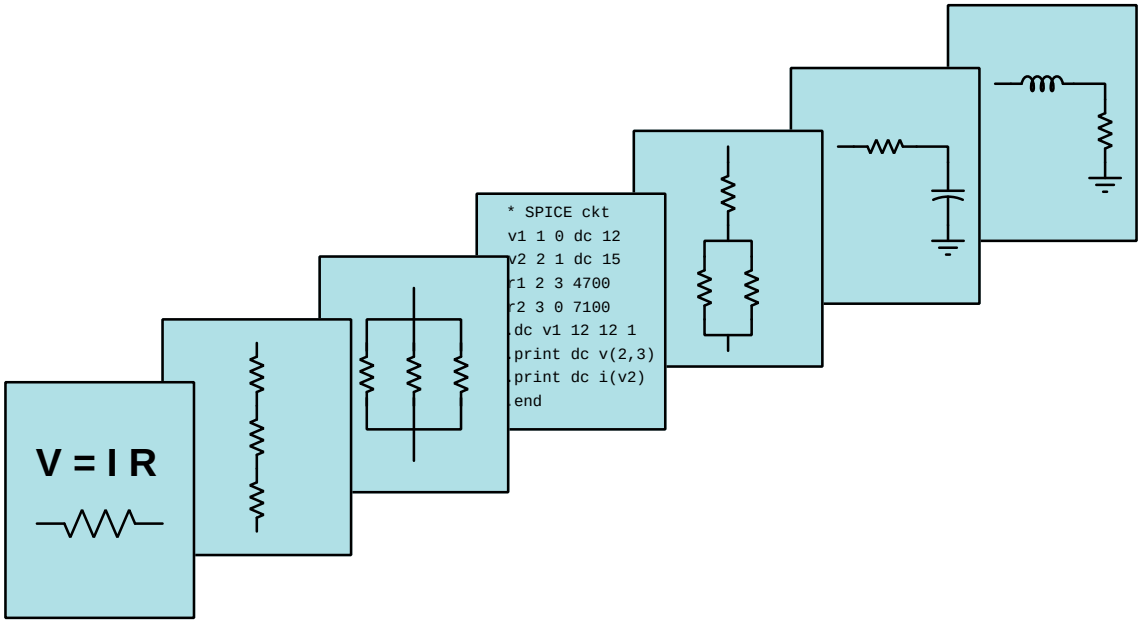
Modular Electronics Learning (ModEL) project



Bridge Circuits

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Contents

1 Introduction **3**
- 1.1 Recommendations for students . . . 3
- 1.2 Challenging concepts related to bridge circuits . . . 5
- 1.3 Recommendations for instructors . . . 7

2 Tutorial **9**
- 2.1 Voltage divider networks . . . 10
- 2.2 Bridge networks . . . 11
- 2.3 Balanced bridge examples . . . 13
- 2.4 Resistance-measuring bridges . . . 14
- 2.5 Sensor bridges . . . 16
- 2.6 Power bridges . . . 21

3 Historical References **23**
- 3.1 Early references to Wheatstone bridges . . . 24

4 Questions **29**
- 4.1 Conceptual reasoning . . . 33
  - 4.1.1 Reading outline and reflections . . . 33
  - 4.1.2 Foundational concepts . . . 35
  - 4.1.3 Edison's bridge . . . 36
  - 4.1.4 Thermistor bridge circuit . . . 37
- 4.2 Quantitative reasoning . . . 38
  - 4.2.1 Miscellaneous physical constants . . . 39
  - 4.2.2 Introduction to spreadsheets . . . 40
  - 4.2.3 Two voltage divider networks . . . 43
  - 4.2.4 Ground-referenced and differential voltage measurements . . . 44
  - 4.2.5 Balancing a resistive bridge . . . 45
  - 4.2.6 Using a primitive bridge to measure resistance . . . 47
  - 4.2.7 Unknown excitation voltage . . . 48
- 4.3 Diagnostic reasoning . . . 49
  - 4.3.1 Thermistor bridge faults . . . 50
  - 4.3.2 Failed photocell bridge . . . 51

5 Projects and Experiments **53**
5.1 Recommended practices . . . 53
5.1.1 Safety first! . . . 54
5.1.2 Other helpful tips . . . 56
5.1.3 Terminal blocks for circuit construction . . . 57
5.1.4 Conducting experiments . . . 61
5.1.5 Constructing projects . . . 65
5.2 Experiment: pencil-trace bridge circuit . . . 66

A Problem-Solving Strategies **69**

B Instructional philosophy **71**
B.1 First principles of learning . . . 72
B.2 Proven strategies for instructors . . . 73
B.3 Proven strategies for students . . . 75
B.4 Design of these learning modules . . . 76

C Tools used **79**

D Creative Commons License **83**

E References **91**

F Version history **93**

Index **94**

Chapter 1

Introduction

1.1 Recommendations for students

A very useful and interesting circuit configuration called a *bridge* finds wide application for measurement and power-conversion purposes. In its full form with a total of five components it is neither series nor parallel, but something else entirely. This module explores the basic principles of bridge circuits.

Important concepts related to bridge circuits include **voltage dividers**, **Kirchhoff's Voltage Law**, mathematical **ratios**, **piezoresistive** sensors, **opens** versus **shorts**, **limiting cases**, properties of **series** versus **parallel** networks, **diodes**, AC-to-DC **rectification** circuits, **transistors**, and DC-to-AC **inversion**.

Here are some good questions to ask of yourself while studying this subject:

- How might an experiment be designed and conducted to demonstrate the behavior of a voltage divider network? What hypothesis (i.e. prediction) might you pose for that experiment, and what result(s) would either support or disprove that hypothesis?
- How might an experiment be designed and conducted to demonstrate the behavior of a Wheatstone bridge network? What hypothesis (i.e. prediction) might you pose for that experiment, and what result(s) would either support or disprove that hypothesis?
- How does a voltage divider function?
- What is voltage and how is it measured?
- What is necessary to make a resistive bridge circuit *balanced*?
- How may a bridge circuit be used to measure resistance?
- How may a resistive bridge circuit be used to measure physical parameters such as mechanical strain?

- What algebraic principles were used in the manipulation of the voltage divider formulae to arrive at a balanced-bridge formula?
- What is a *limiting case*, and how may this problem-solving technique be applied to bridge circuits?
- What is the guaranteed effect of a *short* in a circuit?
- What is the guaranteed effect of an *open* in a circuit?
- Which properties of series networks find application in bridge circuits?
- Which properties of parallel networks find application in bridge circuits?
- When using a Wheatstone bridge circuit to measure an unknown resistance, what aspects of the circuit must be accurately known?
- When using a Wheatstone bridge circuit to measure an unknown resistance, what aspects of the circuit may be inaccurate or unknown without affecting the accuracy of the unknown resistance's measurement?
- How are strain-gauge bridge circuits made insensitive to changes in temperature, and why is this important?
- How does a *bridge rectifier* convert alternating current into direct current?
- How does a *bridge inverter* convert direct current into alternating current?

1.2 Challenging concepts related to bridge circuits

The following list cites concepts related to this module's topic that are easily misunderstood, along with suggestions for properly understanding them:

- **Fractions** – of all the basic mathematical concepts, *fractions* seem to cause my students the greatest amount of trouble. Informal polls I've taken suggest a deficiency in our public-education math curriculum, where many students are encouraged to use electronic calculators to perform simple calculations, avoiding the use of fractions to perform mental arithmetic. Some students tell me fractions simply aren't emphasized much at all. This deficit causes much trouble when they must apply fractions to algebraic expressions, particularly when they must multiply or divide fractions together in order to manipulate an equation to solve for certain variables. As an instructor you should anticipate spending time in class reviewing and applying basic fractional arithmetic: how to add and subtract fractions, how to multiply and divide fractions, and how to reduce compound fractions to simpler forms. With regard to the application of fractions to equation manipulation, I recommend *multiplying by a fraction's reciprocal* in order to cancel out a fraction-multiplication as opposed to *dividing by that fraction*, since the division of one fraction by another naturally creates a compound fraction which then must be reduced in a subsequent step.

- **Voltage polarity symbols** – the "+" and "−" symbols used to denote voltages in circuits only have meaning as *pairs* and do not mean anything on their own.

- **Voltage as a relative quantity** – this is arguably one of the most difficult concepts students first encounter when studying electricity: understanding that voltage is a *difference* in energy levels between two locations, and not something that can exist at any single point. In this respect, voltage is similar to *distance* which is always something existing *between* two specified points.

- **Choosing KVL loops** – the choice of which pathway to "step around" in any electrical network while applying Kirchhoff's Voltage Law is completely arbitrary so long as the path ends where it began. The loop need not follow in the same direction as current, nor need it even follow conductors, nor need it include a source! What you are doing when tracing a KVL loop is taking an imaginary "test charge" on a journey to various points in an electrical network and tallying how much energy that charge would have to gain or lose if it moved from one point to another. The path that imaginary "test charge" takes is utterly irrelevant to the truth of KVL, as is the means by which it might move from point to point along that path. To have practical value, any KVL loop we trace should have but one unknown voltage in it (i.e. one interval between points where we do not yet know the voltage) with all the other voltage values known.

- **Differentiating series from parallel connections** – series networks are defined by having just a single path for current between multiple components, whereas parallel networks are defined as having components span the same two sets of electrically common points. An interesting feature of five-resistor bridge networks is that no two resistors are in series or in parallel, making the bridge configuration truly unique! However, all laws learned for analyzing series and parallel networks still apply: Ohm's Law, Joule's Law, Kirchhoff's Voltage Law, Kirchhoff's Current Law, etc.

- **Schematic annotation as a problem-solving tool** – as an instructor you will find most students new to the study of electronics attempt to solve circuit-analysis problems by inspection, or by plugging given values into familiar equations until something resembling an answer emerges. Such efforts are usually fruitless because it is difficult to track the real-world context of every circuit parameter doing things this way. You will need to demonstrate, encourage, and at times *require* your students to apply the alternative strategy of first sketching a schematic diagram of the circuit and then annotating it with all known values. Once the given information gains context through this annotation, appropriate principles (e.g. Ohm's Law, KVL, KCL) are much easier to identify and then properly apply for that circuit. Then, as calculated values get annotated on the same schematic diagram the next logical steps to follow become easier and easier to identify. I recommend using different colors to label different parameters (e.g. blue for voltage, red for current, black for component values, green for power). Students should have blank "scratch" paper and writing instruments with them at all times, even during discussions, so as to neatly organize information and to articulate and focus their thoughts!

1.3 Recommendations for instructors

This section lists realistic student learning outcomes supported by the content of the module as well as suggested means of assessing (measuring) student learning. The outcomes state what learners should be able to do, and the assessments are specific challenges to prove students have learned.

- **Outcome** – Demonstrate effective technical reading and writing

  Assessment – Students present their outlines of this module's instructional chapters (e.g. Case Tutorial, Tutorial, Historical References, etc.) ideally as an entry to a larger Journal document chronicling their learning. These outlines should exhibit good-faith effort at summarizing major concepts explained in the text.

  Assessment – Students show how resistance values needed to balance a bridge network were obtained by the author in the Tutorial chapter's examples.

  Assessment – Students present their "Conceptual Toolbox" Theory Journal entries listing important concepts applied in the Tutorial text(s). A "Conceptual Toolbox" serves as inspiration when solving problems, each concept suggesting a potential problem-solving action. Challenge students to collaboratively generate a Conceptual Toolbox for the day's topic during the class session and then use it when solving problems together, thus fostering the habit of reasoning from principles rather than recalling procedures!

- **Outcome** – Apply the concept of resistance ratios to bridge network balancing

  Assessment – determine proper resistance values to balance a Wheatstone bridge network; e.g. pose problems in the form of the "Balancing a resistive bridge" Quantitative Reasoning question.

- **Outcome** – Apply Kirchhoff's Voltage Law to unbalanced bridge circuit analysis

  Assessment – predict DC voltage and polarity between specified points within an unbalanced bridge circuit; e.g. pose problems in the form of the "Ground-referenced and differential voltage measurements" Quantitative Reasoning question.

- **Outcome** – Apply the problem-solving strategy of Limiting Cases

  Assessment – predict qualitative changes in voltage for bridge circuits given some stimulus or component value change; e.g. pose problems in the form of the "Thermistor bridge circuit" Conceptual Reasoning question.

- **Outcome** – Independent research

  Assessment – Locate datasheets for load cells and identify the strain-gauge resistance(s) contained within.

  Assessment – Locate datasheets for bridge rectifier ICs and identify the rectifying diodes contained within.

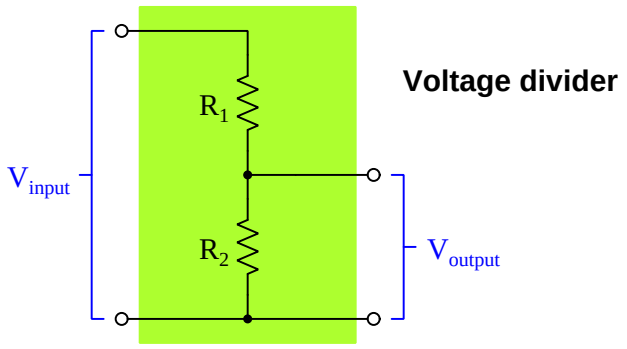
  Assessment – Locate datasheets for H-bridge transistor modules and identify how the individual transistors contained within connect to form a bridge.

Chapter 2

Tutorial

2.1 Voltage divider networks

A very important circuit configuration known as a *voltage divider* consists of series-connected resistances, its purpose being to divide an applied voltage into smaller proportions. A simple example of a voltage divider appears in this schematic diagram, shown by itself with no power source connected, but with terminals provided for input and output voltages:



The characteristic formula for a voltage divider, predicting the amount of voltage dropped across any one of the series resistances R, is as follows:

$$V_{output} = V_{input}\left(\frac{R}{R_{total}}\right)$$

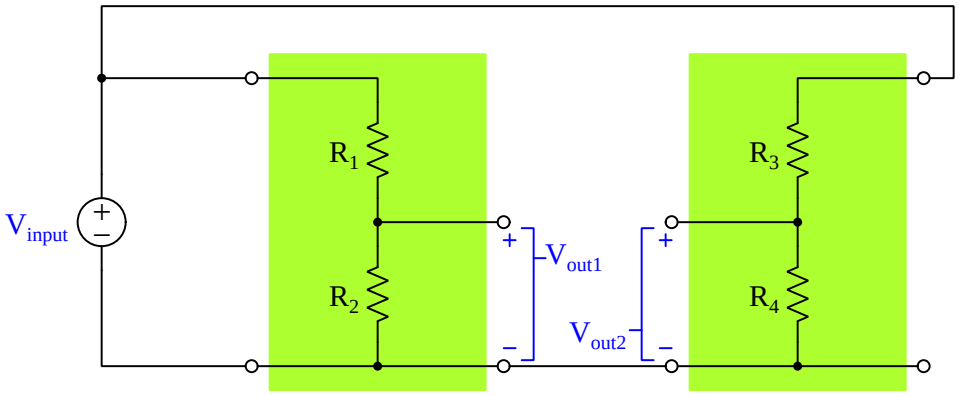
In the case of the two-resistor voltage divider illustrated above, the formula could be written as:

$$V_{output} = V_{input}\left(\frac{R_2}{R_1 + R_2}\right)$$

The ratio of output voltage to input voltage $\left(\frac{V_{output}}{V_{input}}\right)$ is established by the ratio of one resistance to the total resistance $\left(\frac{R}{R_{total}}\right)$, and is completely independent of the applied (input) voltage. Regardless of V_{input}'s magnitude, the ratio $\frac{V_{output}}{V_{input}}$ must always be equal to $\frac{R_2}{R1+R2}$.

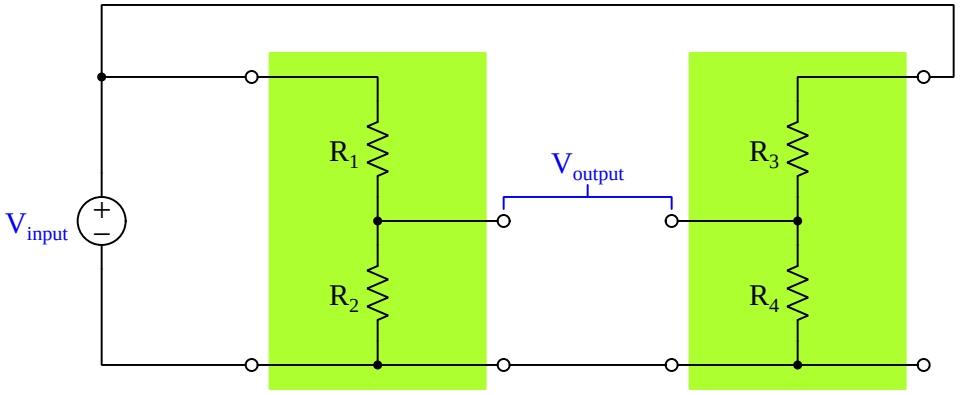
2.2 Bridge networks

Now consider *two* voltage divider networks powered by the same source voltage:



The output of the left-hand divider (V_{out1}) will be the fraction of V_{input} set by the ratio of $\frac{R_2}{R_1+R_2}$, while the output of the right-hand divider (V_{out2}) will be the fraction of V_{input} set by the ratio of $\frac{R_4}{R_3+R_4}$. If the ratios $\frac{R_2}{R_1+R_2}$ and $\frac{R_4}{R_3+R_4}$ happen to be equal to each other, then the two output voltages V_{out1} and V_{out2} will be equal to each other as well, being identical proportions of the same input voltage (V_{input}).

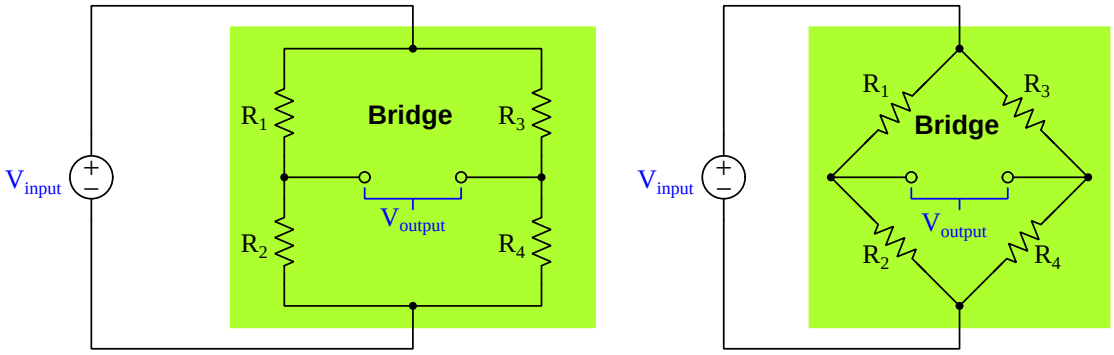
If we define the "output" voltage of this circuit to be the difference of potential between the two open terminals rather than have two output voltages, we see that it must be the *difference* between what was formerly labeled V_{out1} and V_{out2}:



The fact that $V_{output} = V_{out1} - V_{out2}$ is evident if we apply Kirchhoff's Voltage Law around a loop encompassing the two open terminals and the wire connecting the two divider networks together at the bottom. If we imagine the ratios of the two dividers being equal (i.e. $\frac{R_2}{R_1+R_2} = \frac{R_4}{R_3+R_4}$) then

this output voltage must be zero, because there will be no *difference* between the individual output voltages of each divider network.

Such a resistance network, comprised of two voltage dividers with the output voltage signal being a comparison of the two dividers' outputs, is commonly referred to as a *bridge* network, and is typically drawn in one of the following forms:



Note that these two networks are electrically identical. They only differ in orientation (i.e. angled resistors and connecting wires), not in the components or the connections. Although the "diamond" (right) depiction is more common, you should feel free to think of bridge circuits drawn rectangularly (left) if this is helpful.

