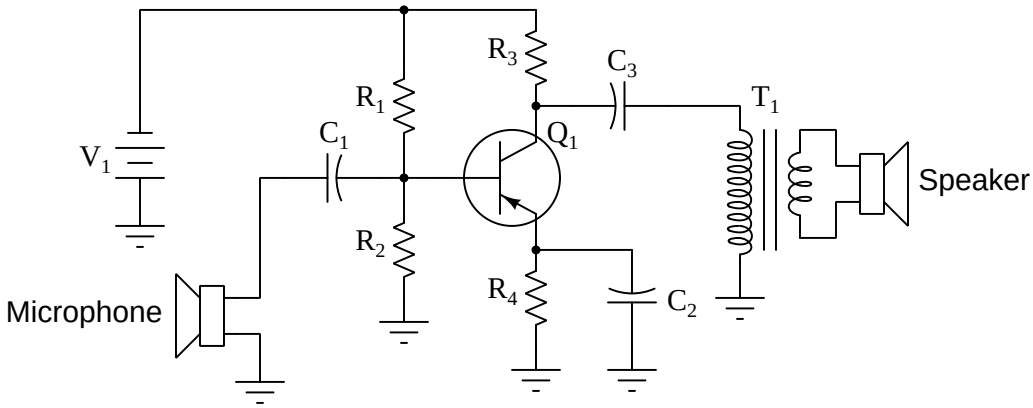
- Microphone coil fails open:
- Capacitor C_1 fails shorted:
- Resistor R_1 fails open:
- Resistor R_2 fails open:
- Capacitor C_3 fails open:
- Transformer T_1 primary winding fails open:

Challenges

- Add a volume control to this circuit.
- Redesign this amplifier circuit to use fewer components (hint: the transformer may be used in a slightly different way).

5.3.5 Faulted amplifier with good quiescent values

Suppose this microphone amplifier circuit used to function fine, but now has stopped outputting any sound at all:



Initial diagnostic measurements show all quiescent (DC) voltages to be normal. From this data, where would you suspect the problem is, and where would you suspect the problem is *not*?

Challenges

- Explain how incorrect resistor values for R_1 or R_2 could potentially cause a lack of sound at the speaker.
- Explain why capacitor C_1 is necessary between the microphone and the transistor.

5.3.6 Misconception on impedance-matching

A student recently learns about amplifier impedances and the importance of matching impedance values to maximize power transfer. From this lesson they conclude it is important for an amplifier's Z_{in} and Z_{out} to be equal values.

Explain why this is a misconception, and replace this erroneous conclusion with a proper one.

Challenges

- Give an example of where we actually *want* an amplifier to have different Z_{in} and Z_{out} values.

Appendix A

Problem-Solving Strategies

The ability to solve complex problems is arguably one of the most valuable skills one can possess, and this skill is particularly important in any science-based discipline.

- Study principles, not procedures. Don't be satisfied with merely knowing how to compute solutions – learn *why* those solutions work.
- Identify what it is you need to solve, identify all relevant data, identify all units of measurement, identify any general principles or formulae linking the given information to the solution, and then identify any "missing pieces" to a solution. Annotate all diagrams with this data.
- Sketch a diagram to help visualize the problem. When building a real system, always devise a plan for that system and analyze its function *before* constructing it.
- Follow the units of measurement and meaning of every calculation. If you are ever performing mathematical calculations as part of a problem-solving procedure, and you find yourself unable to apply each and every intermediate result to some aspect of the problem, it means you don't understand what you are doing. Properly done, every mathematical result should have practical meaning for the problem, and not just be an abstract number. You should be able to identify the proper units of measurement for each and every calculated result, and show where that result fits into the problem.
- Perform "thought experiments" to explore the effects of different conditions for theoretical problems. When troubleshooting real systems, perform *diagnostic tests* rather than visually inspecting for faults, the best diagnostic test being the one giving you the most information about the nature and/or location of the fault with the fewest steps.
- Simplify the problem until the solution becomes obvious, and then use that obvious case as a model to follow in solving the more complex version of the problem.
- Check for exceptions to see if your solution is incorrect or incomplete. A good solution will work for *all* known conditions and criteria. A good example of this is the process of testing scientific hypotheses: the task of a scientist is not to find support for a new idea, but rather to *challenge* that new idea to see if it holds up under a battery of tests. The philosophical

principle of *reductio ad absurdum* (i.e. disproving a general idea by finding a specific case where it fails) is useful here.

