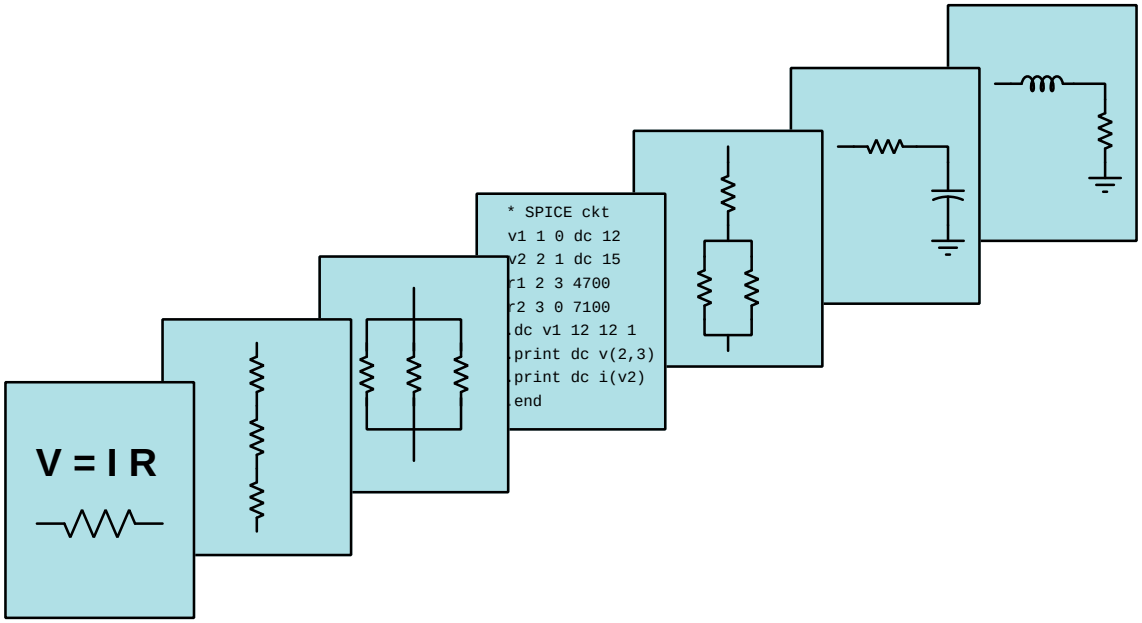
Modular Electronics Learning (ModEL) project



Capacitance and Inductance

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Chapter 1

Introduction

1.1 Recommendations for students

While resistance dissipates electrical energy in the form of heat, capacitance and inductance both *store* and *release* energy, capacitance relying on electric fields and inductance relying on magnetic fields to do so. This module explores these complementary phenomena with a strong emphasis on energy and its conservation as a unifying principle.

Important concepts related to both capacitance and inductance include **energy** in both **kinetic** and **potential** forms, the **Conservation of Energy**, **Newton's First Law of Motion**, **sources** versus **loads**, **resistance**, **voltage**, **current**, **electric fields**, **magnetic fields**, **rates of change**, voltage **rise** and **drop**, effects of **opens** versus **shorts**, **power**, **resonance**, **oscillation**, **AC** versus **DC**, **tank circuits**, and **pulsing** signals.

A useful reading strategy when studying a topic such as this is to write your own summary page of important principles, especially when those principles mirror each other as they often do for capacitance and inductance. For this module I would recommend writing your own summaries of capacitive properties versus inductive properties: how they differ, how they are similar, and the fundamental reasons why those properties are true.

Here are some good questions to ask of yourself while studying this subject:

- How might an experiment be designed and conducted to explore the phenomenon of capacitance? What hypothesis (i.e. prediction) might you pose for that experiment, and what result(s) would either support or disprove that hypothesis?
- How might an experiment be designed and conducted to explore the phenomenon of inductance? What hypothesis (i.e. prediction) might you pose for that experiment, and what result(s) would either support or disprove that hypothesis?
- How might an experiment be designed and conducted to explore the phenomenon of electrical resonance? What hypothesis (i.e. prediction) might you pose for that experiment, and what result(s) would either support or disprove that hypothesis?

- What does it mean to say that energy is always *conserved?*
- How does an electrical load behave?
- How does an electrical source behave?
- What are some examples of electrical components capable of being either sources or loads?
- What is a *field?*
- How do electric and magnetic fields differ from one another?
- What physical systems exhibit capacitance?
- What physical systems exhibit inductance?
- Does capacitance obey Ohm's Law like resistance does?
- Does inductance obey Ohm's Law like resistance does?
- What parameter is most stable – voltage or current – where there exists capacitance?
- What parameter is most stable – voltage or current – where there exists inductance?
- Why does the combination of capacitance and inductance naturally support *oscillation*?
- What mechanical phenomenon is analogous to capacitance?
- What mechanical phenomenon is analogous to inductance?
- Why does the insertion of resistance tend to dampen oscillations in a circuit?

One of the distinguishing characteristics of the Modular Electronics Learning (ModEL) series is an emphasis on *energy* and *conservation laws* as a paradigm for comprehending electric circuits. Viewed from this perspective, challenging concepts such as voltage become clear, and subsequent principles such as Kirchhoff's Laws are seen to be mere extensions of energy and charge conservation principles. Such is the nature of science: distilling complex phenomena down to a small set of relatively simple Laws. Another hallmark of scientific discovery is *symmetry*: instances where seemingly disparate phenomena are found to be in fact closely related to one another. Series versus parallel circuits is one example of the symmetry seen in electric circuits: current is the same through all components in a series network, while voltage is the same across all components in a parallel network; voltages add in series, while current adds in parallel; Kirchhoff's Voltage Law applies to any loop of components, Kirchhoff's Current Law applies to any junction point (node) of two or more conductors.

This module, focusing on the twin phenomena of capacitance and inductance follows this same pattern. Both concepts are defined quite readily in terms of energy storage, the former associated with voltage and the latter associated with current. As for symmetry, capacitance and inductance are really flip-sides of the same coin: capacitance involves the storage of energy within an electric field as a function of voltage, while inductance involves the storage of energy within a magnetic field as a function of current; capacitance tends to stabilize voltage, while inductance stabilizes

current. This symmetry also parallels a symmetry found in classical mechanics: potential energy versus kinetic energy, with the stabilization of voltage and current mirroring Newton's First Law of Motion[1].

[1] Newton's First Law states that an object at rest tends to remain at rest, and that an object in motion tends to remain in motion, unless acted upon by a force. Applying this concept to inductors, we would say that the current through an inductor tends to remain unchanged unless acted upon by a voltage across the inductor; also, that the voltage across a capacitor tends to remain unchanged unless acted upon by a current through the capacitor.

1.2 Challenging concepts related to capacitance and inductance

The following list cites concepts related to this module's topic that are easily misunderstood, along with suggestions for properly understanding them:

- **Rates of change** – when learning the relationships between voltage and current for inductors and capacitors, one must think in terms of how fast a variable is changing. The amount of voltage induced across an inductor is proportional to how *quickly* the current through it changes, not how strong the current is. Likewise, the amount of current "through" a capacitor is proportional to how *quickly* the voltage across it changes. This is the first hurdle in calculus: to comprehend what a rate of change is, and it is far from obvious. A helpful strategy here is to relate electrical rates of change to other, more common, rates of change such as *speed* (i.e. the rate-of-change of physical position over time). Also helpful is to relate rates of change to the *slope* of a line or curve plotted on a graph, showing how *rise/fall over run* describes not only the phenomenon of a rate-of-change but also the appropriate units of measurement.

- **Source and Load identification** – in order to identify a given component as either behaving as a source or as a load, we must know the direction of current passing through it as well as the polarity of the voltage across it. *Both* of these must be known in order to identify a component as either a source or a load, with only one of them being insufficient. Only when we know both the direction of current *and* the voltage polarity may we conclude with confidence whether electric charge carriers are gaining energy or losing energy passing through the component.

- **Current "through" a capacitance** – since a capacitor is by its very construction electrically *open*, having no conductive pathway from one terminal to the other, it seems extremely counter-intuitive to speak of capacitors as ever conducting a current "through" them. A key point about electrical opens prohibiting current is that this prohibition only holds for *continuous* currents. Momentary, transient currents are indeed possible for incomplete circuits, lightning being a spectacular natural example of this. For those who wish to dig deeper into the phenomenon, James Clerk Maxwell's notion of *displacement current* is a helpful start.

- **Ohm's Law** – while normally not a difficult concept to understand or apply, the challenge while studying capacitance and/or inductance is to avoid *mis*-applying Ohm's Law where it does not fit. In reality, Ohm's "Law" is not really a *law* at all, but rather a definition of electrical resistance, and therefore it only applies to situations where there is resistance in a circuit; i.e. where energy leaves the circuit in a form that shall never return to the circuit. Capacitors and inductors alike don't dissipate energy the way resistances do, and therefore Ohm's Law simply does not apply to these new components. Instead, we must apply new mathematical principles such as $I = C\frac{dV}{dt}$ and $V = L\frac{dI}{dt}$.

- **Initial and final values** – circuits containing capacitance and/or inductance exhibit changes in voltage and current over time, which is a property purely resistive circuits do not intrinsically possess. Key to identifying these initial and final values lies in correctly reasoning from the first principles of energy storage in capacitors and inductors: that capacitors store energy in electric fields which translate to voltage, and inductors store energy in magnetic fields which translate to current. Therefore, a de-energized capacitor is guaranteed to have zero voltage, and a

de-energized inductor is guaranteed to have zero current. Conversely, current for a capacitor merely indicates how rapidly it's gaining or losing stored energy; voltage for an inductor merely indicates how rapidly it's gaining or losing energy. A common tendency among students is to try to apply Ohm's Law ($V = IR$), falsely reasoning that current and voltage should be proportional for capacitors and inductors like it is for resistors, but Ohm's Law is not truly a Law in the strict sense of the word because it narrowly applies to resistance, not to capacitance or inductance.

- **Series and parallel capacitance** – after learning how resistance adds in series and diminishes in parallel, and especially after learning how inductance follows the same patters for series and parallel, it comes as quite a surprise to most students that capacitance exhibits the exact opposite pattern: diminishing in series and adding in parallel. One way to dispel this misconception is to appeal to the physical factors affecting capacitance: plate area and dielectric thickness. Paralleled capacitances add because this effectively adds plate area; series-connected capacitances diminish because this effectively increases dielectric thickness. However, a deeper understanding of series and parallel capacitance may be gained by exploring the effects on stored energy ($E_C = \frac{1}{2}CV^2$) as shown in the Case Tutorial section "Example: series versus parallel capacitors". As usual, reasoning from the perspective of *energy* tends to yield the best conceptual understanding of things.

The *Case Tutorial* section on "Example: measuring signal rates of change" is helpful to showcase rates of change in the practical context of measuring these rates on an oscilloscope's display.

1.3 Recommendations for instructors

This section lists realistic student learning outcomes supported by the content of the module as well as suggested means of assessing (measuring) student learning. The outcomes state what learners should be able to do, and the assessments are specific challenges to prove students have learned.

- **Outcome** – Demonstrate effective technical reading and writing

  Assessment – Students present their outlines of this module's instructional chapters (e.g. Case Tutorial, Tutorial, Historical References, etc.) ideally as an entry to a larger Journal document chronicling their learning. These outlines should exhibit good-faith effort at summarizing major concepts explained in the text.

  Assessment – Students present their "Conceptual Toolbox" Theory Journal entries listing important concepts applied in the Tutorial text(s). A "Conceptual Toolbox" serves as inspiration when solving problems, each concept suggesting a potential problem-solving action. Challenge students to collaboratively generate a Conceptual Toolbox for the day's topic during the class session and then use it when solving problems together, thus fostering the habit of reasoning from principles rather than recalling procedures!

- **Outcome** – Quantitatively apply the concept of energy storage to both capacitors and inductors

  Assessment – Properly calculate stored energy given device parameters, and/or device parameters given a necessary amount of energy storage; e.g. pose problems in the form of the "Capacitors and inductors compared against batteries" Quantitative Reasoning question.

- **Outcome** – Apply the concepts of source versus load to capacitors and inductors alike

  Assessment – Show a circuit or portion of a circuit containing a capacitor, an inductor, or both, and challenge students to determine source/load identification given known values of current rate-of-change or voltage rate-of-change.

  Assessment – Show a circuit diagram with capacitors and/or inductors as well as resistors and voltage/current sources, challenging students to apply Kirchhoff's Laws and Ohm's Law to determine all voltages and all currents given instantaneous capacitor voltages and inductor currents; e.g. pose problems in the form of the "Mixed-source circuits with capacitors and inductors" Quantitative Reasoning question.

- **Outcome** – Prove the concept of capacitive and inductive energy storage by experiment

  Assessment – Energize a capacitor and then transfer that energy to a load such as an LED, judging the amount of energy by the brightness and duration of the LED's illumination.

  Assessment – Energize an inductor and then transfer that energy to a load such as an LED, judging the amount of energy by the brightness and duration of the LED's illumination.

  Assessment – Re-run these experiments with multiple energy-storing components connected in series versus parallel to compare their relative energy-storage abilities.

- **Outcome** – Independent research

Assessment – Locate capacitor datasheets and properly interpret some of the information contained in those documents including capacitance tolerance, voltage ratings, temperature coefficient, voltage coefficient, etc.

Assessment – Locate inductor datasheets and properly interpret some of the information contained in those documents including inductance tolerance, current ratings, saturation limits, etc.

Chapter 2

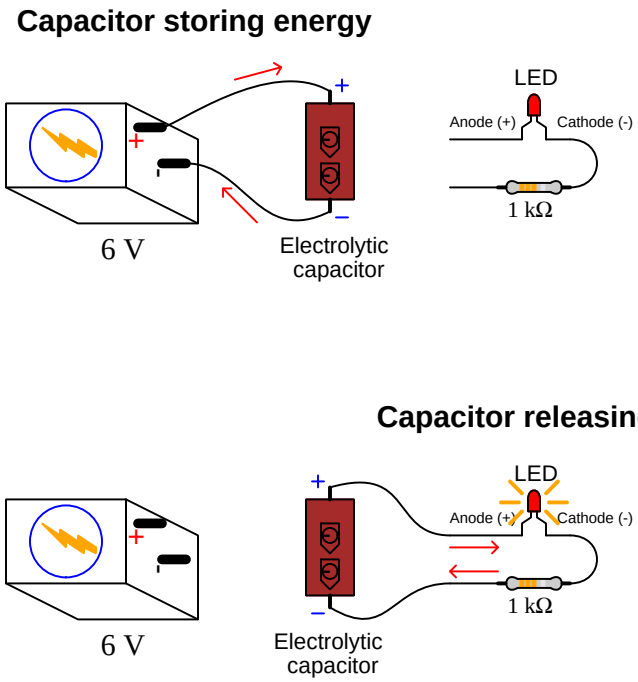
Case Tutorial

The idea behind a *Case Tutorial* is to explore new concepts by way of example. In this chapter you will read less presentation of theory compared to other Tutorial chapters, but by close observation and comparison of the given examples be able to discern patterns and principles much the same way as a scientific experimenter. Hopefully you will find these cases illuminating, and a good supplement to text-based tutorials.

These examples also serve well as challenges following your reading of the other Tutorial(s) in this module – can you explain *why* the circuits behave as they do?

2.1 Example: capacitor storing and releasing energy

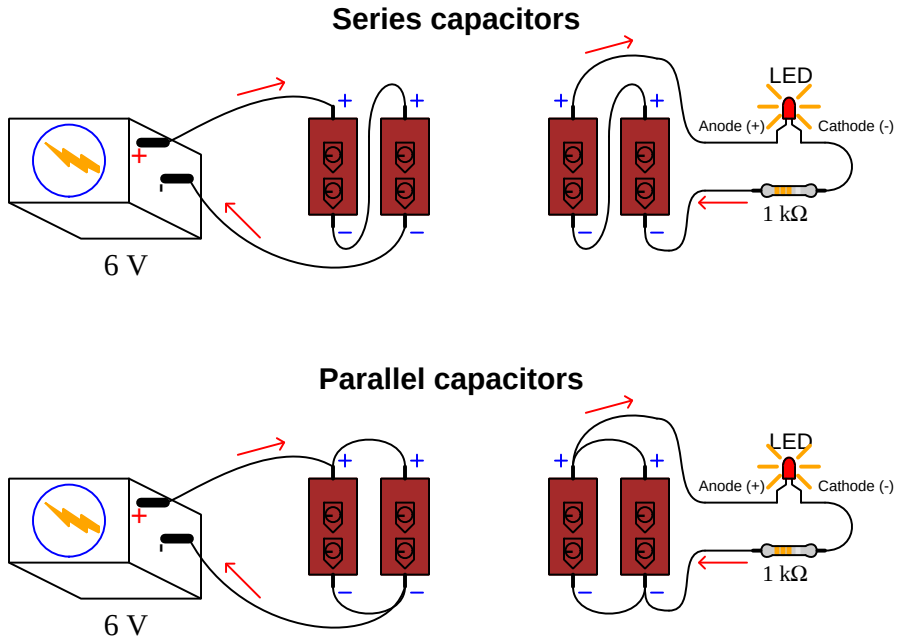
It is easy to use a battery, capacitor[1], resistor, and LED to demonstrate the principle of capacitance storing and releasing energy:



In the upper configuration the capacitor acts as a load. In the lower configuration the capacitor acts as a source.

[1] An aluminum electrolytic capacitor having a value of at least 100 μF and a voltage rating greater than the voltage of the battery generally works well. Beware the polarity requirements of the capacitor, as electrolytic capacitors are polarized devices and will be damaged by applying a reverse-polarity DC voltage!

A recommended variation of this experiment is to try using multiple capacitors of the same rating as the original, energized by the same source (battery) voltage as well. The point of these experiments is to compare how long the LED remains energized when powered by the series capacitor network, versus when powered by the parallel capacitor network, versus when powered by single capacitor, total voltage being equal in all cases:

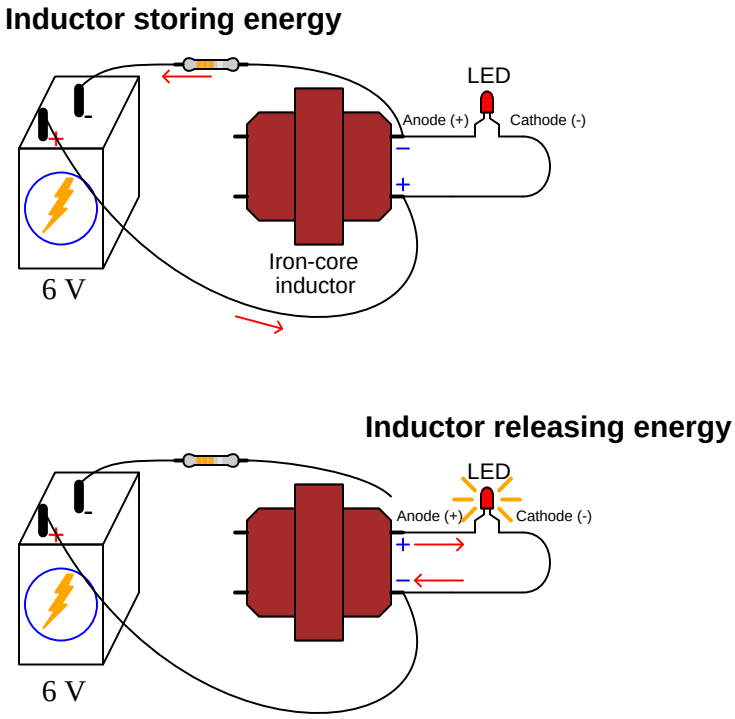


Since capacitance is defined as the storage of energy in the form of an electric field, the comparative results of these experiments will prove whether capacitance is enhanced in parallel or in series. The configuration resulting in the LED remaining energized longest is the one having the most capacitance. If you are experienced in the use of a *digital oscilloscope*, consider simplifying the load to only be a resistor (with no LED) and use the oscilloscope's single-sweep feature to capture the voltage decay over time! An oscilloscope will show you quantitatively how voltage decays over time, and without the non-linearity of an LED being part of the circuit that rate of decay will be a very natural inverse-exponential curve.

An important principle in scientific experimentation is to *control* for all variables in an experiment except the one variable we wish to explore. This is why it is important here to use identically-valued capacitors in all cases, and use the same amount of source voltage too. This way, the only variable is the total amount of capacitance resulting from the number of capacitors and the manner in which they interconnect with each other.

2.2 Example: inductor storing and releasing energy

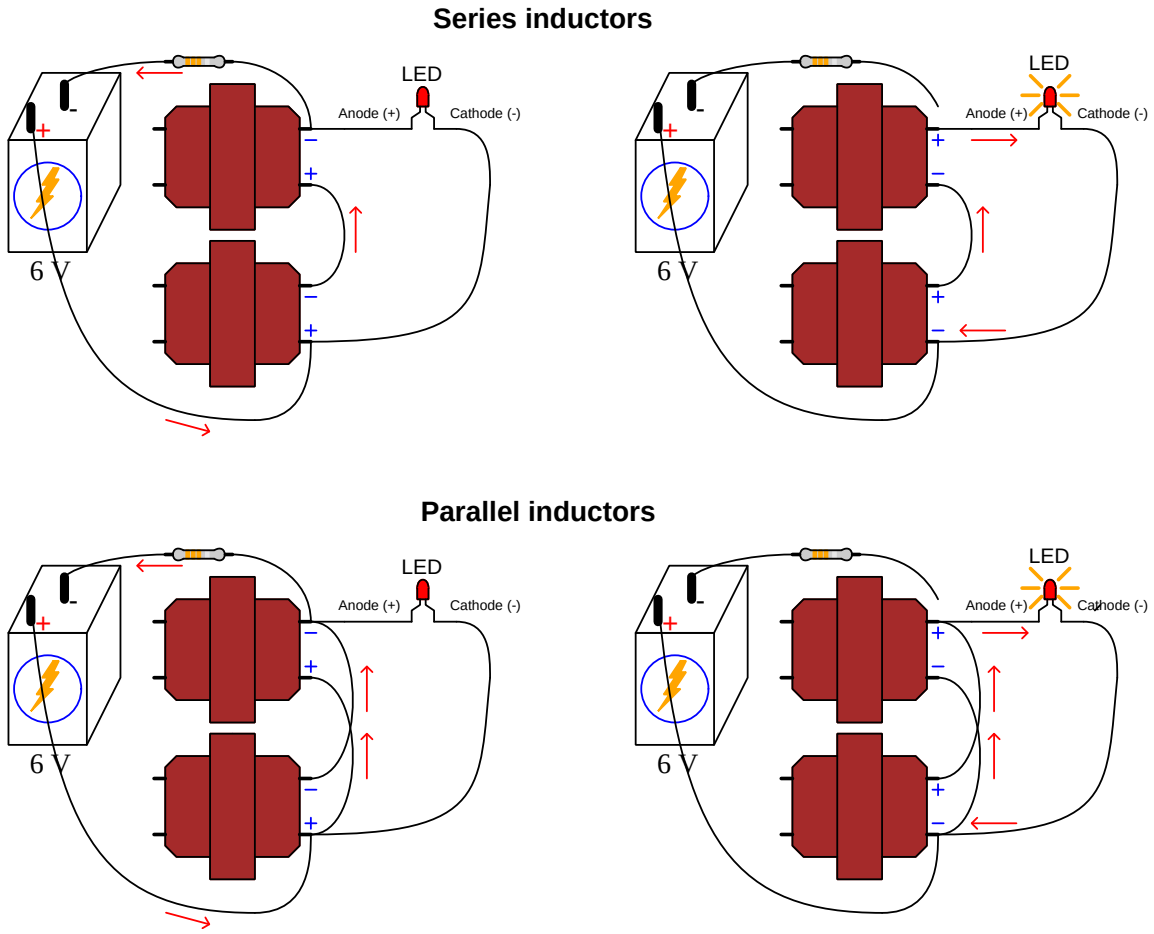
It is easy to use a battery, inductor[2], resistor, and LED to demonstrate the principle of inductance storing and releasing energy:



In the upper configuration the inductor acts as a load. The LED does not illuminate here because the polarity is backwards. In the lower configuration (lifting one wire) the inductor acts as a source, which energizes the LED because the inductor's voltage polarity reverses. The purpose of the resistor is to limit current through the inductor to a value that is safe for the LED, since the LED's initial current will be equal to the inductor's final current. This resistor value will need to be customized for your experiment, so that the series combination of this resistor and the inductor's winding resistance together limit the energization current to an LED-safe value.

[2]The high-voltage winding of a step-down power transformer generally works well for this inductance. If the transformer has dual-voltage capability (e.g. 120/240 Volts), wire it for the highest-voltage configuration in order to have the greatest amount of inductance.

A recommended variation of this experiment is to try using multiple inductors of the same rating as the original, energized by the same source current as well. The point of these experiments is to compare how long the LED remains energized when powered by the series inductor network, versus when powered by the parallel inductor network, versus when powered by single inductor, total current being equal in all cases:



Since inductance is defined the storage of energy in the form of a magnetic field, the comparative results of these experiments will prove whether inductance is enhanced in parallel or in series. The configuration resulting in the LED remaining energized longest is the one having the most inductance. If you are experienced in the use of a *digital oscilloscope*, consider replacing the LED with a second resistor of approximately the same resistance as the first[3] and use the oscilloscope's single-sweep feature to capture this new resistor's voltage drop as it decays over time! By applying Ohm's Law you will be able to infer inductor current from resistor voltage drop measurements at

[3]This choice of resistor value will ensure that the peak voltage developed across this second resistor when the battery (source) is disconnected will be modest, not high enough to pose a threat to the oscilloscope's input.

every point in time. An oscilloscope will show you quantitatively how these quantities decay over time, and without the non-linearity of an LED being part of the circuit that rate of decay will be a very natural inverse-exponential curve.

An important principle in scientific experimentation is to *control* for all variables in an experiment except the one variable we wish to explore. This is why it is important here to use identically-valued inductors in all cases, and use the same amount of source current too. Fixing the source current while using a *voltage* source (e.g. battery) and experiencing different amounts of inductor winding resistance in each case requires the resistor value be customized for each iteration of the experiment, using an ammeter to measure source and adjusting the resistor value in each version of the circuit to achieve the same total current for each. This way, the only variable is the total amount of inductance resulting from the number of inductors and the manner in which they interconnect with each other.

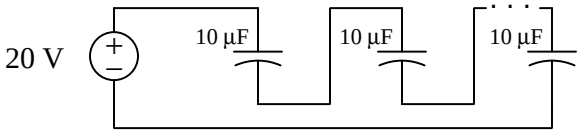
2.3 Example: series versus parallel capacitors

Calculating energy stored within a series network of 10 μF capacitors based on the number of capacitors (n) connected in series, in each case energized by a 20 Volt source. The amount of energy stored within each capacitor is given by the following formula, and the total energy stored is the simple sum of all capacitors' stored energy:

$$E_C = \frac{1}{2}CV^2$$

Where,

E_C = capacitor energy in Joules (J)
C = capacitance in Farads (F)
V = voltage in Volts (V)



Number of capacitors (n)	**Voltage across each capacitor**	**Energy stored in each capacitor**	**Energy stored (total)**
1	20.00 V	2.0000 mJ	2.0000 mJ
2	10.00 V	0.5000 mJ	1.0000 mJ
3	6.667 V	0.2222 mJ	0.6667 mJ
4	5.000 V	0.1250 mJ	0.5000 mJ

Connecting more capacitors in series results in the source voltage (20 Volts) being divided into smaller portions. With less voltage across each capacitor, there is dramatically less energy stored within each owing to the squared voltage term in the $E_C = \frac{1}{2}CV^2$ equation, so that with more capacitors in the series network there is actually less total energy stored for the entire network.

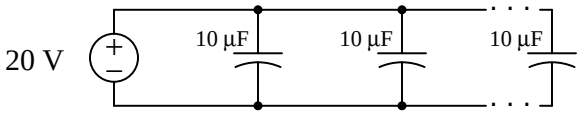
Since capacitance is defined as the ability to store energy within an electric field, we must conclude that connecting capacitors in series results in *less* over-all capacitance.

Calculating energy stored within a parallel network of 10 μF capacitors based on the number of capacitors (n) connected in parallel, in each case energized by a 20 Volt source. The amount of energy stored within each capacitor is given by the following formula, and the total energy stored is the simple sum of all capacitors' stored energy:

$$E_C = \frac{1}{2}CV^2$$

Where,

E_C = capacitor energy in Joules (J)
C = capacitance in Farads (F)
V = voltage in Volts (V)



Number of capacitors (n)	**Voltage across each capacitor**	**Energy stored in each capacitor**	**Energy stored (total)**
1	20.00 V	2.0000 mJ	2.0000 mJ
2	20.00 V	2.0000 mJ	4.0000 mJ
3	20.00 V	2.0000 mJ	6.0000 mJ
4	20.00 V	2.0000 mJ	8.0000 mJ

Connecting more capacitors in parallel results in a direct increase in stored energy, because each capacitor receives the same source voltage (20 Volts) and therefore stores the same amount of energy.

Since capacitance is defined as the ability to store energy within an electric field, we must conclude that connecting capacitors in parallel results in *more* over-all capacitance.