A bridge circuit is said to be *balanced* when its two voltage divider sub-networks have equal voltage-division ratios, thereby causing V_{output} to be zero. With some algebraic manipulation, we may simplify the "balance equation" for this bridge circuit even more:

$$\frac{R_2}{R_1 + R_2} = \frac{R_4}{R_3 + R_4}$$

$$\frac{R_1 + R_2}{R_2} = \frac{R_3 + R_4}{R_4}$$

$$\frac{R_1}{R_2} + \frac{R_2}{R_2} = \frac{R_3}{R_4} + \frac{R_4}{R_4}$$

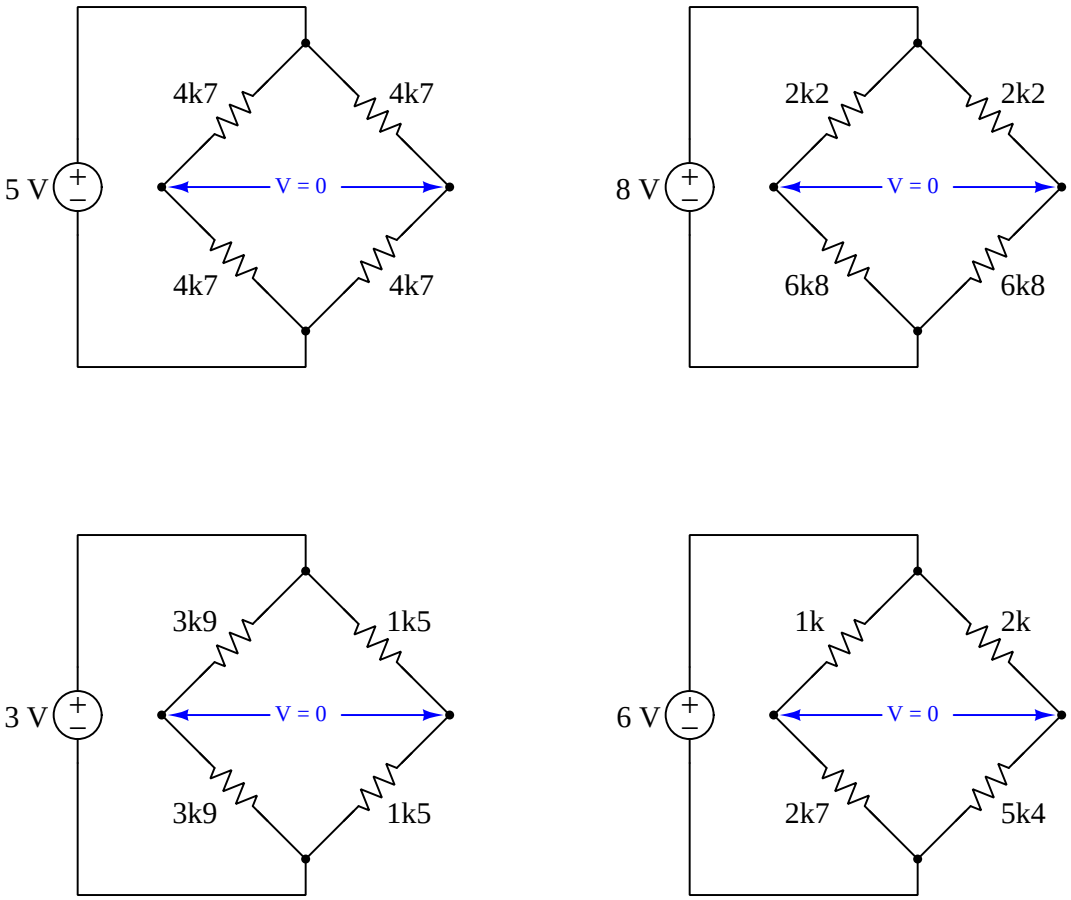
$$\frac{R_1}{R_2} + 1 = \frac{R_3}{R_4} + 1$$

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

With this simple equation, it is possible to use a bridge circuit to compare the value of an unknown resistance against the values of three known resistances. If such a bridge circuit is constructed, energized, and then one of its known resistances varied until a "balanced" condition is achieved, the value of the unknown resistance may be found by solving the balance equation for that one resistance.

2.3 Balanced bridge examples

Each of the following bridge circuits is electrically balanced, and we may tell this is true by comparing the resistance ratios from one side of the bridge network to the other. Examine each of these example circuits, and prove to yourself that the ratios in each case are indeed equal:

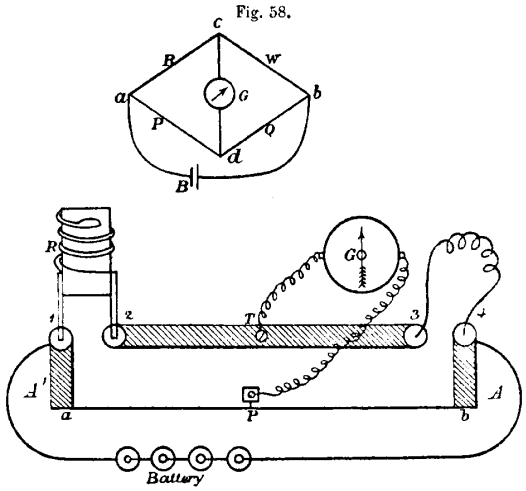


Whether we compare the upper/lower resistance ratio on the left side against the upper/lower resistance ratio on the right side for equality, or we compare left/right resistance ratio on the upper half against the left/right ratio on the lower half, equal ratios mean the bridge is balanced. Note that balance depends *only* on these resistance ratios being equal, and not on the voltage source's magnitude.

Comparing ratios on the lower-right bridge example, we see that $\frac{1000}{2700} = \frac{2000}{5400}$, $\frac{1000}{2700}$ representing the ratio for the two left-side resistors and $\frac{2000}{5400}$ representing the ratio for the two right-side resistors. Alternatively, we may compare resistance values along the other dimensional axis and see that $\frac{1000}{2000} = \frac{2700}{5400}$, $\frac{1000}{2000}$ being the ratio for the two upper resistors and $\frac{2700}{5400}$ being the ratio for the two lower resistors.

2.4 Resistance-measuring bridges

Bridge circuits were first used as a means of measuring unknown resistance values, by providing a relatively easy way to compare known ("standard") resistances against an unknown resistance. An illustration from John Ambrose Fleming's book *Short Lectures to Electrical Artisans* written in 1886 shows a typical resistance-measuring bridge of that era, commonly referred to as a *Wheatstone bridge* after being popularized by the English scientist Sir Charles Wheatstone:



Wheatstone's Bridge.

The unknown resistance in this example is a length of wire connected between terminals 3 and 4 in the pictorial diagram, labeled w in the schematic diagram. A "standard" resistor of known resistance (R) was connected between terminals 1 and 2, labeled R in the schematic and pictorial diagrams. These two resistances – the coiled wire and the standard resistor – formed one of the two voltage-divider networks of the bridge, representing the path b-c-a in the schematic diagram. A length of fine-gauge "German silver" wire[1] stretched between A' and A in the pictorial diagram (path a-d-b in the schematic) with a sliding contact P formed a primitive potentiometer which acted as an adjustable voltage divider for the other half of the bridge network. A very sensitive *galvanometer* (G) acted as the "null" sensor to indicate when the bridge was balanced by showing zero in the balanced condition.

The person operating this apparatus would move the sliding contact (P) along the wire until the galvanometer registered zero, indicating the bridge was then in a balanced condition. Once balanced, the sliding contact's position along the fine-gauge German silver wire could be measured with a precision ruler. So long as that metal wire had a consistent amount of resistance per unit length, the position measurement of that sliding contact along the wire would then indicate a resistance ratio formed by the wire's sections (from one end to the sliding contact, versus from the sliding contact to the other end). That ratio of physical distances must then be the same as the ratio of the unknown versus standard resistances, given an electrically-balanced bridge circuit.

[1] *German silver* is actually a metal alloy of copper, nickel, and zinc with the lustrous appearance of silver but a much greater electrical resistivity than actual silver. Its temperature coefficient of resistance is also low.

If the wire's resistance happened to equal the standard resistor's, the bridge would balance when the slider was moved precisely to the mid-way position along the potentiometer. For any condition where $R_{unknown} \neq R_{standard}$, the bridge would become balanced whenever the slider was moved to that same *proportional* position along the potentiometer's slide-wire. For example, if $R_{wire} = 150$ Ohms and $R_{standard} = 250$ Ohms, the galvanometer would finally register zero when the slider was moved to the position matching that same proportion:

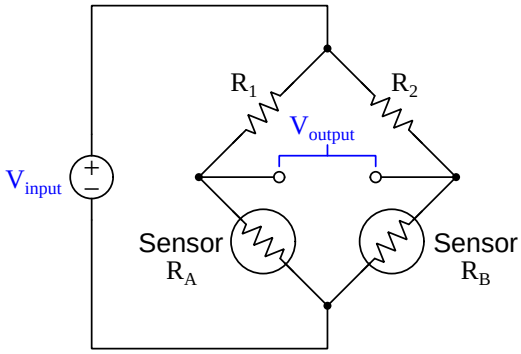
$$\frac{150}{250} = \frac{(P - b) \text{ distance}}{(P - a) \text{ distance}}$$

Although this may seem like a strange and tedious way to measure resistance, bear in mind that accurate electrical meters were scarce or non-existent for most people back in 1886. It was far more practical at that time to build your own standard resistor using wire of known metal composition and gauge than it was to fabricate a meter that could measure voltage or current with comparable accuracy. The galvanometer used in a bridge network did not have to be accurate in the modern sense of the term: it merely had to tell the difference between zero and non-zero. Any sufficiently sensitive meter would suffice for this purpose. Also, the bridge circuit's accuracy did not depend on the power source being stable: even if the battery's voltage decreased or drifted over time you could still make accurate resistance measurements using the bridge because balancing that circuit was strictly a matter of matching *ratios* of resistance, and doing that was as simple as measuring *ratios* of physical distance (length) where the sliding contact touched the slide-wire. *In other words, this system allowed experimenters to accurately measure electrical resistance simply by accurately measuring length!* The beauty of a bridge measurement network is that it is based solely on ratios of component values, and therefore we refer to this as a *ratiometric* system.

As electrical meter technology improved, bridge circuits still remained a viable way to measure resistance with high accuracy. Modern electrical metrology laboratories still use bridge circuits for such purposes when the need for absolute accuracy is paramount. Other types of bridge circuits were developed for ratiometric measurement including measurement of component characteristics such as capacitance and inductance.

2.5 Sensor bridges

Bridge circuits are also useful for comparing physical quantities that result in electrical resistance variations. If we construct a bridge circuit where two of the resistances are resistive *sensors* (e.g. photocells[2], strain gauges[3], thermistors[4], etc.), the output voltage of the bridge circuit becomes a representation of that physical comparison:



If fixed resistors R_1 and R_2 are equal and sensors A and B are identical, then the bridge circuit will be balanced whenever those two sensors experience the same amount of stimulus. However, if one sensor experiences a different amount of stimulus than the other, the bridge circuit will become "unbalanced" resulting in a voltage between its output terminals, the magnitude of that voltage increasing as the sensed difference increases. The polarity of an unbalanced bridge circuit's output voltage indicates which sensor detects more stimulus. For the dual-sensor bridge shown above, an increase in sensor resistance R_A will result in the left-hand output terminal becoming more positive and the right-hand terminal becoming more negative because that increase in R_A causes the left-hand voltage divider sub-network's ratio to increase while the right-hand divider's ratio remains unchanged.

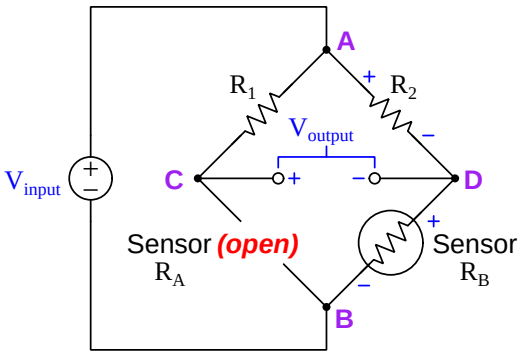
A useful problem-solving technique for determining the output voltage polarity of an unbalanced bridge circuit is to imagine the given change in sensor resistance being extreme rather than modest. For example, if we wish to determine the output voltage polarity in the above bridge circuit given an increase in R_A beyond a balanced condition, we may simply imagine R_A becoming infinite, which is equivalent to R_A becoming *open*. This technique of considering an extreme version of the given condition is called *limiting cases*, since an open resistor is the limiting case for a resistance increase. Likewise, if the scenario were sensor R_A decreasing in resistance, we could apply this same technique by imagining the effects of R_A *shorting*. The power of this problem-solving technique lies in the fact that the circuit generally becomes simpler to analyze once components are eliminated by replacement with either shorts or opens.

[2] A *photocell* exhibits a resistance varying with light exposure.

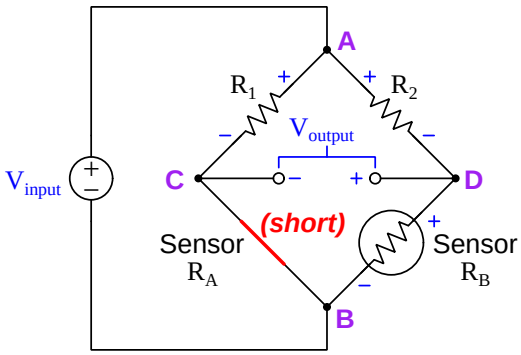
[3] A *strain gauge* is a thin conductor made longer and thinner by the application of physical tension (stretching), and shorter and thicker by physical compression. Thus, a strain gauge's resistance increases with tension and decreases with compression. When bonded to a specimen, a strain gauge is able to sense how far that specimen strains (i.e. stretches or compresses) with applied force, which may be used to infer the amount of physical force being applied to that specimen.

[4] A *thermistor's* resistance varies with temperature.

Here we see the "limiting cases" technique applied to both R_A increasing and R_A decreasing in this bridge circuit:



An *open* R_A is the limiting case of R_A *increasing* resistance

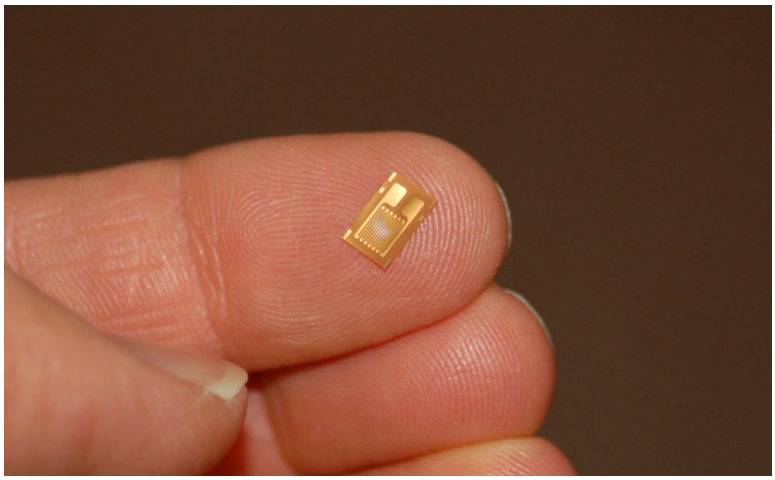


A *shorted* R_A is the limiting case of R_A *decreasing* resistance

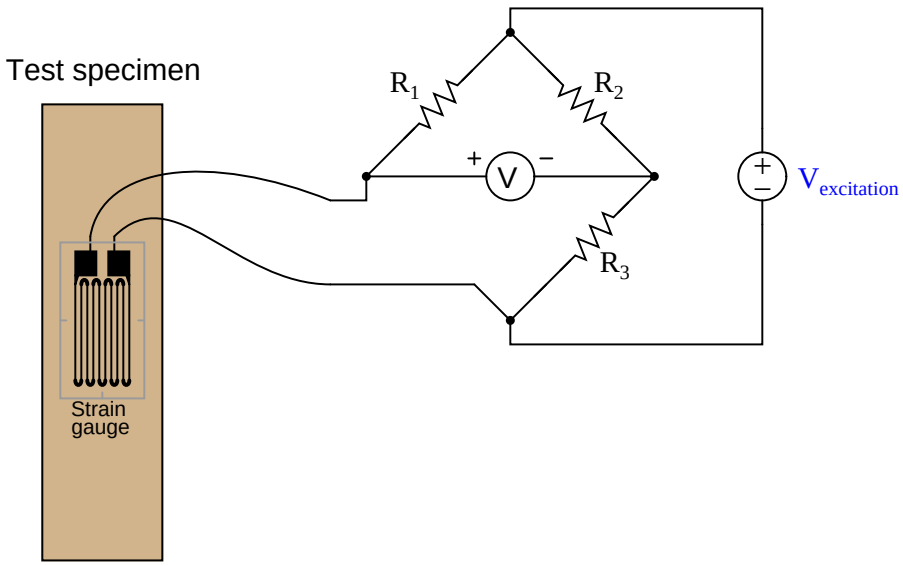
In the R_A increase scenario, we replace sensor R_A with an *open* to simulate the limiting case of more resistance. This has the effect of halting current through resistor R_1, since the open R_A prevents current through the series sub-network of R_1 and R_A. Zero current passing through the finite resistance of R_1 results in zero voltage dropped across R_1, and zero voltage across R_1 makes points A and C in the bridge circuit equipotential. When points A and C become equipotential, the voltage between the output terminals becomes the same as the voltage dropped across resistor R_2 (i.e. the voltage between points A and D), since the right-hand output terminal is already electrically common to D and the left-hand terminal is now equipotential to A. Another perspective is to consider the output terminals as being essentially in *parallel* with R_2 now that R_A is open and R_1 conducts no current, and therefore the output terminals must exhibit the same voltage between them as across resistor R_2.

In the R_A decrease scenario, we replace sensor R_A with a *short* to simulate the limiting case of less resistance. This has the effect of making points B and C electrically common, and therefore equipotential to each other. With points B and C now electrically common, the voltage between the output terminals becomes the same as the voltage dropped across sensor R_B (i.e. the output terminals are now literally in *parallel* with R_B and therefore must share the same voltage).

One of the most common applications for this kind of bridge circuit is in strain measurement, where the mechanical strain of an object is converted into an electrical signal. The sensor used here is a device known as a *strain gauge*: a folded wire designed to stretch and compress with the object under test, altering its electrical resistance accordingly. Strain gauges are typically quite small, as shown by this photograph:



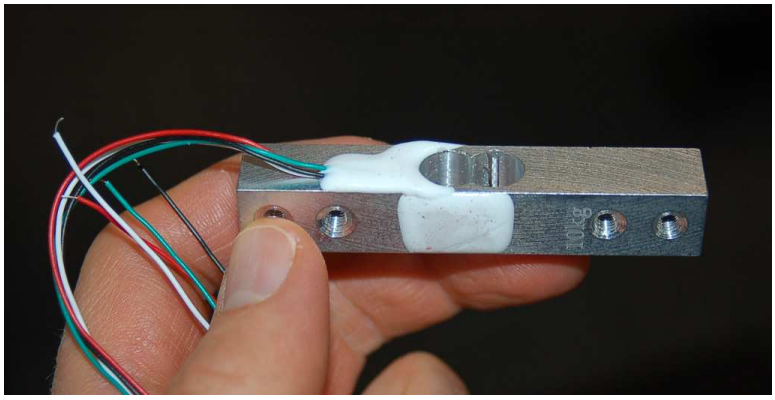
Strain gauges are useful when bonded to metal specimens, providing a means of electrically sensing the strain ("stretching" or "compressing" of that specimen). The following bridge circuit is a typical application for a strain gauge:



When the specimen is stretched along its long axis, the metal wires in the strain gauge stretch with it, increasing their length and decreasing their cross-sectional area, both of which work to

increase the wire's electrical resistance. This stretching is microscopic in scale, but the resistance change is measurable and repeatable within the specimen's elastic limit. In the above circuit example, stretching the specimen will cause the voltmeter to read upscale (as defined by the polarity marks). Compressing the specimen along its long axis has the opposite effect, decreasing the strain gauge resistance and driving the meter downscale.