- Work "backward" from a hypothetical solution to a new set of given conditions.
- Add quantities to problems that are qualitative in nature, because sometimes a little math helps illuminate the scenario.
- Sketch graphs illustrating how variables relate to each other. These may be quantitative (i.e. with realistic number values) or qualitative (i.e. simply showing increases and decreases).
- Treat quantitative problems as qualitative in order to discern the relative magnitudes and/or directions of change of the relevant variables. For example, try determining what happens if a certain variable were to increase or decrease before attempting to precisely calculate quantities: how will each of the dependent variables respond, by increasing, decreasing, or remaining the same as before?
- Consider limiting cases. This works especially well for qualitative problems where you need to determine which direction a variable will change. Take the given condition and magnify that condition to an extreme degree as a way of simplifying the direction of the system's response.
- Check your work. This means regularly testing your conclusions to see if they make sense. This does *not* mean repeating the same steps originally used to obtain the conclusion(s), but rather to use some other means to check validity. Simply repeating procedures often leads to *repeating the same errors* if any were made, which is why alternative paths are better.

Appendix B

Instructional philosophy

B.1 First principles of learning

- **Anyone can learn anything** given appropriate time, effort, resources, challenges, encouragement, and expectations. Dedicating time and investing effort are the student's responsibility; providing resources, challenges, and encouragement are the teacher's responsibility; high expectations are a responsibility shared by both student and teacher.

- **Transfer is not automatic.** The human mind has a natural tendency to compartmentalize information, which means the process of taking knowledge learned in one context and applying it to another usually does not come easy and therefore should never be taken for granted.

- **Learning is iterative.** The human mind rarely learns anything perfectly on the first attempt. Anticipate mistakes and plan for multiple tries to achieve full understanding, using the lessons of those mistakes as feedback to guide future attempts.

- **Information is absorbed, but understanding is created.** Facts and procedures may be memorized easily enough by repeated exposure, but the ability to reliably apply principles to novel scenarios only comes through intense personal effort. This effort is fundamentally creative in nature: casting new concepts in one's own words, running mental experiments to perceive patterns and trends, etc. These acts of making knowledge "one's own" need not be perfect in order to be effective, as the value lies in the activity and not necessarily the finished product. Depth of understanding and permanence of memory are both directly related to mental effort during the learning process – i.e. *easy learning is shallow and fleeting.*

- **Education trumps training.** There is no such thing as an entirely isolated subject, as all fields of knowledge are connected. Training is narrowly-focused and task-oriented. Education is broad-based and principle-oriented. When preparing for a life-long technical career, education beats training every time.

- **Character matters.** Poor habits are more destructive than deficits of knowledge or skill. This is especially true in collective endeavors, where a team's ability to function depends on trust between its members. Simply put, no one wants an untrustworthy person on their team. An essential component of education then, is character development.

- **People learn to be responsible by bearing responsibility.** An irresponsible person is someone who has never *had* to be responsible for anything that mattered enough to them. Just as anyone can learn anything, anyone can become responsible if the personal cost of irresponsibility becomes high enough.

- **What gets measured, gets done.** Accurate and relevant assessment of learning is key to ensuring all students learn. Therefore, it is imperative to measure what matters.

- **Failure is nothing to fear.** Every human being fails, and fails in multiple ways at multiple times. Eventual success only happens when we don't stop trying.

