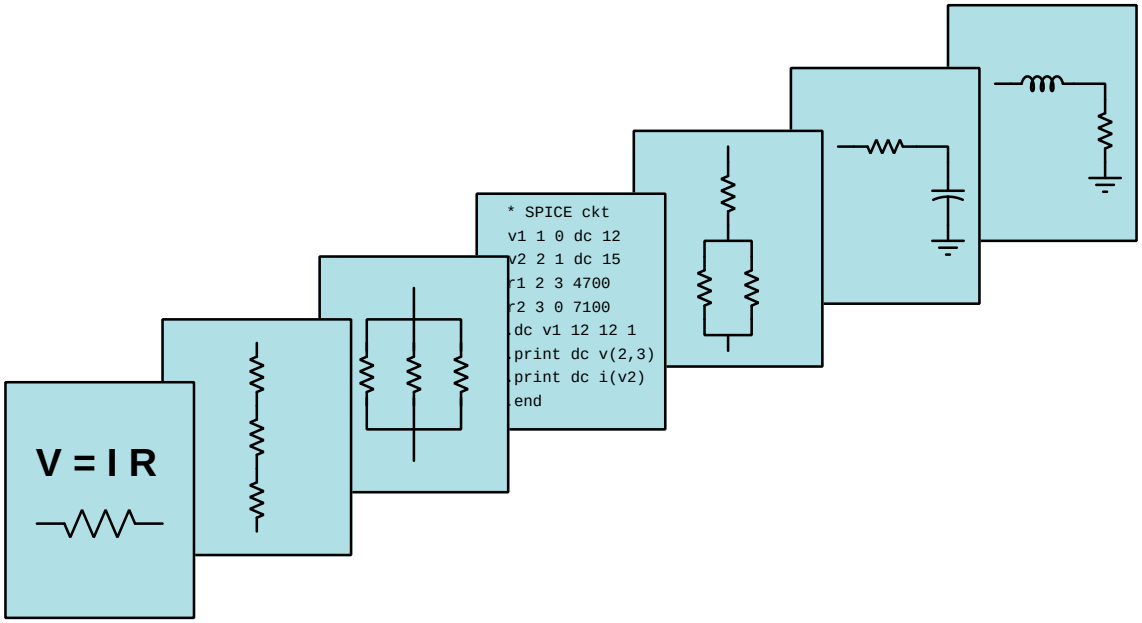
Modular Electronics Learning (ModEL) project



Capacitors and Capacitive Circuits

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Contents

1 Introduction **3**
- 1.1 Recommendations for students . . . 3
- 1.2 Challenging concepts related to capacitance and capacitors . . . 5
- 1.3 Recommendations for instructors . . . 7

2 Case Tutorial **9**
- 2.1 Example: capacitor storing and releasing energy . . . 10
- 2.2 Example: series versus parallel capacitors . . . 12
- 2.3 Example: RC demonstration circuit . . . 14
- 2.4 Example: battery voltage stabilization . . . 15
- 2.5 Example: decoupling capacitors . . . 18
- 2.6 Example: voltage source energizing a capacitor . . . 21
- 2.7 Example: current source energizing a capacitor . . . 24
- 2.8 Example: RC time-delay network values . . . 27
- 2.9 Example: voltage divider and capacitor . . . 30
- 2.10 Example: simplifying complex RC networks using Thévenin's Theorem . . . 31
- 2.11 Example: measuring signal rates of change . . . 34

3 Tutorial **39**
- 3.1 Electric fields and energy storage . . . 39
- 3.2 Voltage, current, energy, and capacitance . . . 43
- 3.3 Applications of capacitance . . . 45
- 3.4 Inverse exponential growth/decay . . . 47
- 3.5 Inverse-exponential calculation examples . . . 50
- 3.6 Capacitor types and limitations . . . 53
- 3.7 Series and parallel capacitance . . . 55

4 Historical References **57**
- 4.1 Early examples of capacitors . . . 58
- 4.2 Fatality resulting from cable capacitance . . . 62

5 Derivations and Technical References **67**
- 5.1 IEC standard component values . . . 68
- 5.2 Capacitor labeling . . . 70

5.3 Capacitor parameters and dielectric types . . . 75
5.4 Capacitor parasitics . . . 79
5.4.1 Model of a real capacitor . . . 79
5.4.2 Parasitic resistance in capacitors . . . 80
5.4.3 Parasitic inductance in capacitors . . . 80
5.4.4 Other parasitic effects in capacitors . . . 81
5.5 Electric field quantities . . . 82
5.6 Capacitor displacement current . . . 84
5.7 Time delay circuits and Thévenin's/Norton's Theorems . . . 89

6 Programming References **93**
6.1 Programming in C++ . . . 94
6.2 Programming in Python . . . 98
6.3 Modeling inverse exponential growth and decay using C++ . . . 103
6.4 Modeling inverse exponential growth and decay using Python . . . 110
6.5 Modeling an energizing capacitor using C++ . . . 112
6.6 Solving RC and LR circuit differential equations using C++ . . . 115
6.6.1 Differential equation solver for an RC circuit and voltage source . . . 116
6.6.2 Differential equation solver for an RC circuit and current source . . . 117
6.6.3 Differential equation solver for an LR circuit and voltage source . . . 118
6.6.4 Differential equation solver for an LR circuit and current source . . . 119

7 Questions **121**
7.1 Conceptual reasoning . . . 125
7.1.1 Reading outline and reflections . . . 126
7.1.2 Foundational concepts . . . 127
7.1.3 Electric flux lines . . . 129
7.1.4 Leyden jar . . . 129
7.1.5 Copper tape . . . 130
7.1.6 Water tank analogy . . . 131
7.1.7 Cable capacitance . . . 132
7.1.8 Capacitor as source versus load . . . 133
7.1.9 Audio power capacitors . . . 134
7.1.10 Capacitor charging circuit . . . 135
7.1.11 Analog electrical model of warming box . . . 138
7.1.12 Explaining the meaning of calculations . . . 139
7.2 Quantitative reasoning . . . 141
7.2.1 Miscellaneous physical constants . . . 142
7.2.2 Introduction to spreadsheets . . . 143
7.2.3 Air capacitor gap . . . 146
7.2.4 Defibrillator capacitor . . . 146
7.2.5 Capacitor labeling . . . 147
7.2.6 Half-life . . . 148
7.2.7 Number of time constants . . . 148
7.2.8 Inverse exponential functions . . . 149
7.2.9 Current versus rate-of-voltage-change . . . 150

7.2.10 Capacitance of three-capacitor network . . . 151
7.2.11 Resistance for specified time constant . . . 151
7.2.12 Safely de-energizing a large capacitor . . . 152
7.2.13 Capacitor charging scenario . . . 153
7.2.14 Capacitor voltage at specified intervals . . . 154
7.2.15 Partial capacitor de-energization . . . 155
7.2.16 Rapid charge and slow discharge . . . 155
7.2.17 Time required to reach specified voltages . . . 156
7.2.18 Measuring capacitance by time delay . . . 157
7.2.19 Intervalometer . . . 159
7.2.20 SPICE analysis of an energizing capacitor . . . 160
7.2.21 SPICE analysis of a de-energizing capacitor . . . 162
7.2.22 Capacitor voltage simulation program . . . 164
7.3 Diagnostic reasoning . . . 167
7.3.1 Dielectric breakdown . . . 167
7.3.2 Incorrect voltage/current curves . . . 168

A Problem-Solving Strategies **169**

B Instructional philosophy **171**
B.1 First principles of learning . . . 172
B.2 Proven strategies for instructors . . . 173
B.3 Proven strategies for students . . . 175
B.4 Design of these learning modules . . . 176

C Tools used **179**

D Creative Commons License **183**

E References **191**

F Version history **193**

Index **198**

Chapter 1

Introduction

1.1 Recommendations for students

Capacitance is the ability to store energy in an electric field, caused by a difference on electric charge between two points separated by an insulating gap. Any device constructed to exhibit this property is called a *capacitor*. The amount of capacitance offered by any capacitor is directly proportional to the overlapping areas of the conductive surfaces (i.e. "plates"), directly proportional to the permittivity of the insulating *dielectric* material separating those plates, and inversely proportional to the distance of their separation. The amount of energy stored in any capacitor is directly proportional to the capacitance (measured in Farads) and directly proportional to the square of the voltage as described by the formula $E_C = \frac{1}{2}CV^2$.

Important concepts related to capacitance include **Conservation of Energy**, **electric fields**, **permittivity**, **dielectric**, **sources** versus **loads**, **"Ohm's Law"** for capacitors, **rates of change** (e.g. "speed"), natural **decay** functions, **time constant** (τ), capacitor **ratings**, properties of **series** networks, properties of **parallel networks**, and **energy storage**.

Here are some good questions to ask of yourself while studying this subject:

- How might an experiment be designed and conducted to explore the phenomenon of capacitance? What hypothesis (i.e. prediction) might you pose for that experiment, and what result(s) would either support or disprove that hypothesis?
- What is a *field*, and how might we measure its presence?
- How may we construct a capacitor?
- What factors influence the amount of capacitance offered by a capacitor?
- Is the amount of energy stored within a capacitor a function of its voltage, its current, or both?
- Which parameter is able to vary most rapidly for a capacitor, voltage or current?
- What is the mathematical relationship between voltage and current for a capacitor?

- What are some practical applications for capacitors?
- Why do voltage and current tend to rise or "decay" in inverse-exponential fashion in a resistor-capacitor circuit?
- What does *time constant* mean?
- What factors influence the maximum voltage or current ratings for a capacitor?
- Why does capacitance add in parallel and diminish in series? Explain in terms of the energy stored by each component versus by the larger network!
- What "parasitic" properties are found in real capacitors other than capacitance?
- Why are some capacitors *polarized* and others are not?

A useful resource for understanding inverse-exponential growth and decay as it relates to a resistor-capacitor network is found in the *Programming References* chapter. There you will find example code showing how these changes in voltage and current occur over time.

The calculation of voltages and currents in capacitor-resistor circuits is universally challenging for students new to the subject. Part of this challenge is a general unfamiliarity with exponential equations, but perhaps the biggest obstacle is the unrealistic expectation of students that *concepts should be completely understandable through explanation*. The truth is that deep concepts almost never clarify in one's mind without *focused effort* and *application*. A very direct and effective way to learn new concepts with focused effort and application is to try solving the exact same problems shown as examples in the text, including *performing the author's calculations on your own until you can see the patterns yourself*, explaining to yourself *the first principles underlying those calculations*, and *annotating the schematic diagrams* with your own notations showing current directions, voltage polarities, etc.

This advice – to actively articulate and solve (or at least attempt to solve) the example problems used by the author – is both simple and effective, but surprisingly difficult to inculcate as a habit. Most of my students seeing subjects like this for the first time find themselves staring at the text, trying to understand the author's words, and shortly giving up when the desired understanding does not spring to mind. This is true even when they have practiced active approaches studying earlier concepts such as series-parallel resistor network analysis. They've been advised what to do, and indeed have done it before, but when presented with a new and daunting concept their behavior reverts to a familiar passivity.

Becoming a proficient technologist therefore consists in much more than just learning new concepts, most importantly learning *new habits of thought and of action*. When you find yourself challenged by a complicated concept, *actively apply it* and you will be surprised by how much better you will comprehend it! Always link each of your analytical steps to trusted principles so that you may explain *why* you're doing the necessary steps, never satisfying yourself with mere procedures. Always bear in mind that your goal is not just to learn how to analyze circuits, but to learn how to tackle *any* new problem and learn *any* new concept more effectively.

1.2 Challenging concepts related to capacitance and capacitors

The following list cites concepts related to this module's topic that are easily misunderstood, along with suggestions for properly understanding them:

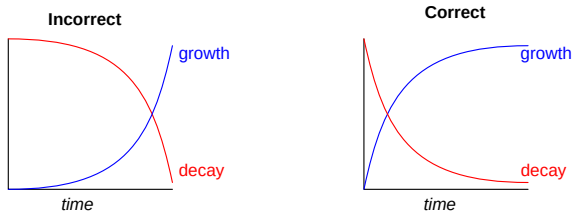
- **Current "through" a capacitance** – since a capacitor is by its very construction electrically *open*, having no conductive pathway from one terminal to the other, it seems extremely counter-intuitive to speak of capacitors as ever conducting a current "through" them. A key point about electrical opens prohibiting current is that this prohibition only holds for *continuous* currents. Momentary, transient currents are indeed possible for incomplete circuits, lightning being a spectacular natural example of this. For those who wish to dig deeper into the phenomenon, James Clerk Maxwell's notion of *displacement current* is a helpful start.

- **Determining initial and final values** – the fact that capacitor voltage cannot instantaneously change but capacitor current can is something many students try to rote-memorize. A far better approach is to reason from first principles. In this case, always begin by analyzing the circuit in terms of energy storage and energy conservation: capacitors store energy in electric fields which are associated with voltage; this means a capacitor's voltage cannot vary without work being done on or done by the capacitor, which always requires some amount of time. Current, on the other hand, is what constitutes power (i.e. energy exchange per unit time) when combined with voltage. Therefore, current can jump to new values instantaneously through a capacitor, such as when a de-energized capacitor is suddenly connected to a source, or a pre-energized capacitor is suddenly connected to a load.

- **Rates of change** – When learning the relationships between voltage and current for inductors and capacitors, one must think in terms of how fast a variable is changing. The amount of voltage induced across an inductor is proportional to how *quickly* the current through it changes, not how strong the current is. Likewise, the amount of current "through" a capacitor is proportional to how *quickly* the voltage across it changes. This is the first hurdle in calculus: to comprehend what a rate of change is, and it is far from obvious. A helpful strategy here is to relate electrical rates of change to other, more common, rates of change such as *speed* (i.e. the rate-of-change of physical position over time).

- **The time-constant equation** – Many students find the time-constant equation difficult because it involves exponents, particularly exponents of Euler's constant e. This exponent is often expressed as a negative quantity, making it even more difficult to understand. The single most popular mathematical mistake I see students make with this equation is failing to properly follow algebraic order of operations. Some students try to overcome this weakness by using calculators which allow parenthetical entries, nesting parentheses in such a way that the calculator performs the proper order of operations. However, if you don't understand order of operations yourself, you will not know where to properly place the parentheses. If you have trouble with algebraic order of operations, there is no solution but to invest the necessary time and learn it!

- **Determining voltage and current trajectories** – an important step in analyzing voltages and currents in DC circuits containing energy-storing components is to qualitatively graph the trajectories of those voltages and currents. This begins with determining initial and final values, but continues to show either decay or growth that is inverse-exponential, the values in question always asymptotically approaching some final value over time. A very common misconception is for students to plot these graphs with the wrong shape, showing voltages and/or currents changing slowly at first and then more rapidly as time progresses. This is a sign of attempting to memorize shapes (and recalling them incorrectly) rather than thinking through how inverse-exponential growth and decay actually works from a mathematical perspective. A gentle way to approach this is to reflect on half-life decay which is based on negative powers of two (i.e. after one half-life there is $\frac{1}{2}$ the original quantity left, after two half-lives there is $\frac{1}{4}$ remaining, etc.) and then extend that concept to $e^{-\frac{t}{\tau}}$.



- **Parasitic effects** – phenomena such as resistance, capacitance, and inductance don't just exist where we want them to, but in fact always exist everywhere they can. All conductors (except for superconductors) have some amount of resistance, all conductors separated by an insulating gap exhibit capacitance, and all conductors possess inductance by virtue of their ability to produce a magnetic field with the passage of current. Often these "parasitic" effects are small enough to ignore, but in some applications they may be severe.

1.3 Recommendations for instructors

This section lists realistic student learning outcomes supported by the content of the module as well as suggested means of assessing (measuring) student learning. The outcomes state what learners should be able to do, and the assessments are specific challenges to prove students have learned.

- **Outcome** – Demonstrate effective technical reading and writing

  Assessment – Students present their outlines of this module's instructional chapters (e.g. Case Tutorial, Tutorial, Historical References, etc.) ideally as an entry to a larger Journal document chronicling their learning. These outlines should exhibit good-faith effort at summarizing major concepts explained in the text.

  Assessment – Students show how inverse-exponential quantitative results were obtained by the author in the Tutorial chapter's examples.

  Assessment – Students present their "Conceptual Toolbox" Theory Journal entries listing important concepts applied in the Tutorial text(s). A "Conceptual Toolbox" serves as inspiration when solving problems, each concept suggesting a potential problem-solving action. Challenge students to collaboratively generate a Conceptual Toolbox for the day's topic during the class session and then use it when solving problems together, thus fostering the habit of reasoning from principles rather than recalling procedures!

- **Outcome** – Apply the concept of inverse exponential functions to capacitor charging and discharging

  Assessment – Calculate voltages and currents at specified times for a given resistor-capacitor network.

  Assessment – Calculate times when specified voltages and currents will reach specified values for a given resistor-capacitor network.

  Note – The importance of having students work through all their own calculations in class cannot be overstated!

- **Outcome** – Demonstrate how to create capacitance

  Assessment – Conduct an experiment showing how to create different amounts of capacitance with materials such as aluminum foil, plastic sheets, glass, paper, etc. using a multimeter capable of capacitance measurement.

- **Outcome** – Prove the concept of capacitive energy storage by experiment

  Assessment – Energize a capacitor and then transfer that energy to a load such as an LED, judging the amount of energy by the brightness and duration of the LED's illumination.

  Assessment – Re-run these experiments with multiple capacitors connected in series versus parallel to compare their relative energy-storage abilities.

- **Outcome** – Independent research

Assessment – Locate capacitor datasheets and properly interpret some of the information contained in those documents including capacitance tolerance, voltage ratings, temperature coefficient, voltage coefficient, etc.

Chapter 2

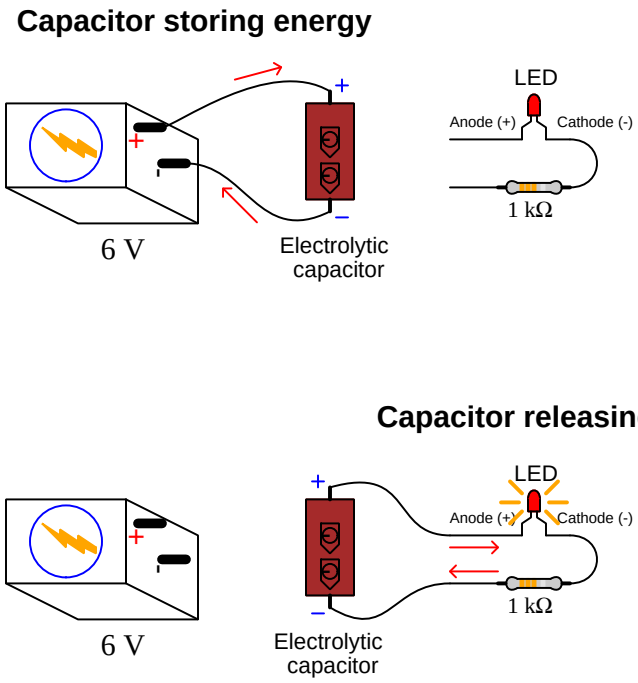
Case Tutorial

The idea behind a *Case Tutorial* is to explore new concepts by way of example. In this chapter you will read less presentation of theory compared to other Tutorial chapters, but by close observation and comparison of the given examples be able to discern patterns and principles much the same way as a scientific experimenter. Hopefully you will find these cases illuminating, and a good supplement to text-based tutorials.

These examples also serve well as challenges following your reading of the other Tutorial(s) in this module – can you explain *why* the circuits behave as they do?

2.1 Example: capacitor storing and releasing energy

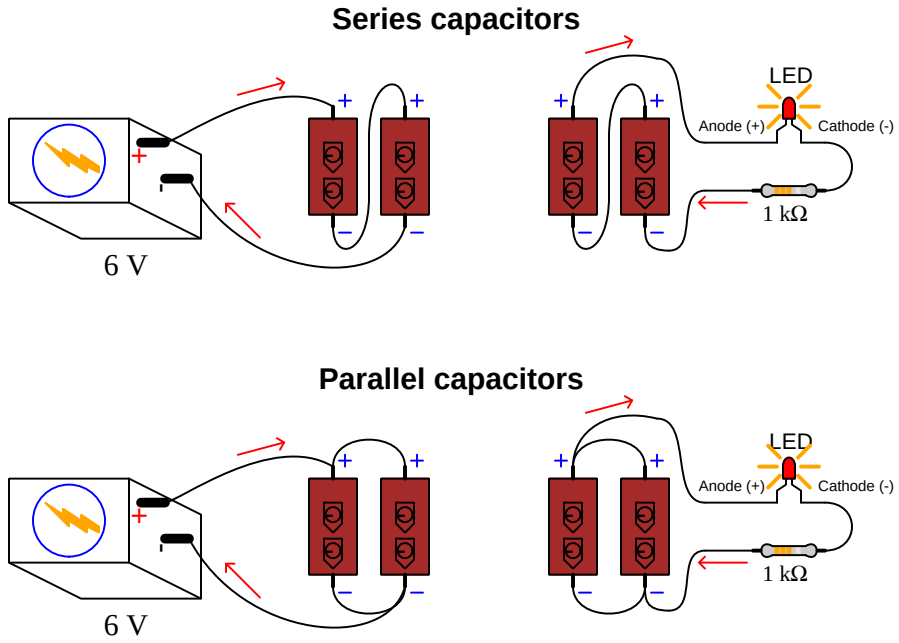
It is easy to use a battery, capacitor[1], resistor, and LED to demonstrate the principle of capacitance storing and releasing energy:



In the upper configuration the capacitor acts as a load. In the lower configuration the capacitor acts as a source.

[1] An aluminum electrolytic capacitor having a value of at least 100 μF and a voltage rating greater than the voltage of the battery generally works well. Beware the polarity requirements of the capacitor, as electrolytic capacitors are polarized devices and will be damaged by applying a reverse-polarity DC voltage!

A recommended variation of this experiment is to try using multiple capacitors of the same rating as the original, energized by the same source (battery) voltage as well. The point of these experiments is to compare how long the LED remains energized when powered by the series capacitor network, versus when powered by the parallel capacitor network, versus when powered by single capacitor, total voltage being equal in all cases:



Since capacitance is defined as the storage of energy in the form of an electric field, the comparative results of these experiments will prove whether capacitance is enhanced in parallel or in series. The configuration resulting in the LED remaining energized longest is the one having the most capacitance. If you are experienced in the use of a *digital oscilloscope*, consider simplifying the load to only be a resistor (with no LED) and use the oscilloscope's single-sweep feature to capture the voltage decay over time! An oscilloscope will show you quantitatively how voltage decays over time, and without the non-linearity of an LED being part of the circuit that rate of decay will be a very natural inverse-exponential curve.

An important principle in scientific experimentation is to *control* for all variables in an experiment except the one variable we wish to explore. This is why it is important here to use identically-valued capacitors in all cases, and use the same amount of source voltage too. This way, the only variable is the total amount of capacitance resulting from the number of capacitors and the manner in which they interconnect with each other.

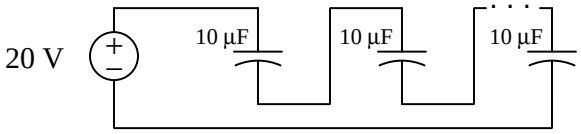
2.2 Example: series versus parallel capacitors

Calculating energy stored within a series network of 10 μF capacitors based on the number of capacitors (n) connected in series, in each case energized by a 20 Volt source. The amount of energy stored within each capacitor is given by the following formula, and the total energy stored is the simple sum of all capacitors' stored energy:

$$E_C = \frac{1}{2}CV^2$$

Where,

E_C = capacitor energy in Joules (J)
C = capacitance in Farads (F)
V = voltage in Volts (V)



Number of capacitors (n)	**Voltage across each capacitor**	**Energy stored in each capacitor**	**Energy stored (total)**
1	20.00 V	2.0000 mJ	2.0000 mJ
2	10.00 V	0.5000 mJ	1.0000 mJ
3	6.667 V	0.2222 mJ	0.6667 mJ
4	5.000 V	0.1250 mJ	0.5000 mJ

Connecting more capacitors in series results in the source voltage (20 Volts) being divided into smaller portions. With less voltage across each capacitor, there is dramatically less energy stored within each owing to the squared voltage term in the $E_C = \frac{1}{2}CV^2$ equation, so that with more capacitors in the series network there is actually less total energy stored for the entire network.

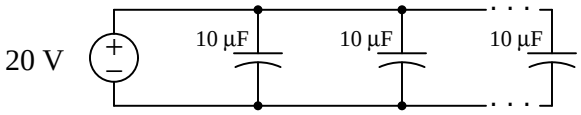
Since capacitance is defined as the ability to store energy within an electric field, we must conclude that connecting capacitors in series results in *less* over-all capacitance.

Calculating energy stored within a parallel network of 10 μF capacitors based on the number of capacitors (n) connected in parallel, in each case energized by a 20 Volt source. The amount of energy stored within each capacitor is given by the following formula, and the total energy stored is the simple sum of all capacitors' stored energy:

$$E_C = \frac{1}{2}CV^2$$

Where,

E_C = capacitor energy in Joules (J)
C = capacitance in Farads (F)
V = voltage in Volts (V)



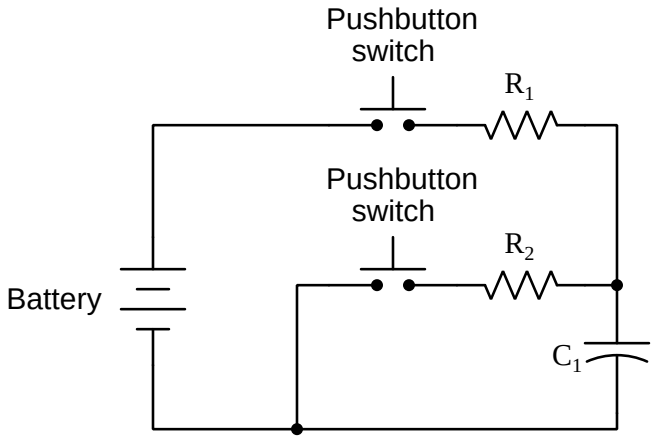
Number of capacitors (n)	**Voltage across each capacitor**	**Energy stored in each capacitor**	**Energy stored (total)**
1	20.00 V	2.0000 mJ	2.0000 mJ
2	20.00 V	2.0000 mJ	4.0000 mJ
3	20.00 V	2.0000 mJ	6.0000 mJ
4	20.00 V	2.0000 mJ	8.0000 mJ

Connecting more capacitors in parallel results in a direct increase in stored energy, because each capacitor receives the same source voltage (20 Volts) and therefore stores the same amount of energy.

Since capacitance is defined as the ability to store energy within an electric field, we must conclude that connecting capacitors in parallel results in *more* over-all capacitance.

2.3 Example: RC demonstration circuit

The following circuit is useful for demonstrating the inverse-exponential behavior of simple resistor-capacitor (RC) networks:



Pressing the upper pushbutton causes the capacitor to "charge up" to a higher voltage level; pressing the lower pushbutton causes it to "discharge" to a lower voltage level. The time constant (τ) is equal to RC for which ever resistor is in effect at the time, meaning you may design the circuit to charge and discharge at different rates if you wish.

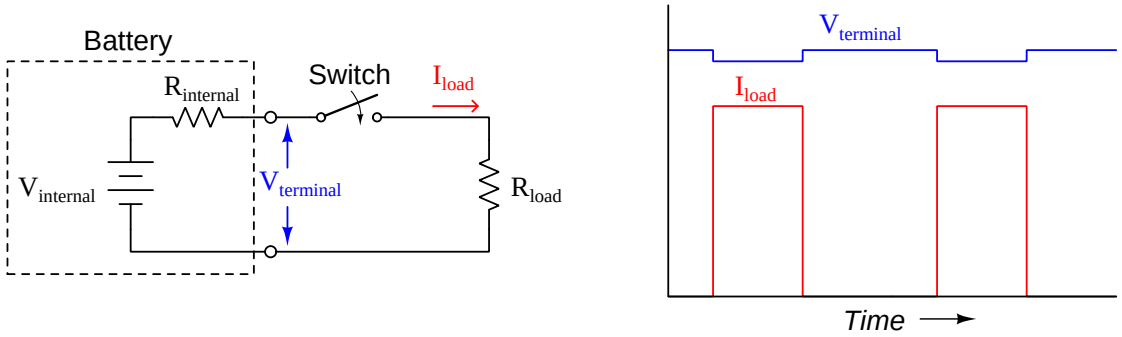
For best results using a multimeter to measure capacitor voltage and/or current, select resistors between 1 kΩ and 100 kΩ in value, and choose a capacitor with a large enough value to make the time constant reasonable for manual measurements (e.g. a τ value of at least 1 second).

Electrolytic capacitors are the most practical choice for the amount of capacitance you will likely require in order to achieve a relatively slow τ value, so beware that these capacitors are *polarized* which means any reversal of voltage polarity will likely destroy it!

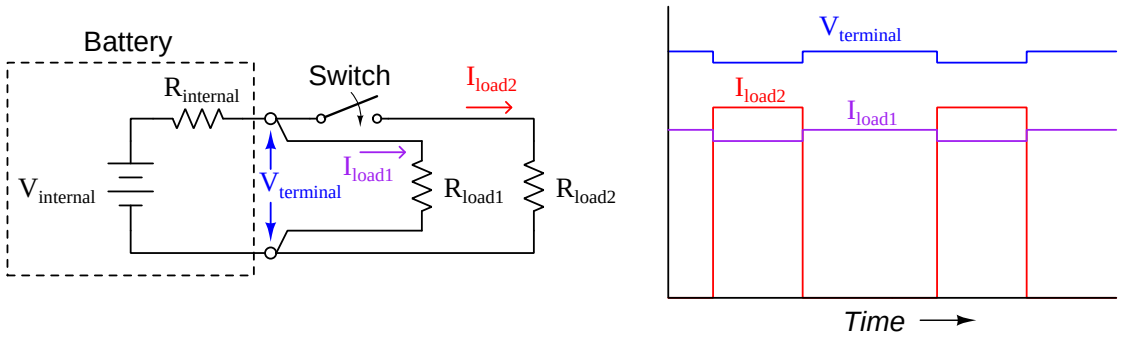
An interesting phenomenon you will likely observe when using an electrolytic capacitor in this circuit is *dielectric relaxation*, also known as "battery action" whereby the capacitor's voltage rebounds a small amount after being discharged.

2.4 Example: battery voltage stabilization

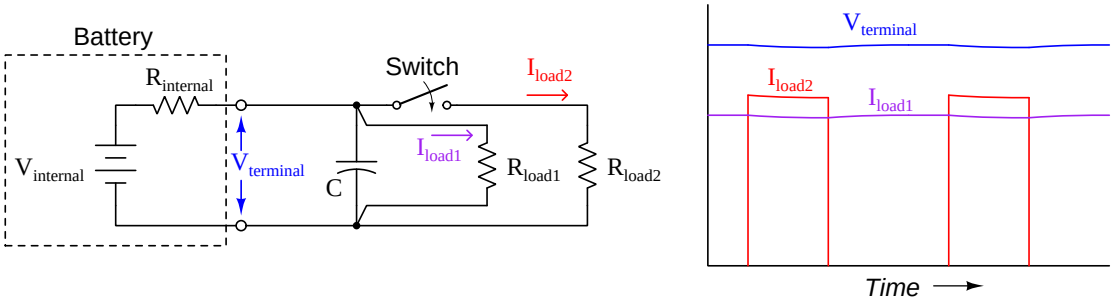
Electrochemical batteries are excellent sources of electrical energy for small-scale electronic projects. They are safer than utility-energized "power supply" units, portable, and readily available. However, they do suffer some limitations, one of them being internal resistance that increases over time as the battery ages. If a battery with high internal resistance is used to energize a load with a pulsing current demand (e.g. where the load is switched on and off), the battery's terminal voltage will rise and fall accordingly:



This variation in battery terminal voltage over time may become problematic if more than one load is energized by the same battery, as a continuously-energizing load will "see" the voltage variations created by the pulsing load:



One way to help stabilize a battery's terminal voltage is to connect a large capacitor in parallel with the battery. When terminal voltage is steady, the capacitor's current will be zero. However, if something acts to disturb that terminal voltage the capacitor will pass current in direct proportion to the rate of voltage change ($I_C = C\frac{dV}{dt}$). In essence, the capacitor acts as a temporary voltage source to "shore up" the battery for brief periods of time while a pulsing load draws current:



If you look closely at the voltage and current graphs you will see a slight inverse-exponential curve at each switching point rather than a "step" as there was before the addition of the capacitor. Not only does the continuously-energized load (R_{load1}) enjoy a more stable voltage, but the pulsed load (R_{load2}) "sees" a voltage closer to that of the battery's internal (ideal) value with the capacitor in place.

Proper sizing of the capacitor depends on the degree to which the loads may tolerate voltage variations and also the time durations of the pulsing load. A simple approximation may be based on an assumed constant-rate voltage rise/fall, the "on" time of the pulsing load, and the current drawn by that pulsing load. For example, if the pulsing load demands 500 mA of current and will be pulsed "on" for 2 milliseconds at a time, and we wish the voltage to fall no more than 250 milliVolts during that "on" time, we could approximate the necessary capacitor size as follows:

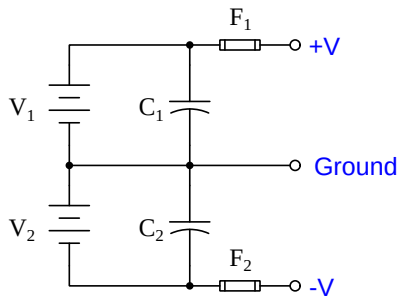
$$C = \frac{I}{\left(\frac{dV}{dt}\right)} = \frac{0.5}{\left(\frac{0.25}{0.002}\right)} = 4000\ \mu\text{F}$$

Since over-sizing the capacitance will result in a more stable voltage and is therefore preferable to under-sizing, it would make sense to select for this application the common capacitor size of 4700 μF.

Capacitors are often connected in parallel to the DC power terminals of high-power audio amplifiers used in automotive sound systems, for this very purpose: to give the bass amplifier a more stable voltage during periods of time when it draws very high current from the vehicle's battery to deliver transient pulses of sound to the speakers. In such applications, where the current demands are quite high, "banks" of parallel-connected capacitors offering *multiple Farads* of total capacitance are typical.

When battery-powering electronic circuits for educational purposes, I recommend a power supply consisting of two alkaline-cell batteries, two capacitors, and two fuses. This provides a "split" power supply with voltage stabilization and overcurrent protection:

Battery-based "split" DC power supply

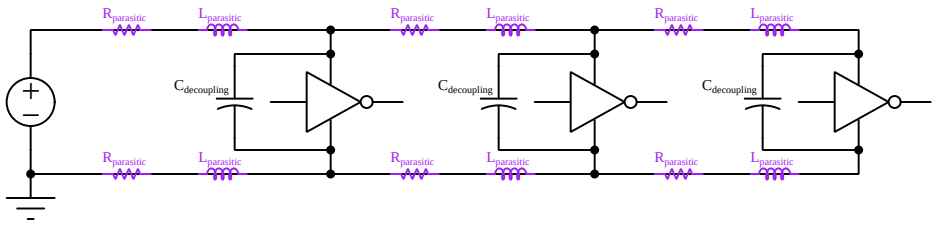


9-Volt "transistor" batteries are good for this purpose, as are 6-Volt "lantern" batteries. The two capacitors should be several thousand microFarads each, and may consist of multiple paralleled capacitors to achieve that amount of capacitance. Fuses should be sized to blow at a current value less than what the battery is able to deliver to a short-circuit (I recommend $\frac{1}{4}$ Ampere AGC-type glass-cartridge fuses for general-purpose use).

2.5 Example: decoupling capacitors

Many forms of electronic circuits are especially sensitive to variations in DC power supply voltage resulting from pulsations of current drawn from the supply over time, the DC terminal voltage tending to "sag" or "droop" as current surges. A highly effective safeguard against this problem is to attach a capacitor as closely as possible to the power supply terminals of the affected circuit, the capacitor serving as a reservoir of electrical energy to temper these transient voltage events.

The nature of the problem may be seen in the following schematic diagram, where we have three separate digital logic "gates" all powered by a common DC voltage source. The wires or printed circuit board traces carrying DC power to these three loads inherently possess both resistance (R) and inductance (L), both of which cause sudden voltage drops when current suddenly increases:



If we imagine the far-right logic gate suddenly drawing current because its output switches state, all the parasitic R and L elements between it and the DC voltage source will manifest additional voltage drop[2], resulting in all three logic gates experiencing a momentary decrease in supply voltage if not for the stabilizing influence of the capacitors connected in parallel to each gate's power supply terminals.

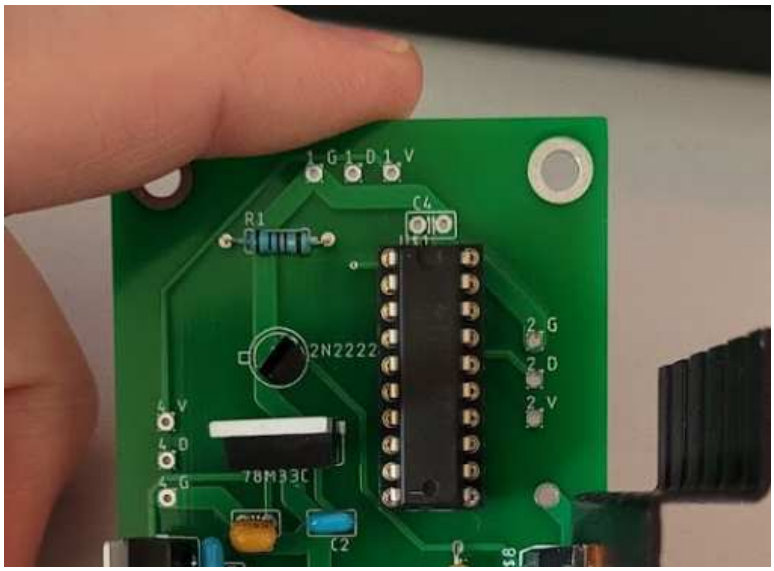
The tendency for DC supply voltage to suddenly dip in a circuit due to effects such as this is sometimes referred to as *ground bounce* or V_{DD} *bounce*[3]. The practice of connecting capacitors in parallel with the integrated circuits (ICs) is called *decoupling*, because it helps prevent the switching action of one load from interfering or "coupling" to another load. Such "bounce" may severely degrade a circuit's performance, causing noise problems in analog circuits and all manner of problems in digital circuits (e.g. erratic signal timing, false counts).

Decoupling is a good practice in any system where we anticipate rapid on/off switching of the components, and it is best to use capacitors with low equivalent series resistance (ESR) so they will be able to source high levels of transient current if necessary. For very high-frequency printed circuit boards a good practice for DC power distribution is to devote two or more of the layers in a multi-layer board as *power planes*: sheets of copper sandwiched between layers of insulating board material acting as "busses" for each rail of the DC power source. Not only does this make DC power easy for each device to access (simply install a "via" which is an inter-layer connector between the desired power plane layer and the device power terminal) but the natural capacitance existing between power planes separated by board insulation contributes to the decoupling offered by discrete capacitors installed adjacent to each device.

[2] Resistance drops voltage proportional to current, which is simply Ohm's Law in action ($V = IR$). Inductance drops voltage in response to a rise in current over time ($V = L\frac{dI}{dt}$), the voltage drop's magnitude being proportional to the *speed* at which current increases.

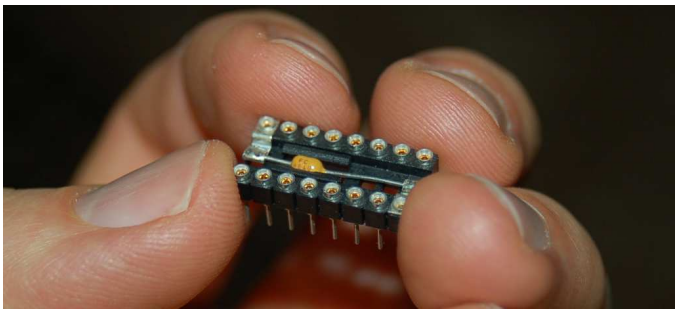
[3] In CMOS digital circuits the positive pole of the DC voltage source is commonly referred to as V_{DD} in honor of its connection to the Drain terminals of many transistors within the IC.

In the following photograph you can see two holes in a printed circuit board ready to accept a small capacitor (C_4) immediately above the 20-pin integrated circuit. When soldered into this incomplete PCB, C_4 will stabilize the DC power supply voltage to that IC so that it experiences less "bounce" on its power supply terminals during switching events:



Note how close to the IC the decoupling capacitor C_4 will be located. The rationale for positioning the decoupling capacitor immediately adjacent to the protected IC is to minimize the amount of length for any copper traces connecting it to the IC's power supply pins, because greater conductor length means more resistance and more inductance, all other factors being equal. Recall how it is the presence of R and L between the load and DC power source that aggravates the "bounce" problem, and so we need our decoupling capacitor to have as little impedance as possible between it and the load whose DC voltage we intend to stabilize.