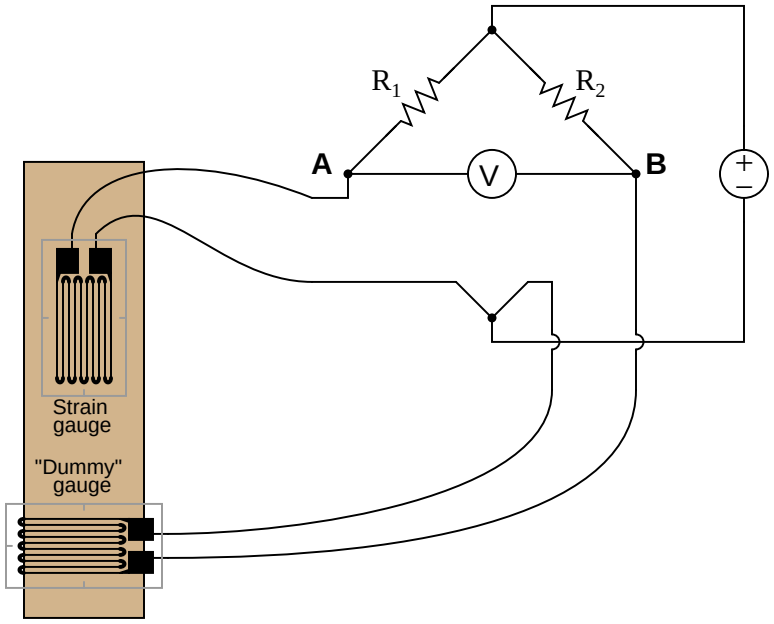
Strain gauges are used to precisely measure the strain (stretching or compressing motion) of mechanical elements. One application for strain gauges is the measurement of strain on machinery components, such as the frame components of an automobile or airplane undergoing design development testing. Another application is in the measurement of force in a device called a *load cell*. A "load cell" is comprised of one or more strain gauges bonded to the surface of a metal structure having precisely known elastic properties. This metal structure will stretch and compress very precisely with applied force, as though it were an extremely stiff spring. The strain gauges bonded to this structure measure the strain, translating applied force into electrical resistance changes. You can see what a load cell looks like in the following photograph:



Strain gauges are not the only dynamic element applicable to bridge circuits. In fact, any resistance-based sensor may be used in a bridge circuit to translate a physical measurement into an electrical (voltage) signal. Thermistors (changing resistance with temperature) and photocells (changing resistance with light exposure) are just two alternatives to strain gauges.

It should be noted that the amount of voltage output by this bridge circuit depends both on the amount of resistance change of the sensor *and* the value of the excitation source. This dependency on source voltage value is a major difference between a sensing bridge circuit and a Wheatstone (balanced) bridge circuit. In a perfectly balanced bridge, the excitation voltage is irrelevant: the output voltage is zero no matter what source voltage value you use. In an unbalanced bridge circuit, however, source voltage value matters! For this reason, these bridge circuits are often rated in terms of how many millivolts of output they produce *per volt of excitation* per unit of physical measurement (microns of strain, newtons of stress, etc.).

An interesting feature of a sensing bridge circuit is its ability to cancel out unwanted variables. In the case of a strain gauge, for example, mechanical strain is not the only variable affecting gauge resistance. Temperature also affects gauge resistance. Since we do not wish our strain gauge to also act as a thermometer (which would make measurements very uncertain – how would we differentiate the effects of changing temperature from the effects of changing strain?), we must find some way to nullify resistance changes due solely to temperature, such that our bridge circuit will respond only to changes in strain. The solution is to creatively use a "dummy" strain gauge as another arm of the bridge:



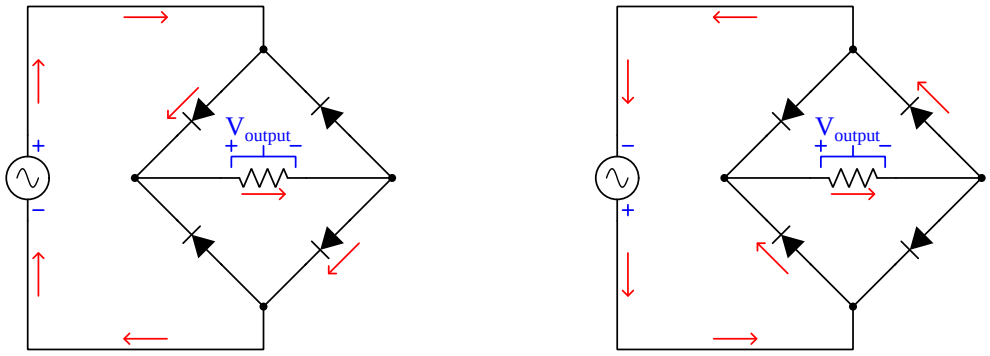
The "dummy" gauge is attached to the specimen in such a way that it maintains the same temperature as the active strain gauge, yet experiences no strain. Thus, any *difference* in gauge resistances must be due solely to specimen strain. The differential nature of the bridge circuit naturally translates the differential resistance of the two gauges into one voltage signal representing strain.

2.6 Power bridges

Bridge circuits are not just useful for measurement, but may be used for other purposes as well if different types of components are inserted into the "arms" of the bridge.

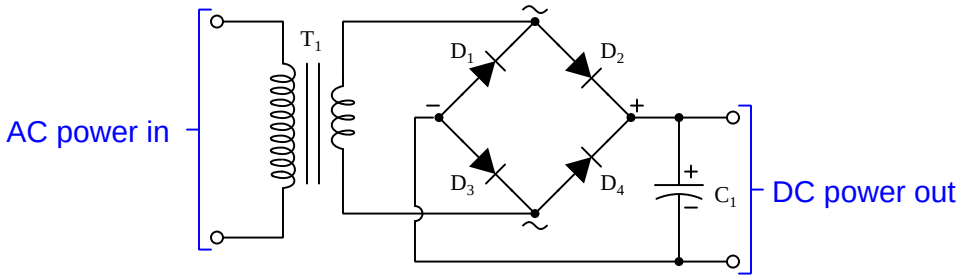
One such bridge circuit is the *bridge rectifier*, used to convert alternating current (AC) into direct current (DC) through the use of *diodes* which are solid-state electronic components acting as one-way valves for electric current:

Bridge rectifier circuit



Note how the four diodes in the bridge "steer" the AC source's current through the load resistor from left to right regardless of source polarity. As the AC source alternates back and forth, current through the resistor always travels in the same direction.

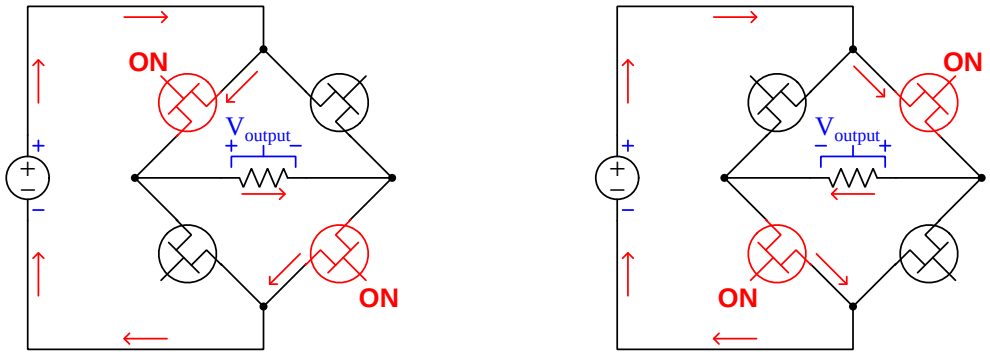
A simple "brute-force" design of AC-to-DC power conversion circuit appears in the following schematic diagram:



The transformer (T_1) is an electromagnetic device converting higher-voltage AC (from a common wall receptacle) into lower-voltage more suitable for powering electronic circuits. The capacitor (C_1) works to "filter" the rectified power so that it fluctuates less over time, resulting in DC power to the load more closely resembling power from a chemical battery or other natural DC source.

Another type of bridge circuit uses solid-state *transistor* switches to convert DC into AC by alternately switching on and off in opposite pairs. These transistors function as on/off switching elements, but unlike regular switches they have no moving parts and are able to turn on and off at extremely rapid rates if necessary. This is the opposite of rectification, called *inversion*:

Bridge inverter circuit



The circuitry generating the controlling signals to trigger these four transistors to turn on and off is not shown in this simplified diagram, only the bridge network itself. Coupled to other components such as capacitors and transformers, such inverter bridge networks are useful for DC-to-AC power conversion, AC motor speed control, and other similar applications.

Chapter 3

Historical References

This chapter is where you will find references to historical texts and technologies related to the module's topic.

Readers may wonder why historical references might be included in any modern lesson on a subject. Why dwell on old ideas and obsolete technologies? One answer to this question is that the initial discoveries and early applications of scientific principles typically present those principles in forms that are unusually easy to grasp. Anyone who first discovers a new principle must necessarily do so from a perspective of ignorance (i.e. if you truly *discover* something yourself, it means you must have come to that discovery with no prior knowledge of it and no hints from others knowledgeable in it), and in so doing the discoverer lacks any hindsight or advantage that might have otherwise come from a more advanced perspective. Thus, discoverers are forced to think and express themselves in less-advanced terms, and this often makes their explanations more readily accessible to others who, like the discoverer, comes to this idea with no prior knowledge. Furthermore, early discoverers often faced the daunting challenge of explaining their new and complex ideas to a naturally skeptical scientific community, and this pressure incentivized clear and compelling communication. As James Clerk Maxwell eloquently stated in the Preface to his book *A Treatise on Electricity and Magnetism* written in 1873,

> It is of great advantage to the student of any subject to read the original memoirs on that subject, for science is always most completely assimilated when it is in its nascent state . . . `[page xi]`

Furthermore, grasping the historical context of technological discoveries is important for understanding how science intersects with culture and civilization, which is ever important because new discoveries and new applications of existing discoveries will always continue to impact our lives. One will often find themselves impressed by the ingenuity of previous generations, and by the high degree of refinement to which now-obsolete technologies were once raised. There is much to learn and much inspiration to be drawn from the technological past, and to the inquisitive mind these historical references are treasures waiting to be (re)-discovered.

3.1 Early references to Wheatstone bridges

The Scottish physicist James Clerk Maxwell (1831-1879) wrote a book entitled *A Treatise on Electricity and Magnetism*, within which he describes two different methods of making comparative resistance measurements.

The first method used a special instrument called a *differential galvanometer*, consisting of two identical wire coils flanking a magnetized iron needle suspended on a thread. The needle will be deflected (i.e. turned to a different position) by the magnetic force generated by current passing through either wire coil. If equal currents are passed through the two coils, in different directions so as to produce opposing magnetic fields, the resultant force on the needle will be zero and no deflection will result. Thus, the differential galvanometer registers any *difference* in value between two electric currents: any inequality makes the needle move, while precisely equal currents makes no motion at all.

When the two coils of a differential galvanometer are connected in series with two resistances to be compared, and each of these series networks powered by a common voltage source (battery), the galvanometer needle will deflect if there is any difference between the two resistance values. If the two resistances are exactly equal and the two galvanometer coils are precisely identical to each other, then no deflection will result. The beauty of this technique is that it remains accurate even if the battery voltage is uncertain or even unstable over time. Maxwell describes the utility of measuring resistance in this manner:

> The merit of the method consists in the fact that the thing observed is the absence of any [sic] deflexion, or in other words, the method is a Null method, one in which the non-existence of a force is asserted from an observation in which the force, it it had been different from zero by more than a certain small amount, would have produced an observable effect. [page 473]
>
> Null methods are of great value where they can be employed, but they can only be employed where we can cause two equal and opposite quantities of the same kind to enter into the experiment together. [page 474]

By testing for the *absence* of galvanometer needle motion rather than some precise amount of needle motion, there is no need for absolute measurements of either voltage or current as there would be if we were inferring resistance from Ohm's Law ($R = \frac{V}{I}$).

A weakness of this differential galvanometer method is the possibility that the two wire coils within the galvanometer might not be precisely identical to one another. If those two wire coils differed at all, either in the number of turns, the amount of wire resistance, their relative positions from the suspended needle, etc., the needle would in fact move slightly when exposed to two identical currents and therefore it would not be reliable as a "null" instrument.

After describing the use of a differential galvanometer to make comparative resistance measurements, Maxwell then proceeds to describe the Wheatstone bridge method which escapes this weakness of differential galvanometers:

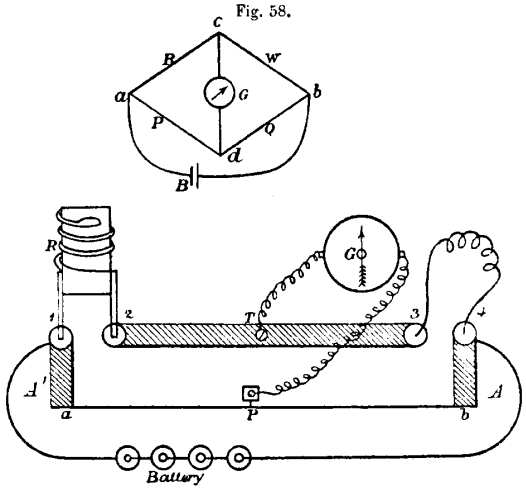
> The other null method, in which Wheatstone's Bridge is used, requires only an ordinary galvanometer, and the observed zero deflexion of the needle is due, not to the opposing action of two currents, but to the non-existence of a current in the wire. Hence we have not merely a null deflexion, but a null current as the phenomena observed, and no errors can arise from want of regularity or change of any kind in the coils of the galvanometer. The galvanometer is only required to be sensitive enough to detect the existence and direction of a current, without in any way determining its value or comparing its value with that of another current. [page 475]

It may seem strange to us living in the twenty-first century that so much trouble would be taken to perform a task as simple as measuring resistance, but this is only because we take for granted the existence of precise, accurate, and traceable standards for such things as voltage, current, and resistance measurement. In Maxwell's time none of these were a given, and the laboratory experimenter needed to devise clever means of making precise measurements using what we would consider primitive equipment. The Wheatstone bridge was a clever solution to the problem of precise resistance measurement, necessitating only an arbitrarily sensitive meter (galvanometer) and some trusted resistance specimen for comparison.

Another early reference to the use of Wheatstone bridge circuits for resistance measurement comes from John Ambrose Fleming's book *Short Lectures to Electrical Artisans* written in 1886. This text gives a fairly detailed explanation of the bridge circuit's function and operation. Bear in mind that this was in an ear before the advent of multimeters, when stable-voltage batteries were rare, when slide-wire potentiometers needed to be fabricated out of resistive wire, and when the most secure means of joining conductors together with a minimum of resistance and stray voltage was by dipping wires into cups of liquid mercury:

> . . . The well-known Wheatstone's bridge is the most usual appliance for comparing resistances.
>
> As is well known, the method was not invented by Wheatstone, but my Mr. Christie, in 1833[1]. It consists essentially of an arrangement of six conductors, joining four points. One of these conductors is a battery circuit, and the other a galvanometer. A Wheatstone bridge, suitable for workshop purposes, may be made as follows: – On a board about 4 feet long fasten down a paper scale of inches and tenths, and over the paper scale stretch a fine uniformly drawn German `[page 142]`

Fig. 58.



Wheatstone's Bridge.

> silver wire. The ends of the wire are soldered to thick copper strips A A' (see Fig. 58), which terminate in mercury cups 1, 4. Between these extreme cups there is a strip of copper B about 1 inch wide and one-sixteenth of an inch thick, having mercury cups 2, 3, at its ends. There is also a terminal T soldered in the middle.
>
> Prepare five resistance coils, as described above, of approximately 1, 5, 10, 50, 100 ohms, and let them, if possible, be adjusted by some one who possesses known standards. These coils are made so that their copper legs will dip into the mercury cups 1, 2. A battery of

[1] `Footnote from Fleming's text:` See Wheatstone's Scientific Papers, p. 129, published by the Physical Society of London.

> half-a-dozen Leclanché cells is also needed, and a delicate astatic galvanometer. [page 143]
>
> Insert the standard coil in the cups 1, 2, the ends of the wire whose resistance is to be measured in the cups 3, 4. Connect the battery to the cups 1, 4, one terminal of the galvanometer to the terminal T, and take the other end of the galvanometer wire in the hand, and just touch it to some point on the German silver wire. The galvanometer will in general make a deflection, but by sliding the galvanometer wire along the German silver wire a point P can be found at which there is no deflection right or left. When this is the case, note the lengths of the German silver wire Pa and Pb on either side of the point of no current. Then, as length Pa is to length Pb, so is the value of the standard of resistance R to the resistance of the wire to be measured to. Hence a simple rule of four sum gives us the required resistance.
>
> To understand the reason for this rule, we see, if we refer to the upper diagram in Fig. 58, that the battery current from B entering at a has two routes open to it to pass to b, one by the resistances R and W, and the other by the resistances P and Q. Down each of these paths there is a gradient of pressure or fall of potential, and when the "bridge is balanced," that is when a point is found on each path, such that no current will flow between them when they are connected by a galvanometer, it follows that the pressure at c is equal to that at d. Now the fall in potential in passing from a to c along R must be equal to the fall in potential in going from a to d along P, and similarly the fall of pressure from c to b is equal to that from d to b. Now just as in the first method given of measuring resistance we see that the ratio of resistance R to resistance W is equal to that ratio of fall of potential between a c and that between c b, and this ratio is equal to that of fall of potential between a d to that between d b, which in its turn is equal to the ratio of the resistance P to Q. Accordingly when the bridge is balanced we have the proportion – [page 144]
>
> As resistance R is to resistance W, so is resistance P to resistance Q, and any three of these being given we can find the fourth. [page 145]

Sliding the potentiometer's lead along the resistive German silver wire (i.e. a home-made *potentiometer*) allowed the user to adjust the bridge to obtain a condition of balance indicated by a zero ("null") reading on the galvanometer. Once balance was obtained, one could read the position of the sliding wire (at point "P") along the graduated scale laid down beside the wire, and read its position as a fraction, or ratio, of the scale's total length. This ratio would then indicate the ratio of the test-wire's resistance to the "standard coil" resistance. Fleming mentions the preparation of five such standard-resistance coils which would be used as standards of comparison against the wire of unknown resistance, implying that one might need to try several of these coils before finding a reasonable balance point on the bridge, and it is here we see the wisdom of using mercury-filled cups as points of contact for the standard and for the test-wire: the liquid mercury would make a very repeatable low-resistance connection with any wire dipped into it, even after repeated trials. Any attempt to use a spring clip, screw, or any other "modern" connection method to attach the standard coils to the bridge would have resulted in slight variations in connection resistance with each trial, whereas a mercury cup would be very consistent because liquid mercury is an excellent conductor of electricity and its ability to flow would guarantee optimum contact with the metal wire every time.

Chapter 4

Questions

This learning module, along with all others in the ModEL collection, is designed to be used in an inverted instructional environment where students independently read[1] the tutorials and attempt to answer questions on their own *prior* to the instructor's interaction with them. In place of lecture[2], the instructor engages with students in Socratic-style dialogue, probing and challenging their understanding of the subject matter through inquiry.

Answers are not provided for questions within this chapter, and this is by design. Solved problems may be found in the Tutorial and Derivation chapters, instead. The goal here is *independence*, and this requires students to be challenged in ways where others cannot think for them. Remember that you always have the tools of *experimentation* and *computer simulation* (e.g. SPICE) to explore concepts!

The following lists contain ideas for Socratic-style questions and challenges. Upon inspection, one will notice a strong theme of *metacognition* within these statements: they are designed to foster a regular habit of examining one's own thoughts as a means toward clearer thinking. As such these sample questions are useful both for instructor-led discussions as well as for self-study.