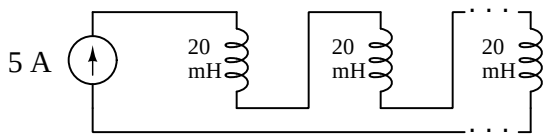
2.4 Example: series versus parallel inductors

Calculating energy stored within a series network of 20 mH inductors based on the number of inductors (n) connected in series, in each case energized by a 5 Ampere source. The amount of energy stored within each inductor is given by the following formula, and the total energy stored is the simple sum of all inductors' stored energy:

$$E_L = \frac{1}{2}LI^2$$

Where,

E_L = inductor energy in Joules (J)
L = inductance in Henrys (H)
I = current in Amperes (A)



Number of inductors (n)	**Current through each inductor**	**Energy stored in each inductor**	**Energy stored (total)**
1	5.000 A	250.00 mJ	250.00 mJ
2	5.000 A	250.00 mJ	500.00 mJ
3	5.000 A	250.00 mJ	750.00 mJ
4	5.000 A	250.00 mJ	1.0000 J

Connecting more inductors in series results in a direct increase in stored energy, because each inductor receives the same source current (5 Amperes) and therefore stores the same amount of energy.

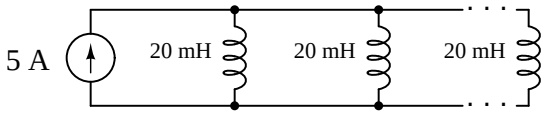
Since inductance is defined as the ability to store energy within a magnetic field, we must conclude that connecting inductors in series results in *more* over-all inductance.

Calculating energy stored within a parallel network of 20 mH inductors based on the number of inductors (n) connected in parallel, in each case energized by a 5 Ampere source. The amount of energy stored within each inductor is given by the following formula, and the total energy stored is the simple sum of all inductors' stored energy:

$$E_L = \frac{1}{2}LI^2$$

Where,

E_L = inductor energy in Joules (J)
L = inductance in Henrys (H)
I = current in Amperes (A)



Number of inductors (n)	**Current through each inductor**	**Energy stored in each inductor**	**Energy stored (total)**
1	5.000 A	250.00 mJ	250.00 mJ
2	2.500 A	62.500 mJ	125.00 mJ
3	1.667 A	27.778 mJ	83.333 mJ
4	1.250 A	15.625 mJ	62.500 mJ

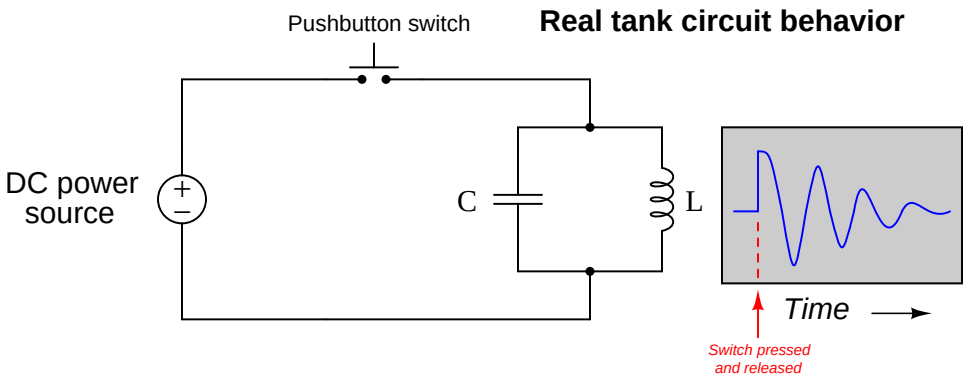
Connecting more inductors in parallel results in the source current (5 Amperes) being divided into smaller portions. With less current through each inductor, there is dramatically less energy stored within each owing to the squared current term in the $E_L = \frac{1}{2}LI^2$ equation, so that with more inductors in the parallel network there is actually less total energy stored for the entire network.

Since inductance is defined as the ability to store energy within a magnetic field, we must conclude that connecting inductors in parallel results in *less* over-all inductance.

2.5 Example: inductor-capacitor resonance

A battery, inductor[4], and capacitor may be used to build a simple circuit demonstrating the phenomenon of resonance. The parallel combination of inductance (L) and capacitance (C) form what is commonly known as a *tank circuit*.

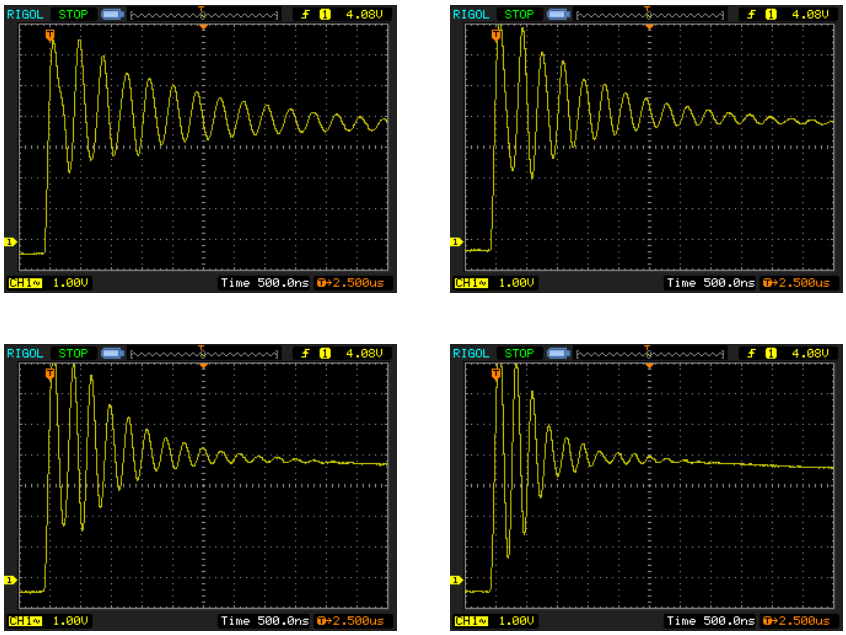
In an ideal tank circuit, oscillations will continue forever after initial energization, but in a real tank circuit dissipative losses in the circuit will cause the oscillations to subside over time:



Oscillations following the release of the pushbutton switch are measurable using an oscilloscope connected in parallel with the inductor and capacitor. These oscillations will be short-lived, and may consist of just a few cycles before they completely dissipate, therefore the oscilloscope will need to be configured for "single-sweep" mode to capture and hold the image for viewing.

[4]The windings of a step-down power transformer generally work well for this inductance if your goal is to create a relatively low frequency of oscillation. Feel free to experiment with using different windings to see their effects!

The following oscillographs were captured using a home-made air-core inductor (18 AWG copper wire bent into a solenoid-shaped coil 60 millimeters in diameter, 95 millimeters long, and having 14 turns) connected in parallel with a variable air-dielectric capacitor having a maximum capacitance of 617 pF. The resulting oscillations captured after pressing the pushbutton show very high frequency values due to these small L and C values. The capacitor was adjusted to a higher value in the upper-left oscillograph, and progressively decreased in capacitance for the other oscillographs moving left to right, top to bottom:



Note how with decreasing capacitance not only does the oscillatory frequency increase, but the oscillations decay more rapidly to zero as well. The hastened decay with less capacitance is the result of less capacitance being able to store less total energy at each peak of the wave (given the same initial voltage), and with less total energy stored it depletes more rapidly with each cycle.

2.6 Example: measuring signal rates of change

Capacitors and inductors relate voltage to current by rates of change. For a capacitor, the amount of current is proportional to how quickly voltage across that capacitor either rises or falls over time ($I = C\frac{dV}{dt}$). For an inductor, the amount of voltage is proportional to how quickly current through that inductor rises or falls over time ($V = L\frac{dI}{dt}$).

For example, a 330 microFarad capacitor experiencing a voltage increasing at a rate of 45 Volts per second will pass 14.85 milliAmperes. If the voltage happens to decrease at an equivalent rate (i.e. $\frac{dV}{dt} = -45$ Volts per second) then the 14.85 milliAmpere current will reverse direction through the capacitor compared to how it flowed with the increasing voltage.

Similarly, a 100 milliHenry inductor experiencing a current increasing at a rate of 5 Amperes per second will induce a voltage of 500 milliVolts. If the current happens to decrease by an equivalent rate (i.e. $\frac{dI}{dt} = -5$ Amperes per second) then the 500 milliVolt voltage induced across the inductor will reverse polarity from what it was during the period of increasing current.

Not only are rates-of-change important in determining how energy-storing devices such as capacitors and inductors will respond in circuits, but rates-of-change are also important for determining how *parasitic* capacitances and inductances will affect intended circuit behavior. Parasitic capacitance exists between any two conducting surfaces separated by an electrically insulating medium, and parasitic inductance exists along any length of conductor. This means any rate-of-change of voltage over time between two separated conductors will cause some amount of current to "pass"[5] between them, and that any rate-of-change of current over time through any single conductor will cause some amount of voltage to drop across its length. In many circuits these parasitic effects are negligible, but in circuits experiencing extremely fast rates of change for voltage and/or current the effects can be significant or even severe.

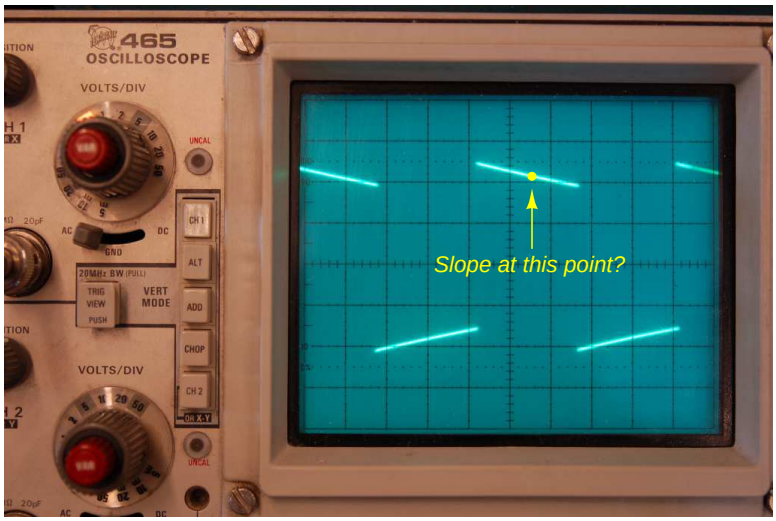
Oscilloscopes are ideal for performing empirical measurements of voltage rates-of-change, and of current rates-of-change given the proper accessories[6]. Some skill is required to do this, though, and here we will explore practical examples to show how it is done.

For any signal plotted in the time domain, where the horizontal axis of the plot is expressed in units of seconds, milliseconds, microseconds, etc., the signal's rate of change at any given point will be the *slope* or *pitch* of the waveform, mathematically defined as its *rise over run*. A great aid to discerning slope at any location on a waveform is to sketch a straight line visually matching the wave's slope at that point, then use locations along that straight line to more easily discern how far it rises (or falls) over some "run" of time. We call this straight line a *tangent line*.

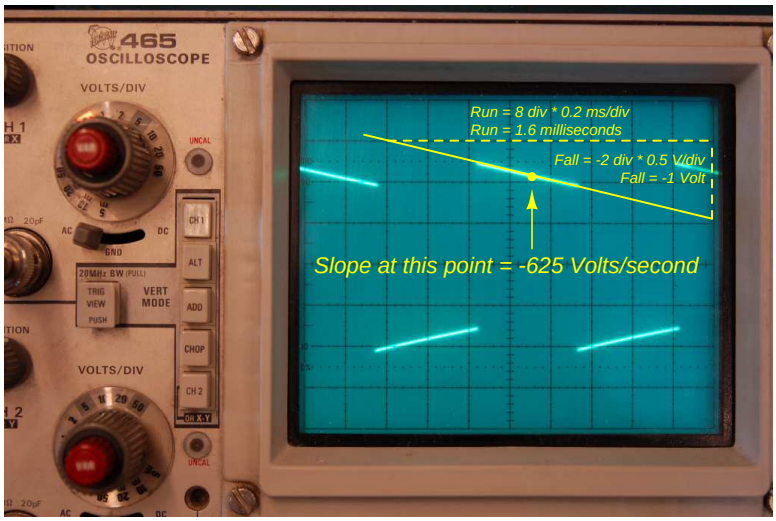
[5] The Scottish physicist James Clerk Maxwell referred to this current as *displacement current*.

[6] For example, a *current probe* converting a sensed current into a voltage the oscilloscope may directly sense, or a *shunt resistor* placed in the circuit developing an oscilloscope-measurable voltage drop for any current passing through.

Here we see an example of a waveform with sloping sections. In the first image we see a specific location on the waveform where we wish to measure voltage rate-of-change ($\frac{dV}{dt}$):



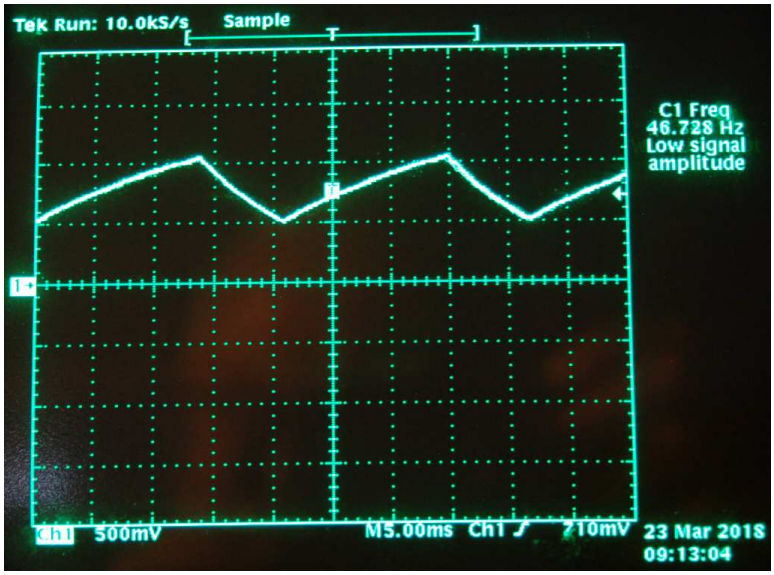
Next we see a tangent line drawn to match the slope of the waveform at the specified location, with convenient points for fall/run measurements taken on that line against the oscilloscope grid's major divisions. In this example, the oscilloscope's vertical sensitivity has been set for 0.5 Volts per division, and the horizontal timebase for 0.2 milliseconds per division:



As we can see, the tangent line falls 2 vertical divisions (-1 Volt) over a timespan of 8 horizontal divisions (1.6 milliseconds), yielding a $\frac{dV}{dt}$ quotient of -625 Volts per second, which may also be expressed as -0.625 Volts per millisecond. The negative sign is important, as it distinguishes this particular rate-of-change as *falling* rather than rising over time.

Such rate-of-change measurements are necessarily approximate, as they require us to visually gauge where a tangent line may be overlaid on the waveform's oscillograph, and also to visually assess the slope of that tangent line using the grid provided on the instrument's display screen. However, in most applications extremely precise rate-of-changes are not necessary, and such techniques suffice quite well.

Below is another example of a waveform with sloping sections, the oscilloscope configured for 500 milliVolts per division on the vertical axis and 5 milliseconds per division on the horizontal:

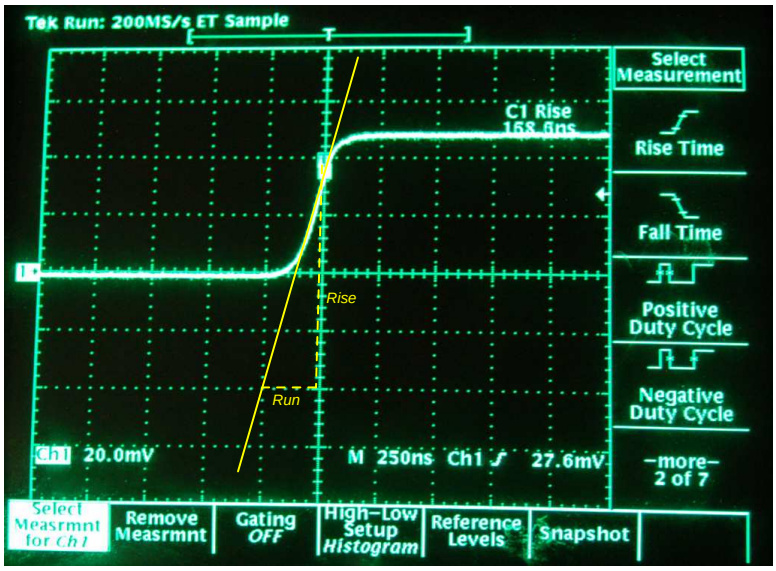


Approximating the slope for each rising section of this wave, we count one division of rise over 2.8 divisions of run, or 500 milliVolts rise over 14 milliseconds of run. This is a rate-of-rise of +35.7 Volts per second.

Approximating the slope for each falling section of this wave, we count one division of fall over 1.4 divisions of run, or −500 milliVolts fall over 7 milliseconds of run. This is a rate-of-fall of −71.4 Volts per second.

If these rates-of-change appear suspiciously large compared to the actual amplitude of the waveform, which barely crests over +1 Volt on the oscillograph, bear in mind that we are calculating *rates of change* for voltage and not absolute values of voltage itself. This is analogous to the distinction between *speed* and *distance*: traveling at a rate of 30 kilometers per hour does not necessarily mean you will travel 30 kilometers, as the actual distance traveled depends on how long that speed is sustained. A voltage rising at a rate of 35.7 Volts per second would indeed rise 35.7 Volts *if given a full second to do so*, but since each rising/falling portion of this waveform is so short in duration the actual amount of rise or fall in each case is only one-half of one Volt. It is therefore perfectly appropriate to consider any $\frac{dV}{dt}$ value as being the *speed* at which a voltage increases or decreases over time, distinct from the actual value of that voltage at any particular moment in time.

Here we see another oscillograph, this one zoomed into the rising edge of a square wave. For this measurement the oscilloscope was configured for 20 milliVolts per division of vertical sensitivity and a timebase of 250 nanoseconds per division on the horizontal:



The tangent line overlaid on this screenshot for the purpose of measuring the pulse edge's rate-of-change rises approximately 4 vertical divisions over a run of 1 division, which is 80 milliVolts of rise over 250 nanoseconds of run. The pulse edge's rate-of-change, therefore, is approximately +320,000 Volts per second, or +320 Volts per millisecond, or +0.32 Volts per microsecond (all equivalent expressions of $\frac{dV}{dt}$).

Chapter 3

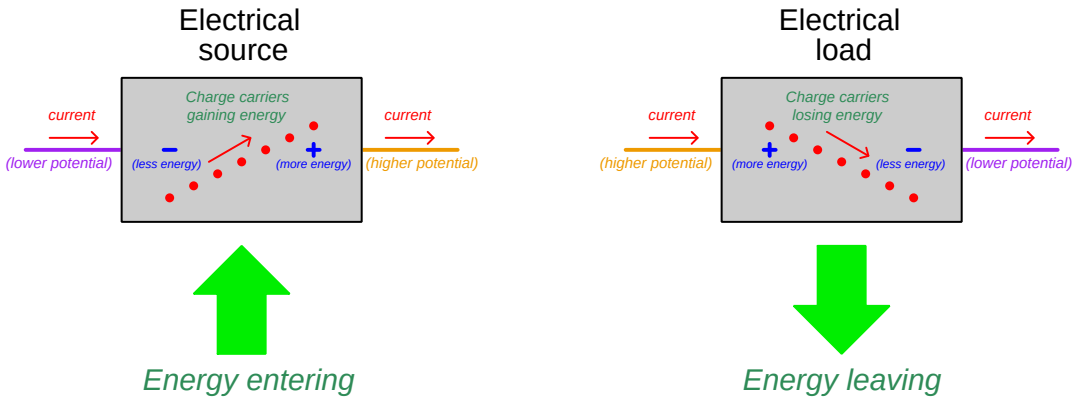
Simplified Tutorial

Energy is the ability to set matter into motion, and electric circuits convey energy by means of moving electric charges. We classify energy into two broad categories of *potential* and *kinetic* depending on whether or not motion is involved: potential energy is energy that *can* cause motion but is not currently causing motion, whereas kinetic energy is energy existing in moving objects. Examples of potential energy include weights suspended above ground level, compressed springs, compressed gases, unreacted chemical fuels, and separated electric charge carriers. Examples of kinetic energy include hot masses (the molecules being in constant motion), projectiles in flight, spinning objects, and moving charge carriers.

A remarkable feature of all energy is that it is *conserved*, which means it is eternal. No matter what happens to energy or what form it takes, the sum total of all energy in any isolated[1] system remains constant over time. Energy therefore cannot appear from nowhere nor vanish into nowhere – it must always be accounted.

[1] An "isolated" system is one where no energy is allowed to enter or leave.

If an electrical component transfers energy from one form to another, we classify that component as either a *source* or a *load*. A "source" transfers energy from some external store into the charge carriers of an electric circuit. A "load" extracts energy from charge carriers within a circuit and transfers that energy somewhere else. The conventions of + and − symbols for voltage[2] and "conventional flow" arrow notation for current[3] makes this clear:

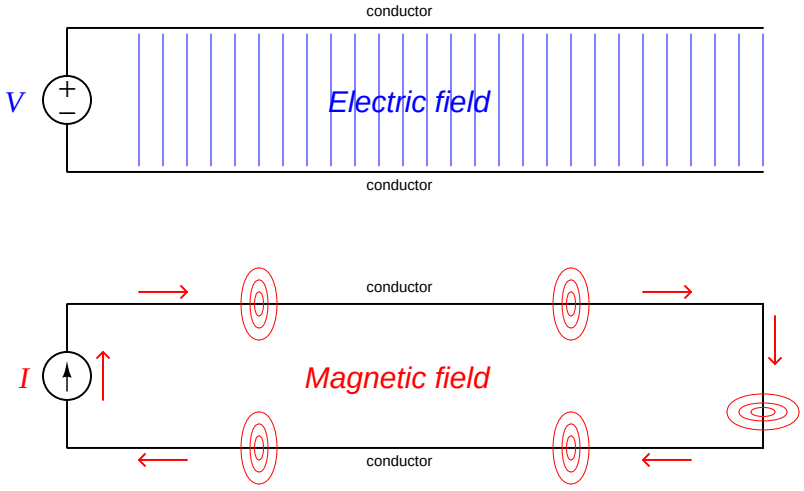


Resistors always behave as loads, converting energy into heat which then leaves the circuit as it dissipates into the surrounding environment. Secondary-cell batteries are able to behave as either sources (while discharging) or loads (while charging), exchanging energy between passing charge carriers and the chemical solution within the battery's cells.

[2] Voltage is defined as the amount of potential energy possessed by each electric charge, the unit of the *Volt* defined as one Joule of energy per Coulomb (equivalent to 6.2415×10^{18} electrons) of electric charges.

[3] Current is defined as the rate of charge carrier drift, the unit of the *Ampere* defined as one Coulomb of electric charges passing by per second of time.

Electric and magnetic fields are also able to store energy, electric fields existing wherever oppositely-charged carriers are separated over insulating space and magnetic fields existing wherever electric charge carriers move. Electric fields are associated with potential energy because the separated charges *would* move closer together if given the opportunity. Magnetic fields are associated with kinetic energy because they occur only when charge carriers are in a state of motion:



We refer to the ability to store energy in an electric field *capacitance*, while the ability to store energy in a magnetic field is called *inductance*. Capacitance is measured in the unit of the *Farad* (F) and is symbolized by the variable C. Inductance is measured in the unit of the *Henry* (H) and is symbolized by the variable L. The amount of energy stored by either mechanism may be calculated as follows:

$$E_C = \frac{1}{2}CV^2 \qquad\qquad E_L = \frac{1}{2}LI^2$$

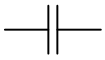
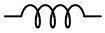
Where,

E_C = Energy stored in capacitance (Joules)
C = Capacitance (Farads)
V = Voltage across the capacitance (Volts)

E_L = Energy stored in inductance (Joules)
L = Inductance (Henrys)
I = Current through the inductance (Amperes)

An electrical component designed to enhance capacitance is called as *capacitor*, and an electrical component designed to enhance inductance is called an *inductor*. Capacitors typically take the form of two metal plates separated by a small insulating gap, while inductors typically take the form of a coiled wire (often wrapped around an iron core).

Comparisons between capacitance and inductance may be seen in the following table. It is difficult to ignore the symmetry between these two phenomena:

Capacitance (C)	**Inductance** (L)
 NEMA symbol IEC symbol	 NEMA symbol IEC symbol
An *electric field* is created by a *voltage* existing between two conductors	A *magnetic field* is created by a *current* passing through a conductor
Capacitance is enhanced by the area of the conductors, and a narrow spacing between them	Inductance is enhanced by the length of the conductor, and forming a coil shape with multiple turns
An electric field stores energy as a function of voltage: $E_C = \frac{1}{2}CV^2$	A magnetic field stores energy as a function of current: $E_L = \frac{1}{2}LI^2$

Applications of capacitance and inductance abound – what follows is a brief summary of uses.

Both capacitors and inductors may be used to very rapidly store and release energy because their rate of energy transfer is not limited by chemical reaction speed as is the case with secondary-cell batteries. Energy transfer rate is important for certain applications such as high-energy physics experiments and kinetic energy weapons (e.g. "rail guns").

Capacitors may be used to stabilize voltage in a circuit, while inductors may be used to stabilize current. The principle here is that energy is always conserved, and since capacitors store energy in an electric field which manifests as a voltage and inductors store energy in a magnetic field which manifests as a current, these electrical parameters cannot vary without energy transfer[4] to or from the component. The greater the energy storage ability of that component, the more work will be required to make voltage or current vary, and therefore the more stable those parameters will be in the circuit.

Capacitance and inductance may also be used to generate voltages and currents that vary over time. Any energy-storing component connected to a load will gradually transfer its energy to that load, and the transfer will be marked by changes in voltage and/or current over time. This phenomenon may be used to create precise *time delays*.

When capacitance and inductance exists together, energy may transfer back and forth between C and L to form alternating *oscillations* of voltage and current (i.e. AC) over time. This is called *resonance*, and since capacitance and inductance both exist to some degree in all circuits this means unwanted oscillations (sometimes called "ringing") may occur.

[4] The principle might be easier to grasp if you imagine a universe in which energy was *not* conserved. In such a universe, a capacitor's voltage could increase or decrease with no energy transfer at all, and the same for an inductor's current. This would be analogous to a cup of water suddenly possessing more or less water than before (i.e. water level varying with no transfer of water into or out of the cup), in a universe with no Conservation of Mass.

Chapter 4

Full Tutorial

4.1 Review of sources and loads

Energy is the ability to set matter into motion, and is broadly grouped into two different types: *potential* and *kinetic*. Potential energy is that which has the *potential* to set matter into motion but exists without motion being present. Kinetic energy is present in moving objects, which of course have the ability to move other objects by collision. Examples of potential energy include weights suspended above ground level, compressed springs, compressed gases, and unreacted chemical fuels. Examples of kinetic energy include hot masses (the molecules being in constant motion), projectiles in flight, and spinning objects.

One of the fundamental properties of energy is that it cannot spontaneously appear or disappear. The potential energy stored in a compressed spring, for example, cannot simply vanish without a trace: in order for the spring to relax and relinquish energy, that energy must be transferred to some other form such as the motion of another object. Likewise, a relaxed spring never becomes compressed all on its own: some external energy source must invest the work necessary to compress the spring (and in doing so, energy is transferred from that external source to the spring). Kinetic energy is similarly accounted: a moving object does not slow down unless and until it imparts energy to something else; nor do objects suddenly begin moving without the action of a motivating force. We refer to this absolute accounting of energy as the *Conservation of Energy*. This is one of the foundational Laws of physics, having no known exception.

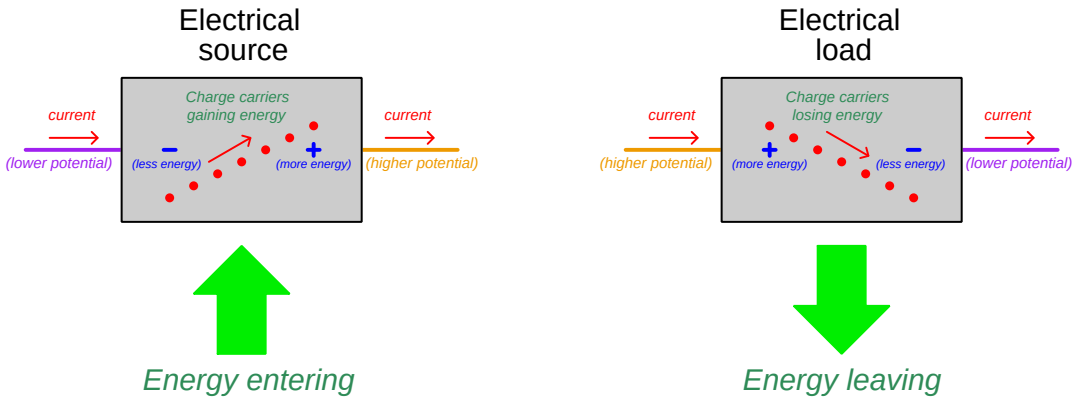
An early expression of energy conservation is Isaac Newton's First Law of Motion: *An object at rest tends to stay at rest; an object in motion tends to stay in motion.* An object at rest is by definition not moving, and so possesses no kinetic energy. Therefore, that object will remain at rest unless and until some other form of energy comes to bear on it to make it move. An object in motion possesses kinetic energy, and will continue to possess that energy (i.e. continue to move) unless energy transfers to or from the moving object by way of interaction with something else.