B.2 Proven strategies for instructors

- Assume every student is capable of learning anything they desire given the proper conditions. Treat them as capable adults by granting real responsibility and avoiding artificial incentives such as merit or demerit points.
- Create a consistent culture of high expectations across the entire program of study. Demonstrate and encourage patience, persistence, and a healthy sense of self-skepticism. Anticipate and de-stigmatize error. Teach respect for the capabilities of others as well as respect for one's own fallibility.
- Replace lecture with "inverted" instruction, where students first encounter new concepts through reading and then spend class time in Socratic dialogue with the instructor exploring those concepts and solving problems individually. There is a world of difference between observing someone solve a problem versus actually solving a problem yourself, and so the point of this form of instruction is to place students in a position where they *cannot* passively observe.
- Require students to read extensively, write about what they learn, and dialogue with you and their peers to sharpen their understanding. Apply Francis Bacon's advice that "reading maketh a full man; conference a ready man; and writing an exact man". These are complementary activities helping students expand their confidence and abilities.
- Use artificial intelligence (AI) to challenge student understanding rather than merely provide information. Find productive ways for AI to critique students' clarity of thought and of expression, for example by employing AI as a Socratic-style interlocutor or as a reviewer of students' journals. Properly applied, AI has the ability to expand student access to critical review well outside the bounds of their instructor's reach.
- Build frequent and rapid feedback into the learning process so that students know at all times how well they are learning, to identify problems early and fix them before they grow. Model the intellectual habit of self-assessing and self-correcting your own understanding (i.e. a cognitive *feedback loop*), encouraging students to do the same.
- Use "mastery" as the standard for every assessment, which means the exam or experiment or project must be done with 100% competence in order to pass. Provide students with multiple opportunity for re-tries (different versions of the assessment every time).
- Require students to devise their own hypotheses and procedures on all experiments, so that the process is truly a scientific one. Have students assess their proposed experimental procedures for risk and devise mitigations for those risks. Let nothing be pre-designed about students' experiments other than a stated task (i.e. what principle the experiment shall test) at the start and a set of demonstrable knowledge and skill objectives at the end.
- Have students build as much of their lab equipment as possible: building power sources, building test assemblies[1], and building complete working systems (no kits!). In order to provide

[1] In the program I teach, every student builds their own "Development Board" consisting of a metal chassis with DIN rail, terminal blocks, and an AC-DC power supply of their own making which functions as a portable lab environment they can use at school as well as take home.

this same "ground-up" experience for every new student, this means either previous students take their creations with them, or the systems get disassembled in preparation for the new students, or the systems grow and evolve with each new student group.

- Incorporate external accountability for you and for your students, continuously improving the curriculum and your instructional methods based on proven results. Have students regularly network with active professionals through participation in advisory committee meetings, service projects, tours, jobshadows, internships, etc. Practical suggestions include requiring students to design and build projects for external clients (e.g. community groups, businesses, different departments within the institution), and also requiring students attend all technical advisory committee meetings and to dialogue with the industry representatives at those meetings.
- Repeatedly explore difficult-to-learn concepts across multiple courses, so that students have multiple opportunities to build their understanding.
- Relate all new concepts, whenever possible, to previous concepts and to relevant physical laws. Challenge each and every student, every day, to *reason* from concept to concept and to explain the logical connections between. Challenge students to verify their conclusions by multiple approaches (e.g. double-checking their work using different methods). Ask *"Why?"* often.
- Maintain detailed records on each student's performance and share these records privately with them. These records should include academic performance as well as professionally relevant behavioral tendencies.
- Hold mandatory "check-in" meetings between all program faculty and each new student during their first term. Offer these to all other students as an option, except for any students continuing to manifest unprofessional behaviors, poor academic performance, or who have some other need for a face-to-face meeting with faculty.
- Address problems while they are small, before they grow larger. This is equally true for tutoring technical concepts as it is for helping students build professional habits.
- Build rigorous quality control into the curriculum to ensure every student masters every important concept, and that the mastery is retained over time. This includes (1) review questions added to every exam to re-assess knowledge taught in previous terms, (2) cumulative exams at the end of every term to re-assess all important concepts back to the very beginning of the program, and (3) review assessments in practical (hands-on) coursework to ensure critically-important skills were indeed taught and are still retained. What you will find by doing this is that it actually boosts retention of students by ensuring that important knowledge gets taught and is retained over long spans of time. In the absence of such quality control, student learning and retention tends to be spotty and this contributes to drop-out and failure rates later in their education.
- Finally, *never rush learning.* Education is not a race. Give your students ample time to digest complex ideas, as you continually remind yourself of just how long it took you to achieve mastery! Long-term retention and the consistently correct application of concepts are always the result of *focused effort over long periods of time* which means there are no shortcuts to learning.