[1] Technical reading is an essential academic skill for any technical practitioner to possess for the simple reason that the most comprehensive, accurate, and useful information to be found for developing technical competence is in textual form. Technical careers in general are characterized by the need for continuous learning to remain current with standards and technology, and therefore any technical practitioner who cannot read well is handicapped in their professional development. An excellent resource for educators on improving students' reading prowess through intentional effort and strategy is the book textitReading For Understanding – How Reading Apprenticeship Improves Disciplinary Learning in Secondary and College Classrooms by Ruth Schoenbach, Cynthia Greenleaf, and Lynn Murphy.

[2] Lecture is popular as a teaching method because it is easy to implement: any reasonably articulate subject matter expert can talk to students, even with little preparation. However, it is also quite problematic. A good lecture always makes complicated concepts seem easier than they are, which is bad for students because it instills a false sense of confidence in their own understanding; reading and re-articulation requires more cognitive effort and serves to verify comprehension. A culture of teaching-by-lecture fosters a debilitating dependence upon direct personal instruction, whereas the challenges of modern life demand independent and critical thought made possible only by gathering information and perspectives from afar. Information presented in a lecture is ephemeral, easily lost to failures of memory and dictation; text is forever, and may be referenced at any time.

General challenges following tutorial reading

- Summarize as much of the text as you can in one paragraph of your own words. A helpful strategy is to explain ideas as you would for an intelligent child: as simple as you can without compromising too much accuracy.
- Simplify a particular section of the text, for example a paragraph or even a single sentence, so as to capture the same fundamental idea in fewer words.
- Where did the text make the most sense to you? What was it about the text's presentation that made it clear?
- Identify where it might be easy for someone to misunderstand the text, and explain why you think it could be confusing.
- Identify any new concept(s) presented in the text, and explain in your own words.
- Identify any familiar concept(s) such as physical laws or principles applied or referenced in the text.
- Devise a proof of concept experiment demonstrating an important principle, physical law, or technical innovation represented in the text.
- Devise an experiment to disprove a plausible misconception.
- Did the text reveal any misconceptions you might have harbored? If so, describe the misconception(s) and the reason(s) why you now know them to be incorrect.
- Describe any useful problem-solving strategies applied in the text.
- Devise a question of your own to challenge a reader's comprehension of the text.

General follow-up challenges for assigned problems

- Identify where any fundamental laws or principles apply to the solution of this problem, especially before applying any mathematical techniques.
- Devise a thought experiment to explore the characteristics of the problem scenario, applying known laws and principles to mentally model its behavior.
- Describe in detail your own strategy for solving this problem. How did you identify and organized the given information? Did you sketch any diagrams to help frame the problem?
- Is there more than one way to solve this problem? Which method seems best to you?
- Show the work you did in solving this problem, even if the solution is incomplete or incorrect.
- What would you say was the most challenging part of this problem, and why was it so?
- Was any important information missing from the problem which you had to research or recall?
- Was there any extraneous information presented within this problem? If so, what was it and why did it not matter?
- Examine someone else's solution to identify where they applied fundamental laws or principles.
- Simplify the problem from its given form and show how to solve this simpler version of it. Examples include eliminating certain variables or conditions, altering values to simpler (usually whole) numbers, applying a limiting case (i.e. altering a variable to some extreme or ultimate value).
- For quantitative problems, identify the real-world meaning of all intermediate calculations: their units of measurement, where they fit into the scenario at hand. Annotate any diagrams or illustrations with these calculated values.
- For quantitative problems, try approaching it qualitatively instead, thinking in terms of "increase" and "decrease" rather than definite values.
- For qualitative problems, try approaching it quantitatively instead, proposing simple numerical values for the variables.
- Were there any assumptions you made while solving this problem? Would your solution change if one of those assumptions were altered?
- Identify where it would be easy for someone to go astray in attempting to solve this problem.
- Formulate your own problem based on what you learned solving this one.

General follow-up challenges for experiments or projects

- In what way(s) was this experiment or project easy to complete?
- Identify some of the challenges you faced in completing this experiment or project.

- Show how thorough documentation assisted in the completion of this experiment or project.
- Which fundamental laws or principles are key to this system's function?
- Identify any way(s) in which one might obtain false or otherwise misleading measurements from test equipment in this system.
- What will happen if (component X) fails (open/shorted/etc.)?
- What would have to occur to make this system unsafe?

4.1 Conceptual reasoning

These questions are designed to stimulate your analytic and synthetic thinking[3]. In a Socratic discussion with your instructor, the goal is for these questions to prompt an extended dialogue where assumptions are revealed, conclusions are tested, and understanding is sharpened. Your instructor may also pose additional questions based on those assigned, in order to further probe and refine your conceptual understanding.

Questions that follow are presented to challenge and probe your understanding of various concepts presented in the tutorial. These questions are intended to serve as a guide for the Socratic dialogue between yourself and the instructor. Your instructor's task is to ensure you have a sound grasp of these concepts, and the questions contained in this document are merely a means to this end. Your instructor may, at his or her discretion, alter or substitute questions for the benefit of tailoring the discussion to each student's needs. The only absolute requirement is that each student is challenged and assessed at a level equal to or greater than that represented by the documented questions.

It is far more important that you convey your *reasoning* than it is to simply convey a correct answer. For this reason, you should refrain from researching other information sources to answer questions. What matters here is that *you* are doing the thinking. If the answer is incorrect, your instructor will work with you to correct it through proper reasoning. A correct answer without an adequate explanation of how you derived that answer is unacceptable, as it does not aid the learning or assessment process.

You will note a conspicuous lack of answers given for these conceptual questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your conceptual answers, where applicable, is to use circuit simulation software to explore the effects of changes made to circuits. For example, if one of these conceptual questions challenges you to predict the effects of altering some component parameter in a circuit, you may check the validity of your work by simulating that same parameter change within software and seeing if the results agree.

4.1.1 Reading outline and reflections

"Reading maketh a full man; conference a ready man; and writing an exact man" – Francis Bacon

Francis Bacon's advice is a blueprint for effective education: reading provides the learner with knowledge, writing focuses the learner's thoughts, and critical dialogue equips the learner to confidently communicate and apply their learning. Independent acquisition and application of knowledge is a powerful skill, well worth the effort to cultivate. To this end, students should read

[3] *Analytical* thinking involves the "disassembly" of an idea into its constituent parts, analogous to dissection. *Synthetic* thinking involves the "assembly" of a new idea comprised of multiple concepts, analogous to construction. Both activities are high-level cognitive skills, extremely important for effective problem-solving, necessitating frequent challenge and regular practice to fully develop.

these educational resources closely, journal their own reflections on the reading, and discuss in detail their findings with classmates and instructor(s). You should be able to do all of the following after reading any instructional text:

☑ Briefly SUMMARIZE THE TEXT in the form of a journal entry documenting your learning as you progress through the course of study. Share this summary in dialogue with your classmates and instructor. Journaling is an excellent self-test of thorough reading because you cannot clearly express what you have not read or did not comprehend.

☑ Demonstrate ACTIVE READING STRATEGIES, including verbalizing your impressions as you read, simplifying long passages to convey the same ideas using fewer words, annotating text and illustrations with your own interpretations, working through mathematical examples shown in the text, cross-referencing passages with relevant illustrations and/or other passages, identifying problem-solving strategies applied by the author, etc. Technical reading is a special case of problem-solving, and so these strategies work precisely because they help solve any problem: paying attention to your own thoughts (metacognition), eliminating unnecessary complexities, identifying what makes sense, paying close attention to details, drawing connections between separated facts, and noting the successful strategies of others.

☑ Identify IMPORTANT THEMES, especially GENERAL LAWS and PRINCIPLES, expounded in the text and express them in the simplest of terms as though you were teaching an intelligent child. This emphasizes connections between related topics and develops your ability to communicate complex ideas to anyone.

☑ Form YOUR OWN QUESTIONS based on the reading, and then pose them to your instructor and classmates for their consideration. Anticipate both correct and incorrect answers, the incorrect answer(s) assuming one or more plausible misconceptions. This helps you view the subject from different perspectives to grasp it more fully.

☑ Devise EXPERIMENTS to test claims presented in the reading, or to disprove misconceptions. Predict possible outcomes of these experiments, and evaluate their meanings: what result(s) would confirm, and what would constitute disproof? Running mental simulations and evaluating results is essential to scientific and diagnostic reasoning.

☑ Specifically identify any points you found CONFUSING. The reason for doing this is to help diagnose misconceptions and overcome barriers to learning.

4.1.2 Foundational concepts

Correct analysis and diagnosis of electric circuits begins with a proper understanding of some basic concepts. The following is a list of some important concepts referenced in this module's full tutorial. Define each of them in your own words, and be prepared to illustrate each of these concepts with a description of a practical example and/or a live demonstration.

Energy

Conservation of Energy

Limiting cases as a problem-solving strategy

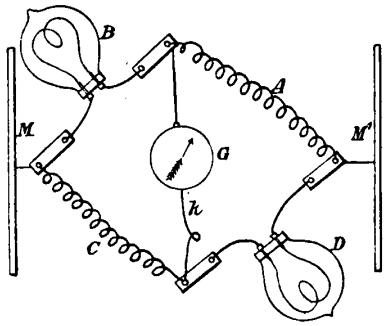
OTHER CONCEPT

4.1.3 Edison's bridge

Operators of electric generating stations must ensure that the voltage output by their generators is relatively constant, in order to ensure proper operation of all customer loads. This was especially important in the early days of electrical power systems when electric lamps were somewhat experimental and prone to poor performance, as unsteady source voltage would cause electric lamps to either burn too dimly (causing customer dissatisfaction) or fail prematurely (causing a different form of customer dissatisfaction). Therefore, operators of generating stations needed to monitor the output voltage of their generators carefully and make adjustments to maintain that voltage at some specified value.

John Ambrose Fleming, on page 145 of his book *Short Lectures to Electrical Artisans*, describes a special bridge circuit invented by Thomas Edison for determining when the voltage output by a DC generator was at a particular value. Edison's bridge consisted of two carbon-filament lamps and two coils of German silver wire. Copper bus-bars M and M' are connected to the generator's terminals to power this bridge, while G is a sensitive *galvanometer* indicating whether or not the bridge circuit is in a condition of balance, a balanced condition proving that the generator output voltage was proper. A normally open telegraph-style "key" switch (k) could be pressed at any time by the power station operator to complete the galvanometer circuit and perform a voltage check:

Fig. 59.



Edison's Bridge for preserving Constant Electromotive Force on Electric Light Circuits.

Explain why Edison's bridge circuit could only be balanced at one particular source voltage value. An important fact to know is that the carbon filaments in the lamps decreased in resistance as they warmed, while German silver increased in resistance with rising temperature.

Explain why this particular bridge circuit is sensitive to changes in supply voltage, while most Wheatstone bridges are not.

Assuming M is the positive bus and M' is the negative bus, identify the direction of current through the galvanometer when the key was pressed and the generator voltage was excessive.

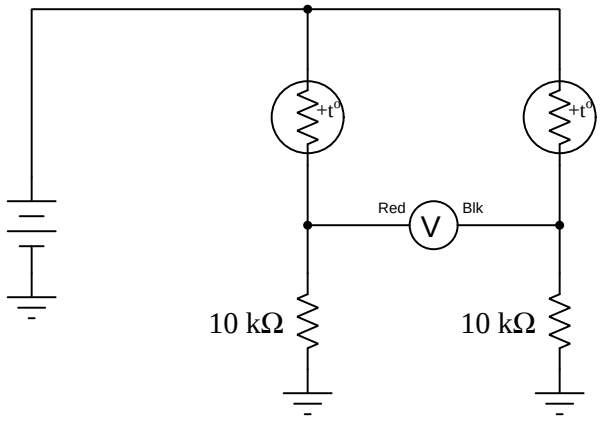
Challenges

- Would a modern version of Edison's bridge function as a voltage indicator, using tungsten-metal lamp filaments instead of carbon lamp filaments? Explain why or why not.

- Why install a momentary-contact key switch to connect the galvanometer to this bridge circuit, instead of having the galvanometer connected all the time?

4.1.4 Thermistor bridge circuit

A *thermistor* is a special resistor that dramatically changes resistance with changes in temperature. Consider the circuit shown below, with two identical thermistors:



The "+t°" label in each one shows that they both have positive α coefficients[4].

How much voltage would you expect the voltmeter to register when the two thermistors are at the exact same temperature? Which thermistor would have to become hotter in order to cause the voltmeter to read a significant *negative* voltage?

Challenges

- Can this circuit be used as a thermometer to measure room temperature? Why or why not?

[4]The "alpha" coefficient for any conductive substance is the degree to which its electrical resistance varies with changes in temperature. A positive α value means resistance increases with temperature; a negative value means resistance decreases with temperature.

4.2 Quantitative reasoning

These questions are designed to stimulate your computational thinking. In a Socratic discussion with your instructor, the goal is for these questions to reveal your mathematical approach(es) to problem-solving so that good technique and sound reasoning may be reinforced. Your instructor may also pose additional questions based on those assigned, in order to observe your problem-solving firsthand.

Mental arithmetic and estimations are strongly encouraged for all calculations, because without these abilities you will be unable to readily detect errors caused by calculator misuse (e.g. keystroke errors).

You will note a conspicuous lack of answers given for these quantitative questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. My advice is to use circuit simulation software such as SPICE to check the correctness of quantitative answers. Refer to those learning modules within this collection focusing on SPICE to see worked examples which you may use directly as practice problems for your own study, and/or as templates you may modify to run your own analyses and generate your own practice problems.

Completely worked example problems found in the Tutorial may also serve as "test cases[5]" for gaining proficiency in the use of circuit simulation software, and then once that proficiency is gained you will never need to rely[6] on an answer key!

[5] In other words, set up the circuit simulation software to analyze the same circuit examples found in the Tutorial. If the simulated results match the answers shown in the Tutorial, it confirms the simulation has properly run. If the simulated results disagree with the Tutorial's answers, something has been set up incorrectly in the simulation software. Using every Tutorial as practice in this way will quickly develop proficiency in the use of circuit simulation software.

[6] This approach is perfectly in keeping with the instructional philosophy of these learning modules: *teaching students to be self-sufficient thinkers*. Answer keys can be useful, but it is even more useful to your long-term success to have a set of tools on hand for checking your own work, because once you have left school and are on your own, there will no longer be "answer keys" available for the problems you will have to solve.

4.2.1 Miscellaneous physical constants

Note: constants shown in **bold** type are *exact*, not approximations. Values inside of parentheses show one standard deviation (σ) of uncertainty in the final digits: for example, the magnetic permeability of free space value given as $1.25663706212(19) \times 10^{-6}$ H/m represents a center value (i.e. the location parameter) of $1.25663706212 \times 10^{-6}$ Henrys per meter with one standard deviation of uncertainty equal to $0.0000000000019 \times 10^{-6}$ Henrys per meter.

Avogadro's number (N_A) = **6.02214076** $\times 10^{23}$ **per mole** (mol^{-1})

Boltzmann's constant (k) = **1.380649** $\times 10^{-23}$ **Joules per Kelvin** (J/K)

Electronic charge (e) = **1.602176634** $\times 10^{-19}$ **Coulomb** (C)

Faraday constant (F) = **96,485.33212...** $\times 10^{4}$ **Coulombs per mole** (C/mol)

Magnetic permeability of free space (μ_0) = $1.25663706212(19) \times 10^{-6}$ Henrys per meter (H/m)

Electric permittivity of free space (ϵ_0) = $8.8541878128(13) \times 10^{-12}$ Farads per meter (F/m)

Characteristic impedance of free space (Z_0) = 376.730313668(57) Ohms (Ω)

Gravitational constant (G) = $6.67430(15) \times 10^{-11}$ cubic meters per kilogram-seconds squared ($m^3/kg\text{-}s^2$)

Molar gas constant (R) = **8.314462618... Joules per mole-Kelvin** (J/mol-K) = 0.08205746(14) liters-atmospheres per mole-Kelvin

Planck constant (h) = **6.62607015** $\times 10^{-34}$ **joule-seconds** (J-s)

Stefan-Boltzmann constant (σ) = **5.670374419...** $\times 10^{-8}$ **Watts per square meter-Kelvin**^{4} ($W/m^2 \cdot K^4$)

Speed of light in a vacuum (c) = **299,792,458 meters per second** (m/s) = 186282.4 miles per second (mi/s)

Note: All constants taken from NIST data "Fundamental Physical Constants – Complete Listing", from `http://physics.nist.gov/constants`, National Institute of Standards and Technology (NIST), 2018 CODATA Adjustment.

4.2.2 Introduction to spreadsheets

A powerful computational tool you are encouraged to use in your work is a *spreadsheet*. Available on most personal computers (e.g. Microsoft Excel), *spreadsheet* software performs numerical calculations based on number values and formulae entered into cells of a grid. This grid is typically arranged as lettered columns and numbered rows, with each cell of the grid identified by its column/row coordinates (e.g. cell `B3`, cell `A8`). Each cell may contain a string of text, a number value, or a mathematical formula. The spreadsheet automatically updates the results of all mathematical formulae whenever the entered number values are changed. This means it is possible to set up a spreadsheet to perform a series of calculations on entered data, and those calculations will be re-done by the computer any time the data points are edited in any way.

For example, the following spreadsheet calculates average speed based on entered values of distance traveled and time elapsed:

	A	B	C	D
1	Distance traveled	46.9	Kilometers	
2	Time elapsed	1.18	Hours	
3	Average speed	= B1 / B2	km/h	
4				
5				

Text labels contained in cells `A1` through `A3` and cells `C1` through `C3` exist solely for readability and are not involved in any calculations. Cell `B1` contains a sample distance value while cell `B2` contains a sample time value. The formula for computing speed is contained in cell `B3`. Note how this formula begins with an "equals" symbol (`=`), references the values for distance and speed by lettered column and numbered row coordinates (`B1` and `B2`), and uses a forward slash symbol for division (`/`). The coordinates `B1` and `B2` function as *variables*[7] would in an algebraic formula.

When this spreadsheet is executed, the numerical value `39.74576` will appear in cell `B3` rather than the formula `= B1 / B2`, because 39.74576 is the computed speed value given 46.9 kilometers traveled over a period of 1.18 hours. If a different numerical value for distance is entered into cell `B1` or a different value for time is entered into cell `B2`, cell `B3`'s value will automatically update. All you need to do is set up the given values and any formulae into the spreadsheet, and the computer will do all the calculations for you.

Cell `B3` may be referenced by other formulae in the spreadsheet if desired, since it is a variable just like the given values contained in `B1` and `B2`. This means it is possible to set up an entire chain of calculations, one dependent on the result of another, in order to arrive at a final value. The arrangement of the given data and formulae need not follow any pattern on the grid, which means you may place them anywhere.

[7] Spreadsheets may also provide means to attach text labels to cells for use as variable names (Microsoft Excel simply calls these labels "names"), but for simple spreadsheets such as those shown here it's usually easier just to use the standard coordinate naming for each cell.

Common[8] arithmetic operations available for your use in a spreadsheet include the following:

- Addition (`+`)
- Subtraction (`-`)
- Multiplication (`*`)
- Division (`/`)
- Powers (`^`)
- Square roots (`sqrt()`)
- Logarithms (`ln()` , `log10()`)

Parentheses may be used to ensure[9] proper order of operations within a complex formula. Consider this example of a spreadsheet implementing the *quadratic formula*, used to solve for roots of a polynomial expression in the form of $ax^2 + bx + c$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

	A	B
1	x_1	= (-B4 + sqrt((B4^2) - (4*B3*B5))) / (2*B3)
2	x_2	= (-B4 - sqrt((B4^2) - (4*B3*B5))) / (2*B3)
3	a =	9
4	b =	5
5	c =	-2

This example is configured to compute roots[10] of the polynomial $9x^2 + 5x - 2$ because the values of 9, 5, and -2 have been inserted into cells `B3`, `B4`, and `B5`, respectively. Once this spreadsheet has been built, though, it may be used to calculate the roots of *any* second-degree polynomial expression simply by entering the new a, b, and c coefficients into cells `B3` through `B5`. The numerical values appearing in cells `B1` and `B2` will be automatically updated by the computer immediately following any changes made to the coefficients.