Electricity is the transfer of energy by electric charges. Charges may act as couriers of energy, picking up energy from electrical components called *sources* and delivering energy to electrical devices called *loads*. A simple example circuit of a battery powering a lamp through metal wires illustrates this energy transfer: chemical compounds inside the battery react in such a way as to transition from high-energy states into lower-energy states, and in doing so deliver this energy to mobile charge carriers (free electrons and ions) inside the battery. Free charge carriers inside the metal wires (electrons) receive this energy from the battery and deliver it to the lamp, where the charge carriers encounter *resistance* to motion. This resistance drains the charge carriers of the energy they absorbed from the battery, the energy manifesting as light and heat at the lamp. The end-result is that the battery loses energy over time, while the lamp dissipates that energy into light and heat which leaves the circuit entirely as it radiates into the surrounding environment. Mobile charge carriers inside the metal wires merely act to convey that energy from the battery's chemical interior to the lamp. The amount of energy absorbed by or released by each electric charge as it moves[1] from one location to another is called *voltage*, while the rate of charge motion through any electrically conductive pathway is called *current*[2].

[1] Voltage may exist between two points between which there is no actual path for charge carriers to move. In such a case, we define voltage as the amount of energy that *would be* gained or lost by a "test charge" carried across the gap by some means of conveyance.

[2] Since electric charges are so miniscule, we number them in larger units called *Coulombs*, each Coulomb equivalent to 6.2415×10^{18} electrons. One Volt of electric potential is thus defined as one Joule of potential energy gained or lost

Regardless of the types of energy sources, charge carriers, or uses, we denote the difference of charge carrier potential energy between any two points using − and + "polarity" symbols, the − (negative) symbol representing the point of lesser energy and the + (positive) symbol representing the point of greater energy. Electric current, following this same convention, is shown flowing from − to + when passing through a source, and flowing from + to − when passing through a load:

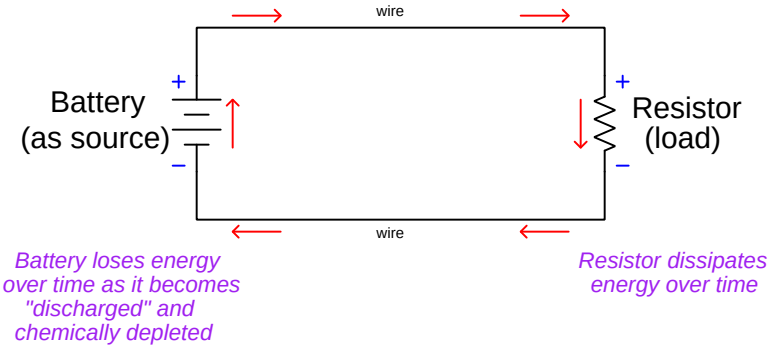


Please take note of the relationship between voltage (polarity) and current (direction of charge carrier motion) for sources and loads. It is the *combination* of voltage polarity and current direction distinguishing a source from a load in any annotated electric circuit. Specifically, the orientation of the polarity marks in relation to current arrows denotes whether charge carriers are gaining energy or losing energy as they pass through the component in question.

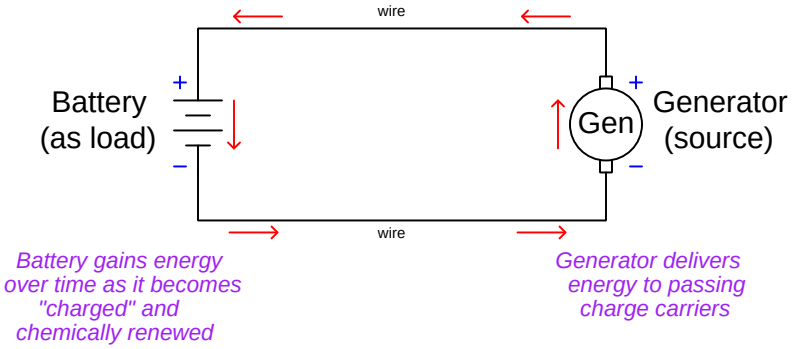
Some electrical components always function as sources (e.g. primary-cell batteries), while some other electrical components always function as loads (e.g. resistors). Others yet are able to alternately function as either sources or loads (e.g. secondary-cell, or "rechargeable" batteries), being able to supply energy to passing charge carriers, or extract energy from them.

per Coulomb of charge carriers. One Ampere of electric current is defined as the passage of one Coulomb of charge carriers per second of time.

A rechargeable battery, for example, acts as an electrical source when current flows from its negative terminal through to its positive terminal, but will act as an electrical load when current is made to flow through the other direction. When acting as a source the battery's internal store of energy depletes over time as that energy is delivered to the load and dissipated there:



If current is forced the other direction through the rechargeable battery by another electrical source, the battery will function as a load which means its internal store of energy becomes renewed over time as it absorbs energy from the other source:



Note how voltage polarity remains fixed while current switches direction when the battery transitions between acting as a source and acting as a load. The reason for this lies with how a battery stores energy: in chemical form, which is a function of electron potentials within the molecules of the battery's electrodes and the electrolyte separating them[3]. Since energy is always conserved, and stored energy within a chemical battery manifests as voltage, it is voltage that is the conserved parameter while current changes direction based on the accumulation or release of energy.

Secondary-cell batteries are not unique in their ability to function as either electrical sources or electrical loads. Other components are known to do this as well, including electromechanical motors which may function as electrical loads (as motors, extracting energy from charge carriers to drive a

[3] An *electrode* is simply a conductive terminal designed to make contact with something else. In the case of a battery, the electrodes are rods and/or plates immersed in a chemical electrolyte solution.

mechanism) or as electrical sources (as generators, taking energy from a prime mover and infusing that energy into charge carriers). The drive motor of an electric vehicle is a good example of a machine alternately behaving as an electrical load (accelerating the vehicle by using energy from the battery) or an electrical source (decelerating the vehicle by delivering energy to the battery).

4.2 Energy storage in electric fields

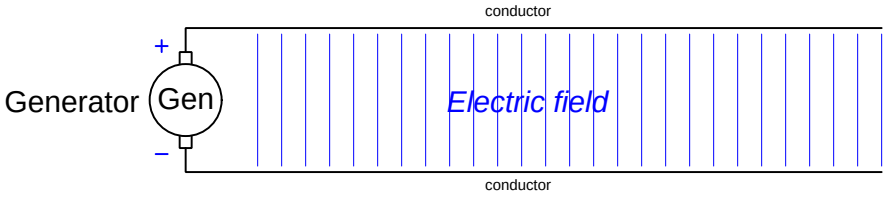
An interesting phenomenon in electric circuits is the association between *fields* and energy. A "field" is an abstract concept, denoting invisible lines of interaction through space. Gravity is an example of a field, acting upon objects possessing mass. Electric fields act upon objects possessing electric charge. Magnetic fields act upon *moving* electrical charges. There are two fundamental types of fields encountered in electric circuits: *electric fields* spanning the space between two points of different electric potential (i.e. two points between which there exists a voltage), and *magnetic fields* circulating in the space surrounding any current-carrying conductor. This means any region of empty space separating two conductors is capable of storing energy by virtue of the electric field established by voltage between the conductors, and any region of empty space around a conductor is capable of storing energy by virtue of the magnetic field established by current passing through the conductor. In summary, fields acts as reservoirs of energy which may be filled or emptied: electric fields by altering the amount of voltage and magnetic fields by altering the amount of current.

The term used to describe energy storage within an electric field is called *capacitance*, and the term used to describe energy storage within a magnetic field is called *inductance*. These two terms together embody a rather remarkable idea: that simple conductors may function as energy-storing components in and of themselves. The degree to which any given length of wire can store energy by participating in the formation of either an electric or a magnetic field is typically so miniscule that it is ignored for all practical purposes. This is why the twin effects of capacitance and inductance are completely overlooked when performing calculations on basic resistive circuits: the amount of energy stored along the wires connecting a battery to a lamp is negligible compared to the energy being supplied by the battery and dissipated by the lamp. However, there are some applications where the energy associated with electric and magnetic fields around conductors becomes significant, and of course we may design components intended to accentuate these effects.

Let's explore capacitance and inductance through *thought experiments*: imaginary scenarios where we apply the consequences of known principles. We will take what we know about electric and magnetic fields, sources and loads, as well as voltage and current, to see how capacitance and inductance both work as circuit elements. For the capacitance thought experiment, we will consider a case of two parallel conductors separated by an air gap, connected to a generator of varying output voltage. For the inductance thought experiment, we will consider a case of a long wire loop connected to a generator of varying output current. In both cases we will assume wires with zero electrical resistance[4], just to eliminate any effects not directly the result of capacitance or inductance.

[4]Such conductors do exist, and they are called *superconductors*. A superconducting wire possesses absolutely no electrical resistance whatsoever, allowing electric charge carriers to pass from one end to the other without losing any energy at all. Electric charge carriers drifting through a normal metallic conductor experience collisions along the way, each collision robbing some energy from the charge carrier and dissipating it in the form of heat. In fact, these collisions through a normal conductor constitute the basis for *Ohm's Law* ($V = IR$): the faster the drift velocity (increased I), the more energy will be lost by each electric charge carrier during each collision (increased V drop) assuming an equal number of collisions (constant R).

First, the capacitance thought experiment. The two conductors represent an electrically *parallel* structure with the terminals of the generator – two sets of electrically common points. Therefore, the voltage measurable between any two points across the gap must be equal to the generator's output voltage at any given time:

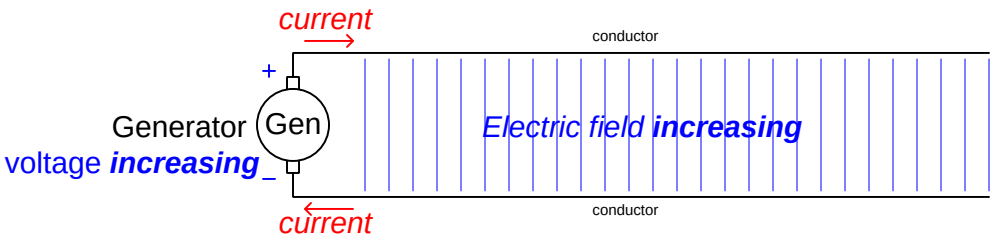


Electric fields are always present in the space between points at differing potential. That is to say, an electric field exists whenever there is voltage between two points[5]. As voltage increases, so does the intensity of the electric field.

Consider what will happen if the generator's voltage output increases. As previously described, this increases the intensity of the electric field spanning the gap between the two conductors. At higher voltage levels, the energy difference between charge carriers located on either side of the gap increases: the higher-energy charges literally carry more energy than they did before. This means the two-conductor structure now harbors more potential energy than before.

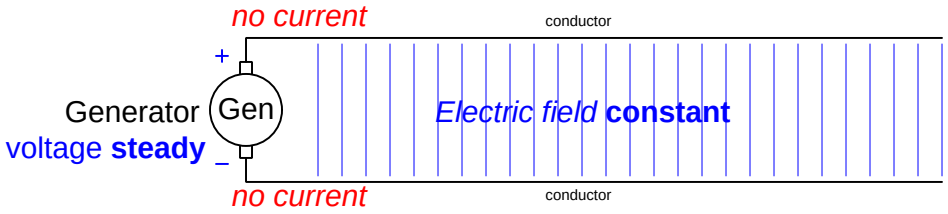
If this new, stronger electric field represents a greater potential energy difference between the two conductors' charge carriers, it must mean it absorbed energy from the generator during the time the generator's output voltage increased because that additional energy could not have come from nowhere. In other words, the two parallel conductors must behave as a *load* during the time the generator's output voltage increases.

Consider what we know about loads and how they behave in a circuit: in order for a component to be a load, charge carriers must flow into its positive terminal and out of its negative terminal. Drawing current arrows in conventional flow notation to represent this action of charge carriers flowing into the positive conductor and out of the negative conductor:



[5]This makes sense based on our definition of voltage, as being *potential energy per unit electric charge*. Potential energy is fundamentally a result of displacement and force along the same axis. In order for there to be an electric potential between two points, electric charges must be at different energy levels from one point to the other. The distance separating those points represents the *displacement* necessary for potential energy, and so the *force* must arise from an electric field tugging at the charges. Viewed from the perspective of energy, the intensity of the electric field must be *proportional* to voltage so long as the distance separating the charges (displacement) remains constant: with displacement fixed, the only variable influencing the potential energy of each charge is the force exerted on each charge by the electric field. Therefore, an increased voltage across the same separation distance must result in a stronger electric field.

If the generator's output now stabilizes at some fixed voltage value, this load action ceases. The two conductors now have a stronger electric field between them than before, and their respective charge carriers exist at a greater potential energy difference than before, but since there is no dissipation of energy into heat, light, or any other form, this energy is simply *stored*. Therefore, any current that existed while the generator's output voltage was rising now ceases.



The fact that current existed while the generator's voltage increased, but no current exists now, means that some electric charges were *displaced* during that time and remain in a state of tension. One may think of this as some charges having been drawn out of one wire and pushed into the other by the generator's voltage: an example of *static* (i.e. stationary) electricity.

We may define capacitance in terms of how much charge gets displaced for a given amount of voltage applied between the two conductors:

$$C = \frac{Q}{V}$$

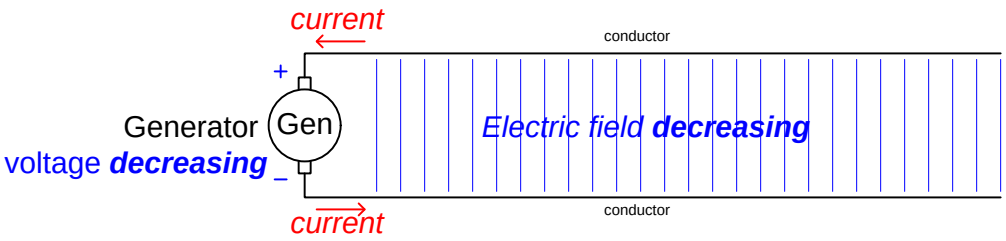
Where,

Q = Charge displaced (Coulombs)

C = Capacitance (Farads)

V = Applied voltage (Volts)

If the two conductors behave as a temporary load while the generator's voltage increases, it stands to reason they will behave as a temporary *source*[6] while the generator's voltage decreases. Decreased voltage represents a weaker electric field, and correspondingly less potential energy difference per electric charge carrier along the two conductors. Since energy cannot disappear into nowhere, this reduced energy must mean the assembly releases energy when experiencing a decreasing voltage, with a corresponding current flowing out of the positive conductor and into the negative conductor:



The amount of current flowing into or out of this capacitance is a direct function of how quickly voltage varies over time. This may be expressed using calculus "derivative" notation:

$$I = C\frac{dV}{dt}$$

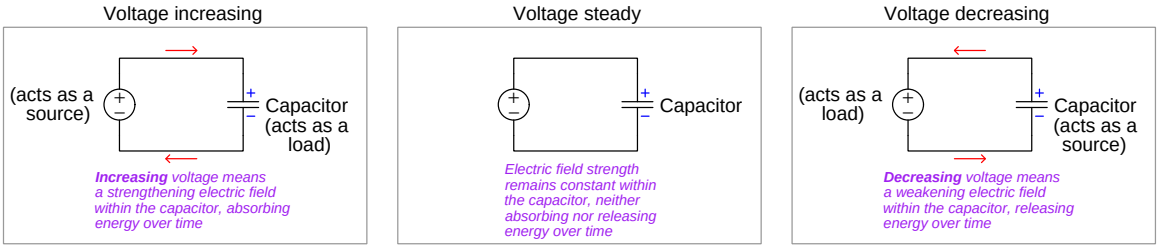
Where,
I = Current (Amperes)
C = Capacitance (Farads)
$\frac{dV}{dt}$ = Rate of voltage rise/fall over time (Volts per second)

Unlike current through a resistance, which is a simple proportion ($I = \frac{V}{R}$), current through a capacitance is a function of *time*. When the voltage is steady ($\frac{dV}{dt} = 0$) there will be zero current. When voltage rises ($\frac{dV}{dt} = +$), current flows as through a load. When voltage falls ($\frac{dV}{dt} = -$), current flows as through a source.

The $I = C\frac{dV}{dt}$ formula may in fact be derived from the charge/capacitance formula $Q = CV$ seen earlier. If we realize that current (I) is simply the rate at which electric charges (Q) move, then I must be mathematically equal to $\frac{dQ}{dt}$. If we "differentiate" both sides of the $Q = CV$ formula with respect to time, Q becomes $\frac{dQ}{dt}$ and V becomes $\frac{dV}{dt}$ while C remains unchanged since it is a constant for any given pair of conductors. Therefore, if $Q = CV$ then $\frac{dQ}{dt} = I = C\frac{dV}{dt}$.

[6] If the two conductors release energy, they must do so to the generator, which means the generator will temporarily act as a *motor* and be powered by that released energy!

This loading/sourcing effect of two parallel conductors is vanishingly small when ordinary wires are involved, but the effect may be magnified if we build a special component with large-area conductors called *plates* separated by a small gap to intensify the electric field. Such a component is called a *capacitor*, and its NEMA[7] symbol is a pair of parallel lines (representing the two conductive plates). Drawn as a discrete component, connected to a variable-voltage source, we may show its behavior for conditions of increasing, steady, and decreasing voltage:



Like a secondary-cell battery, note how the capacitor's voltage polarity remains fixed while current switches direction when the capacitor transitions between acting as a source and acting as a load. The reason for this lies with how a capacitor stores energy: in an electric field, which is a function of voltage between the two plates. **Since energy is always conserved, and stored energy within a capacitor manifests as voltage, it is voltage that is conserved while current changes direction based on the accumulation or release of energy.**

This energy storage may be quantified[8] by the following formula, relating stored energy (E)[9] to the amount of voltage between the capacitor plates (V) and the capacitance (C) reflecting the energy-storing capability of the electric field's space:

$$E_C = \frac{1}{2}CV^2$$

Where,

E_C = Energy stored in the capacitor (Joules)

C = Capacitance (Farads)

V = Voltage between the two capacitor plates (Volts)

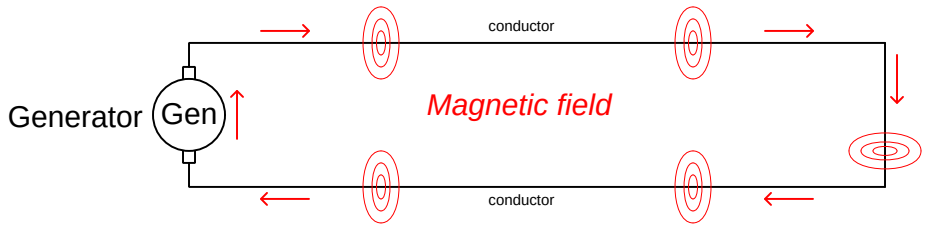
[7] The National Electrical Manufacturers Association, a standards organization in North America. The European (IEC) symbol for a capacitor is very similar to the NEMA symbol, the only difference being thicker line segments used to represent the two plates.

[8] A calorimetric (heat-based) procedure for quantifying the energy stored by an energized capacitor is to first "charge" or energize the capacitor by connecting it in parallel with a voltage source of known voltage. Then, disconnect the capacitor from that source and subsequently connect it to a resistor of known mass and specific heat, where the temperature rise created by the dissipated energy may be computed to arrive at an energy value in Joules.

[9] Throughout this tutorial series the variable E will be taken to mean *energy*. Many texts on electric circuit theory use E to represent voltage output by a source (often labeled as *electro-motive force*, or *EMF*), as opposed to V representing voltage dropped by a load. I make no such distinction here, principally because the concept of energy and its conservation is so central to my approach of teaching electric circuit theory. Also, the use of the word "force" to describe voltage is not scientifically accurate and therefore will never be used to describe voltage in these tutorials. Although forces exerted on electric charges by electric fields are certainly real, force alone is insufficient to define energy and therefore inadequate in describing the energy possessed or gained or lost by a charge.

4.3 Energy storage in magnetic fields

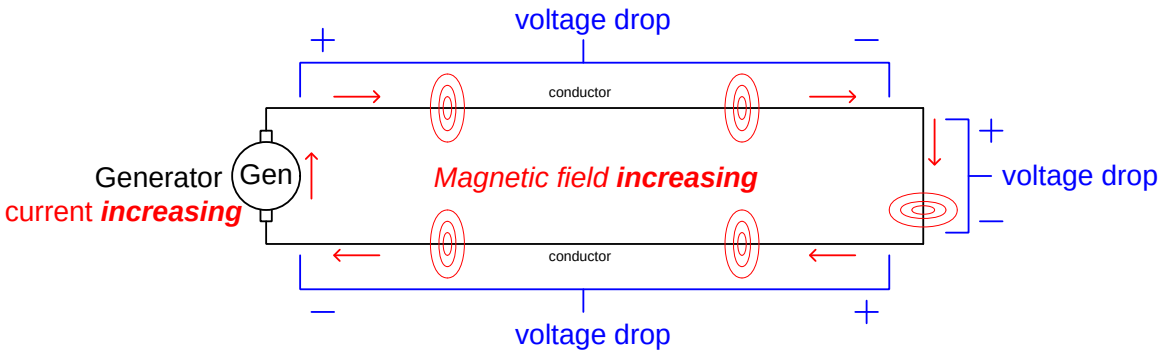
Next, we will explore inductance through a similar thought experiment. A wire loop connected to the terminals of a generator represents an electrically *series* structure – a single conductive pathway through which electric charge carriers may flow. Therefore, the current measurable at any point in this loop must be equal to the generator's output current at any given time:



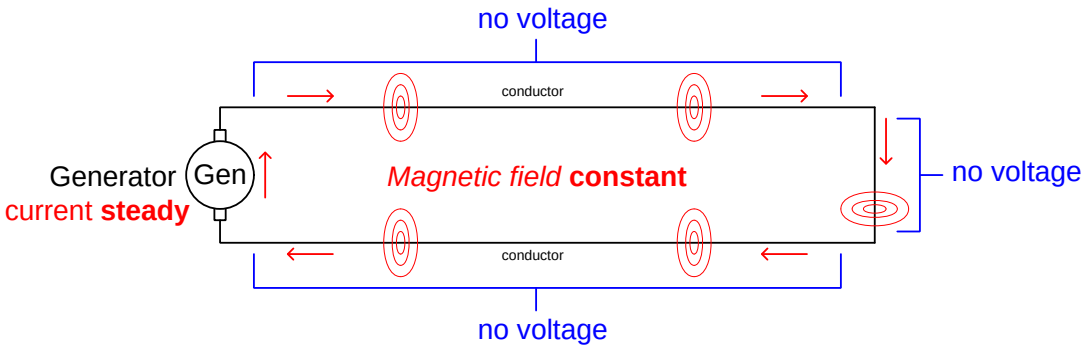
Magnetic fields are always present around a current-carrying conductor. That is to say, a magnetic field exists whenever there is electric charge motion. As current increases, so does the intensity of the magnetic field.

Consider what will happen if the generator's current output increases. As previously described, this increases the intensity of the magnetic field surrounding each conductor. Like electric fields, magnetic fields represent an investment of energy, and so a stronger magnetic field represents an increase in stored energy. If this new, stronger magnetic field represents a greater energy level, it must mean the field absorbed energy from the generator during the time the generator's output current increased because that additional energy could not have come from nowhere. In other words, the conductive loop behaves as a *load* during the time the generator's output current increases.

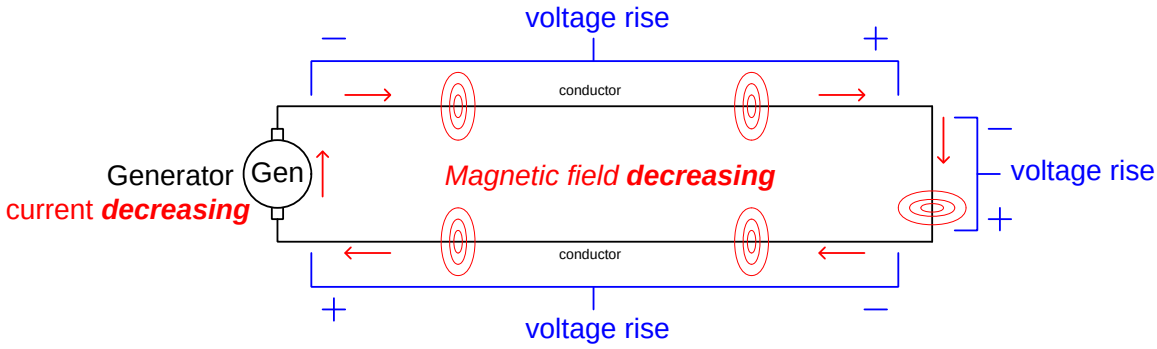
Consider what we know about loads and how they behave in a circuit: in order for a component to be a load, it must manifest a *voltage drop* from one terminal to the other, as charge carriers flow into its positive terminal at higher energy and leave its negative terminal at lower energy. Drawing polarity marks across each straight-line length of conductor to represent the depletion of energy from charge carriers as they deliver energy to the magnetic field:



If the generator's output now stabilizes at some fixed current value, this load action ceases. The conductive loop now has a stronger magnetic field than before, representing a greater level of energy, but since there is no dissipation of energy into heat, light, or any other form, this energy is simply *stored*. Therefore, any voltage drops[10] that existed while the generator's output current was rising now vanish.



If the conductive loop behaves as a temporary load while the generator's current increases, it stands to reason it will behave as a temporary *source*[11] while the generator's current decreases. Decreased current represents a weaker magnetic field, and correspondingly less energy stored in that field. Since energy cannot disappear into nowhere, this reduced energy must mean the assembly releases energy when experiencing a decreasing current, with a corresponding voltage rise as charge carriers leave with more energy than they entered:



[10] Again, we are assuming the use of superconducting wires for this thought experiment. Normal wires will exhibit some voltage drop due to resistance.

[11] If the loop releases energy, it must do so to the generator, which means the generator will temporarily act as a *motor* and be powered by that released energy!

The amount of voltage manifest along the conductive wires is a direct function of how quickly current varies over time. This may be expressed using calculus "derivative" notation:

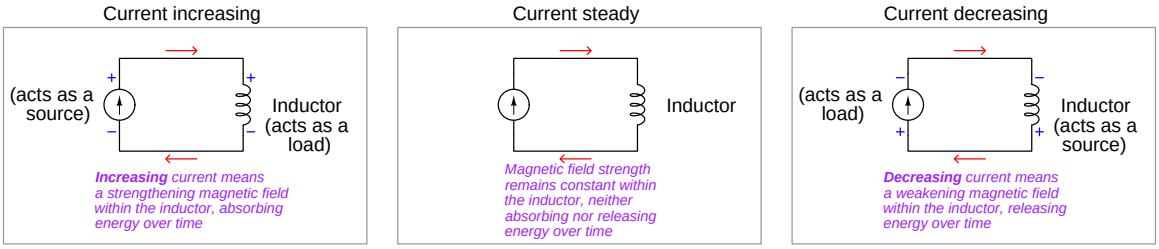
$$V = L\frac{dI}{dt}$$

Where,

V = Voltage (Volts)
L = Inductance (Henrys)
$\frac{dI}{dt}$ = Rate of current rise/fall over time (Amperes per second)

Unlike voltage dropped across a resistance, which is a simple proportion ($V = IR$), voltage across an inductance is a function of *time*. When the current is steady ($\frac{dI}{dt} = 0$) there will be zero voltage drop or rise. When current rises ($\frac{dI}{dt} = +$), voltage drops as across a load. When current falls ($\frac{dI}{dt} = -$), voltage rises as across a source.

This loading/sourcing effect of a conductive loop is miniscule when ordinary wires are involved, but the effect may be magnified if we build a special component with a wire *coil* designed to intensify the magnetic field. Such a component is called an *inductor*, and its NEMA symbol[12] actually resembles a wire coil. Drawn as a discrete component, connected to a variable-current source, we may show its behavior for conditions of increasing, steady, and decreasing current:



Note how the inductor's current direction remains fixed while voltage switches polarity when the inductor transitions between acting as a source and acting as a load. The reason for this lies with how an inductor stores energy: in a magnetic field, which is a function of *current*. **Since energy is always conserved, and stored energy within an inductor manifests as current, it is current that is conserved while voltage changes polarity based on the accumulation or release of energy.**

[12] The European (IEC) symbol for an inductor is a solid-color rectangle, identical in shape to the IEC resistor symbol (a hollow rectangle).

This energy storage may be quantified[13] by the following formula, relating stored energy (E) to the amount of current through the inductor's wire (I) and inductance (L) reflecting the energy-storing capability of the magnetic field's space:

$$E_L = \frac{1}{2}LI^2$$

Where,
E_L = Energy stored in the inductor (Joules)
L = Inductance (Henrys)
I = Current passing through the inductor (Amperes)

It is worth pausing at this juncture to note the similarities and differences between resistance, capacitance, and inductance. Resistance is the relationship of voltage to current irrespective of time. Capacitance is the relationship of current to the rate at which voltage varies over time. Inductance is the relationship of voltage to the rate at which current varies over time:

$$V = IR \qquad I = C\frac{dV}{dt} \qquad V = L\frac{dI}{dt}$$

Resistances only dissipate energy and never store or release energy. Capacitance and inductance alike only[14] store and release energy and never dissipate energy:

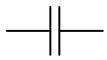
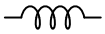
$$E_C = \frac{1}{2}CV^2 \qquad E_L = \frac{1}{2}LI^2$$

[13] The same calorimetric (heat-based) procedure for quantifying the energy stored by an energized capacitor is applicable to inductors as well: first "charge" or energize the inductor by connecting it in series with a current source of known current. Then, simultaneously disconnect the inductor from that source and connect it to a resistor of known mass and specific heat, where the temperature rise created by the dissipated energy may be computed to arrive at an energy value in Joules.

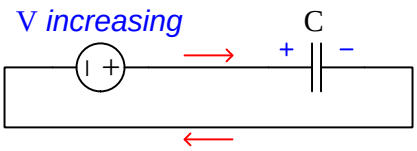
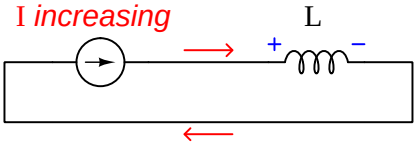
[14] This is true for ideal capacitors and inductors, but real components invariably contain some resistance or the equivalent of resistance resulting in *some* dissipation of energy into heat.