B.3 Proven strategies for students

The single most important piece of advice I have for any student of any subject is to take responsibility for your own development in all areas of life including mental development. Expecting others in your life to entirely guide your own development is a recipe for disappointment. This is just as true for students enrolled in formal learning institutions as it is for auto-didacts pursuing learning entirely on their own. Learning to think in new ways is key to being able to gainfully use information, to make informed decisions about your life, and to best serve those you care about. With this in mind, I offer the following advice to students:

- **Approach all learning as valuable**. No matter what course you take, no matter who you learn from, no matter the subject, there is something useful in every learning experience. If you don't see the value of every new experience, you are not looking closely enough!
- **Continually challenge yourself**. Let other people take shortcuts and find easy answers to easy problems. The purpose of education is to stretch your mind, in order to shape it into a more powerful tool. This doesn't come by taking the path of least resistance. An excellent analogy for an empowering education is productive physical exercise: becoming stronger, more flexible, and more persistent only comes through intense personal effort.
- **Master the use of language**. This includes reading extensively, writing every day, listening closely, and speaking articulately. To a great extent language channels and empowers thought, so the better you are at wielding language the better you will be at grasping abstract concepts and articulating them not only for your benefit but for others as well.
- **Do not limit yourself to the resources given to you**. Read books that are not on the reading list. Run experiments that aren't assigned to you. Form study groups outside of class. Take an entrepreneurial approach to your own education, as though it were a business you were building for your future benefit.
- **Express and share what you learn**. Take every opportunity to teach what you have learned to others, as this will not only help them but will also strengthen your own understanding[2].
- Realize that **no one can give you understanding**, just as no one can give you physical fitness. These both must be *built*.
- **Above all, recognize that learning is hard work, and that a certain level of frustration is unavoidable**. There are times when you will struggle to grasp some of these concepts, and that struggle is a natural thing. Take heart that it will yield with persistent and varied[3] effort, and never give up! That concepts don't immediately come to you is not a sign of something wrong, but rather of something right: that you have found a worthy challenge!

[2] On a personal note, I was surprised to learn just how much my own understanding of electronics and related subjects was strengthened by becoming a teacher. When you are tasked every day with helping other people grasp complex topics, it catalyzes your own learning by giving you powerful incentives to study, to articulate your thoughts, and to reflect deeply on the process of learning.

[3] As the old saying goes, "Insanity is trying the same thing over and over again, expecting different results." If you find yourself stumped by something in the text, you should attempt a different approach. Alter the thought experiment, change the mathematical parameters, do whatever you can to see the problem in a slightly different light, and then the solution will often present itself more readily.

B.4 Design of these learning modules

"The unexamined circuit is not worth energizing" – Socrates (if he had taught electricity)

These learning modules, although useful for self-study, were designed to be used in a formal learning environment where a subject-matter expert challenges students to digest the content and exercise their critical thinking abilities in the answering of questions and in the construction and testing of working circuits. Every effort has been made to embed the following instructional and assessment philosophies within:

- The first goal of education is to enhance clear and independent thought, in order that every student reach their fullest potential in a highly complex and inter-dependent world. Robust reasoning is *always* more important than particulars of any subject matter, because its application is universal.
- Literacy is fundamental to independent learning and thought because text continues to be the most efficient way to communicate complex ideas over space and time. Those who cannot read with ease are limited in their ability to acquire knowledge and perspective.
- Articulate communication is fundamental to work that is complex and interdisciplinary.
- Faulty assumptions and poor reasoning are best corrected through challenge, not presentation. The rhetorical technique of *reductio ad absurdum* (disproving an assertion by exposing an absurdity) works well to discipline student's minds, not only to correct the problem at hand but also to learn how to detect and correct future errors.
- Important principles should be repeatedly explored and widely applied throughout a course of study, not only to reinforce their importance and help ensure their mastery, but also to showcase the interconnectedness and utility of knowledge.