[8] Modern spreadsheet software offers a bewildering array of mathematical functions you may use in your computations. I recommend you consult the documentation for your particular spreadsheet for information on operations other than those listed here.

[9] Spreadsheet programs, like text-based programming languages, are designed to follow standard order of operations by default. However, my personal preference is to use parentheses even where strictly unnecessary just to make it clear to any other person viewing the formula what the intended order of operations is.

[10] Reviewing some algebra here, a *root* is a value for x that yields an overall value of zero for the polynomial. For this polynomial ($9x^2 + 5x - 2$) the two roots happen to be $x = 0.269381$ and $x = -0.82494$, with these values displayed in cells `B1` and `B2`, respectively upon execution of the spreadsheet.

Alternatively, one could break up the long quadratic formula into smaller pieces like this:

$$y = \sqrt{b^2 - 4ac} \qquad z = 2a$$

$$x = \frac{-b \pm y}{z}$$

	A	B	C
1	x_1	= (-B4 + C1) / C2	= sqrt((B4^2) - (4*B3*B5))
2	x_2	= (-B4 - C1) / C2	= 2*B3
3	a =	9	
4	b =	5	
5	c =	-2	

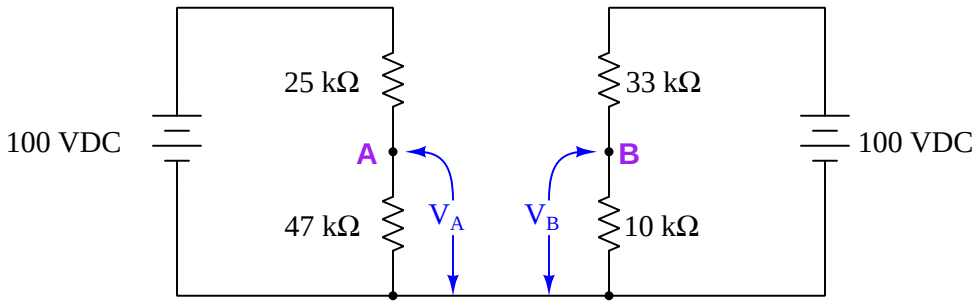
Note how the square-root term (y) is calculated in cell `C1`, and the denominator term (z) in cell `C2`. This makes the two final formulae (in cells `B1` and `B2`) simpler to interpret. The positioning of all these cells on the grid is completely arbitrary[11] – all that matters is that they properly reference each other in the formulae.

Spreadsheets are particularly useful for situations where the same set of calculations representing a circuit or other system must be repeated for different initial conditions. The power of a spreadsheet is that it automates what would otherwise be a tedious set of calculations. One specific application of this is to simulate the effects of various components within a circuit failing with abnormal values (e.g. a shorted resistor simulated by making its value nearly zero; an open resistor simulated by making its value extremely large). Another application is analyzing the behavior of a circuit design given new components that are out of specification, and/or aging components experiencing drift over time.

[11] My personal preference is to locate all the "given" data in the upper-left cells of the spreadsheet grid (each data point flanked by a sensible name in the cell to the left and units of measurement in the cell to the right as illustrated in the first distance/time spreadsheet example), sometimes coloring them in order to clearly distinguish which cells contain entered data versus which cells contain computed results from formulae. I like to place all formulae in cells below the given data, and try to arrange them in logical order so that anyone examining my spreadsheet will be able to figure out *how* I constructed a solution. This is a general principle I believe all computer programmers should follow: *document and arrange your code to make it easy for other people to learn from it.*

4.2.3 Two voltage divider networks

Calculate the output voltages of these two voltage divider circuits (V_A and V_B), and then calculate the voltage between points **A** and **B** (V_{AB}):



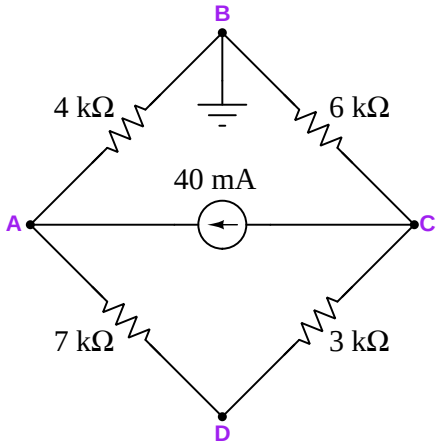
Challenges

- What would change if the wire connecting the two voltage divider circuits together were removed?

4.2.4 Ground-referenced and differential voltage measurements

In electrical and electronic systems, we often refer to voltage measurements as being either *ground-referenced* or *differential*. A ground-referenced voltage measurement is one taken between a specified test point in the circuit and a point in the circuit labeled "ground". A differential voltage measurement is one taken between two non-grounded test points. Ground-referenced voltages are often symbolized by V with a single subscript (e.g. V_C, voltage between C and ground), where the subscripted test point connects to the voltmeter's red lead and ground connects to the meter's black lead. Differential voltages are symbolized by dual subscripts, with the first representing the red lead and the second the black lead (e.g. V_{BC}, red on B and black on C).

Examine the following bridge circuit with test point B grounded, and then calculate the specified voltages:



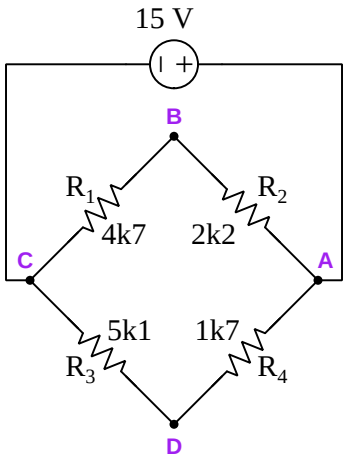
- V_{BA} =
- V_{BD} =
- V_{AC} =
- V_A =
- V_B =
- V_C =
- V_D =

Challenges

- What would happen to all the voltages if test point D were *also* grounded?
- Demonstrate Kirchhoff's Voltage Law in the loop DABCD, after having calculated all voltages.

4.2.5 Balancing a resistive bridge

Explain what the term *unbalanced* means as it applies to the following bridge circuit:



Calculate the following quantities in the circuit, with source and resistor values as shown. Interpret the subscript test-point letters by the convention that the first letter represents the *red* voltmeter test lead and the second letter the *black* test lead:

- $V_{AB} =$
- $V_{BC} =$
- $V_{DC} =$
- $V_{DA} =$
- $V_{BD} =$
- $I_{source} =$

Next, calculate the necessary resistor value for each of the four resistors to balance this bridge circuit, assuming in each case that you only change *that* resistor's value and leave the other three as shown in the schematic diagram:

- $R_1 =$
- $R_2 =$
- $R_3 =$
- $R_4 =$

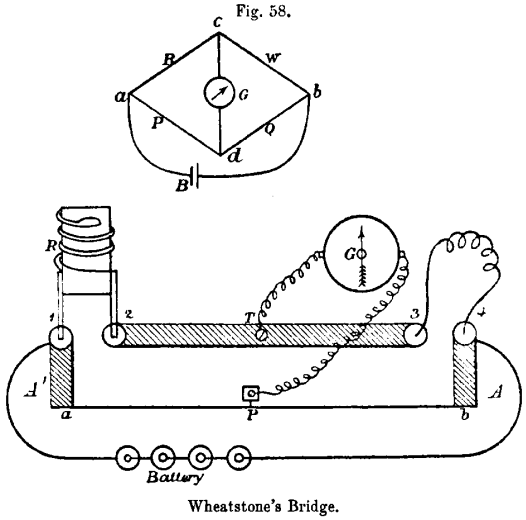
Finally, identify how some of the foundational concepts you've studied apply to this circuit: **Ohm's Law**, **Kirchhoff's Voltage Law**, properties of **series** and **parallel** networks, **ratios**, **voltage dividers**. Feel free to include any other foundational concepts not listed here.

Challenges

- Identify ways to estimate balancing resistor values without using a calculator.
- Demonstrate how Kirchhoff's Voltage Law is true for any "loop" chosen in this bridge circuit (e.g. loop CADC).
- Does a bridge circuit's state of balance depend on the source voltage? Why or why not?
- Suppose we were to connect test point B to ground. Given this condition, calculate V_D.

4.2.6 Using a primitive bridge to measure resistance

The following schematic shows a primitive Wheatstone bridge circuit *circa 1886* used to measure an unknown resistance connected between terminals **3** and **4**, using a standard resistor connected between terminals **1** and **2** and a resistive wire stretched between terminals **a** and **b** with a sliding contact **P** acting as the wiper on a potentiometer. A very sensitive meter called a galvanometer (**G**) served as a balance detector to indicate when the bridge was adjusted for a balanced condition:



Wheatstone's Bridge.

Calculate the resistance of the unknown resistor in each of the following test cases where the standard resistor values and potentiometer/slide dimensions are given to make the galvanometer register zero:

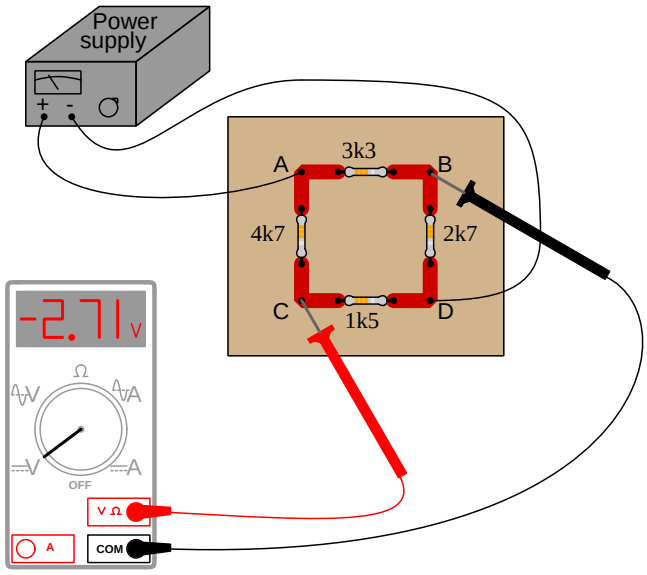
- $R_{standard}$ = 550 Ω ; Distance **a**-**b** = 38 cm ; Distance **P**-**a** = 12 cm
- $R_{standard}$ = 2.6 kΩ ; Distance **a**-**b** = 38 cm ; Distance **P**-**a** = 20 cm
- $R_{standard}$ = 1.1 kΩ ; Distance **a**-**b** = 38 cm ; Distance **P**-**b** = 11 cm
- $R_{standard}$ = 300 Ω ; Distance **a**-**b** = 38 cm ; Distance **P**-**b** = 25 cm
- $R_{standard}$ = 725 Ω ; Distance **P**-**b** = 24 cm ; Distance **P**-**a** = 14 cm
- $R_{standard}$ = 5.2 kΩ ; Distance **P**-**a** = 10 cm ; Distance **P**-**b** = 28 cm

Challenges

- Is the battery voltage relevant at all? Why or why not?

4.2.7 Unknown excitation voltage

Calculate the amount of "excitation" voltage applied to this unbalanced bridge circuit by the DC power supply:



Challenges

- Is this bridge circuit balanced or not? How can we tell?
- Explain the relationship between V_{meter} and V_{supply} for an unbalanced bridge circuit.
- Explain the relationship between V_{meter} and V_{supply} for a balanced bridge circuit.

4.3 Diagnostic reasoning

These questions are designed to stimulate your deductive and inductive thinking, where you must apply general principles to specific scenarios (deductive) and also derive conclusions about the failed circuit from specific details (inductive). In a Socratic discussion with your instructor, the goal is for these questions to reinforce your recall and use of general circuit principles and also challenge your ability to integrate multiple symptoms into a sensible explanation of what's wrong in a circuit. Your instructor may also pose additional questions based on those assigned, in order to further challenge and sharpen your diagnostic abilities.

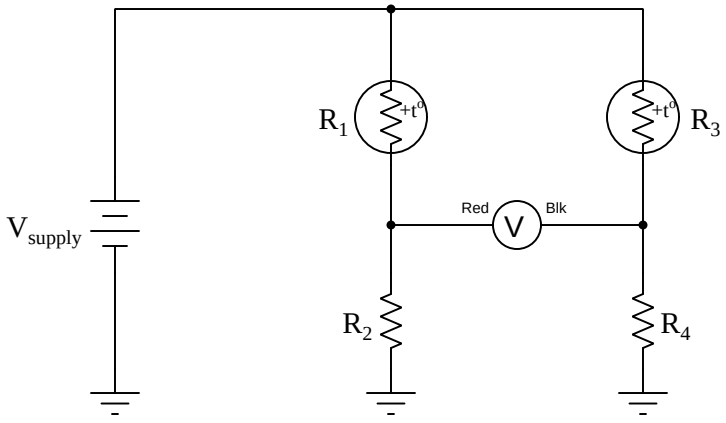
As always, your goal is to fully *explain* your analysis of each problem. Simply obtaining a correct answer is not good enough – you must also demonstrate sound reasoning in order to successfully complete the assignment. Your instructor's responsibility is to probe and challenge your understanding of the relevant principles and analytical processes in order to ensure you have a strong foundation upon which to build further understanding.

You will note a conspicuous lack of answers given for these diagnostic questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your diagnostic answers, where applicable, is to use circuit simulation software to explore the effects of faults placed in circuits. For example, if one of these diagnostic questions requires that you predict the effect of an open or a short in a circuit, you may check the validity of your work by simulating that same fault (substituting a very high resistance in place of that component for an open, and substituting a very low resistance for a short) within software and seeing if the results agree.

4.3.1 Thermistor bridge faults

Predict how the operation of this thermistor bridge circuit will be affected as a result of the following faults. Consider each fault independently (i.e. one at a time, no coincidental faults):



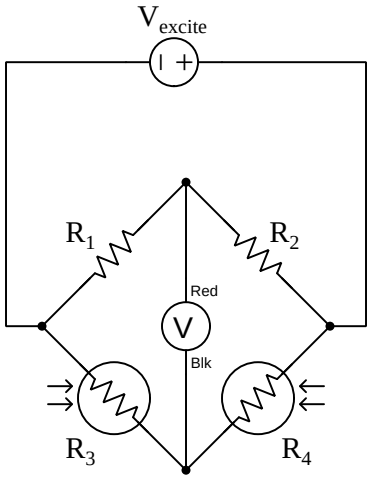
- Thermistor R_1 fails open:
- Thermistor R_3 fails open:
- Solder bridge (short) across thermistor R_3:
- Resistor R_2 fails open:
- Resistor R_4 fails open:

Challenges

- Identify how you could use a multimeter to diagnose any of these faults.

4.3.2 Failed photocell bridge

This bridge circuit is suppose to generate an output voltage proportional to the *difference* between light exposure on the two photocells:



However something has failed in this circuit, because the voltmeter is "pegged" fully negative and will not change with varying light exposures on the two cells. Identify at least two possible failures that could cause the voltmeter to over-range in the negative direction.

Challenges

- Which polarity should the voltmeter register (i.e. a positive value or a negative value) if photocell R_4 receives more light than photocell R_3, assuming photocell resistance *decreases* with light exposure?
- Identify how you could use a multimeter to diagnose any of these faults.

Chapter 5

Projects and Experiments

The following project and experiment descriptions outline things you can build to help you understand circuits. With any real-world project or experiment there exists the potential for physical harm. *Electricity can be very dangerous in certain circumstances, and you should follow proper safety precautions at all times!*

5.1 Recommended practices

This section outlines some recommended practices for all circuits you design and construct.

5.1.1 Safety first!

Electricity, when passed through the human body, causes uncomfortable sensations and in large enough measures[1] will cause muscles to involuntarily contract. The overriding of your nervous system by the passage of electrical current through your body is particularly dangerous in regard to your heart, which is a vital muscle. Very large amounts of current can produce serious internal burns in addition to all the other effects.

Cardio-pulmonary resuscitation (CPR) is the standard first-aid for any victim of electrical shock. This is a very good skill to acquire if you intend to work with others on dangerous electrical circuits. You should never perform tests or work on such circuits unless someone else is present who is proficient in CPR.

As a general rule, any voltage in excess of 30 Volts poses a definitive electric shock hazard, because beyond this level human skin does not have enough resistance to safely limit current through the body. "Live" work of any kind with circuits over 30 volts should be avoided, and if unavoidable should only be done using electrically insulated tools and other protective equipment (e.g. insulating shoes and gloves). If you are unsure of the hazards, or feel unsafe at any time, stop all work and distance yourself from the circuit!

A policy I strongly recommend for students learning about electricity is to *never come into electrical contact*[2] *with an energized conductor, no matter what the circuit's voltage*[3] *level!* Enforcing this policy may seem ridiculous when the circuit in question is powered by a single battery smaller than the palm of your hand, but it is precisely this instilled habit which will save a person from bodily harm when working with more dangerous circuits. Experience has taught me that students who learn early on to be careless with safe circuits have a tendency to be careless later with dangerous circuits!

In addition to the electrical hazards of shock and burns, the construction of projects and running of experiments often poses other hazards such as working with hand and power tools, potential

[1] Professor Charles Dalziel published a research paper in 1961 called "The Deleterious Effects of Electric Shock" detailing the results of electric shock experiments with both human and animal subjects. The threshold of perception for human subjects holding a conductor in their hand was in the range of 1 milliampere of current (less than this for alternating current, and generally less for female subjects than for male). Loss of muscular control was exhibited by half of Dalziel's subjects at less than 10 milliamperes alternating current. Extreme pain, difficulty breathing, and loss of all muscular control occurred for over 99% of his subjects at direct currents less than 100 milliamperes and alternating currents less than 30 milliamperes. In summary, it doesn't require much electric current to induce painful and even life-threatening effects in the human body! Your first and best protection against electric shock is maintaining an insulating barrier between your body and the circuit in question, such that current from that circuit will be unable to flow through your body.

[2] By "electrical contact" I mean either directly touching an energized conductor with any part of your body, or indirectly touching it through a conductive tool. The only physical contact you should ever make with an energized conductor is via an electrically insulated tool, for example a screwdriver with an electrically insulated handle, or an insulated test probe for some instrument.

[3] Another reason for consistently enforcing this policy, even on low-voltage circuits, is due to the dangers that even some low-voltage circuits harbor. A single 12 Volt automobile battery, for example, can cause a surprising amount of damage if short-circuited simply due to the high current levels (i.e. very low internal resistance) it is capable of, even though the voltage level is too low to cause a shock through the skin. Mechanics wearing metal rings, for example, are at risk from severe burns if their rings happen to short-circuit such a battery! Furthermore, even when working on circuits that are simply too low-power (low voltage and low current) to cause any bodily harm, touching them while energized can pose a threat to the circuit components themselves. In summary, it's generally wise (and *always* a good habit to build) to "power down" *any* circuit before making contact between it and your body.

contact with high temperatures, potential chemical exposure, etc. You should never proceed with a project or experiment if you are unaware of proper tool use or lack basic protective measures (e.g. personal protective equipment such as safety glasses) against such hazards.