4.4 Capacitance and inductance compared

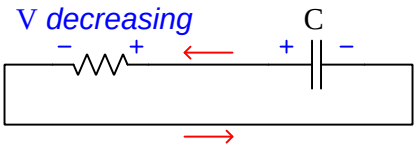
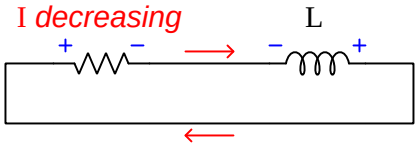
It should be clear by now that capacitance and inductance are symmetrical phenomena – in both cases they represent energy storage within invisible fields, but in complementary ways:

Capacitance (C)	**Inductance** (L)
 NEMA symbol IEC symbol	 NEMA symbol IEC symbol
An *electric field* is created by a *voltage* existing between two conductors	A *magnetic field* is created by a *current* passing through a conductor
Capacitance is enhanced by the area of the conductors, and a narrow spacing between them	Inductance is enhanced by the length of the conductor, and forming a coil shape with multiple turns
An electric field stores energy as a function of voltage: $E_C = \frac{1}{2}CV^2$	A magnetic field stores energy as a function of current: $E_L = \frac{1}{2}LI^2$
Acts as a *load* when *voltage increases* over time ; current flows into + terminal and out of − terminal	Acts as a *load* when *current increases* over time ; voltage drop + where current enters and − where current exits
Maintains a constant voltage when passively storing energy	Maintains a constant current when passively storing energy
Acts as a *source* when *voltage decreases* over time ; current flows out of + terminal and into − terminal	Acts as a *source* when *current decreases* over time ; voltage rise − where current enters and + where current exits
Current is able to *instantly* cease or reverse direction while voltage cannot; current switches direction when capacitor transitions between source and load	Voltage is able to *instantly* cease or reverse polarity while current cannot; voltage switches polarity when inductor transitions between source and load
The amount of current is proportional to the *rate* at which energy is being absorbed by or released from the electric field	The amount of voltage is proportional to the *rate* at which energy is being absorbed by or released from the magnetic field
Current is equal to capacitance times the *speed* that voltage varies over time: $I = C\frac{dV}{dt}$	Voltage is equal to inductance times the *speed* that current varies over time: $V = L\frac{dI}{dt}$

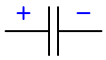
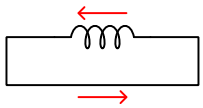
The relationships between energy, electric fields, magnetic fields, voltage, and current are foundational to understanding how capacitors and inductors behave in circuits. Consider what happens when each of these components behaves as an electrical load, extracting energy from passing charge carriers:

Capacitor as a load	**Inductor as a load**
	
An electric field stores energy as a function of voltage: $E_C = \frac{1}{2}CV^2$	A magnetic field stores energy as a function of current: $E_L = \frac{1}{2}LI^2$
Energy extracted from the charge carriers is absorbed by the electric field which makes it grow more intense	Energy extracted from the charge carriers is absorbed by the magnetic field which makes it grow more intense
Increasing electric field = capacitor gaining energy = increasing voltage	Increasing magnetic field = inductor gaining energy = increasing current
Acts as a *load* when *voltage increases* over time ; current flows into + terminal and out of − terminal: $I = C\frac{dV}{dt}$ = positive	Acts as a *load* when *current increases* over time ; voltage drop + where current enters and − where current exits: $V = L\frac{dI}{dt}$ = positive
Work is being *done on* the capacitor	Work is being *done on* the inductor
The amount of current is proportional to the *rate* at which energy is being absorbed by the electric field	The amount of voltage is proportional to the *rate* at which energy is being absorbed by the magnetic field

These same principles also explain how capacitors and inductors function as sources, imparting energy to passing charge carriers. Resistors are shown connected as loads to dissipate energy:

Capacitor as a source	**Inductor as a source**
	
An electric field stores energy as a function of voltage: $E_C = \frac{1}{2}CV^2$	A magnetic field stores energy as a function of current: $E_L = \frac{1}{2}LI^2$
Energy imparted to the charge carriers is released by the electric field which makes it grow weaker	Energy imparted to the charge carriers is released by the magnetic field which makes it grow weaker
Decreasing electric field = capacitor losing energy = decreasing voltage	Decreasing magnetic field = inductor losing energy = decreasing current
Acts as a *source* when *voltage decreases* over time ; current flows out of + terminal and into − terminal: $I = C\frac{dV}{dt}$ = negative	Acts as a *source* when *current decreases* over time ; voltage rise − where current enters and + where current exits: $V = L\frac{dI}{dt}$ = negative
Work is being *done by* the capacitor	Work is being *done by* the inductor
The amount of current is proportional to the *rate* at which energy is being released by the electric field	The amount of voltage is proportional to the *rate* at which energy is being released by the magnetic field

Furthermore, these principles explain why capacitors and inductors behave as they do when disconnected from any external source or load. In order to place a capacitor into a condition where it neither gains nor loses energy but merely stores[15] it, we must form an open circuit. An inductor requires a short-circuit[16] to do the same:

Capacitor passively storing energy	**Inductor passively storing energy**
	
An electric field stores energy as a function of voltage: $E_C = \frac{1}{2}CV^2$	A magnetic field stores energy as a function of current: $E_L = \frac{1}{2}LI^2$
Voltage is constant because the electric field is constant, neither gaining nor losing energy	Current is constant because the magnetic field is constant, neither gaining nor losing energy
Zero current ensures no charges convey energy out of the capacitor (i.e. zero rate of energy loss): $I = C\frac{dV}{dt} = 0$	Zero voltage ensures charges lose no energy as they circulate (i.e. zero rate of energy loss): $V = L\frac{dI}{dt} = 0$
An open-circuit condition guarantees zero current	A (perfect) short-circuit condition guarantees zero voltage drop
Current is able to *instantly* stop while voltage cannot	Voltage is able to *instantly* cease while current cannot

The amount of energy which may be stored within a practical inductor or capacitor is relatively little compared to a chemical battery of similar size. This means we cannot use capacitors or inductors as equivalent alternatives to secondary-cell batteries. However, in applications where extremely high *power* is required, capacitors and inductors perform the task of energy storage better than batteries. The rate at which energy may be delivered (i.e. the definition of *power*)[17] by a chemical battery is limited by the reaction rate of the chemical compounds stored within. Capacitors and inductors, by contrast, are virtually unlimited by comparison. This means a capacitor or inductor is able to deliver its entire store of energy in a very short time span (small fractions of a second) to create an extremely powerful impulse.

[15] Real capacitors are not perfect energy-storage devices due to the fact that dielectric materials are not perfect insulators. Real capacitors behave as though very high resistance is connected in parallel with a perfect capacitor, that high resistance providing a means to very slowly discharge any stored energy. However, capacitors can be remarkably effective, and for this reason pose a potential safety hazard to personnel working with them. A capacitor "charged" or energized to a high voltage level is capable of delivering its energy to an unsuspecting person in the form of an electric shock! Safe handling of capacitors requires temporary connection of a resistor to the capacitor's terminals to dissipate any hazardous levels of stored energy.

[16] Lossless energy storage is possible for an inductor only when the inductor and its shorting wire are both superconducting (i.e. a *perfect* short-circuit possessing no electrical resistance whatsoever). Actual experiments have been been performed on superconducting loops to demonstrate this principle in action: such loops, once "energized" with an initial current, maintain that current indefinitely in the absence of any external source or load.

[17] Recall that the metric unit of measurement for power is the *Watt*, defined as one Joule of energy per second of time. Energy is a measure of how much work may be done; power is a measure of how *quickly* work may be done.

4.5 Applications of capacitance and inductance

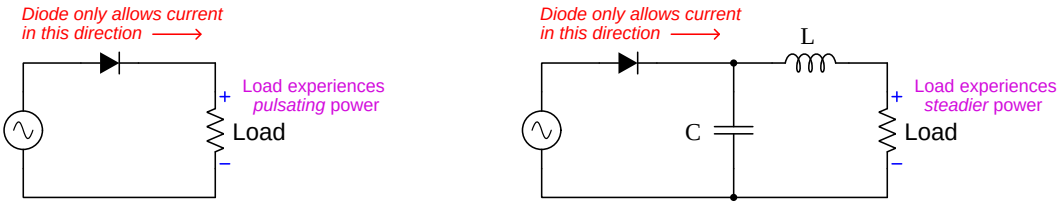
As previously mentioned, Isaac Newton's First Law of Motion (*"An object at rest tends to stay at rest; an object in motion tends to stay in motion"*) is based on the known fact that energy is always conserved, in this specific case energy in its *kinetic* (motion) form. An object at rest possesses no kinetic energy, and cannot spontaneously move because that would require kinetic energy appearing from nowhere; an object already in motion possesses a certain amount of kinetic energy, and cannot spontaneously change velocity because that would likewise require kinetic energy either appearing from nowhere or disappearing into nowhere.

By extension we may declare our own First Laws of capacitors and inductors, based on the conserved energy in these devices:

- *A de-energized capacitor/inductor tends to remain de-energized*
- *An energized capacitor tends to maintain a constant voltage*
- *An energized inductor tends to maintain a constant current*

Whatever form energy takes within an object or component, that is the quantity which cannot spontaneously appear or disappear and thus is conserved (i.e. resists change over time). For an object with mass where kinetic energy is a function of velocity, velocity is the conserved quantity. For a capacitor where stored energy is a function of voltage, voltage is the conserved quantity. For an inductor where stored energy is a function of current, current is the conserved quantity. This does not mean a capacitor's voltage cannot rise or fall, or that an inductor's current cannot rise or fall, any more than it means a moving object cannot speed up or slow down – it just means the conserved quantity cannot vary without *work* being done.

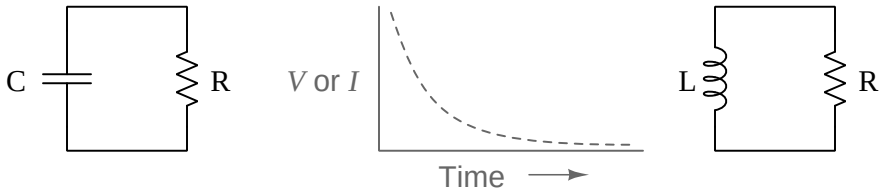
For this reason capacitors are often used as voltage-stabilization devices in circuits, and inductors as current-stabilization devices. A common application is in the design of AC-DC power supply circuits, where an alternating current (AC) voltage source is *rectified* into direct current (DC) using components called *diodes* which act as one-way valves for electric current. The result of converting an AC voltage into DC through rectification is a series of energy pulses to the load rather than a steady flow of energy, but these pulsations may be steadied by the addition of capacitance and inductance to the circuit:



The simpler circuit on the left-hand side provides power to the load during only about half the time (while the AC source is positive on top and negative on the bottom). During the other half of the time, the diode blocks the source's reverse polarity from reaching the load. The result is a pulsed delivery of power. If a more constant delivery of power is desired, a capacitor connected

in parallel between the two power conductors works to maintain a more stable voltage, while an inductor connected in series works to maintain a more stable current to the load.

The necessity of energy transfer to vary the voltage across a capacitance or the current through an inductance may be exploited to create time-delayed changes in voltage or current. For example, an energized capacitor or inductor connected to a resistive load will result in the stored energy being gradually dissipated by the resistor into heat while both voltage and current diminish over time:



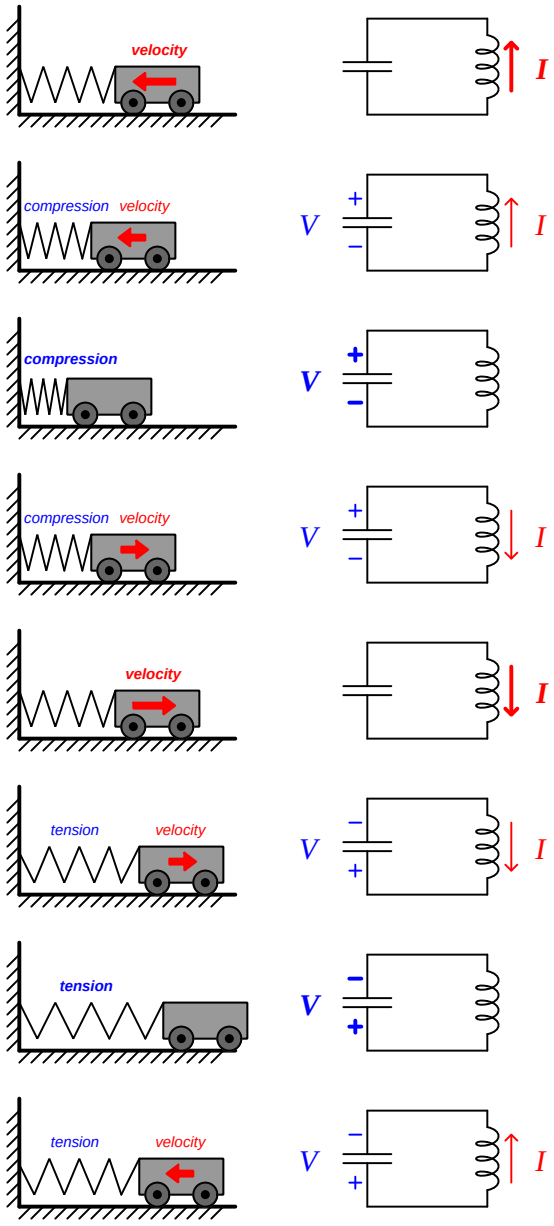
Perhaps the most interesting application of capacitance and inductance is a phenomenon known as *resonance*, which occurs when capacitance and inductance are coupled together. Capacitors and inductors absorb, store, and release energy in complementary forms. This is analogous to springs and masses, which store mechanical energy in complementary forms as well: springs storing potential energy in the form of tension and displacement, and masses storing kinetic energy in the form of motion. Any mass connected to a spring will exhibit resonance: an ability to sustain alternating motion for sometime after being stimulated. A spring-loaded mass moves to and fro because the spring and mass exchange energy back and forth with each other.

A cart with mass stores energy in kinetic form, while a spring with elasticity stores energy in potential form. Similarly, an inductor stores energy in its magnetic field as a function of current (kinetic charge carriers), while a capacitor stores energy in its electric field as a function of voltage (potential energy per charge). Coupling these complementary devices together means they now have the ability to exchange energy back and forth, and will do so if initially energized by some external source. For the spring-coupled cart it means the assembly will tend to move back and forth cyclicly; for the capacitor-inductor network it means the circuit will *oscillate* (i.e. produce alternating current, or AC).

The rate at which systems like this will oscillate depends on the characteristics of its elements. A stiffer spring will cause the cart to move back and forth more rapidly. A more massive cart will cause the cart to alternate more slowly. With capacitors and inductors it is simpler: increasing the size of either component will slow down the rate of oscillation[18].

[18] Resonant frequency for a simple capacitor-inductor network is predicted by the formula $f = \frac{1}{2\pi\sqrt{LC}}$ where L is inductance in Henrys, C is capacitance in Farads, and f is the frequency of oscillation in cycles per second or Hertz.

The following series of illustrations show a step-by-step sequence of events for an oscillating spring-and-cart mechanism alongside an oscillating capacitor-inductor circuit:



The spring's ability to store potential energy is analogous to the capacitor's ability to store energy in an electric field. The cart's ability to store kinetic energy is analogous to the inductor's ability to store energy in a magnetic field. Compression and tension within the mechanical spring is analogous to voltage across the capacitor; velocity of the cart is analogous to current through the inductor.

Even the mathematical expressions of stored energy for the spring and cart resemble the formulae for energy stored in the capacitor and inductor:

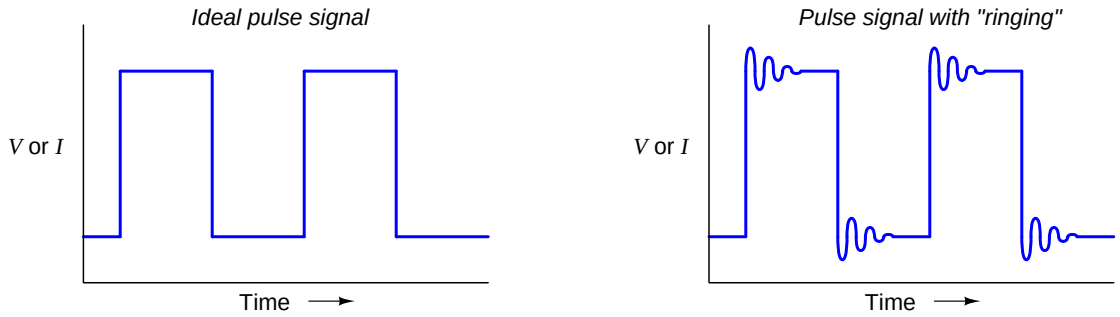
Potential energy (measured in Joules)	**Kinetic energy** (measured in Joules)
Potential energy stored in a tensed spring: $$E_p = \frac{1}{2}kx^2$$ where k is spring stiffness (measured in Newtons per meter) and x is spring displacement (measured in meters)	Kinetic energy stored in a moving mass: $$E_k = \frac{1}{2}mv^2$$ where m is mass (measured in kilograms) and v is velocity (rate of mass motion, measured in meters per second)
Energy stored in a capacitor: $$E_C = \frac{1}{2}CV^2$$ where C is capacitance (measured in Farads) and V is voltage (measured in Volts)	Energy stored in an inductor: $$E_L = \frac{1}{2}LI^2$$ where L is inductance (measured in Henrys) and I is current (rate of charge motion, measured in Amperes)

In the absence of any friction (resistance), both systems will oscillate ceaselessly as they exchange energy back and forth without losses. Mechanical systems made of springs and masses are used to create vibrations, an example being the string of a musical instrument such as a guitar: the elasticity and mass of the spring serves both roles in one component, so that the assembly *resonates* at a desired pitch. Capacitor-inductor circuits (often called *tank circuits*) are similarly used to create alternating current (AC) signals of a specific frequency[19].

[19] *Frequency* is the rate at which oscillations occur over time, generally measured in *cycles per second* (Hertz).

4.6 Parasitic capacitance and inductance

Capacitance and inductance, like elasticity and mass in mechanical systems, are intrinsic phenomena and therefore exist in *every* circuit whether desired or not. The existence of such "parasitic" capacitance and inductance in circuits means resonance may occur where it is unintended, just as unintended resonance may occur in mechanical assemblies[20]. An example of unintended electrical resonance called *ringing* is often encountered in *pulse* circuits where voltage or current is suddenly switched on and off to some load: instead of the graph of this voltage or current over time resembling a square-edge as desired, some oscillations will be seen at the leading edges of the pulse.



Whether the unintended resonance is electrical or mechanical, the solution is the same: insert enough energy dissipation into the system to dampen any oscillations that may otherwise occur. For mechanical systems this means incorporating sufficient *friction*[21]; for electrical circuits this means incorporating sufficient *resistance*.

For most electric circuits lacking capacitors and inductors, the amount of capacitance present between wires and the amount of inductance along the length of wires is so small compared to the energy-dissipating elements that these effects are negligible, and unintended resonance is simply not possible. However, for some specific types of circuits, especially those dealing with extremely high-frequency (fast-alternating) AC signals such as *radio* circuits, special care must be taken positioning all conductors to mitigate parasitic capacitance and inductance.

[20] Perhaps the most obvious example of resonance in a mechanical assembly is undesirable "bouncing" in a suspension system, but in fact nearly every mechanical assembly is capable of resonance. Rotating machines exhibit *critical speeds* where the shaft rotation period matches the resonance period of the rotating element, causing vibration to naturally occur at these speeds. Struts and other supporting members for some mass will tend to oscillate at a natural frequency, the mass acting to store kinetic energy and the strut's elasticity acting to store potential energy. The infamous Tacoma Narrows bridge was a spectacular example of unintended resonance in a mechanical structure, earning it the nickname "Galloping Gertie" by motorists due to wind-induced oscillations prior to its catastrophic collapse on the seventh of November in the year 1940.

[21] The so-called *shock absorber* is a component designed to do exactly this in vehicle suspension systems, to prevent bouncing (oscillations) that would otherwise result as a consequence of the vehicle's elastic suspension system and its mass.

Chapter 5

Historical References

This chapter is where you will find references to historical texts and technologies related to the module's topic.

Readers may wonder why historical references might be included in any modern lesson on a subject. Why dwell on old ideas and obsolete technologies? One answer to this question is that the initial discoveries and early applications of scientific principles typically present those principles in forms that are unusually easy to grasp. Anyone who first discovers a new principle must necessarily do so from a perspective of ignorance (i.e. if you truly *discover* something yourself, it means you must have come to that discovery with no prior knowledge of it and no hints from others knowledgeable in it), and in so doing the discoverer lacks any hindsight or advantage that might have otherwise come from a more advanced perspective. Thus, discoverers are forced to think and express themselves in less-advanced terms, and this often makes their explanations more readily accessible to others who, like the discoverer, comes to this idea with no prior knowledge. Furthermore, early discoverers often faced the daunting challenge of explaining their new and complex ideas to a naturally skeptical scientific community, and this pressure incentivized clear and compelling communication. As James Clerk Maxwell eloquently stated in the Preface to his book *A Treatise on Electricity and Magnetism* written in 1873,

> It is of great advantage to the student of any subject to read the original memoirs on that subject, for science is always most completely assimilated when it is in its nascent state . . . `[page xi]`

Furthermore, grasping the historical context of technological discoveries is important for understanding how science intersects with culture and civilization, which is ever important because new discoveries and new applications of existing discoveries will always continue to impact our lives. One will often find themselves impressed by the ingenuity of previous generations, and by the high degree of refinement to which now-obsolete technologies were once raised. There is much to learn and much inspiration to be drawn from the technological past, and to the inquisitive mind these historical references are treasures waiting to be (re)-discovered.

5.1 Early examples of capacitors

Capacitance is the ability to store energy within an electric field, and a *capacitor* is a device built expressly for that purpose. Every capacitor consists in one form or another of two conductive surfaces separated from each other by one or more layers of insulating material called a *dielectric*.

One of the earliest forms of capacitor was the so-called *Leyden jar* used extensively by electrical experimenters in the 18th and 19th centuries, which was nothing more than a glass jar coated inside and out with metal foil or metal leaf. The thickness of the glass constituted the dielectric, while each coating of metal (one outside the jar, one inside) comprised the two conductive surfaces. This construction leveraged existing glassware and metal-gilding technology of the day, and the fact that the device was a *jar* evoked the mental image of that jar holding the mysterious and invisible electric fluid that early electrical experimenters imagined constituted electricity.

The following photograph taken from the book *Wireless Telegraphy and Telephony* by Alfred P. Morgan and published in 1920 shows four Leyden jars suitable for use in high-voltage radio communication circuits of the day:

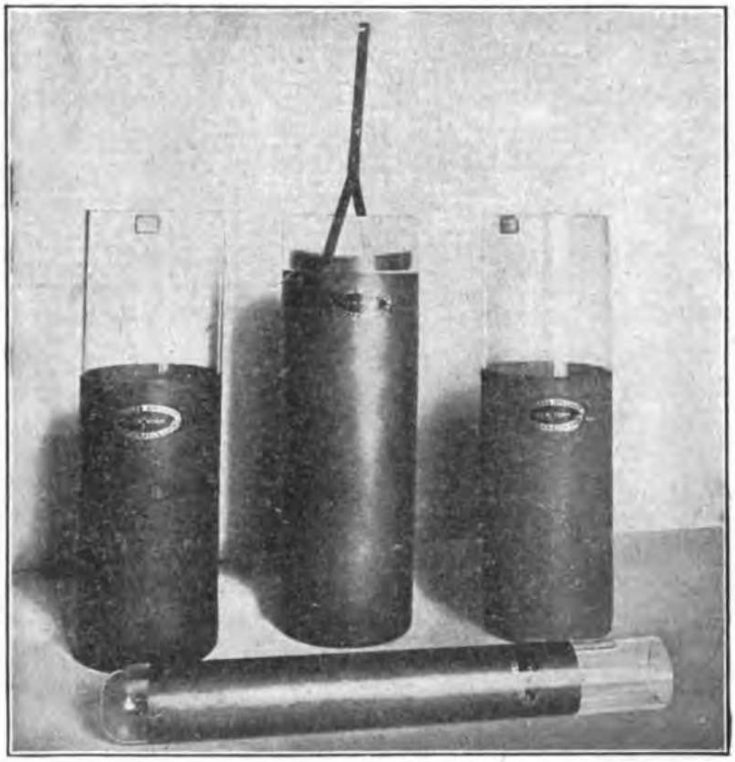


FIG. 6.—Navy type of Leyden jars. Coated with copper deposited upon the surface of the glass.

This next photograph shows historical examples of Leyden jars in a museum of radio and electricity:

A large bank of large Leyden jars nearly as tall as a man is seen in the following photograph of an early German radio transmitting station, taken from the same book. In this circuit 360 Leyden jars are connected together, most likely in parallel to maximize total capacitance:

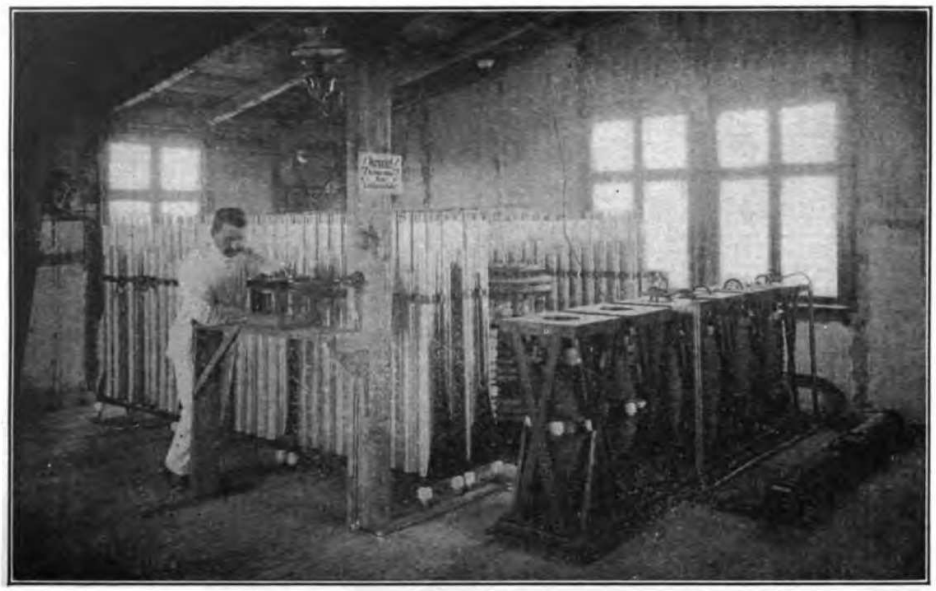


Fig. 116.—Transmitting equipment of the high power station at Nauen, twenty-five miles northwest of Berlin, Germany, showing six induction coils (in the foreground) arranged to charge the Leyden jars (composed of 360 units).

A typical feature of Leyden jars was a fairly high voltage rating, owing to the high dielectric strength of the insulating glass. For applications where the voltages encountered were lower, different types of dielectrics could be used, and be made thinner as well. The term "Leyden jar" was used only for jar-style capacitors, with other designs coming to be known as *condensers* or *condensors*. A common form of condenser construction in the early 20th century was a multi-layered device of metal foil and dielectric sheets of glass, mica, or paper. Two illustrations from the same book show a design where metal foil and dielectric sheets are interleaved, the alternating metal foil layers protruding from opposite sides of the assembly for convenient connection to form two electrically distinct metal surfaces:

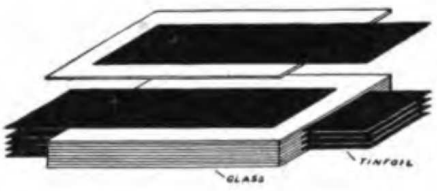


Fig. 46.—Diagram showing construction of condenser.

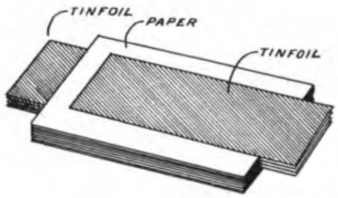


Fig. 80.—Fixed condenser.

Modern multi-layer ceramic capacitors (MLCCs) employ the same basic form of construction,

with layers of metal interleaved between layers of insulating ceramic.

Early capacitor technology also included variable capacitors (called *variable condensers*) that worked on the principle of moving one set of conductive surfaces relative to the other, thereby altering the amount of overlapping area and causing the capacitance to vary with physical position. Two examples of variable condenser design are illustrated in the same book:

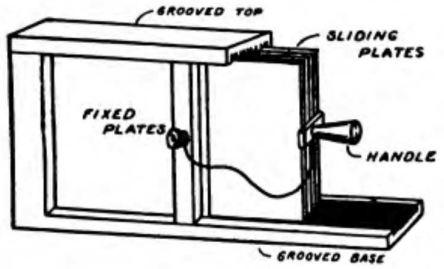


Fig. 84.—Sliding plate variable condenser.