These learning modules were expressly designed to be used in an "inverted" teaching environment[4] where students first read the introductory and tutorial chapters on their own, then individually attempt to answer the questions and construct working circuits according to the experiment and project guidelines. The instructor never lectures, but instead meets regularly with each individual student to review their progress, answer questions, identify misconceptions, and challenge the student to new depths of understanding through further questioning. Regular meetings between instructor and student should resemble a Socratic[5] dialogue, where questions serve as scalpels to dissect topics and expose assumptions. The student passes each module only after consistently demonstrating their ability to logically analyze and correctly apply all major concepts in each question or project/experiment. The instructor must be vigilant in probing each student's understanding to ensure they are truly *reasoning* and not just *memorizing*. This is why "Challenge" points appear throughout, as prompts for students to think deeper about topics and as starting points for instructor queries. Sometimes these challenge points require additional knowledge that hasn't been covered in the series to answer in full. This is okay, as the major purpose of the Challenges is to stimulate analysis and synthesis on the part of each student.

The instructor must possess enough mastery of the subject matter and awareness of students' reasoning to generate their own follow-up questions to practically any student response. Even completely correct answers given by the student should be challenged by the instructor for the purpose of having students practice articulating their thoughts and defending their reasoning. Conceptual errors committed by the student should be exposed and corrected not by direct instruction, but rather by reducing the errors to an absurdity[6] through well-chosen questions and thought experiments posed by the instructor. Becoming proficient at this style of instruction requires time and dedication, but the positive effects on critical thinking for both student and instructor are spectacular.

An inspection of these learning modules reveals certain unique characteristics. One of these is a bias toward thorough explanations in the tutorial chapters. Without a live instructor to explain concepts and applications to students, the text itself must fulfill this role. This philosophy results in lengthier explanations than what you might typically find in a textbook, each step of the reasoning process fully explained, including footnotes addressing common questions and concerns students raise while learning these concepts. Each tutorial seeks to not only explain each major concept in sufficient detail, but also to explain the logic of each concept and how each may be developed

[4] In a traditional teaching environment, students first encounter new information via *lecture* from an expert, and then independently apply that information via *homework*. In an "inverted" course of study, students first encounter new information via *homework*, and then independently apply that information under the scrutiny of an expert. The expert's role in lecture is to simply *explain*, but the expert's role in an inverted session is to *challenge*, *critique*, and if necessary *explain* where gaps in understanding still exist.

[5] Socrates is a figure in ancient Greek philosophy famous for his unflinching style of questioning. Although he authored no texts, he appears as a character in Plato's many writings. The essence of Socratic philosophy is to leave no question unexamined and no point of view unchallenged. While purists may argue a topic such as electric circuits is too narrow for a true Socratic-style dialogue, I would argue that the essential thought processes involved with scientific reasoning on *any* topic are not far removed from the Socratic ideal, and that students of electricity and electronics would do very well to challenge assumptions, pose thought experiments, identify fallacies, and otherwise employ the arsenal of critical thinking skills modeled by Socrates.

[6] This rhetorical technique is known by the Latin phrase *reductio ad absurdum*. The concept is to expose errors by counter-example, since only one solid counter-example is necessary to disprove a universal claim. As an example of this, consider the common misconception among beginning students of electricity that voltage cannot exist without current. One way to apply *reductio ad absurdum* to this statement is to ask how much current passes through a fully-charged battery connected to nothing (i.e. a clear example of voltage existing without current).

from "first principles". Again, this reflects the goal of developing clear and independent thought in students' minds, by showing how clear and logical thought was used to forge each concept. Students benefit from witnessing a model of clear thinking in action, and these tutorials strive to be just that.