Some other safety-related practices should be followed as well:

- All power conductors extending outward from the project must be *firmly* strain-relieved (e.g. "cord grips" used on line power cords), so that an accidental tug or drop will not compromise circuit integrity.
- All electrical connections must be sound and appropriately made (e.g. soldered wire joints rather than twisted-and-taped; terminal blocks rather than solderless breadboards for high-current or high-voltage circuits). Use "touch-safe" terminal connections with recessed metal parts to minimize risk of accidental contact.
- Always provide overcurrent protection in any circuit you build. *Always.* This may be in the form of a fuse, a circuit breaker, and/or an electronically current-limited power supply.
- Always ensure circuit conductors are rated for more current than the overcurrent protection limit. *Always.* A fuse does no good if the wire or printed circuit board trace will "blow" before it does!
- Always bond metal enclosures to Earth ground for any line-powered circuit. *Always.* Ensuring an equipotential state between the enclosure and Earth by making the enclosure electrically common with Earth ground ensures no electric shock can occur simply by one's body bridging between the Earth and the enclosure.
- Avoid building a high-energy circuit when a low-energy circuit will suffice. For example, I always recommend beginning students power their first DC resistor circuits using small batteries rather than with line-powered DC power supplies. The intrinsic energy limitations of a dry-cell battery make accidents highly unlikely.
- Use line power receptacles that are GFCI (Ground Fault Current Interrupting) to help avoid electric shock from making accidental contact with a "hot" line conductor.
- Always wear eye protection when working with tools or live systems having the potential to eject material into the air. Examples of such activities include soldering, drilling, grinding, cutting, wire stripping, working on or near energized circuits, etc.
- Always use a step-stool or stepladder to reach high places. Never stand on something not designed to support a human load.
- When in doubt, *ask an expert.* If anything even seems remotely unsafe to you, do not proceed without consulting a trusted person fully knowledgeable in electrical safety.

5.1.2 Other helpful tips

Experience has shown the following practices to be very helpful, especially when students make their own component selections, to ensure the circuits will be well-behaved:

- Avoid resistor values less than 1 kΩ or greater than 100 kΩ, unless such values are definitely necessary[4]. Resistances below 1 kΩ may draw excessive current if directly connected to a voltage source of significant magnitude, and may also complicate the task of accurately measuring current since any ammeter's non-zero resistance inserted in series with a low-value circuit resistor will significantly alter the total resistance and thereby skew the measurement. Resistances above 100 kΩ may complicate the task of measuring voltage since any voltmeter's finite resistance connected in parallel with a high-value circuit resistor will significantly alter the total resistance and thereby skew the measurement. Similarly, AC circuit impedance values should be between 1 kΩ and 100 kΩ, and for all the same reasons.

- Ensure all electrical connections are low-resistance and physically rugged. For this reason, one should avoid *compression splices* (e.g. "butt" connectors), solderless breadboards[5], and wires that are simply twisted together.

- Build your circuit with **testing** in mind. For example, provide convenient connection points for test equipment (e.g. multimeters, oscilloscopes, signal generators, logic probes).

- Design permanent projects with **maintenance** in mind. The more convenient you make maintenance tasks, the more likely they will get done.

- **Always document and save your work**. Circuits lacking schematic diagrams are more difficult to troubleshoot than documented circuits. Similarly, circuit construction is simpler when a schematic diagram precedes construction. Experimental results are easier to interpret when comprehensively recorded. Consider modern videorecording technology for this purpose where appropriate.

- **Record your steps** when troubleshooting. **Talk to yourself** when solving problems. These simple steps clarify thought and simplify identification of errors.

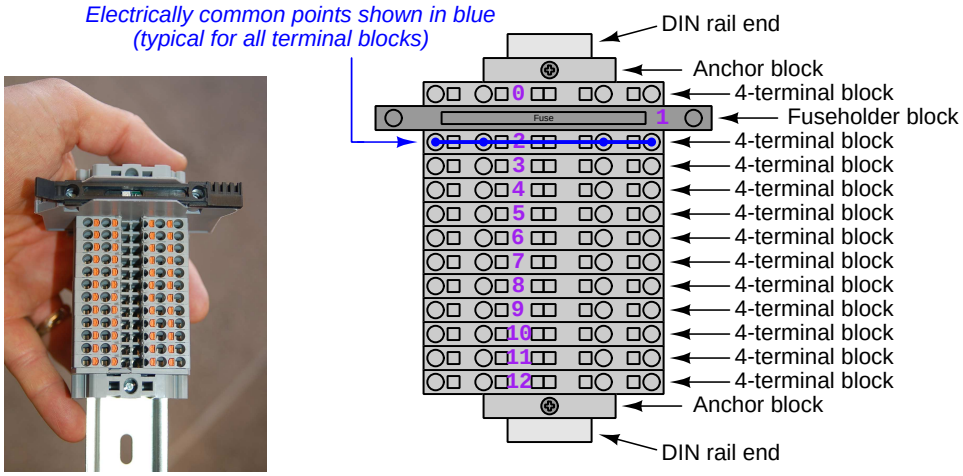
[4] An example of a necessary resistor value much less than 1 kΩ is a *shunt resistor* used to produce a small voltage drop for the purpose of sensing current in a circuit. Such shunt resistors must be low-value in order not to impose an undue load on the rest of the circuit. An example of a necessary resistor value much greater than 100 kΩ is an electrostatic *drain resistor* used to dissipate stored electric charges from body capacitance for the sake of preventing damage to sensitive semiconductor components, while also preventing a path for current that could be dangerous to the person (i.e. shock).

[5] Admittedly, solderless breadboards are very useful for constructing complex electronic circuits with many components, especially DIP-style integrated circuits (ICs), but they tend to give trouble with connection integrity after frequent use. An alternative for projects using low counts of ICs is to solder IC sockets into prototype printed circuit boards (PCBs) and run wires from the soldered pins of the IC sockets to terminal blocks where reliable temporary connections may be made.

5.1.3 Terminal blocks for circuit construction

Terminal blocks are the standard means for making electric circuit connections in industrial systems, and are also quite useful for student-built experiments, providing a robust alternative to solderless breadboards[6]. Terminal blocks provide highly reliable connections rated for significant voltage and current magnitudes, and they force the builder to think carefully about component placement which is an important mental practice. Terminal blocks designed to mount on standard 35 mm DIN rail[7] come in many types and capabilities.

Every student of electricity should build their own terminal block array consisting of several terminal blocks where each block has at least 4 connection points all electrically common to each other[8] and at least one terminal block that is a fuse holder for overcurrent protection. A pair of anchoring blocks hold all terminal blocks securely on the DIN rail, preventing them from sliding off the rail. Each of the terminals should bear a unique number starting from 0. An example is shown in the following photograph and illustration:



Screwless terminal blocks (using internal spring clips to clamp wire and component lead ends) are preferred over screw-based terminal blocks, as they reduce assembly and disassembly time, and also minimize repetitive wrist stress from twisting screwdrivers. Some screwless terminal blocks require the use of a special tool to release the spring clip, while others such as the Phoenix Contact model 3211019 blocks shown in the photograph provide buttons[9] for this task which may be pressed using the tip of any suitable tool such as a small screwdriver or multimeter test probe tip.

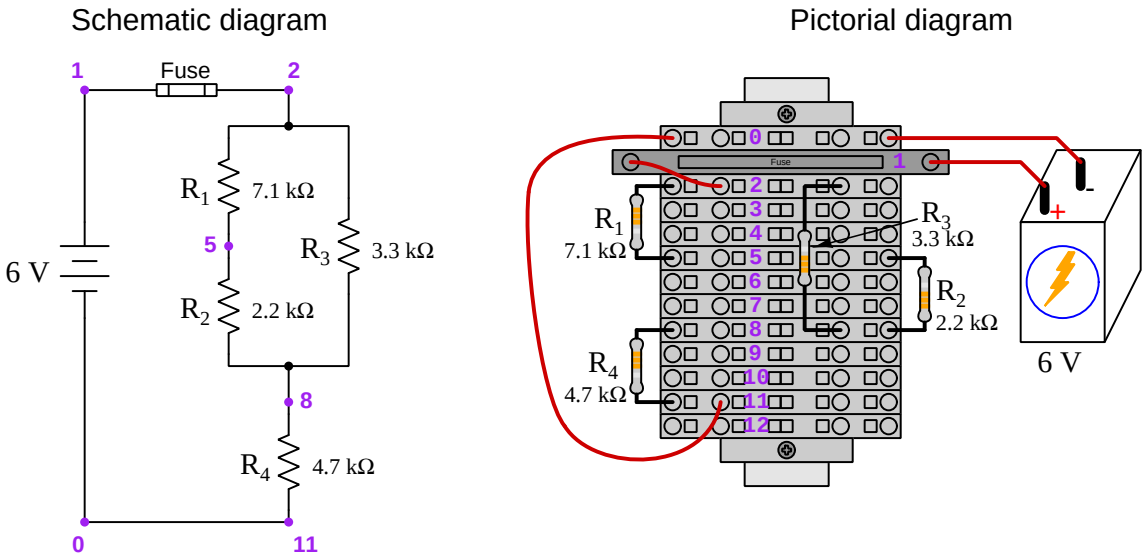
[6] Solderless breadboard are preferable for complicated electronic circuits with multiple integrated "chip" components, but for simpler circuits I find terminal blocks much more practical. An alternative to solderless breadboards for "chip" circuits is to solder chip sockets onto a PCB and then use wires to connect the socket pins to terminal blocks. This also accommodates *surface-mount* components, which solderless breadboards do not.

[7] DIN rail is a metal rail designed to serve as a mounting point for a wide range of electrical and electronic devices such as terminal blocks, fuses, circuit breakers, relay sockets, power supplies, data acquisition hardware, etc.

[8] Sometimes referred to as *equipotential*, *same-potential*, or *potential distribution* terminal blocks.

[9] The small orange-colored squares seen in the above photograph are buttons for this purpose, and may be actuated by pressing with any tool of suitable size.

The following example shows how such a terminal block array might be used to construct a series-parallel resistor circuit consisting of four resistors and a battery:



Numbering on the terminal blocks provides a very natural translation to SPICE[10] netlists, where component connections are identified by terminal number:

```
* Series-parallel resistor circuit
v1 1 0 dc 6
r1 2 5 7100
r2 5 8 2200
r3 2 8 3300
r4 8 11 4700
rfuse 1 2 0.01
rwire 0 11 0.01
.op
.end
```

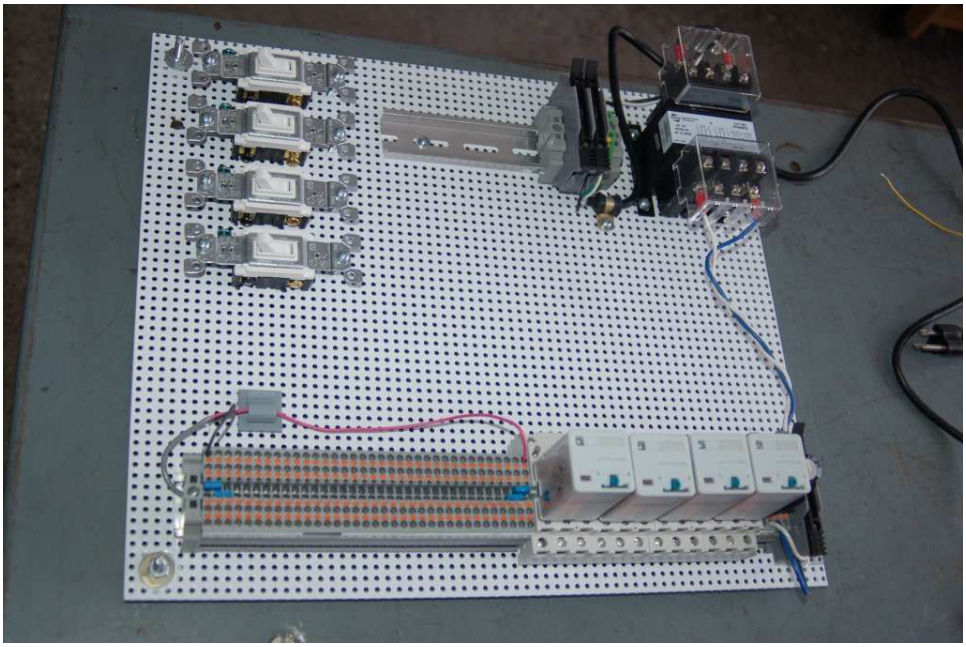
Note the use of two resistances `rfuse` and `rwire` to describe fuse resistance between terminals 1 and 2 and "jumper" wire resistance between terminals 0 and 11, respectively. Being resistances, SPICE requires a resistance value for each, and here we see they have both been set to an arbitrarily low value of 0.01 Ohm appropriate to their real-life resistance values.

Listing all components and wires along with their numbered terminals happens to be a useful documentation method for any circuit built on terminal blocks, independent of SPICE. Such a

[10] SPICE is computer software designed to analyze electrical and electronic circuits. Circuits are described for the computer in the form of *netlists* which are text files listing each component type, connection node numbers, and component values.

"wiring sequence" may be thought of as a *non-graphical description* of an electric circuit, and is exceptionally easy to follow.

An example of a more elaborate terminal block array is shown in the following photograph, with terminal blocks and "ice-cube" style electromechanical relays mounted to DIN rail, which is turn mounted to a perforated subpanel[11]. This "terminal block board" hosts an array of thirty five undedicated terminal block sections, four SPDT toggle switches, four DPDT "ice-cube" relays, a step-down control power transformer, bridge rectifier and filtering capacitor, and several fuses for overcurrent protection:



Four plastic-bottomed "feet" support the subpanel above the benchtop surface, and an unused section of DIN rail stands ready to accept other components. Safety features include electrical bonding of the AC line power cord's ground to the metal subpanel (and all metal DIN rails), mechanical strain relief for the power cord to isolate any cord tension from wire connections, clear plastic finger guards covering the transformer's screw terminals, as well as fused overcurrent protection for the 120 Volt AC line power and the transformer's 12 Volt AC output. The perforated holes happen to be on $\frac{1}{4}$ inch centers with a diameter suitable for tapping with 6-32 machine screw threads, their presence making it very easy to attach other sections of DIN rail, printed circuit boards, or specialized electrical components directly to the grounded metal subpanel. Such a "terminal block board" is an inexpensive[12] yet highly flexible means to construct physically robust circuits using industrial wiring practices.

[11] An electrical *subpanel* is a thin metal plate intended for mounting inside an electrical enclosure. Components are attached to the subpanel, and the subpanel in turn bolts inside the enclosure. Subpanels allow circuit construction outside the confines of the enclosure, which speeds assembly. In this particular usage there is no enclosure, as the subpanel is intended to be used as an open platform for the convenient construction of circuits on a benchtop by students. In essence, this is a modern version of the traditional *breadboard* which was literally a wooden board such as might be used for cutting loaves of bread, but which early electrical and electronic hobbyists used as platforms for the construction of circuits.

[12] At the time of this writing (2019) the cost to build this board is approximately $250 US dollars.

5.1.4 Conducting experiments

An *experiment* is an exploratory act, a test performed for the purpose of assessing some proposition or principle. Experiments are the foundation of the *scientific method*, a process by which careful observation helps guard against errors of speculation. All good experiments begin with an *hypothesis*, defined by the American Heritage Dictionary of the English Language as:

> An assertion subject to verification or proof, as (a) A proposition stated as a basis for argument or reasoning. (b) A premise from which a conclusion is drawn. (c) A conjecture that accounts, within a theory or ideational framework, for a set of facts and that can be used as a basis for further investigation.

Stated plainly, an hypothesis is an *educated guess* about cause and effect. The correctness of this initial guess matters little, because any well-designed experiment will reveal the truth of the matter. In fact, *incorrect* hypotheses are often the most valuable because the experiments they engender lead us to surprising discoveries. One of the beautiful aspects of science is that it is more focused on the process of *learning* than about the status of *being correct*[13]. In order for an hypothesis to be valid, it must be testable[14], which means it must be a claim possible to refute given the right data. Hypotheses impossible to critique are useless.

Once an hypothesis has been formulated, an experiment must be designed to test that hypothesis. A well-designed experiment requires careful regulation of all relevant variables, both for personal safety and for prompting the hypothesized results. If the effects of one particular variable are to be tested, the experiment must be run multiple times with different values of (only) that particular variable. The experiment set up with the "baseline" variable set is called the *control*, while the experiment set up with different value(s) is called the *test* or *experimental*.

For some hypotheses a viable alternative to a physical experiment is a *computer-simulated experiment* or even a *thought experiment*. Simulations performed on a computer test the hypothesis against the physical laws encoded within the computer simulation software, and are particularly useful for students learning new principles for which simulation software is readily available[15].

[13] Science is more about clarifying our view of the universe through a systematic process of error detection than it is about proving oneself to be right. Some scient*ists* may happen to have large egos – and this may have more to do with the ways in which large-scale scientific research is *funded* than anything else – but scientific *method* itself is devoid of ego, and if embraced as a practical philosophy is quite an effective stimulant for humility. Within the education system, scientific method is particularly valuable for helping students break free of the crippling fear of *being wrong*. So much emphasis is placed in formal education on assessing correct retention of facts that many students are fearful of saying or doing anything that might be perceived as a mistake, and of course making mistakes (i.e. having one's hypotheses disproven by experiment) is an indispensable tool for learning. Introducing science in the classroom – *real* science characterized by individuals forming actual hypotheses and testing those hypotheses by experiment – helps students become self-directed learners.

[14] This is the principle of *falsifiability*: that a scientific statement has value only insofar as it is liable to disproof given the requisite experimental evidence. Any claim that is unfalsifiable – that is, a claim which can *never* be disproven by any evidence whatsoever – could be completely wrong and we could never know it.

[15] A very pertinent example of this is learning how to analyze electric circuits using simulation software such as SPICE. A typical experimental cycle would proceed as follows: (1) Find or invent a circuit to analyze; (2) Apply your analytical knowledge to that circuit, predicting all voltages, currents, powers, etc. relevant to the concepts you are striving to master; (3) Run a simulation on that circuit, collecting "data" from the computer when complete; (4) Evaluate whether or not your hypotheses (i.e. predicted voltages, currents, etc.) agree with the computer-generated results; (5) If so, your analyses are (provisionally) correct – if not, examine your analyses and the computer simulation again to determine the source of error; (6) Repeat this process as many times as necessary until you achieve mastery.

Thought experiments are useful for detecting inconsistencies within your own understanding of some subject, rather than testing your understanding against physical reality.

Here are some general guidelines for conducting experiments:

- The clearer and more specific the hypothesis, the better. Vague or unfalsifiable hypotheses are useless because they will fit *any* experimental results, and therefore the experiment cannot teach you anything about the hypothesis.
- Collect as much data (i.e. information, measurements, sensory experiences) generated by an experiment as is practical. This includes the time and date of the experiment, too!
- *Never* discard or modify data gathered from an experiment. If you have reason to believe the data is unreliable, write notes to that effect, but never throw away data just because you think it is untrustworthy. It is quite possible that even "bad" data holds useful information, and that someone else may be able to uncover its value even if you do not.
- Prioritize *quantitative* data over *qualitative* data wherever practical. Quantitative data is more specific than qualitative, less prone to subjective interpretation on the part of the experimenter, and amenable to an arsenal of analytical methods (e.g. statistics).
- Guard against your own bias(es) by making your experimental results available to others. This allows other people to scrutinize your experimental design and collected data, for the purpose of detecting and correcting errors you may have missed. Document your experiment such that others may independently replicate it.
- Always be looking for sources of error. No physical measurement is perfect, and so it is impossible to achieve *exact* values for any variable. Quantify the amount of uncertainty (i.e. the "tolerance" of errors) whenever possible, and be sure your hypothesis does not depend on precision better than this!
- Always remember that scientific confirmation is provisional – no number of "successful" experiments will prove an hypothesis true for all time, but a single experiment can disprove it. Put into simpler terms, *truth is elusive but error is within reach.*
- Remember that scientific method is about *learning*, first and foremost. An unfortunate consequence of scientific triumph in modern society is that science is often viewed by non-practitioners as an unerring source of truth, when in fact science is an ongoing process of challenging existing ideas to probe for errors and oversights. This is why it is perfectly acceptable to have a failed hypothesis, and why the only truly failed experiment is one where nothing was learned.