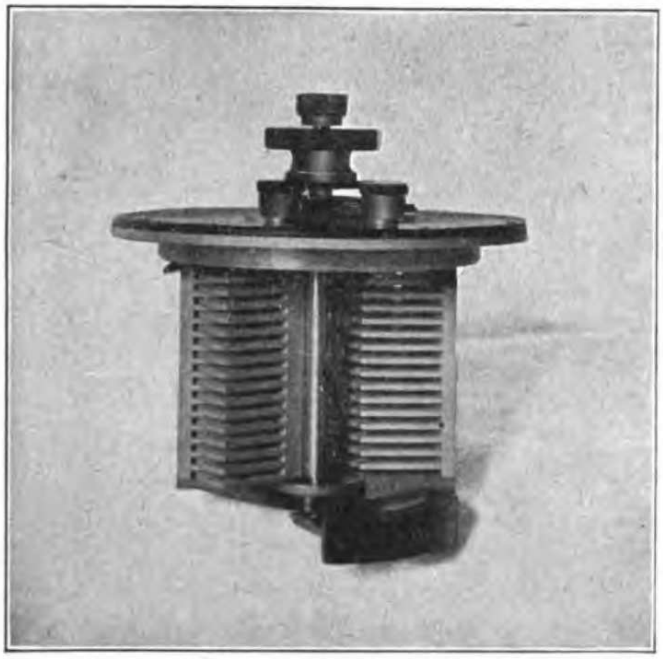


Fig. 83.—Dr. Seibt's rotary variable condenser. The plates are turned from a solid casting and the separation between is only .01 inch.

5.2 Early examples of inductors

Inductance is the ability to store energy within a magnetic field, and an *inductor* is a device built expressly for that purpose. Any length of wire exhibits inductance simply due to the magnetic field that necessarily forms around it when electrical charges pass through as a current, but most inductors maximize this energy-storage effect by wrapping the conductor into a coil shape, and often wrapped around a ferromagnetic *core material* to enhance the magnetic field.

The following photograph taken from the book *Wireless Telegraphy and Telephony* by Alfred P. Morgan and published in 1920 shows a coil of wire wrapped around a wooden frame to form what that author refers to as a *helix*, suitable for use in radio communication circuits of the day. In modern parlance we would refer to this as an *air-core inductor*:

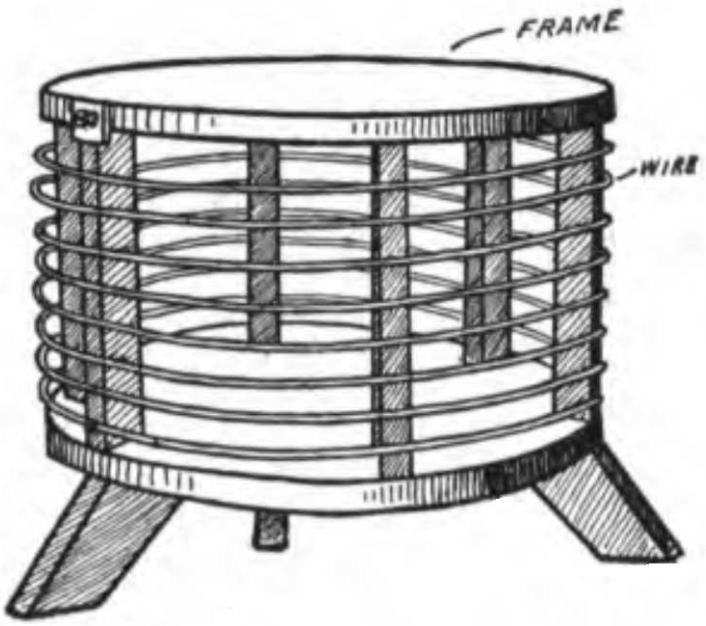


FIG. 48.—Helix.

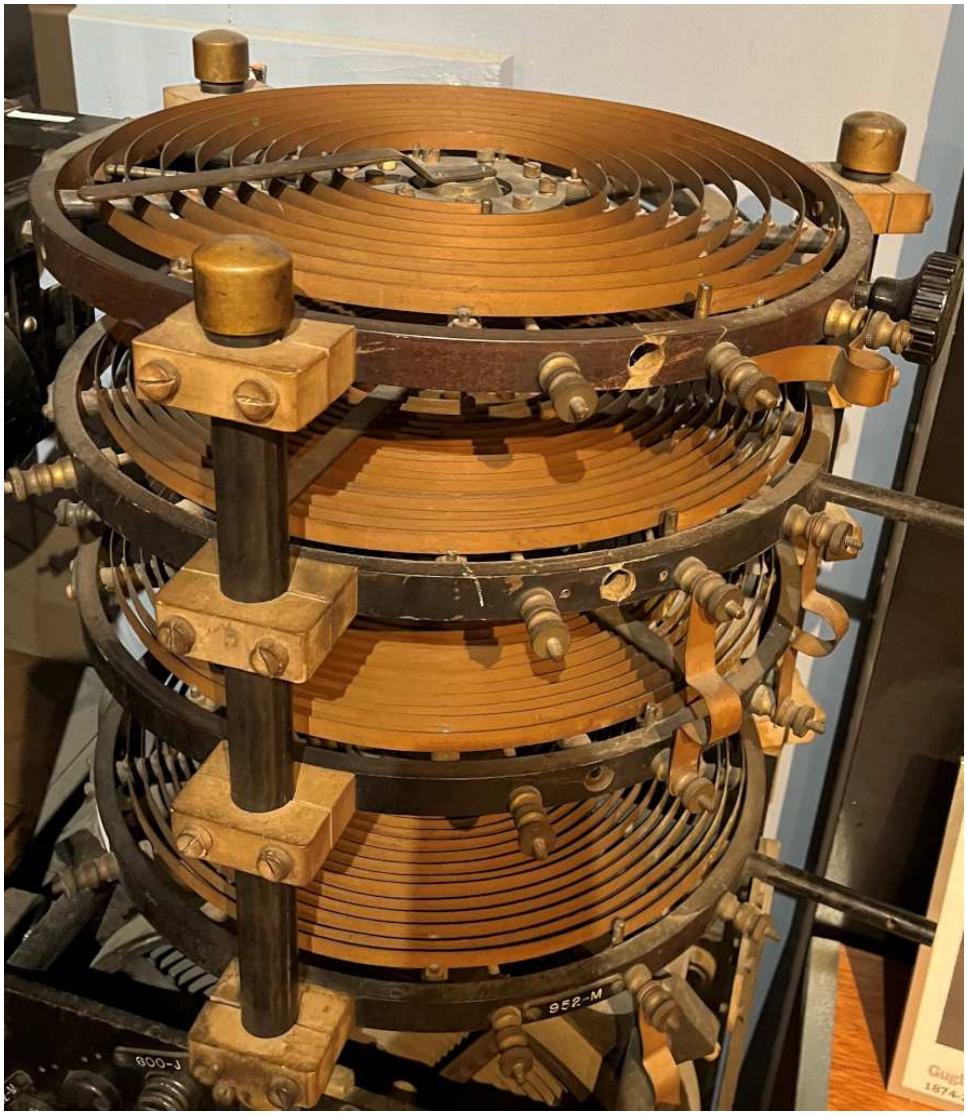
Coils form stronger magnetic fields than straight conductors because the close proximity of each "turn" of the coil results in the circular magnetic fields around each turn reinforcing the fields around adjacent turns, all the turns together acting to produce one large magnetic field with lines running through the center and looping around the outside of the coil assembly.

This next photograph comes from page 84 of *Proceedings of The Institute of Radio Engineers* (Volume 2, Number 1) dated March 1914, showing a *spiral* inductor made from heavy metal strip suitable for use in a high-power radio transmitter circuit:

Figure 15—Twenty-five Microhenry Inductance Coil Unit, 200 Amperes.

As the caption indicates, this air-cored inductor is rated at 25 μH and may carry upwards of 200 Amperes of radio-frequency AC current.

A set of spiral-wound air-core inductors for radio system use are seen in the following photograph:



The close proximity of these inductors means there will be some *magnetic coupling* between them, such that they will function as a *transformer* linking different inductive circuits together.

Early inductor technology also included variable inductors that worked on the principle of a moving "slide" contact that would connect to the coil at different locations when moved along its length. An example of a "dual-slide" inductor is illustrated in the 1920 *Wireless Telegraphy and Telephony* book by Alfred P. Morgan:

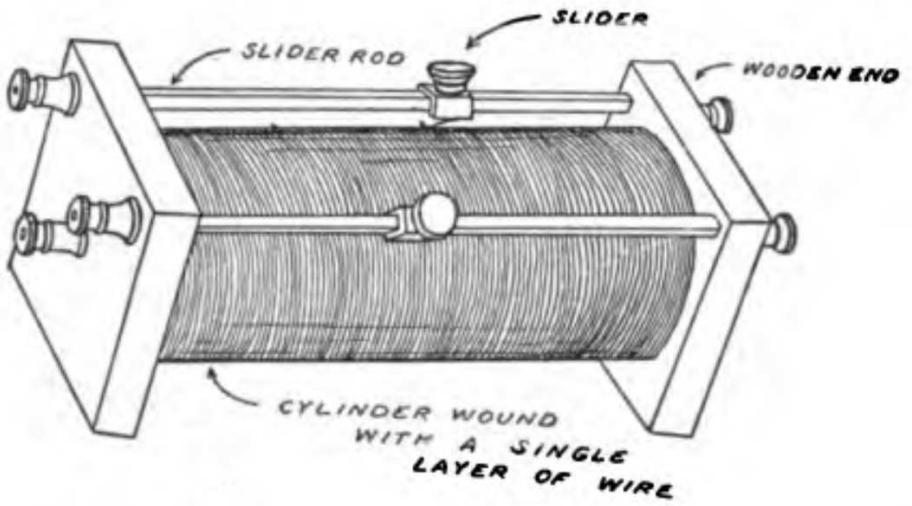


FIG. 78.—Tuning coil of the double slide type.

An ingenious design of variable inductor called the *Variometer* used two coils of wire, one stationary and the other mounted on a pivot such that the angular relation of one coil to the other could be varied by turning a knob. With the two coils connected in series with each other, changing the relative angle between them would allow their respective magnetic fields to reinforce or cancel one another to varying degrees, thereby adjusting the amount of total inductance offered by the assembly.

The book *The Wireless Experiments Manual* written by Elmer E. Bucher and published in 1920 gives instructions on the construction and usage of a variometer. Below we see an illustration of a Variometer using cardboard as forms to hold the wire coils:

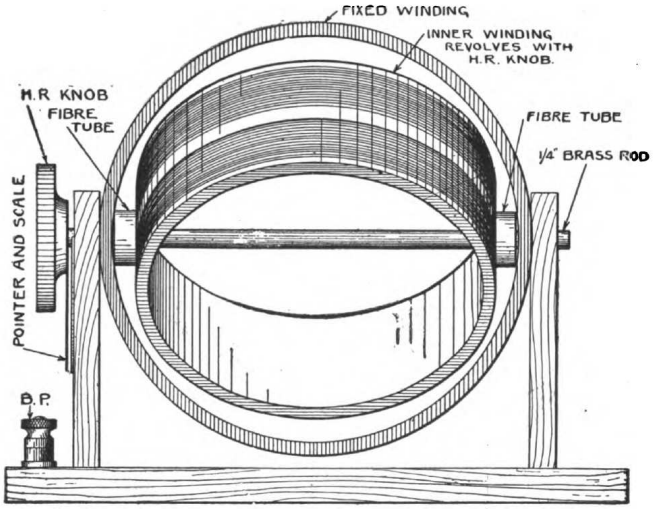


Fig. 145. View of the assembled variometer of Fig. 144.

A commercial Variometer of that era is shown in a photograph from the same book:

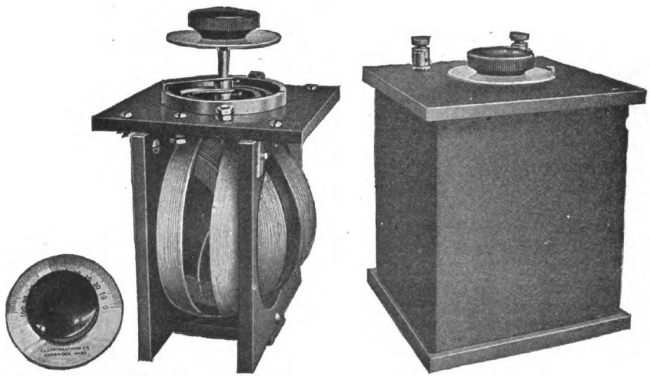
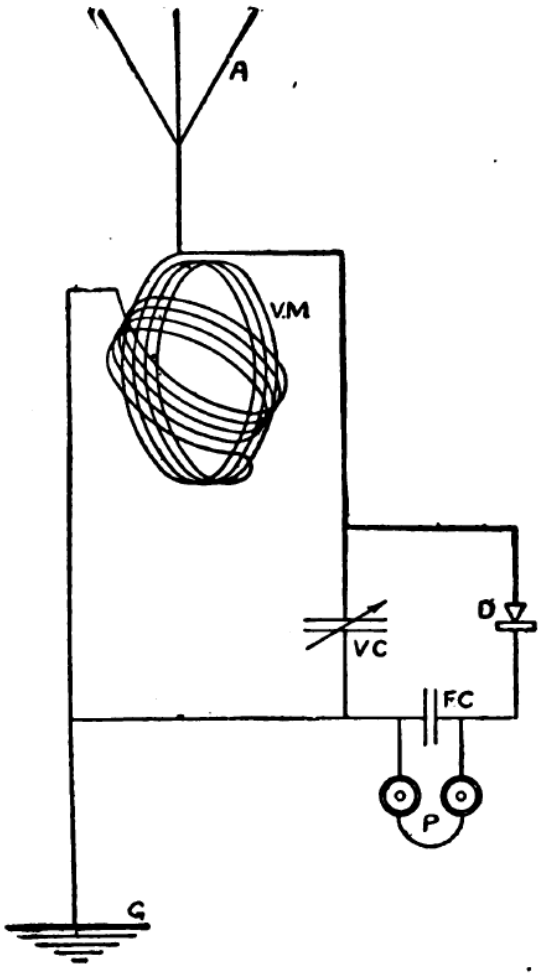


Fig. 150a. Clapp-Eastham-type S.S. Variometer for receiving circuits.

Next we see a schematic diagram from Bucher's manual showing a Variometer (labeled as *VM*) connected in series with the antenna (*A*) of a radio receiver circuit, the two series-connected coils clearly shown in the diagram:



In this receiver circuit the *VC* is a variable capacitor (or variable *condenser* as it was more commonly known in 1920), *FC* is a fixed capacitor, *D* is a diode (or *detector*), and *P* was a pair of audio headphones for listening to the demodulated radio signal. Together, the parallel-connected variable inductor and variable capacitor formed a *resonant tuning circuit* that could be used to select one radio frequency signal from other signals present at the antenna.

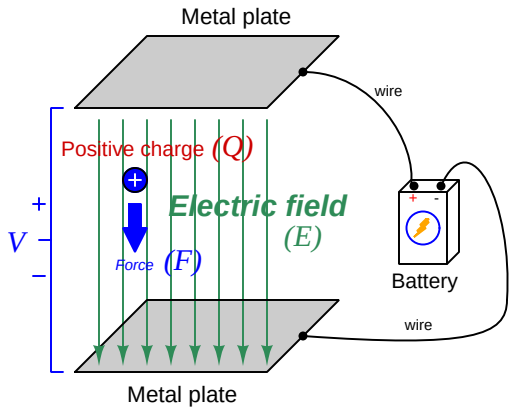
Chapter 6

Derivations and Technical References

This chapter is where you will find mathematical derivations too detailed to include in the tutorial, and/or tables and other technical reference material.

6.1 Electric field quantities

A useful definition of electric field (E) is in terms of the force (F) exerted on an electric charge (Q) influenced by that field:



$$\vec{F} = Q\vec{E}$$

Where,

$\vec{F}$ = Force exerted on the charge (Newtons)
Q = Charge quantity (Coulombs[1])
$\vec{E}$ = Electric field (Newtons per Coulomb)

The small "arrow" symbols above the variables for force and electric field in the equation denote those variables as *vector quantities*, having both magnitude and direction. Charge is a *scalar quantity* having only magnitude but no direction, and as a scalar quantity when multiplied by the electric field vector it simply magnifies the magnitude but does not alter the direction. Therefore, the force and electric field vectors always point in the same direction.

Alternatively electric field may be defined in terms of the voltage between the end-points and the distance separating them, in which case we may express the electric field in units of *Volts per meter* as an alternative to *Newtons per Coulomb*:

$$\vec{E} = \frac{V}{\vec{d}}$$

This measurement of electric field strength is very important for quantifying the *breakdown* of electrical insulators: the point at which the electric field becomes so powerful that otherwise immobile charges within the insulating substance are torn free to constitute a current and that substance is no longer an insulator. For rating the dielectric strength of insulating materials, we often see electric fields expressed in units of *kiloVolts per millimeter* rather than Volts per meter just to make the numerical quantities easier to manage (1 kV/mm = 1 *million* V/m).

[1] One Coulomb of electric charge is equal to 6.2415×10^{18} electrons.

The vector arrows shown in the previous illustration representing the electric field between two metal plates actually represent electric *flux* (Φ_E). The electric field (E) is related to electric flux by area (A), the field being a measurement of how densely-packed those flux lines are per unit area:

$$\vec{E} = \frac{\Phi_E}{\vec{A}}$$

Where,
$\vec{E}$ = Electric field, or electric flux density (Newtons per Coulomb)
Φ_E = Electric flux (Newton-meter squared per Coulomb)
$\vec{A}$ = Area over which flux is distributed (square meters)

The mere presence of an unbalanced electric charge at any point in space is sufficient to generate lines of electric flux, the total magnitude of that flux predicted by the following equation:

$$\Phi_E = \frac{Q}{\epsilon}$$

Where,
Φ_E = Electric flux (Newton-meter squared per Coulomb)
Q = Charge quantity (Coulombs)
ϵ = Electric permittivity of the surrounding space (Coulombs squared per Newton-meter squared, approximately 8.85×10^{-12} for empty space)

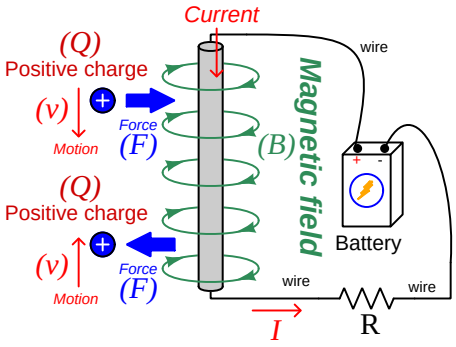
By convention, these flux vectors point *away* from positive charges and point *toward* negative charges, their direction indicating force exerted on any positively-charged particle influenced by that field[2]. As the equation states, the amount of flux depends on how much charge exists at each particle as well as the permittivity of the surrounding space:

For example, identical charges suspended in a vacuum versus in a substance such as oil will have different amounts of flux associated with them as a result of oil and vacuum having different permittivity values. Perfectly empty space has the least amount of permittivity, which means anything else (gas, liquid, or solid matter) has greater ϵ which acts to diminish the amount of electric flux surrounding any charged particle. Superconducting materials have infinite permittivity, which means they forbid the existence of any electric field inside their bulk.

[2]Conversely, the flux vectors point exactly opposite the direction of force applied to any *negatively*-charged particle within that field. This makes sense of the aphorism that *like charges repel and opposite charges attract*. If you consider the two charges shown in this illustration, the positive charge will be pulled in the direction of the flux vectors pointing toward the negative charge, as the negative charge will also be pulled opposite the direction of the flux vectors pointing away from the positive charge (i.e. the negative charge will be pulled *toward* the positive charge) – thus the positive and negative charges feel mutual attraction.

6.2 Magnetic field quantities

A useful definition of magnetic field (B) is in terms of the force (F, called the *Lorentz force*) exerted on a moving electric charge (Q) influenced by that field:



$$\vec{F} = Q\vec{v} \times \vec{B}$$

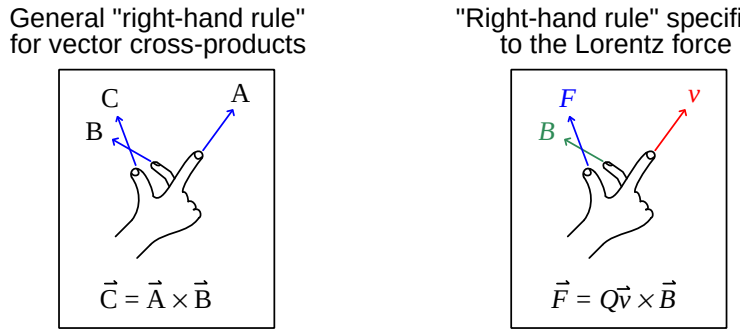
Where,

$\vec{F}$ = Force exerted on the charge (Newtons)
Q = Charge quantity (Coulombs[3])
$\vec{v}$ = Velocity of moving charge (meters per second)
$\vec{B}$ = Magnetic field (Tesla, Webers per square meter, or Newtons per Ampere-meter)

The small "arrow" symbols above the variables for force and velocity and magnetic field in the equation denote those variables as *vector quantities*, having both magnitude and direction. Charge is a *scalar quantity* having only magnitude but no direction, and as a scalar quantity when multiplied by the velocity vector it simply magnifies the magnitude but does not alter the direction. The "cross-product" ($\times$) is a specific form of vector multiplication, and it results in a product at right angles to the vector directions of both terms. Therefore, the force and velocity and electric field vectors never all point in the same direction.

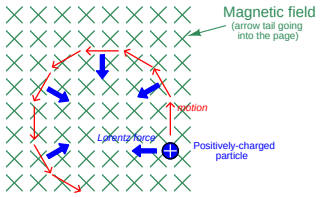
[3] One Coulomb of electric charge is equal to 6.2415×10^{18} electrons.

Vector cross-products conveniently relate to the fingers of the right hand, which is where the "right-hand rule" originates:



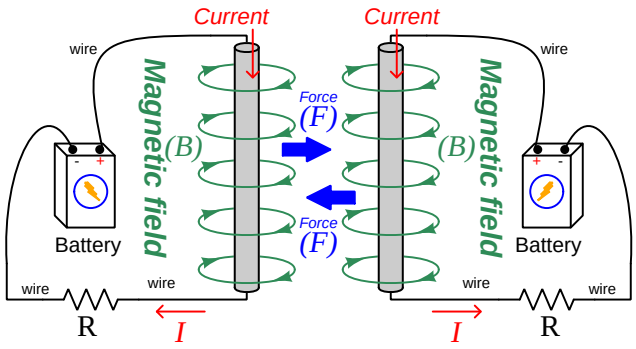
When holding the index finger, middle finger, and thumb of your right hand perpendicular to each other, your index finger points in the direction of the velocity vector ($\vec{v}$), your middle finger in the direction of the magnetic field vector ($\vec{B}$), and your thumb in the direction of the force vector ($\vec{F}$). A simple mnemonic I use to remember these relationships of fingers to vectors is that the **I**ndex finger points in the direction of current[4] (I), the **M**iddle finger points in the direction of the **m**agnetic field (B), and the **Th**umb points in the direction of the **th**rust (i.e. force) acting upon the moving charge.

The Lorentz force's effect on electrically-charged particles in motion has many applications, from redirecting the paths of charged-particle beams in particle accelerator machines, to bending the trajectory of electron beams in a cathode-ray tube (CRT), to forcing electrons to travel in spiral or circular trajectories inside of magnetron (microwave oscillator) tubes. An illustration of a positively-charged particle curving perpendicular to a magnetic field appears here:



[4]Some textbooks speak of a "left-hand rule" which is intended to make sense of electric charge motion (current) in terms of *electron flow*. As we know, electrons are the only real mobile charge carriers within metal conductors, and so technically "electron flow" notation is most physically accurate when describing the motion of electric charges in metallic circuits. However, the right-hand rule is a *mathematical definition* for vector cross products, the concept of the cross product arising in the late 18th century when electrical science was still in its infancy. Early explorers of electricity used the established mathematical tools of their time and applied it to their work with electric currents and magnetism. At that time, charge carriers in metal wires were assumed to be "positive" and this is how the motion of positively-charged carriers became associated with the first vector of the cross-product. As a result of this assumption which was later proven false, we have two different conventions for denoting the motion of electricity: electron-flow which is *physically* accurate (for metal wires, at least), and conventional flow which is *mathematically* accurate. This, perhaps more than any other reason, is why educational programs designed for mathematically rigorous fields (e.g. electrical engineering) exclusively use conventional flow notation rather than electron flow notation to denote the direction of current.

If the moving charge in question is not a single charged particle but rather part an electric *current* passing through a conductor parallel to the first, both conductors will experience a mutually-attracting force given by the following equation:



$$\vec{F} = I\vec{l} \times \vec{B}$$

Where,

$\vec{F}$ = Force exerted on both conductors (Newtons)

I = Current (Amperes)

$\vec{l}$ = Length of wire (meters)

$\vec{B}$ = Magnetic field (Tesla, or Webers per square meter, or Newtons per Ampere-meter)

The point-charge Lorentz force equation and the two-conductor Lorentz force equation are not that different from one another. Dimensional analysis validates this: the Lorentz force on a moving charge uses that charge quantity (Coulombs) multiplied by the point-charge's velocity in meters per second to give Coulomb-meters per second for the first term:

$$Q\vec{v} = [\mathrm{C}]\left[\frac{\mathrm{m}}{\mathrm{s}}\right] = \left[\frac{\mathrm{C} \cdot \mathrm{m}}{\mathrm{s}}\right]$$

The Lorentz force on a current-carrying conductor uses the current (Amperes, which is Coulombs per second) multiplied by length in meters, for the same composite units of Coulomb-meters per second:

$$I\vec{l} = \left[\frac{\mathrm{C}}{\mathrm{s}}\right][\mathrm{m}] = \left[\frac{\mathrm{C} \cdot \mathrm{m}}{\mathrm{s}}\right]$$

This dimensional equivalence makes conceptual sense as well: an electrically-charged particle moving through empty space is an electric current in its own right, and an electric current flowing through a conductor is just a collection of charged particles moving through space (just not *empty* space). In either case, the basis for the Lorentz force remains the same: the moving charge(s) create their own magnetic field, which reacts with the magnetic field of the original current-carrying wire to produce forces acting on both.

If the two currents flow in the same direction, their mutual forces *attract*. If the two currents flow in opposite directions, their mutual forces *repel*. This is the basis of electric motors: causing

mechanical motion by electro-magnetic attraction and repulsion. It also represents an interesting contrast with electric fields:

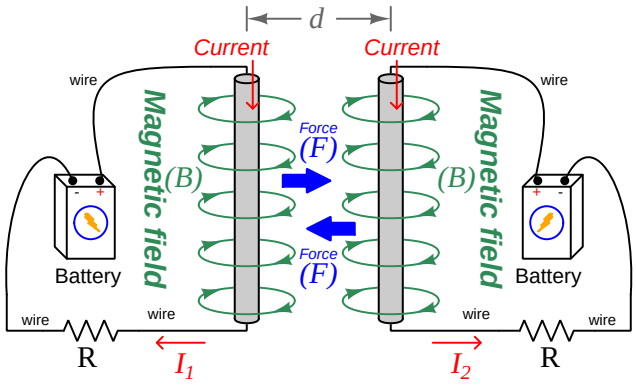
With electric fields, opposite *charges* attract and like *charges* repel.

With magnetic fields, opposite *poles* attract and like *poles* repel.

With parallel currents, opposite *directions* repel and like *directions* attract[5].

Two parallel current-carrying conductors of length l and separated by a distance d will generate a mutual force proportional to both their currents:

$$F = l\frac{\mu I_1 I_2}{2\pi d}$$



[5]That is, assuming it's *like* charges moving in these directions! If the charges in question are opposite each other – for example electrons in one circuit and holes in another – then like directions will repel and opposite directions will attract!

The circular loops surrounding the current-carrying conductors in the previous illustrations represent the magnetic lines of *flux* (Φ_B) surrounding each of those conductors. The magnetic field (B) is related to magnetic flux by area (A), the field being a measurement of how densely-packed those flux lines are per unit area. For this reason, magnetic field (B) is more properly known as *magnetic flux density*:

$$\vec{B} = \frac{\Phi_B}{\vec{A}}$$

Where,

$\vec{B}$ = Magnetic field or flux density (Tesla, Webers per square meter, or Newtons per Ampere-meter)

Φ_B = Magnetic flux (Webers)

$\vec{A}$ = Area over which flux is distributed (square meters)

An older unit of measurement for magnetic flux density B is the *Gauss* which is much smaller than a Tesla, with one Tesla equivalent to 10,000 Gauss. To put things into perspective, the Earth's natural magnetic field has a strength of approximately one-half of one Gauss[6].

Magnetic field strength is an inverse function of distance from any current-carrying wire, and also depends on the magnetic permeability of the space adjacent to the wire:

$$B = \frac{\mu I}{2\pi d}$$

Where,

B = Magnetic field or flux density (Tesla, Webers per square meter, or Newtons per Ampere-meter)

μ = Magnetic permeability of the surrounding space (Tesla-meters per Ampere, $4\pi \times 10^{-7}$ for empty space)

I = Current (Amperes)

d = Distance from conductor (meters)

[6] Using the online *Magnetic Field Calculator* application provided by NOAA (the National Oceanic and Atmospheric Administration) at `https://ngdc.noaa.gov/geomag/calculators/magcalc.shtml#igrfwmm`, applying the World Magnetic Model WMM modeling algorithm for years 2019-2024, the total magnetic field strength at my home is 53,584.4 nano-Tesla (53,584.4 nT or 0.535844 Gauss), and presently (May 2020) decaying at a rate of -104.1 nT per year.