Another feature of these learning modules is that they do not treat topics in isolation. Rather, important concepts are introduced early in the series, and appear repeatedly as stepping-stones toward other concepts in subsequent modules. This helps to avoid the "compartmentalization" of knowledge, demonstrating the inter-connectedness of concepts and simultaneously reinforcing them. Each module is fairly complete in itself, reserving the beginning of its tutorial to a review of foundational concepts.

To high standards of education,

Tony R. Kuphaldt

Appendix C

Tools used

I am indebted to the developers of many open-source software applications in the creation of these learning modules. The following is a list of these applications with some commentary on each.

You will notice a theme common to many of these applications: a bias toward *code*. Although I am by no means an expert programmer in any computer language, I understand and appreciate the flexibility offered by code-based applications where the user (you) enters commands into a plain ASCII text file, which the software then reads and processes to create the final output. Code-based computer applications are by their very nature *extensible*, while WYSIWYG (What You See Is What You Get) applications are generally limited to whatever user interface the developer makes for you.

The `GNU/Linux` computer operating system

> There is so much to be said about Linus Torvalds' `Linux` and Richard Stallman's `GNU` project. First, to credit just these two individuals is to fail to do justice to the *mob* of passionate volunteers who contributed to make this amazing software a reality. I first learned of `Linux` back in 1996, and have been using this operating system on my personal computers almost exclusively since then. It is *free*, it is completely *configurable*, and it permits the continued use of highly efficient `Unix` applications and scripting languages (e.g. shell scripts, Makefiles, `sed`, `awk`) developed over many decades. `Linux` not only provided me with a powerful computing platform, but its open design served to inspire my life's work of creating open-source educational resources.

Bram Moolenaar's `Vim` text editor

> Writing code for any code-based computer application requires a *text editor*, which may be thought of as a word processor strictly limited to outputting plain-ASCII text files. Many good text editors exist, and one's choice of text editor seems to be a deeply personal matter within the programming world. I prefer `Vim` because it operates very similarly to `vi` which is ubiquitous on `Unix`/`Linux` operating systems, and because it may be entirely operated via keyboard (i.e. no mouse required) which makes it fast to use.

Donald Knuth's TEX typesetting system

Developed in the late 1970's and early 1980's by computer scientist extraordinaire Donald Knuth to typeset his multi-volume magnum opus *The Art of Computer Programming*, this software allows the production of formatted text for screen-viewing or paper printing, all by writing plain-text code to describe how the formatted text is supposed to appear. TEX is not just a markup language for documents, but it is also a Turing-complete programming language in and of itself, allowing useful algorithms to be created to control the production of documents. Simply put, TEX *is a programmer's approach to word processing*. Since TEX is controlled by code written in a plain-text file, this means anyone may read that plain-text file to see exactly how the document was created. This openness afforded by the code-based nature of TEX makes it relatively easy to learn how other people have created their own TEX documents. By contrast, examining a beautiful document created in a conventional WYSIWYG word processor such as Microsoft `Word` suggests nothing to the reader about *how* that document was created, or what the user might do to create something similar. As Mr. Knuth himself once quipped, conventional word processing applications should be called WYSIAYG (What You See Is *All* You Get).

Leslie Lamport's LATEX extensions to TEX

Like all true programming languages, TEX is inherently extensible. So, years after the release of TEX to the public, Leslie Lamport decided to create a massive extension allowing easier compilation of book-length documents. The result was LATEX, which is the markup language used to create all ModEL module documents. You could say that TEX is to LATEX as `C` is to `C++`. This means it is permissible to use any and all TEX commands within LATEX source code, and it all still works. Some of the features offered by LATEX that would be challenging to implement in TEX include automatic index and table-of-content creation.

Tim Edwards' `Xcircuit` drafting program

This wonderful program is what I use to create all the schematic diagrams and illustrations (but not photographic images or mathematical plots) throughout the ModEL project. It natively outputs PostScript format which is a true vector graphic format (this is why the images do not pixellate when you zoom in for a closer view), and it is so simple to use that I have never had to read the manual! Object libraries are easy to create for `Xcircuit`, being plain-text files using PostScript programming conventions. Over the years I have collected a large set of object libraries useful for drawing electrical and electronic schematics, pictorial diagrams, and other technical illustrations.