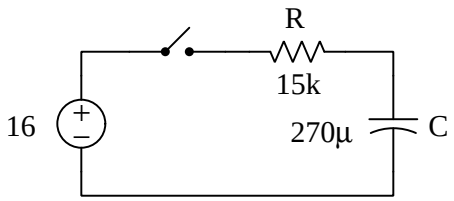
A clever way of incorporating decoupling capacitors into circuits is found in the following photograph of a 14-pin DIP socket. Here the decoupling capacitor is built-in, connected between pins 7 and 14 which are commonly the negative and positive DC power pins for a great many TTL and CMOS logic ICs:



As a general rule, the best decoupling capacitor types are ceramic, mica, and polystyrene due to their naturally low equivalent series resistance (ESR). Tantalum electrolytic capacitors, although blessed with relatively high capacitance in small packages, have fairly high ESR which limits their effectiveness as decoupling capacitors.

2.6 Example: voltage source energizing a capacitor

Note: these calculations all assume an ideal capacitor having no leakage resistance, no equivalent series resistance, no soakage, etc. The switch begins in an open state and is closed at time $t = 0$, the capacitor being completely de-energized prior to switch closure.



Switch closure time	V_C	V_R	I
−1 seconds	0 V	0 V	0 mA
0 seconds	0 V	16 V	1.0667 mA
1 second	3.501 V	12.499 V	0.8333 mA
2 seconds	6.235 V	9.765 V	0.6510 mA
3 seconds	8.372 V	7.628 V	0.5085 mA
4 seconds	10.041 V	5.959 V	0.3973 mA
5 seconds	11.345 V	4.655 V	0.3104 mA
6 seconds	12.363 V	3.637 V	0.2425 mA
7 seconds	13.159 V	2.841 V	0.1894 mA
8 seconds	13.781 V	2.219 V	0.1480 mA
∞	16 V	0 V	0 mA

When the capacitor contains no energy (prior to and at time $t = 0$), it contains zero electric field and consequently zero voltage. As it gradually gains energy from the source following the switch's closure, its electric field increases in strength and so does its voltage. Thus, V_C follows the inverse-exponential growth equation from 0 Volts at $t = 0$ building up to a maximum of 16 Volts at infinite time:

$$V_C = (16 \text{ V}) \left(1 - e^{-\frac{t}{\tau}}\right)$$

Once capacitor voltage is calculated for each time value, all other circuit parameters may be calculated from those V_C values following Kirchhoff's Voltage Law and Ohm's Law.

Here is a step-by-step conceptual analysis of this circuit:

1. **Zero capacitor energy** before the switch is closed, stated as a given condition. Since capacitors store energy in electric fields, and electric fields directly relate to voltage, we may conclude that the capacitor's voltage must be zero as well before the switch is closed.

2. **Zero circuit current** before the switch is closed, since opens guarantee zero current and this is a series circuit which means current must be equal for all components.

3. **Zero capacitor energy** at the precise moment the switch closes, since energy transfer takes time and so far no time has yet elapsed with the switch in the closed state. Again, this means zero capacitor voltage at the precise moment of switch closure.

4. **Full resistor voltage** (16 Volts) at the precise moment of switch closure, as per Kirchhoff's Voltage Law, positive on the left-hand side of the resistor and negative on the right-hand side.

5. **Full resistor current** (0.9375 milliAmperes) at the precise moment of switch closure, as per Ohm's Law, flowing from left to right through the resistor.

6. **Full current through all components** (0.9375 milliAmperes clockwise) at the precise moment of switch closure, since this is a series circuit and current must be equal through all series-connected components.

7. **Increasing capacitor energy** over time as the source forces current "through" it and transfers energy into it. Since capacitors store energy in electric fields, and electric fields directly relate to voltage, we may conclude that the capacitor's voltage must increase as well over time, positive on top and negative on bottom because the capacitor is acting as a load now.

8. **Decreasing resistor voltage** over time as the capacitor's voltage increases, as per Kirchhoff's Voltage Law.

9. **Decreasing resistor current** over time as the resistor's voltage decreases, as per Ohm's Law.

10. **Decreasing circuit current** over time, being a series circuit where all components share the same current value at each point in time.

11. **Full capacitor energy** after the switch has been closed a long while, following the increasing trend established after the switch closed.

12. **Full capacitor voltage** after the switch has been closed a long while and the entirety of the source voltage is dropped across it (16 Volts). This means the capacitor now possesses the maximum amount of energy it can for this circuit, since capacitor energy is directly related to electric field strength and capacitor voltage.

13. **Zero resistor voltage** after the switch has been closed a long while, as per Kirchhoff's Voltage Law with the capacitor dropping the entire source voltage.

14. **Zero resistor current** after the switch has been closed a long while, as per Ohm's Law.

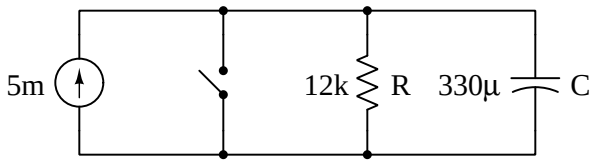
15. **Zero circuit current** after the switch has been closed a long while, all components being connected in series and therefore sharing the same current.

16. *SUMMARY:* capacitor voltage grows from zero to 16 Volts when the switch closes; resistor voltage spikes to 16 Volts when the switch closes and then decays to zero over time; circuit current spikes to 0.9375 milliAmperes when the switch closes and then decays to zero over time. *We are now ready to commence inverse-exponential calculations of voltages and currents over time!*

Many students have a bad tendency to skip these conceptual steps in lieu of jumping immediately to the inverse-exponential formulae which they know will ultimately give them the answers they seek for voltages and currents at certain times. Remember, it is important to always let the physical concepts guide the application of mathematics, lest you fall into the common error of blindly plugging numbers into equations in the hope of getting the right answers! Your schematic diagram should be fully annotated with current arrows and voltage polarities before any calculations are performed!

2.7 Example: current source energizing a capacitor

Note: these calculations all assume an ideal capacitor having no leakage resistance, no equivalent series resistance, no soakage, etc. The switch begins in a closed state and is opened at time $t = 0$.



Switch opening time	I_C	I_R	V
-1 seconds	0 mA	0 mA	0 V
0 seconds	5 mA	0 mA	0 V
1 second	3.8842 mA	1.1158 mA	13.39 V
2 seconds	3.0174 mA	1.9826 mA	23.79 V
3 seconds	2.3440 mA	2.6560 mA	31.88 V
4 seconds	1.8209 mA	3.1791 mA	38.15 V
5 seconds	1.4146 mA	3.5854 mA	43.03 V
6 seconds	1.0989 mA	3.9011 mA	46.81 V
7 seconds	0.8536 mA	4.1464 mA	49.76 V
8 seconds	0.6631 mA	4.3369 mA	52.04 V
∞	0 mA	5 mA	60 V

The switch's closed contact status prior to and at time $t = 0$ ensures a zero-voltage condition at the capacitor, preventing any electric field from developing within the capacitor and therefore ensuring a zero-energy state for the capacitor. After the switch opens the capacitor is free to accept energy from the current source and so its electric field increases in strength and so does its voltage. Thus, V_C follows the inverse-exponential growth equation from 0 Volts at $t = 0$ building up to a maximum of 60 Volts at infinite time:

$$V_C = (60 \text{ V}) \left(1 - e^{-\frac{t}{\tau}}\right)$$

Once capacitor voltage is calculated for each time value, all other circuit parameters may be calculated from those V_C values following Kirchhoff's Current Law and Ohm's Law.

Here is a step-by-step conceptual analysis of this circuit:

1. **Zero switch voltage** throughout before the switch is opened, since the closed switch prohibits any voltage developing across it.
2. **Zero circuit voltage** throughout before the switch is opened, due to the fact that all components in this circuit are connected in parallel with each other, and parallel components must share the same voltage.
3. **Zero capacitor energy** before the switch is opened, since the zero-voltage condition established by the closed switch ensures a zero-energy state for the capacitor since capacitors store energy in electric fields and electric fields directly relate to voltage.
4. **Zero resistor current** before the switch is opened, as per Ohm's Law.
5. **Maximum switch current** before the switch is opened, as per Kirchhoff's Current Law since the source's 5 milliAmpere current has nowhere else to go.
6. **Zero capacitor energy** at the precise moment the switch opens, since energy transfer takes time and so far no time has yet elapsed with the switch in the open state. Again, this means zero capacitor voltage at the precise moment of switch opening.
7. **Zero resistor voltage** at the precise moment of switch opening since the resistor is in parallel with – and must share the same voltage as – the capacitor which still has zero voltage across it.
8. **Zero resistor current** at the precise moment of switch opening, as per Ohm's Law.
9. **Zero switch current** at the precise moment of switch opening, since the guaranteed effect of an open is no current.
10. **Full current through the capacitor** (5 milliAmperes downward) at the precise moment of switch opening, as per Kirchhoff's Current Law since the source's current has nowhere else to go.
11. **Increasing capacitor energy** over time as the source forces current "through" it and transfers energy into it. Since capacitors store energy in electric fields, and electric fields directly relate to voltage, we may conclude that the capacitor's voltage must increase as well over time, positive on top and negative on bottom because the capacitor is acting as a load now.
12. **Increasing resistor voltage** over time as the capacitor's voltage increases, since they are in parallel with each other and therefore must share the same voltage.
13. **Increasing resistor current** over time as the resistor's voltage increases, as per Ohm's Law.
14. **Decreasing capacitor current** over time, as per Kirchhoff's Current Law with the resistor drawing more of the source's fixed 5 milliAmperes, leaving less current for the capacitor.

15. **Full resistor current** after the switch has been open a long while, following the increasing trend established after the switch opened.

16. **Full resistor voltage** after the switch has been open a long while, as per Ohm's Law.

17. **Full capacitor voltage** after the switch has been open a long while, being in parallel with the resistor and therefore sharing the same full voltage. This means the capacitor now possesses the maximum amount of energy it can for this circuit, since capacitor energy is directly related to electric field strength and capacitor voltage.

18. **Zero capacitor current** after the switch has been open a long while, as per Kirchhoff's Current Law with the resistor passing the entirety of the source current.

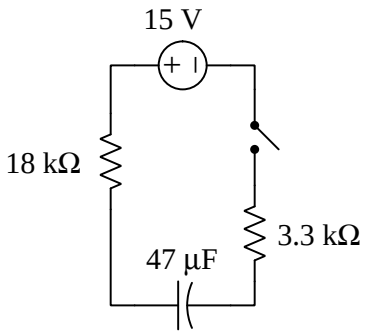
19. *SUMMARY:* capacitor voltage grows from zero to 60 Volts when the switch opens; resistor voltage also grows from zero to 60 Volts over time; capacitor current spikes to 5 milliAmperes when the switch opens and then decays to zero over time. *We are now ready to commence inverse-exponential calculations of voltages and currents over time!*

Many students have a bad tendency to skip these conceptual steps in lieu of jumping immediately to the inverse-exponential formulae which they know will ultimately give them the answers they seek for voltages and currents at certain times. Remember, it is important to always let the physical concepts guide the application of mathematics, lest you fall into the common error of blindly plugging numbers into equations in the hope of getting the right answers! Your schematic diagram should be fully annotated with current arrows and voltage polarities before any calculations are performed.

2.8 Example: RC time-delay network values

Example #1

Determine the trajectories of capacitor voltage and current in this circuit once the switch closes ($t = 0$), assuming zero stored energy in the capacitor prior to switch closure, as well as values for V_C and I_C at $t = 1$ second, $t = 2$ seconds, and $t = 3$ seconds.



Before the switch closes ($t < 0$) the capacitor will have zero voltage because it is completely de-energized, and capacitors store energy in electric fields manifesting as voltage. Current will also be zero because the open switch prohibits current.

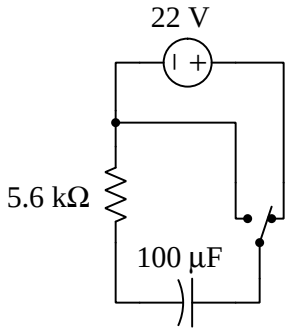
At the moment of switch closure ($t = 0$) the capacitor still has zero voltage because there has been no time for any energy to transfer into it from the 15 Volt source, and energy cannot simply appear from nowhere. By Kirchhoff's Voltage Law this means the entirety of the 15 Volt source drops across the circuit's total series resistance of 21.3 kΩ, resulting in 0.70423 mA of current moving counter-clockwise in the circuit.

As the switch remains closed ($t > 0$) the capacitor will act as a load absorbing energy from the 15 Volt source, which means capacitor voltage will grow over time until it equalizes with the source's voltage value. At that point the resistors will have no voltage dropped across them (by Kirchhoff's Voltage Law) and this means current will ultimately decay down to zero. Once current reaches its final value of zero, the capacitor will no longer be a load (nor a source) and its energy value and voltage value will remain constant at zero.

- At $t = 1$ second: $V_C = 9.4757$ Volts and $I_C = 0.25935$ milliAmperes
- At $t = 2$ seconds: $V_C = 12.966$ Volts and $I_C = 0.095516$ milliAmperes
- At $t = 3$ seconds: $V_C = 14.251$ Volts and $I_C = 0.035177$ milliAmperes

Example #2

Determine the trajectories of capacitor voltage and current in this circuit once the switch toggles position ($t = 0$), assuming the switch has been in the position shown for a very long time prior to toggling, as well as values for V_C and I_C at $t = 1$ second, $t = 2$ seconds, and $t = 3$ seconds.



Before the switch toggles ($t < 0$) the capacitor will have 22 Volts across it because it is connected in parallel with the 22 V source. Current will also be zero because with the source and capacitor voltages equal there will be no differential to move current across wire resistance.

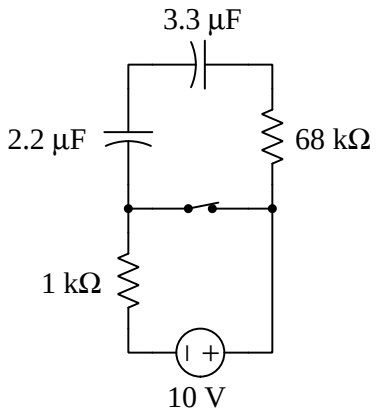
At the moment of switch toggling ($t = 0$) the capacitor still has 22 Volts because there has been no time for any energy to transfer out of it to the load resistor, and energy cannot simply vanish into nothing. With the switch in its new position, though, this voltage will be applied across the resistor which will yield a current by Ohm's Law (3.9286 mA) current moving counter-clockwise.

As the switch remains in its new position ($t > 0$) the capacitor will act as a source delivering energy to the resistor's load, which means capacitor voltage will decay over time until it reaches zero and is fully de-energized. At that point the resistors will also have no voltage dropped them (being in parallel with the capacitor) and this means current will ultimately decay down to zero as well. Once current reaches its final value of zero, the capacitor's voltage remains at zero Volts because it is acting neither as a source nor a load.

- At $t = 1$ second: $V_C = 3.6889$ Volts and $I_C = 0.65873$ milliAmperes
- At $t = 2$ seconds: $V_C = 0.61854$ Volts and $I_C = 0.11045$ milliAmperes
- At $t = 3$ seconds: $V_C = 0.10372$ Volts and $I_C = 0.018521$ milliAmperes

Example #3

Determine the trajectories of capacitor voltage and current in this circuit once the switch opens ($t = 0$), assuming the switch was in its closed position for a very long time, as well as values for V_C and I_C at $t = 0.1$ second, $t = 0.2$ seconds, and $t = 0.3$ seconds.



Before the switch opens ($t < 0$) the capacitors should collectively have no voltage across them, having had ample time to exhaust their stored energy in the 68 kΩ resistor. Current will also be zero in this de-energized state.

At the moment of switch opening ($t = 0$) the capacitors will still have no voltage across them because there has been no time for any energy to transfer into them from the 10 Volt source, and we know stored energy in any capacitor manifests as voltage across that capacitor's terminals. With the switch in its new position, though, the capacitors will be able to absorb energy from the source over time. Kirchhoff's Voltage Law tells us the two resistors (taken together as a 69 kΩ series equivalent resistance) must drop the full voltage of the 10 Volt source, and by Ohm's Law this must yield a counter-clockwise current in the circuit of 144.93 microAmperes.

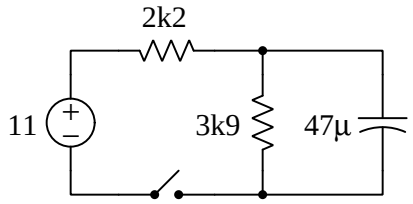
As the switch remains open ($t > 0$) the capacitors will act as loads absorbing energy from the 10 Volt source, which means both capacitors' voltage will grow over time until they collectively (in series) reach 10 Volts. At that point the resistors will also have no voltage dropped them (by Kirchhoff's Voltage Law) and this means current will ultimately decay down to zero.

It is helpful to simplify this circuit by considering it as one with a single capacitor (1.32 μF) and a single resistor (69 kΩ), having a time constant of $t = 0.09108$ seconds.

- At $t = 0.1$ second: $V_C = 6.6644$ Volts and $I_C = 48.342$ microAmperes
- At $t = 0.2$ seconds: $V_C = 8.8874$ Volts and $I_C = 16.125$ microAmperes
- At $t = 0.3$ seconds: $V_C = 9.6289$ Volts and $I_C = 5.3786$ microAmperes

2.9 Example: voltage divider and capacitor

This circuit is more challenging to analyze, given the presence of *two* resistors rather than just one. As usual, an ideal capacitor is assumed having no internal resistance or soakage characteristics:



First, we close the switch and determine capacitor voltage and capacitor current at 30 millisecond intervals, then after letting the capacitor reach its full state of energization we open the switch and let it de-energize again:

```
Switch closes!
At time = 0 msec,   V_C = 0 V       and I_C = 5 mA
At time = 30 msec,  V_C = 2.5655 V  and I_C = 3.17604 mA
At time = 60 msec,  V_C = 4.19513 V and I_C = 2.01745 mA
At time = 90 msec,  V_C = 5.23028 V and I_C = 1.2815 mA
At time = 120 msec, V_C = 5.88782 V and I_C = 0.814022 mA
At time = 150 msec, V_C = 6.30549 V and I_C = 0.517074 mA
At time = 180 msec, V_C = 6.5708 V  and I_C = 0.32845 mA
At time = 210 msec, V_C = 6.73933 V and I_C = 0.208634 mA
At time = 240 msec, V_C = 6.84638 V and I_C = 0.132526 mA
At time = 270 msec, V_C = 6.91438 V and I_C = 0.084182 mA
At time = 300 msec, V_C = 6.95757 V and I_C = 0.0534731 mA

Switch opens!
At time = 0 msec,   V_C = 7.03279 V and I_C = 1.80328 mA
At time = 30 msec,  V_C = 5.97101 V and I_C = 1.53103 mA
At time = 60 msec,  V_C = 5.06954 V and I_C = 1.29988 mA
At time = 90 msec,  V_C = 4.30417 V and I_C = 1.10363 mA
At time = 120 msec, V_C = 3.65435 V and I_C = 0.937013 mA
At time = 150 msec, V_C = 3.10264 V and I_C = 0.795548 mA
At time = 180 msec, V_C = 2.63422 V and I_C = 0.675441 mA
At time = 210 msec, V_C = 2.23652 V and I_C = 0.573466 mA
At time = 240 msec, V_C = 1.89886 V and I_C = 0.486887 mA
At time = 270 msec, V_C = 1.61218 V and I_C = 0.41338 mA
At time = 300 msec, V_C = 1.36878 V and I_C = 0.35097 mA
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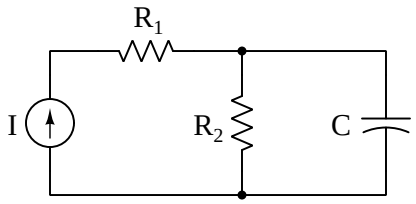
2.10 Example: simplifying complex RC networks using Thévenin's Theorem

Determining the time constant (τ) and initial/final voltages for an elementary RC network is usually a straightforward process, especially time constant which is simply the product $\tau = RC$. However, for networks containing multiple resistors it may not be clear upon first inspection exactly how those resistors interact with the capacitor. In some multi-resistor networks *every* resistor impacts the time constant, while in others only some do.

An extremely useful tool for this application is Thévenin's Theorem, applying it to the original RC network in such a way that the capacitor is identified as the load, reducing the rest of the original network to a single voltage source (V_{Th}) and a single equivalent resistance (R_{Th}). That Thévenin equivalent voltage limits the capacitor's maximum voltage at any time (assuming the capacitor was not pre-charged to some greater potential prior to connection to the network), and that Thévenin resistance combined with the "load" capacitance directly defines time constant ($\tau = R_{Th}C$).

The following RC network examples showcase the utility of Thévenin's Theorem to help make comprehensible what might otherwise defy intuition. You, as the reader, are strongly encouraged to Thévenize each of these example networks on your own in order to see for yourself how the equations for time constant and maximum voltage emerge. Simply regard the capacitor in each case as the load, remove that load from the network and determine R_{Th} by replacing the source with its ideal internal resistance value (shorts for voltage sources and opens for current sources) and then determining total resistance as measured from the load terminals (where the capacitor used to be), and then determine load terminal voltage with the source activated once again.

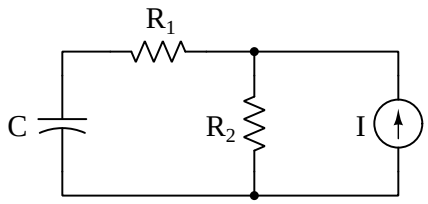
Example #1



$$R_{Th} = R_2 \qquad \tau = R_2C \qquad V_{Th} = IR_2$$

In this network the value of R_1 is irrelevant both to time constant and to maximum capacitor voltage.

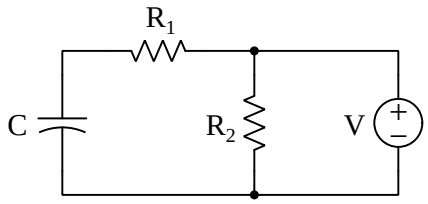
Example #2



$$R_{Th} = R_1 + R_2 \qquad \tau = (R_1 + R_2)C \qquad V_{Th} = IR_2$$

In this network the value of R_1 is irrelevant to maximum capacitor voltage but does affect time constant.

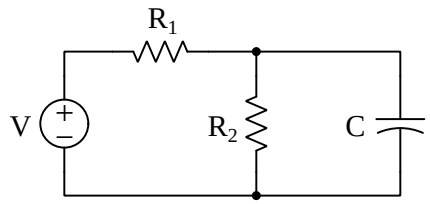
Example #3



$$R_{Th} = R_1 \qquad \tau = R_1C \qquad V_{Th} = V$$

In this network the value of R_2 is irrelevant both to time constant and to maximum capacitor voltage, while R_1 only affects time constant.

Example #4



$$R_{Th} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \qquad \tau = \frac{C}{\frac{1}{R_1} + \frac{1}{R_2}} \qquad V_{Th} = \frac{V R_2}{R_1 + R_2}$$

In this network both the values of R_1 and R_2 affect time constant and maximum capacitor voltage. That maximum voltage (V_{Th}) is derived using the voltage divider equation, since R_1 and R_2 effectively form a voltage-division network with voltage source V from the perspective of the load terminals.

2.11 Example: measuring signal rates of change

Capacitors and inductors relate voltage to current by rates of change. For a capacitor, the amount of current is proportional to how quickly voltage across that capacitor either rises or falls over time ($I = C\frac{dV}{dt}$). For an inductor, the amount of voltage is proportional to how quickly current through that inductor rises or falls over time ($V = L\frac{dI}{dt}$).

For example, a 330 microFarad capacitor experiencing a voltage increasing at a rate of 45 Volts per second will pass 14.85 milliAmperes. If the voltage happens to decrease at an equivalent rate (i.e. $\frac{dV}{dt} = -45$ Volts per second) then the 14.85 milliAmpere current will reverse direction through the capacitor compared to how it flowed with the increasing voltage.

Similarly, a 100 milliHenry inductor experiencing a current increasing at a rate of 5 Amperes per second will induce a voltage of 500 milliVolts. If the current happens to decrease by an equivalent rate (i.e. $\frac{dI}{dt} = -5$ Amperes per second) then the 500 milliVolt voltage induced across the inductor will reverse polarity from what it was during the period of increasing current.

Not only are rates-of-change important in determining how energy-storing devices such as capacitors and inductors will respond in circuits, but rates-of-change are also important for determining how *parasitic* capacitances and inductances will affect intended circuit behavior. Parasitic capacitance exists between any two conducting surfaces separated by an electrically insulating medium, and parasitic inductance exists along any length of conductor. This means any rate-of-change of voltage over time between two separated conductors will cause some amount of current to "pass"[4] between them, and that any rate-of-change of current over time through any single conductor will cause some amount of voltage to drop across its length. In many circuits these parasitic effects are negligible, but in circuits experiencing extremely fast rates of change for voltage and/or current the effects can be significant or even severe.

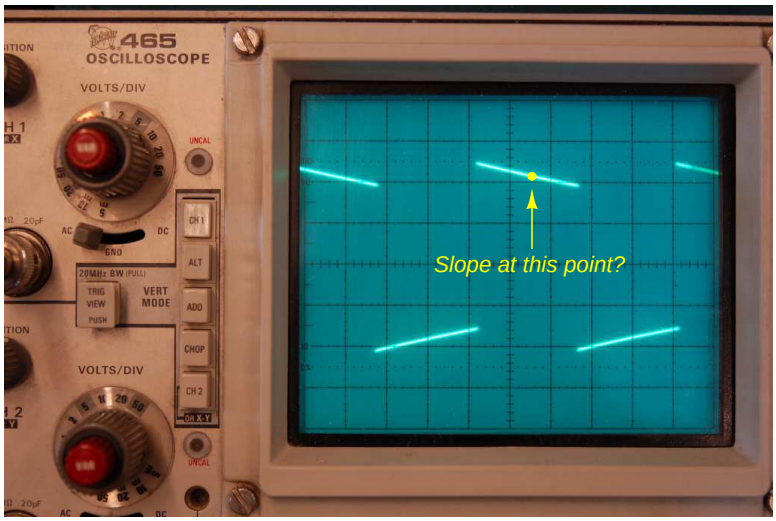
Oscilloscopes are ideal for performing empirical measurements of voltage rates-of-change, and of current rates-of-change given the proper accessories[5]. Some skill is required to do this, though, and here we will explore practical examples to show how it is done.

For any signal plotted in the time domain, where the horizontal axis of the plot is expressed in units of seconds, milliseconds, microseconds, etc., the signal's rate of change at any given point will be the *slope* or *pitch* of the waveform, mathematically defined as its *rise over run*. A great aid to discerning slope at any location on a waveform is to sketch a straight line visually matching the wave's slope at that point, then use locations along that straight line to more easily discern how far it rises (or falls) over some "run" of time. We call this straight line a *tangent line*.

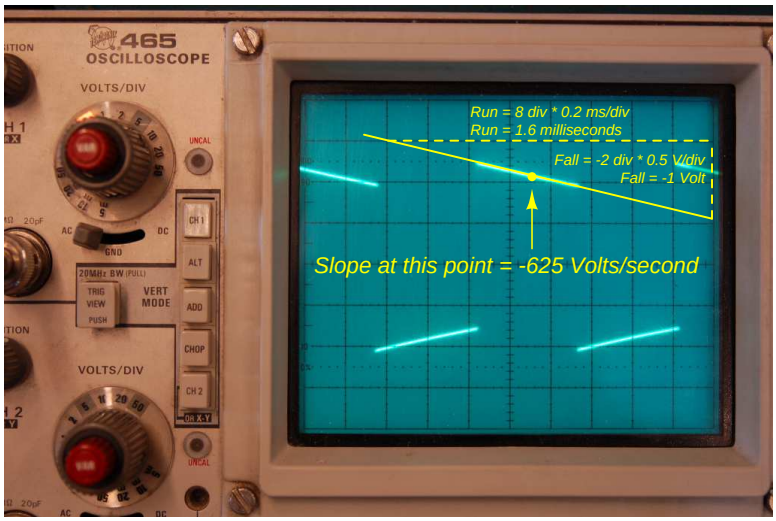
[4] The Scottish physicist James Clerk Maxwell referred to this current as *displacement current*.

[5] For example, a *current probe* converting a sensed current into a voltage the oscilloscope may directly sense, or a *shunt resistor* placed in the circuit developing an oscilloscope-measurable voltage drop for any current passing through.

Here we see an example of a waveform with sloping sections. In the first image we see a specific location on the waveform where we wish to measure voltage rate-of-change ($\frac{dV}{dt}$):



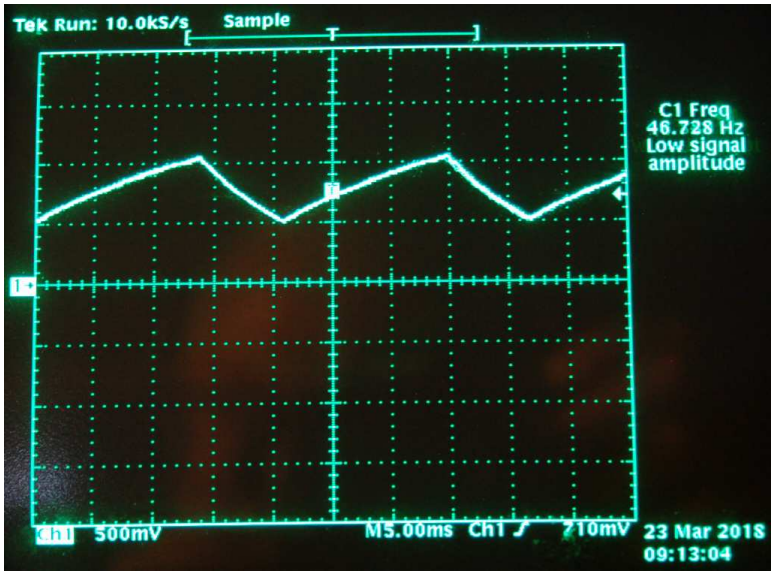
Next we see a tangent line drawn to match the slope of the waveform at the specified location, with convenient points for fall/run measurements taken on that line against the oscilloscope grid's major divisions. In this example, the oscilloscope's vertical sensitivity has been set for 0.5 Volts per division, and the horizontal timebase for 0.2 milliseconds per division:



As we can see, the tangent line falls 2 vertical divisions (-1 Volt) over a timespan of 8 horizontal divisions (1.6 milliseconds), yielding a $\frac{dV}{dt}$ quotient of -625 Volts per second, which may also be expressed as -0.625 Volts per millisecond. The negative sign is important, as it distinguishes this particular rate-of-change as *falling* rather than rising over time.

Such rate-of-change measurements are necessarily approximate, as they require us to visually gauge where a tangent line may be overlaid on the waveform's oscillograph, and also to visually assess the slope of that tangent line using the grid provided on the instrument's display screen. However, in most applications extremely precise rate-of-changes are not necessary, and such techniques suffice quite well.

Below is another example of a waveform with sloping sections, the oscilloscope configured for 500 milliVolts per division on the vertical axis and 5 milliseconds per division on the horizontal:

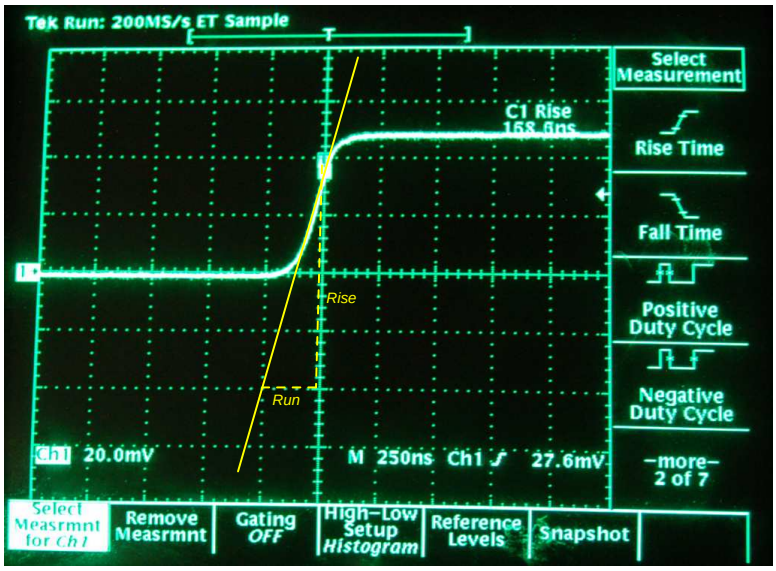


Approximating the slope for each rising section of this wave, we count one division of rise over 2.8 divisions of run, or 500 milliVolts rise over 14 milliseconds of run. This is a rate-of-rise of +35.7 Volts per second.

Approximating the slope for each falling section of this wave, we count one division of fall over 1.4 divisions of run, or −500 milliVolts fall over 7 milliseconds of run. This is a rate-of-fall of −71.4 Volts per second.

If these rates-of-change appear suspiciously large compared to the actual amplitude of the waveform, which barely crests over +1 Volt on the oscillograph, bear in mind that we are calculating *rates of change* for voltage and not absolute values of voltage itself. This is analogous to the distinction between *speed* and *distance*: traveling at a rate of 30 kilometers per hour does not necessarily mean you will travel 30 kilometers, as the actual distance traveled depends on how long that speed is sustained. A voltage rising at a rate of 35.7 Volts per second would indeed rise 35.7 Volts *if given a full second to do so*, but since each rising/falling portion of this waveform is so short in duration the actual amount of rise or fall in each case is only one-half of one Volt. It is therefore perfectly appropriate to consider any $\frac{dV}{dt}$ value as being the *speed* at which a voltage increases or decreases over time, distinct from the actual value of that voltage at any particular moment in time.

Here we see another oscillograph, this one zoomed into the rising edge of a square wave. For this measurement the oscilloscope was configured for 20 milliVolts per division of vertical sensitivity and a timebase of 250 nanoseconds per division on the horizontal:



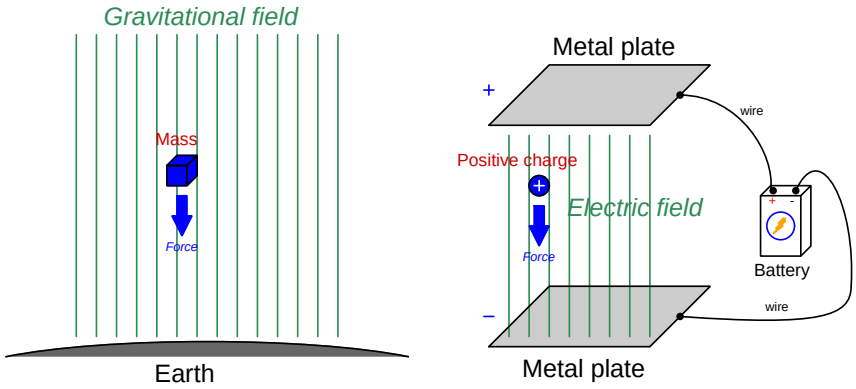
The tangent line overlaid on this screenshot for the purpose of measuring the pulse edge's rate-of-change rises approximately 4 vertical divisions over a run of 1 division, which is 80 milliVolts of rise over 250 nanoseconds of run. The pulse edge's rate-of-change, therefore, is approximately +320,000 Volts per second, or +320 Volts per millisecond, or +0.32 Volts per microsecond (all equivalent expressions of $\frac{dV}{dt}$).

Chapter 3

Tutorial

3.1 Electric fields and energy storage

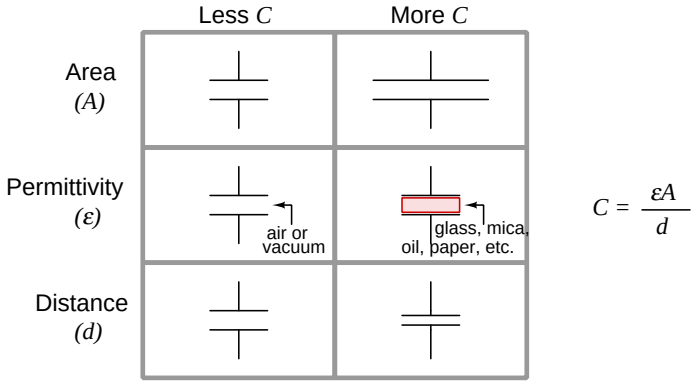
An *electric field* is an invisible web of interaction between electrically charged matter, existing whenever opposite charges are separated from each other over space. Any electrically charged matter within an electric field will experience a force acting upon it from that field, similar to how any mass within a gravitational field experiences a force we call *weight*:



An electric charge experiences a force when exposed to an electric field, just as a mass experiences force (called "weight") when exposed to a gravitational field

Just as any weight suspended within a location where it *could* fall given the proper conditions possesses potential energy, opposing electric charge carriers separated by an insulating medium also possess electrical potential energy. This is the basis of *capacitance*: the ability to store energy in an electric field. The amount of energy stored by any capacitance is a function of voltage: all other factors being equal, more voltage existing between two surfaces means more energy stored in the electric field.

Electrical components called *capacitors*[1] exploit this phenomenon, constructed of two conductive surfaces separated by a layer of electrical insulation called a *dielectric*. The amount of capacitance exhibited by any capacitor is directly proportional to the overlapping area of the conductive surfaces (in square meters), directly proportional to the dielectric *permittivity*[2] (ϵ) of the dielectric material, and inversely proportional to the separation distance between plates (in meters):



Energy stored in any capacitance is a function of the capacitance and the amount of voltage impressed across it as described below:

$$E_C = \frac{1}{2}CV^2$$

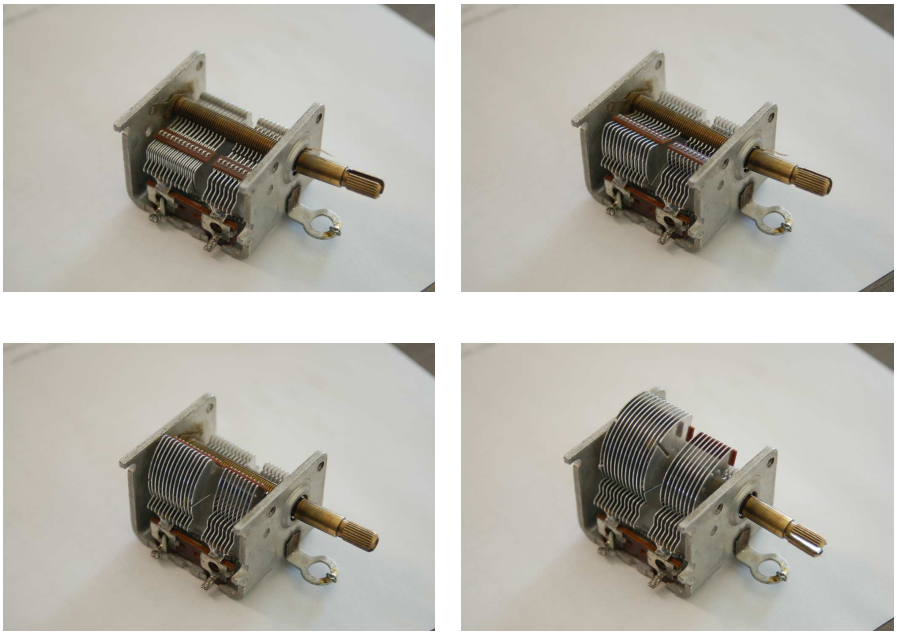
Where,
E_C = energy in Joules (J)
C = capacitance in Farads (F)
V = voltage in Volts (V)

Note the similarities between this equation and those describing potential energy stored within a mechanical spring ($E_p = \frac{1}{2}kx^2$) and kinetic energy stored within a moving mass ($E_k = \frac{1}{2}mv^2$). Just as the amount of potential energy stored within a spring varies with the square of the spring's displacement (x), potential energy stored within a capacitor varies with the square of the voltage across it.

[1] An obsolete term for capacitor is *condensor* or *condenser*.

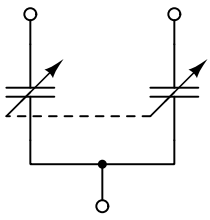
[2] Permittivity, simply defined, is a measure of how effective a dielectric substance is at electrostatically storing energy for any given electric field strength. It may be measured in absolute terms, expressed in units of Farads per meter, or it may be expressed as a ratio relative to the permittivity of a perfect vacuum.

One way to make a *variable* capacitor is to arrange the plates in such a way that their overlap area may be easily adjusted. In the example below we see a variable capacitor using a set of stationary plates versus a set of movable plates rotated by a shaft, those two sets of metal plates separated by an air gap. From left to right and top to bottom, we see a sequence beginning with maximum overlap (i.e. maximum capacitance) and progressing to less and less overlap (i.e. less capacitance):



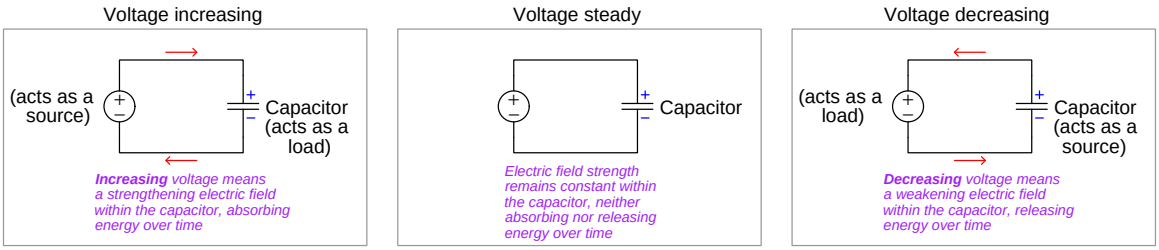
Since the separation distance between the plates is fixed, and so is the nature of the dielectric (air), the dominant variable affecting capacitance is the effective plate area which is determined by their overlap area.

This variable capacitor is actually *two* capacitors in one, sharing a common terminal. Its schematic diagram looks like this:



The two individual capacitor terminals appear on the left-hand side of the device shown in the photos, while the common terminal is below the rotary shaft.

Energy is always conserved, which means it must always be accounted for in any system and cannot simply appear or vanish. Another way of stating this same fact is to say that the only way for the amount of stored energy in any system to vary is if energy *transfers* to or from some other place[3]. Since energy is a function of voltage for any capacitance, the amount of voltage across a fixed capacitance's terminals cannot vary unless energy is added to or removed from that capacitance. That energy transfer occurs when charge carriers enter or exit a capacitor. **In other words, given that energy is always conserved and that stored energy within a capacitor manifests as voltage, it must be voltage that is conserved while current changes direction based on the accumulation or release of energy.**



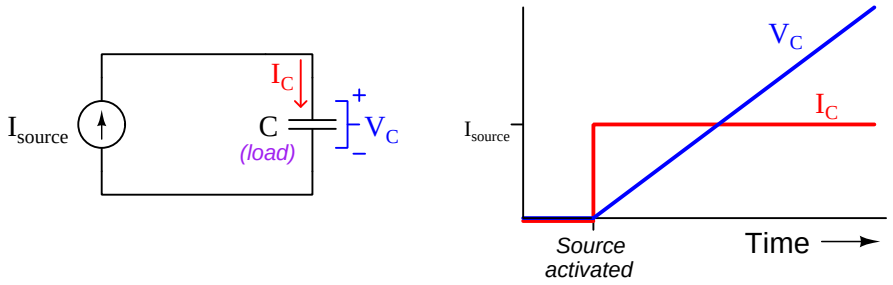
Thus, a capacitance maintains a steady store of energy when its voltage is constant and its current[4] is zero. In this condition of steady voltage and zero current the capacitance simply behaves as an *open*. However, if voltage increases across a capacitance it means its stored energy must also increase, which in turn means the capacitance must act as a load as it absorbs energy from some external source. Conversely, if voltage decreases across a capacitance it means its stored energy must also decrease, which in turn means the capacitance must act as a *source* as it releases energy to some external load.

[3] A close analogy based on the Conservation of Mass is that of a container holding water. Since water has mass, the only way for the amount of water in that container to vary is if water leaves it or enters it. A container's water mass cannot simply vary without any cause, because mass is a conserved quantity.

[4] At first it may seem as though current should be impossible because a capacitor is an *open* circuit. However, recall that open circuits do not prohibit current; they only prohibit *continuous current*.

3.2 Voltage, current, energy, and capacitance

If we connect a DC current source to a de-energized capacitor, we find that the voltage across the capacitor will rise at a steady rate determined by the amount of capacitance and the amount of current. Capacitor voltage must begin at zero because capacitors store energy by means of electric fields, and since voltage is a function of electric field strength this means a de-energized capacitor is guaranteed to have zero voltage. As energy gradually transfers from the current source to the capacitor, its electric field strength increases and along with that so must its voltage:



Since energy transfer in or out of a capacitor requires movement of charge carriers, charge carrier flow (i.e. current magnitude) dictates energy transfer rate and therefore how rapidly capacitor voltage varies. This may be expressed mathematically as follows:

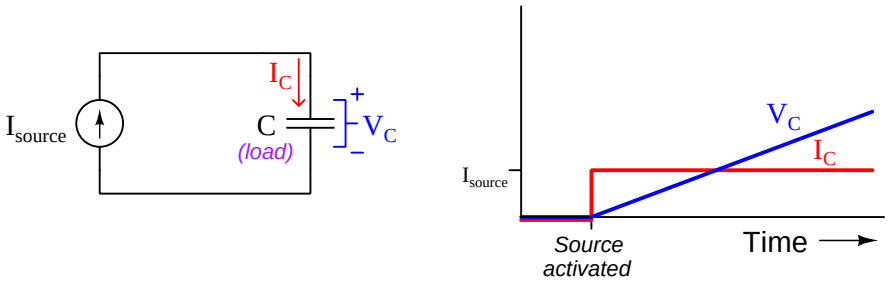
$$I = C\frac{dV}{dt}$$

Where,

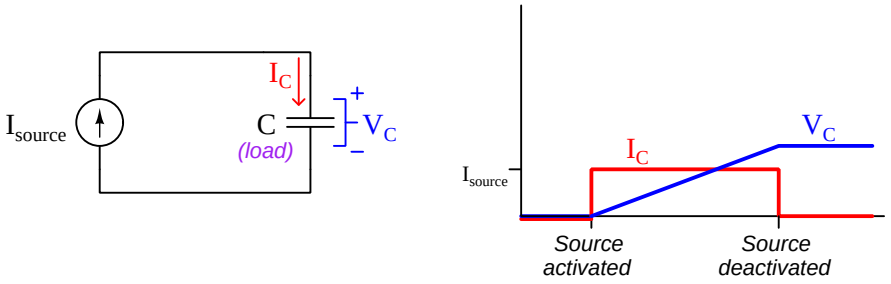
I = current in Amperes (A)
C = capacitance in Farads (F)
$\frac{dV}{dt}$ = rate-of-change of voltage in Volts per second (V/s)

This formula may be thought of as "Ohm's Law" for capacitance. Unlike a resistor where voltage and current are always in direct and immediate proportion to each other, the *speed* of a capacitor's voltage is directly proportional to its current. The presence of time in the characteristic equation makes capacitance behave quite different from resistance, and any circuit containing a substantial amount of capacitance must be analyzed with *time* as one of its variables.

If we were to repeat this experiment on the same size capacitor, but using a current source with only half as much current, we would find the voltage still rises linearly over time but at half the rate. The decreased current means the rate of energy transfer to the capacitor from the source is less, and with that the rate of voltage rise must also be less because (once again) capacitor energy and capacitor voltage are directly related to each other:



De-activating the current source at some point in time prior to the end of the experiment results in the capacitor holding its store of energy within its electric field and therefore maintaining a constant voltage:

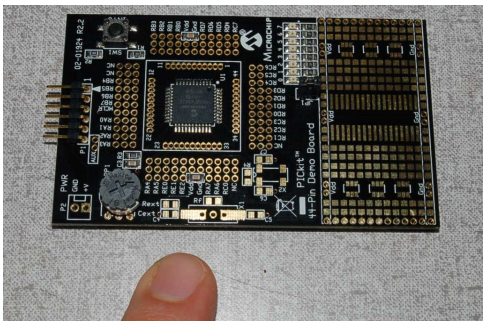


Thinking in terms of *energy storage* is essential for proper reasoning regarding voltage and current for any capacitance. Capacitance is defined as the storage of energy within an *electric field*, and electric fields directly relate to *voltage*. Voltage cannot instantaneously jump from one value to another for any capacitance because in order for capacitive voltage to vary at all there must be a transfer of energy either to or from somewhere else, and this always requires time to occur. Current, on the other hand, may very well immediately jump from one value to another for a capacitance because current merely dictates the *rate* at which energy transfers in or out rather than the absolute amount of energy stored. Joule's Law is helpful here, *power* being the product of voltage and current ($P = IV$): for any given amount of voltage, the rate of energy transfer (i.e. power) depends on how much current there is.

3.3 Applications of capacitance

Capacitors find many applications in DC circuits. Perhaps the most obvious use of capacitance is to store energy in a manner similar to that of a secondary-cell ("rechargeable") battery. Connecting a capacitor across the terminals of a DC voltage source will cause that capacitor to energize and absorb energy from that source. Subsequently disconnecting that capacitor from the source results in the capacitor holding that stored energy, and then when the capacitor is connected to a load it delivers that stored energy to the load. The following photograph shows an array of six so-called *supercapacitors*[5] ganged together to serve as a replacement for the lead-acid storage battery in a passenger vehicle:

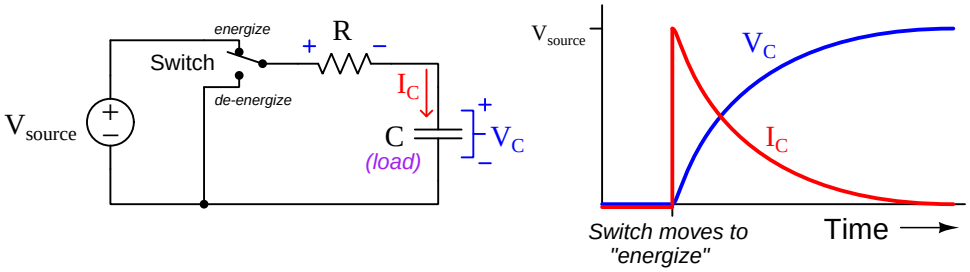
Another use of capacitors is *voltage stabilization*, exploiting the principle that a capacitance's voltage cannot vary unless energy transfer occurs. A capacitor connected in parallel with any component will force that component's voltage to be more stable than it might be otherwise, the capacitor acting as a load when voltage rises and acting as a source when voltage falls. This is analogous to an *accumulator*[6] acting to stabilize the fluid pressure within a hydraulic system by serving as a load when fluid pressure increases and as a source when fluid pressure decreases. An examination of integrated circuit (IC) "chips" on a *printed circuit board* (PCB) typically reveals a capacitor connected in parallel with the power supply terminals on each IC for the purpose of mitigating voltage fluctuations that might make the IC misbehave. The following photograph shows two such capacitors, each soldered between the `Vdd` and `Gnd` terminals of a microcontroller PCB:



[5] This term is well-deserved, and refers to the unusually high capacitance exhibited by each capacitor. One Farad is a rather large amount of capacitance by any standard, and each one of the supercapacitors shown is rated at *650 Farads*.

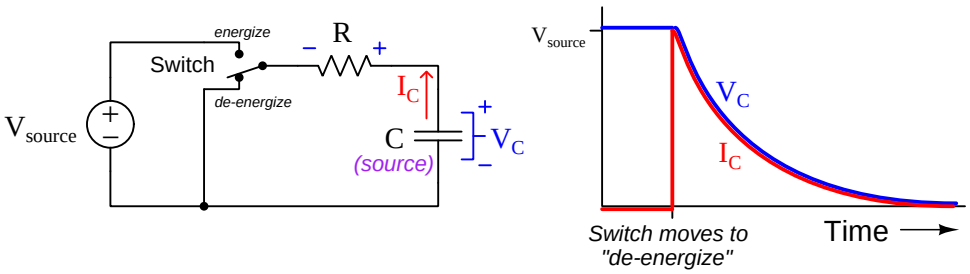
[6] An "accumulator" is a vessel partially full of liquid and partially full of gas, the gas acting as a compressible spring on the liquid.

Yet another use of capacitors is creating time-delayed voltages and currents. Consider this example circuit showing a capacitor connected to a DC voltage source through a single-pole double-throw (SPDT) switch and a resistor:



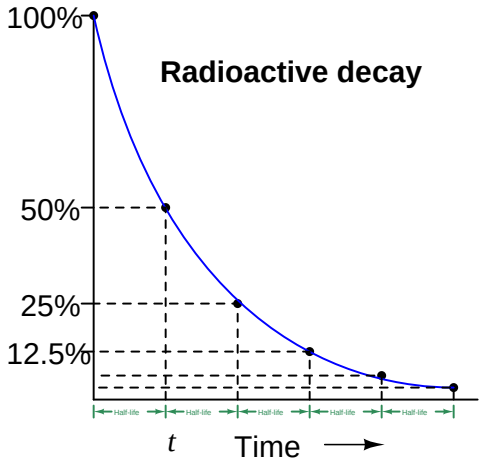
We know capacitors store energy by means of electric fields, and that the strength of a capacitor's electric field is directly related to voltage between its plates. If we assume the capacitor begins in a fully de-energized state, its voltage therefore must be zero when the switch initially moves to the "energize" position. With zero V_C, Kirchhoff's Voltage Law demands the resistor drop the full source voltage, and from Ohm's Law ($I = \frac{V}{R}$) we can tell current must start at its maximum value. A high current value results in a high rate-of-change for capacitor voltage ($I = C\frac{dV}{dt}$), and so V_C increases rapidly. As V_C increases, less of the source's voltage will be dropped across the resistor in accordance with Kirchhoff's Voltage Law, and by Ohm's Law this means less current in the circuit. Less current results in a slower rate-of-rise for voltage ($I = C\frac{dV}{dt}$), and so V_C increases not linearly but rather it rises at a slower and slower pace until it finally levels off at the source voltage value. As V_C rises, V_R falls, and so current must fall as well in accordance with Ohm's Law, eventually settling at zero but at a slower and slower speed over time.

Moving the SPDT switch to the "de-energize" position causes the capacitor to behave as a source now rather than as a load, delivering its stored energy to the resistor. As the capacitor releases energy its electric field weakens and its voltage decreases, causing current to decrease as well (Ohm's Law, $I = \frac{V}{R}$), resulting in a slower and slower descent for both voltage and current over time:



3.4 Inverse exponential growth/decay

The voltage and current curves plotted in the previous section for circuits containing capacitors and resistors are actually common to a wide range of phenomena. For example, if we plot the temperature of a hot object as it cools down to equal ambient air temperature, we will obtain a graph having the same shape as the one showing a capacitor de-energizing through a resistor. Another example of similar mathematical behavior is found in radioactive materials, where atoms randomly disintegrate to become other types of elements. An easily-comprehended way to quantify the decay of a radioactive substance is by a measure called *half-life*, which is the amount of time required for one-half of the remaining atoms to disintegrate:



The proportion of original radioactive atoms remaining after each half-life period may be calculated using reciprocated powers of two: after the first half-life period only $\frac{1}{2}$ of the original atoms are intact; after the second half-life period only $\frac{1}{4}$ (or $\frac{1}{2^2}$) of the original atoms are intact; after the third half-life period only $\frac{1}{8}$ (or $\frac{1}{2^3}$) of the original atoms remain, etc. An alternative way to express this mathematically is to use *negative powers of two* such as 2^{-2} to represent $\frac{1}{4}$ and 2^{-3} to represent $\frac{1}{8}$.

Simple powers of two work well to describe the proportion of original atoms remaining after whole intervals of the half-life period, but if we wish to calculate the proportion at any arbitrary time we will need a more sophisticated formula. Here we let t represent time and h represent the half-life of the substance:

$$\text{Proportion remaining} = 2^{-\frac{t}{h}}$$

The fraction $\frac{t}{h}$ simply declares *how many half-lives have elapsed*, and the negative sign ensures the power-of-two will have a fractional value between one and zero. For example, we know that the radioactive gas Tritium has a half-life of 12.3 years. If we take a pure sample of Tritium and keep it in a sealed vessel for exactly 4 years, we would expect 79.82 percent of the original Tritium atoms to remain in that vessel after those 4 years, the decayed atoms having converted into Helium atoms:

$$\text{Tritium proportion remaining} = 2^{-\frac{4}{12.3}}$$

$$\text{Tritium proportion remaining} = 2^{-0.3252}$$

$$\text{Tritium proportion remaining} = 0.7982 = 79.82\%$$

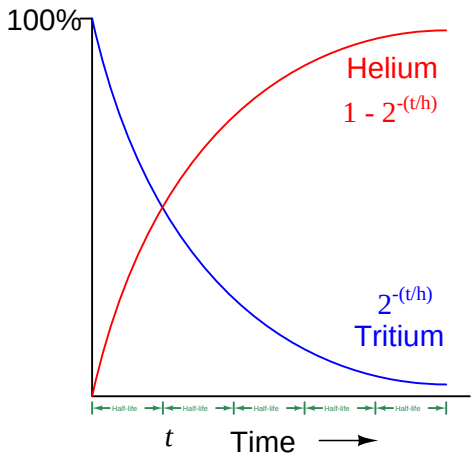
We may modify this equation to tell us the proportion of Tritium atoms that have decayed into Helium over time t as well, which is simply the mathematical complement of what remains:

$$\text{Helium proportion} = 1 - 2^{-\frac{4}{12.3}}$$

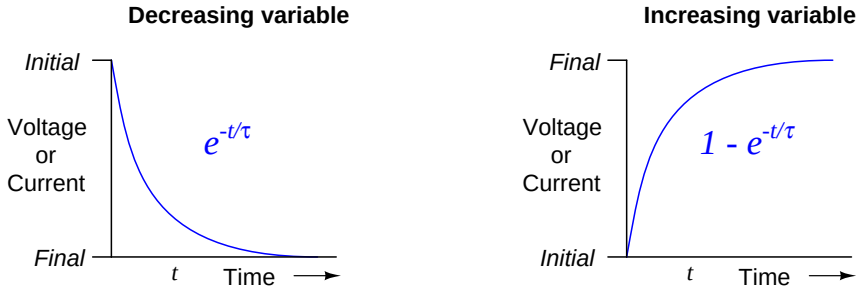
$$\text{Helium proportion} = 1 - 0.7982$$

$$\text{Helium proportion} = 0.2018 = 20.18\%$$

If we could continue this test through the end of time, eventually the Tritium proportion would reach zero and the Helium proportion would reach 100%.



When analyzing resistor-capacitor networks we find that negative powers of e work better than negative powers of two to model growth and decay, e being a mathematical constant approximately equal to 2.718. Instead of working with *half-life* quantities we instead measure the growth/decay in terms of *time constant* which is symbolized by the Greek letter "tau" (τ) and is simply equal to resistance times capacitance ($\tau = RC$). Any decaying value in one of these networks follows the proportion of $e^{-\frac{t}{\tau}}$ while any growing value follows $1 - e^{-\frac{t}{\tau}}$ as shown in the following graphs:



To illustrate how each of these formulae work, we will calculate the values of $e^{-\frac{t}{\tau}}$ and $1 - e^{-\frac{t}{\tau}}$ for several whole-numbered time constant periods beginning with zero representing the system at its start time and progressing to increasingly *negative* values:

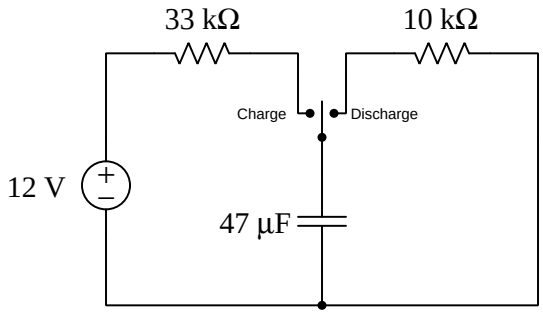
$-\frac{t}{\tau}$	$e^{-\frac{t}{\tau}}$	$1 - e^{-\frac{t}{\tau}}$
0	1	0
-1	0.3679	0.6321
-2	0.1353	0.8647
-3	0.04979	0.9502
-4	0.01832	0.9817
-5	0.006738	0.9933

After one time constant ($t = \tau$) the decaying variable will be at 36.79% of its original value while the growing variable reaches 63.21% of its final value. After five time constants' worth of time, all values have settled to well within 1% of their final values.

It is important to realize that the formulae $e^{-\frac{t}{\tau}}$ and $1-e^{-\frac{t}{\tau}}$ merely describe *fractional proportions* of some initial or final quantity and not necessarily the quantity itself. For example, the voltage across a resistor-capacitor network losing energy over time will *follow* the $e^{-\frac{t}{\tau}}$ "decay" formula as it goes from a value of 1 at the starting time ($t = 0$) and ends up at 0 after an infinite amount of time has passed, but to calculate the actual amount of voltage in units of Volts we must incorporate a multiplier describing that initial voltage. For example, a resistor-capacitor network initially energized to a potential difference of 20 Volts with a time constant of 3 seconds would be most fully described by the decay formula $V_C = 20e^{-\frac{t}{3}}$ where V_C is the capacitor's voltage and t is the discharge time in seconds. Similarly, a resistor-capacitor-source network starting at a capacitor voltage of zero at $t = 0$ and building up to a maximum of 35 Volts with a time constant of 400 milliseconds would be most fully described by the growth formula $V_C = 35(1 - e^{-\frac{t}{0.4}})$.

3.5 Inverse-exponential calculation examples

Suppose the capacitor in the following circuit begins in a completely de-energized state, with the SPDT[7] switch in the position shown:



What will happen if we flip the switch to the "Charge" position? To answer this question, we should first consider the capacitor from the perspective of energy storage: we know capacitors store energy using electric fields, and that electric field strength is proportional to voltage, therefore a capacitor with no energy must exhibit no voltage, and as it gains energy from the source its voltage must rise over time. Therefore, we know we must use the inverse-exponential *growth* equation $(1 - e^{-\frac{t}{\tau}})$ to predict how the capacitor's voltage will change over time. Since we know the voltage source is rated at 12 Volts, this will be the capacitor's final voltage value, the growth equation taking the following form:

$$V_C = (12\text{ V})(1 - e^{-\frac{t}{\tau}})$$

We also know the circuit's time constant (symbolized by the Greek letter "tau" τ) will be equal to the product of 33 kiloOhms and 47 microFarads, which is 1.551 seconds. If we evaluate this equation at 1 second, 2 seconds, and 3 seconds we predict capacitor voltage values of 5.702 Volts, 8.695 Volts, and 10.266 Volts, respectively.

[7] "SPDT" refers to *single-pole, double-throw.*

Multiple ways exist to calculate current in this circuit. One way is to take each of the calculated capacitor voltage values at specific times and use Kirchhoff's Voltage Law to compute the resistor's voltage drop at each of those times, then use those voltage values and Ohm's Law to calculate resistor current values. Being a series network, of course, the resistor's current value at any given time will be the same as current through any other component in the network.

Alternatively, we could select the appropriate inverse-exponential equations to directly calculate resistor voltage and circuit current. Knowing that the capacitor begins in a completely de-energized state and gradually absorbs energy over time, we know its voltage must begin at zero and rise over time. If capacitor voltage begins at zero and rises to full source voltage over time, then Kirchhoff's Voltage Law demands the 33 kΩ resistor's voltage must begin at the full source value of 12 Volts and decay to zero over time. At 12 Volts, the resistor's current must be $\frac{12\,\text{V}}{33000\,\Omega}$ or 363.6 μA. Thus, the inverse-exponential *decay* equation is the right one to use for resistor voltage as well as current with the switch in the "Charge" position:

$$V_R = (12\,\text{V})(e^{-\frac{t}{\tau}})$$

$$I = (363.6\,\mu\text{A})(e^{-\frac{t}{\tau}})$$

If we evaluate this voltage equation at 1 second, 2 seconds, and 3 seconds we get resistor voltage values of 6.298 Volts, 3.305 Volts, and 1.734 Volts, respectively. If we evaluate this current equation at 1 second, 2 seconds, and 3 seconds we get current values of 190.8 microAmperes, 100.1 microAmperes, and 52.56 microAmperes, respectively. These are the exact same results we would have obtained had we simply calculated capacitor voltages using the inverse-exponential growth equation the same times along with Kirchhoff's Voltage Law and Ohm's Law.

What happens if we flip the SPDT switch to the "Discharge" position after letting the capacitor equalize in voltage to the DC source? In this scenario the capacitor begins in a state of energization, but then delivers energy to the 10 kΩ resistor over time, causing its internal store of energy to deplete and eventually settle at zero. Since capacitors store energy within electric fields, and voltage is directly related to electric field strength, this means the capacitor's voltage must begin at 12 Volts and decay to zero over time when the switch is flipped to the "Discharge" position.

With the switch in that position, the capacitor is placed in parallel with the 10 kΩ resistor, ensuring both experience the same voltage. Thus, the resistor's voltage also begins at 12 Volts and decays to zero when the switch moves to the "Discharge" position. Ohm's Law tells us the resistor current must begin at 1.2 mA ($\frac{12\,\text{V}}{10000\,\Omega}$) and follow the trajectory of the inverse-exponential decay equation just like voltage.

$$V_C = (12\text{ V})(e^{-\frac{t}{\tau}})$$

$$I = (1.2\text{ mA})(e^{-\frac{t}{\tau}})$$

The circuit's time constant will be different in the "Discharge" switch position compared to the "Charge" position, since the two resistor values are unequal. In the "Discharge" position the rates of change will be quicker than before:

$$\tau = (10 \times 10^3)(47 \times 10^{-6}) = 0.47 \text{ seconds}$$

Evaluating the voltage and current decay equations at 1 second, 2 seconds, and 3 seconds using the new τ value yields results of 1.429 Volts at 0.1429 milliAmperes, 0.1703 Volts at 17.03 microAmperes, and 20.28 milliVolts at 2.028 microAmperes, respectively.

3.6 Capacitor types and limitations

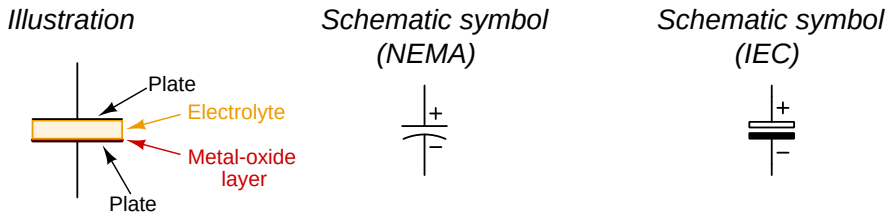
Capacitors are broadly categorized by the type of dielectric material used. Air, polyester plastic, ceramic, mica, and glass are among the different materials employed as dielectric media between capacitor plates. These dielectric materials vary in *permittivity* (i.e. the degree to which they enhance capacitance) and *dielectric strength* (i.e. the amount of electric field strength they can tolerate without "breaking down" and conducting current). As shown in the following equation, permittivity directly relates to capacitance for any given plate area and separation distance:

$$C = \frac{A\epsilon}{d}$$

Where,

C = Capacitance in Farads (F)
A = Overlapping area of plates in square meters (m^2)
ϵ = Absolute[8] permittivity of dielectric material in Farads per meter (F/m)
d = Distance of separation between plates in meters (m)

One class of dielectric deserving of special attention is *electrolytic*, referring to the way in which this capacitor is constructed. Recall that for a given plate area and permittivity, the amount of capacitance is inversely proportional to the distance separating the capacitor plates. This means we must make the dielectric layer extremely thin in order to achieve high capacitance with reasonable plate area. One way to create a super-thin dielectric layer is to *anodize* a metal surface which forms an electrically insulating (i.e. dielectric) metal-oxide film on the metal's surface. This thin layer of metal-oxide serves as the dielectric, while an electrolyte paste or fluid serves as a conductive extension of the far plate up to the oxide's surface. Most electrolytic capacitors are considered *polarized* because applying sufficient DC voltage in the wrong polarity will remove this anodized metal-oxide layer and cause the capacitor to fail in a shorted manner. Polarized capacitors are always marked as to which terminal is the positive, and may be symbolized in schematic diagrams using a curved line rather than a straight line as the negative plate:



[8] Material permittivity is often specified as *relative* to a vacuum, in which case ϵ_r is equal to 1 for a vacuum and greater than 1 for all other substances. The permittivity of free space (ϵ_0 is approximately $8.8541878128 \times 10^{-12}$ Farads per meter) must be multiplied by any given relative permittivity value in order to arrive at the absolute permittivity of the substance in question.

The most elementary capacitor ratings are *capacitance* and *maximum voltage*. Capacitance is typically expressed either in Farads (F), microFarads (μF), nanoFarads (nF), or picoFarads (pF). Interestingly, one never encounters a capacitor rated in milliFarads, although there is nothing mathematically improper with the use of that metric prefix. Also, it is common to see the capital letter M used to represent *micro*, although that it technically improper[9] as a metric prefix. Typically both capacitance and maximum voltage ratings will be found printed in text on the body of the capacitor itself. Capacitance expressed with a decimal point (e.g. 2.2) but no other markings refers to microFarads by default. Capacitance expressed without a decimal point (e.g. 33) and no other marking refers to picoFarads by default. A three-digit number with no decimal point (e.g. 473) and no other markings is read in a manner similar to the first three bands of a four-band resistor color code: the three numerals representing Digit, Digit, and Multiplier of picoFarads. For example a "473" capacitor is 47×10^3 picoFarads, or 0.047 μF. Maximum voltage ratings are always followed by the letter "V". Other alphabetical letters are used to designate tolerance (e.g. D = $\pm$0.5%, F = $\pm$1%, G = $\pm$2%, H = $\pm$3%, J = $\pm$5%, K = $\pm$10%, M = $\pm$20%, etc.).

Other capacitor ratings include *leakage resistance* which is the resistance of the dielectric in Ohms (ideally infinite) and *equivalent series resistance* or *ESR* which is the effect of all dissipative losses within the capacitor modeled as a single resistance value. The wire leads of a capacitor also harbor *parasitic inductance*[10] along their lengths.

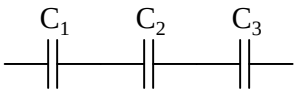
For any given type of dielectric, the physical size of a capacitor is proportional to its capacitance as well as to its voltage rating. A capacitor having more capacitance but the same voltage rating must be made with more plate area and the same dielectric thickness, and this makes it larger. A capacitor having a higher voltage rating but the same capacitance must be made with thicker dielectric (which by itself reduces capacitance), and then also more plate area to make up for the degradation in capacitance caused by the thicker dielectric – this again makes it larger.

[9] A capital "M" of course should mean *mega* (1×10^6), but this is never the case for capacitance!

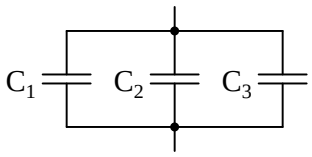
[10] Inductance is the ability to store energy in a magnetic field surrounding a conductor. A "parasitic" property is one that exists as an artifact of a component's construction, and is not necessarily desired. For a capacitor, any stray inductance is considered parasitic, as is any resistance other than the (ideally) infinite resistance of the dielectric.

3.7 Series and parallel capacitance

Like resistors, capacitors may be connected together to form networks with different total capacitance than any single capacitor. Unlike resistors, though, connecting capacitors in series diminishes total capacitance, while connecting capacitors in parallel accumulates total capacitance:



$$C_{series} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$



$$C_{parallel} = C_1 + C_2 + C_3$$

This may be understood in terms of geometry: stacking capacitors in series with each other is equivalent to making one capacitor with a wider gap between its plates (thus diminishing capacitance), while ganging capacitors in parallel with each other is equivalent to making one capacitor with larger plate area (thus increasing capacitance).

This may also be understood in terms of energy storage, and proven with a few simple thought experiments. Imagine a 3 Farad capacitor energized with 2 Volts between its plates, storing 6 Joules of energy as predicted by $E_C = \frac{1}{2}CV^2$. Now imagine three of these same capacitors connected in parallel so all of them share the same 2 Volts. Total stored energy must be the sum of the three individual capacitors' energies and therefore is 18 Joules, *three times* that of the single capacitor. Now imagine three of these same capacitors connected in series, dividing the 2 Volt total voltage into thirds. Each 3-Farad capacitor dropping $\frac{2}{3}$ Volt will store only $\frac{2}{3}$ Joule of energy with the three together storing only 2 Joules of energy in total, just *one third* that of the single capacitor.

Chapter 4

Historical References

This chapter is where you will find references to historical texts and technologies related to the module's topic.

Readers may wonder why historical references might be included in any modern lesson on a subject. Why dwell on old ideas and obsolete technologies? One answer to this question is that the initial discoveries and early applications of scientific principles typically present those principles in forms that are unusually easy to grasp. Anyone who first discovers a new principle must necessarily do so from a perspective of ignorance (i.e. if you truly *discover* something yourself, it means you must have come to that discovery with no prior knowledge of it and no hints from others knowledgeable in it), and in so doing the discoverer lacks any hindsight or advantage that might have otherwise come from a more advanced perspective. Thus, discoverers are forced to think and express themselves in less-advanced terms, and this often makes their explanations more readily accessible to others who, like the discoverer, comes to this idea with no prior knowledge. Furthermore, early discoverers often faced the daunting challenge of explaining their new and complex ideas to a naturally skeptical scientific community, and this pressure incentivized clear and compelling communication. As James Clerk Maxwell eloquently stated in the Preface to his book *A Treatise on Electricity and Magnetism* written in 1873,

> It is of great advantage to the student of any subject to read the original memoirs on that subject, for science is always most completely assimilated when it is in its nascent state . . . `[page xi]`

Furthermore, grasping the historical context of technological discoveries is important for understanding how science intersects with culture and civilization, which is ever important because new discoveries and new applications of existing discoveries will always continue to impact our lives. One will often find themselves impressed by the ingenuity of previous generations, and by the high degree of refinement to which now-obsolete technologies were once raised. There is much to learn and much inspiration to be drawn from the technological past, and to the inquisitive mind these historical references are treasures waiting to be (re)-discovered.

4.1 Early examples of capacitors

Capacitance is the ability to store energy within an electric field, and a *capacitor* is a device built expressly for that purpose. Every capacitor consists in one form or another of two conductive surfaces separated from each other by one or more layers of insulating material called a *dielectric*.

One of the earliest forms of capacitor was the so-called *Leyden jar* used extensively by electrical experimenters in the 18th and 19th centuries, which was nothing more than a glass jar coated inside and out with metal foil or metal leaf. The thickness of the glass constituted the dielectric, while each coating of metal (one outside the jar, one inside) comprised the two conductive surfaces. This construction leveraged existing glassware and metal-gilding technology of the day, and the fact that the device was a *jar* evoked the mental image of that jar holding the mysterious and invisible electric fluid that early electrical experimenters imagined constituted electricity.

The following photograph taken from the book *Wireless Telegraphy and Telephony* by Alfred P. Morgan and published in 1920 shows four Leyden jars suitable for use in high-voltage radio communication circuits of the day:

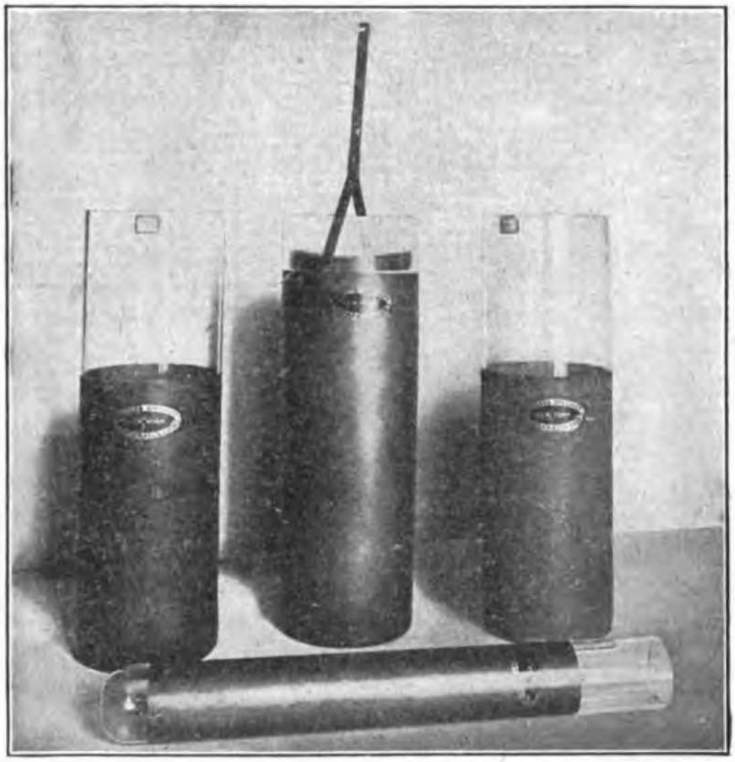


FIG. 6.—Navy type of Leyden jars. Coated with copper deposited upon the surface of the glass.

This next photograph shows historical examples of Leyden jars in a museum of radio and electricity:

A large bank of large Leyden jars nearly as tall as a man is seen in the following photograph of an early German radio transmitting station, taken from the same book. In this circuit 360 Leyden jars are connected together, most likely in parallel to maximize total capacitance:

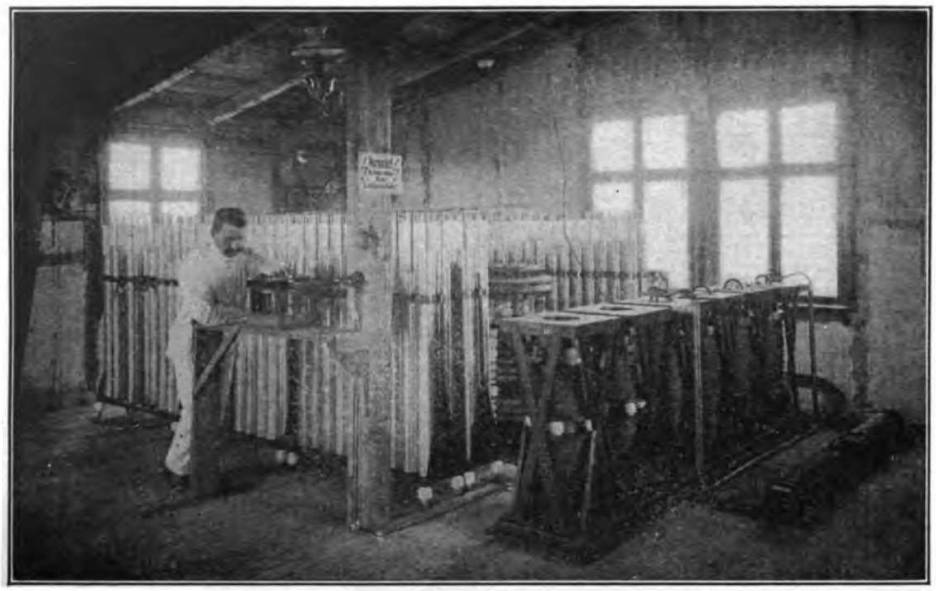


Fig. 116.—Transmitting equipment of the high power station at Nauen, twenty-five miles northwest of Berlin, Germany, showing six induction coils (in the foreground) arranged to charge the Leyden jars (composed of 360 units).

A typical feature of Leyden jars was a fairly high voltage rating, owing to the high dielectric strength of the insulating glass. For applications where the voltages encountered were lower, different types of dielectrics could be used, and be made thinner as well. The term "Leyden jar" was used only for jar-style capacitors, with other designs coming to be known as *condensers* or *condensors*. A common form of condenser construction in the early 20th century was a multi-layered device of metal foil and dielectric sheets of glass, mica, or paper. Two illustrations from the same book show a design where metal foil and dielectric sheets are interleaved, the alternating metal foil layers protruding from opposite sides of the assembly for convenient connection to form two electrically distinct metal surfaces:

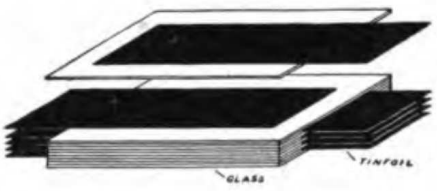


Fig. 46.—Diagram showing construction of condenser.

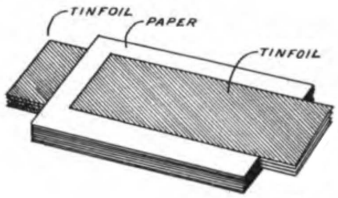


Fig. 80.—Fixed condenser.

Modern multi-layer ceramic capacitors (MLCCs) employ the same basic form of construction,

with layers of metal interleaved between layers of insulating ceramic.

Early capacitor technology also included variable capacitors (called *variable condensers*) that worked on the principle of moving one set of conductive surfaces relative to the other, thereby altering the amount of overlapping area and causing the capacitance to vary with physical position. Two examples of variable condenser design are illustrated in the same book:

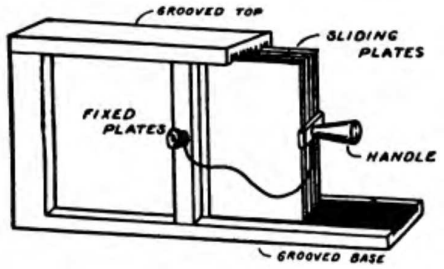


Fig. 84.—Sliding plate variable condenser.

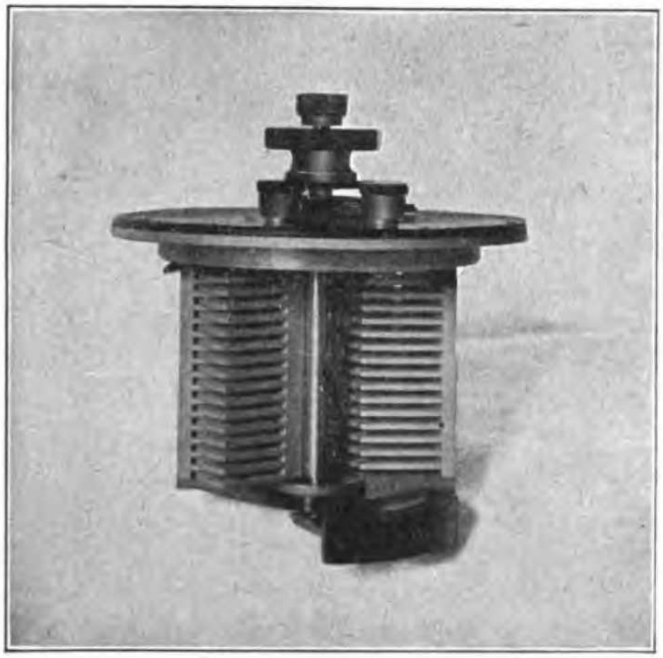


Fig. 83.—Dr. Seibt's rotary variable condenser. The plates are turned from a solid casting and the separation between is only .01 inch.

4.2 Fatality resulting from cable capacitance

The following article appeared in pages 988-989 of *The Electrician*, number 1351, published 8 April 1904, contained in volume 52 of the same, documenting the accidental death of a power station worker. It stands as a grim reminder of the value of safety protocol, and also of the hazards intrinsic to capacitance appearing in long-distance power cables.

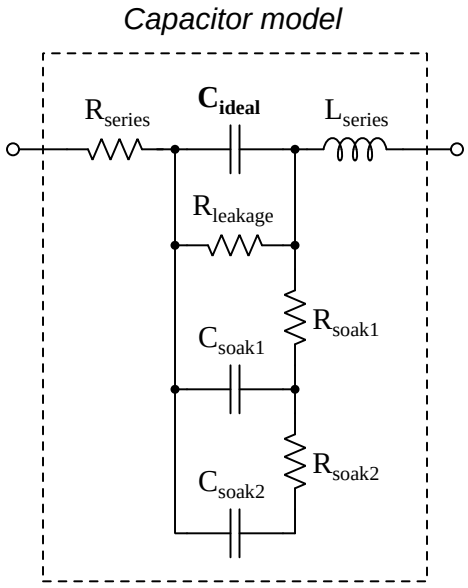
> The unfortunate fatality which occurred on March 28th at the Charing Cross and Strand Electricity Supply Corporation's generating station at Bow was surrounded by very peculiar circumstances, and was of such a nature as to admit of only one probable explanation. It appears that the victim, a switchboard attendant named George Crates, aged 22, went on duty as usual at 6:30 a.m., and at 1 o'clock was assisting in shutting down the generating plant in order that some repairs might be carried out. While so engaged he received a severe shock, and, in spite of efforts to restore respiration, he died before the arrival of the medical man who had been summoned. It will be remembered that the Bow station is employed for the generation of three-phase current at 10,000 volts, energy being transmitted to the company's sub-stations situated in various parts of London, where it is transformed to continuous current by means of motor-generators, and distributed in the ordinary way. Six cables are used for the transmission of energy at high voltage to the City and West End, and their total length is about 36 miles. Each has three cores, and the sectional areas of the cores are about 0.1 sq. in. After all the sub-stations had disconnected their motor-generators, the engineer-in-charge at Bow proceeded to reduce the generator fields and bring the engines to a dead-slow speed. This accomplished, he then broke the generator fields, and finally opened opened the main switches. The voltmeter at the generating station was observed to be standing at zero, and, signals being given that all was clear, Crates commenced to remove the cable fuses. These fuses are mounted in heavy glass tubes, with about 8in. between the metal fittings at each end. All went well until the sixth fuse had been removed, but when the seventh – apparently the first fuse of the third cable – was being taken out there was a report and a flash, and Crates fell backward with the fuse in his hand. Evidence given by the medical attendant was to the effect that the deceased had died from an electric shock, and that his face was scorched and his hair singed. `[page 988]`
>
> Naturally, such an occurrence appears extraordinary, not to say mysterious, at first sight. According to the particulars at our disposal the generators were disconnected entirely, and, as they are star-wound with the centre of the star earthed, when they had been switched off the earth connection on the cable conductors was, of course, removed. Even if all the motor-generators in the sub stations had not been switched off as was stated, they could not have been sending current back, for we are informed that the voltmeter indicated zero pressure and no sparks were observed when the first six fuses were removed. Hence it was fair to presume the cables to be absolutely dead. Moreover, if they had been alive, Crates must have suffered very severe burns, but, as a matter of fact, we have it on the authority of those who saw the poor fellow after the occurrence that they did not notice any singed hair as described by the doctor ; what they did see, however, were slight red marks on the hands and arm and a round red mark between the eyes. These marks could be compared to nettle-rash better than anything else. Assuming, therefore, that the cables were not connected to any source of electrical energy, the only remaining

hypothesis is that the fatality was due to a "residual" electrostatic charge, and, however improbable this may appear, it must be accepted as the explanation. To render such an explanation feasible, the switchboard connections would have to possess a high insulation resistance to earth to prevent any charge leaking away with great rapidity. There were special insulated tongs provided for removing the fuses, and there were regulations in force which made their use compulsory, but on this occasion they were not used, and we are not surprised; it would be the most natural thing in the world for an attendant, knowing that all machines were disconnected, to presume everything safe and dispense with the use of the tongs.