The relation of magnetic *flux* to current through a conductor follows a similar equation:

$$\Phi = \frac{\mu A I}{2\pi d}$$

Where,

Φ = Magnetic flux (Webers)

μ = Magnetic permeability of the surrounding space (Tesla-meters per Ampere, $4\pi \times 10^{-7}$ for empty space)

A = Area over which flux is distributed (square meters)

I = Current (Amperes)

d = Distance from conductor (meters)

As this equation makes clear, the amount of magnetic flux surrounding a current-carrying conductor depends not only on the amount of current, but also on the sampled area, the distance from the wire, and also the surrounding material. Most[7] substances (gas, liquid, solid) have permeability values greater than that of empty space, and so this means magnetic flux is usually *enhanced* by the presence of matter around the current-carrying conductor.

The total magnetic flux enclosed by a circular wire loop follows a similar equation:

$$\Phi = \frac{\pi \mu I r}{2}$$

Where,

Φ = Magnetic flux (Webers)

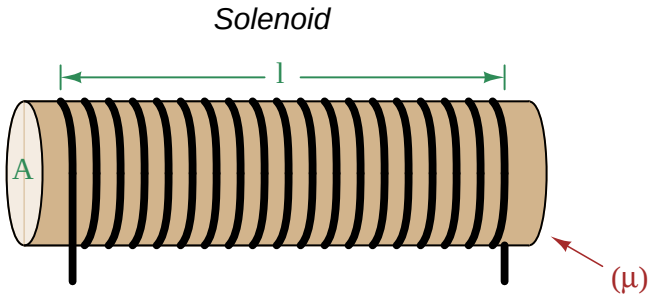
μ = Magnetic permeability of the surrounding space (Tesla-meters per Ampere, $4\pi \times 10^{-7}$ for empty space)

I = Current (Amperes)

r = Radius of circular loop (meters)

[7] Interestingly, superconducting materials *forbid* magnetic fields inside of their bulk, and so the permeability value of any superconductor must be zero!

A common form of electromagnet known as a *solenoid* takes the form of a wire coil wrapped in such a way as to form a long[8] cylinder, often wrapped around a plastic frame, and often with a ferromagnetic material such as iron in the center:



The amount of magnetic flux, and the flux density, within the interior of a current-carrying solenoid are given by the following formulae:

$$\Phi = \frac{\mu N A I}{l} \qquad\qquad B = \frac{\mu N I}{l}$$

Where,

Φ = Magnetic flux (Webers)

B = Magnetic field or flux density (Tesla, Webers per square meter, or Newtons per Ampere-meter)

μ = Magnetic permeability of the surrounding space (Tesla-meters per Ampere, $4\pi \times 10^{-7}$ for empty space)

N = Number of turns of wire in the coil

A = Cross-sectional area of solenoid coil (square meters)

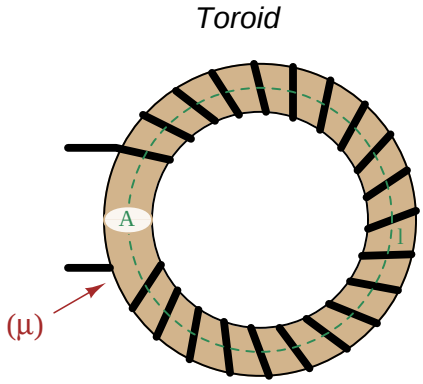
I = Current (Amperes)

l = Length of solenoid coil (meters)

These formulae have interesting implications for solenoid design. Note how a shorter (i.e. smaller length l) solenoid identical in all other respects will generate a stronger magnetic field for a given current. Note also how the flux density (B) remains constant with increasing cross-sectional area (A) if all other factors are equal, and that this necessarily means a greater amount of total magnetic flux (Φ) for a greater area A.

[8] These magnetic field formulae apply perfectly to a solenoid coil that is closely-packed (i.e. each turn adjacent to the next) and infinitely long. Therefore, they only approximate real solenoid behavior. This fact may be understood by performing a thought experiment where we decrease the solenoid coil's length to zero, in which case the formulae predict an *infinite* amount of magnetism for any amount of current at all, which of course cannot be true.

Another common form of electromagnet known as a *toroid* is really just a solenoid bent in a circle so that its two ends meet[9] cylinder, often wrapped around a plastic frame, and often with a ferromagnetic material such as iron in the center. Toroids have the unusual property of *containing* their magnetic flux lines extremely well, unlike solenoids, wires, and simple coils which all radiate magnetic fields. They find application as energy-storage devices, or as electromagnets suitable for applying magnetic fields to specimens placed *inside* the toroid's cross-section:



The amount of magnetic flux, and the flux density, within the interior of a current-carrying toroid are identical to that within an otherwise identical otherwise identical solenoid having a length (l) equal to the toroid's circumference:

$$\Phi = \frac{\mu N A I}{l} \qquad\qquad B = \frac{\mu N I}{l}$$

Where,

Φ = Magnetic flux (Webers)

B = Magnetic field or flux density (Tesla, Webers per square meter, or Newtons per Ampere-meter)

μ = Magnetic permeability of the surrounding space (Tesla-meters per Ampere, $4\pi \times 10^{-7}$ for empty space)

N = Number of turns of wire in the coil

A = Cross-sectional area of toroid (square meters)

I = Current (Amperes)

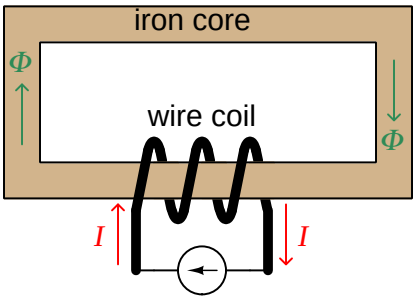
l = Circumference of toroid (meters)

If we wish to substitute toroid radius (r) for circumferential length (l), the formulae become the following:

$$\Phi = \frac{\mu N A I}{2\pi r} \qquad\qquad B = \frac{\mu N I}{2\pi r}$$

[9] Again, the magnetic field formulae are only accurate for a toroidal coil that is closely-packed (i.e. each turn adjacent to the next) and infinitely long, and therefore only approximate real toroid behavior. This fact may be understood by performing an equivalent thought experiment as before where we decrease the toroid's circumference to zero and absurdly end up with *infinite* magnetism for a finite current.

Many applications of electromagnetism involve conductive *coils* wrapped around some form of ferromagnetic core material, the purpose of that core being to provide a higher-permeability pathway for the magnetic flux than would exist otherwise through air, and the purpose of the wire coil being to intensify the amount of magnetism developed by the electric current beyond what would be possible with a straight current-carrying wire. These magnetic cores typically form a closed loop, or *magnetic circuit* for the lines of magnetic flux to naturally form a closed path. A simple example appears here:



The amount of magnetic flux (Φ) present in the magnetic "circuit" formed by the iron core depends on many factors. First and foremost is the amount of electric current (in Amperes) passing through the wire coil and the number of turns that coil makes around the iron core. The product of this current and the number of turns is called the *magnetomotive force* or *mmf* of the magnetic circuit, analogous to "electromotive force" or "emf" often used as a synonym for voltage in an electric circuit. Not surprisingly, the standard metric unit of measurement for magnetomotive force is the *Ampere-turn*.

However, magnetomotive force alone does not fully describe the current's effect on magnetism within the iron core. The total length of the magnetic circuit is also an important factor, since a longer path distributes that magnetomotive force over a greater distance. The quotient of magnetomotive force and magnetic circuit length is called *magnetic field intensity*, symbolized by the variable H and expressed in units of *Ampere-turns per meter*.

Magnetic permeability (μ) relates magnetic field intensity (H) to the magnetic flux density (B) within the core material, such that a greater permeability will result in higher flux density for any given amount of field intensity. Permeability is a property of the core material and not its geometry, mathematically defined as the ratio of flux density to field intensity: $\mu = \frac{B}{H}$

Magnetic reluctance ($\Re$) relates magnetomotive force (mmf) to magnetic flux (Φ), and is related not only to the core material's permeability but also its geometry. It is mathematically defined as the ratio of magnetomotive force to magnetic flux: $\Re = \frac{\text{mmf}}{\Phi}$

If all this seems confusing, you are in good company. Not only are there many magnetic variables, some related to physical geometry and others not, but there are *two different sets of metric units* appropriate for expressing each! The older units were based on the centimeter-gram-second (CGS) version of the metric system, while the newer units are based on the meter-kilogram-second or SI (Système International) version of the metric system.

Quantity	**Symbol**	**SI unit**	**CGS unit**
Magnetomotive force	mmf	Ampere-turn (A-t)	Gilbert (Gb)
Flux	Φ	Weber (Wb)	Maxwell (Mx)
Field intensity	H	Ampere-turns per meter (A-t/m)	Oersted (Oe)
Flux density	B	Tesla (T)	Gauss (G)
Permeability	μ	Tesla-meters per Ampere-turn (T-m/A-t)	Gauss per Oersted (G/Oe)
Reluctance	$\Re$	Ampere-turns per Weber (A-t/Wb)	Gilberts per Maxwell (G/Mx)

Magnetomotive force (mmf) and magnetic flux (Φ) may be thought of as the "raw" measures of magnetism, with Ampere-turns and Webers being their respective SI metric units. Reluctance ($\Re$) is the ratio of the two for any given magnetic circuit with known dimensions and core material. Simply put, reluctance tells you how many Ampere-turns of magnetomotive force will be necessary to create one Weber of magnetic flux in a given space.

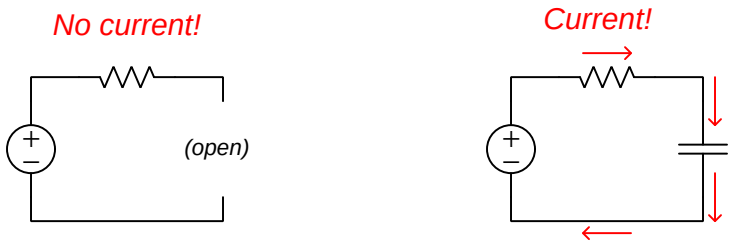
Magnetic field intensity (H) and magnetic flux density (B) may be thought of as the "normalized" measures of magnetism, with Ampere-turns per meter and Tesla being their respective SI metric units. H and B relate to mmf and flux by the physical dimensions of the magnetic circuit (length and cross-sectional area, respectively). Permeability is the ratio of the two for any given magnetic core material. Simply put, permeability tells you how many Tesla of magnetic field (i.e. flux density, or Webers of flux per square meter or cross-sectional core area) you will obtain for one Ampere-turn per meter of magnetic field intensity applied to a given core material.

Conversion between the newer SI and the older CGS metric units are as follows:

Quantity	**Conversion equivalence**
Magnetomotive force (mmf)	1 Ampere-turn = $\frac{4\pi}{10}$ Gilberts
Magnetic flux (Φ)	1 Weber = 10^8 Maxwells
Magnetic field intensity (H)	1 Ampere-turn/meter = $\frac{4\pi}{1000}$ Oersteds
Magnetic flux density (B)	1 Tesla = 10^4 Gauss
Permeability (μ)	1 Tesla-meter/Ampere-turn = $\frac{10^7}{4\pi}$ Gauss/Oersteds
Reluctance ($\Re$)	1 Ampere-turn/Weber = $\frac{4\pi}{10^9}$ Gilberts/Maxwell

6.3 Capacitor displacement current

A common point of confusion regarding capacitors is their ability to permit an electric current despite the lack of a continuous conductive pathway between their plates. Any capacitance, after all, is really just a pair of conductive surfaces separated by an insulator (called a *dielectric*). With no true *circuit* (i.e. a loop of electrically conductive material), how can an electric current possibly flow?



The key to understanding this is to realize that a "broken" loop is prohibited only from carrying a *continuous* current, but *temporary* motion of electric charge carriers is entirely possible where no true circuit exists. Most students learn that a break anywhere in a simple circuit halts current *everywhere* in that circuit, and so it is easy to conclude that broken circuits cannot *ever* sustain current. And, since a capacitor is essentially a "break" it makes sense someone might conclude that no current could ever pass through the wires of a capacitor.

Yet we know of examples where electricity is able to flow – albeit momentarily – in the absence of a complete circuit. Lightning is perhaps the most impressive example, where a sudden surge of electric current passes between clouds and the Earth (or from cloud to cloud). Anyone who has ever felt the shock of "static electricity" when touching a metal object has experienced a miniature version of this same phenomenon: a rush of electric charge carriers between yourself and another object where no continuous circuit exists.

It is comforting to know that some very intelligent people wrestled with this same conceptual problem. Among them was James Clerk Maxwell, perhaps most famous for his laws mathematically relating electricity and magnetism. It was well-known in Maxwell's time that an electric current traveling through a conductive wire produced a magnetic field, and so Maxwell concluded magnetic fields must be the result of electric charge carriers in motion. As he formulated his theory, he came to conclude that an electric field existing across an insulating medium must also be capable of producing a magnetic field, *if that electric field varied over time*. This conclusion was later vindicated in rather spectacular fashion by the discovery of electromagnetic waves in open space, consisting of mutually-supporting electric and magnetic field waves (one wave supporting the other as they alternate polarities and propagate at the speed of light). Even though no electricity can pass through an insulator, something akin to an electric current must exist within the volume of an insulating body (or across any stretch of open space, which is also electrically insulating) in order to preserve the dependency of magnetism on electricity. Maxwell called this current-through-an-insulator a *displacement current*, contrasting it against the "regular" mode of current through conductors.

Maxwell described his theory in multiple publications, one of them being the *Philosophical Magazine*, Volume 23, fourth series, published in the year 1862. Contained in an article of this

journal entitled "On Physical Lines of Force, Part III – The Theory of Molecular Vortices applied to Statical Electricity", Maxwell contrasts electric currents through conductors against "displacement" current through insulators:

> When a difference of tension exists in different parts of any body, the electricity passes, or tends to pass, from places of greater to places of smaller tension. If the body is a conductor, an actual passage of electricity takes place; and if the difference of tensions is kept up, the current continues to flow with a velocity proportional inversely to the resistance, or directly to the conductivity of the body.
>
> The electric resistance has a very wide range of values, that of the metals being the smallest, and that of glass being so great that a charge of electricity has been preserved in a glass vessel for years without penetrating the thickness of the glass.
>
> Bodies which do not permit a current of electricity to flow through them are called insulators. But though electricity does [page 13]
>
> not flow through them, electrical effects are propagated through them, and the amount of these effects differs according to the nature of the body; so that equally good insulators may act differently as dielectrics.
>
> Here then we have two independent qualities of bodies, one by which they allow of the passage of electricity through them, and the other by which they allow of electrical action being transmitted through them without any electricity being allowed to pass. A conducting body may be compared to a porous membrane which opposes more or less resistance to the passage of a fluid, while a dielectric is like an elastic membrane which may be impervious to the fluid, but transmits the pressure of the fluid on one side to that on the other. [page 14]

In these few paragraphs, Maxwell summarized *Ohm's Law*, *electrical resistivity and conductance*, and *dielectric permittivity*. Ohm's Law is evident in his statement that "current continues to flow with a velocity proportional inversely to the resistance". Maxwell's characterization of that quality of matter by which electrical effects might pass through insulating materials without there being an actual flow of current is what we now call *permittivity*.

Maxwell continues his line of thought by explaining what happens inside of conductors and insulators when exposed to differences of electrical potential:

> As long as electromagnetic force acts on a conductor, it produces a current which, as it meet with resistance, occasions a continual transformation of electrical energy into heat, which is incapable of being restored again as electrical energy by any reversion of the process.
>
> Electromotive force acting on a dielectric produces a state of polarization of its parts similar in distribution to the polarity of the particles of iron under the influence of a magnet, and, like the magnetic polarization, capable of being described as a state in which every particle has its poles in opposite conditions.
>
> In a dielectric under induction, we may conceive that the electricity in each molecule is so displaced that one side is rendered positively, and the other negatively electrical, but

> that the electricity remains entirely connected with the molecule, and does not pass from one molecule to another.
>
> The effect of this action on the whole dielectric mass is to produce a general displacement of the electricity in a certain direction. This displacement does not amount to a current, because when it has attained a certain value it remains constant, but it is the commencement of a current, and its variations constitute currents in the positive or negative direction, according as the displacement is increasing or diminishing. The amount of the displacement depends on the nature of the body, and on the electromotive force; so that if h is the displacement, R the electromotive force, and E a coefficient depending on the nature of the dielectric,
>
> $$R = -4\pi E^2 h \ ;$$
>
> and if r is the value of the electric current due to displacement,
>
> $$r = \frac{dh}{dt}$$
>
> [page 14]

Here we see Maxwell first describe the action of a resistance as an electrical *load*, transforming electrical energy into a form (heat) that leaves the circuit never to return. Next, he describes the action of an electric field (i.e. an impressed voltage) across an insulating body "polarizing" the molecules of that body in a manner similar to iron filings turning about under the influence of a magnetic field. The impressed voltage's effect of distorting these molecules amounts to a slight shift of the electric charges within those molecules, which he describes as being "a commencement of a current". This slight charge motion constitutes a temporary current "through" the dielectric, the rate-of-change of this displacement over time ($\frac{dh}{dt}$ in calculus terminology) must constitute an electric current.

Using modern notation, we may express Maxwell's sentiments in the two following formulae:

$$Q = CV \qquad\qquad I = C\frac{dV}{dt}$$

Where,
Q = Charge displaced (Coulombs)
I = Displacement current (Amperes)
C = Capacitance (Farads)
V = Applied voltage (Volts)
$\frac{dV}{dt}$ = Rate of voltage rise/fall (Volts per second)

$Q = CV$ expresses the amount of electric charge (Q) displaced in the circuit as voltage (V) is applied to a capacitance (C). Re-arranged, this formula serves to mathematically define capacitance as the amount of charge displaced per unit of applied voltage ($C = \frac{Q}{V}$). If we consider a capacitor under the influence of a *changing* voltage, that voltage's rate-of-change ($\frac{dV}{dt}$) results in a temporary current (I) through the capacitive circuit as described by Maxwell.

Returning to Maxwell's explanation, he claims that this displacement current's existence stands independent of how dielectrics might function internally. The model of dielectric materials being "elastic" in response to an applied voltage is just an analogy, however useful it may be:

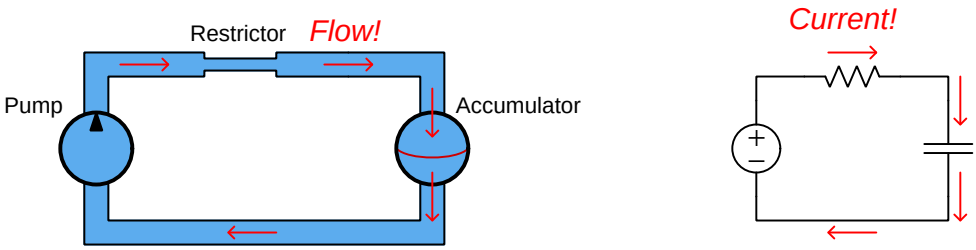
> These relations are independent of any theory about the internal mechanism of dielectrics; but when we find electromotive force producing electric displacement in a dielectric, and when we find the dielectric recovering from its state of electric displacement with an equal electromotive force, we cannot help regarding the phenomena as those of an elastic body, yielding to a pressure, and recovering its form when the pressure is removed. [page 15]

Dielectric substances accentuate capacitance, by virtue of their constituent molecules becoming distorted by electric fields. This might be thought of in terms of each distorted molecule acting as a microscopic spring, being stretched or compressed by an external force. Springs, as we know when stretched or compressed, store energy in potential form. Likewise, a substance having a large permittivity value allows for greater storage of energy by an electric field (i.e. greater capacitance), all other factors being equal. However, it is fascinating to realize that capacitance and displacement currents exist all the same when the insulating medium in question is a pure *vacuum*. This is where the model of displacement current as molecular distortion loses its explanatory power, because in a vacuum there are no molecules at all for the electric field to distort.

This is not to say that Maxwell's description of molecular distortion is wrong. It simply means dielectric action is *part* of the model and not the entirety of it. Even when there is no insulating matter between two conductors, the empty space of a perfect vacuum still supports electric fields, still stores energy, and still generates a magnetic field when a time-varying electric field is impressed across it. It's just that a pure vacuum doesn't exhibit *as much* energy storage, or *as much* displacement current, for any given electric field.

In summary, this is how we may conserve our notion of electric current requiring a complete loop in which to travel: the insulating gap of a capacitance supports a *displacement current* that is not a real motion of electric charges from one point to another, but nevertheless functions similar to one. Displacement currents are always transient, never continuous. They exist in proportion to the rate-of-change over time of an electric field, whether accentuated by the molecules of a dielectric substance or not. Current throughout the rest of the circuit (i.e. through conductors connected in series with the capacitance) follow the transient nature of the displacement current. What a *capacitor* does is magnify the effect of displacement current by means of intensifying the electric field and enhancing its girth by using a large overlapping area between conductors.

We may model this phenomenon in the form of a *hydraulic circuit* where a pump attempts to move liquid through some pipes, but there is a device called an *accumulator* at the far end of the circuit. This "accumulator" consists of a metal vessel divided into two chambers by an elastic diaphragm, such that forcing liquid into one chamber has the effect of distending this diaphragm into the other chamber where liquid gets forced out:



If the diaphragm's tension reaches a point where it is able to fully oppose the pump's pressure, the flow of liquid in the hydraulic circuit will cease. However, up until that point, the pump will be able to move liquid around the hydraulic circuit as the elastic diaphragm continues to stretch. So, we see here that even in a case where an impermeable diaphragm prevents continuous liquid motion around the hydraulic circuit, under conditions of changing pressure we are still able to create temporary flow in this circuit lacking a continuous pathway for flow. This is what we see happening in the RC electric circuit: activating the source causes the unenergized capacitor to begin to receive energy, and we will have current in the circuit. As the capacitor's energy level increases, and its voltage drop increases accordingly, there will come a point in time when the capacitor's voltage is equal in value to and opposite in polarity to the source at which point current finally ceases.

The accumulator diaphragm's action also mimics what happens inside a dielectric material sandwiched between two capacitor plates. The electric field created between those plates causes the positive and negative charges within the atoms of the dielectric to be pulled in opposite directions, essentially "distorting" the atoms within that material and thus storing energy in a manner not unlike the stretching of a spring by mechanical force or the distortion of an elastic diaphragm by fluid pressure. This is why the addition of a dielectric material between a capacitor's plates (as opposed to vacuum) results in greater capacitance: the atomic elasticity of that material functions as an energy-storage mechanism.

6.4 Capacitor parameters and dielectric types

Capacitors come in many different types, the primary distinction being the type of substance used for the dielectric (i.e. the insulating layer between the capacitor plates). Rather than reproduce tables of dielectric material types and various performance ratings for each, here we will list several important parameters and for each of these cite the best and worst dielectric materials.

- **Rated voltage**: *the maximum continuous DC voltage across the capacitor based on the dielectric's breakdown limit*
- **Capacitance density**: *the amount of capacitance per unit volume, generally a desirable feature* – "supercapacitors" exploiting such effects as double-layer and pseudo-capacitance are the undisputed champions for this parameter. For conventional dielectrics, aluminum and tantalum electrolytic are best. Worst is vacuum or air.
- **Insulation resistance**: *the desirable ability of the dielectric to oppose "leakage current" through the capacitor* – polystyrene film is best; tantalum electrolytic is worst.
- **ESR and dissipation factor**: *mechanisms of energy dissipation within the capacitor, any of which are undesirable* – polystyrene, mica, and NP0 ceramic is best; tantalum electrolytic is worst.
- **Dielectric absorption**: *the undesirable tendency of a dielectric to maintain a DC bias after rapid discharging* – polypropylene and polystyrene are best; tantalum electrolytic is worst.
- **Stability**: *the degree to which a capacitor's capacitance value undesirably drifts over long periods of time* – NP0 ceramic is best; several dielectric materials tie for worst (High-K ceramic, reduced titanate, multi-layer glass with "V" characteristic).
- **Temperature coefficient**: *the degree to which a capacitor's capacitance value undesirably changes with temperature* – NP0 ceramic is best; High-K ceramic is worst.
- **Voltage coefficient**: *the degree to which a capacitor's capacitance value undesirably changes with applied voltage* – electrolytics are best; Class 2 ceramic and plastic-film are worst. Any dielectric material with *paraelectric* or *ferroelectric* properties (e.g. Class 2 ceramics such as barium titanate) exhibit this trait because their permittivity values vary significantly as the electric field increases and decreases in strength.
- **Polarization**: *whether or not the capacitor requires unidirectional voltage polarity* – all electrolytics are polarized; others are not.

The principal mechanism of energy dissipation within a capacitor is *dielectric loss*, where molecules within the dielectric material shift with a varying electric field. This voltage-induced molecular movement results in the dielectric becoming warm, thus converting electrical energy into thermal energy where it does not return to the circuit. Interestingly, this phenomenon is actually exploited in certain industrial processes such as the heating of plastics via high-voltage and high-frequency AC applied across the insulating material. In a capacitor, though, this effect is universally undesirable. Not only does dielectric loss constitute energy diverted from the circuit, but the heating will cause the capacitor's own capacitance to drift and may even destroy the device if severe enough.

The following table attempts to rank ten different dielectric types according to five important performance parameters. Please note that these rankings are based on some *worst-case* data I found, and that the performance metrics of specific capacitor models may likely vary from the values and orders shown below. As always, manufacture literature and/or empirical measurement are your best sources of information:

Rank	**Temp coeff.**	**Stability**	**Absorption**	**ESR**	**Insul. Resis.**
Best	NP0 = 0.3%	NP0 = 0.1%	PolyP = 0.05%	NP0 = 0.1%	PolyS = 10^6 MΩ
	PolyS = 1%	Mica = 0.1%	PolyS = 0.05%	Mica = 0.1%	PolyP = 10^5 MΩ
	Mica = 1.8%	PolyS = 2%	PolyC = 0.35%	PolyS = 0.1%	NP0 = 10^5 MΩ
	PolyC = 2%	PolyP = 3%	PolyE = 0.5%	PolyP = 0.35%	PolyC = 10^4 MΩ
	PolyP = 2.5%	PolyC = 5%	NP0 = 0.6%	PolyC = 1%	HiK = 10^3 MΩ
	Tant = 8%	PolyE = 10%	Mica = 0.7%	PolyE = 2%	PolyE = 10^3 MΩ
	Alum = 10%	Tant = 10%		HiK = 3%	RedT = 10^2 MΩ
	PolyE = 12%	Alum = 10%		Alum = 8%	Mica = 10^2 MΩ
	RedT = 30%	RedT = 20%	Alum	RedT = 10%	Alum = 10^2 MΩ
Worst	HiK = 80%	HiK = 20%	Tant	Tant = 24%	Tant = 10^1 MΩ

Abbreviations for dielectric types:

- NP0 = Multi-layer NP0 “Class 1” type ceramic
- HiK = Multi-layer High-K “Class 2” type ceramic
- RedT = Reduced titanate
- PolyE = Polyester film (“mylar”)
- PolyC = Polycarbonate film
- PolyP = Polypropylene film
- PolyS = Polystyrene film
- Tant = Solid tantalum electrolytic
- Alum = Aluminum electrolytic

Also note that conservative design principles may help minimize undesirable capacitor traits. For example, if the capacitor is known to have a large temperature coefficient, placing it in an area of the circuit board where you would expect temperature to be most stable will mitigate this shortcoming. Similarly, if the capacitor has a large voltage coefficient a good mitigation strategy is to over-rate the capacitor in terms of voltage (i.e. select a capacitor with a much higher voltage rating than the expected voltage for your application).

Multi-layer ceramic capacitors are extremely popular given their non-polarized performance making them suitable to reversing DC polarities as well as AC, and their ready availability in surface-mount device (SMD) packages. However, the two major classes (NP0 as "Class 1" and High-K as "Class 2") differ widely in performances. Class 1 ceramic capacitors are extremely stable in terms of their capacitance varying with age, with applied voltage, and with temperature. Class 2 ceramic capacitors sacrifice these desirable traits in favor of greater capacitance per physical volume. We typically find Class 1 ceramic capacitors used within signal filter and timing networks where their greater stability is worth their increased size and weight. Popular applications for Class 2 ceramic capacitors include DC power decoupling and radio-frequency interference (RFI) reduction where the precise amount of capacitance is less important than a certain minimum value of capacitance. Interestingly, Class 2 ceramic dielectrics also exhibit *piezoelectric* properties, meaning they produce voltage when physically strained and will physically strain when subjected to voltage. This means vibrations (including sound) will introduce AC noise voltage into the circuit at those same frequencies, and applied AC will correspondingly produce mechanical vibrations within the capacitor!

An important factor in ceramic dielectric types is temperature stability, i.e. how much capacitance is expected to vary as the device's temperature changes. Alphanumeric codes typically not appearing on the devices themselves specify the device's temperature dependence, the table below showing codes for Class 1 (NP0) ceramic dielectric materials:

Base temp. coefficient	**Temp. coefficient multiplier**	**Temp. coefficient tolerance**
C = 0.0 ppm/deg C	0 = −1	G = ± 30 ppm
B = 0.3 ppm/deg C	1 = −10	H = ± 60 ppm
L = 0.8 ppm/deg C	2 = −100	H = ± 120 ppm
A = 0.9 ppm/deg C	3 = −1000	K = ± 250 ppm
M = 1.0 ppm/deg C	4 = +1	L = ± 500 ppm
P = 1.5 ppm/deg C	6 = +10	M = ± 1000 ppm
R = 2.2 ppm/deg C	7 = +100	N = ± 2500 ppm
S = 3.3 ppm/deg C	8 = +1000	
T = 4.7 ppm/deg C		
V = 5.6 ppm/deg C		
U = 7.5 ppm/deg C		

C0G is the most common Class 1 dielectric type, with the best temperature stability of them all. Type C0G dielectric is so popular, in fact, that the designation is often used interchangeably with NP0 as a shorthand for "Class 1" dielectric.