`Gimp` graphic image manipulation program

Essentially an open-source clone of Adobe's `PhotoShop`, I use `Gimp` to resize, crop, and convert file formats for all of the photographic images appearing in the ModEL modules. Although `Gimp` does offer its own scripting language (called `Script-Fu`), I have never had occasion to use it. Thus, my utilization of `Gimp` to merely crop, resize, and convert graphic images is akin to using a sword to slice bread.

`SPICE` circuit simulation program

`SPICE` is to circuit analysis as TEX is to document creation: it is a form of markup language designed to describe a certain object to be processed in plain-ASCII text. When the plain-text "source file" is compiled by the software, it outputs the final result. More modern circuit analysis tools certainly exist, but I prefer `SPICE` for the following reasons: it is *free*, it is *fast*, it is *reliable*, and it is a fantastic tool for *teaching* students of electricity and electronics how to write simple code. I happen to use rather old versions of `SPICE`, version 2g6 being my "go to" application when I only require text-based output. `NGSPICE` (version 26), which is based on Berkeley `SPICE` version 3f5, is used when I require graphical output for such things as time-domain waveforms and Bode plots. In all `SPICE` example netlists I strive to use coding conventions compatible with all `SPICE` versions.

Andrew D. Hwang's `ePiX` mathematical visualization programming library

This amazing project is a `C++` library you may link to any `C/C++` code for the purpose of generating PostScript graphic images of mathematical functions. As a completely free and open-source project, it does all the plotting I would otherwise use a Computer Algebra System (CAS) such as `Mathematica` or `Maple` to do. It should be said that `ePiX` is *not* a Computer Algebra System like `Mathematica` or `Maple`, but merely a mathematical *visualization* tool. In other words, it won't determine integrals for you (you'll have to implement that in your own `C/C++` code!), but it can graph the results, and it does so beautifully. What I really admire about `ePiX` is that it is a `C++` programming library, which means it builds on the existing power and toolset available with that programming language. Mr. Hwang could have probably developed his own stand-alone application for mathematical plotting, but by creating a `C++` library to do the same thing he accomplished something much greater.

`gnuplot` mathematical visualization software

Another open-source tool for mathematical visualization is `gnuplot`. Interestingly, this tool is *not* part of Richard Stallman's GNU project, its name being a coincidence. For this reason the authors prefer "gnu" *not* be capitalized at all to avoid confusion. This is a much "lighter-weight" alternative to a spreadsheet for plotting tabular data, and the fact that it easily outputs directly to an X11 console or a file in a number of different graphical formats (including PostScript) is very helpful. I typically set my `gnuplot` output format to default (X11 on my Linux PC) for quick viewing while I'm developing a visualization, then switch to PostScript file export once the visual is ready to include in the document(s) I'm writing. As with my use of `Gimp` to do rudimentary image editing, my use of `gnuplot` only scratches the surface of its capabilities, but the important points are that it's *free* and that it *works well*.

`Python` programming language

Both Python and C++ find extensive use in these modules as instructional aids and exercises, but I'm listing Python here as a *tool* for myself because I use it almost daily as a *calculator*. If you open a Python interpreter console and type `from math import *` you can type mathematical expressions and have it return results just as you would on a hand calculator. Complex-number (i.e. *phasor*) arithmetic is similarly supported if you include the complex-math library (`from cmath import *`). Examples of this are shown in the Programming References chapter (if included) in each module. Of course, being a fully-featured programming language, Python also supports conditionals, loops, and other structures useful for calculation of quantities. Also, running in a console environment where all entries and returned values show as text in a chronologically-ordered list makes it easy to copy-and-paste those calculations to document exactly how they were performed.

Appendix D

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Appendix E

Version history

This is a list showing all significant additions, corrections, and other edits made to this learning module. Each entry is referenced by calendar date in reverse chronological order (newest version first), which appears on the front cover of every learning module for easy reference. Any contributors to this open-source document are listed here as well.