Since the generator fields were switched off before the main circuit was broken, there will have been direct connection between each core of the cables and earth through the dynamo armature for a few instants, and this should have sufficed to discharge the cables. Hence, it can only be assumed that, during the time that elapsed between the switching-off and the fatal shock, a "residual charge" must have accumulated in the cables. This phenomenon is familiar to those who perform experiments with condensers, and is even not unknown in the testing room of a cable factory. It must be remembered that the combined capacity of the cables was considerable.

Whether the shock was very severe or not there is nothing to show, and as there was no post mortem examination it is not apparent whether the deceased suffered from any affection of the heart, but assuming this to be the case, then, of course, a comparatively slight shock might have proved fatal. Indeed, there is every reason to believe that many fatalities alleged to be due to electric shock are actually attributable to ordinary heart failure or heart disease, so that although it may be the shock which has actually produced death, any shock – whether electrical, mechanical or mental – would have caused a similar result. Although the sad event is greatly to be deplored, yet it is pleasing to know that no blame can be attached to any of the company's employees; at the same time the incident serves as a warning to generating station engineers. No matter how safe everything may seem, a high-tension cable – and especially an extra high-tension cable – should always be treated as a source of potential danger; before any work is done on such a cable the conductors should be connected to earth, and the stringent regulations in force at the electricity works should not be relaxed in the slightest degree. [page 989]

The "residual charge" referred to by the author of this article is the *soakage* effect represented within the following capacitor model:

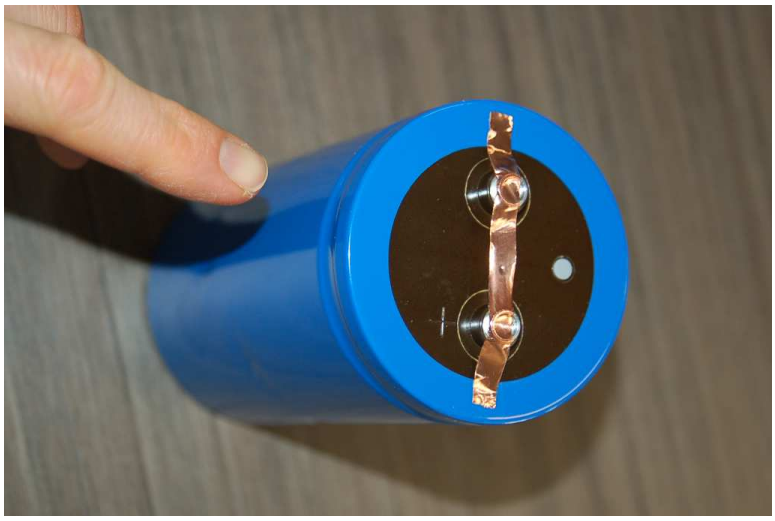


In this case, the power cables, being 36 miles long, possessed significant parasitic capacitance from cable to cable as well as from cable to Earth ground. This parasitic capacitance resulting from the dielectric effect of the cable insulation, has not only a bulk capacitive property but also its own parasitics as shown in the above capacitor model. One of these sub-parasitic effects is *soakage*, whereby some stored energy remains in the polarization of the dielectric molecules even when the measured voltage between the capacitance points is zero. Upon disconnection from a load, this soakage manifests as a rising voltage between those same terminals which can achieve some fraction of the original applied voltage. The rate of rise depends on the soakage capacitance and soakage resistances (shown in the model above), as well as the voltage which the capacitance was previously charged to.

Note the recommendations given by the author at the end of the article to ensure safety in such situations:

- Always connect dangerous conductors to Earth ground (and by extension, to each other as well) to ensure a zero-voltage condition. This is called *safety bonding* in modern electrical practice, and it is standard in high-voltage work.
- Always follow safety procedures, even when you don't think they are necessary! Consider the maxim that *all safety regulations are written in blood.*

Soakage is the reason why large capacitors should always be stored with a shorting-wire or copper tape forcing the terminals to be electrically common with each other as shown in the photograph below:



Aluminum electrolytic capacitors are well-known for exhibiting particularly strong soakage effects. Experimentally demonstrating this phenomenon is as simple as taking any electrolytic capacitor, connecting a voltmeter to its terminals, temporarily shorting those terminals to force a zero-voltage condition (as verified on the voltmeter), then removing the short and observing the voltmeter's indication gradually rising.

Chapter 5

Derivations and Technical References

This chapter is where you will find mathematical derivations too detailed to include in the tutorial, and/or tables and other technical reference material.

5.1 IEC standard component values

Components such as resistors, inductors, and capacitors are manufactured in several *standard values*, described by IEC standard 60063. Rather than having a single series of standard values, the IEC publishes lists called *E series* based on the number of unique values spanning a single *decade* (i.e. a 10:1 range).

The shortest of these series, called *E3* contains just three values: 10, 22, and 47. The next series is called *E6* with six unique values: 10, 15, 22, 33, 47, and 68. These values represent *significant values* for components, meaning the decimal point may be freely moved to create values spanning multiple decades. For example, "33" simply means one can expect to find components manufactured in values of 33, 3.3, 0.33, and 0.033 as well as 330, 3.3 k, 33 k, etc.

Although this may seem like a strange standard for component manufacturers to follow, there is a compelling logic to it. The terms of each series are closer-spaced at the low end than at the high end, and this allows for *series* and/or *parallel* combinations of components to achieve most any desired value. For example, in the E6 series we only have values with the significant figures 10, 15, 22, 33, 47, and 68, but this doesn't mean we are limited to *total* values with these significant figures. For example, if we needed 80 Ohms of resistance we could connect a 33 Ohm and 47 Ohm resistor together in series. 50 Ohms could be made from two 68 Ohm resistors in parallel (making 34 Ohms) plus a 15 Ohm and 1 Ohm resistor in series.

On the next page is a table showing the four most common E-series specified by IEC standard 60063.

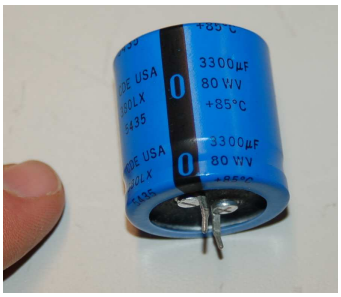
E3	**E6**	**E12**	**E24**
10	10	10	10
			11
		12	12
			13
	15	15	15
			16
		18	18
			20
22	22	22	22
			24
		27	27
			30
	33	33	33
			36
		39	39
			43
47	47	47	47
			51
		56	56
			62
	68	68	68
			75
		82	82
			91

E48, *E96*, and *E192* series are also found in the IEC 60063 standard, used for components with tighter tolerance ratings than typical.

5.2 Capacitor labeling

Capacitors store electrical energy using electric fields, the amount of energy stored being proportional to the square of the applied voltage. The ratio of stored energy to voltage (squared) is called *capacitance* (C), measured in the unit of the *Farad* (F), and it is the primary characteristic of any capacitor. Unfortunately, the labeling of capacitance on many capacitors is rather confusing. Color codes used to be popular for denoting capacitance value, following a very similar pattern to resistors, but numerical labels are now the norm. However, these labels tend to be rather terse, and this can cause interpretational difficulties.

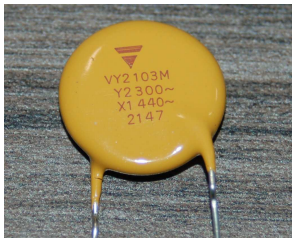
Physically large capacitors having sufficient surface area to allow many characters of printed text are the best in this regard.



The "supercapacitor" seen in the left-hand photograph is the clearest of them all: 650 Farads of capacitance with a working voltage rating of 2.7 Volts DC. The capacitor in the center photograph is also very unambiguous, having ratings of 3300 microFarads and 80 Volts DC ("WV" = *working voltage*). However the capacitor seen in the right-hand photograph, despite being physically larger than either of the others, is more cryptic: 16500 microFarads of capacitance with a working voltage rating of 50 Volts DC. Here we must simply assume that the "16500" figure refers to *micro*Farads.

Polarity is another important parameter for electrolytic-type capacitors such as these. For the supercapacitor (left photo) we see a stripe and letter "P" denoting the positive terminal of the capacitor; the 3300 microFarad capacitor (center photo) bears a black stripe and "−" symbol pointing toward its negative terminal; the 16500 microFarad capacitor (right photo) has polarity markings molded into the end-cap near its screw terminals (not visible in this photo).

Capacitors connected to AC line ("mains") voltage within line-powered appliances and other electronic devices must be *safety-rated*, as they are exposed to routine voltage transients such as those commonly experienced on electric power grids during lightning storms, and also because any short-circuit failure of a line-connected component would be catastrophic. For this reason, such capacitors often bear X and Y voltage ratings such as those seen on the following capacitor:



An X rating refers to the capacitor connected in parallel with the AC line's "hot" and "neutral" conductors: that is, directly across the line voltage. A Y rating refers to the capacitor connected between an Earth-potential conductor and either the "hot" or "neutral" conductor of an AC line source. These two different applications have their own safety concerns: a capacitor in an "X" application is at risk of catastrophically failing and possibly igniting nearby flammable materials if it develops an internal short-circuit, whereas a capacitor in a "Y" application is at risk of impressing dangerous electrical potential onto a metallic surface it internally fails shorted.

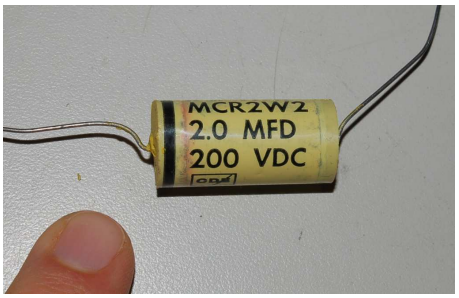
Immediately following the X or Y designator is a single numerical digit representing the rating sub-class, based on the peak impulse (transient) voltage:

Sub-class	**Maximum transient service voltage**
X1	± Between 2.5 kV and 4 kV
X2	± Less than 2.5 kV
X3	± Less than 1.2 kV

Sub-class	**Maximum transient test voltage**
Y1	± 8 kV
Y2	± 5 kV
Y3	± (none)
Y4	± 2.5 kV

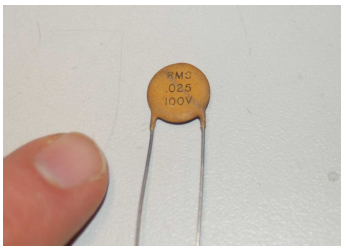
Any voltage value following an X or Y class/sub-class rating refers to the continuous AC voltage that capacitor is rated to handle in such a safety-critical application. For example, the `X1 440~` label on the capacitor shown in the photograph tells us that capacitor is rated for continuous duty across the poles of a 440 Volt AC source and that its service transient (impulse) rating is somewhere between 2500 Volts and 4000 Volts. This is the sort of capacitor we might find connected in parallel with the "hot" and "neutral" conductors of a 120 Volt AC power cord for suppression of high-frequency "noise".

Returning to our exploration of general capacitor labeling:



On the left we have a 2500 microFarad electrolytic capacitor rated for 50 Volts DC (positive on left), a capital letter "U" employed to symbolize *micro*. On the right we have a 2 microFarad plastic-film capacitor rated at 200 Volts DC, this time with the capital letter "M"[1] representing *micro*. Since plastic-film capacitors are non-polarized, the stripe on the left-hand side must represent something other than polarity: here it represents the terminal connected to the outer-most layer of metal foil[2] inside the capacitor.

Smaller capacitors become slightly more consistent, though no less confusing, in their labeling. Consider these examples:



From left to right we have (left) a 0.082 microFarad capacitor rated for 100 Volts DC[3], (center) a 0.025 microFarad capacitor rated for 100 Volts DC, and (right) a 0.68 microFarad capacitor rated for 160 Volts DC. The rule here is, any numerical capacitance value with a decimal point should be read as *micro*Farads. Note how the capacitor in the right-hand photograph uses a *comma* rather than a *point* between the "0" and the "68" because this one is of European manufacture, and it is conventional in European technical literature to use commas rather than points for decimal numbers.

[1] In the metric system a lower-case "m" is supposed to represent the prefix *milli* ($\times 10^{-3}$) and an upper-case "M" is supposed to represent the prefix *mega* ($\times 10^{6}$). However, since no technology yet invented is able to pack a *mega*Farad of capacitance into a single component, and for some unknown reason the prefix "milli" is never used for capacitance, we are expected to deduce that this letter "M" must represent *micro*. This, of course, flies in the face of standard metric notation, but it is nevertheless common to see in capacitor labeling.

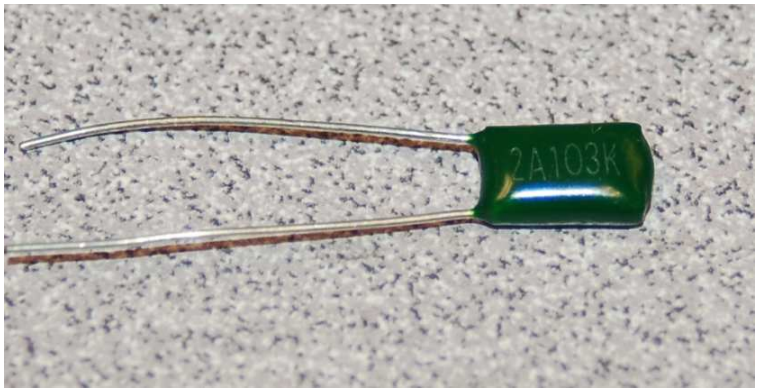
[2] This might be important to know for reasons of interference and signal coupling in densely-packed circuits. For a plastic-case capacitor such as this, the outer-most metal foil layer may comprise a parasitic capacitance with some adjacent component, and this in turn may degrade the circuit's performance. Best practice is to connect the "stripe" terminal of such a capacitor to ground or a power supply "rail" or some other node having a stable electrical potential.

[3] The second "0" of the number 100 happens to be partially rubbed off.

For capacitors with smaller capacitance values, the default metric prefix becomes *pico* ($\times 10^{-12}$) rather than micro. We can tell the difference between a "micro" capacitor and a "pico" capacitor by the presence or lack, respectively, of a decimal point. For example, a capacitor labeled 2.2 would be 2.2 microFarads in size, but a capacitor labeled 22 would be 22 picoFarads in size.

For decimal values of picoFarads, a lower-case letter "p" may be used as a decimal point; e.g. 2p2 would be 2.2 picoFarads. Similarly, the lower-case letter "n" may be used as a decimal point as well (e.g. 2n2 would be 2.2 nanoFarads). Confusingly, some manufacturers use a capital letter "R" as a generic decimal point for the capacitance value, the default metric prefix once again being *pico* (e.g. 2R2 would be 2.2 picoFarads).

A very common standard for small capacitor labeling is the use of three-digit numerical codes following the same digit-digit-multiplier format as resistor color codes, combined with the default assumption of *pico*Farads. For example, 332 would represent 33×10^2 picoFarads, which is equal to 3.3 nanoFarads. The following photograph shows a mylar film capacitor with a "103" code representing 10×10^3 picoFarads, which is equivalent to 10 nanoFarads:



Note the "2A" preceding the "103" capacitance rating: this is nothing but part of the manufacturer's part number. This is an unfortunate convention, of mixing manufacturer-specific part numbering with standardized value coding, and we see another example of it in the following photograph:

This Vishay capacitor also has a rating of 10 nanoFarads, but here we see the "103" capacitance code preceded by "YV2" which is unique to Vishay's part-numbering system. The last character

("M") specifies this capacitor's tolerance of ±20%, which is the next topic we will explore.

Capacitor *tolerance*, like resistor tolerance, specifies how far the particular device's capacitance may deviate from its advertised (nominal) value. In one of the previous examples, we saw a 0.082 microFarad capacitor with a ±10% tolerance, printed exactly as such on its body. However, for physically small capacitors where not enough surface area exists to print percentage figures, such as the 10 nF Vishay capacitor previously shown, we must use special letter codes to designate tolerance. These letter codes are also used for other components such as resistors, the following table showing several of them:

Letter code	**Tolerance**
A	± 0.05 pF
B	± 0.1 pF
C	± 0.25 pF
D	± 0.5 pF
E	± 0.5%
F	± 1%
G	± 2%
H	± 3%
J	± 5%
K	± 10%
L	± 15%
M	± 20%
N	± 30%
P	± −0% to +100%
S	± −20% to +50%
W	± −0% to +200%
X	± −20% to +40%
Z	± −20% to +80%

Note how the first four of these codes refer to absolute tolerances in picoFarads, while the rest represent tolerances in percent. When printed on a capacitor's body, the tolerance code typically follows the numerical capacitance code. For example, the photograph on the previous page showed a green-colored capacitor with "103K" on its body, which is 10×10^3 picoFarads ± 10%.

5.3 Capacitor parameters and dielectric types

Capacitors come in many different types, the primary distinction being the type of substance used for the dielectric (i.e. the insulating layer between the capacitor plates). Rather than reproduce tables of dielectric material types and various performance ratings for each, here we will list several important parameters and for each of these cite the best and worst dielectric materials.

- **Rated voltage**: *the maximum continuous DC voltage across the capacitor based on the dielectric's breakdown limit*
- **Capacitance density**: *the amount of capacitance per unit volume, generally a desirable feature* – "supercapacitors" exploiting such effects as double-layer and pseudo-capacitance are the undisputed champions for this parameter. For conventional dielectrics, aluminum and tantalum electrolytic are best. Worst is vacuum or air.
- **Insulation resistance**: *the desirable ability of the dielectric to oppose "leakage current" through the capacitor* – polystyrene film is best; tantalum electrolytic is worst.
- **ESR and dissipation factor**: *mechanisms of energy dissipation within the capacitor, any of which are undesirable* – polystyrene, mica, and NP0 ceramic is best; tantalum electrolytic is worst.
- **Dielectric absorption**: *the undesirable tendency of a dielectric to maintain a DC bias after rapid discharging* – polypropylene and polystyrene are best; tantalum electrolytic is worst.
- **Stability**: *the degree to which a capacitor's capacitance value undesirably drifts over long periods of time* – NP0 ceramic is best; several dielectric materials tie for worst (High-K ceramic, reduced titanate, multi-layer glass with "V" characteristic).
- **Temperature coefficient**: *the degree to which a capacitor's capacitance value undesirably changes with temperature* – NP0 ceramic is best; High-K ceramic is worst.
- **Voltage coefficient**: *the degree to which a capacitor's capacitance value undesirably changes with applied voltage* – electrolytics are best; Class 2 ceramic and plastic-film are worst. Any dielectric material with *paraelectric* or *ferroelectric* properties (e.g. Class 2 ceramics such as barium titanate) exhibit this trait because their permittivity values vary significantly as the electric field increases and decreases in strength.
- **Polarization**: *whether or not the capacitor requires unidirectional voltage polarity* – all electrolytics are polarized; others are not.

The principal mechanism of energy dissipation within a capacitor is *dielectric loss*, where molecules within the dielectric material shift with a varying electric field. This voltage-induced molecular movement results in the dielectric becoming warm, thus converting electrical energy into thermal energy where it does not return to the circuit. Interestingly, this phenomenon is actually exploited in certain industrial processes such as the heating of plastics via high-voltage and high-frequency AC applied across the insulating material. In a capacitor, though, this effect is universally undesirable. Not only does dielectric loss constitute energy diverted from the circuit, but the heating will cause the capacitor's own capacitance to drift and may even destroy the device if severe enough.

The following table attempts to rank ten different dielectric types according to five important performance parameters. Please note that these rankings are based on some *worst-case* data I found, and that the performance metrics of specific capacitor models may likely vary from the values and orders shown below. As always, manufacture literature and/or empirical measurement are your best sources of information:

Rank	**Temp coeff.**	**Stability**	**Absorption**	**ESR**	**Insul. Resis.**
Best	NP0 = 0.3%	NP0 = 0.1%	PolyP = 0.05%	NP0 = 0.1%	PolyS = 10^6 MΩ
	PolyS = 1%	Mica = 0.1%	PolyS = 0.05%	Mica = 0.1%	PolyP = 10^5 MΩ
	Mica = 1.8%	PolyS = 2%	PolyC = 0.35%	PolyS = 0.1%	NP0 = 10^5 MΩ
	PolyC = 2%	PolyP = 3%	PolyE = 0.5%	PolyP = 0.35%	PolyC = 10^4 MΩ
	PolyP = 2.5%	PolyC = 5%	NP0 = 0.6%	PolyC = 1%	HiK = 10^3 MΩ
	Tant = 8%	PolyE = 10%	Mica = 0.7%	PolyE = 2%	PolyE = 10^3 MΩ
	Alum = 10%	Tant = 10%		HiK = 3%	RedT = 10^2 MΩ
	PolyE = 12%	Alum = 10%		Alum = 8%	Mica = 10^2 MΩ
	RedT = 30%	RedT = 20%	Alum	RedT = 10%	Alum = 10^2 MΩ
Worst	HiK = 80%	HiK = 20%	Tant	Tant = 24%	Tant = 10^1 MΩ

Abbreviations for dielectric types:

- NP0 = Multi-layer NP0 “Class 1” type ceramic
- HiK = Multi-layer High-K “Class 2” type ceramic
- RedT = Reduced titanate
- PolyE = Polyester film (“mylar”)
- PolyC = Polycarbonate film
- PolyP = Polypropylene film
- PolyS = Polystyrene film
- Tant = Solid tantalum electrolytic
- Alum = Aluminum electrolytic

Also note that conservative design principles may help minimize undesirable capacitor traits. For example, if the capacitor is known to have a large temperature coefficient, placing it in an area of the circuit board where you would expect temperature to be most stable will mitigate this shortcoming. Similarly, if the capacitor has a large voltage coefficient a good mitigation strategy is to over-rate the capacitor in terms of voltage (i.e. select a capacitor with a much higher voltage rating than the expected voltage for your application).

Multi-layer ceramic capacitors are extremely popular given their non-polarized performance making them suitable to reversing DC polarities as well as AC, and their ready availability in surface-mount device (SMD) packages. However, the two major classes (NP0 as "Class 1" and High-K as "Class 2") differ widely in performances. Class 1 ceramic capacitors are extremely stable in terms of their capacitance varying with age, with applied voltage, and with temperature. Class 2 ceramic capacitors sacrifice these desirable traits in favor of greater capacitance per physical volume. We typically find Class 1 ceramic capacitors used within signal filter and timing networks where their greater stability is worth their increased size and weight. Popular applications for Class 2 ceramic capacitors include DC power decoupling and radio-frequency interference (RFI) reduction where the precise amount of capacitance is less important than a certain minimum value of capacitance. Interestingly, Class 2 ceramic dielectrics also exhibit *piezoelectric* properties, meaning they produce voltage when physically strained and will physically strain when subjected to voltage. This means vibrations (including sound) will introduce AC noise voltage into the circuit at those same frequencies, and applied AC will correspondingly produce mechanical vibrations within the capacitor!

An important factor in ceramic dielectric types is temperature stability, i.e. how much capacitance is expected to vary as the device's temperature changes. Alphanumeric codes typically not appearing on the devices themselves specify the device's temperature dependence, the table below showing codes for Class 1 (NP0) ceramic dielectric materials:

Base temp. coefficient	**Temp. coefficient multiplier**	**Temp. coefficient tolerance**
C = 0.0 ppm/deg C	0 = −1	G = ± 30 ppm
B = 0.3 ppm/deg C	1 = −10	H = ± 60 ppm
L = 0.8 ppm/deg C	2 = −100	H = ± 120 ppm
A = 0.9 ppm/deg C	3 = −1000	K = ± 250 ppm
M = 1.0 ppm/deg C	4 = +1	L = ± 500 ppm
P = 1.5 ppm/deg C	6 = +10	M = ± 1000 ppm
R = 2.2 ppm/deg C	7 = +100	N = ± 2500 ppm
S = 3.3 ppm/deg C	8 = +1000	
T = 4.7 ppm/deg C		
V = 5.6 ppm/deg C		
U = 7.5 ppm/deg C		

C0G is the most common Class 1 dielectric type, with the best temperature stability of them all. Type C0G dielectric is so popular, in fact, that the designation is often used interchangeably with NP0 as a shorthand for "Class 1" dielectric.

Class 2 ceramic dielectric materials have a completely different alphanumeric coding scheme for representing temperature dependence, as shown in the table below. Rather than specify a temperature coefficient in parts per million, Class 2 dielectrics specify lower and upper bounds of their operating temperature range plus the percentage of rated capacitance value we may expect the capacitance to vary over that range:

Low temperature	**High temperature**	**Capacitance change over range**
X = −55 deg C	4 = +65 deg C	P = ±10%
Y = −30 deg C	5 = +85 deg C	R = ±15%
Z = +10 deg C	6 = +105 deg C	S = ±22%
	7 = +125 deg C	T = +22/−33%
	8 = +150 deg C	U = +22/−56%
	9 = +200 deg C	V = +22/−82%

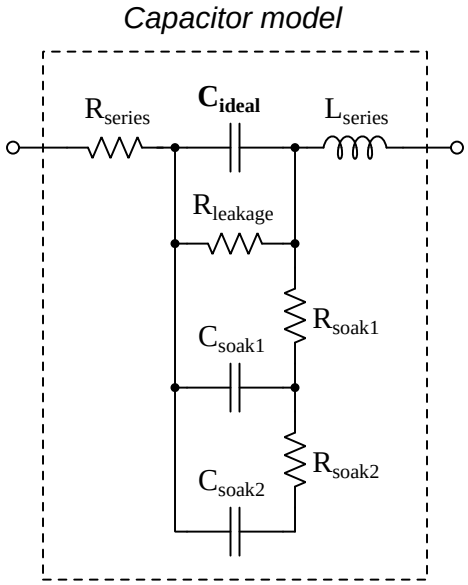
Note how all three parameters together define temperature coefficient for a given ceramic capacitor. For example, a capacitor with X5R dielectric will drift more per degree of temperature variation than a capacitor with X7R dielectric despite the fact these two dielectric types share the same "R" letter code representing ±15%, because the X5R has a significantly narrower temperature range than the X7R for that same drift percentage. If a type-X7R capacitor drifts 15% in its capacitance as temperature rises from −55 degrees to +125 degrees, but a type-X5R capacitor drifts the same amount with less temperature rise (−55 degrees to just +85 degrees), it means the type-X5R capacitor is more sensitive to changes in temperature and therefore X7R would be a better choice than X5R for applications requiring a more stable capacitance value.

Common Class 2 dielectrics include X5R, X7R, and Y5V.

5.4 Capacitor parasitics

5.4.1 Model of a real capacitor

An ideal capacitor exhibits only capacitance, with no inductance, resistance, or other characteristics to interfere. Real capacitors exhibit all these phenomena to varying degrees, and we collectively refer to these undesirable traits as *parasitic effects*. The following diagram models some of the parasitic effects observed in real capacitors:



In addition to the capacitance the capacitor is supposed to exhibit (C_{ideal}), the capacitor also has parasitic resistance (R_{series}, also known as *Equivalent Series Resistance*, or *ESR*), parasitic inductance (L_{series}), and additional energy storage in the form of *soakage* (also known as *dielectric absorption*) whereby the dielectric substance itself absorbs and releases energy after relatively long periods of time compared to the main (ideal) capacitance.

Some of these parasitic effects – such as leakage resistance and soakage – affect the capacitor's performance in DC applications. Most of the other parasitic effects cause problems in AC and pulsed applications. For example, the effective series capacitor-inductor combination formed by C_{ideal} and L_{series} will cause *resonance* to occur at a particular AC frequency, resulting in much less reactance at that frequency than what would be predicted by the capacitive reactance formula $X_C = \frac{1}{2\pi fC}$.

Next we will explore common mechanisms for each of these effects.

5.4.2 Parasitic resistance in capacitors

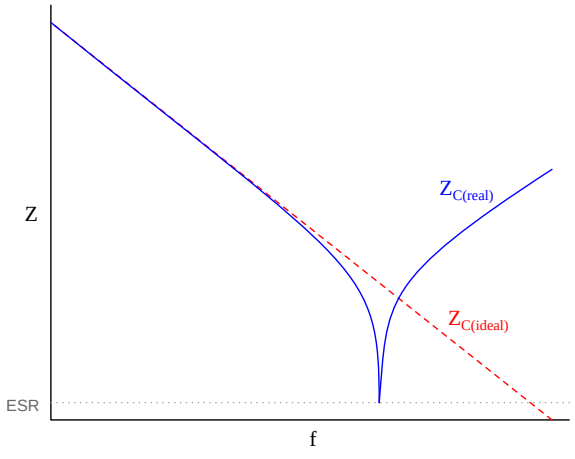
Wire resistance, of course, plays a part in this parasitic effect, but this is generally very small due to the short conductor lengths we typically see with capacitors. More significant is *dielectric losses* – energy dissipation caused by the stressing and relaxation of dipoles within the dielectric material – which act like resistance because energy ends up leaving the component (in the form of heat) and not returning to the circuit. Electrolytic capacitors have an additional source of parasitic resistance, in the form of the electrolytic gel substance used to make electrical contact from the metal-foil "plate" to the surface of the dielectric layer.

Another form of parasitic resistance within a capacitor behaves like a resistor connected in parallel with the ideal capacitance ($R_{leakage}$), resulting from the dielectric not being a perfectly insulating medium. This parasitic characteristic results in a small current passing through the capacitor even when the voltage across the capacitor is steady (i.e. $\frac{dV}{dt}$ is zero).

5.4.3 Parasitic inductance in capacitors

Any time a magnetic field forms around a current-carrying conductor, energy is stored in that magnetic field. We call this magnetic-based energy-storing capability *inductance*, and of course all capacitors must have some inductance due to the wire leads serving as connection points to the capacitor's metal plates. Much of a capacitor's parasitic inductance may be minimized by maintaining short lead lengths as it attaches to a printed-circuit board.

Parasitic inductance is a problem for capacitors in AC applications because inductive reactance (X_L) tends to cancel out capacitive reactance (X_C). If we plot the impedance of a capacitor as a function of frequency, we would expect an ideal capacitor to manifest a straight-line descent on a logarithmic plot. However, what we see is that at a certain frequency the series parasitic inductance resonates with the capacitance leaving only parasitic resistance (ESR), and then past that frequency the inductive effects overshadow the capacitance:

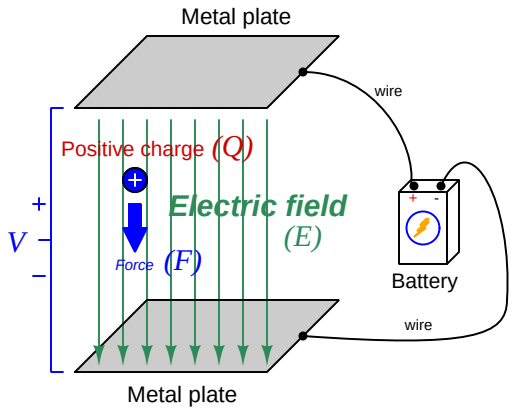


5.4.4 Other parasitic effects in capacitors

Soakage is an interesting effect resulting from dipole relaxation within the dielectric material itself, and may be modeled (as shown) by a series of resistor-capacitor subnetworks. This effect is especially prominent in aluminum electrolytic capacitors, and may be easily demonstrated by discharging a capacitor (by briefly connecting a shorting wire across the capacitor's terminals) and then monitoring the capacitor's DC voltage slowly "recover" with no connection to an external source.

5.5 Electric field quantities

A useful definition of electric field (E) is in terms of the force (F) exerted on an electric charge (Q) influenced by that field:



$$\vec{F} = Q\vec{E}$$

Where,

$\vec{F}$ = Force exerted on the charge (Newtons)
Q = Charge quantity (Coulombs[4])
$\vec{E}$ = Electric field (Newtons per Coulomb)

The small "arrow" symbols above the variables for force and electric field in the equation denote those variables as *vector quantities*, having both magnitude and direction. Charge is a *scalar quantity* having only magnitude but no direction, and as a scalar quantity when multiplied by the electric field vector it simply magnifies the magnitude but does not alter the direction. Therefore, the force and electric field vectors always point in the same direction.

Alternatively electric field may be defined in terms of the voltage between the end-points and the distance separating them, in which case we may express the electric field in units of *Volts per meter* as an alternative to *Newtons per Coulomb*:

$$\vec{E} = \frac{V}{\vec{d}}$$

This measurement of electric field strength is very important for quantifying the *breakdown* of electrical insulators: the point at which the electric field becomes so powerful that otherwise immobile charges within the insulating substance are torn free to constitute a current and that substance is no longer an insulator. For rating the dielectric strength of insulating materials, we often see electric fields expressed in units of *kiloVolts per millimeter* rather than Volts per meter just to make the numerical quantities easier to manage (1 kV/mm = 1 *million* V/m).

[4] One Coulomb of electric charge is equal to 6.2415×10^{18} electrons.

The vector arrows shown in the previous illustration representing the electric field between two metal plates actually represent electric *flux* (Φ_E). The electric field (E) is related to electric flux by area (A), the field being a measurement of how densely-packed those flux lines are per unit area:

$$\vec{E} = \frac{\Phi_E}{\vec{A}}$$

Where,
$\vec{E}$ = Electric field, or electric flux density (Newtons per Coulomb)
Φ_E = Electric flux (Newton-meter squared per Coulomb)
$\vec{A}$ = Area over which flux is distributed (square meters)

The mere presence of an unbalanced electric charge at any point in space is sufficient to generate lines of electric flux, the total magnitude of that flux predicted by the following equation:

$$\Phi_E = \frac{Q}{\epsilon}$$

Where,
Φ_E = Electric flux (Newton-meter squared per Coulomb)
Q = Charge quantity (Coulombs)
ϵ = Electric permittivity of the surrounding space (Coulombs squared per Newton-meter squared, approximately 8.85×10^{-12} for empty space)

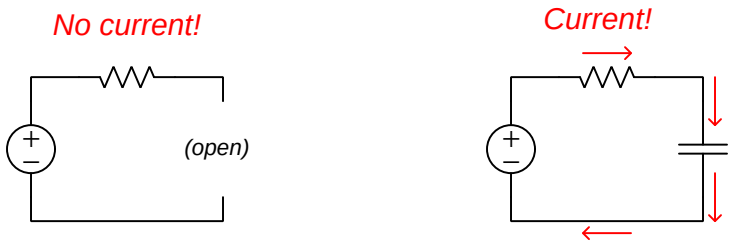
By convention, these flux vectors point *away* from positive charges and point *toward* negative charges, their direction indicating force exerted on any positively-charged particle influenced by that field[5]. As the equation states, the amount of flux depends on how much charge exists at each particle as well as the permittivity of the surrounding space:

For example, identical charges suspended in a vacuum versus in a substance such as oil will have different amounts of flux associated with them as a result of oil and vacuum having different permittivity values. Perfectly empty space has the least amount of permittivity, which means anything else (gas, liquid, or solid matter) has greater ϵ which acts to diminish the amount of electric flux surrounding any charged particle. Superconducting materials have infinite permittivity, which means they forbid the existence of any electric field inside their bulk.

[5] Conversely, the flux vectors point exactly opposite the direction of force applied to any *negatively*-charged particle within that field. This makes sense of the aphorism that *like charges repel and opposite charges attract*. If you consider the two charges shown in this illustration, the positive charge will be pulled in the direction of the flux vectors pointing toward the negative charge, as the negative charge will also be pulled opposite the direction of the flux vectors pointing away from the positive charge (i.e. the negative charge will be pulled *toward* the positive charge) – thus the positive and negative charges feel mutual attraction.

5.6 Capacitor displacement current

A common point of confusion regarding capacitors is their ability to permit an electric current despite the lack of a continuous conductive pathway between their plates. Any capacitance, after all, is really just a pair of conductive surfaces separated by an insulator (called a *dielectric*). With no true *circuit* (i.e. a loop of electrically conductive material), how can an electric current possibly flow?



The key to understanding this is to realize that a "broken" loop is prohibited only from carrying a *continuous* current, but *temporary* motion of electric charge carriers is entirely possible where no true circuit exists. Most students learn that a break anywhere in a simple circuit halts current *everywhere* in that circuit, and so it is easy to conclude that broken circuits cannot *ever* sustain current. And, since a capacitor is essentially a "break" it makes sense someone might conclude that no current could ever pass through the wires of a capacitor.

Yet we know of examples where electricity is able to flow – albeit momentarily – in the absence of a complete circuit. Lightning is perhaps the most impressive example, where a sudden surge of electric current passes between clouds and the Earth (or from cloud to cloud). Anyone who has ever felt the shock of "static electricity" when touching a metal object has experienced a miniature version of this same phenomenon: a rush of electric charge carriers between yourself and another object where no continuous circuit exists.

It is comforting to know that some very intelligent people wrestled with this same conceptual problem. Among them was James Clerk Maxwell, perhaps most famous for his laws mathematically relating electricity and magnetism. It was well-known in Maxwell's time that an electric current traveling through a conductive wire produced a magnetic field, and so Maxwell concluded magnetic fields must be the result of electric charge carriers in motion. As he formulated his theory, he came to conclude that an electric field existing across an insulating medium must also be capable of producing a magnetic field, *if that electric field varied over time*. This conclusion was later vindicated in rather spectacular fashion by the discovery of electromagnetic waves in open space, consisting of mutually-supporting electric and magnetic field waves (one wave supporting the other as they alternate polarities and propagate at the speed of light). Even though no electricity can pass through an insulator, something akin to an electric current must exist within the volume of an insulating body (or across any stretch of open space, which is also electrically insulating) in order to preserve the dependency of magnetism on electricity. Maxwell called this current-through-an-insulator a *displacement current*, contrasting it against the "regular" mode of current through conductors.

Maxwell described his theory in multiple publications, one of them being the *Philosophical Magazine*, Volume 23, fourth series, published in the year 1862. Contained in an article of this

journal entitled "On Physical Lines of Force, Part III – The Theory of Molecular Vortices applied to Statical Electricity", Maxwell contrasts electric currents through conductors against "displacement" current through insulators:

> When a difference of tension exists in different parts of any body, the electricity passes, or tends to pass, from places of greater to places of smaller tension. If the body is a conductor, an actual passage of electricity takes place; and if the difference of tensions is kept up, the current continues to flow with a velocity proportional inversely to the resistance, or directly to the conductivity of the body.
>
> The electric resistance has a very wide range of values, that of the metals being the smallest, and that of glass being so great that a charge of electricity has been preserved in a glass vessel for years without penetrating the thickness of the glass.
>
> Bodies which do not permit a current of electricity to flow through them are called insulators. But though electricity does [page 13]
>
> not flow through them, electrical effects are propagated through them, and the amount of these effects differs according to the nature of the body; so that equally good insulators may act differently as dielectrics.
>
> Here then we have two independent qualities of bodies, one by which they allow of the passage of electricity through them, and the other by which they allow of electrical action being transmitted through them without any electricity being allowed to pass. A conducting body may be compared to a porous membrane which opposes more or less resistance to the passage of a fluid, while a dielectric is like an elastic membrane which may be impervious to the fluid, but transmits the pressure of the fluid on one side to that on the other. [page 14]

In these few paragraphs, Maxwell summarized *Ohm's Law*, *electrical resistivity and conductance*, and *dielectric permittivity*. Ohm's Law is evident in his statement that "current continues to flow with a velocity proportional inversely to the resistance". Maxwell's characterization of that quality of matter by which electrical effects might pass through insulating materials without there being an actual flow of current is what we now call *permittivity*.

Maxwell continues his line of thought by explaining what happens inside of conductors and insulators when exposed to differences of electrical potential:

> As long as electromagnetic force acts on a conductor, it produces a current which, as it meet with resistance, occasions a continual transformation of electrical energy into heat, which is incapable of being restored again as electrical energy by any reversion of the process.
>
> Electromotive force acting on a dielectric produces a state of polarization of its parts similar in distribution to the polarity of the particles of iron under the influence of a magnet, and, like the magnetic polarization, capable of being described as a state in which every particle has its poles in opposite conditions.
>
> In a dielectric under induction, we may conceive that the electricity in each molecule is so displaced that one side is rendered positively, and the other negatively electrical, but

> that the electricity remains entirely connected with the molecule, and does not pass from one molecule to another.
>
> The effect of this action on the whole dielectric mass is to produce a general displacement of the electricity in a certain direction. This displacement does not amount to a current, because when it has attained a certain value it remains constant, but it is the commencement of a current, and its variations constitute currents in the positive or negative direction, according as the displacement is increasing or diminishing. The amount of the displacement depends on the nature of the body, and on the electromotive force; so that if h is the displacement, R the electromotive force, and E a coefficient depending on the nature of the dielectric,
>
> $$R = -4\pi E^2 h \ ;$$
>
> and if r is the value of the electric current due to displacement,
>
> $$r = \frac{dh}{dt}$$
>
> [page 14]

Here we see Maxwell first describe the action of a resistance as an electrical *load*, transforming electrical energy into a form (heat) that leaves the circuit never to return. Next, he describes the action of an electric field (i.e. an impressed voltage) across an insulating body "polarizing" the molecules of that body in a manner similar to iron filings turning about under the influence of a magnetic field. The impressed voltage's effect of distorting these molecules amounts to a slight shift of the electric charges within those molecules, which he describes as being "a commencement of a current". This slight charge motion constitutes a temporary current "through" the dielectric, the rate-of-change of this displacement over time ($\frac{dh}{dt}$ in calculus terminology) must constitute an electric current.

Using modern notation, we may express Maxwell's sentiments in the two following formulae:

$$Q = CV \qquad\qquad I = C\frac{dV}{dt}$$

Where,
Q = Charge displaced (Coulombs)
I = Displacement current (Amperes)
C = Capacitance (Farads)
V = Applied voltage (Volts)
$\frac{dV}{dt}$ = Rate of voltage rise/fall (Volts per second)

$Q = CV$ expresses the amount of electric charge (Q) displaced in the circuit as voltage (V) is applied to a capacitance (C). Re-arranged, this formula serves to mathematically define capacitance as the amount of charge displaced per unit of applied voltage ($C = \frac{Q}{V}$). If we consider a capacitor under the influence of a *changing* voltage, that voltage's rate-of-change ($\frac{dV}{dt}$) results in a temporary current (I) through the capacitive circuit as described by Maxwell.

Returning to Maxwell's explanation, he claims that this displacement current's existence stands independent of how dielectrics might function internally. The model of dielectric materials being "elastic" in response to an applied voltage is just an analogy, however useful it may be:

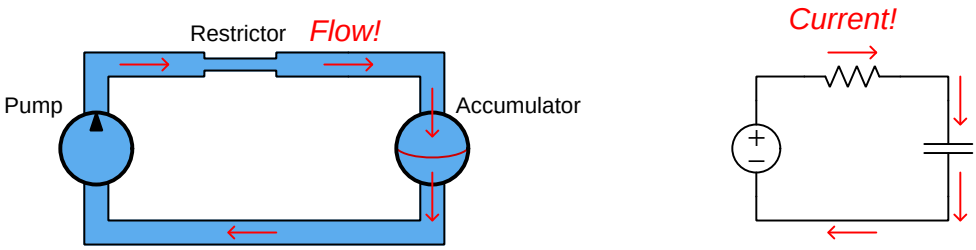
> These relations are independent of any theory about the internal mechanism of dielectrics; but when we find electromotive force producing electric displacement in a dielectric, and when we find the dielectric recovering from its state of electric displacement with an equal electromotive force, we cannot help regarding the phenomena as those of an elastic body, yielding to a pressure, and recovering its form when the pressure is removed. [page 15]

Dielectric substances accentuate capacitance, by virtue of their constituent molecules becoming distorted by electric fields. This might be thought of in terms of each distorted molecule acting as a microscopic spring, being stretched or compressed by an external force. Springs, as we know when stretched or compressed, store energy in potential form. Likewise, a substance having a large permittivity value allows for greater storage of energy by an electric field (i.e. greater capacitance), all other factors being equal. However, it is fascinating to realize that capacitance and displacement currents exist all the same when the insulating medium in question is a pure *vacuum*. This is where the model of displacement current as molecular distortion loses its explanatory power, because in a vacuum there are no molecules at all for the electric field to distort.

This is not to say that Maxwell's description of molecular distortion is wrong. It simply means dielectric action is *part* of the model and not the entirety of it. Even when there is no insulating matter between two conductors, the empty space of a perfect vacuum still supports electric fields, still stores energy, and still generates a magnetic field when a time-varying electric field is impressed across it. It's just that a pure vacuum doesn't exhibit *as much* energy storage, or *as much* displacement current, for any given electric field.

In summary, this is how we may conserve our notion of electric current requiring a complete loop in which to travel: the insulating gap of a capacitance supports a *displacement current* that is not a real motion of electric charges from one point to another, but nevertheless functions similar to one. Displacement currents are always transient, never continuous. They exist in proportion to the rate-of-change over time of an electric field, whether accentuated by the molecules of a dielectric substance or not. Current throughout the rest of the circuit (i.e. through conductors connected in series with the capacitance) follow the transient nature of the displacement current. What a *capacitor* does is magnify the effect of displacement current by means of intensifying the electric field and enhancing its girth by using a large overlapping area between conductors.

We may model this phenomenon in the form of a *hydraulic circuit* where a pump attempts to move liquid through some pipes, but there is a device called an *accumulator* at the far end of the circuit. This "accumulator" consists of a metal vessel divided into two chambers by an elastic diaphragm, such that forcing liquid into one chamber has the effect of distending this diaphragm into the other chamber where liquid gets forced out:



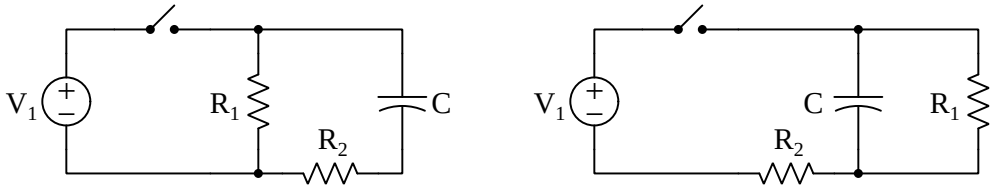
If the diaphragm's tension reaches a point where it is able to fully oppose the pump's pressure, the flow of liquid in the hydraulic circuit will cease. However, up until that point, the pump will be able to move liquid around the hydraulic circuit as the elastic diaphragm continues to stretch. So, we see here that even in a case where an impermeable diaphragm prevents continuous liquid motion around the hydraulic circuit, under conditions of changing pressure we are still able to create temporary flow in this circuit lacking a continuous pathway for flow. This is what we see happening in the RC electric circuit: activating the source causes the unenergized capacitor to begin to receive energy, and we will have current in the circuit. As the capacitor's energy level increases, and its voltage drop increases accordingly, there will come a point in time when the capacitor's voltage is equal in value to and opposite in polarity to the source at which point current finally ceases.

The accumulator diaphragm's action also mimics what happens inside a dielectric material sandwiched between two capacitor plates. The electric field created between those plates causes the positive and negative charges within the atoms of the dielectric to be pulled in opposite directions, essentially "distorting" the atoms within that material and thus storing energy in a manner not unlike the stretching of a spring by mechanical force or the distortion of an elastic diaphragm by fluid pressure. This is why the addition of a dielectric material between a capacitor's plates (as opposed to vacuum) results in greater capacitance: the atomic elasticity of that material functions as an energy-storage mechanism.

5.7 Time delay circuits and Thévenin's/Norton's Theorems

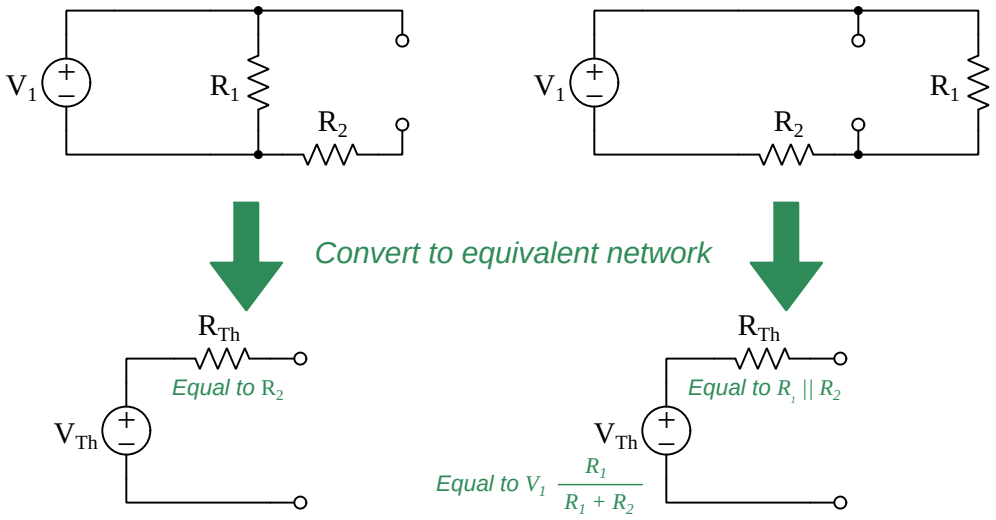
Circuits containing resistors along with capacitors or inductors typically result in voltage and current values that vary over time. Determining starting and final values for these parameters, as well as calculating the "time constant" ($\tau = RC = \frac{L}{R}$) can be challenging especially if the circuit in question consists of more than just a single resistor, single source, and single reactive (C or L).

Take for example the following two RC circuits, each one containing one capacitor but two resistors:



If the capacitor begins in a completely de-energized state with the switch in the open position, capacitor voltage for each circuit is guaranteed to begin at 0 Volts because energy stored in any capacitance always manifests as a voltage across that capacitance. When we close the switch, we would expect the capacitor in both circuits to begin energizing as they absorb energy from the voltage source V_1, and this means their voltages will rise over time. However, what is not as clear upon first inspection is what the final voltage will be for either capacitor after the switch has been left in its closed state for a long time, as well as how we might calculate the time constant $\tau = RC$ given the fact each circuit has *two* resistors and not just one.

Obviously, an RC circuit containing just one resistor and one capacitor instead of two resistors and one capacitor would be simpler to analyze, so why don't we just apply either Thévenin's Theorem or Norton's Theorem to each circuit in order to reduce it to just a single source and single resistance? Choosing the capacitor to be the load, we will convert the rest of each circuit into its Thévenin equivalent:

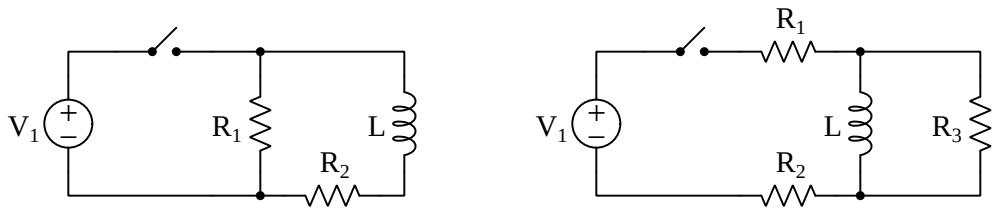


Recall that determining either the Thévenin or the Norton equivalent resistance of any complex network consists of disabling all sources in the original circuit and determining resistance as measured from load terminal to load terminal. In the left-hand circuit the disabled V_1 is replaced by a short which completely bypasses R_1 and leaves only R_2 remaining. In the right-hand circuit the disabled V_1 (again, replaced by a short) causes R_1 and R_2 to be in parallel with each other from the perspective of the two load terminals. Thus, the time constant for the left-hand circuit is simply $\tau = R_2C$ while the time constant for the right-hand circuit will be $\tau = \frac{C}{\frac{1}{R_1} + \frac{1}{R_2}}$.

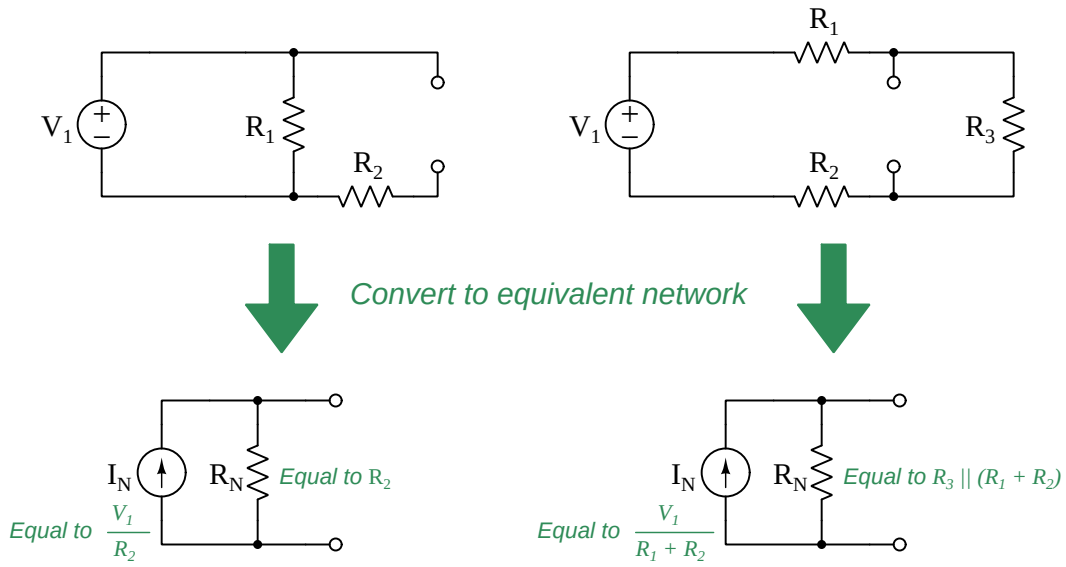
Recall that determining the Thévenin equivalent voltage of any complex network consists of calculating voltage from one load terminal to the other with all original sources active. Here we see that the left-hand circuit's open-circuit load terminal voltage must simply be V_1, while the right-hand circuit's open-circuit terminal voltage will be the result of V_1 powering a two-resistor voltage divider.

Re-attaching the capacitor C to the load terminals of the Thévenin equivalent networks results in a pair of simpler circuits to analyze, where the time constant in each case is $\tau = R_{Th}C$ and the final capacitor voltage after full energization is V_{Th}.

Norton's Theorem, of course, works just as well as Thévenin's Theorem for this purpose. To illustrate, we will apply Norton's Theorem to a pair of resistor-inductor circuits to determine how multiple resistors affect the inductors' energization:



Removing the inductor from each circuit and converting the remaining networks into Norton equivalents:



The process for determining Norton equivalent resistance is exactly the same as it is for Thévenin equivalent resistance: simply disable all original sources and determine what an ohmmeter would register if connected between the two load terminals. In the left-hand circuit we see once again that R_1 is of no effect to the Norton equivalent value of R_N, while in the right-hand circuit R_N is a series-parallel combination of the three original resistors.

Recall that determining the Norton equivalent current of any complex network consists of calculating current passing through a shorting wire placed between the two load terminals with all the original sources active. In the left-hand circuit we see that this short-circuit current is the limited only by R_2 and so $I_N = \frac{V_1}{R_2}$. In the right-hand circuit we see only R_1 and R_2 serving to limit current, and so $I_N = \frac{V_1}{R_1+R_2}$.

Re-attaching the inductor L to the load terminals of the Norton equivalent networks results in a pair of simpler circuits to analyze, where the time constant in each case is $\tau = \frac{L}{R_N}$ and the final inductor current after full energization is I_N.

Chapter 6

Programming References

A powerful tool for mathematical modeling is text-based *computer programming*. This is where you type coded commands in text form which the computer is able to interpret. Many different text-based languages exist for this purpose, but we will focus here on just two of them, *C++* and *Python*.

6.1 Programming in C++

One of the more popular text-based computer programming languages is called *C++*. This is a *compiled* language, which means you must create a plain-text file containing C++ code using a program called a *text editor*, then execute a software application called a *compiler* to translate your "source code" into instructions directly understandable to the computer. Here is an example of "source code" for a very simple C++ program intended to perform some basic arithmetic operations and print the results to the computer's console:

```
#include <iostream>
using namespace std;

int main (void)
{
  float x, y;

  x = 200;
  y = -560.5;

  cout << "This simple program performs basic arithmetic on" << endl;
  cout << "the two numbers " << x << " and " << y << " and then" << endl;
  cout << "displays the results on the computer's console." << endl;

  cout << endl;

  cout << "Sum = " << x + y << endl;
  cout << "Difference = " << x - y << endl;
  cout << "Product = " << x * y << endl;
  cout << "Quotient of " << x / y << endl;

  return 0;
}
```

Computer languages such as C++ are designed to make sense when read by human programmers. The general order of execution is left-to-right, top-to-bottom just the same as reading any text document written in English. Blank lines, indentation, and other "whitespace" is largely irrelevant in C++ code, and is included only to make the code more pleasing[1] to view.

[1] Although not included in this example, *comments* preceded by double-forward slash characters (`//`) may be added to source code as well to provide explanations of what the code is supposed to do, for the benefit of anyone reading it. The compiler application will ignore all comments.

Let's examine the C++ source code to explain what it means:

- `#include <iostream>` and `using namespace std;` are set-up instructions to the compiler giving it some context in which to interpret your code. The code specific to your task is located between the brace symbols ({ and }, often referred to as "curly-braces").

- `int main (void)` labels the "Main" function for the computer: the instructions within this function (lying between the { and } symbols) it will be commanded to execute. Every complete C++ program contains a `main` function at minimum, and often additional functions as well, but the `main` function is where execution always begins. The `int` declares this function will return an *integer* number value when complete, which helps to explain the purpose of the `return 0;` statement at the end of the `main` function: providing a numerical value of zero at the program's completion as promised by `int`. This returned value is rather incidental to our purpose here, but it is fairly standard practice in C++ programming.

- Grouping symbols such as (parentheses) and {braces} abound in C, C++, and other languages (e.g. Java). Parentheses typically group data to be processed by a function, called *arguments* to that function. Braces surround lines of executable code belonging to a particular function.

- The `float` declaration reserves places in the computer's memory for two *floating-point* variables, in this case the variables' names being `x` and `y`. In most text-based programming languages, variables may be named by single letters or by combinations of letters (e.g. `xyz` would be a single variable).

- The next two lines assign numerical values to the two variables. Note how each line terminates with a semicolon character (`;`) and how this pattern holds true for most of the lines in this program. In C++ semicolons are analogous to periods at the ends of English sentences. This demarcation of each line's end is necessary because C++ ignores whitespace on the page and doesn't "know" otherwise where one line ends and another begins.

- All the other instructions take the form of a `cout` command which prints characters to the "standard output" stream of the computer, which in this case will be text displayed on the console. The double-less-than symbols (`<<`) show data being sent *toward* the `cout` command. Note how verbatim text is enclosed in quotation marks, while variables such as `x` or mathematical expressions such as `x - y` are not enclosed in quotations because we want the computer to display the numerical values represented, not the literal text.

- Standard arithmetic operations (add, subtract, multiply, divide) are represented as `+`, `-`, `*`, and `/`, respectively.

- The `endl` found at the end of every `cout` statement marks the end of a line of text printed to the computer's console display. If not for these `endl` inclusions, the displayed text would resemble a run-on sentence rather than a paragraph. Note the `cout << endl;` line, which does nothing but create a blank line on the screen, for no reason other than esthetics.

After saving this *source code* text to a file with its own name (e.g. `myprogram.cpp`), you would then *compile* the source code into an *executable* file which the computer may then run. If you are using a console-based compiler such as *GCC* (very popular within variants of the Unix operating system[2], such as Linux and Apple's OS X), you would type the following command and press the Enter key:

```
g++ -o myprogram.exe myprogram.cpp
```

This command instructs the `GCC` compiler to take your source code (`myprogram.cpp`) and create with it an executable file named `myprogram.exe`. Simply typing `./myprogram.exe` at the command-line will then execute your program:

```
./myprogram.exe
```

If you are using a graphic-based C++ development system such as Microsoft Visual Studio[3], you may simply create a new console application "project" using this software, then paste or type your code into the example template appearing in the editor window, and finally run your application to test its output.

As this program runs, it displays the following text to the console:

```
This simple program performs basic arithmetic on
the two numbers 200 and -560.5 and then
displays the results on the computer's console.

Sum = -360.5
Difference = 760.5
Product = -112100
Quotient of -0.356824
```

As crude as this example program is, it serves the purpose of showing how easy it is to write and execute simple programs in a computer using the C++ language. As you encounter C++ example programs (shown as source code) in any of these modules, feel free to directly copy-and-paste the source code text into a text editor's screen, then follow the rest of the instructions given here (i.e. save to a file, compile, and finally run your program). You will find that it is generally easier to

[2] A very functional option for users of Microsoft Windows is called *Cygwin*, which provides a Unix-like console environment complete with all the customary utility applications such as GCC!

[3] Using Microsoft Visual Studio community version 2017 at the time of this writing to test this example, here are the steps I needed to follow in order to successfully compile and run a simple program such as this: (1) Start up Visual Studio and select the option to create a New Project; (2) Select the Windows Console Application template, as this will perform necessary set-up steps to generate a console-based program which will save you time and effort as well as avoid simple errors of omission; (3) When the editing screen appears, type or paste the C++ code within the `main()` function provided in the template, deleting the "Hello World" `cout` line that came with the template; (4) Type or paste any preprocessor directives (e.g. `#include` statements, `namespace` statements) necessary for your code that did not come with the template; (5) Lastly, under the Debug drop-down menu choose either Start Debugging (F5 hot-key) or Start Without Debugging (Ctrl-F5 hotkeys) to compile ("Build") and run your new program. Upon execution a console window will appear showing the output of your program.

learn computer programming by closely examining others' example programs and modifying them than it is to write your own programs starting from a blank screen.

6.2 Programming in Python

Another text-based computer programming language called *Python* allows you to type instructions at a terminal prompt and receive immediate results without having to compile that code. This is because Python is an *interpreted* language: a software application called an *interpreter* reads your source code, translates it into computer-understandable instructions, and then executes those instructions in one step.

The following shows what happens on my personal computer when I start up the Python interpreter on my personal computer, by typing `python3`[4] and pressing the Enter key:

```
Python 3.7.2 (default, Feb 19 2019, 18:15:18)
[GCC 4.1.2] on linux
Type "help", "copyright", "credits" or "license" for more information.
>>>
```

The `>>>` symbols represent the prompt within the Python interpreter "shell", signifying readiness to accept Python commands entered by the user.

Shown here is an example of the same arithmetic operations performed on the same quantities, using a Python interpreter. All lines shown preceded by the `>>>` prompt are entries typed by the human programmer, and all lines shown without the `>>>` prompt are responses from the Python interpreter software:

```
>>> x = 200
>>> y = -560.5
>>> x + y
-360.5
>>> x - y
760.5
>>> x * y
-112100.0
>>> x / y
-0.35682426404995538
>>> quit()
```

[4] Using version 3 of Python, which is the latest at the time of this writing.

More advanced mathematical functions are accessible in Python by first entering the line `from math import *` which "imports" these functions from Python's math *library* (with functions identical to those available for the C programming language, and included on any computer with Python installed). Some examples show some of these functions in use, demonstrating how the Python interpreter may be used as a scientific calculator:

```
>>> from math import *
>>> sin(30.0)
-0.98803162409286183
>>> sin(radians(30.0))
0.49999999999999994
>>> pow(2.0, 5.0)
32.0
>>> log10(10000.0)
4.0
>>> e
2.7182818284590451
>>> pi
3.1415926535897931
>>> log(pow(e,6.0))
6.0
>>> asin(0.7071068)
0.78539819000368838
>>> degrees(asin(0.7071068))
45.000001524425265
>>> quit()
```

Note how trigonometric functions assume angles expressed in *radians* rather than *degrees*, and how Python provides convenient functions for translating between the two. Logarithms assume a base of e unless otherwise stated (e.g. the `log10` function for common logarithms).

The interpreted (versus compiled) nature of Python, as well as its relatively simple syntax, makes it a good choice as a person's first programming language. For complex applications, interpreted languages such as Python execute slower than compiled languages such as C++, but for the very simple examples used in these learning modules speed is not a concern.

Another Python math library is `cmath`, giving Python the ability to perform arithmetic on complex numbers. This is very useful for AC circuit analysis using *phasors*[5] as shown in the following example. Here we see Python's interpreter used as a scientific calculator to show series and parallel impedances of a resistor, capacitor, and inductor in a 60 Hz AC circuit:

```
>>> from math import *
>>> from cmath import *
>>> r = complex(400,0)
>>> f = 60.0
>>> xc = 1/(2 * pi * f * 4.7e-6)
>>> zc = complex(0,-xc)
>>> xl = 2 * pi * f * 1.0
>>> zl = complex(0,xl)
>>> r + zc + zl
(400-187.38811239154882j)
>>> 1/(1/r + 1/zc + 1/zl)
(355.837695813625+125.35793777619385j)
>>> polar(r + zc + zl)
(441.717448903332, -0.4381072059213295)
>>> abs(r + zc + zl)
441.717448903332
>>> phase(r + zc + zl)
-0.4381072059213295
>>> degrees(phase(r + zc + zl))
-25.10169387356105
```

When entering a value in rectangular form, we use the `complex()` function where the arguments are the real and imaginary quantities, respectively. If we had opted to enter the impedance values in polar form, we would have used the `rect()` function where the first argument is the magnitude and the second argument is the angle in radians. For example, we could have set the capacitor's impedance (`zc`) as $X_C \angle -90^o$ with the command `zc = rect(xc,radians(-90))` rather than with the command `zc = complex(0,-xc)` and it would have worked the same.

Note how Python defaults to rectangular form for complex quantities. Here we defined a 400 Ohm resistance as a complex value in rectangular form (400 +j0 Ω), then computed capacitive and inductive reactances at 60 Hz and defined each of those as complex (phasor) values ($0 - jX_c$ Ω and $0 + jX_l$ Ω, respectively). After that we computed total impedance in series, then total impedance in parallel. Polar-form representation was then shown for the series impedance (441.717 Ω $\angle -25.102^o$). Note the use of different functions to show the polar-form series impedance value: `polar()` takes the complex quantity and returns its polar magnitude and phase angle in *radians*; `abs()` returns just the polar magnitude; `phase()` returns just the polar angle, once again in radians. To find the polar phase angle in degrees, we nest the `degrees()` and `phase()` functions together.

The utility of Python's interpreter environment as a scientific calculator should be clear from these examples. Not only does it offer a powerful array of mathematical functions, but also unlimited

[5] A "phasor" is a voltage, current, or impedance represented as a complex number, either in rectangular or polar form.

assignment of variables as well as a convenient text record[6] of all calculations performed which may be easily copied and pasted into a text document for archival.

It is also possible to save a set of Python commands to a text file using a text editor application, and then instruct the Python interpreter to execute it at once rather than having to type it line-by-line in the interpreter's shell. For example, consider the following Python program, saved under the filename `myprogram.py`:

```
x = 200
y = -560.5

print("Sum")
print(x + y)

print("Difference")
print(x - y)

print("Product")
print(x * y)

print("Quotient")
print(x / y)
```

As with C++, the interpreter will read this source code from left-to-right, top-to-bottom, just the same as you or I would read a document written in English. Interestingly, whitespace *is* significant in the Python language (unlike C++), but this simple example program makes no use of that.

To execute this Python program, I would need to type `python myprogram.py` and then press the Enter key at my computer console's prompt, at which point it would display the following result:

```
Sum
-360.5
Difference
760.5
Product
-112100.0
Quotient
-0.35682426405
```

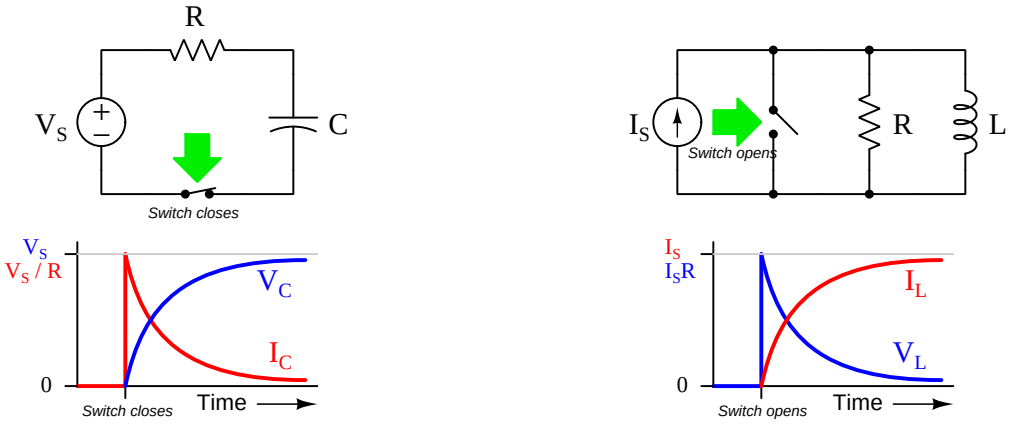
As you can see, syntax within the Python programming language is simpler than C++, which is one reason why it is often a preferred language for beginning programmers.

[6] Like many command-line computing environments, Python's interpreter supports "up-arrow" recall of previous entries. This allows quick recall of previously typed commands for editing and re-evaluation.

If you are interested in learning more about computer programming in *any* language, you will find a wide variety of books and free tutorials available on those subjects. Otherwise, feel free to learn by the examples presented in these modules.

6.3 Modeling inverse exponential growth and decay using C++

Many different types of physical systems, including capacitor and inductor circuits, often exhibit a phenomenon known as *inverse exponential* growth and decay, where variables asymptotically[7] approach final values over time. Consider the following two circuits, one capacitive and the other inductive, both components receiving an input of energy from a source:



Capacitors store energy as a function of voltage (manifest as an *electric field*), while inductors store energy as a function of current (manifest as a *magnetic field*). In both circuits, the reactive component receives energy from the source over time, which explains the capacitor's rising voltage and the inductor's rising current. The other variable in each circuit (i.e. current for the capacitor and voltage for the inductor) is a function of the *rate* at the energy-related variable grows, and so these variables approach zero as the energy-related variables level off at some terminal value.

In each case, the decreasing variable is said to *decay to zero* over time, and its path has the same shape as an inverse exponential function, e^{-t}. As time progresses (i.e. t grows larger), the value of e^{-t} diminishes. The full formula for each circuit includes a term for the initial (peak) value as well as the *time constant* (τ) of the circuit:

$$I_C = I_0 e^{-\frac{t}{\tau}}$$

$$V_L = V_0 e^{-\frac{t}{\tau}}$$

In each case, the increasing variable is said to experience *inverse exponential growth*, and its shape appears to be a mirror-image of the decaying variable. Mathematically, the inverse growth function is simply the complement of the decay function, $1 - e^{-t}$. As time progresses (i.e. larger t), the value of e^{-t} approaches zero and the value of $1 - e^{-t}$ approaches 1. The amount of time equal to one time constant (τ) is the time required for the variable in question to go 63.2% ($1 - e^{-1}$) of

[7] An asymptote is a mathematical concept, consisting of a fixed value which some other variable approaches closer and closer but never fully attains.

the way from its starting value to its final (asymptote) value. For any variable decaying to zero, this is the time required to reach 36.8% (e^{-1}) of its starting value.

These exponential functions are not limited to describing capacitor and inductor circuits, but are actually useful for a wide range of physical systems. The decay of a hot object's temperature[8], for example, follows the same e^{-t} trajectory.

We may employ a simple computer program to repeatedly calculate percentage values for such rising and decaying variables as time (`t`) progresses from zero to ten (measured in whole-multiples of time constants). This is a very good application for a computer, as it relieves us of both the tedium and the many opportunities for error faced with repeatedly calculating e^{-t} and $1 - e^{-t}$.

The "source code" for this computer program, written in the C++ language, is shown here containing all the instructions necessary to tell the computer how to perform these repeated calculations and display the results on the console:

```
#include <iostream>
#include <cmath>
using namespace std;

int main (void)
{
  float initial, x, t;

  initial = 100.0;      // This sets the initial value to 100%

  cout.precision(4);
  cout.setf(ios::fixed, ios::floatfield);

  cout << "Time \t\t" << "Falling \t" << "Rising" << endl;

  for (t = 0.0; t <= 10 ; t = t + 1.0)
  {
    x = initial * exp(-t);
    cout << t << " tau \t";
    cout << x << "\% \t";
    cout << 100.0 - x << "\%" << endl;
  }

  return 0;
}
```

[8] In fact, this is referred to as *Newton's Cooling Law*.

Let's analyze the lines of code within this short C++ program, touching on the following programming principles:

- Reading/execution order
- Preprocessor directives (`#include`) and `namespace`
- The `main` function
- Delimiter characters (e.g. { } ;)
- Variable declarations
- `float` and `int` data types and variable names
- Variable initialization and assignment (e.g. `=`)
- The `cout` instruction
- `for` loops
- Mathematical functions
- Return

As with most text-based programming languages, C and C++ alike are read by the compiler software and executed in the same order one reads an English-language document: left to right character by character, and top to bottom line by line.

The first three lines (`#include` and `using namespace`) direct the compiler software on how to interpret much of what follows. The `using` line tells the compiler to use the "standard" namespace[9] convention for C++, while the two previous lines (called preprocessor directives) request the contents of the `iostream` and `math` header files to be included when compiling the code into an executable file. For example, the `cout` instruction is defined within `iostream` while the `exp` function is defined within `math`.

All C++ programs have a `main` function marking the starting point of execution. All of our circuit-modeling code appears between the `main` function's curly-brace ({ }) symbols, those symbols denoting the boundaries of the `main` function. This particular `main` function is defined here as returning an *integer* number value (`int`) when complete, and it requires no input of information (`void`).

In C, C++, and some other languages such as Java, parentheses and "curly-brace" symbols serve to *group* certain types of information together, much the same as parentheses and brackets group terms together in mathematical formulae. Parentheses immediately following a function name enclose *arguments* to that function: i.e. data that function must act upon. Curly-brace symbols define the starting and ending points for lines of code to be executed within a given function.

[9] Namespaces are a concept found in C++ but not in its predecessor language C. Without getting too detailed, namespaces allow advanced programmers to redefine the meaning of certain key instructions and functions within the language for their own purposes. In this simple program we are doing no such thing, and so we instruct the compiler to use the *standard* (`std`) namespace of C++.

The line beginning with `float` reserves space in the computer's memory for three *floating-point* variables. "Floating-point" is a type of digital numeration useful for expressing non-integer values both large and small. C++ permits variable names ranging from single letters (e.g. `x`) to entire words (e.g. `initial`).

This declaration line ends with a semicolon delimiter (`;`) telling the compiler where the line ends, which is important because neither C nor C++ takes heed of "whitespace" in the source file. You might think of the C/C++ semicolon as being the programming-language equivalent of a period at the end of an English sentence. If not for these semicolons, the compiler would treat all the lines as being a single, continuous line in the same way you would read unterminated lines in a page of text as part of a long sentence. You will notice most of the lines of code end with semicolons, exceptions being `main` (because its curly-brace symbols serve the same purpose) and the `#include` directives which are technically set-up instructions for the compiler and not executed at run-time.

Immediately following the declaration line is a line of code *initializing* one of those variables with a numerical value, in this case setting the `initial` value to `100.0` which will represent 100% of source voltage or current in a capacitive or inductive circuit. In C and C++, a single "equals" symbol (`=`) signifies *assignment* rather than the conventional mathematical meaning of *equality*. Whenever you see a single equals symbol in C or C++, think "*set equal to*".

Several lines of code in this short program begin with `cout` which is the C++ instruction for outputting text to the computer's *console* display. When `cout` is accompanied by the "put to" operator (`<<`) it means that instruction will output either verbatim text or text-control characters (enclosed in quotation marks) or the numerical values of any variables referenced by `cout`. Text-control characters include `t` (tab) instructing a horizontal shift in text and `endl` (end-line) signifying the end of line line and the beginning of another as displayed on the computer's console.

Those `cout` lines lacking "put to" (`<<`) operators instruct subsequent instances of `cout` how to behave. For example, `cout.precision(4)` tells `cout` to display every numerical output at four decimal places to the right of the decimal point. In the next line of code, `cout.setf(ios::fixed, ios::floatfield)` instructs `cout` to used a fixed-point notation when displaying numbers. Both of these formatting instructions will make the program's displayed text look neater on the computer's console, but neither is strictly necessary if all we care about is function and not appearance.

All the interesting operations occur within the `for` loop, where time (`t`) is incremented from 0 to 10 in steps of 1, and the computer calculates e^{-t} with the function `exp(-t)`. In C and C++, a `for` loop instructs the computer to repeat certain lines of code (enclosed in another set of curly-braces) while a particular variable changes from one value to another. In this particular case the variable in question is `t`, starting at a value of `0.0` and incrementing by `1.0` with each new iteration of the `for` loop. The loop repeats so long as `t` is equal to or less than `10.0`.

Three `cout` lines contained within the `for` loop work together to generate a single line of printed text. The first `cout` prints the time variable (`t`), the next `cout` prints the value of the "falling" variable `x`, and the last `cout` prints the value of the "rising" variable which is simply the complement of `x`. A single `endl` control character is printed with that last `cout` instruction to terminate the line and prepare for a new line of text with the next iteration of the `for` loop.

Finally, the last line of code in this program returns an integer value of zero as the `main` function successfully completes. This is not strictly necessary, but is a good programming practice.

When compiled and executed, this program prints the percentage values of the rising and falling quantities over a timespan of zero to ten time constants (i.e. 0 to 10τ). It is instructive to compare this text output to the lines of code previously shown, and to follow the execution path of the code in your mind. For example, a careful examination of the code should reveal to you why we only see one line reading `Time Falling Rising` but several lines of numbers:

```
Time            Falling         Rising
0.0000 tau      100.0000%       0.0000%
1.0000 tau      36.7879%        63.2121%
2.0000 tau      13.5335%        86.4665%
3.0000 tau      4.9787%         95.0213%
4.0000 tau      1.8316%         98.1684%
5.0000 tau      0.6738%         99.3262%
6.0000 tau      0.2479%         99.7521%
7.0000 tau      0.0912%         99.9088%
8.0000 tau      0.0335%         99.9665%
9.0000 tau      0.0123%         99.9877%
10.0000 tau     0.0045%         99.9955%
```

At the very start ($t = 0$) the falling variable is at its maximum value of 100% and the rising variable is at zero. After one time constants' worth of time ($t = \tau$) the falling variable has fallen well over half-way to a value of approximately 36.8% of its initial value. At that same time the rising variable has risen well over half-way to just over 63.2%. After ten time constants' worth of time ($t = 10\tau$) we can see both variables are a small fraction of a percent away from their final values.

The first point in time where we see the variables come within one percent of their final values is at $t = 5\tau$. This is why students are commonly taught that the variables have "settled" for all practical purposes after *five* time constants. It is important to understand, however, that there is nothing truly significant about 5τ. *Five* just happens to be the first whole-number multiple of τ where the variables approach their final values to within the completely arbitrary threshold of 1%. Just to be clear on this point: the rising and falling variables indeed settle to within 1% of their final values after five time constants' worth of time has passed, but their exact value at 5τ is not 1%, nor do they reach the 1% threshold after exactly 5τ, and they certainly are not *fully* settled by then. The rest of this section is devoted to overturning these misconceptions by modeling the inverse exponential functions in greater detail.

We could alter our program to take smaller steps in time, incrementing t by steps of 0.4 instead than by steps of 1, if we wished to take a closer look at exactly when these falling and rising variables come within 1% of their final values. We need only alter one line of code to do this, the conditions controlling execution of the `for` loop:

```
for (t = 0.0; t < 10 ; t = t + 0.4)
```

Now when we re-compile and run this program, we obtain a much more precise table of numbers:

```
Time            Falling         Rising
0.0000 tau      100.0000%       0.0000%
0.4000 tau      67.0320%        32.9680%
0.8000 tau      44.9329%        55.0671%
1.2000 tau      30.1194%        69.8806%
1.6000 tau      20.1897%        79.8103%
2.0000 tau      13.5335%        86.4665%
2.4000 tau      9.0718%         90.9282%
2.8000 tau      6.0810%         93.9190%
3.2000 tau      4.0762%         95.9238%
3.6000 tau      2.7324%         97.2676%
4.0000 tau      1.8316%         98.1684%
4.4000 tau      1.2277%         98.7723%
4.8000 tau      0.8230%         99.1770%
5.2000 tau      0.5517%         99.4483%
5.6000 tau      0.3698%         99.6302%
6.0000 tau      0.2479%         99.7521%
6.4000 tau      0.1662%         99.8338%
6.8000 tau      0.1114%         99.8886%
7.2000 tau      0.0747%         99.9253%
7.6000 tau      0.0500%         99.9500%
8.0000 tau      0.0335%         99.9665%
8.4000 tau      0.0225%         99.9775%
8.8000 tau      0.0151%         99.9849%
9.2000 tau      0.0101%         99.9899%
9.6000 tau      0.0068%         99.9932%
10.0000 tau     0.0045%         99.9955%
```

This simulation proves 5τ is not the true point at which we get within 1% of final value, since we see the values come within 1% after only 4.8 time constants.

We may focus our computer simulation even tighter by altering the `for` loop conditions once again for finer resolution (and more limited range, so as to not generate an enormously long table of values):

```
for (t = 4.55; t < 4.7 ; t = t + 0.01)
```

The result, when compiled and run again is as follows:

```
Time            Falling         Rising
4.5500 tau      1.0567%         98.9433%
4.5600 tau      1.0462%         98.9538%
4.5700 tau      1.0358%         98.9642%
4.5800 tau      1.0255%         98.9745%
4.5900 tau      1.0153%         98.9847%
4.6000 tau      1.0052%         98.9948%
4.6100 tau      0.9952%         99.0048%
4.6200 tau      0.9853%         99.0147%
4.6300 tau      0.9755%         99.0245%
4.6400 tau      0.9658%         99.0342%
4.6500 tau      0.9562%         99.0438%
4.6600 tau      0.9466%         99.0534%
4.6700 tau      0.9372%         99.0628%
4.6800 tau      0.9279%         99.0721%
4.6900 tau      0.9187%         99.0813%
```

With this closer view we see the values come within 1% of final closer to *4.61* time constants. In other words, 5τ is a rather coarse rounding-off of the true time value for settling within 1%.

Of course, the *best* way to determine how long it takes for these variables to come within 1% of their final values is to algebraically solve for t with the decay function set equal to 0.01 (i.e. 1%):

$$0.01 = e^{-t}$$

$$\ln 0.01 = -t$$

$$t = -\ln 0.01$$

$$t = 4.605170186$$

It should be clear by now that the frequently-taught rule of "settled within five time constants" is a rounded-off estimation based on an arbitrary threshold and should be regarded as such. In any timing applications requiring precision, the rule of 5τ just isn't good enough and you will need to actually calculate t.

6.4 Modeling inverse exponential growth and decay using Python

This very simple Python script shows the value of the e^{-x} and $1 - e^{-x}$ functions, using an infinite loop to obtain different values of x from the human user of the script:

```
from math import *

while True:
    x = float(input("x = "))
    print("e^(-x) =", exp(-x))
    print("1 - e^(-x) =", 1 - exp(-x))
    print(" ")
```

To execute this script on a computer with Python installed, save the script as a plain-text file to some filename such as `script.py` and then from the command line interface type the command `python3 script.py` to execute that script using version 3 of the Python language.

Here is an example showing the script's output with user-entered values of 0, 1, 2, 3.5, and 6 for x:

```
x = 0
e^(-x) = 1.0
1 - e^(-x) = 0.0

x = 1
e^(-x) = 0.36787944117144233
1 - e^(-x) = 0.6321205588285577

x = 2
e^(-x) = 0.1353352832366127
1 - e^(-x) = 0.8646647167633873

x = 3.5
e^(-x) = 0.0301973834223185
1 - e^(-x) = 0.9698026165776815

x = 6
e^(-x) = 0.002478752176666 3585
1 - e^(-x) = 0.9975212478233336
```

The mathematical instruction in Python for raising e to some exponent is the *exponential* function `exp()`. Whatever value is inside the parentheses of this function is the exponent. Therefore, `exp(-1)` instructs the computer to calculate e^{-1}.

6.5 Modeling an energizing capacitor using C++

Here is an example C++ program intended to calculate voltage and current for an energizing capacitor:

```
#include <iostream>
#include <cmath>
using namespace std;

int main (void)
{
  float v1, r1, c1, i, vr1, vc1, tau, t, incr;

  t = 0.0;
  incr = 0.05;
  v1 = 15.00;
  r1 = 3.3e3;
  c1 = 22e-6;

  cout << "+---V1---R1---C1---+ " << endl;
  cout << "|                   | " << endl;
  cout << "+-------------------+ " << endl;

  cout << "V1 = " << v1 << " Volts" << endl;
  cout << "R1 = " << r1 << " Ohms" << endl;
  cout << "C1 = " << c1 << " Farads" << endl;

  tau = r1 * c1;

  // This "FOR" loop repeats until the capacitor's
  // voltage reaches 99% of the source voltage
  for (t = 0.00 ; vc1 < (0.99 * v1) ; t = t + incr)
  {
    vc1 = v1 * (1 - exp(-t / tau));
    vr1 = v1 - vc1;
    i = vr1 / r1;

    cout << endl;
    cout << "Time = " << t << " Seconds" << endl;
    cout << "   I = " << i << " Amperes" << endl;
    cout << " VR1 = " << vr1 << " Volts" << endl;
    cout << " VC1 = " << vc1 << " Volts" << endl;
  }
```

```
  return 0;
}
```

Note the use of a `for` "loop" in the code, commanding the computer to repeat a set of instructions so long as some condition is met. In this case, the condition is that the capacitor's voltage is less than 99% of the source voltage, as described within the text of the code by the *comment* lines (those two lines of text preceded by double forward-slash characters). Note also the use of the *exponential* function (`exp`) used to compute $e^{\frac{-t}{\tau}}$, which requires an additional "include" statement at the beginning of the code (`#include <cmath>`) which gives the compiler access to descriptions of mathematical functions more advanced than those defined within `iostream`.

When compiled and executed, this program generates the following output:

```
+---V1---R1---C1---+
|                  |
+------------------+
V1 = 15 Volts
R1 = 3300 Ohms
C1 = 2.2e-05 Farads

Time = 0 Seconds
   I = 0.00454545 Amperes
 VR1 = 15 Volts
 VC1 = 0 Volts

Time = 0.05 Seconds
   I = 0.00228284 Amperes
 VR1 = 7.53339 Volts
 VC1 = 7.46661 Volts

Time = 0.1 Seconds
   I = 0.0011465 Amperes
 VR1 = 3.78346 Volts
 VC1 = 11.2165 Volts

Time = 0.15 Seconds
   I = 0.000575804 Amperes
 VR1 = 1.90015 Volts
 VC1 = 13.0998 Volts

Time = 0.2 Seconds
   I = 0.000289184 Amperes
 VR1 = 0.954306 Volts
```

```
 VC1 = 14.0457 Volts

Time = 0.25 Seconds
   I = 0.000145236 Amperes
 VR1 = 0.479278 Volts
 VC1 = 14.5207 Volts

Time = 0.3 Seconds
   I = 7.29411e-05 Amperes
 VR1 = 0.240705 Volts
 VC1 = 14.7593 Volts

Time = 0.35 Seconds
   I = 3.66329e-05 Amperes
 VR1 = 0.120889 Volts
 VC1 = 14.8791 Volts
```

6.6 Solving RC and LR circuit differential equations using C++

A *differential equation* is a mathematical equation containing both a variable and at least one of its derivatives (i.e. its rate-of-change). Differential equations are extremely useful for describing a wide range of physical phenomena including the energization and de-energization of energy-storing components such as capacitors and inductors. While differential equations require calculus techniques to solve analytically, their solutions may be approximated quite closely using repeated calculations of simple arithmetic. Computers excel at doing such tasks, and so here we will view multiple programs written in C++ to solve for voltages and currents in resistor-capacitor and resistor-inductor networks.

Each of these programs outputs data in comma-separated variable (CSV) text format, suitable for plotting using mathematical visualization software such as `gnuplot` or spreadsheets such as Microsoft `Excel`.

6.6.1 Differential equation solver for an RC circuit and voltage source

```
#include <iostream>
#include <cmath>
using namespace std;

/* Schematic:
  +-----R-----+
  |           |
  Vs          C   Ic = C dVc/dt
  |           |
  +-----------+
*/

int main (void)
{
  double Ic, C = 33e-6, R = 10e3, Vr, Vs = 0.0, Vc = 10.0, dVc, t, dt = 0.0001;

  cout << "Time , Vc" << endl;

  for (t = 0 ; t < 2.0 ; t = t + dt)
  {
    Vr = Vs - Vc;         // Kirchhoff's Voltage Law
    Ic = Vr / R;          // Ohm's Law
    dVc = dt * (Ic / C);  // I = C dV/dt "Ohm's Law" for capacitors
    Vc = Vc + dVc;        // Integrating capacitor voltage from small changes
    cout << t << " , " << Vc << endl;
  }

  return 0;
}
```

With the initialized values shown in the `double` line of this program, it will compute capacitor voltages at 0.1 millisecond intervals assuming an initial capacitor voltage of 10 Volts and discharging toward zero volts (i.e. the voltage source value `Vs` is set to 0.0). Of course, any combination of `Vs` and initial `Vc` values will work perfectly well, as this program will show the gradual progression from initial to final voltage over time.

6.6.2 Differential equation solver for an RC circuit and current source

```
#include <iostream>
#include <cmath>
using namespace std;

/* Schematic:
  +-----+-----+
  |     |     |
  Is    R     C   I = C dVc/dt
  |     |     |
  +-----+-----+
*/

int main (void)
{
  double Ic, C = 33e-6, R = 10e3, Is = 1e-3, Ir, Vc = 0.0, dVc, t, dt = 0.0001;

  cout << "Time , Vc" << endl;

  for (t = 0 ; t < 2.0 ; t = t + dt)
  {
    Ir = Vc / R;          // Ohm's Law
    Ic = Is - Ir;         // Kirchhoff's Current Law
    dVc = dt * (Ic / C);  // I = C dV/dt "Ohm's Law" for capacitors
    Vc = Vc + dVc;        // Integrating capacitor voltage from small changes
    cout << t << " , " << Vc << endl;
  }

  return 0;
}
```

With the initialized values shown in the `double` line of this program, it will compute capacitor voltages at 0.1 millisecond intervals assuming an initial capacitor voltage of 0 Volts and charging toward 10 Volts (i.e. the source current of 1 milliAmpere passing entirely through the 10 kΩ resistor once the capacitor reaches full voltage). Of course, any combination of source current (`Is`) and initial capacitor voltage `Vc` values will work perfectly well, as this program will show the gradual progression from initial to final voltage over time.

6.6.3 Differential equation solver for an LR circuit and voltage source

```
#include <iostream>
#include <cmath>
using namespace std;

/* Schematic:
  +-----R-----+
  |           |
  Vs          L   Vl = L dIl/dt
  |           |
  +-----------+
*/

int main (void)
{
  double Vl, L = 50.0, R = 100.0, Vs = 100.0, Vr, Il = 0.0, dIl, t, dt = 0.0001;

  cout << "Time , Il" << endl;

  for (t = 0 ; t < 2.0 ; t = t + dt)
  {
    Vr = Il * R;          // Ohm's Law
    Vl = Vs - Vr;         // Kirchhoff's Voltage Law
    dIl = dt * (Vl / L);  // V = L dI/dt "Ohm's Law" for capacitors
    Il = Il + dIl;        // Integrating inductor current from small changes
    cout << t << " , " << Il << endl;
  }

  return 0;
}
```

With the initialized values shown in the `double` line of this program, it will compute inductor currents at 0.1 millisecond intervals assuming zero initial inductor current and charging toward 1 Ampere (i.e. the voltage source value `Vs` of 100 Volts divided by 100 Ohms). Of course, any combination of `Vs` and initial `Il` values will work perfectly well, as this program will show the gradual progression from initial to final current over time.

6.6.4 Differential equation solver for an LR circuit and current source

```
#include <iostream>
#include <cmath>
using namespace std;

/* Schematic:
  +-----+-----+
  |     |     |
  Is    R     L   Vl = L dIl/dt
  |     |     |
  +-----+-----+
*/

int main (void)
{
  double Vl, L = 50.0, R = 100.0, Is = 100e-3, Ir, Il = 0.0, dIl, t, dt = 0.0001;

  cout << "Time , Il" << endl;

  for (t = 0 ; t < 2.0 ; t = t + dt)
  {
    Ir = Is - Il;         // Kirchhoff's Current Law
    Vl = Ir * R;          // Ohm's Law
    dIl = dt * (Vl / L);  // V = L dI/dt "Ohm's Law" for capacitors
    Il = Il + dIl;        // Integrating inductor current from small changes
    cout << t << " , " << Il << endl;
  }