Class 2 ceramic dielectric materials have a completely different alphanumeric coding scheme for representing temperature dependence, as shown in the table below. Rather than specify a temperature coefficient in parts per million, Class 2 dielectrics specify lower and upper bounds of their operating temperature range plus the percentage of rated capacitance value we may expect the capacitance to vary over that range:

Low temperature	**High temperature**	**Capacitance change over range**
X = −55 deg C	4 = +65 deg C	P = ±10%
Y = −30 deg C	5 = +85 deg C	R = ±15%
Z = +10 deg C	6 = +105 deg C	S = ±22%
	7 = +125 deg C	T = +22/−33%
	8 = +150 deg C	U = +22/−56%
	9 = +200 deg C	V = +22/−82%

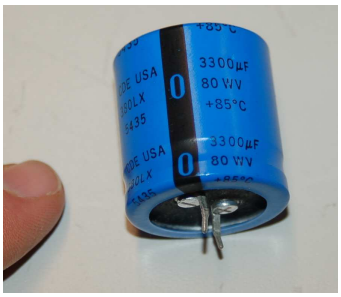
Note how all three parameters together define temperature coefficient for a given ceramic capacitor. For example, a capacitor with X5R dielectric will drift more per degree of temperature variation than a capacitor with X7R dielectric despite the fact these two dielectric types share the same "R" letter code representing ±15%, because the X5R has a significantly narrower temperature range than the X7R for that same drift percentage. If a type-X7R capacitor drifts 15% in its capacitance as temperature rises from −55 degrees to +125 degrees, but a type-X5R capacitor drifts the same amount with less temperature rise (−55 degrees to just +85 degrees), it means the type-X5R capacitor is more sensitive to changes in temperature and therefore X7R would be a better choice than X5R for applications requiring a more stable capacitance value.

Common Class 2 dielectrics include X5R, X7R, and Y5V.

6.5 Capacitor labeling

Capacitors store electrical energy using electric fields, the amount of energy stored being proportional to the square of the applied voltage. The ratio of stored energy to voltage (squared) is called *capacitance* (C), measured in the unit of the *Farad* (F), and it is the primary characteristic of any capacitor. Unfortunately, the labeling of capacitance on many capacitors is rather confusing. Color codes used to be popular for denoting capacitance value, following a very similar pattern to resistors, but numerical labels are now the norm. However, these labels tend to be rather terse, and this can cause interpretational difficulties.

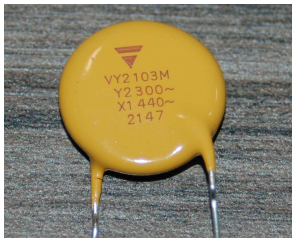
Physically large capacitors having sufficient surface area to allow many characters of printed text are the best in this regard.



The "supercapacitor" seen in the left-hand photograph is the clearest of them all: 650 Farads of capacitance with a working voltage rating of 2.7 Volts DC. The capacitor in the center photograph is also very unambiguous, having ratings of 3300 microFarads and 80 Volts DC ("WV" = *working voltage*). However the capacitor seen in the right-hand photograph, despite being physically larger than either of the others, is more cryptic: 16500 microFarads of capacitance with a working voltage rating of 50 Volts DC. Here we must simply assume that the "16500" figure refers to *micro*Farads.

Polarity is another important parameter for electrolytic-type capacitors such as these. For the supercapacitor (left photo) we see a stripe and letter "P" denoting the positive terminal of the capacitor; the 3300 microFarad capacitor (center photo) bears a black stripe and "−" symbol pointing toward its negative terminal; the 16500 microFarad capacitor (right photo) has polarity markings molded into the end-cap near its screw terminals (not visible in this photo).

Capacitors connected to AC line ("mains") voltage within line-powered appliances and other electronic devices must be *safety-rated*, as they are exposed to routine voltage transients such as those commonly experienced on electric power grids during lightning storms, and also because any short-circuit failure of a line-connected component would be catastrophic. For this reason, such capacitors often bear X and Y voltage ratings such as those seen on the following capacitor:



An X rating refers to the capacitor connected in parallel with the AC line's "hot" and "neutral" conductors: that is, directly across the line voltage. A Y rating refers to the capacitor connected between an Earth-potential conductor and either the "hot" or "neutral" conductor of an AC line source. These two different applications have their own safety concerns: a capacitor in an "X" application is at risk of catastrophically failing and possibly igniting nearby flammable materials if it develops an internal short-circuit, whereas a capacitor in a "Y" application is at risk of impressing dangerous electrical potential onto a metallic surface it internally fails shorted.

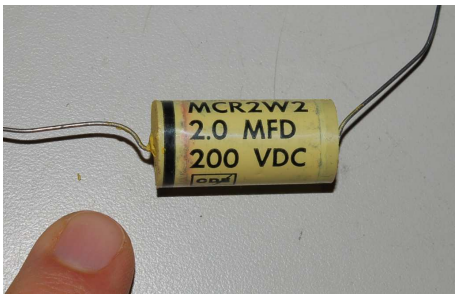
Immediately following the X or Y designator is a single numerical digit representing the rating sub-class, based on the peak impulse (transient) voltage:

Sub-class	**Maximum transient service voltage**
X1	± Between 2.5 kV and 4 kV
X2	± Less than 2.5 kV
X3	± Less than 1.2 kV

Sub-class	**Maximum transient test voltage**
Y1	± 8 kV
Y2	± 5 kV
Y3	± (none)
Y4	± 2.5 kV

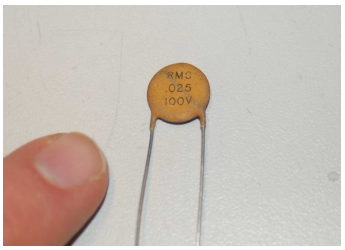
Any voltage value following an X or Y class/sub-class rating refers to the continuous AC voltage that capacitor is rated to handle in such a safety-critical application. For example, the `X1 440~` label on the capacitor shown in the photograph tells us that capacitor is rated for continuous duty across the poles of a 440 Volt AC source and that its service transient (impulse) rating is somewhere between 2500 Volts and 4000 Volts. This is the sort of capacitor we might find connected in parallel with the "hot" and "neutral" conductors of a 120 Volt AC power cord for suppression of high-frequency "noise".

Returning to our exploration of general capacitor labeling:



On the left we have a 2500 microFarad electrolytic capacitor rated for 50 Volts DC (positive on left), a capital letter "U" employed to symbolize *micro*. On the right we have a 2 microFarad plastic-film capacitor rated at 200 Volts DC, this time with the capital letter "M"[10] representing *micro*. Since plastic-film capacitors are non-polarized, the stripe on the left-hand side must represent something other than polarity: here it represents the terminal connected to the outer-most layer of metal foil[11] inside the capacitor.

Smaller capacitors become slightly more consistent, though no less confusing, in their labeling. Consider these examples:



From left to right we have (left) a 0.082 microFarad capacitor rated for 100 Volts DC[12], (center) a 0.025 microFarad capacitor rated for 100 Volts DC, and (right) a 0.68 microFarad capacitor rated for 160 Volts DC. The rule here is, any numerical capacitance value with a decimal point should be read as *micro*Farads. Note how the capacitor in the right-hand photograph uses a *comma* rather than a *point* between the "0" and the "68" because this one is of European manufacture, and it is conventional in European technical literature to use commas rather than points for decimal numbers.

[10] In the metric system a lower-case "m" is supposed to represent the prefix *milli* ($\times 10^{-3}$) and an upper-case "M" is supposed to represent the prefix *mega* ($\times 10^{6}$). However, since no technology yet invented is able to pack a *mega*Farad of capacitance into a single component, and for some unknown reason the prefix "milli" is never used for capacitance, we are expected to deduce that this letter "M" must represent *micro*. This, of course, flies in the face of standard metric notation, but it is nevertheless common to see in capacitor labeling.

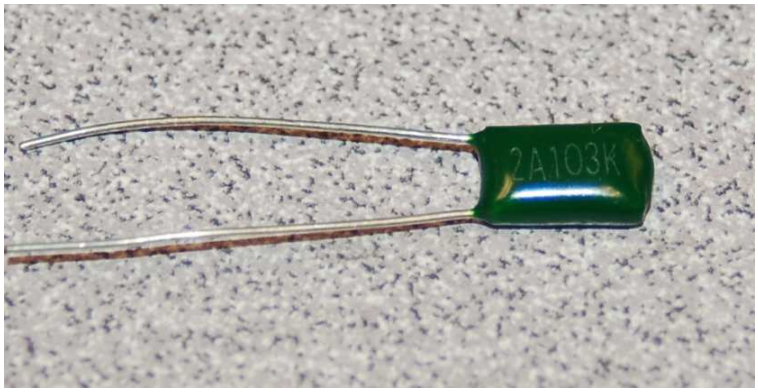
[11] This might be important to know for reasons of interference and signal coupling in densely-packed circuits. For a plastic-case capacitor such as this, the outer-most metal foil layer may comprise a parasitic capacitance with some adjacent component, and this in turn may degrade the circuit's performance. Best practice is to connect the "stripe" terminal of such a capacitor to ground or a power supply "rail" or some other node having a stable electrical potential.

[12] The second "0" of the number 100 happens to be partially rubbed off.

For capacitors with smaller capacitance values, the default metric prefix becomes *pico* ($\times 10^{-12}$) rather than micro. We can tell the difference between a "micro" capacitor and a "pico" capacitor by the presence or lack, respectively, of a decimal point. For example, a capacitor labeled 2.2 would be 2.2 microFarads in size, but a capacitor labeled 22 would be 22 picoFarads in size.

For decimal values of picoFarads, a lower-case letter "p" may be used as a decimal point; e.g. 2p2 would be 2.2 picoFarads. Similarly, the lower-case letter "n" may be used as a decimal point as well (e.g. 2n2 would be 2.2 nanoFarads). Confusingly, some manufacturers use a capital letter "R" as a generic decimal point for the capacitance value, the default metric prefix once again being *pico* (e.g. 2R2 would be 2.2 picoFarads).

A very common standard for small capacitor labeling is the use of three-digit numerical codes following the same digit-digit-multiplier format as resistor color codes, combined with the default assumption of *pico*Farads. For example, 332 would represent 33×10^2 picoFarads, which is equal to 3.3 nanoFarads. The following photograph shows a mylar film capacitor with a "103" code representing 10×10^3 picoFarads, which is equivalent to 10 nanoFarads:



Note the "2A" preceding the "103" capacitance rating: this is nothing but part of the manufacturer's part number. This is an unfortunate convention, of mixing manufacturer-specific part numbering with standardized value coding, and we see another example of it in the following photograph:

This Vishay capacitor also has a rating of 10 nanoFarads, but here we see the "103" capacitance code preceded by "YV2" which is unique to Vishay's part-numbering system. The last character

("M") specifies this capacitor's tolerance of ±20%, which is the next topic we will explore.

Capacitor *tolerance*, like resistor tolerance, specifies how far the particular device's capacitance may deviate from its advertised (nominal) value. In one of the previous examples, we saw a 0.082 microFarad capacitor with a ±10% tolerance, printed exactly as such on its body. However, for physically small capacitors where not enough surface area exists to print percentage figures, such as the 10 nF Vishay capacitor previously shown, we must use special letter codes to designate tolerance. These letter codes are also used for other components such as resistors, the following table showing several of them:

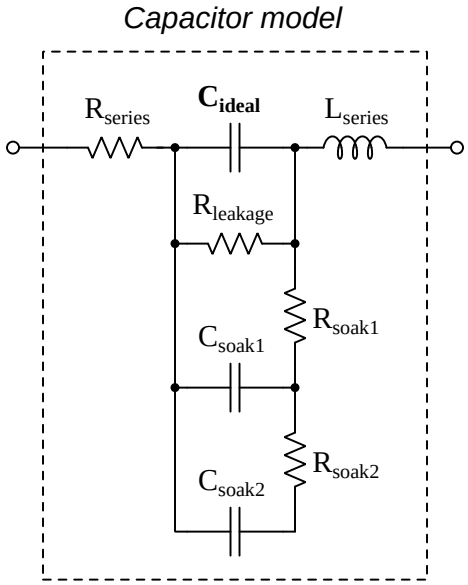
Letter code	**Tolerance**
A	± 0.05 pF
B	± 0.1 pF
C	± 0.25 pF
D	± 0.5 pF
E	± 0.5%
F	± 1%
G	± 2%
H	± 3%
J	± 5%
K	± 10%
L	± 15%
M	± 20%
N	± 30%
P	± −0% to +100%
S	± −20% to +50%
W	± −0% to +200%
X	± −20% to +40%
Z	± −20% to +80%

Note how the first four of these codes refer to absolute tolerances in picoFarads, while the rest represent tolerances in percent. When printed on a capacitor's body, the tolerance code typically follows the numerical capacitance code. For example, the photograph on the previous page showed a green-colored capacitor with "103K" on its body, which is 10×10^3 picoFarads ± 10%.

6.6 Capacitor parasitics

6.6.1 Model of a real capacitor

An ideal capacitor exhibits only capacitance, with no inductance, resistance, or other characteristics to interfere. Real capacitors exhibit all these phenomena to varying degrees, and we collectively refer to these undesirable traits as *parasitic effects*. The following diagram models some of the parasitic effects observed in real capacitors:



In addition to the capacitance the capacitor is supposed to exhibit (C_{ideal}), the capacitor also has parasitic resistance (R_{series}, also known as *Equivalent Series Resistance*, or *ESR*), parasitic inductance (L_{series}), and additional energy storage in the form of *soakage* (also known as *dielectric absorption*) whereby the dielectric substance itself absorbs and releases energy after relatively long periods of time compared to the main (ideal) capacitance.

Some of these parasitic effects – such as leakage resistance and soakage – affect the capacitor's performance in DC applications. Most of the other parasitic effects cause problems in AC and pulsed applications. For example, the effective series capacitor-inductor combination formed by C_{ideal} and L_{series} will cause *resonance* to occur at a particular AC frequency, resulting in much less reactance at that frequency than what would be predicted by the capacitive reactance formula $X_C = \frac{1}{2\pi fC}$.

Next we will explore common mechanisms for each of these effects.

6.6.2 Parasitic resistance in capacitors

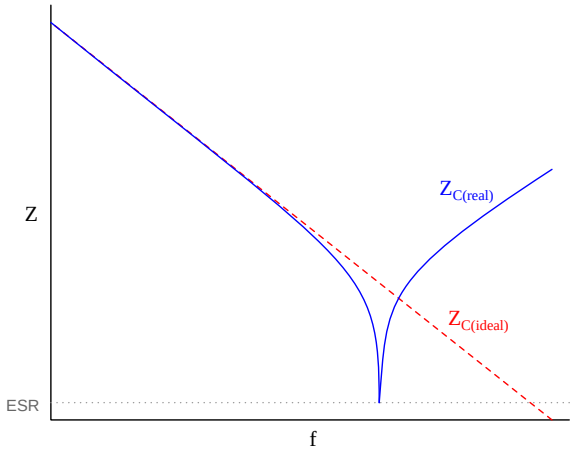
Wire resistance, of course, plays a part in this parasitic effect, but this is generally very small due to the short conductor lengths we typically see with capacitors. More significant is *dielectric losses* – energy dissipation caused by the stressing and relaxation of dipoles within the dielectric material – which act like resistance because energy ends up leaving the component (in the form of heat) and not returning to the circuit. Electrolytic capacitors have an additional source of parasitic resistance, in the form of the electrolytic gel substance used to make electrical contact from the metal-foil "plate" to the surface of the dielectric layer.

Another form of parasitic resistance within a capacitor behaves like a resistor connected in parallel with the ideal capacitance ($R_{leakage}$), resulting from the dielectric not being a perfectly insulating medium. This parasitic characteristic results in a small current passing through the capacitor even when the voltage across the capacitor is steady (i.e. $\frac{dV}{dt}$ is zero).

6.6.3 Parasitic inductance in capacitors

Any time a magnetic field forms around a current-carrying conductor, energy is stored in that magnetic field. We call this magnetic-based energy-storing capability *inductance*, and of course all capacitors must have some inductance due to the wire leads serving as connection points to the capacitor's metal plates. Much of a capacitor's parasitic inductance may be minimized by maintaining short lead lengths as it attaches to a printed-circuit board.

Parasitic inductance is a problem for capacitors in AC applications because inductive reactance (X_L) tends to cancel out capacitive reactance (X_C). If we plot the impedance of a capacitor as a function of frequency, we would expect an ideal capacitor to manifest a straight-line descent on a logarithmic plot. However, what we see is that at a certain frequency the series parasitic inductance resonates with the capacitance leaving only parasitic resistance (ESR), and then past that frequency the inductive effects overshadow the capacitance:



6.6.4 Other parasitic effects in capacitors

Soakage is an interesting effect resulting from dipole relaxation within the dielectric material itself, and may be modeled (as shown) by a series of resistor-capacitor subnetworks. This effect is especially prominent in aluminum electrolytic capacitors, and may be easily demonstrated by discharging a capacitor (by briefly connecting a shorting wire across the capacitor's terminals) and then monitoring the capacitor's DC voltage slowly "recover" with no connection to an external source.

6.7 Inductor parameters and core types

Inductors come in many different types, the primary distinctions being the type of substance used for the magnetic core and the physical shape of that core. Several important performance parameters exist for inductors, listed below:

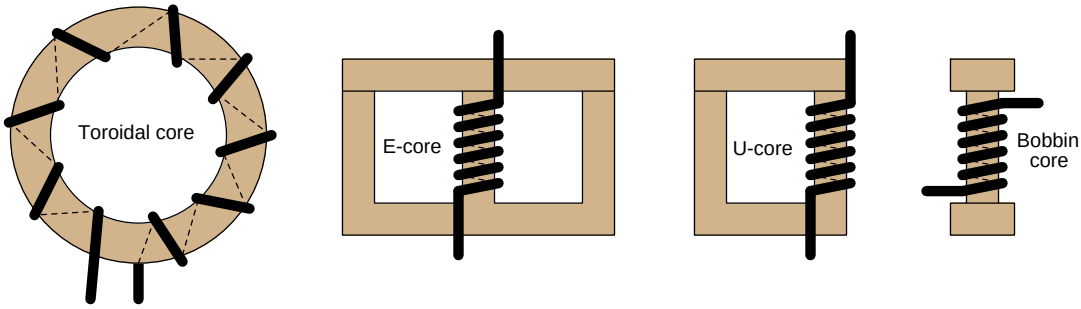
- **Insulation resistance**: *the desirable ability of the wire insulation to oppose "breakdown" resulting from high voltage applied across that insulation, either from the wire to the core material, from one wire to another (e.g. primary-to-secondary in a transformer), or from one turn of wire to an adjacent turn of wire*
- **DC resistance**: *Ohmic resistance of the wire used to make the winding(s) of the inductor, generally undesirable because this constitutes a mechanism of energy dissipation within the inductor*
- **Rated current**: *the maximum continuous DC current through the inductor based on wire heating from I^2R dissipation*
- **Saturation current**: *the amount of current at which point the core begins to magnetically saturate*[13]*, and its inductance correspondingly decreases, usually based on a specified percentage of inductance decrease compared to at zero current*
- **Distributed (parasitic) capacitance**: *wire insulation separating winding turns from each other and the winding from the core material (if electrically conductive) constitute parasitic capacitance in the inductor*
- **Self-resonance**: *the frequency at which an inductor will self-resonate based on its inductance reacting with its own parasitic capacitance*
- **Leakage inductance**: *the amount of inductance inherent to magnetic flux escaping the core (i.e. "leaking" flux), which may couple with adjacent conductors*
- **Stability**: *the degree to which an inductor's value undesirably drifts over long periods of time*
- **Temperature coefficient**: *the degree to which an inductor's value undesirably changes with temperature*
- **Rated temperature**: *the maximum operating temperature at which an inductor will still perform within specification, limited by insulation type, thermal expansion coefficients of the wire and core materials, and the Curie temperature*[14] *of the core material*

[13] Ferromagnetic materials assist in the development of magnetic fields as their internal magnetic "domains" align one-by-one with any externally-applied field such as that from an energized coil of wire. If we keep increasing the external magnetic field, we will reach a point where all the ferromagnetic core material's domains are fully aligned with that field, at which point the core can offer no further assistance to increasing field strength. Here the core is said to be *magnetically saturated*, like a sponge fully saturated with water. Any further increases in magnetic field strength (e.g. increasing coil current beyond the saturation point) will proceed unaided by the core.

[14] The Curie temperature is that temperature beyond which a ferromagnetic material begins to lose its magnetic properties.

Multiple energy-dissipation mechanisms exist within inductors, and may be broadly divided into two categories: "copper" (wire) losses and "iron" (core) losses. All wires made of non-superconducting materials exhibit some electrical resistance, and this directly converts electrical energy into thermal energy ($P = I^2R$) where it will never return to the circuit. At very high frequencies where the *skin effect* causes AC current to avoid the center of the conductor and only travel near the surface (skin), the wire's effective cross-sectional area decreases and this leads to greater I^2R losses. Ferromagnetic core materials experience molecular-level motion[15] when the magnetic field varies in intensity, and this represents another way for electrical energy to become converted into heat and never return to the circuit. Magnetic materials dissipate energy in this way proportional to the amount of hysteresis they exhibit: magnetically "soft" materials having narrow B-H hysteresis curves are more energy-efficient as inductor and transformer cores than magnetically "hard" materials having wide B-H hysteresis curves. Furthermore, if the core material happens to be electrically conductive there will be some amount of electric current induced within the core itself as inductor current varies, as a result of electromagnetic induction. These *eddy currents* encounter electrical resistance in the core material, also resulting in I^2R losses.

A variety of core shapes exist for different sizes and purposes of inductor:

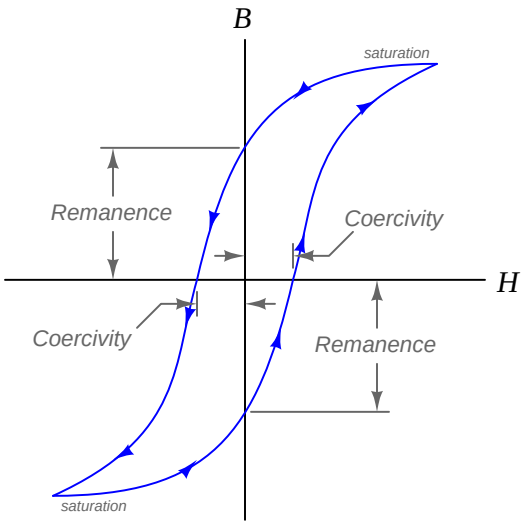


Toroidal cores are exceptionally good at confining the magnetic field to the inductor (i.e. very low "leakage" flux) but are more expensive to manufacture.

Inductor core materials exist to enhance the amount of magnetic flux resulting from electric current passing through the winding(s), as well as to "contain" the magnetic flux so that it remains inside the inductor case and does not link with any conductors outside of the inductor. Some inductors have no solid core material, and are called *air-core* inductors. These have very low inductance values and suffer from unwanted "coupling" with adjacent conductors but enjoy immunity from magnetic saturation. Most inductors use some form of *ferromagnetic* material such as iron or iron alloy.

[15] This effect when viewed on a macroscopic scale is called *magnetostriction*, and accounts for why ferromagnetic inductor and transformer cores may "hum" when their respective windings are energized by AC in the audio-frequency range.

Most inductor designs attempt to minimize energy losses in the magnetic core, one strategy for this being to minimize magnetic hysteresis losses. Hysteresis losses occur when the magnetic domains inside the material have a high degree of *remanence*, necessitating work to be done when magnetizing, unmagnetizing, and re-magnetizing the core material as inductor current varies and/or reverses direction. The amount of energy dissipated in this manner by a magnetic core is proportional to the area enclosed by the B-H curve:



Choosing a magnetic material with a naturally narrow B-H curve helps minimize hysteresis losses. So does "over-building" the core to have greater cross-sectional area than is absolutely necessary, in an effort to spread out the magnetic flux over more material and therefore decrease flux density (B) so the core remains as far from saturation as possible.

Another strategy for minimizing energy losses in the core focuses on the mechanism of *eddy currents* which are electric currents induced in the core material by way of electromagnetic induction. This is a problem when the core material happens to be electrically conductive as well as ferromagnetic (e.g. iron is a good example of this), as these circulating electric currents do no useful work yet still encounter electrical resistance in the core material causing heat dissipation which is energy extracted from the circuit never to return to it. An effective mitigation is to build the core in such a way that it electrically interrupts these currents by not providing a continuous pathway for them to circulate. If the core material is iron, constructing the core as a stack of thin iron sheets coated in (insulating) enamel accomplishes this goal, and we call this a *laminated iron core*. Another strategy is to use powdered iron rather than solid iron. Yet another is to use a material that is ferromagnetic but electrically insulating, for example *ferrite* which is a chemical oxide of iron rather than being metallic iron.

For the end-user rather than the designer of manufactured inductors, an effective strategy for minimizing energy losses is to simply over-rate the inductor in terms of current (i.e. select an inductor with a much higher current rating than the expected current for your application). Using an inductor with a much higher current rating than your expected design current means that inductor will have larger-gauge wire windings, as well as an over-sized magnetic core capable of handling much stronger magnetic fields than you will ever impress upon it. Over-sized windings minimize resistive

energy losses, while the over-sized core effectively shortens and narrows the B-H curve which in turn minimizes hysteresis losses.

6.8 Inductor labeling

Inductors store electrical energy using magnetic fields, the amount of energy stored being proportional to the square of the applied current. The ratio of stored energy to current (squared) is called *inductance* (L), measured in the unit of the *Henry* (H), and it is the primary characteristic of any inductor. Color codes used to be popular for denoting inductance value, following a very similar pattern to resistors, but numerical labels are now the norm.

The color code used to mark inductors follows the same chromatic and numerical sequence used for resistors[16], with the first and second bands representing significant digits, the third band representing a power-of-ten multiplier, and the fourth band representing tolerance. However, rather than directly indicating inductance in Henrys, the implied metric prefix is *micro*.

Band color	First digit value	Second digit value	Multiplier	Tolerance
Black	0	0	10^0	± 20%
Brown	1	1	10^1	± 1%
Red	2	2	10^2	± 2%
Orange	3	3	10^3	± 3%
Yellow	4	4	10^4	± 4%
Green	5	5	10^5	
Blue	6	6	10^6	
Violet	7	7	10^7	
Grey	8	8	10^8	
White	9	9	10^9	
Gold			10^{-1}	± 5%
Silver			10^{-2}	± 10%
None				± 20%

For example, an inductor marked with colored bands Yellow, Violet, Orange, and Gold would be 47×10^3 microHenrys, or 47 mH.

Inductors labeled with numerical markings follow the same digit-digit-multiplier pattern as the color code: two significant digits followed by a power-of-ten multiplier, with an assumed base prefix of *micro*. For example, 331 would represent 33×10^1 microHenrys, which is 330 μH or 0.33 mH. If a decimal point is required for a numerically-labeled inductor, a capital letter "R" is used. For example, instead of labeling a 0.01 μH inductor as 0.01, it would be shown as R01.

[16] A useful mnemonic for associating these colors with decimal digits 0 through 9 and the percentages 5-10-20% is as follows: *"Better Be Right Or Your Great Big Venture Goes Wrong. Get Started Now."*.

Very small inductors are sometimes numerically labeled on the basis of *nano*Henrys rather than microHenrys. In such cases a capital letter "N" represents the decimal point for the nanoHenry value: for example, 3N9 represents 3.9 nH and 27N represents 27 nH.

Inductor *tolerance* is another important parameter which is often printed on the body of the inductor. For physically small inductors where not enough surface area exists to print percentage figures, we must resort to other means for expressing tolerance. To this end, a system of letter-codes has been developed, shown here in the following table:

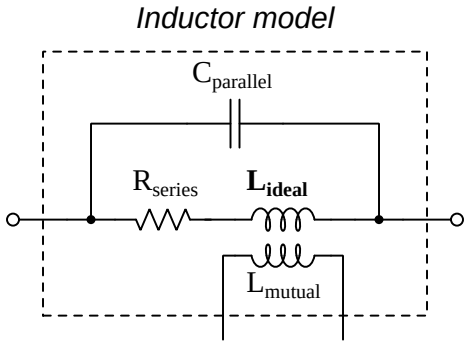
Letter code	**Tolerance**
B	± 0.15 nH
C	± 0.2 nH
S	± 0.3 nH
D	± 0.5 nH
F	± 1%
G	± 2%
H	± 3%
J	± 5%
K	± 10%
L	± 15%
M	± 20%
V	± 25%
N	± 30%

Note how the first four of these codes refer to absolute tolerances in nanoHenrys, while the rest represent tolerances in percent. When printed on a inductor's body, the tolerance code typically follows the numerical inductance code. For example, an inductor with "821J" on its body would be 820 μH ± 5%.

6.9 Inductor parasitics

6.9.1 Model of a real inductor

An ideal inductor exhibits only inductance, with no capacitance, resistance, or other characteristics to interfere. Real inductors exhibit all these phenomena to varying degrees, and we collectively refer to these undesirable traits as *parasitic effects*. The following diagram models some of the parasitic effects observed in real inductors:



In addition to the inductance the inductor is supposed to exhibit (L_{ideal}), the inductor also has parasitic resistance (R_{series}, also known as *Equivalent Series Resistance*, or *ESR*), parasitic capacitance ($C_{parallel}$), and mutual inductance (L_{mutual}) with nearby wires and components.

Some of these parasitic effects – such as equivalent series resistance – affect the inductor's performance in DC applications. Most of the other parasitic effects cause problems in AC and pulsed applications. For example, the effective inductor-capacitor "tank circuit" formed by L_{ideal} and $C_{parallel}$ will cause *resonance* to occur at a particular AC frequency, resulting in much more reactance at that frequency than what would be predicted by the inductive reactance formula $X_L = 2\pi fL$.