The following is an example of a well-planned and executed experiment, in this case a physical experiment demonstrating Ohm's Law.

```
Planning Time/Date = 09:30 on 12 February 2019

HYPOTHESIS: the current through any resistor should be exactly proportional
to the voltage impressed across it.

PROCEDURE: connect a resistor rated 1 k Ohm and 1/4 Watt to a variable-voltage
DC power supply.  Use an ammeter in series to measure resistor current and
a voltmeter in parallel to measure resistor voltage.

RISKS AND MITIGATION: excessive power dissipation may harm the resistor and/
or pose a burn hazard, while excessive voltage poses an electric shock hazard.
30 Volts is a safe maximum voltage for laboratory practices, and according to
Joule's Law a 1000 Ohm resistor will dissipate 0.25 Watts at 15.81 Volts
(P = V^2 / R), so I will remain below 15 Volts just to be safe.

Experiment Time/Date = 10:15 on 12 February 2019

DATA COLLECTED:
  (Voltage)    (Current)         (Voltage)   (Current)
   0.000 V   =  0.000 mA          8.100    =  7.812 mA
   2.700 V   =  2.603 mA          10.00 V  =  9.643 mA
   5.400 V   =  5.206 mA          14.00 V  =  13.49 mA

Analysis Time/Date = 10:57 on 12 February 2019

ANALYSIS: current definitely increases with voltage, and although I expected
exactly one milliAmpere per Volt the actual current was usually less than
that.  The voltage/current ratios ranged from a low of 1036.87 (at 8.1 Volts)
to a high of 1037.81 (at 14 Volts), but this represents a variance of only
-0.0365% to +0.0541% from the average, indicating a very consistent
proportionality -- results consistent with Ohm's Law.

ERROR SOURCES: one major source of error is the resistor's value itself.  I
did not measure it, but simply assumed color bands of brown-black-red meant
exactly 1000 Ohms.  Based on the data I think the true resistance is closer
to 1037 Ohms.  Another possible explanation is multimeter calibration error.
However, neither explains the small positive and negative variances from the
average.  This might be due to electrical noise, a good test being to repeat
the same experiment to see if the variances are the same or different.  Noise
should generate slightly different results every time.
```

The following is an example of a well-planned and executed *virtual* experiment, in this case demonstrating Ohm's Law using a computer (SPICE) simulation.

```
Planning Time/Date = 12:32 on 14 February 2019

HYPOTHESIS: for any given resistor, the current through that resistor should be
exactly proportional to the voltage impressed across it.

PROCEDURE: write a SPICE netlist with a single DC voltage source and single
1000 Ohm resistor, then use NGSPICE version 26 to perform a "sweep" analysis
from 0 Volts to 25 Volts in 5 Volt increments.

    * SPICE circuit
    v1 1 0 dc
    r1 1 0 1000
    .dc v1 0 25 5
    .print dc v(1) i(v1)
    .end

RISKS AND MITIGATION: none.

DATA COLLECTED:
       DC transfer characteristic  Thu Feb 14 13:05:08  2019
    ------------------------------------------------------------
    Index   v-sweep         v(1)            v1#branch
    ------------------------------------------------------------
    0       0.000000e+00    0.000000e+00    0.000000e+00
    1       5.000000e+00    5.000000e+00    -5.00000e-03
    2       1.000000e+01    1.000000e+01    -1.00000e-02
    3       1.500000e+01    1.500000e+01    -1.50000e-02
    4       2.000000e+01    2.000000e+01    -2.00000e-02
    5       2.500000e+01    2.500000e+01    -2.50000e-02

Analysis Time/Date = 13:06 on 14 February 2019

ANALYSIS: perfect agreement between data and hypothesis -- current is precisely
1/1000 of the applied voltage for all values.  Anything other than perfect
agreement would have probably meant my netlist was incorrect.  The negative
current values surprised me, but it seems this is just how SPICE interprets
normal current through a DC voltage source.

ERROR SOURCES: none.
```

As gratuitous as it may seem to perform experiments on a physical law as well-established as Ohm's Law, even the examples listed previously demonstrate opportunity for real learning. In the physical experiment example, the student should identify and explain why their data does not perfectly agree with the hypothesis, and this leads them naturally to consider sources of error. In the computer-simulated experiment, the student is struck by SPICE's convention of denoting regular current through a DC voltage source as being *negative* in sign, and this is also useful knowledge for future simulations. Scientific experiments are most interesting when things *do not* go as planned!

Aside from verifying well-established physical laws, simple experiments are extremely useful as educational tools for a wide range of purposes, including:

- Component familiarization (e.g. *Which terminals of this switch connect to the NO versus NC contacts?*)
- System testing (e.g. *How heavy of a load can my AC-DC power supply source before the semiconductor components reach their thermal limits?*)
- Learning programming languages (e.g. *Let's try to set up an "up" counter function in this PLC!*)

Above all, the priority here is to inculcate the habit of hypothesizing, running experiments, and analyzing the results. This experimental cycle not only serves as an excellent method for self-directed learning, but it also works exceptionally well for troubleshooting faults in complex systems, and for these reasons should be a part of every technician's and every engineer's education.

5.1.5 Constructing projects

Designing, constructing, and testing projects is a very effective means of practical education. Within a formal educational setting, projects are generally chosen (or at least vetted) by an instructor to ensure they may be reasonably completed within the allotted time of a course or program of study, and that they sufficiently challenge the student to learn certain important principles. In a self-directed environment, projects are just as useful as a learning tool but there is some risk of unwittingly choosing a project beyond one's abilities, which can lead to frustration.

Here are some general guidelines for managing projects:

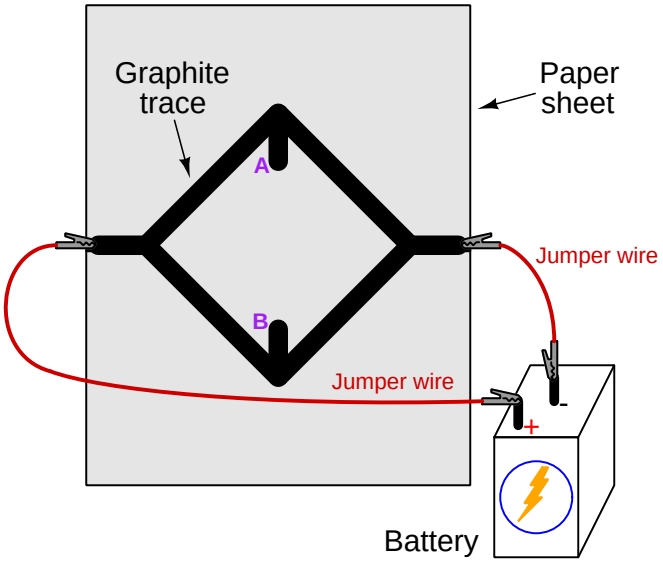
- Define your goal(s) before beginning a project: what do you wish to achieve in building it? What, exactly, should the completed project *do*?
- Analyze your project prior to construction. Document it in appropriate forms (e.g. schematic diagrams), predict its functionality, anticipate all associated risks. In other words, *plan ahead*.
- Set a reasonable budget for your project, and stay within it.
- Identify any deadlines, and set reasonable goals to meet those deadlines.
- Beware of *scope creep*: the tendency to modify the project's goals before it is complete.
- Document your progress! An easy way to do this is to use photography or videography: take photos and/or videos of your project as it progresses. Document failures as well as successes, because both are equally valuable from the perspective of learning.

5.2 Experiment: pencil-trace bridge circuit

In this experiment you will sketch a bridge circuit on a piece of paper using a soft-graphite pencil, energize it using a battery, and measure its voltage using a digital multimeter (DMM).

- A clean sheet of white notebook paper
- A soft-lead pencil (no harder than #2 "lead" hardness rating)
- Rubber eraser
- A digital multimeter (DMM) capable of measuring DC voltage
- 6 or 9 Volt battery for powering the bridge circuit
- Alligator-clip style jumper wires to connect the battery to the paper

Use the pencil to draw an open bridge circuit on the paper sheet with connection points on the paper's edge to receive the alligator clips, like this:



Attach the alligator clip jumper wires as shown, one to each end of the bridge circuit, being sure to make contact between the metal jaw of each alligator clip and the graphite mark. Connect the other ends of the jumper wires to the two terminals of your battery.

Each graphite trace drawn on the paper possesses a certain amount of electrical resistance. That resistance may be decreased by drawing the line darker and/or wider, and increased by using an eraser to make the line lighter and/or thinner.

Measure voltage between test points **A** and **B**. Experiment with using your pencil and eraser to make this voltage increase and decrease. Identify any patterns you see (e.g. "V_{AB} increases when I . . . "), to the point where you are able to adjust V_{AB} at will.

Challenges

- Does the physical size of the sketched bridge matter? Are small bridges easier to manipulate than large bridges?
- Predict the effect(s) of reversing the battery's polarity.
- Predict the effect(s) of moving the alligator clip farther off the paper so that its teeth contact less of the graphite trace.
- Predict the effect(s) of shorting past a portion of one of the bridge's "arms" (sides of the diamond-shape) using another jumper wire.
- Predict the effect of sliding one of your voltmeter's test leads to some point on the graphite trace other than the marked test point.

Appendix A

Problem-Solving Strategies

The ability to solve complex problems is arguably one of the most valuable skills one can possess, and this skill is particularly important in any science-based discipline.

- Study principles, not procedures. Don't be satisfied with merely knowing how to compute solutions – learn *why* those solutions work.
- Identify what it is you need to solve, identify all relevant data, identify all units of measurement, identify any general principles or formulae linking the given information to the solution, and then identify any "missing pieces" to a solution. Annotate all diagrams with this data.
- Sketch a diagram to help visualize the problem. When building a real system, always devise a plan for that system and analyze its function *before* constructing it.
- Follow the units of measurement and meaning of every calculation. If you are ever performing mathematical calculations as part of a problem-solving procedure, and you find yourself unable to apply each and every intermediate result to some aspect of the problem, it means you don't understand what you are doing. Properly done, every mathematical result should have practical meaning for the problem, and not just be an abstract number. You should be able to identify the proper units of measurement for each and every calculated result, and show where that result fits into the problem.
- Perform "thought experiments" to explore the effects of different conditions for theoretical problems. When troubleshooting real systems, perform *diagnostic tests* rather than visually inspecting for faults, the best diagnostic test being the one giving you the most information about the nature and/or location of the fault with the fewest steps.
- Simplify the problem until the solution becomes obvious, and then use that obvious case as a model to follow in solving the more complex version of the problem.
- Check for exceptions to see if your solution is incorrect or incomplete. A good solution will work for *all* known conditions and criteria. A good example of this is the process of testing scientific hypotheses: the task of a scientist is not to find support for a new idea, but rather to *challenge* that new idea to see if it holds up under a battery of tests. The philosophical

principle of *reductio ad absurdum* (i.e. disproving a general idea by finding a specific case where it fails) is useful here.

- Work "backward" from a hypothetical solution to a new set of given conditions.
- Add quantities to problems that are qualitative in nature, because sometimes a little math helps illuminate the scenario.
- Sketch graphs illustrating how variables relate to each other. These may be quantitative (i.e. with realistic number values) or qualitative (i.e. simply showing increases and decreases).
- Treat quantitative problems as qualitative in order to discern the relative magnitudes and/or directions of change of the relevant variables. For example, try determining what happens if a certain variable were to increase or decrease before attempting to precisely calculate quantities: how will each of the dependent variables respond, by increasing, decreasing, or remaining the same as before?
- Consider limiting cases. This works especially well for qualitative problems where you need to determine which direction a variable will change. Take the given condition and magnify that condition to an extreme degree as a way of simplifying the direction of the system's response.
- Check your work. This means regularly testing your conclusions to see if they make sense. This does *not* mean repeating the same steps originally used to obtain the conclusion(s), but rather to use some other means to check validity. Simply repeating procedures often leads to *repeating the same errors* if any were made, which is why alternative paths are better.

Appendix B

Instructional philosophy

B.1 First principles of learning

- **Anyone can learn anything** given appropriate time, effort, resources, challenges, encouragement, and expectations. Dedicating time and investing effort are the student's responsibility; providing resources, challenges, and encouragement are the teacher's responsibility; high expectations are a responsibility shared by both student and teacher.

- **Transfer is not automatic.** The human mind has a natural tendency to compartmentalize information, which means the process of taking knowledge learned in one context and applying it to another usually does not come easy and therefore should never be taken for granted.

- **Learning is iterative.** The human mind rarely learns anything perfectly on the first attempt. Anticipate mistakes and plan for multiple tries to achieve full understanding, using the lessons of those mistakes as feedback to guide future attempts.

- **Information is absorbed, but understanding is created.** Facts and procedures may be memorized easily enough by repeated exposure, but the ability to reliably apply principles to novel scenarios only comes through intense personal effort. This effort is fundamentally creative in nature: casting new concepts in one's own words, running mental experiments to perceive patterns and trends, etc. These acts of making knowledge "one's own" need not be perfect in order to be effective, as the value lies in the activity and not necessarily the finished product. Depth of understanding and permanence of memory are both directly related to mental effort during the learning process – i.e. *easy learning is shallow and fleeting.*

- **Education trumps training.** There is no such thing as an entirely isolated subject, as all fields of knowledge are connected. Training is narrowly-focused and task-oriented. Education is broad-based and principle-oriented. When preparing for a life-long technical career, education beats training every time.

- **Character matters.** Poor habits are more destructive than deficits of knowledge or skill. This is especially true in collective endeavors, where a team's ability to function depends on trust between its members. Simply put, no one wants an untrustworthy person on their team. An essential component of education then, is character development.

- **People learn to be responsible by bearing responsibility.** An irresponsible person is someone who has never *had* to be responsible for anything that mattered enough to them. Just as anyone can learn anything, anyone can become responsible if the personal cost of irresponsibility becomes high enough.

- **What gets measured, gets done.** Accurate and relevant assessment of learning is key to ensuring all students learn. Therefore, it is imperative to measure what matters.

- **Failure is nothing to fear.** Every human being fails, and fails in multiple ways at multiple times. Eventual success only happens when we don't stop trying.

B.2 Proven strategies for instructors

- Assume every student is capable of learning anything they desire given the proper conditions. Treat them as capable adults by granting real responsibility and avoiding artificial incentives such as merit or demerit points.
- Create a consistent culture of high expectations across the entire program of study. Demonstrate and encourage patience, persistence, and a healthy sense of self-skepticism. Anticipate and de-stigmatize error. Teach respect for the capabilities of others as well as respect for one's own fallibility.
- Replace lecture with "inverted" instruction, where students first encounter new concepts through reading and then spend class time in Socratic dialogue with the instructor exploring those concepts and solving problems individually. There is a world of difference between observing someone solve a problem versus actually solving a problem yourself, and so the point of this form of instruction is to place students in a position where they *cannot* passively observe.
- Require students to read extensively, write about what they learn, and dialogue with you and their peers to sharpen their understanding. Apply Francis Bacon's advice that "reading maketh a full man; conference a ready man; and writing an exact man". These are complementary activities helping students expand their confidence and abilities.
- Use artificial intelligence (AI) to challenge student understanding rather than merely provide information. Find productive ways for AI to critique students' clarity of thought and of expression, for example by employing AI as a Socratic-style interlocutor or as a reviewer of students' journals. Properly applied, AI has the ability to expand student access to critical review well outside the bounds of their instructor's reach.
- Build frequent and rapid feedback into the learning process so that students know at all times how well they are learning, to identify problems early and fix them before they grow. Model the intellectual habit of self-assessing and self-correcting your own understanding (i.e. a cognitive *feedback loop*), encouraging students to do the same.
- Use "mastery" as the standard for every assessment, which means the exam or experiment or project must be done with 100% competence in order to pass. Provide students with multiple opportunity for re-tries (different versions of the assessment every time).
- Require students to devise their own hypotheses and procedures on all experiments, so that the process is truly a scientific one. Have students assess their proposed experimental procedures for risk and devise mitigations for those risks. Let nothing be pre-designed about students' experiments other than a stated task (i.e. what principle the experiment shall test) at the start and a set of demonstrable knowledge and skill objectives at the end.
- Have students build as much of their lab equipment as possible: building power sources, building test assemblies[1], and building complete working systems (no kits!). In order to provide

[1] In the program I teach, every student builds their own "Development Board" consisting of a metal chassis with DIN rail, terminal blocks, and an AC-DC power supply of their own making which functions as a portable lab environment they can use at school as well as take home.

this same "ground-up" experience for every new student, this means either previous students take their creations with them, or the systems get disassembled in preparation for the new students, or the systems grow and evolve with each new student group.

- Incorporate external accountability for you and for your students, continuously improving the curriculum and your instructional methods based on proven results. Have students regularly network with active professionals through participation in advisory committee meetings, service projects, tours, jobshadows, internships, etc. Practical suggestions include requiring students to design and build projects for external clients (e.g. community groups, businesses, different departments within the institution), and also requiring students attend all technical advisory committee meetings and to dialogue with the industry representatives at those meetings.
- Repeatedly explore difficult-to-learn concepts across multiple courses, so that students have multiple opportunities to build their understanding.
- Relate all new concepts, whenever possible, to previous concepts and to relevant physical laws. Challenge each and every student, every day, to *reason* from concept to concept and to explain the logical connections between. Challenge students to verify their conclusions by multiple approaches (e.g. double-checking their work using different methods). Ask *"Why?"* often.
- Maintain detailed records on each student's performance and share these records privately with them. These records should include academic performance as well as professionally relevant behavioral tendencies.
- Hold mandatory "check-in" meetings between all program faculty and each new student during their first term. Offer these to all other students as an option, except for any students continuing to manifest unprofessional behaviors, poor academic performance, or who have some other need for a face-to-face meeting with faculty.
- Address problems while they are small, before they grow larger. This is equally true for tutoring technical concepts as it is for helping students build professional habits.
- Build rigorous quality control into the curriculum to ensure every student masters every important concept, and that the mastery is retained over time. This includes (1) review questions added to every exam to re-assess knowledge taught in previous terms, (2) cumulative exams at the end of every term to re-assess all important concepts back to the very beginning of the program, and (3) review assessments in practical (hands-on) coursework to ensure critically-important skills were indeed taught and are still retained. What you will find by doing this is that it actually boosts retention of students by ensuring that important knowledge gets taught and is retained over long spans of time. In the absence of such quality control, student learning and retention tends to be spotty and this contributes to drop-out and failure rates later in their education.
- Finally, *never rush learning.* Education is not a race. Give your students ample time to digest complex ideas, as you continually remind yourself of just how long it took you to achieve mastery! Long-term retention and the consistently correct application of concepts are always the result of *focused effort over long periods of time* which means there are no shortcuts to learning.

B.3 Proven strategies for students

The single most important piece of advice I have for any student of any subject is to take responsibility for your own development in all areas of life including mental development. Expecting others in your life to entirely guide your own development is a recipe for disappointment. This is just as true for students enrolled in formal learning institutions as it is for auto-didacts pursuing learning entirely on their own. Learning to think in new ways is key to being able to gainfully use information, to make informed decisions about your life, and to best serve those you care about. With this in mind, I offer the following advice to students:

- **Approach all learning as valuable**. No matter what course you take, no matter who you learn from, no matter the subject, there is something useful in every learning experience. If you don't see the value of every new experience, you are not looking closely enough!
- **Continually challenge yourself**. Let other people take shortcuts and find easy answers to easy problems. The purpose of education is to stretch your mind, in order to shape it into a more powerful tool. This doesn't come by taking the path of least resistance. An excellent analogy for an empowering education is productive physical exercise: becoming stronger, more flexible, and more persistent only comes through intense personal effort.
- **Master the use of language**. This includes reading extensively, writing every day, listening closely, and speaking articulately. To a great extent language channels and empowers thought, so the better you are at wielding language the better you will be at grasping abstract concepts and articulating them not only for your benefit but for others as well.
- **Do not limit yourself to the resources given to you**. Read books that are not on the reading list. Run experiments that aren't assigned to you. Form study groups outside of class. Take an entrepreneurial approach to your own education, as though it were a business you were building for your future benefit.
- **Express and share what you learn**. Take every opportunity to teach what you have learned to others, as this will not only help them but will also strengthen your own understanding[2].
- Realize that **no one can give you understanding**, just as no one can give you physical fitness. These both must be *built*.
- **Above all, recognize that learning is hard work, and that a certain level of frustration is unavoidable**. There are times when you will struggle to grasp some of these concepts, and that struggle is a natural thing. Take heart that it will yield with persistent and varied[3] effort, and never give up! That concepts don't immediately come to you is not a sign of something wrong, but rather of something right: that you have found a worthy challenge!