  return 0;
}
```

With the initialized values shown in the `double` line of this program, it will compute inductor currents at 0.1 millisecond intervals assuming zero initial inductor current and charging toward 100 milliAmperes. Of course, any combination of `Is` and initial `Il` values will work perfectly well, as this program will show the gradual progression from initial to final current over time.

Chapter 7

Questions

This learning module, along with all others in the ModEL collection, is designed to be used in an inverted instructional environment where students independently read[1] the tutorials and attempt to answer questions on their own *prior* to the instructor's interaction with them. In place of lecture[2], the instructor engages with students in Socratic-style dialogue, probing and challenging their understanding of the subject matter through inquiry.

Answers are not provided for questions within this chapter, and this is by design. Solved problems may be found in the Tutorial and Derivation chapters, instead. The goal here is *independence*, and this requires students to be challenged in ways where others cannot think for them. Remember that you always have the tools of *experimentation* and *computer simulation* (e.g. SPICE) to explore concepts!

The following lists contain ideas for Socratic-style questions and challenges. Upon inspection, one will notice a strong theme of *metacognition* within these statements: they are designed to foster a regular habit of examining one's own thoughts as a means toward clearer thinking. As such these sample questions are useful both for instructor-led discussions as well as for self-study.

[1] Technical reading is an essential academic skill for any technical practitioner to possess for the simple reason that the most comprehensive, accurate, and useful information to be found for developing technical competence is in textual form. Technical careers in general are characterized by the need for continuous learning to remain current with standards and technology, and therefore any technical practitioner who cannot read well is handicapped in their professional development. An excellent resource for educators on improving students' reading prowess through intentional effort and strategy is the book textitReading For Understanding – How Reading Apprenticeship Improves Disciplinary Learning in Secondary and College Classrooms by Ruth Schoenbach, Cynthia Greenleaf, and Lynn Murphy.

[2] Lecture is popular as a teaching method because it is easy to implement: any reasonably articulate subject matter expert can talk to students, even with little preparation. However, it is also quite problematic. A good lecture always makes complicated concepts seem easier than they are, which is bad for students because it instills a false sense of confidence in their own understanding; reading and re-articulation requires more cognitive effort and serves to verify comprehension. A culture of teaching-by-lecture fosters a debilitating dependence upon direct personal instruction, whereas the challenges of modern life demand independent and critical thought made possible only by gathering information and perspectives from afar. Information presented in a lecture is ephemeral, easily lost to failures of memory and dictation; text is forever, and may be referenced at any time.

General challenges following tutorial reading

- Summarize as much of the text as you can in one paragraph of your own words. A helpful strategy is to explain ideas as you would for an intelligent child: as simple as you can without compromising too much accuracy.
- Simplify a particular section of the text, for example a paragraph or even a single sentence, so as to capture the same fundamental idea in fewer words.
- Where did the text make the most sense to you? What was it about the text's presentation that made it clear?
- Identify where it might be easy for someone to misunderstand the text, and explain why you think it could be confusing.
- Identify any new concept(s) presented in the text, and explain in your own words.
- Identify any familiar concept(s) such as physical laws or principles applied or referenced in the text.
- Devise a proof of concept experiment demonstrating an important principle, physical law, or technical innovation represented in the text.
- Devise an experiment to disprove a plausible misconception.
- Did the text reveal any misconceptions you might have harbored? If so, describe the misconception(s) and the reason(s) why you now know them to be incorrect.
- Describe any useful problem-solving strategies applied in the text.
- Devise a question of your own to challenge a reader's comprehension of the text.

General follow-up challenges for assigned problems

- Identify where any fundamental laws or principles apply to the solution of this problem, especially before applying any mathematical techniques.
- Devise a thought experiment to explore the characteristics of the problem scenario, applying known laws and principles to mentally model its behavior.
- Describe in detail your own strategy for solving this problem. How did you identify and organized the given information? Did you sketch any diagrams to help frame the problem?
- Is there more than one way to solve this problem? Which method seems best to you?
- Show the work you did in solving this problem, even if the solution is incomplete or incorrect.
- What would you say was the most challenging part of this problem, and why was it so?
- Was any important information missing from the problem which you had to research or recall?
- Was there any extraneous information presented within this problem? If so, what was it and why did it not matter?
- Examine someone else's solution to identify where they applied fundamental laws or principles.
- Simplify the problem from its given form and show how to solve this simpler version of it. Examples include eliminating certain variables or conditions, altering values to simpler (usually whole) numbers, applying a limiting case (i.e. altering a variable to some extreme or ultimate value).
- For quantitative problems, identify the real-world meaning of all intermediate calculations: their units of measurement, where they fit into the scenario at hand. Annotate any diagrams or illustrations with these calculated values.
- For quantitative problems, try approaching it qualitatively instead, thinking in terms of "increase" and "decrease" rather than definite values.
- For qualitative problems, try approaching it quantitatively instead, proposing simple numerical values for the variables.
- Were there any assumptions you made while solving this problem? Would your solution change if one of those assumptions were altered?
- Identify where it would be easy for someone to go astray in attempting to solve this problem.
- Formulate your own problem based on what you learned solving this one.

General follow-up challenges for experiments or projects

- In what way(s) was this experiment or project easy to complete?
- Identify some of the challenges you faced in completing this experiment or project.

- Show how thorough documentation assisted in the completion of this experiment or project.
- Which fundamental laws or principles are key to this system's function?
- Identify any way(s) in which one might obtain false or otherwise misleading measurements from test equipment in this system.
- What will happen if (component X) fails (open/shorted/etc.)?
- What would have to occur to make this system unsafe?

7.1 Conceptual reasoning

These questions are designed to stimulate your analytic and synthetic thinking[3]. In a Socratic discussion with your instructor, the goal is for these questions to prompt an extended dialogue where assumptions are revealed, conclusions are tested, and understanding is sharpened. Your instructor may also pose additional questions based on those assigned, in order to further probe and refine your conceptual understanding.

Questions that follow are presented to challenge and probe your understanding of various concepts presented in the tutorial. These questions are intended to serve as a guide for the Socratic dialogue between yourself and the instructor. Your instructor's task is to ensure you have a sound grasp of these concepts, and the questions contained in this document are merely a means to this end. Your instructor may, at his or her discretion, alter or substitute questions for the benefit of tailoring the discussion to each student's needs. The only absolute requirement is that each student is challenged and assessed at a level equal to or greater than that represented by the documented questions.

It is far more important that you convey your *reasoning* than it is to simply convey a correct answer. For this reason, you should refrain from researching other information sources to answer questions. What matters here is that *you* are doing the thinking. If the answer is incorrect, your instructor will work with you to correct it through proper reasoning. A correct answer without an adequate explanation of how you derived that answer is unacceptable, as it does not aid the learning or assessment process.

You will note a conspicuous lack of answers given for these conceptual questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your conceptual answers, where applicable, is to use circuit simulation software to explore the effects of changes made to circuits. For example, if one of these conceptual questions challenges you to predict the effects of altering some component parameter in a circuit, you may check the validity of your work by simulating that same parameter change within software and seeing if the results agree.

[3] *Analytical* thinking involves the "disassembly" of an idea into its constituent parts, analogous to dissection. *Synthetic* thinking involves the "assembly" of a new idea comprised of multiple concepts, analogous to construction. Both activities are high-level cognitive skills, extremely important for effective problem-solving, necessitating frequent challenge and regular practice to fully develop.

7.1.1 Reading outline and reflections

"Reading maketh a full man; conference a ready man; and writing an exact man" – Francis Bacon

Francis Bacon's advice is a blueprint for effective education: reading provides the learner with knowledge, writing focuses the learner's thoughts, and critical dialogue equips the learner to confidently communicate and apply their learning. Independent acquisition and application of knowledge is a powerful skill, well worth the effort to cultivate. To this end, students should read these educational resources closely, journal their own reflections on the reading, and discuss in detail their findings with classmates and instructor(s). You should be able to do all of the following after reading any instructional text:

☑ Briefly SUMMARIZE THE TEXT in the form of a journal entry documenting your learning as you progress through the course of study. Share this summary in dialogue with your classmates and instructor. Journaling is an excellent self-test of thorough reading because you cannot clearly express what you have not read or did not comprehend.

☑ Demonstrate ACTIVE READING STRATEGIES, including verbalizing your impressions as you read, simplifying long passages to convey the same ideas using fewer words, annotating text and illustrations with your own interpretations, working through mathematical examples shown in the text, cross-referencing passages with relevant illustrations and/or other passages, identifying problem-solving strategies applied by the author, etc. Technical reading is a special case of problem-solving, and so these strategies work precisely because they help solve any problem: paying attention to your own thoughts (metacognition), eliminating unnecessary complexities, identifying what makes sense, paying close attention to details, drawing connections between separated facts, and noting the successful strategies of others.

☑ Identify IMPORTANT THEMES, especially GENERAL LAWS and PRINCIPLES, expounded in the text and express them in the simplest of terms as though you were teaching an intelligent child. This emphasizes connections between related topics and develops your ability to communicate complex ideas to anyone.

☑ Form YOUR OWN QUESTIONS based on the reading, and then pose them to your instructor and classmates for their consideration. Anticipate both correct and incorrect answers, the incorrect answer(s) assuming one or more plausible misconceptions. This helps you view the subject from different perspectives to grasp it more fully.

☑ Devise EXPERIMENTS to test claims presented in the reading, or to disprove misconceptions. Predict possible outcomes of these experiments, and evaluate their meanings: what result(s) would confirm, and what would constitute disproof? Running mental simulations and evaluating results is essential to scientific and diagnostic reasoning.

☑ Specifically identify any points you found CONFUSING. The reason for doing this is to help diagnose misconceptions and overcome barriers to learning.

7.1.2 Foundational concepts

Correct analysis and diagnosis of electric circuits begins with a proper understanding of some basic concepts. The following is a list of some important concepts referenced in this module's full tutorial. Define each of them in your own words, and be prepared to illustrate each of these concepts with a description of a practical example and/or a live demonstration.

Thought experiments as a problem-solving strategy

Electric field

Energy

Conservation of Energy

Capacitance

Dielectric

Permittivity

Capacitor

Electrical source

Electrical load

"Ohm's Law" for capacitance

Voltage stabilization

Ohm's Law

Kirchhoff's Voltage Law

Time constant

Dielectric strength

Polarized capacitor

Farad

Leakage resistance

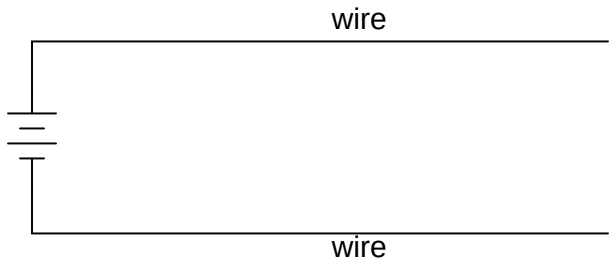
Equivalent series resistance (ESR)

Capacitor physical size

Series versus parallel capacitances

7.1.3 Electric flux lines

Suppose two wires, separated by an air gap, are connected to opposite terminals on a voltage source (such as a battery). An electric field will develop in the space between the two wires: an invisible web of interaction, similar in some ways to a magnetic field. In this diagram, draw the invisible "lines of flux" for this electric field, showing their physical range:



Challenges

- Explain how electric flux lines differ in geometry from magnetic flux lines.

7.1.4 Leyden jar

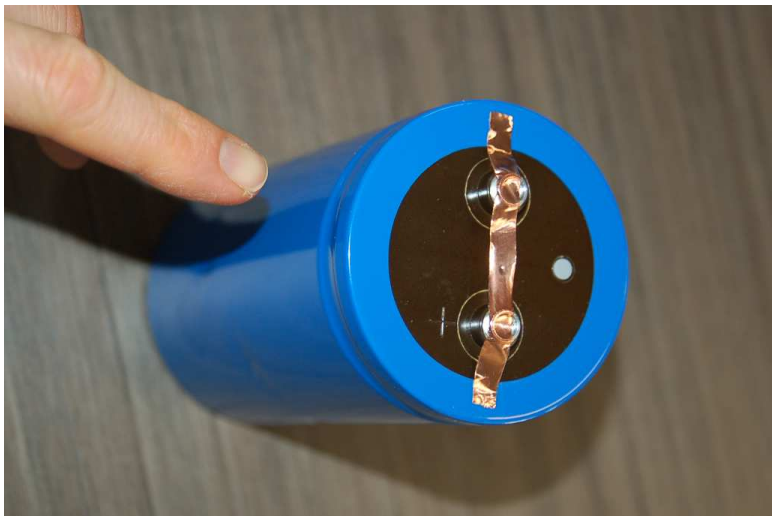
What is a *Leyden Jar*, and how it its construction similar to the construction of all *capacitors*?

Challenges

- Identify how you could construct your own Leyden jar.

7.1.5 Copper tape

What is the purpose of this copper tape, stretched between the terminals of a large electrolytic capacitor maintained in storage?

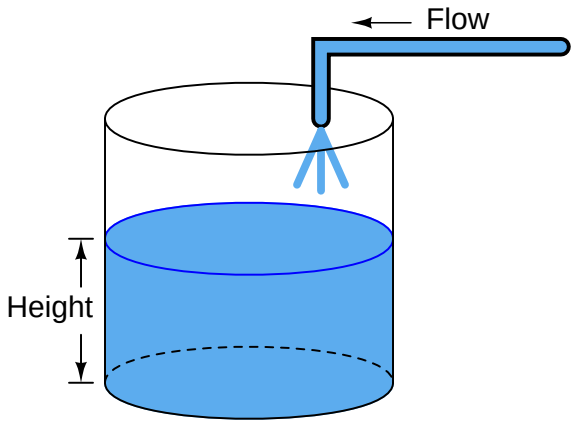


Challenges

- Explain what you should do prior to placing this copper tape on the capacitor, for your own safety.

7.1.6 Water tank analogy

If we introduce a constant flow of water into a cylindrical tank with water, the height of water inside that tank will rise at a constant rate over time:



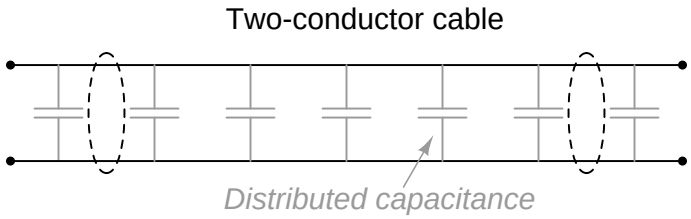
Like the water tank, electrical **capacitance** also exhibits the phenomenon of integration with respect to time. Which electrical quantity (voltage or current) dictates the rate-of-change over time of which other quantity (voltage or current) in a capacitance? Or, to re-phrase the question, which quantity (voltage or current), when maintained at a constant value, results in which other quantity (current or voltage) steadily ramping either up or down over time?

Challenges

- Can you think of a way we could exploit the similarity of capacitive voltage/current integration to *simulate* the behavior of a water tank's filling, or any other physical process described by the same mathematical relationship?

7.1.7 Cable capacitance

Capacitance exists between any two conductors separated by an insulating medium. Given this fact, it makes sense that a length of two-conductor electrical cable will have capacitance distributed naturally along its length:



Devise a means of proving the existence of this "stray" capacitance in any substantial length of two-conductor cable.

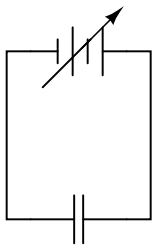
Challenges

- Devise at least two more methods to prove the existence of capacitance between the conductors of a long cable.

7.1.8 Capacitor as source versus load

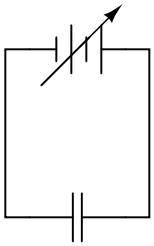
Suppose a capacitor is connected directly to an adjustable-voltage source, and the voltage of that source is steadily *increased* over time. We know that an increasing voltage across a capacitor will produce an electric field of increasing strength. Does this increase in electric field constitute an *accumulation* of energy in the capacitor, or a *release* of energy from the capacitor? In this scenario, does the capacitor act as a *load* or as a *source* of electrical energy?

Voltage *increasing*



Now, suppose the adjustable voltage source is steadily *decreased* over time. We know this will result in an electric field of decreasing strength in the capacitor. Does this decrease in electric field constitute an *accumulation* of energy in the capacitor, or a *release* of energy from the capacitor? In this scenario, does the capacitor act as a *load* or as a *source* of electrical energy?

Voltage *decreasing*



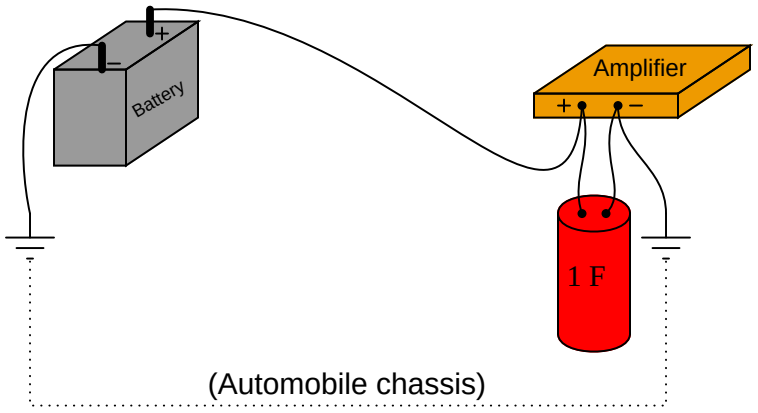
For each of these scenarios, label the direction of current in the circuit.

Challenges

- Identify factors influencing the current's *magnitude* in each circuit.

7.1.9 Audio power capacitors

Very large capacitors (typically in excess of 1 Farad!) are often used in the DC power wiring of high-power audio amplifier systems installed in automobiles. The capacitors are connected in parallel with the amplifier's DC power terminals, as close to the amplifier as possible, like this:



What is the purpose of having a capacitor connected in parallel with the amplifier's power terminals? What benefit does this give to the audio system, overall?

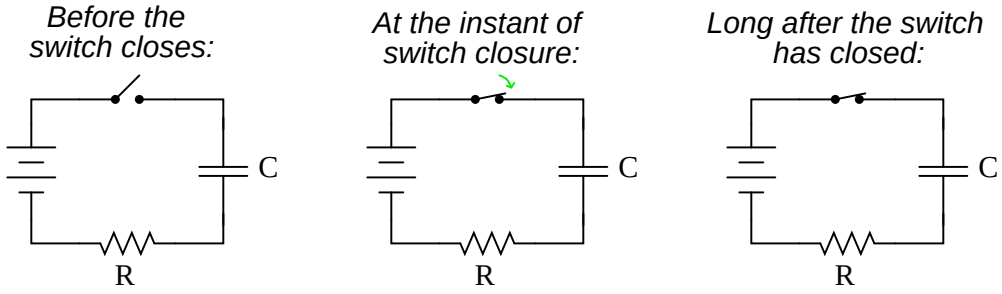
Challenges

- A similar application for capacitors is called *decoupling* in the DC power circuitry of digital electronic circuits. Research this topic and explain the similarity to the audio power application.

7.1.10 Capacitor charging circuit

A practical strategy for quantitatively analyzing capacitor circuits is to first *qualitatively* determine the trajectories of energy, voltage, and current values over time based on first principles (e.g. Ohm's Law, Kirchhoff's Laws, and the fact that capacitive energy storage is a function of capacitor voltage). Once you have done this, you will better know which equation(s) to use for quantitative analysis. A common mistake made in these types of problems is to plug numbers into equations without first reasoning *why* those equations are appropriate, so the point of this exercise is to build sound problem-solving habits!

Try qualitatively determining the following parameters in this simple resistor-capacitor circuit at three different times: (1) just before the switch closes, (2) at the instant the switch contacts touch, and (3) after the switch has been closed for a long time. Assume that the capacitor begins in a completely de-energized state:



Express your answers qualitatively: "maximum," "minimum," or perhaps "zero" if you know that to be the case.

Before the switch closes:

E_C (energy stored in capacitor) =
V_C =
V_R =
V_{switch} =
I =

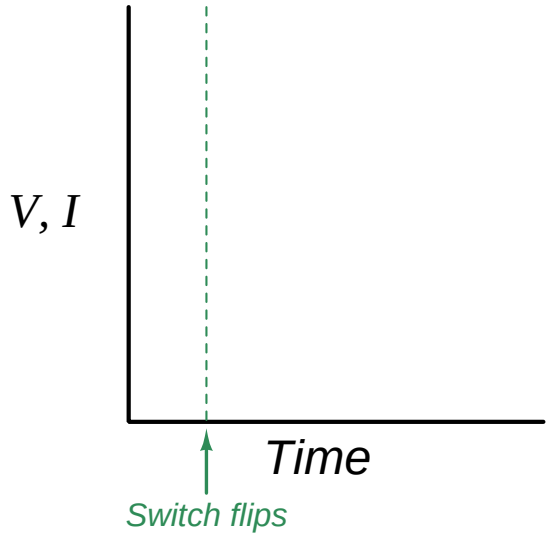
At the instant of switch closure:

E_C (energy stored in capacitor) =
V_C =
V_R =
V_{switch} =
I =

Long after the switch has closed:

E_C (energy stored in capacitor) =
V_C =
V_R =
V_{switch} =
I =

Also sketch a qualitative graph showing V_C and I in this circuit over time:



Note: you may find the following equations conceptually useful:

$$E_C = \frac{1}{2}CV^2 \qquad\qquad I = C\frac{dV}{dt}$$

Also, identify how the following foundational concepts apply to this circuit:

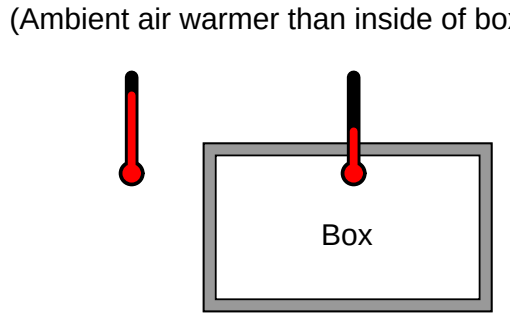
- How does the Conservation of Energy help explain the capacitor's voltage at the instant of switch closure?
- How does Kirchhoff's Voltage Law apply to the circuit before the switch closes?
- How does Kirchhoff's Voltage Law apply to the circuit after the switch closes?
- How does the "Ohm's Law" formula for capacitance apply to the circuit after the switch has been left in the closed state for a very long time?

Challenges

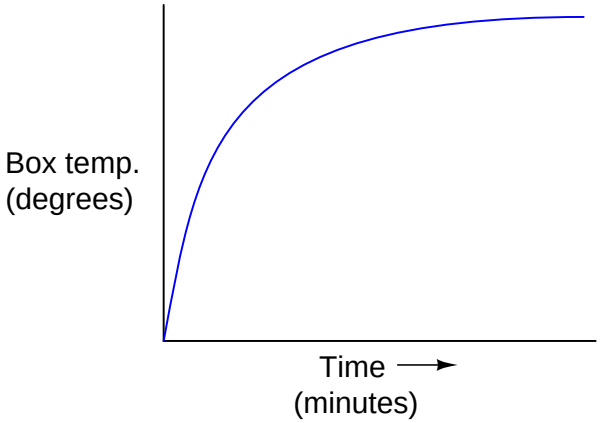
- Explain how the *Conservation of Energy* relates to changes in *voltage* and *current* for a capacitor.
- In this experiment, is the capacitor absorbing energy or releasing energy? How can we tell?
- Explain what will happen in the circuit when the switch is re-opened after the switch has been closed for a long period of time.
- Explain how drawing multiple versions of a circuit and annotating conditions before and after switch actuation is a helpful problem-solving strategy for any time-delay circuit.

7.1.11 Analog electrical model of warming box

Suppose we were measuring the interior temperature of an insulated box recently removed from a refrigerator, as it was being warmed by the ambient air around it:



Graphing the box's temperature over time, we see a curve that looks something like this:



An engineer approaches you and says she wants you to build an electrical circuit that *models* this thermal system, otherwise known as an electrical *analog* circuit. What kind of circuit would you consider building for the engineer, to make a realistic electrical analogue of the box's temperature? Be as specific as you can in your answer.

Challenges

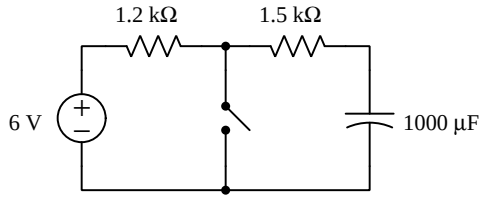
- Identify how your analog circuit could be adjusted to represent a greater amount of insulation for the box.
- Identify how your analog circuit could be adjusted to represent a greater amount of mass stored inside the box.
- Identify how your analog circuit could be adjusted to represent a greater room temperature.

7.1.12 Explaining the meaning of calculations

An unfortunate tendency among beginning students in any quantitative discipline is to perform calculations without regard for the real-world meanings of the values, and also to follow mathematical formulae without considering the general principles embodied in each. To ignore concepts while performing calculations is a serious error for a variety of reasons, not the least of which being an increased likelihood of computing results that turn out to be nonsense.

In the spirit of honoring concepts, I present to you a quantitative problem where all the calculations have been done for you, but all variable labels, units, and other identifying data have been stripped away. Your task is to *assign proper meaning* to each of the numbers, identify the correct unit of measurement in each case, apply any appropriate metric prefixes to those values, explain the significance of each value by describing where it "fits" into the circuit being analyzed, and identify the general principle employed at each step.

Here is the schematic diagram of the resistor-capacitor circuit:



Here are all the calculations performed in order from first to last. Conditions for these calculations are that the switch began in the open state and was left open for a long time, after which it was closed at time $t = 0$ seconds. The first several calculations work toward finding voltages and currents at $t = 2$ seconds after closure, the remaining calculations toward finding the time required for the capacitor voltage to decay down to 1 Volt:

1. $(1500) \times (1000 \times 10^{-6}) = 1.5$
2. $e^{\frac{-2}{1.5}} = 0.263597$
3. $6 \times 0.263597 = 1.58158$
4. $\frac{1.58158}{1500} = 1.05439 \times 10^{-3}$
5. $\frac{6}{1200} = 5.000 \times 10^{-3}$
6. $(1.05439 \times 10^{-3}) + (5.000 \times 10^{-3})$
7. $\frac{-1.05439 \times 10^{-3}}{1000 \times 10^{-6}} = -1.05439$
8. $\frac{1}{6} = 0.166667$
9. $\ln 0.166667 = -1.79176$
10. $(-1.79176) \times (-1.5) = 2.68764$

Explain what each value means in the circuit, identify its unit of measurement and appropriate metric prefix, and identify the general principle used to compute it!

Challenges

- Why is it conceptually proper that the mathematical sign of the calculated result for step 7 negative rather than positive?

7.2 Quantitative reasoning

These questions are designed to stimulate your computational thinking. In a Socratic discussion with your instructor, the goal is for these questions to reveal your mathematical approach(es) to problem-solving so that good technique and sound reasoning may be reinforced. Your instructor may also pose additional questions based on those assigned, in order to observe your problem-solving firsthand.

Mental arithmetic and estimations are strongly encouraged for all calculations, because without these abilities you will be unable to readily detect errors caused by calculator misuse (e.g. keystroke errors).

You will note a conspicuous lack of answers given for these quantitative questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. My advice is to use circuit simulation software such as SPICE to check the correctness of quantitative answers. Refer to those learning modules within this collection focusing on SPICE to see worked examples which you may use directly as practice problems for your own study, and/or as templates you may modify to run your own analyses and generate your own practice problems.

Completely worked example problems found in the Tutorial may also serve as "test cases[4]" for gaining proficiency in the use of circuit simulation software, and then once that proficiency is gained you will never need to rely[5] on an answer key!

[4] In other words, set up the circuit simulation software to analyze the same circuit examples found in the Tutorial. If the simulated results match the answers shown in the Tutorial, it confirms the simulation has properly run. If the simulated results disagree with the Tutorial's answers, something has been set up incorrectly in the simulation software. Using every Tutorial as practice in this way will quickly develop proficiency in the use of circuit simulation software.

[5] This approach is perfectly in keeping with the instructional philosophy of these learning modules: *teaching students to be self-sufficient thinkers*. Answer keys can be useful, but it is even more useful to your long-term success to have a set of tools on hand for checking your own work, because once you have left school and are on your own, there will no longer be "answer keys" available for the problems you will have to solve.

7.2.1 Miscellaneous physical constants

Note: constants shown in **bold** type are *exact*, not approximations. Values inside of parentheses show one standard deviation (σ) of uncertainty in the final digits: for example, the magnetic permeability of free space value given as $1.25663706212(19) \times 10^{-6}$ H/m represents a center value (i.e. the location parameter) of $1.25663706212 \times 10^{-6}$ Henrys per meter with one standard deviation of uncertainty equal to $0.0000000000019 \times 10^{-6}$ Henrys per meter.

Avogadro's number (N_A) = **6.02214076** $\times 10^{23}$ **per mole** (mol^{-1})

Boltzmann's constant (k) = **1.380649** $\times 10^{-23}$ **Joules per Kelvin** (J/K)

Electronic charge (e) = **1.602176634** $\times 10^{-19}$ **Coulomb** (C)

Faraday constant (F) = **96,485.33212...** $\times 10^{4}$ **Coulombs per mole** (C/mol)

Magnetic permeability of free space (μ_0) = $1.25663706212(19) \times 10^{-6}$ Henrys per meter (H/m)

Electric permittivity of free space (ϵ_0) = $8.8541878128(13) \times 10^{-12}$ Farads per meter (F/m)

Characteristic impedance of free space (Z_0) = 376.730313668(57) Ohms (Ω)

Gravitational constant (G) = $6.67430(15) \times 10^{-11}$ cubic meters per kilogram-seconds squared ($m^3/kg\text{-}s^2$)

Molar gas constant (R) = **8.314462618... Joules per mole-Kelvin** (J/mol-K) = 0.08205746(14) liters-atmospheres per mole-Kelvin

Planck constant (h) = **6.62607015** $\times 10^{-34}$ **joule-seconds** (J-s)

Stefan-Boltzmann constant (σ) = **5.670374419...** $\times 10^{-8}$ **Watts per square meter-Kelvin**^{4} ($W/m^2 \cdot K^4$)

Speed of light in a vacuum (c) = **299,792,458 meters per second** (m/s) = 186282.4 miles per second (mi/s)

Note: All constants taken from NIST data "Fundamental Physical Constants – Complete Listing", from `http://physics.nist.gov/constants`, National Institute of Standards and Technology (NIST), 2018 CODATA Adjustment.

7.2.2 Introduction to spreadsheets

A powerful computational tool you are encouraged to use in your work is a *spreadsheet*. Available on most personal computers (e.g. Microsoft Excel), *spreadsheet* software performs numerical calculations based on number values and formulae entered into cells of a grid. This grid is typically arranged as lettered columns and numbered rows, with each cell of the grid identified by its column/row coordinates (e.g. cell `B3`, cell `A8`). Each cell may contain a string of text, a number value, or a mathematical formula. The spreadsheet automatically updates the results of all mathematical formulae whenever the entered number values are changed. This means it is possible to set up a spreadsheet to perform a series of calculations on entered data, and those calculations will be re-done by the computer any time the data points are edited in any way.

For example, the following spreadsheet calculates average speed based on entered values of distance traveled and time elapsed:

	A	B	C	D
1	Distance traveled	46.9	Kilometers	
2	Time elapsed	1.18	Hours	
3	Average speed	= B1 / B2	km/h	
4				
5				

Text labels contained in cells `A1` through `A3` and cells `C1` through `C3` exist solely for readability and are not involved in any calculations. Cell `B1` contains a sample distance value while cell `B2` contains a sample time value. The formula for computing speed is contained in cell `B3`. Note how this formula begins with an "equals" symbol (`=`), references the values for distance and speed by lettered column and numbered row coordinates (`B1` and `B2`), and uses a forward slash symbol for division (`/`). The coordinates `B1` and `B2` function as *variables*[6] would in an algebraic formula.

When this spreadsheet is executed, the numerical value `39.74576` will appear in cell `B3` rather than the formula `= B1 / B2`, because 39.74576 is the computed speed value given 46.9 kilometers traveled over a period of 1.18 hours. If a different numerical value for distance is entered into cell `B1` or a different value for time is entered into cell `B2`, cell `B3`'s value will automatically update. All you need to do is set up the given values and any formulae into the spreadsheet, and the computer will do all the calculations for you.

Cell `B3` may be referenced by other formulae in the spreadsheet if desired, since it is a variable just like the given values contained in `B1` and `B2`. This means it is possible to set up an entire chain of calculations, one dependent on the result of another, in order to arrive at a final value. The arrangement of the given data and formulae need not follow any pattern on the grid, which means you may place them anywhere.

[6] Spreadsheets may also provide means to attach text labels to cells for use as variable names (Microsoft Excel simply calls these labels "names"), but for simple spreadsheets such as those shown here it's usually easier just to use the standard coordinate naming for each cell.

Common[7] arithmetic operations available for your use in a spreadsheet include the following:

- Addition (`+`)
- Subtraction (`-`)
- Multiplication (`*`)
- Division (`/`)
- Powers (`^`)
- Square roots (`sqrt()`)
- Logarithms (`ln()` , `log10()`)

Parentheses may be used to ensure[8] proper order of operations within a complex formula. Consider this example of a spreadsheet implementing the *quadratic formula*, used to solve for roots of a polynomial expression in the form of $ax^2 + bx + c$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

	A	B
1	x_1	= (-B4 + sqrt((B4^2) - (4*B3*B5))) / (2*B3)
2	x_2	= (-B4 - sqrt((B4^2) - (4*B3*B5))) / (2*B3)
3	a =	9
4	b =	5
5	c =	-2

This example is configured to compute roots[9] of the polynomial $9x^2 + 5x - 2$ because the values of 9, 5, and -2 have been inserted into cells `B3`, `B4`, and `B5`, respectively. Once this spreadsheet has been built, though, it may be used to calculate the roots of *any* second-degree polynomial expression simply by entering the new a, b, and c coefficients into cells `B3` through `B5`. The numerical values appearing in cells `B1` and `B2` will be automatically updated by the computer immediately following any changes made to the coefficients.

[7] Modern spreadsheet software offers a bewildering array of mathematical functions you may use in your computations. I recommend you consult the documentation for your particular spreadsheet for information on operations other than those listed here.

[8] Spreadsheet programs, like text-based programming languages, are designed to follow standard order of operations by default. However, my personal preference is to use parentheses even where strictly unnecessary just to make it clear to any other person viewing the formula what the intended order of operations is.

[9] Reviewing some algebra here, a *root* is a value for x that yields an overall value of zero for the polynomial. For this polynomial ($9x^2 + 5x - 2$) the two roots happen to be $x = 0.269381$ and $x = -0.82494$, with these values displayed in cells `B1` and `B2`, respectively upon execution of the spreadsheet.

Alternatively, one could break up the long quadratic formula into smaller pieces like this:

$$y = \sqrt{b^2 - 4ac} \qquad z = 2a$$

$$x = \frac{-b \pm y}{z}$$

	A	B	C
1	x_1	= (-B4 + C1) / C2	= sqrt((B4^2) - (4*B3*B5))
2	x_2	= (-B4 - C1) / C2	= 2*B3
3	a =	9	
4	b =	5	
5	c =	-2	

Note how the square-root term (y) is calculated in cell C1, and the denominator term (z) in cell C2. This makes the two final formulae (in cells B1 and B2) simpler to interpret. The positioning of all these cells on the grid is completely arbitrary[10] – all that matters is that they properly reference each other in the formulae.

Spreadsheets are particularly useful for situations where the same set of calculations representing a circuit or other system must be repeated for different initial conditions. The power of a spreadsheet is that it automates what would otherwise be a tedious set of calculations. One specific application of this is to simulate the effects of various components within a circuit failing with abnormal values (e.g. a shorted resistor simulated by making its value nearly zero; an open resistor simulated by making its value extremely large). Another application is analyzing the behavior of a circuit design given new components that are out of specification, and/or aging components experiencing drift over time.

[10] My personal preference is to locate all the "given" data in the upper-left cells of the spreadsheet grid (each data point flanked by a sensible name in the cell to the left and units of measurement in the cell to the right as illustrated in the first distance/time spreadsheet example), sometimes coloring them in order to clearly distinguish which cells contain entered data versus which cells contain computed results from formulae. I like to place all formulae in cells below the given data, and try to arrange them in logical order so that anyone examining my spreadsheet will be able to figure out *how* I constructed a solution. This is a general principle I believe all computer programmers should follow: *document and arrange your code to make it easy for other people to learn from it.*

7.2.3 Air capacitor gap

How far away from each other would two metal plates, 2 square meters in area each, have to be in order to create a capacitance of 1 μF? Assume that the plates are separated by air.

Challenges

- A common mistake is to calculate a value of 2 million meters (2×10^6 meters). Explain how someone might make this mistake.

7.2.4 Defibrillator capacitor

A *defibrillator* is a medical device used to deliver an electric shock to a patient suffering from *cardiac fibrillation*, which is an ineffectual heart rhythm. The amount of electrical energy necessary to shock a person's heart back to normal rhythm varies with body weight and a variety of other factors, but typically ranges from 70 to 360 Joules[11].

Suppose you are part of a team designing a new model of defibrillator, using a capacitor to store the energy delivered to the patient. Assuming the capacitor will reach a voltage of 1200 Volts when fully energized, how large of a capacitor will be necessary for this design?

Suppose the design of this new defibrillator varies the amount of delivered energy by controlling the voltage value of the energized capacitor. How many Volts would the capacitor need to be energized to in order to store the 90 Joules of energy?

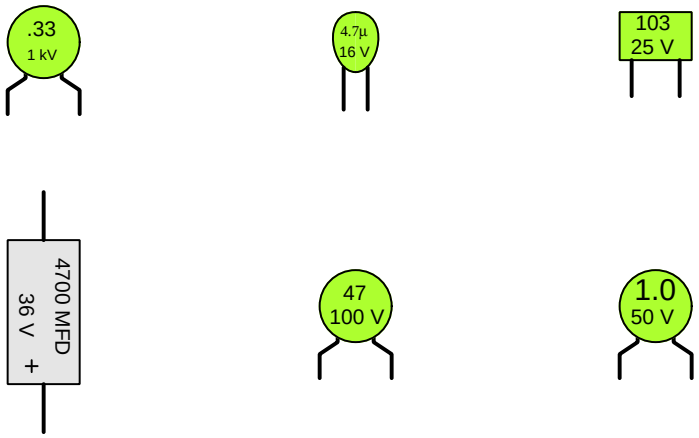
Challenges

- This illustration should be enough to convince you that energized capacitors can be very dangerous. Explain how you could test a capacitor for hazardous levels of energy before handling it with your bare hands.

[11] If defibrillation is done to a patient during chest surgery, where the shock may be administered directly to the heart muscle and not through the chest, this energy figure is significantly lower!

7.2.5 Capacitor labeling

Identify the ratings of the following capacitors:



Challenges

- Explain why the voltage rating of a capacitor is important. What do you suppose might happen if this rating is exceeded?

7.2.6 Half-life

When a radioactive substance decays, it does so in a way that is mathematically identical to that of a capacitor de-energizing as it powers a resistor: the quantity begins at a high level, falling at a fast rate, with the rate slowing down as the quantity approaches zero. The amount of time required for a radioactive sample to decay to a level where just one-half of its quantity remains is called the *half-life* of that substance. For example, 16 grams of a radioactive substance having a half-life of 3 minutes will decay to 8 grams after the first three minutes, 4 grams after the next three minutes, 2 grams after the next three minutes, and so on.

Demonstrate how the mathematical decay of voltage for a de-energizing capacitor may be measured in terms of *half-life*, instead of measuring that decay in terms of *time constant* (τ).

Challenges

- A simple way to demonstrate half-life is to flip a large number of coins, removing from the group any coin(s) that come up "heads" rather than "tails". Since the probability of either outcome is one-half (0.5), this means that the population of the coin group should decay by one-half with every flip. Perform an experiment to demonstrate this principle, and observe how closely the outcome matches theory.

7.2.7 Number of time constants

Determine the number of time constants (τ) that 7.5 seconds is equal to in each of the following resistor-capacitor circuits:

- R = 10 kΩ, C = 220 μF ; 7.5 sec =
- R = 33 kΩ, C = 470 μF ; 7.5 sec =
- R = 1.5 kΩ, C = 100 μF ; 7.5 sec =
- R = 790 Ω, C = 9240 nF ; 7.5 sec =
- R = 100 kΩ, C = 33 pF ; 7.5 sec =

Challenges

- For each of these RC networks, calculate the percentage of decay for voltage or current in a de-energizing condition following 7.5 seconds after disconnection of the source.

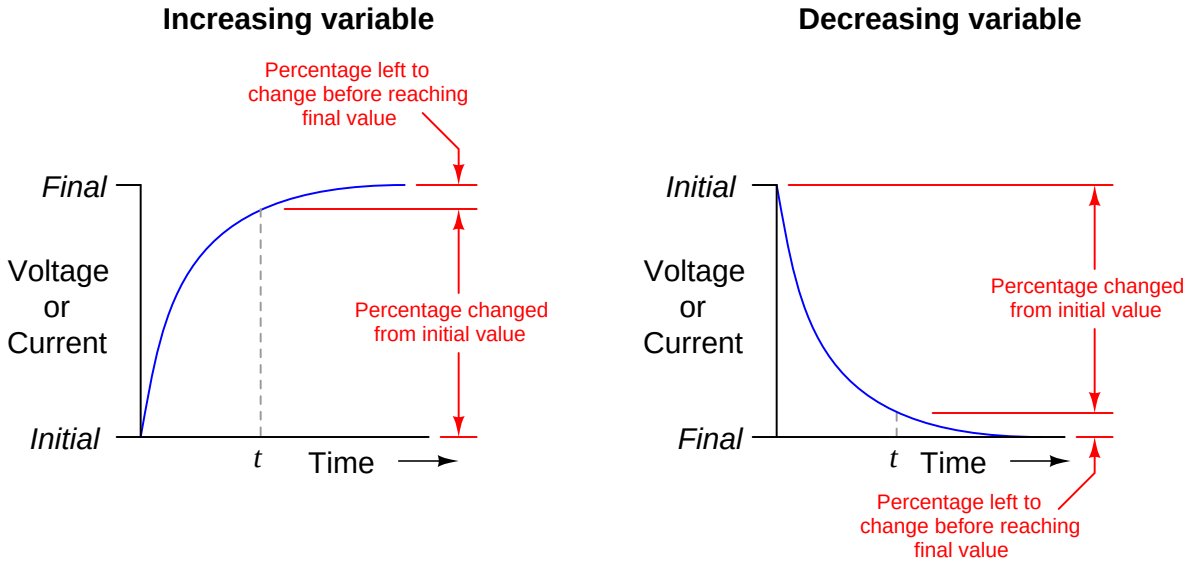
7.2.8 Inverse exponential functions

The following two expressions are frequently used to calculate values of changing variables (voltage and current) in RC and LR timing circuits:

$$e^{-\frac{t}{\tau}} \qquad \text{or} \qquad 1 - e^{-\frac{t}{\tau}}$$

One of these expressions describes the percentage that a changing value in an RC or LR circuit *has gone* from the starting time. The other expression describes how far that same variable *has left to go* before it reaches its ultimate value (at $t = \infty$).

The question is, which expression represents which quantity? This is often a point of confusion, because students have a tendency to try to correlate these expressions to the quantities by rote memorization. Does the expression $e^{-\frac{t}{\tau}}$ represent the amount a variable has changed, or how far it has left to go until it stabilizes? What about the other expression $1 - e^{-\frac{t}{\tau}}$? More importantly, *how can we figure this out so we don't have to rely on memory?*



Challenges

- A useful mathematical problem-solving technique is to set the independent variable in this case, t, to some simple value such as 0 or 1 to see what happens. What would setting $t = 0$ simulate in either of these functions?

7.2.9 Current versus rate-of-voltage-change

A 470 μF capacitor is subjected to an applied voltage that changes at a rate of 200 Volts per second. How much current will there be "through" this capacitor?

Now suppose two 470 μF capacitors are connected to form a series network, and the whole network subjected to the same rate-of-change of voltage. How much current would you expect now?

Now suppose two 470 μF capacitors are connected to form a parallel network, and the whole network subjected to the same rate-of-change of voltage. How much current would you expect now?

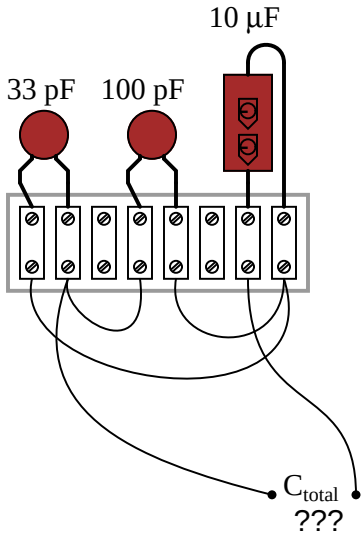
Also, explain why I placed quotation marks around the word *through* in the previous sentence. Why can't this word be used in its fullest sense when describing electric current in a capacitor circuit?

Challenges

- Based on the currents calculated for each capacitor network, prove that C adds in parallel and diminishes in series.

7.2.10 Capacitance of three-capacitor network

Calculate the total capacitance in this collection of capacitors, as measured between the two wires:



Challenges

- Suppose one of the terminal block's hold-down screws were to come loose on one of the leads for the middle capacitor, making a bad (open) connection. What effect would this have on the total capacitance?

7.2.11 Resistance for specified time constant

Calculate the resistor value which when connected in series with a 33 μF capacitor will provide a *time constant* (τ) of 10 seconds. Express your answer in the form of a five-band precision resistor color code (with a tolerance of +/- 0.1%).

Challenges

- What advantage does a five-band resistor color code enjoy over a four-band?

7.2.12 Safely de-energizing a large capacitor

An electronic service technician prepares to work on a high-voltage power supply circuit containing one large capacitor. On the side of this capacitor are the following specifications:

3000 WVDC $\quad 0.75\mu$F

Obviously this device poses a certain amount of danger, even with the AC line power secured (lock-out/tag-out). Discharging this capacitor by directly shorting its terminals with a screwdriver or some other piece of metal might be dangerous due to the quantity of the stored charge. What needs to be done is to discharge this capacitor at a modest rate.

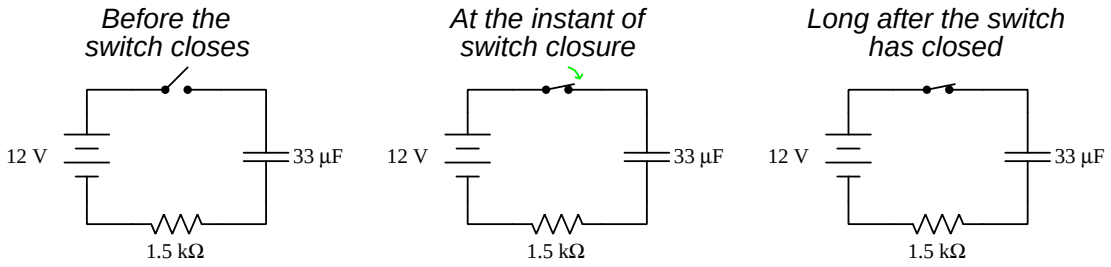
The technician realizes that she can discharge the capacitor at any rate desired by connecting a resistor in parallel with it (holding the resistor with electrically-insulated pliers, of course, to avoid having to make bodily contact with either capacitor terminal). What size resistor should she use, if she wants to discharge the capacitor to less than 1% charge in 15 seconds? State your answer using the standard 4-band resistor color code (tolerance = +/- 10%).

Challenges

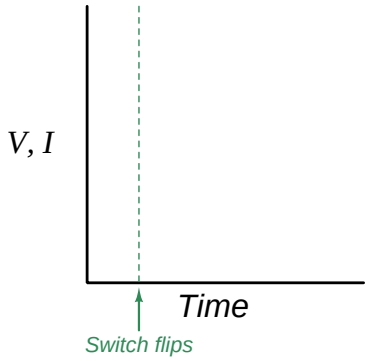
- Identify some hazards that might result from accidently choosing a resistor too small.
- Identify some hazards that might result from accidently choosing a resistor too large.

7.2.13 Capacitor charging scenario

Annotate all voltages and currents in the following circuit under the three conditions shown: before the switch closes, at the exact instant of switch closure, and after a long time with the switch closed. Explain your reasoning for each annotated quantity, based on trusted principles of electric circuits such as the guaranteed effects of opens versus shorts, series network properties, Ohm's Law, Kirchhoff's Voltage Law, and properties of capacitance (e.g. $E_C = \frac{1}{2}CV^2$, $I_C = C\frac{dV}{dt}$):



Next, sketch a qualitative graph showing V_C and I in this circuit over time:



Also, identify how the following foundational concepts apply to this circuit:

- How does the Conservation of Energy help explain the capacitor's voltage at the instant of switch closure?
- How does Kirchhoff's Voltage Law apply to the circuit before the switch closes?
- How does Kirchhoff's Voltage Law apply to the circuit after the switch closes?
- How does the "Ohm's Law" formula for capacitance apply to the circuit after the switch has been left in the closed state for a very long time?

Challenges

- Explain how the *Conservation of Energy* relates to changes in *voltage* and *current* for a capacitor.
- In this experiment, is the capacitor absorbing energy or releasing energy? How can we tell?
- Explain what will happen in the circuit when the switch is re-opened after the switch has been closed for a long period of time.
- Explain how drawing multiple versions of a circuit and annotating conditions before and after switch actuation is a helpful problem-solving strategy for any time-delay circuit.

7.2.14 Capacitor voltage at specified intervals

Suppose a capacitor is charged to a voltage of exactly 100 Volts, then connected to a resistor so it discharges slowly. Calculate the amount of voltage remaining across the capacitor terminals at the following points in time:

- 1 time constant (τ) after connecting the resistor:
- 2 time constants (2τ) after connecting the resistor:
- 3 time constants (3τ) after connecting the resistor:
- 4 time constants (4τ) after connecting the resistor:
- 5 time constants (5τ) after connecting the resistor:

Challenges

- Write an equation solving for these voltages at the specified times.

7.2.15 Partial capacitor de-energization

Calculate the voltage across a 470 μF capacitor after discharging through a 10 kΩ resistor for 9 seconds, if the capacitor's original voltage (at $t = 0$) was 24 Volts.

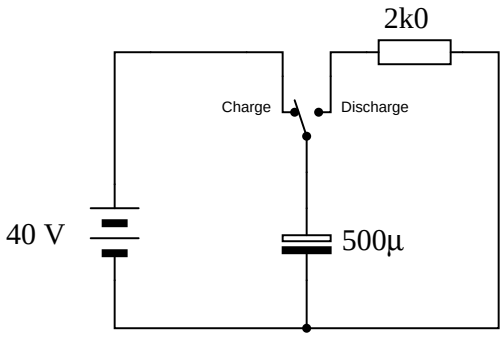
Also, express this amount of time (9 seconds) in terms of how many *time constants* have elapsed.

Challenges

- If the resistance was 15 kΩ instead of 10 kΩ, how would the voltage after 9 seconds be affected?

7.2.16 Rapid charge and slow discharge

The following circuit allows a capacitor to be rapidly charged and slowly discharged:



Suppose that the switch was left in the "charge" position for some substantial amount of time. Then, someone moves the switch to the "discharge" position to let the capacitor discharge. Calculate the amount of capacitor voltage and capacitor current at exactly 3 seconds after moving the switch to the "discharge" position.

$V_C =$ _____ @ $t = 3$ seconds

$I_C =$ _____ @ $t = 3$ seconds

Also, show the direction of discharge current in this circuit.

Challenges

- How would the voltage and current values be affected if this circuit had *two* 500 μF capacitors in parallel with each other instead of just the one in the circuit?
- How would the voltage and current values be affected if this circuit had *two* 500 μF capacitors in series with each other instead of just the one in the circuit?

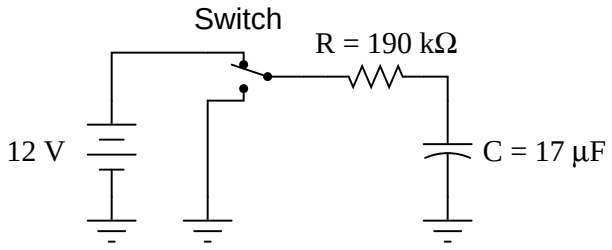
7.2.17 Time required to reach specified voltages

Calculate the amount of time it takes for a 33 μF capacitor to charge from 0 Volts to 20 Volts, if powered by a 24 Volt battery through a 10 kΩ resistor.

Calculate the amount of time it takes for a 10 μF capacitor to discharge from 18 Volts to 7 Volts if its ultimate (final) voltage when fully discharged will be 0 Volts, and it is discharging through a 22 kΩ resistor.

A 470 μF capacitor begins in a charged state of 270 Volts, and discharges through a 100 kΩ resistor. How long will it take before the capacitor's voltage will fall to a relatively safe value (30 Volts or less)?

Determine the amount of time needed for the capacitor voltage (V_C) to fall to the specified levels after the switch is thrown to the "discharge" position, assuming it had first been charged to full battery voltage:



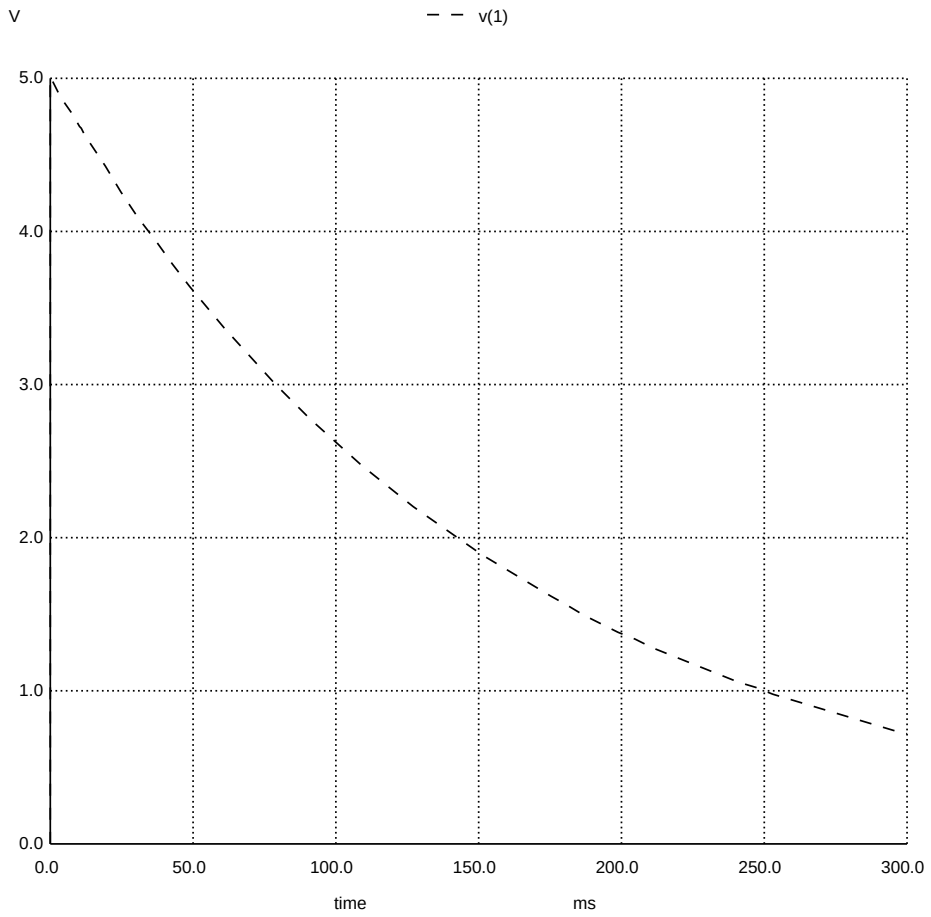
- 10 Volts =
- 8 Volts =
- 6 Volts =
- 4 Volts =
- 2 Volts =

Challenges

- What will happen to all of these time values if the resistor in each circuit is increased in value?

7.2.18 Measuring capacitance by time delay

A capacitor of unknown value is connected to a precision 5 Volt DC power supply and then allowed to discharge through a 3.3 kΩ resistor. The following image shows the decay of voltage over time:



Calculate the approximate size of the capacitor in microFarads based on the information you see here.

Challenges

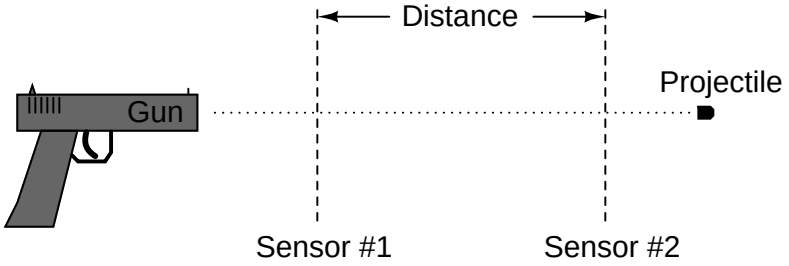
- A common tendency among students new to electric circuit analysis is to first "think mathematically" rather than first "think conceptually". Before plugging any numbers into any formulae, one should always identify electrical principles such as sources and loads, directions of current, and voltage polarities, usually sketching a diagram of a circuit to see where all these

concepts fit. Demonstrate how to "think conceptually" when solving this problem, rather than immediately trying to compute values.

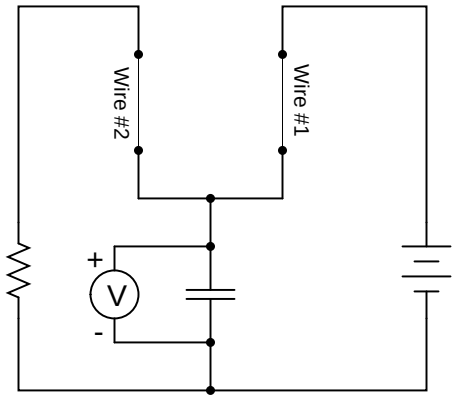
- How would the plot of voltage over time change if the resistor were made smaller?
- What happened to the energy stored in the capacitor when it was energized to a potential of 5 Volts?

7.2.19 Intervalometer

An *intervalometer* is a device that measures the interval of time between two events. Such devices are commonly used to measure the speed of projectiles, given a known distance between two sensors:



A crude intervalometer may be constructed using two thin wires as sensors, which are broken by the passage of the projectile. The two wires are connected in an RC circuit as such:



In order for this circuit to function properly as an intervalometer, which wire does the projectile need to break first? Explain why. Also, the voltmeter used in this instrument must be one with as high an input resistance as possible for best accuracy. Explain why this is necessary as well.

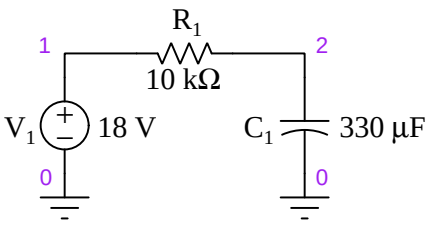
Which will produce a greater voltage indication after the test, a fast projectile or a slow projectile? Explain your answer.

Challenges

- The voltmeter itself introduces a source of error in this intervalometer. Identify the error, and determine if this error makes the intervalometer register falsely high or falsely low with regard to projectile speed.

7.2.20 SPICE analysis of an energizing capacitor

The following resistor-capacitor circuit will be simulated by a computer program called SPICE. The text-based "netlist" used to describe the circuit to SPICE is shown immediately to the right of the schematic diagram. Numbered points on the diagram identify "nodes" used by SPICE to declare how the components connect to one another:




```
* SPICE circuit
v1 1 0 dc 18
r1 1 2 10e3
c1 2 0 330e-6 ic=0
.tran 1 15 uic
.width out=80
.plot tran v(2)
.end
```

In this analysis, SPICE will plot capacitor voltage (between node 2 and ground) over a timespan of 15 seconds in increments of 1 second each. The capacitor begins (i.e. "initial condition" `ic=`) with zero voltage across it, which means no stored energy. When run, the simulation results are as follows:

```
Legend:  + = v(2)
--------------------------------------------------------------------------------
 time       v(2)     0.00e+00                 1.00e+01                 2.00e+01
----------------------|------------------------|------------------------|
 0.000e+00  0.000e+00 +                        .                         .
 1.000e+00  4.708e+00 .          +             .                         .
 2.000e+00  8.178e+00 .                    +   .                         .
 3.000e+00  1.075e+01 .                        .+                        .
 4.000e+00  1.264e+01 .                        .     +                   .
 5.000e+00  1.405e+01 .                        .         +               .
 6.000e+00  1.508e+01 .                        .           +             .
 7.000e+00  1.584e+01 .                        .             +           .
 8.000e+00  1.641e+01 .                        .               +         .
 9.000e+00  1.682e+01 .                        .                +        .
 1.000e+01  1.713e+01 .                        .                +        .
 1.100e+01  1.736e+01 .                        .                 +       .
 1.200e+01  1.753e+01 .                        .                 +       .
 1.300e+01  1.765e+01 .                        .                  +      .
 1.400e+01  1.774e+01 .                        .                  +      .
 1.500e+01  1.781e+01 .                        .                  +      .
----------------------|------------------------|------------------------|
 time       v(2)     0.00e+00                 1.00e+01                 2.00e+01
```

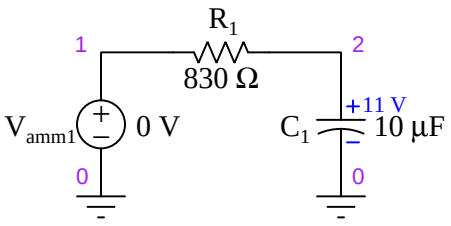
Verify any of the simulated capacitor voltage values using your own manual calculations.

Challenges

- Explain how you may use a SPICE simulation such as this for your own practice, instead of relying on questions and answers provided to you in a textbook.
- Explain how you could use this simulation to practice calculating capacitor or resistor values from the simulated results. In other words, imagine this was real data from a real capacitor-resistor circuit experiment, and you needed to compute either the resistor value or the capacitor value from all the other data.

7.2.21 SPICE analysis of a de-energizing capacitor

The following resistor-capacitor circuit will be simulated by a computer program called SPICE. The text-based "netlist" used to describe the circuit to SPICE is shown immediately to the right of the schematic diagram. Numbered points on the diagram identify "nodes" used by SPICE to declare how the components connect to one another:




```
* SPICE circuit
vamm1 1 0
r1 1 2 830
c1 2 0 10e-6 ic=11
.tran 1.5e-3 21e-3 uic
.width out=80
.plot tran i(vamm1)
.end
```

In this analysis, SPICE will plot circuit current (measured using the "dummy" voltage source V_{amm1}) over a timespan of 21 milliseconds in increments of 1.5 milliseconds each. The capacitor begins (i.e. "initial condition" `ic=`) with 11 Volts, with no other energy source in the circuit. When run, the simulation results are as follows:

```
Legend:  + = vamm1#branch
--------------------------------------------------------------------------------
 time       vamm1#br 0.00e+00                 1.00e-02                 2.00e-02
----------------------|-------------------------|-------------------------|
 0.000e+00  0.000e+00 +                          .                          .
 1.500e-03  1.106e-02 .                          . +                        .
 3.000e-03  9.233e-03 .                        + .                          .
 4.500e-03  7.708e-03 .                    +     .                          .
 6.000e-03  6.433e-03 .                +         .                          .
 7.500e-03  5.369e-03 .             +            .                          .
 9.000e-03  4.482e-03 .          +               .                          .
 1.050e-02  3.740e-03 .        +                 .                          .
 1.200e-02  3.122e-03 .      +                   .                          .
 1.350e-02  2.605e-03 .     +                    .                          .
 1.500e-02  2.175e-03 .    +                     .                          .
 1.650e-02  1.815e-03 .   +                      .                          .
 1.800e-02  1.515e-03 .  +                       .                          .
 1.950e-02  1.264e-03 .  +                       .                          .
 2.100e-02  1.055e-03 . +                        .                          .
----------------------|-------------------------|-------------------------|
 time       vamm1#br 0.00e+00                 1.00e-02                 2.00e-02
```

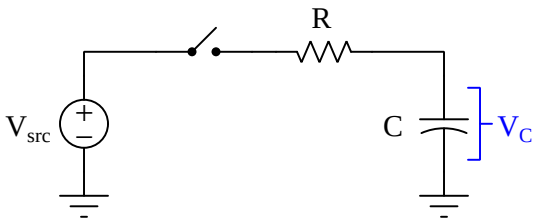
Verify any of the simulated current values using your own manual calculations.

Challenges

- Explain how you may use a SPICE simulation such as this for your own practice, instead of relying on questions and answers provided to you in a textbook.
- Explain how you could use this simulation to practice calculating capacitor or resistor values from the simulated results. In other words, imagine this was real data from a real capacitor-resistor circuit experiment, and you needed to compute either the resistor value or the capacitor value from all the other data.

7.2.22 Capacitor voltage simulation program

When a capacitor is connected to a constant-voltage source through a series resistance, its voltage will gradually reach equilibrium with the constant-voltage source over time. The rate at which the capacitor's voltage approaches the constant voltage value depends on the resistor's resistance as well as the capacitor's capacitance:



As soon as the switch closes (at time = 0) the resistor's voltage drop will become equal to the difference between V_{src} and V_C. This potential difference results in a current (by Ohm's Law, $I = \frac{V}{R}$), and this current in turn determines how rapidly the capacitor's voltage will rise or fall over time ($I = C\frac{dV_c}{dt}$). Setting all this up in an equation:

$$\frac{V_{src} - V_C}{R} = C\frac{dV_C}{dt}$$

If we manipulate this equation to solve for the change in capacitor voltage (dV_C) for each interval of time (dt), we get this:

$$dV_C = \left(\frac{V_{src} - V_C}{RC}\right) dt$$

This type of equation, where a variable's value helps determine how rapidly that variable's value changes, is known as a *differential equation*. In this particular case, the voltage for the capacitor (V_C) plays a role in determining how rapidly it will vary over time ($\frac{dV_C}{dt}$).

What this particular differential equation tells us is that over short intervals of time (dt), the capacitor's voltage will either increase or decrease by an amount equal to $\frac{V_{src}-V_C}{RC}$ over each time interval.

Differential equations are useful for describing many phenomena in the natural world, and techniques for solving them can become quite complex[12]. When electronics students begin their studies of RC networks without the benefit of calculus, they use inverse-exponential equations that are actually solutions to this differential equation. Having inverse-exponential equations ready to use on circuits like this saves us from having to know and apply calculus to solve the differential equation(s). However, if we know how to program a computer to perform a great many repeated calculations, it becomes possible to evaluate the differential equation in its raw form without having to know and apply calculus.

[12] Entire university-level courses exist to teach students how to solve differential equations!

The following computer program written in the C language simulates the capacitor's voltage over a one-second span of time, assuming an initial capacitor voltage of 0 Volts, a source voltage value of 10 Volts, a 10 kΩ resistor, a 22 μF capacitor, and a computation time interval of 0.1 milliseconds:

```
#include <stdio.h>

int main (void)
{
  float t, dVc;
  float Vc = 0.0;
  float Vsrc = 10.0;
  float R = 10e3;
  float C = 22e-6;
  float dt = 0.1e-3;

  printf("Time (sec) , Capacitor voltage (Volts)\n");

  for (t = 0.0 ; t < 1.0 ; t = t + dt)
  {
    printf("%f , %f\n", t, Vc);

    dVc = (Vsrc - Vc) / (R * C) * dt;
    Vc = Vc + dVc;
  }

  return 0;
}
```

Answer the following questions about this simulation program:

- How could this simulation be modified for a longer span of time, perhaps 2 seconds instead of 1 second?
- What purpose is served by the calculation `Vc = Vc + dVc;`?
- Based on your understanding of RC circuits, how much voltage should the capacitor accumulate after the switch has been closed for exactly one time-constant's worth of time?
- Is the value of `dt` significant to the simulation? Why or why not?
- Will this program properly simulate capacitor voltage over time if the capacitor begins at a non-zero voltage value when the switch closes?

Challenges

- What is the justification for the `printf` occurring *before* the calculation of `dVc` and of `Vc` within the `for()` loop, rather than *after*?

7.3 Diagnostic reasoning

These questions are designed to stimulate your deductive and inductive thinking, where you must apply general principles to specific scenarios (deductive) and also derive conclusions about the failed circuit from specific details (inductive). In a Socratic discussion with your instructor, the goal is for these questions to reinforce your recall and use of general circuit principles and also challenge your ability to integrate multiple symptoms into a sensible explanation of what's wrong in a circuit. Your instructor may also pose additional questions based on those assigned, in order to further challenge and sharpen your diagnostic abilities.

As always, your goal is to fully *explain* your analysis of each problem. Simply obtaining a correct answer is not good enough – you must also demonstrate sound reasoning in order to successfully complete the assignment. Your instructor's responsibility is to probe and challenge your understanding of the relevant principles and analytical processes in order to ensure you have a strong foundation upon which to build further understanding.

You will note a conspicuous lack of answers given for these diagnostic questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your diagnostic answers, where applicable, is to use circuit simulation software to explore the effects of faults placed in circuits. For example, if one of these diagnostic questions requires that you predict the effect of an open or a short in a circuit, you may check the validity of your work by simulating that same fault (substituting a very high resistance in place of that component for an open, and substituting a very low resistance for a short) within software and seeing if the results agree.

7.3.1 Dielectric breakdown

A common mode of capacitor failure is *dielectric breakdown*, where the insulating medium (i.e. the "dielectric") material between a capacitor's plates becomes compromised and is no longer an effective insulator.

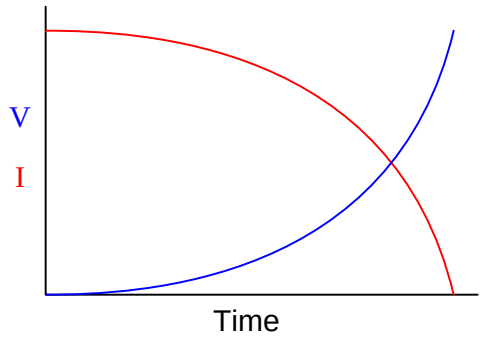
Devise a simple method to test a capacitor for this fault.

Challenges

- Which do you think is more likely to cause dielectric breakdown in a capacitor, excessive voltage or excessive current?

7.3.2 Incorrect voltage/current curves

Explain what is wrong with these voltage and current plots for a capacitor energized by a constant-voltage source through a series resistance:



Furthermore, try to explain *why* someone might make this mistake when predicting capacitor voltage and current.

Challenges

- Devise a "thought experiment" whereby this error would become obviously absurd (i.e. *reductio ad absurdum*).

Appendix A

Problem-Solving Strategies

The ability to solve complex problems is arguably one of the most valuable skills one can possess, and this skill is particularly important in any science-based discipline.

- Study principles, not procedures. Don't be satisfied with merely knowing how to compute solutions – learn *why* those solutions work.
- Identify what it is you need to solve, identify all relevant data, identify all units of measurement, identify any general principles or formulae linking the given information to the solution, and then identify any "missing pieces" to a solution. Annotate all diagrams with this data.
- Sketch a diagram to help visualize the problem. When building a real system, always devise a plan for that system and analyze its function *before* constructing it.
- Follow the units of measurement and meaning of every calculation. If you are ever performing mathematical calculations as part of a problem-solving procedure, and you find yourself unable to apply each and every intermediate result to some aspect of the problem, it means you don't understand what you are doing. Properly done, every mathematical result should have practical meaning for the problem, and not just be an abstract number. You should be able to identify the proper units of measurement for each and every calculated result, and show where that result fits into the problem.
- Perform "thought experiments" to explore the effects of different conditions for theoretical problems. When troubleshooting real systems, perform *diagnostic tests* rather than visually inspecting for faults, the best diagnostic test being the one giving you the most information about the nature and/or location of the fault with the fewest steps.
- Simplify the problem until the solution becomes obvious, and then use that obvious case as a model to follow in solving the more complex version of the problem.
- Check for exceptions to see if your solution is incorrect or incomplete. A good solution will work for *all* known conditions and criteria. A good example of this is the process of testing scientific hypotheses: the task of a scientist is not to find support for a new idea, but rather to *challenge* that new idea to see if it holds up under a battery of tests. The philosophical

principle of *reductio ad absurdum* (i.e. disproving a general idea by finding a specific case where it fails) is useful here.

- Work "backward" from a hypothetical solution to a new set of given conditions.
- Add quantities to problems that are qualitative in nature, because sometimes a little math helps illuminate the scenario.
- Sketch graphs illustrating how variables relate to each other. These may be quantitative (i.e. with realistic number values) or qualitative (i.e. simply showing increases and decreases).
- Treat quantitative problems as qualitative in order to discern the relative magnitudes and/or directions of change of the relevant variables. For example, try determining what happens if a certain variable were to increase or decrease before attempting to precisely calculate quantities: how will each of the dependent variables respond, by increasing, decreasing, or remaining the same as before?
- Consider limiting cases. This works especially well for qualitative problems where you need to determine which direction a variable will change. Take the given condition and magnify that condition to an extreme degree as a way of simplifying the direction of the system's response.
- Check your work. This means regularly testing your conclusions to see if they make sense. This does *not* mean repeating the same steps originally used to obtain the conclusion(s), but rather to use some other means to check validity. Simply repeating procedures often leads to *repeating the same errors* if any were made, which is why alternative paths are better.

Appendix B

Instructional philosophy

B.1 First principles of learning

- **Anyone can learn anything** given appropriate time, effort, resources, challenges, encouragement, and expectations. Dedicating time and investing effort are the student's responsibility; providing resources, challenges, and encouragement are the teacher's responsibility; high expectations are a responsibility shared by both student and teacher.

- **Transfer is not automatic.** The human mind has a natural tendency to compartmentalize information, which means the process of taking knowledge learned in one context and applying it to another usually does not come easy and therefore should never be taken for granted.

- **Learning is iterative.** The human mind rarely learns anything perfectly on the first attempt. Anticipate mistakes and plan for multiple tries to achieve full understanding, using the lessons of those mistakes as feedback to guide future attempts.

- **Information is absorbed, but understanding is created.** Facts and procedures may be memorized easily enough by repeated exposure, but the ability to reliably apply principles to novel scenarios only comes through intense personal effort. This effort is fundamentally creative in nature: casting new concepts in one's own words, running mental experiments to perceive patterns and trends, etc. These acts of making knowledge "one's own" need not be perfect in order to be effective, as the value lies in the activity and not necessarily the finished product. Depth of understanding and permanence of memory are both directly related to mental effort during the learning process – i.e. *easy learning is shallow and fleeting.*

- **Education trumps training.** There is no such thing as an entirely isolated subject, as all fields of knowledge are connected. Training is narrowly-focused and task-oriented. Education is broad-based and principle-oriented. When preparing for a life-long technical career, education beats training every time.

- **Character matters.** Poor habits are more destructive than deficits of knowledge or skill. This is especially true in collective endeavors, where a team's ability to function depends on trust between its members. Simply put, no one wants an untrustworthy person on their team. An essential component of education then, is character development.

- **People learn to be responsible by bearing responsibility.** An irresponsible person is someone who has never *had* to be responsible for anything that mattered enough to them. Just as anyone can learn anything, anyone can become responsible if the personal cost of irresponsibility becomes high enough.

- **What gets measured, gets done.** Accurate and relevant assessment of learning is key to ensuring all students learn. Therefore, it is imperative to measure what matters.

- **Failure is nothing to fear.** Every human being fails, and fails in multiple ways at multiple times. Eventual success only happens when we don't stop trying.

B.2 Proven strategies for instructors

- Assume every student is capable of learning anything they desire given the proper conditions. Treat them as capable adults by granting real responsibility and avoiding artificial incentives such as merit or demerit points.
- Create a consistent culture of high expectations across the entire program of study. Demonstrate and encourage patience, persistence, and a healthy sense of self-skepticism. Anticipate and de-stigmatize error. Teach respect for the capabilities of others as well as respect for one's own fallibility.
- Replace lecture with "inverted" instruction, where students first encounter new concepts through reading and then spend class time in Socratic dialogue with the instructor exploring those concepts and solving problems individually. There is a world of difference between observing someone solve a problem versus actually solving a problem yourself, and so the point of this form of instruction is to place students in a position where they *cannot* passively observe.
- Require students to read extensively, write about what they learn, and dialogue with you and their peers to sharpen their understanding. Apply Francis Bacon's advice that "reading maketh a full man; conference a ready man; and writing an exact man". These are complementary activities helping students expand their confidence and abilities.
- Use artificial intelligence (AI) to challenge student understanding rather than merely provide information. Find productive ways for AI to critique students' clarity of thought and of expression, for example by employing AI as a Socratic-style interlocutor or as a reviewer of students' journals. Properly applied, AI has the ability to expand student access to critical review well outside the bounds of their instructor's reach.
- Build frequent and rapid feedback into the learning process so that students know at all times how well they are learning, to identify problems early and fix them before they grow. Model the intellectual habit of self-assessing and self-correcting your own understanding (i.e. a cognitive *feedback loop*), encouraging students to do the same.
- Use "mastery" as the standard for every assessment, which means the exam or experiment or project must be done with 100% competence in order to pass. Provide students with multiple opportunity for re-tries (different versions of the assessment every time).
- Require students to devise their own hypotheses and procedures on all experiments, so that the process is truly a scientific one. Have students assess their proposed experimental procedures for risk and devise mitigations for those risks. Let nothing be pre-designed about students' experiments other than a stated task (i.e. what principle the experiment shall test) at the start and a set of demonstrable knowledge and skill objectives at the end.
- Have students build as much of their lab equipment as possible: building power sources, building test assemblies[1], and building complete working systems (no kits!). In order to provide

[1] In the program I teach, every student builds their own "Development Board" consisting of a metal chassis with DIN rail, terminal blocks, and an AC-DC power supply of their own making which functions as a portable lab environment they can use at school as well as take home.

this same "ground-up" experience for every new student, this means either previous students take their creations with them, or the systems get disassembled in preparation for the new students, or the systems grow and evolve with each new student group.

- Incorporate external accountability for you and for your students, continuously improving the curriculum and your instructional methods based on proven results. Have students regularly network with active professionals through participation in advisory committee meetings, service projects, tours, jobshadows, internships, etc. Practical suggestions include requiring students to design and build projects for external clients (e.g. community groups, businesses, different departments within the institution), and also requiring students attend all technical advisory committee meetings and to dialogue with the industry representatives at those meetings.
- Repeatedly explore difficult-to-learn concepts across multiple courses, so that students have multiple opportunities to build their understanding.
- Relate all new concepts, whenever possible, to previous concepts and to relevant physical laws. Challenge each and every student, every day, to *reason* from concept to concept and to explain the logical connections between. Challenge students to verify their conclusions by multiple approaches (e.g. double-checking their work using different methods). Ask *"Why?"* often.
- Maintain detailed records on each student's performance and share these records privately with them. These records should include academic performance as well as professionally relevant behavioral tendencies.
- Hold mandatory "check-in" meetings between all program faculty and each new student during their first term. Offer these to all other students as an option, except for any students continuing to manifest unprofessional behaviors, poor academic performance, or who have some other need for a face-to-face meeting with faculty.
- Address problems while they are small, before they grow larger. This is equally true for tutoring technical concepts as it is for helping students build professional habits.
- Build rigorous quality control into the curriculum to ensure every student masters every important concept, and that the mastery is retained over time. This includes (1) review questions added to every exam to re-assess knowledge taught in previous terms, (2) cumulative exams at the end of every term to re-assess all important concepts back to the very beginning of the program, and (3) review assessments in practical (hands-on) coursework to ensure critically-important skills were indeed taught and are still retained. What you will find by doing this is that it actually boosts retention of students by ensuring that important knowledge gets taught and is retained over long spans of time. In the absence of such quality control, student learning and retention tends to be spotty and this contributes to drop-out and failure rates later in their education.
- Finally, *never rush learning.* Education is not a race. Give your students ample time to digest complex ideas, as you continually remind yourself of just how long it took you to achieve mastery! Long-term retention and the consistently correct application of concepts are always the result of *focused effort over long periods of time* which means there are no shortcuts to learning.

B.3 Proven strategies for students

The single most important piece of advice I have for any student of any subject is to take responsibility for your own development in all areas of life including mental development. Expecting others in your life to entirely guide your own development is a recipe for disappointment. This is just as true for students enrolled in formal learning institutions as it is for auto-didacts pursuing learning entirely on their own. Learning to think in new ways is key to being able to gainfully use information, to make informed decisions about your life, and to best serve those you care about. With this in mind, I offer the following advice to students:

- **Approach all learning as valuable**. No matter what course you take, no matter who you learn from, no matter the subject, there is something useful in every learning experience. If you don't see the value of every new experience, you are not looking closely enough!
- **Continually challenge yourself**. Let other people take shortcuts and find easy answers to easy problems. The purpose of education is to stretch your mind, in order to shape it into a more powerful tool. This doesn't come by taking the path of least resistance. An excellent analogy for an empowering education is productive physical exercise: becoming stronger, more flexible, and more persistent only comes through intense personal effort.
- **Master the use of language**. This includes reading extensively, writing every day, listening closely, and speaking articulately. To a great extent language channels and empowers thought, so the better you are at wielding language the better you will be at grasping abstract concepts and articulating them not only for your benefit but for others as well.
- **Do not limit yourself to the resources given to you**. Read books that are not on the reading list. Run experiments that aren't assigned to you. Form study groups outside of class. Take an entrepreneurial approach to your own education, as though it were a business you were building for your future benefit.
- **Express and share what you learn**. Take every opportunity to teach what you have learned to others, as this will not only help them but will also strengthen your own understanding[2].
- Realize that **no one can give you understanding**, just as no one can give you physical fitness. These both must be *built*.
- **Above all, recognize that learning is hard work, and that a certain level of frustration is unavoidable**. There are times when you will struggle to grasp some of these concepts, and that struggle is a natural thing. Take heart that it will yield with persistent and varied[3] effort, and never give up! That concepts don't immediately come to you is not a sign of something wrong, but rather of something right: that you have found a worthy challenge!

[2] On a personal note, I was surprised to learn just how much my own understanding of electronics and related subjects was strengthened by becoming a teacher. When you are tasked every day with helping other people grasp complex topics, it catalyzes your own learning by giving you powerful incentives to study, to articulate your thoughts, and to reflect deeply on the process of learning.

[3] As the old saying goes, "Insanity is trying the same thing over and over again, expecting different results." If you find yourself stumped by something in the text, you should attempt a different approach. Alter the thought experiment, change the mathematical parameters, do whatever you can to see the problem in a slightly different light, and then the solution will often present itself more readily.

B.4 Design of these learning modules

"The unexamined circuit is not worth energizing" – Socrates (if he had taught electricity)

These learning modules, although useful for self-study, were designed to be used in a formal learning environment where a subject-matter expert challenges students to digest the content and exercise their critical thinking abilities in the answering of questions and in the construction and testing of working circuits. Every effort has been made to embed the following instructional and assessment philosophies within:

- The first goal of education is to enhance clear and independent thought, in order that every student reach their fullest potential in a highly complex and inter-dependent world. Robust reasoning is *always* more important than particulars of any subject matter, because its application is universal.
- Literacy is fundamental to independent learning and thought because text continues to be the most efficient way to communicate complex ideas over space and time. Those who cannot read with ease are limited in their ability to acquire knowledge and perspective.
- Articulate communication is fundamental to work that is complex and interdisciplinary.
- Faulty assumptions and poor reasoning are best corrected through challenge, not presentation. The rhetorical technique of *reductio ad absurdum* (disproving an assertion by exposing an absurdity) works well to discipline student's minds, not only to correct the problem at hand but also to learn how to detect and correct future errors.
- Important principles should be repeatedly explored and widely applied throughout a course of study, not only to reinforce their importance and help ensure their mastery, but also to showcase the interconnectedness and utility of knowledge.

These learning modules were expressly designed to be used in an "inverted" teaching environment[4] where students first read the introductory and tutorial chapters on their own, then individually attempt to answer the questions and construct working circuits according to the experiment and project guidelines. The instructor never lectures, but instead meets regularly with each individual student to review their progress, answer questions, identify misconceptions, and challenge the student to new depths of understanding through further questioning. Regular meetings between instructor and student should resemble a Socratic[5] dialogue, where questions serve as scalpels to dissect topics and expose assumptions. The student passes each module only after consistently demonstrating their ability to logically analyze and correctly apply all major concepts in each question or project/experiment. The instructor must be vigilant in probing each student's understanding to ensure they are truly *reasoning* and not just *memorizing*. This is why "Challenge" points appear throughout, as prompts for students to think deeper about topics and as starting points for instructor queries. Sometimes these challenge points require additional knowledge that hasn't been covered in the series to answer in full. This is okay, as the major purpose of the Challenges is to stimulate analysis and synthesis on the part of each student.

The instructor must possess enough mastery of the subject matter and awareness of students' reasoning to generate their own follow-up questions to practically any student response. Even completely correct answers given by the student should be challenged by the instructor for the purpose of having students practice articulating their thoughts and defending their reasoning. Conceptual errors committed by the student should be exposed and corrected not by direct instruction, but rather by reducing the errors to an absurdity[6] through well-chosen questions and thought experiments posed by the instructor. Becoming proficient at this style of instruction requires time and dedication, but the positive effects on critical thinking for both student and instructor are spectacular.

An inspection of these learning modules reveals certain unique characteristics. One of these is a bias toward thorough explanations in the tutorial chapters. Without a live instructor to explain concepts and applications to students, the text itself must fulfill this role. This philosophy results in lengthier explanations than what you might typically find in a textbook, each step of the reasoning process fully explained, including footnotes addressing common questions and concerns students raise while learning these concepts. Each tutorial seeks to not only explain each major concept in sufficient detail, but also to explain the logic of each concept and how each may be developed

[4] In a traditional teaching environment, students first encounter new information via *lecture* from an expert, and then independently apply that information via *homework*. In an "inverted" course of study, students first encounter new information via *homework*, and then independently apply that information under the scrutiny of an expert. The expert's role in lecture is to simply *explain*, but the expert's role in an inverted session is to *challenge*, *critique*, and if necessary *explain* where gaps in understanding still exist.

[5] Socrates is a figure in ancient Greek philosophy famous for his unflinching style of questioning. Although he authored no texts, he appears as a character in Plato's many writings. The essence of Socratic philosophy is to leave no question unexamined and no point of view unchallenged. While purists may argue a topic such as electric circuits is too narrow for a true Socratic-style dialogue, I would argue that the essential thought processes involved with scientific reasoning on *any* topic are not far removed from the Socratic ideal, and that students of electricity and electronics would do very well to challenge assumptions, pose thought experiments, identify fallacies, and otherwise employ the arsenal of critical thinking skills modeled by Socrates.

[6] This rhetorical technique is known by the Latin phrase *reductio ad absurdum*. The concept is to expose errors by counter-example, since only one solid counter-example is necessary to disprove a universal claim. As an example of this, consider the common misconception among beginning students of electricity that voltage cannot exist without current. One way to apply *reductio ad absurdum* to this statement is to ask how much current passes through a fully-charged battery connected to nothing (i.e. a clear example of voltage existing without current).

from "first principles". Again, this reflects the goal of developing clear and independent thought in students' minds, by showing how clear and logical thought was used to forge each concept. Students benefit from witnessing a model of clear thinking in action, and these tutorials strive to be just that.

Another feature of these learning modules is that they do not treat topics in isolation. Rather, important concepts are introduced early in the series, and appear repeatedly as stepping-stones toward other concepts in subsequent modules. This helps to avoid the "compartmentalization" of knowledge, demonstrating the inter-connectedness of concepts and simultaneously reinforcing them. Each module is fairly complete in itself, reserving the beginning of its tutorial to a review of foundational concepts.

To high standards of education,

Tony R. Kuphaldt

Appendix C

Tools used

I am indebted to the developers of many open-source software applications in the creation of these learning modules. The following is a list of these applications with some commentary on each.

You will notice a theme common to many of these applications: a bias toward *code*. Although I am by no means an expert programmer in any computer language, I understand and appreciate the flexibility offered by code-based applications where the user (you) enters commands into a plain ASCII text file, which the software then reads and processes to create the final output. Code-based computer applications are by their very nature *extensible*, while WYSIWYG (What You See Is What You Get) applications are generally limited to whatever user interface the developer makes for you.

The `GNU/Linux` computer operating system

> There is so much to be said about Linus Torvalds' `Linux` and Richard Stallman's `GNU` project. First, to credit just these two individuals is to fail to do justice to the *mob* of passionate volunteers who contributed to make this amazing software a reality. I first learned of `Linux` back in 1996, and have been using this operating system on my personal computers almost exclusively since then. It is *free*, it is completely *configurable*, and it permits the continued use of highly efficient `Unix` applications and scripting languages (e.g. shell scripts, Makefiles, `sed`, `awk`) developed over many decades. `Linux` not only provided me with a powerful computing platform, but its open design served to inspire my life's work of creating open-source educational resources.

Bram Moolenaar's `Vim` text editor

> Writing code for any code-based computer application requires a *text editor*, which may be thought of as a word processor strictly limited to outputting plain-ASCII text files. Many good text editors exist, and one's choice of text editor seems to be a deeply personal matter within the programming world. I prefer `Vim` because it operates very similarly to `vi` which is ubiquitous on `Unix`/`Linux` operating systems, and because it may be entirely operated via keyboard (i.e. no mouse required) which makes it fast to use.

Donald Knuth's TEX typesetting system

Developed in the late 1970's and early 1980's by computer scientist extraordinaire Donald Knuth to typeset his multi-volume magnum opus *The Art of Computer Programming*, this software allows the production of formatted text for screen-viewing or paper printing, all by writing plain-text code to describe how the formatted text is supposed to appear. TEX is not just a markup language for documents, but it is also a Turing-complete programming language in and of itself, allowing useful algorithms to be created to control the production of documents. Simply put, TEX *is a programmer's approach to word processing*. Since TEX is controlled by code written in a plain-text file, this means anyone may read that plain-text file to see exactly how the document was created. This openness afforded by the code-based nature of TEX makes it relatively easy to learn how other people have created their own TEX documents. By contrast, examining a beautiful document created in a conventional WYSIWYG word processor such as Microsoft `Word` suggests nothing to the reader about *how* that document was created, or what the user might do to create something similar. As Mr. Knuth himself once quipped, conventional word processing applications should be called WYSIAYG (What You See Is *All* You Get).

Leslie Lamport's LATEX extensions to TEX

Like all true programming languages, TEX is inherently extensible. So, years after the release of TEX to the public, Leslie Lamport decided to create a massive extension allowing easier compilation of book-length documents. The result was LATEX, which is the markup language used to create all ModEL module documents. You could say that TEX is to LATEX as `C` is to `C++`. This means it is permissible to use any and all TEX commands within LATEX source code, and it all still works. Some of the features offered by LATEX that would be challenging to implement in TEX include automatic index and table-of-content creation.

Tim Edwards' `Xcircuit` drafting program

This wonderful program is what I use to create all the schematic diagrams and illustrations (but not photographic images or mathematical plots) throughout the ModEL project. It natively outputs PostScript format which is a true vector graphic format (this is why the images do not pixellate when you zoom in for a closer view), and it is so simple to use that I have never had to read the manual! Object libraries are easy to create for `Xcircuit`, being plain-text files using PostScript programming conventions. Over the years I have collected a large set of object libraries useful for drawing electrical and electronic schematics, pictorial diagrams, and other technical illustrations.

`Gimp` graphic image manipulation program

Essentially an open-source clone of Adobe's `PhotoShop`, I use `Gimp` to resize, crop, and convert file formats for all of the photographic images appearing in the ModEL modules. Although `Gimp` does offer its own scripting language (called `Script-Fu`), I have never had occasion to use it. Thus, my utilization of `Gimp` to merely crop, resize, and convert graphic images is akin to using a sword to slice bread.

`SPICE` circuit simulation program

`SPICE` is to circuit analysis as TEX is to document creation: it is a form of markup language designed to describe a certain object to be processed in plain-ASCII text. When the plain-text "source file" is compiled by the software, it outputs the final result. More modern circuit analysis tools certainly exist, but I prefer `SPICE` for the following reasons: it is *free*, it is *fast*, it is *reliable*, and it is a fantastic tool for *teaching* students of electricity and electronics how to write simple code. I happen to use rather old versions of `SPICE`, version 2g6 being my "go to" application when I only require text-based output. `NGSPICE` (version 26), which is based on Berkeley `SPICE` version 3f5, is used when I require graphical output for such things as time-domain waveforms and Bode plots. In all `SPICE` example netlists I strive to use coding conventions compatible with all `SPICE` versions.

Andrew D. Hwang's `ePiX` mathematical visualization programming library

This amazing project is a `C++` library you may link to any `C/C++` code for the purpose of generating PostScript graphic images of mathematical functions. As a completely free and open-source project, it does all the plotting I would otherwise use a Computer Algebra System (CAS) such as `Mathematica` or `Maple` to do. It should be said that `ePiX` is *not* a Computer Algebra System like `Mathematica` or `Maple`, but merely a mathematical *visualization* tool. In other words, it won't determine integrals for you (you'll have to implement that in your own `C/C++` code!), but it can graph the results, and it does so beautifully. What I really admire about `ePiX` is that it is a `C++` programming library, which means it builds on the existing power and toolset available with that programming language. Mr. Hwang could have probably developed his own stand-alone application for mathematical plotting, but by creating a `C++` library to do the same thing he accomplished something much greater.

`gnuplot` mathematical visualization software

Another open-source tool for mathematical visualization is `gnuplot`. Interestingly, this tool is *not* part of Richard Stallman's GNU project, its name being a coincidence. For this reason the authors prefer "gnu" *not* be capitalized at all to avoid confusion. This is a much "lighter-weight" alternative to a spreadsheet for plotting tabular data, and the fact that it easily outputs directly to an X11 console or a file in a number of different graphical formats (including PostScript) is very helpful. I typically set my `gnuplot` output format to default (X11 on my Linux PC) for quick viewing while I'm developing a visualization, then switch to PostScript file export once the visual is ready to include in the document(s) I'm writing. As with my use of `Gimp` to do rudimentary image editing, my use of `gnuplot` only scratches the surface of its capabilities, but the important points are that it's *free* and that it *works well*.

`Python` programming language

Both Python and C++ find extensive use in these modules as instructional aids and exercises, but I'm listing Python here as a *tool* for myself because I use it almost daily as a *calculator*. If you open a Python interpreter console and type `from math import *` you can type mathematical expressions and have it return results just as you would on a hand calculator. Complex-number (i.e. *phasor*) arithmetic is similarly supported if you include the complex-math library (`from cmath import *`). Examples of this are shown in the Programming References chapter (if included) in each module. Of course, being a fully-featured programming language, Python also supports conditionals, loops, and other structures useful for calculation of quantities. Also, running in a console environment where all entries and returned values show as text in a chronologically-ordered list makes it easy to copy-and-paste those calculations to document exactly how they were performed.

Appendix D

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Appendix E

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Appendix F

Version history

This is a list showing all significant additions, corrections, and other edits made to this learning module. Each entry is referenced by calendar date in reverse chronological order (newest version first), which appears on the front cover of every learning module for easy reference. Any contributors to this open-source document are listed here as well.

15 October 2025 – took one of the Conceptual Reasoning questions ("Capacitor charging circuit") and modified to be more quantitative, then made this a new Quantitative Reasoning question ("Capacitor charging scenario"). Also edited some instructor comments.

4 September 2025 – minor edits to the "Example: simplifying complex RC networks using Thévenin's Theore" Case Tutorial section.

22 May 2025 – added more calculation steps to the "Explaining the meaning of calculations" Conceptual Reasoning question where we're algebraically manipulating the decay equation to solve for time.

26-27 March 2025 – added comments to the Introduction chapter urging instructors to have students show their in-class work solving problems. Also added more examples to the "Example: RC time-delay network values" Case Tutorial section.

20 March 2025 – added a new Case Tutorial section called "Example: RC time-delay network values" with multiple circuit examples for student practice. Also added a new Conceptual Reasoning question asking students to explain the conceptual meaning of all calculations done for an RC time-delay circuit.

11 March 2025 – corrected a grammatical error ("Evaluating they") courtesy of Jason Durrett.

3 March 2025 – corrected a grammatical error ("we will may") courtesy of Caleb Wing.

10 January 2025 – added a new Case Tutorial section, this one on applying Thévenin's Theorem to networks containing multiple resistors in order to simplify to a single-resistor single-capacitor RC network.

11 December 2024 – added another section to the Historical References chapter on an article from *The Electrician* in April of 1904 documenting a fatality most likely resulting from residual charge built up on power conductors dozens of miles in length.

14 November 2024 – added a new bullet-point to the list of challenging concepts in the Introduction chapter, on proper graphing of trajectories.

8 November 2024 – added a new Programming Reference section on solving differential equations using C++.

23-24 October 2024 – added a new Technical References section on applying Thévenin's and Norton's Theorems to time-delay circuits. Also added a hint to the "Capacitor charging circuit" Conceptual Reasoning question.

18 October 2024 – completely re-wrote the "Inverse exponential growth/decay" section of the Tutorial to begin with a discussion of radioactive half-life.

17 October 2024 – edited the "Capacitor charging circuit" Conceptual Reasoning question to emphasize why this is a good pre-analysis strategy for any quantitative analysis.

16 September 2024 – edited the Introduction chapter to have a separate section for challenging concepts.

21 July 2024 – created a new Historical References section on old capacitor technology.

2 July 2024 – added an enumerated list of conceptual analysis steps to the "Example: voltage source energizing a capacitor" Case Tutorial section, showing students exactly how and why to approach problems such as this.

1 July 2024 – minor edits to analogies used within the Tutorial. Also corrected mathematical errors in the limit equations shown in that chapter.

26-28 June 2024 – divided the Introduction chapter into two sections, one for students and one for instructors, and added content to the instructor section recommending learning outcomes and measures. Also made minor edits to the Tutorial's discussion on capacitance acting as source, load, or open. Also edited image_0451.

5 May 2024 – added a graphing requirement to the "Capacitor charging circuit" Conceptual Reasoning question.

12 March 2024 – corrected a typographical error in a table of inverse-exponential calculated values, courtesy of Matthew Oleson.

10 March 2024 – added a list of challenging concepts to the Introduction chapter.

4 February 2024 – simplified explanations of how to determine what grows versus what decays based on an analysis of stored energy. Also added some clarifying details to some of the Case Tutorial examples.

14 November 2023 – added a new Case Tutorial section on empirically determining signal rates of change.

24 October 2023 – added Challenge question to the "Capacitor charging circuit" Conceptual Reasoning question.

20 October 2023 – modified the "Inverse exponential growth/decay" section of the Tutorial chapter to include a table of pre-calculated values for e^x and $1 - e^x$ showing these two formulae as the exponential term x becomes increasingly negative.

17 August 2023 – created a new Technical Reference section on capacitive displacement current from two separate Technical References, and shared this new reference among multiple modules.

6 July 2023 – added a Quantitative Reasoning question on simulating capacitor voltage using a computer program written in C to evaluate a differential equation.

7 February 2023 – added caveats to Case Tutorial examples about the capacitors being ideal (with internal resistance or soakage).

27 November 2022 – placed questions at the top of the itemized list in the Introduction chapter prompting students to devise experiments related to the tutorial content.

17-19 October 2022 – elaborated in the Tutorial on how to incorporate initial and/or final values to the inverse-exponential formulae. Also added a Programming References section showing an extremely simple Python3 script for modeling inverse-exponential functions. Also added a Technical Reference section modeling capacitor current in terms of hydraulic fluid motion in a system containing a rubber-bladder accumulator, as well as a new Tutorial section showing a worked example of an RC charging/discharging circuit.

22-23 July 2022 – minor edits to Tutorial giving a more thorough definition of a "time constant". Also expanded upon a question in the Introduction chapter, designed to prompt more reasoning based on foundational concepts. Also added a Conceptual Reasoning question on the use of copper tape across a capacitor's terminals.

21 July 2022 – added some more explanatory text to the Tutorial describing the operation of the variable capacitor shown there.

13 June 2022 – added a new Case Tutorial section showing energy storage calculations for series versus parallel capacitor networks.

22 April 2022 – added photographs of a variable capacitor showing different degrees of plate overlap.

31 March 2022 – added "Electrical field quantities" to the Derivations and Technical References chapter.

16 March 2022 – added some content to the Tutorial discussing the calculation of capacitance based on permittivity, plate area, and separation distance.

28 December 2021 – added a Technical Reference section on capacitor parasitics.

3 December 2021 – changed "permeability" to "permittivity", which was a copy-and-paste error from a module written for inductors.

20-21 October 2021 – deleted the Projects and Experiments chapter. Also added some clarifying text in the Tutorial about how the Conservation of Energy relates to voltage and current for these devices.

1 August 2021 – added Case Tutorial on the use of decoupling capacitors to stabilize IC power supply voltage.

23 July 2021 – added a Case Tutorial section showing a simple charge/discharge circuit useful for exploring the inverse-exponential behavior of resistor-capacitor networks. Also added foundational concept review to the "Capacitor charging circuit" Conceptual Reasoning question.

16 July 2021 – added footnote to the Tutorial explaining what permittivity is.

9 July 2021 – improved description of electrolytic capacitor manufacturing found in the Tutorial chapter.

4 July 2021 – divided Tutorial chapter into sections and made some minor edits to the text for clarity.

2 June 2021 – added a Case Tutorial chapter with sections showing how to store and release energy from capacitors.

8 May 2021 – commented out or deleted empty chapters.

13 April 2021 – added "energy" as a parameter for students to qualitatively assess in the "Capacitor charging circuit" Conceptual Reasoning question.

6 April 2021 – added some commentary to Tutorial on units of measurement for time constant (τ).

5 April 2021 – added some commentary to the "Capacitor parameters and dielectric types" Technical Reference on dissipative losses for capacitors.

18 March 2021 – corrected one instance of "volts" that should have been capitalized "Volts".

9 February 2021 – added References for capacitor labeling source documents, to accompany techref_caplabel.

2 January 2021 – added a Technical Reference section on capacitor labeling.

19 December 2020 – added a Case Tutorial example showing the use of a capacitor to stabilize power supply voltage for battery-sourced circuits.

26 October 2020 – added Quantitative questions.

19 October 2020 – corrected typo in the "SPICE analysis of an energizing capacitor" Quantitative question, saying that a de-energized capacitor has zero current when it should have said that an initial condition of zero meant zero *voltage*.

26 September 2020 – added voltage divider RC circuit to the Case Tutorial chapter.

4 September 2020 – incorporated the "Inverse exponential functions" question content into the Tutorial, explaining the formulae for decaying and growing quantities in full detail.

29 August 2020 – significantly edited the Introduction chapter to make it more suitable as a pre-study guide and to provide cues useful to instructors leading "inverted" teaching sessions. Also, added brief comment and footnote in the Tutorial on parasitic inductance.

26 August 2020 – added some Case Tutorial examples showing circuits energized by voltage and current sources, respectively.

17 August 2020 – minor edit to the Tutorial text in order to shorten a paragraph, for typesetting purposes.

17 April 2020 – added a Technical Reference section on IEC standard component values (file techref_IEC60063.latex).

13 April 2020 – added Technical Reference section on capacitor types.

6 April 2020 – minor typographical error corrections, courtesy of Ron Felix.

19 February 2020 – added another Quantitative Reasoning problem.

4 February 2020 – added some Quantitative Reasoning problems.

2 January 2020 – removed from from C++ code execution output, to clearly distinguish it from the source code listing which is still framed.

1 January 2020 – added explanatory text for the C++ code in reference_invexp_decay.latex. Also changed `main ()` to `main (void)` in another C++ programming example.

10 December 2019 – added more questions. Also, corrected error in "Cable capacitance" question. Added letter-code tolerance ratings to Tutorial.

2 November 2019 – added more questions.

26 August 2019 – added mention of difficult concepts to the Introduction.

13 April 2019 – improved the explanation of what happens in RC circuits with inverse-exponential trends of voltage and current. Also introduced the term "speed" to refer to rate-of-change for capacitor voltage (i.e. voltage "speed" = $\frac{dV}{dt}$).

8 April 2019 – minor edit to wording within an illustration (replaced "creates" with "means" to avoid the implication of a unidirectional causality).

24 March 2019 – added a Quantitative Reasoning questions.

13 March 2019 – added a new experiment, demonstrating capacitive time delays, and also added reference to inverse exponential functions in the Tutorial.

12 February 2019 – added a number of new entries to the Foundational Concepts subsection.

7 January 2019 – added a challenge question to a quantitative problem. Also, added illustrative examples to the Tutorial, showing what happens when a capacitor is energized by a constant current. Finally, added a Derivations and Technical References chapter, with an exposition of exponential decay functions in it.

17 December 2018 – added an example C++ program showing calculations for an energizing resistor-capacitor circuit.

October 2018 – clarified a number of details, with constructive criticism from Ron Felix.

August 2018 – wrote an Introduction where none existed before.

July 2018 – document first created.

Index

V_{DD} bounce, 18
τ, 103
"Ohm's Law" for a capacitor, 43

Accumulator, hydraulic, 45
Adding quantities to a qualitative problem, 170
Ampere, unit, 86
Annotating diagrams, 169
Asymptote, 103

Battery action, 14
Bounce, ground, 18
Breakdown, dielectric, 82

C++, 94, 104
Capacitance, 3, 39, 58, 70
Capacitor, 3, 58
Capacitor, decoupling, 18
Checking for exceptions, 170
Checking your work, 170
Chip, 45
Class 1 ceramic dielectrics, 77
Class 2 ceramic dielectrics, 77
Code, computer, 179
Coefficient, temperature, 77
Color code, resistor, 54
Compiler, C++, 94
Component values, IEC standard, 68
Computer programming, 93, 104
Conservation of Energy, 42
Conservation of Mass, 42
Coulomb, 82, 83
Coulomb, unit, 86
Current probe, 34

Decay, exponential, 103
Decoupling, 18
Decoupling capacitor, 18
Dielectric, 3, 40, 53, 58, 84
Dielectric breakdown, 82
Dielectric relaxation, 14
Dielectric strength, 53, 82
Dimensional analysis, 169
Displacement current, 34, 84
Distance versus speed, 36

Edwards, Tim, 180
Electric field, 39, 103
Electromagnetic wave, 84
Energy, kinetic, 40
Energy, potential, 39, 40
Equivalent Series Resistance, 54, 79
Equivalent series resistance, 18, 20
ESR, 18, 20, 54, 79
Exponential decay, 103

Farad, 70
Farad, unit, 3, 40, 86
Field, electric, 39, 103
Field, gravitational, 39
Field, magnetic, 103

Graph values to solve a problem, 170
Gravitational field, 39
Greenleaf, Cynthia, 121
Ground bounce, 18

How to teach with these modules, 177
Hwang, Andrew D., 181

IC, 45
Identify given data, 169
Identify relevant principles, 169
IEC 60063 standard, 68
IEC standard component values, 68
Inductance, 18, 54

Integrated circuit, 45
Intermediate results, 169
Interpreter, Python, 98
Inverse exponential curve, 46
Inverse exponential decay, 103
Inverted instruction, 177

Java, 95, 105
Joule's Law, 44
Joule, unit, 40

Kirchhoff's Current Law, 24
Kirchhoff's Voltage Law, 21, 27, 29, 46, 51
Knuth, Donald, 180

Lamport, Leslie, 180
Leakage resistance, 54
Leyden jar, 58
Limiting cases, 170
Load, 42

Magnetic field, 103
Maxwell, James Clerk, 57, 84
Metacognition, 126
MLCC, 61
Moolenaar, Bram, 179
Multi-layer ceramic capacitor, 61
Murphy, Lynn, 121
Mylar dielectric, 76

Newton, 82, 83
Newton's Cooling Law, 104

Ohm's Law, 18, 21, 24, 27–29, 46, 51, 52
Open, 42
Open-source, 179
Oscilloscope, 11

Parallel capacitance, 55
Parasitic, 54
Parasitic effect, 34, 79
PCB, 45
Permittivity, 3, 40, 53, 83
Philosophical Magazine, 85
Piezoelectricity, 77
Pitch, 34
Power plane, PCB, 18
Printed circuit board, 45
Problem-solving: annotate diagrams, 169
Problem-solving: check for exceptions, 170
Problem-solving: checking work, 170
Problem-solving: dimensional analysis, 169
Problem-solving: graph values, 170
Problem-solving: identify given data, 169
Problem-solving: identify relevant principles, 169
Problem-solving: interpret intermediate results, 169
Problem-solving: limiting cases, 170
Problem-solving: qualitative to quantitative, 170
Problem-solving: quantitative to qualitative, 170
Problem-solving: reductio ad absurdum, 170
Problem-solving: simplify the system, 169
Problem-solving: thought experiment, 55, 169
Problem-solving: track units of measurement, 169
Problem-solving: visually represent the system, 169
Problem-solving: work in reverse, 170
Programming, computer, 93, 104
Python, 98

Qualitatively approaching a quantitative problem, 170

Reading Apprenticeship, 121
Reductio ad absurdum, 170, 176, 177
Resistance, 18
Resistor color code, 54
Rise over run, 34

Schoenbach, Ruth, 121
Scientific method, 126
Series capacitance, 55
Simplifying a system, 169
Slope, 34
Soakage, capacitor, 79, 81
Socrates, 176
Socratic dialogue, 177
Source, 42
Source code, 94
SPDT, 50
SPDT switch, 46
Speed versus distance, 36

SPICE, 121
Stallman, Richard, 179
Standard component values, IEC, 68
Supercapacitor, 45
Superconductor, 83

Tangent line, 34
Temperature coefficient of capacitance, 77
Thévenin's Theorem, 31
Thought experiment, 55, 169
Time constant, 103
Tolerance codes, 54
Torvalds, Linus, 179

Units of measurement, 169

Via, PCB, 18
Visualizing a system, 169
Volt, unit, 40, 86

Weight, 39
Whitespace, C++, 94, 95
Whitespace, Python, 101
Work in reverse to solve a problem, 170
WYSIWYG, 179, 180