22 September 2026 – minor grammatical edits to the Tutorial.

16 September 2026 – removed a gratuitous equals symbol from the "Common-emitter calculations" Quantitative Reasoning question.

14-17 September 2025 – added reference to dynamic emitter resistance to the "Challenging concepts related to single-stage BJT amplifiers" section of the Introduction chapter. Also added some comments about common-base amplifier applications.

26 September 2024 – added some instructor notes.

15-16 September 2024 – divided the Introduction chapter into sections, one with recommendations for students, one with a listing of challenging concepts, and one with recommendations for instructors.

14 February 2024 – added some more explanatory text to the Tutorial section on common-based amplifier on the concept of "swamping" the effects of dynamic emitter resistance by inserting an external emitter resistor into the amplifier circuit. Also added some coloring to image_3487. Also harmonized all representations of dynamic emitter resistance to be r'_E rather than r'_e, because previously it was written one way in some places and the other way in other places. Also corrected a typo where I said "they he" in a question, courtesy of Gavin Koppel.

18-20 September 2023 – added an introductory section label ("Amplifier fundamentals") as well as a new section ("BJT amplifier configurations") to the Tutorial, and made minor textual edits to the rest of the Tutorial.

19 May 2023 – added some instructor notes as well as a new Quantitative Reasoning question on

simulating BJT circuit approximations.

28 November 2022 – placed questions at the top of the itemized list in the Introduction chapter prompting students to devise experiments related to the tutorial content.

20-22 February 2022 – added a Case Tutorial chapter showing common-collector, common-emitter, and common-base amplifier configurations operating with varying amounts of bias, as well as signal loading due to common-emitter input impedance and output impedance.

14-16 February 2022 – minor edits to the Introduction chapter, and some new Challenge questions added. Also added a sub-question to the "Common-emitter calculations" Quantitative Reasoning question asking students to identify where foundation concepts were used in their calculations.

22 September 2021 – minor edits to the Tutorial regarding impedance matching and resistance transformation.

21 September 2021 – added another column to the table in the "Common-emitter calculations" Quantitative Reasoning question.

11 September 2021 – minor edits to some questions, in order to prompt students to think more about foundational concepts.

10 May 2021 – commented out or deleted empty chapters.

22 March 2021 – added more digits to Boltzmann's constant, and clarified that Kelvin is 273.15 more than Celsius.

18 December 2020 – minor typo correction in Tutorial.

16 December 2020 – minor addition to footnote in the design example section, explaining why the SPICE simulation's capacitor required an initial condition setting.

30 November 2020 – minor edits to the Tutorial and to some questions.

19 November 2020 – minor additions to the Introduction chapter.

18 November 2020 – significantly edited the Introduction chapter to make it more suitable as a pre-study guide and to provide cues useful to instructors leading "inverted" teaching sessions.

8 May 2020 – added new Tutorial section showing an amplifier design example.

28 April 2020 – capitalized all instances of the word "Volt".

27 April 2020 – edited Tutorial to elaborate more on impedance matching, adding subsections for the three basic BJT amplifier configurations. Corrected backwards polarized capacitors in the last three of the Diagnostic Reasoning problems.

23 April 2020 – elaborated on the use of beta to calculate transistor terminal currents in the

Tutorial, and also on the practical importance of resistance transformation.

22 April 2020 – added more Challenge questions.

15 February 2020 – added more problems.

29 January 2020 – added Foundational Concepts to the list in the Conceptual Reasoning section.

18 December 2019 – minor edits to diagnostic questions, replacing "no multiple faults" with "no coincidental faults".

17 December 2019 – added Technical Reference on the topic of decibels.

12 November 2019 – continued writing content for the Tutorial.

11 November 2019 – continued writing content for the Tutorial, and added several Conceptual, Quantitative, and Diagnostic questions.

7 November 2019 – continued writing content for the Tutorial, as well as added a Technical Derivation for dynamic resistance of a PN junction.

4 November 2019 – continued writing content for the Tutorial.

23 October 2019 – document first created.

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