[2] On a personal note, I was surprised to learn just how much my own understanding of electronics and related subjects was strengthened by becoming a teacher. When you are tasked every day with helping other people grasp complex topics, it catalyzes your own learning by giving you powerful incentives to study, to articulate your thoughts, and to reflect deeply on the process of learning.

[3] As the old saying goes, "Insanity is trying the same thing over and over again, expecting different results." If you find yourself stumped by something in the text, you should attempt a different approach. Alter the thought experiment, change the mathematical parameters, do whatever you can to see the problem in a slightly different light, and then the solution will often present itself more readily.

B.4 Design of these learning modules

"The unexamined circuit is not worth energizing" – Socrates (if he had taught electricity)

These learning modules, although useful for self-study, were designed to be used in a formal learning environment where a subject-matter expert challenges students to digest the content and exercise their critical thinking abilities in the answering of questions and in the construction and testing of working circuits. Every effort has been made to embed the following instructional and assessment philosophies within:

- The first goal of education is to enhance clear and independent thought, in order that every student reach their fullest potential in a highly complex and inter-dependent world. Robust reasoning is *always* more important than particulars of any subject matter, because its application is universal.
- Literacy is fundamental to independent learning and thought because text continues to be the most efficient way to communicate complex ideas over space and time. Those who cannot read with ease are limited in their ability to acquire knowledge and perspective.
- Articulate communication is fundamental to work that is complex and interdisciplinary.
- Faulty assumptions and poor reasoning are best corrected through challenge, not presentation. The rhetorical technique of *reductio ad absurdum* (disproving an assertion by exposing an absurdity) works well to discipline student's minds, not only to correct the problem at hand but also to learn how to detect and correct future errors.
- Important principles should be repeatedly explored and widely applied throughout a course of study, not only to reinforce their importance and help ensure their mastery, but also to showcase the interconnectedness and utility of knowledge.

These learning modules were expressly designed to be used in an "inverted" teaching environment[4] where students first read the introductory and tutorial chapters on their own, then individually attempt to answer the questions and construct working circuits according to the experiment and project guidelines. The instructor never lectures, but instead meets regularly with each individual student to review their progress, answer questions, identify misconceptions, and challenge the student to new depths of understanding through further questioning. Regular meetings between instructor and student should resemble a Socratic[5] dialogue, where questions serve as scalpels to dissect topics and expose assumptions. The student passes each module only after consistently demonstrating their ability to logically analyze and correctly apply all major concepts in each question or project/experiment. The instructor must be vigilant in probing each student's understanding to ensure they are truly *reasoning* and not just *memorizing*. This is why "Challenge" points appear throughout, as prompts for students to think deeper about topics and as starting points for instructor queries. Sometimes these challenge points require additional knowledge that hasn't been covered in the series to answer in full. This is okay, as the major purpose of the Challenges is to stimulate analysis and synthesis on the part of each student.

The instructor must possess enough mastery of the subject matter and awareness of students' reasoning to generate their own follow-up questions to practically any student response. Even completely correct answers given by the student should be challenged by the instructor for the purpose of having students practice articulating their thoughts and defending their reasoning. Conceptual errors committed by the student should be exposed and corrected not by direct instruction, but rather by reducing the errors to an absurdity[6] through well-chosen questions and thought experiments posed by the instructor. Becoming proficient at this style of instruction requires time and dedication, but the positive effects on critical thinking for both student and instructor are spectacular.

An inspection of these learning modules reveals certain unique characteristics. One of these is a bias toward thorough explanations in the tutorial chapters. Without a live instructor to explain concepts and applications to students, the text itself must fulfill this role. This philosophy results in lengthier explanations than what you might typically find in a textbook, each step of the reasoning process fully explained, including footnotes addressing common questions and concerns students raise while learning these concepts. Each tutorial seeks to not only explain each major concept in sufficient detail, but also to explain the logic of each concept and how each may be developed

[4] In a traditional teaching environment, students first encounter new information via *lecture* from an expert, and then independently apply that information via *homework*. In an "inverted" course of study, students first encounter new information via *homework*, and then independently apply that information under the scrutiny of an expert. The expert's role in lecture is to simply *explain*, but the expert's role in an inverted session is to *challenge*, *critique*, and if necessary *explain* where gaps in understanding still exist.

[5] Socrates is a figure in ancient Greek philosophy famous for his unflinching style of questioning. Although he authored no texts, he appears as a character in Plato's many writings. The essence of Socratic philosophy is to leave no question unexamined and no point of view unchallenged. While purists may argue a topic such as electric circuits is too narrow for a true Socratic-style dialogue, I would argue that the essential thought processes involved with scientific reasoning on *any* topic are not far removed from the Socratic ideal, and that students of electricity and electronics would do very well to challenge assumptions, pose thought experiments, identify fallacies, and otherwise employ the arsenal of critical thinking skills modeled by Socrates.

[6] This rhetorical technique is known by the Latin phrase *reductio ad absurdum*. The concept is to expose errors by counter-example, since only one solid counter-example is necessary to disprove a universal claim. As an example of this, consider the common misconception among beginning students of electricity that voltage cannot exist without current. One way to apply *reductio ad absurdum* to this statement is to ask how much current passes through a fully-charged battery connected to nothing (i.e. a clear example of voltage existing without current).

from "first principles". Again, this reflects the goal of developing clear and independent thought in students' minds, by showing how clear and logical thought was used to forge each concept. Students benefit from witnessing a model of clear thinking in action, and these tutorials strive to be just that.

Another feature of these learning modules is that they do not treat topics in isolation. Rather, important concepts are introduced early in the series, and appear repeatedly as stepping-stones toward other concepts in subsequent modules. This helps to avoid the "compartmentalization" of knowledge, demonstrating the inter-connectedness of concepts and simultaneously reinforcing them. Each module is fairly complete in itself, reserving the beginning of its tutorial to a review of foundational concepts.

To high standards of education,

Tony R. Kuphaldt

Appendix C

Tools used

I am indebted to the developers of many open-source software applications in the creation of these learning modules. The following is a list of these applications with some commentary on each.

You will notice a theme common to many of these applications: a bias toward *code*. Although I am by no means an expert programmer in any computer language, I understand and appreciate the flexibility offered by code-based applications where the user (you) enters commands into a plain ASCII text file, which the software then reads and processes to create the final output. Code-based computer applications are by their very nature *extensible*, while WYSIWYG (What You See Is What You Get) applications are generally limited to whatever user interface the developer makes for you.

The `GNU/Linux` computer operating system

> There is so much to be said about Linus Torvalds' `Linux` and Richard Stallman's `GNU` project. First, to credit just these two individuals is to fail to do justice to the *mob* of passionate volunteers who contributed to make this amazing software a reality. I first learned of `Linux` back in 1996, and have been using this operating system on my personal computers almost exclusively since then. It is *free*, it is completely *configurable*, and it permits the continued use of highly efficient `Unix` applications and scripting languages (e.g. shell scripts, Makefiles, `sed`, `awk`) developed over many decades. `Linux` not only provided me with a powerful computing platform, but its open design served to inspire my life's work of creating open-source educational resources.

Bram Moolenaar's `Vim` text editor

> Writing code for any code-based computer application requires a *text editor*, which may be thought of as a word processor strictly limited to outputting plain-ASCII text files. Many good text editors exist, and one's choice of text editor seems to be a deeply personal matter within the programming world. I prefer `Vim` because it operates very similarly to `vi` which is ubiquitous on `Unix`/`Linux` operating systems, and because it may be entirely operated via keyboard (i.e. no mouse required) which makes it fast to use.

Donald Knuth's TEX typesetting system

Developed in the late 1970's and early 1980's by computer scientist extraordinaire Donald Knuth to typeset his multi-volume magnum opus *The Art of Computer Programming*, this software allows the production of formatted text for screen-viewing or paper printing, all by writing plain-text code to describe how the formatted text is supposed to appear. TEX is not just a markup language for documents, but it is also a Turing-complete programming language in and of itself, allowing useful algorithms to be created to control the production of documents. Simply put, TEX *is a programmer's approach to word processing*. Since TEX is controlled by code written in a plain-text file, this means anyone may read that plain-text file to see exactly how the document was created. This openness afforded by the code-based nature of TEX makes it relatively easy to learn how other people have created their own TEX documents. By contrast, examining a beautiful document created in a conventional WYSIWYG word processor such as Microsoft `Word` suggests nothing to the reader about *how* that document was created, or what the user might do to create something similar. As Mr. Knuth himself once quipped, conventional word processing applications should be called WYSIAYG (What You See Is *All* You Get).

Leslie Lamport's LATEX extensions to TEX

Like all true programming languages, TEX is inherently extensible. So, years after the release of TEX to the public, Leslie Lamport decided to create a massive extension allowing easier compilation of book-length documents. The result was LATEX, which is the markup language used to create all ModEL module documents. You could say that TEX is to LATEX as `C` is to `C++`. This means it is permissible to use any and all TEX commands within LATEX source code, and it all still works. Some of the features offered by LATEX that would be challenging to implement in TEX include automatic index and table-of-content creation.

Tim Edwards' `Xcircuit` drafting program

This wonderful program is what I use to create all the schematic diagrams and illustrations (but not photographic images or mathematical plots) throughout the ModEL project. It natively outputs PostScript format which is a true vector graphic format (this is why the images do not pixellate when you zoom in for a closer view), and it is so simple to use that I have never had to read the manual! Object libraries are easy to create for `Xcircuit`, being plain-text files using PostScript programming conventions. Over the years I have collected a large set of object libraries useful for drawing electrical and electronic schematics, pictorial diagrams, and other technical illustrations.

`Gimp` graphic image manipulation program

Essentially an open-source clone of Adobe's `PhotoShop`, I use `Gimp` to resize, crop, and convert file formats for all of the photographic images appearing in the ModEL modules. Although `Gimp` does offer its own scripting language (called `Script-Fu`), I have never had occasion to use it. Thus, my utilization of `Gimp` to merely crop, resize, and convert graphic images is akin to using a sword to slice bread.

`SPICE` circuit simulation program

`SPICE` is to circuit analysis as TEX is to document creation: it is a form of markup language designed to describe a certain object to be processed in plain-ASCII text. When the plain-text "source file" is compiled by the software, it outputs the final result. More modern circuit analysis tools certainly exist, but I prefer `SPICE` for the following reasons: it is *free*, it is *fast*, it is *reliable*, and it is a fantastic tool for *teaching* students of electricity and electronics how to write simple code. I happen to use rather old versions of `SPICE`, version 2g6 being my "go to" application when I only require text-based output. `NGSPICE` (version 26), which is based on Berkeley `SPICE` version 3f5, is used when I require graphical output for such things as time-domain waveforms and Bode plots. In all `SPICE` example netlists I strive to use coding conventions compatible with all `SPICE` versions.

Andrew D. Hwang's `ePiX` mathematical visualization programming library

This amazing project is a `C++` library you may link to any `C/C++` code for the purpose of generating PostScript graphic images of mathematical functions. As a completely free and open-source project, it does all the plotting I would otherwise use a Computer Algebra System (CAS) such as `Mathematica` or `Maple` to do. It should be said that `ePiX` is *not* a Computer Algebra System like `Mathematica` or `Maple`, but merely a mathematical *visualization* tool. In other words, it won't determine integrals for you (you'll have to implement that in your own `C/C++` code!), but it can graph the results, and it does so beautifully. What I really admire about `ePiX` is that it is a `C++` programming library, which means it builds on the existing power and toolset available with that programming language. Mr. Hwang could have probably developed his own stand-alone application for mathematical plotting, but by creating a `C++` library to do the same thing he accomplished something much greater.

`gnuplot` mathematical visualization software

Another open-source tool for mathematical visualization is `gnuplot`. Interestingly, this tool is *not* part of Richard Stallman's GNU project, its name being a coincidence. For this reason the authors prefer "gnu" *not* be capitalized at all to avoid confusion. This is a much "lighter-weight" alternative to a spreadsheet for plotting tabular data, and the fact that it easily outputs directly to an X11 console or a file in a number of different graphical formats (including PostScript) is very helpful. I typically set my `gnuplot` output format to default (X11 on my Linux PC) for quick viewing while I'm developing a visualization, then switch to PostScript file export once the visual is ready to include in the document(s) I'm writing. As with my use of `Gimp` to do rudimentary image editing, my use of `gnuplot` only scratches the surface of its capabilities, but the important points are that it's *free* and that it *works well*.

`Python` programming language

Both Python and C++ find extensive use in these modules as instructional aids and exercises, but I'm listing Python here as a *tool* for myself because I use it almost daily as a *calculator*. If you open a Python interpreter console and type `from math import *` you can type mathematical expressions and have it return results just as you would on a hand calculator. Complex-number (i.e. *phasor*) arithmetic is similarly supported if you include the complex-math library (`from cmath import *`). Examples of this are shown in the Programming References chapter (if included) in each module. Of course, being a fully-featured programming language, Python also supports conditionals, loops, and other structures useful for calculation of quantities. Also, running in a console environment where all entries and returned values show as text in a chronologically-ordered list makes it easy to copy-and-paste those calculations to document exactly how they were performed.

Appendix D

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Appendix E

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Appendix F

Version history

This is a list showing all significant additions, corrections, and other edits made to this learning module. Each entry is referenced by calendar date in reverse chronological order (newest version first), which appears on the front cover of every learning module for easy reference. Any contributors to this open-source document are listed here as well.

23-24 September 2026 – minor edits to the Tutorial chapter to improve readability.

17 September 2025 – added an approximate date for the bridge circuit depicted in the "Using a primitive bridge to measure resistance" Quantitative Reasoning question.

2 October 2024 – added another "Challenging concepts" bullet-item to the list in the Introduction chapter.

27 September 2024 – added a new Quantitative Reasoning question on using a primitive resistance measuring bridge.

5 September 2024 – divided the Introduction chapter into sections, one with recommendations for students, one with a listing of challenging concepts, and one with recommendations for instructors.

9 February 2024 – minor edits made to the Tutorial.

30 March 2023 – minor grammatical edit made to the Tutorial.

27 November 2022 – placed questions at the top of the itemized list in the Introduction chapter prompting students to devise experiments related to the tutorial content.

1 November 2022 – elaborated more on diodes and transistors in the "Power bridges" section of the Tutorial, for the purpose of foreshadowing these devices for students prior to studying solid-state theory.

27 September 2022 – added more clarifying references to the Tutorial chapter on "Wheatstone" bridge circuits, as well as more index entries.

17 February 2022 – minor edits to the Tutorial chapter regarding German silver wire not actually being made of silver, as well as some additional questions in the Introduction chapter.

7 February 2022 – added a new Challenge question to the "Balancing a resistive bridge" Quantitative Reasoning problem challenging students to use mental math.

27-28 September 2021 – added text, illustrations, and photographs showing strain gauges and load cells as applications for resistive bridge circuits. Also divided Tutorial into separate sections. Also added some foundational concept review to the "Balancing a resistive bridge" Quantitative Reasoning question.

18 February 2021 – added some instructor notes. Also corrected wrong type of quotation marks. Finally, added some missing +/− symbols in image_1057.eps.

6 October 2020 – minor edits to the "Balancing a resistive bridge" Quantitative reasoning question.

24 September 2020 – added more content to the Introduction chapter.

6 September 2020 – significantly edited the Introduction chapter to make it more suitable as a pre-study guide and to provide cues useful to instructors leading "inverted" teaching sessions.

4 September 2020 – corrected date error in the Historical References section on James Clerk Maxwell.

13 January 2020 – renamed Simplified Tutorial to Tutorial, since I haven't written a Full Tutorial yet.

26 August 2019 – added mention of difficult concepts to the Introduction.

16 June 2019 – minor edits to diagnostic questions, replacing "no multiple faults" with "no coincidental faults".

30 May 2019 – corrected typographical error in "Ground-referenced and differential voltage measurements" question.

25 May 2019 – added more Quantitative Reasoning questions.

23 May 2019 – added Experiment to built a pencil-trace bridge circuit.

19 May 2019 – minor typographical error correction ("ad" should have been "and" in the Historical References chapter). Also, added questions and made minor edits to the Simplified Tutorial.

14 May 2019 – corrected capitalization in one of the problem titles.

5 November 2018 – retitled Historical References section(s) so as to not be redundant to the "Historical References" chapter.

September 2018 – document first created.

Index

Adding quantities to a qualitative problem, 70
Annotating diagrams, 69

Breadboard, solderless, 56, 57
Breadboard, traditional, 60
Bridge network, 12
Bridge rectifier, 21
Bridge, Wheatstone, 14, 19, 24–26
Brute-force power supply, 21

Capacitor, 21
Cardio-Pulmonary Resuscitation, 54
Checking for exceptions, 70
Checking your work, 70
Code, computer, 79
CPR, 54

Dalziel, Charles, 54
Differential galvanometer, 24
Differential voltage, 44
Digital multimeter, 66
Dimensional analysis, 69
DIN rail, 57
Diode, 21
DIP, 56
Divider, voltage, 10
DMM, 66

Edwards, Tim, 80
Electric shock, 54
Electrically common points, 55
Enclosure, electrical, 60
Equipotential points, 55, 57
Experiment, 61
Experimental guidelines, 62

Galvanometer, 24
Graph values to solve a problem, 70
Greenleaf, Cynthia, 29
Ground-referenced voltage, 44

How to teach with these modules, 77
Hwang, Andrew D., 81

IC, 56
Identify given data, 69
Identify relevant principles, 69
Intermediate results, 69
Inverted instruction, 77
Inverter, 22

Knuth, Donald, 80

Lamport, Leslie, 80
Limiting case, 16
Limiting cases, 70
Load cell, 19

Maxwell, James Clerk, 23
Metacognition, 34
Moolenaar, Bram, 79
Multimeter, 66
Murphy, Lynn, 29

Open, 16
Open-source, 79

Parallel network properties, 17
Photocell, 16
Potential distribution, 57
Problem-solving: annotate diagrams, 69
Problem-solving: check for exceptions, 70
Problem-solving: checking work, 70
Problem-solving: dimensional analysis, 69
Problem-solving: graph values, 70
Problem-solving: identify given data, 69

Problem-solving: identify relevant principles, 69
Problem-solving: interpret intermediate results, 69
Problem-solving: limiting cases, 16, 70
Problem-solving: qualitative to quantitative, 70
Problem-solving: quantitative to qualitative, 70
Problem-solving: reductio ad absurdum, 70
Problem-solving: simplify the system, 69
Problem-solving: thought experiment, 62, 69
Problem-solving: track units of measurement, 69
Problem-solving: visually represent the system, 69
Problem-solving: work in reverse, 70
Project management guidelines, 65
Properties of parallel networks, 17
Properties of series networks, 17

Qualitatively approaching a quantitative problem, 70

Ratiometric, 15
Reading Apprenticeship, 29
Rectifier, 21
Reductio ad absurdum, 70, 76, 77

Safety, electrical, 54
Schoenbach, Ruth, 29
Scientific method, 34, 61
Scope creep, 65
Sensor, 16
Series network properties, 17
Short, 16
Shunt resistor, 56
Simplifying a system, 69
Socrates, 76
Socratic dialogue, 77
Solderless breadboard, 56, 57
Solid state component, 21
SPICE, 29, 62
SPICE netlist, 58
Stallman, Richard, 79
Strain gauge, 16, 18
Subpanel, 60
Surface mount, 57

Terminal block, 55–58, 60
Thermistor, 16, 37
Thought experiment, 62, 69
Torvalds, Linus, 79
Transformer, 21
Transistor, 22

Units of measurement, 69

Visualizing a system, 69
Voltage divider, 10
Voltage, differential, 44
Voltage, ground-referenced, 44

Wheatstone bridge, 14, 19, 24–26
Wheatstone, Charles, 14
Wiring sequence, 59
Work in reverse to solve a problem, 70
WYSIWYG, 79, 80