Next we will explore common mechanisms for each of these effects.

6.9.2 Parasitic resistance in inductors

Wire resistance plays a dominant role in this parasitic effect due to the typically long lengths of wire necessary to wind the coil that forms most inductors. Wire resistance is not the only dissipative mechanism at work, though. Other losses include magnetic hysteresis of the iron core material as well as *eddy currents* induced in the iron core. An "eddy" current is a circulating electric current induced within the iron core of an inductor, made possible by the fact that iron is an electrically-conductive material as well as being ferromagnetic. These circulating currents do no useful work, and dissipate energy in the form of heating the iron. They may be minimized by forming the iron core from pieces of iron that are electrically insulated from one another, e.g. forming the iron core from *laminated sheets* or *powdered particles* of iron where each sheet or particle is electrically insulated from the next by a layer of non-conductive material on its outer surface.

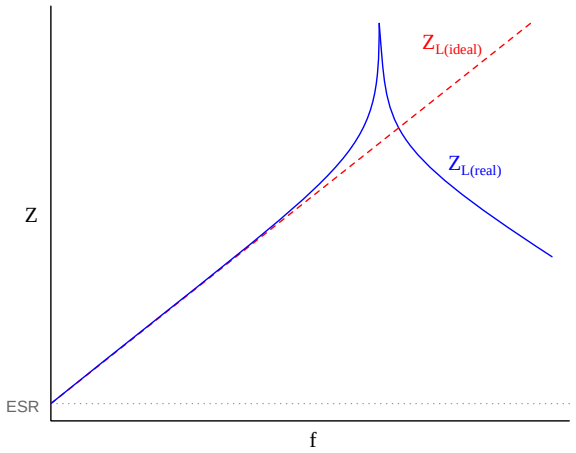
The series resistance of an inductor is always frequency-dependent. In DC conditions (i.e. frequency of zero Hertz) there will be the basic wire resistance of the coil at play. As frequency increases from zero, however, both the magnetic core losses from hysteresis and eddy currents also increase which add to the DC resistance to form a larger ESR. At extremely high frequencies the *skin effect*[17] further adds to the inductor's ESR.

[17] At high frequencies, electric current travels more toward the outer surface of a conductor rather than through the conductor's entire cross-section, effectively decreasing the conductor's cross-sectional area (gauge) as frequency rises.

6.9.3 Parasitic capacitance in inductors

Any time an electric field forms between two conductors, energy is stored in that electric field. We call this electric-based energy-storing capability *capacitance*, and of course all inductors must have some capacitance due to the insulating media between wire leads as well as between adjacent turns of wire within the coil (and between the wire turns and the iron core).

Parasitic capacitance is a problem for inductors in AC applications because capacitive reactance (X_C) tends to cancel out inductive reactance (X_L). If we plot the impedance of an inductor as a function of frequency, we would expect an ideal inductor to manifest a straight-line ascent on a logarithmic plot. However, what we see is that at a certain frequency the parallel parasitic capacitance resonates with the inductance to create a nearly-infinite impedance, and then past that frequency the capacitive effects overshadow the inductance:



Precious little may be done to eliminate parasitic capacitance within any inductor, whereas parasitic inductance is fairly easy to minimize within a capacitor. This explains why when faced with an equivalent choice between a circuit design using capacitors and a circuit design using inductors, capacitors nearly always win. Simply put, it is easier to make a nearly-ideal capacitor than it is to make a nearly-ideal inductor.

This also explains why the self-resonant frequency of most inductors is much lower than the self-resonant frequency of most capacitors: all other factors being equal, an inductor will have more parasitic capacitance in it than an equivalent capacitor will have parasitic inductance within it, making the LC product greater for the inductor than for the capacitor.

6.9.4 Other parasitic effects in inductors

Mutual inductance occurs whenever adjacent conductors' magnetic fields link with one another, which is difficult to avoid especially in physically dense circuit layouts. This parasitic effect may be minimized by proper placement of inductive components (e.g. keeping them spaced as far apart from each other as possible, orienting their axes perpendicular to each other rather than parallel) as well as by core designs with strong magnetic field containment (e.g. toroidal cores contain their magnetic fields better than rectangular cores).

6.9.5 Inductor testing

The following list shows typical tests for inductors at the time of manufacture:

- **DC winding resistance** – the simplest of these to measure, using a multimeter
- **Winding inductance/impedance** – performed by exciting the winding with an AC source and comparing voltage/current, typically using either an LCR meter or an impedance analyzer
- **Equivalent winding resistance** – based on the ratio of AC voltage to AC current (only the portions of each in-phase with each other) representing winding resistance as well as energy losses in the magnetic core
- **Core saturation** – performed by exciting the winding with a sinusoidal AC current and measuring self-induced winding voltage until a non-sinusoidal waveshape is detected, or more crudely by determining when the self-induced voltage no longer increases proportionately to increased excitation current
- **Insulation resistance** – performed with a "Hi-Pot" (high-potential) insulation tester, generally connecting one terminal to either or both ends of the winding and the other tester terminal to the conductive core material, this infers insulation resistance by applying (typically) several thousand Volts DC while measuring the small current passing through the stressed insulation

A more thorough battery of tests used during transformer design development might include the following:

- **BH loop** – performed by exciting the winding with an AC voltage and measuring the self-induced voltage across it, the phasor portion of that induced voltage shifted by 90 degrees from the current waveform representing the rate-of-change of magnetic flux which may be mathematically integrated to infer the amount of magnetic flux, those two variables being plotted using an oscilloscope's X-Y mode much like "curve tracing" for a semiconductor component ; often performed at different temperatures to fully characterize the magnetic properties of the core material
- **High-voltage impulse** – similar to an insulation resistance test, this uses a high-voltage DC pulse applied across the winding insulation to stress it
- **Volt-second** – performed by applying a DC voltage pulse to the winding while measuring current through it, the ideal response being a linear-ramping current whose magnitude is proportional to the Volt-second product of the applied pulse
- **Overload** – subjecting a winding to dramatic overcurrent to thermally and mechanically stress the inductor as it might be in an overloaded power system scenario
- **External magnetic field rejection** – subjecting the entire inductor to an external (alternating) magnetic field and measuring any voltage induced across any winding, usually tested with the magnetic field applied across different physical axes
- **Vibration** – not an electrical test per se, but a useful stressor when followed by repeated electrical tests to ensure electrical/magnetic properties have not substantially changed following the vibration

Chapter 7

Programming References

A powerful tool for mathematical modeling is text-based *computer programming*. This is where you type coded commands in text form which the computer is able to interpret. Many different text-based languages exist for this purpose, but we will focus here on just two of them, *C++* and *Python*.

7.1 Programming in C++

One of the more popular text-based computer programming languages is called *C++*. This is a *compiled* language, which means you must create a plain-text file containing C++ code using a program called a *text editor*, then execute a software application called a *compiler* to translate your "source code" into instructions directly understandable to the computer. Here is an example of "source code" for a very simple C++ program intended to perform some basic arithmetic operations and print the results to the computer's console:

```
#include <iostream>
using namespace std;

int main (void)
{
  float x, y;

  x = 200;
  y = -560.5;

  cout << "This simple program performs basic arithmetic on" << endl;
  cout << "the two numbers " << x << " and " << y << " and then" << endl;
  cout << "displays the results on the computer's console." << endl;

  cout << endl;

  cout << "Sum = " << x + y << endl;
  cout << "Difference = " << x - y << endl;
  cout << "Product = " << x * y << endl;
  cout << "Quotient of " << x / y << endl;

  return 0;
}
```

Computer languages such as C++ are designed to make sense when read by human programmers. The general order of execution is left-to-right, top-to-bottom just the same as reading any text document written in English. Blank lines, indentation, and other "whitespace" is largely irrelevant in C++ code, and is included only to make the code more pleasing[1] to view.

[1] Although not included in this example, *comments* preceded by double-forward slash characters (`//`) may be added to source code as well to provide explanations of what the code is supposed to do, for the benefit of anyone reading it. The compiler application will ignore all comments.

Let's examine the C++ source code to explain what it means:

- `#include <iostream>` and `using namespace std;` are set-up instructions to the compiler giving it some context in which to interpret your code. The code specific to your task is located between the brace symbols ({ and }, often referred to as "curly-braces").

- `int main (void)` labels the "Main" function for the computer: the instructions within this function (lying between the { and } symbols) it will be commanded to execute. Every complete C++ program contains a `main` function at minimum, and often additional functions as well, but the `main` function is where execution always begins. The `int` declares this function will return an *integer* number value when complete, which helps to explain the purpose of the `return 0;` statement at the end of the `main` function: providing a numerical value of zero at the program's completion as promised by `int`. This returned value is rather incidental to our purpose here, but it is fairly standard practice in C++ programming.

- Grouping symbols such as (parentheses) and {braces} abound in C, C++, and other languages (e.g. Java). Parentheses typically group data to be processed by a function, called *arguments* to that function. Braces surround lines of executable code belonging to a particular function.

- The `float` declaration reserves places in the computer's memory for two *floating-point* variables, in this case the variables' names being `x` and `y`. In most text-based programming languages, variables may be named by single letters or by combinations of letters (e.g. `xyz` would be a single variable).

- The next two lines assign numerical values to the two variables. Note how each line terminates with a semicolon character (`;`) and how this pattern holds true for most of the lines in this program. In C++ semicolons are analogous to periods at the ends of English sentences. This demarcation of each line's end is necessary because C++ ignores whitespace on the page and doesn't "know" otherwise where one line ends and another begins.

- All the other instructions take the form of a `cout` command which prints characters to the "standard output" stream of the computer, which in this case will be text displayed on the console. The double-less-than symbols (`<<`) show data being sent *toward* the `cout` command. Note how verbatim text is enclosed in quotation marks, while variables such as `x` or mathematical expressions such as `x - y` are not enclosed in quotations because we want the computer to display the numerical values represented, not the literal text.

- Standard arithmetic operations (add, subtract, multiply, divide) are represented as `+`, `-`, `*`, and `/`, respectively.

- The `endl` found at the end of every `cout` statement marks the end of a line of text printed to the computer's console display. If not for these `endl` inclusions, the displayed text would resemble a run-on sentence rather than a paragraph. Note the `cout << endl;` line, which does nothing but create a blank line on the screen, for no reason other than esthetics.

After saving this *source code* text to a file with its own name (e.g. `myprogram.cpp`), you would then *compile* the source code into an *executable* file which the computer may then run. If you are using a console-based compiler such as *GCC* (very popular within variants of the Unix operating system[2], such as Linux and Apple's OS X), you would type the following command and press the Enter key:

```
g++ -o myprogram.exe myprogram.cpp
```

This command instructs the `GCC` compiler to take your source code (`myprogram.cpp`) and create with it an executable file named `myprogram.exe`. Simply typing `./myprogram.exe` at the command-line will then execute your program:

```
./myprogram.exe
```

If you are using a graphic-based C++ development system such as Microsoft Visual Studio[3], you may simply create a new console application "project" using this software, then paste or type your code into the example template appearing in the editor window, and finally run your application to test its output.

As this program runs, it displays the following text to the console:

```
This simple program performs basic arithmetic on
the two numbers 200 and -560.5 and then
displays the results on the computer's console.

Sum = -360.5
Difference = 760.5
Product = -112100
Quotient of -0.356824
```

As crude as this example program is, it serves the purpose of showing how easy it is to write and execute simple programs in a computer using the C++ language. As you encounter C++ example programs (shown as source code) in any of these modules, feel free to directly copy-and-paste the source code text into a text editor's screen, then follow the rest of the instructions given here (i.e. save to a file, compile, and finally run your program). You will find that it is generally easier to

[2] A very functional option for users of Microsoft Windows is called *Cygwin*, which provides a Unix-like console environment complete with all the customary utility applications such as GCC!

[3] Using Microsoft Visual Studio community version 2017 at the time of this writing to test this example, here are the steps I needed to follow in order to successfully compile and run a simple program such as this: (1) Start up Visual Studio and select the option to create a New Project; (2) Select the Windows Console Application template, as this will perform necessary set-up steps to generate a console-based program which will save you time and effort as well as avoid simple errors of omission; (3) When the editing screen appears, type or paste the C++ code within the `main()` function provided in the template, deleting the "Hello World" `cout` line that came with the template; (4) Type or paste any preprocessor directives (e.g. `#include` statements, `namespace` statements) necessary for your code that did not come with the template; (5) Lastly, under the Debug drop-down menu choose either Start Debugging (F5 hot-key) or Start Without Debugging (Ctrl-F5 hotkeys) to compile ("Build") and run your new program. Upon execution a console window will appear showing the output of your program.

learn computer programming by closely examining others' example programs and modifying them than it is to write your own programs starting from a blank screen.

7.2 Programming in Python

Another text-based computer programming language called *Python* allows you to type instructions at a terminal prompt and receive immediate results without having to compile that code. This is because Python is an *interpreted* language: a software application called an *interpreter* reads your source code, translates it into computer-understandable instructions, and then executes those instructions in one step.

The following shows what happens on my personal computer when I start up the Python interpreter on my personal computer, by typing `python3`[4] and pressing the Enter key:

```
Python 3.7.2 (default, Feb 19 2019, 18:15:18)
[GCC 4.1.2] on linux
Type "help", "copyright", "credits" or "license" for more information.
>>>
```

The `>>>` symbols represent the prompt within the Python interpreter "shell", signifying readiness to accept Python commands entered by the user.

Shown here is an example of the same arithmetic operations performed on the same quantities, using a Python interpreter. All lines shown preceded by the `>>>` prompt are entries typed by the human programmer, and all lines shown without the `>>>` prompt are responses from the Python interpreter software:

```
>>> x = 200
>>> y = -560.5
>>> x + y
-360.5
>>> x - y
760.5
>>> x * y
-112100.0
>>> x / y
-0.35682426404995538
>>> quit()
```

[4] Using version 3 of Python, which is the latest at the time of this writing.

More advanced mathematical functions are accessible in Python by first entering the line `from math import *` which "imports" these functions from Python's math *library* (with functions identical to those available for the C programming language, and included on any computer with Python installed). Some examples show some of these functions in use, demonstrating how the Python interpreter may be used as a scientific calculator:

```
>>> from math import *
>>> sin(30.0)
-0.98803162409286183
>>> sin(radians(30.0))
0.49999999999999994
>>> pow(2.0, 5.0)
32.0
>>> log10(10000.0)
4.0
>>> e
2.7182818284590451
>>> pi
3.1415926535897931
>>> log(pow(e,6.0))
6.0
>>> asin(0.7071068)
0.78539819000368838
>>> degrees(asin(0.7071068))
45.000001524425265
>>> quit()
```

Note how trigonometric functions assume angles expressed in *radians* rather than *degrees*, and how Python provides convenient functions for translating between the two. Logarithms assume a base of e unless otherwise stated (e.g. the `log10` function for common logarithms).

The interpreted (versus compiled) nature of Python, as well as its relatively simple syntax, makes it a good choice as a person's first programming language. For complex applications, interpreted languages such as Python execute slower than compiled languages such as C++, but for the very simple examples used in these learning modules speed is not a concern.

Another Python math library is `cmath`, giving Python the ability to perform arithmetic on complex numbers. This is very useful for AC circuit analysis using *phasors*[5] as shown in the following example. Here we see Python's interpreter used as a scientific calculator to show series and parallel impedances of a resistor, capacitor, and inductor in a 60 Hz AC circuit:

```
>>> from math import *
>>> from cmath import *
>>> r = complex(400,0)
>>> f = 60.0
>>> xc = 1/(2 * pi * f * 4.7e-6)
>>> zc = complex(0,-xc)
>>> xl = 2 * pi * f * 1.0
>>> zl = complex(0,xl)
>>> r + zc + zl
(400-187.38811239154882j)
>>> 1/(1/r + 1/zc + 1/zl)
(355.837695813625+125.35793777619385j)
>>> polar(r + zc + zl)
(441.717448903332, -0.4381072059213295)
>>> abs(r + zc + zl)
441.717448903332
>>> phase(r + zc + zl)
-0.4381072059213295
>>> degrees(phase(r + zc + zl))
-25.10169387356105
```

When entering a value in rectangular form, we use the `complex()` function where the arguments are the real and imaginary quantities, respectively. If we had opted to enter the impedance values in polar form, we would have used the `rect()` function where the first argument is the magnitude and the second argument is the angle in radians. For example, we could have set the capacitor's impedance (`zc`) as $X_C \angle -90^o$ with the command `zc = rect(xc,radians(-90))` rather than with the command `zc = complex(0,-xc)` and it would have worked the same.

Note how Python defaults to rectangular form for complex quantities. Here we defined a 400 Ohm resistance as a complex value in rectangular form (400 +j0 Ω), then computed capacitive and inductive reactances at 60 Hz and defined each of those as complex (phasor) values ($0 - jX_c$ Ω and $0 + jX_l$ Ω, respectively). After that we computed total impedance in series, then total impedance in parallel. Polar-form representation was then shown for the series impedance (441.717 Ω $\angle -25.102^o$). Note the use of different functions to show the polar-form series impedance value: `polar()` takes the complex quantity and returns its polar magnitude and phase angle in *radians*; `abs()` returns just the polar magnitude; `phase()` returns just the polar angle, once again in radians. To find the polar phase angle in degrees, we nest the `degrees()` and `phase()` functions together.

The utility of Python's interpreter environment as a scientific calculator should be clear from these examples. Not only does it offer a powerful array of mathematical functions, but also unlimited

[5] A "phasor" is a voltage, current, or impedance represented as a complex number, either in rectangular or polar form.

assignment of variables as well as a convenient text record[6] of all calculations performed which may be easily copied and pasted into a text document for archival.

It is also possible to save a set of Python commands to a text file using a text editor application, and then instruct the Python interpreter to execute it at once rather than having to type it line-by-line in the interpreter's shell. For example, consider the following Python program, saved under the filename `myprogram.py`:

```
x = 200
y = -560.5

print("Sum")
print(x + y)

print("Difference")
print(x - y)

print("Product")
print(x * y)

print("Quotient")
print(x / y)
```

As with C++, the interpreter will read this source code from left-to-right, top-to-bottom, just the same as you or I would read a document written in English. Interestingly, whitespace *is* significant in the Python language (unlike C++), but this simple example program makes no use of that.

To execute this Python program, I would need to type `python myprogram.py` and then press the Enter key at my computer console's prompt, at which point it would display the following result:

```
Sum
-360.5
Difference
760.5
Product
-112100.0
Quotient
-0.35682426405
```

As you can see, syntax within the Python programming language is simpler than C++, which is one reason why it is often a preferred language for beginning programmers.

[6] Like many command-line computing environments, Python's interpreter supports "up-arrow" recall of previous entries. This allows quick recall of previously typed commands for editing and re-evaluation.

If you are interested in learning more about computer programming in *any* language, you will find a wide variety of books and free tutorials available on those subjects. Otherwise, feel free to learn by the examples presented in these modules.

7.3 Solving RC and LR circuit differential equations using C++

A *differential equation* is a mathematical equation containing both a variable and at least one of its derivatives (i.e. its rate-of-change). Differential equations are extremely useful for describing a wide range of physical phenomena including the energization and de-energization of energy-storing components such as capacitors and inductors. While differential equations require calculus techniques to solve analytically, their solutions may be approximated quite closely using repeated calculations of simple arithmetic. Computers excel at doing such tasks, and so here we will view multiple programs written in C++ to solve for voltages and currents in resistor-capacitor and resistor-inductor networks.

Each of these programs outputs data in comma-separated variable (CSV) text format, suitable for plotting using mathematical visualization software such as `gnuplot` or spreadsheets such as Microsoft `Excel`.

7.3.1 Differential equation solver for an RC circuit and voltage source

```
#include <iostream>
#include <cmath>
using namespace std;

/* Schematic:
  +-----R-----+
  |           |
  Vs          C   Ic = C dVc/dt
  |           |
  +-----------+
*/

int main (void)
{
  double Ic, C = 33e-6, R = 10e3, Vr, Vs = 0.0, Vc = 10.0, dVc, t, dt = 0.0001;

  cout << "Time , Vc" << endl;

  for (t = 0 ; t < 2.0 ; t = t + dt)
  {
    Vr = Vs - Vc;         // Kirchhoff's Voltage Law
    Ic = Vr / R;          // Ohm's Law
    dVc = dt * (Ic / C);  // I = C dV/dt "Ohm's Law" for capacitors
    Vc = Vc + dVc;        // Integrating capacitor voltage from small changes
    cout << t << " , " << Vc << endl;
  }

  return 0;
}
```

With the initialized values shown in the `double` line of this program, it will compute capacitor voltages at 0.1 millisecond intervals assuming an initial capacitor voltage of 10 Volts and discharging toward zero volts (i.e. the voltage source value `Vs` is set to 0.0). Of course, any combination of `Vs` and initial `Vc` values will work perfectly well, as this program will show the gradual progression from initial to final voltage over time.

7.3.2 Differential equation solver for an RC circuit and current source

```
#include <iostream>
#include <cmath>
using namespace std;

/* Schematic:
  +-----+-----+
  |     |     |
  Is    R     C   I = C dVc/dt
  |     |     |
  +-----+-----+
*/

int main (void)
{
  double Ic, C = 33e-6, R = 10e3, Is = 1e-3, Ir, Vc = 0.0, dVc, t, dt = 0.0001;

  cout << "Time , Vc" << endl;

  for (t = 0 ; t < 2.0 ; t = t + dt)
  {
    Ir = Vc / R;          // Ohm's Law
    Ic = Is - Ir;         // Kirchhoff's Current Law
    dVc = dt * (Ic / C);  // I = C dV/dt "Ohm's Law" for capacitors
    Vc = Vc + dVc;        // Integrating capacitor voltage from small changes
    cout << t << " , " << Vc << endl;
  }

  return 0;
}
```

With the initialized values shown in the `double` line of this program, it will compute capacitor voltages at 0.1 millisecond intervals assuming an initial capacitor voltage of 0 Volts and charging toward 10 Volts (i.e. the source current of 1 milliAmpere passing entirely through the 10 kΩ resistor once the capacitor reaches full voltage). Of course, any combination of source current (`Is`) and initial capacitor voltage `Vc` values will work perfectly well, as this program will show the gradual progression from initial to final voltage over time.

7.3.3 Differential equation solver for an LR circuit and voltage source

```
#include <iostream>
#include <cmath>
using namespace std;

/* Schematic:
  +-----R-----+
  |           |
  Vs          L   Vl = L dIl/dt
  |           |
  +-----------+
*/

int main (void)
{
  double Vl, L = 50.0, R = 100.0, Vs = 100.0, Vr, Il = 0.0, dIl, t, dt = 0.0001;

  cout << "Time , Il" << endl;

  for (t = 0 ; t < 2.0 ; t = t + dt)
  {
    Vr = Il * R;          // Ohm's Law
    Vl = Vs - Vr;         // Kirchhoff's Voltage Law
    dIl = dt * (Vl / L);  // V = L dI/dt "Ohm's Law" for capacitors
    Il = Il + dIl;        // Integrating inductor current from small changes
    cout << t << " , " << Il << endl;
  }

  return 0;
}
```

With the initialized values shown in the `double` line of this program, it will compute inductor currents at 0.1 millisecond intervals assuming zero initial inductor current and charging toward 1 Ampere (i.e. the voltage source value `Vs` of 100 Volts divided by 100 Ohms). Of course, any combination of `Vs` and initial `Il` values will work perfectly well, as this program will show the gradual progression from initial to final current over time.

7.3.4 Differential equation solver for an LR circuit and current source

```
#include <iostream>
#include <cmath>
using namespace std;

/* Schematic:
  +-----+-----+
  |     |     |
  Is    R     L   Vl = L dIl/dt
  |     |     |
  +-----+-----+
*/

int main (void)
{
  double Vl, L = 50.0, R = 100.0, Is = 100e-3, Ir, Il = 0.0, dIl, t, dt = 0.0001;

  cout << "Time , Il" << endl;

  for (t = 0 ; t < 2.0 ; t = t + dt)
  {
    Ir = Is - Il;         // Kirchhoff's Current Law
    Vl = Ir * R;          // Ohm's Law
    dIl = dt * (Vl / L);  // V = L dI/dt "Ohm's Law" for capacitors
    Il = Il + dIl;        // Integrating inductor current from small changes
    cout << t << " , " << Il << endl;
  }

  return 0;
}
```

With the initialized values shown in the `double` line of this program, it will compute inductor currents at 0.1 millisecond intervals assuming zero initial inductor current and charging toward 100 milliAmperes. Of course, any combination of `Is` and initial `Il` values will work perfectly well, as this program will show the gradual progression from initial to final current over time.

Chapter 8

Questions

This learning module, along with all others in the ModEL collection, is designed to be used in an inverted instructional environment where students independently read[1] the tutorials and attempt to answer questions on their own *prior* to the instructor's interaction with them. In place of lecture[2], the instructor engages with students in Socratic-style dialogue, probing and challenging their understanding of the subject matter through inquiry.

Answers are not provided for questions within this chapter, and this is by design. Solved problems may be found in the Tutorial and Derivation chapters, instead. The goal here is *independence*, and this requires students to be challenged in ways where others cannot think for them. Remember that you always have the tools of *experimentation* and *computer simulation* (e.g. SPICE) to explore concepts!

The following lists contain ideas for Socratic-style questions and challenges. Upon inspection, one will notice a strong theme of *metacognition* within these statements: they are designed to foster a regular habit of examining one's own thoughts as a means toward clearer thinking. As such these sample questions are useful both for instructor-led discussions as well as for self-study.

[1] Technical reading is an essential academic skill for any technical practitioner to possess for the simple reason that the most comprehensive, accurate, and useful information to be found for developing technical competence is in textual form. Technical careers in general are characterized by the need for continuous learning to remain current with standards and technology, and therefore any technical practitioner who cannot read well is handicapped in their professional development. An excellent resource for educators on improving students' reading prowess through intentional effort and strategy is the book textitReading For Understanding – How Reading Apprenticeship Improves Disciplinary Learning in Secondary and College Classrooms by Ruth Schoenbach, Cynthia Greenleaf, and Lynn Murphy.

[2] Lecture is popular as a teaching method because it is easy to implement: any reasonably articulate subject matter expert can talk to students, even with little preparation. However, it is also quite problematic. A good lecture always makes complicated concepts seem easier than they are, which is bad for students because it instills a false sense of confidence in their own understanding; reading and re-articulation requires more cognitive effort and serves to verify comprehension. A culture of teaching-by-lecture fosters a debilitating dependence upon direct personal instruction, whereas the challenges of modern life demand independent and critical thought made possible only by gathering information and perspectives from afar. Information presented in a lecture is ephemeral, easily lost to failures of memory and dictation; text is forever, and may be referenced at any time.

General challenges following tutorial reading

- Summarize as much of the text as you can in one paragraph of your own words. A helpful strategy is to explain ideas as you would for an intelligent child: as simple as you can without compromising too much accuracy.
- Simplify a particular section of the text, for example a paragraph or even a single sentence, so as to capture the same fundamental idea in fewer words.
- Where did the text make the most sense to you? What was it about the text's presentation that made it clear?
- Identify where it might be easy for someone to misunderstand the text, and explain why you think it could be confusing.
- Identify any new concept(s) presented in the text, and explain in your own words.
- Identify any familiar concept(s) such as physical laws or principles applied or referenced in the text.
- Devise a proof of concept experiment demonstrating an important principle, physical law, or technical innovation represented in the text.
- Devise an experiment to disprove a plausible misconception.
- Did the text reveal any misconceptions you might have harbored? If so, describe the misconception(s) and the reason(s) why you now know them to be incorrect.
- Describe any useful problem-solving strategies applied in the text.
- Devise a question of your own to challenge a reader's comprehension of the text.

General follow-up challenges for assigned problems

- Identify where any fundamental laws or principles apply to the solution of this problem, especially before applying any mathematical techniques.
- Devise a thought experiment to explore the characteristics of the problem scenario, applying known laws and principles to mentally model its behavior.
- Describe in detail your own strategy for solving this problem. How did you identify and organized the given information? Did you sketch any diagrams to help frame the problem?
- Is there more than one way to solve this problem? Which method seems best to you?
- Show the work you did in solving this problem, even if the solution is incomplete or incorrect.
- What would you say was the most challenging part of this problem, and why was it so?
- Was any important information missing from the problem which you had to research or recall?
- Was there any extraneous information presented within this problem? If so, what was it and why did it not matter?
- Examine someone else's solution to identify where they applied fundamental laws or principles.
- Simplify the problem from its given form and show how to solve this simpler version of it. Examples include eliminating certain variables or conditions, altering values to simpler (usually whole) numbers, applying a limiting case (i.e. altering a variable to some extreme or ultimate value).
- For quantitative problems, identify the real-world meaning of all intermediate calculations: their units of measurement, where they fit into the scenario at hand. Annotate any diagrams or illustrations with these calculated values.
- For quantitative problems, try approaching it qualitatively instead, thinking in terms of "increase" and "decrease" rather than definite values.
- For qualitative problems, try approaching it quantitatively instead, proposing simple numerical values for the variables.
- Were there any assumptions you made while solving this problem? Would your solution change if one of those assumptions were altered?
- Identify where it would be easy for someone to go astray in attempting to solve this problem.
- Formulate your own problem based on what you learned solving this one.

General follow-up challenges for experiments or projects

- In what way(s) was this experiment or project easy to complete?
- Identify some of the challenges you faced in completing this experiment or project.

- Show how thorough documentation assisted in the completion of this experiment or project.
- Which fundamental laws or principles are key to this system's function?
- Identify any way(s) in which one might obtain false or otherwise misleading measurements from test equipment in this system.
- What will happen if (component X) fails (open/shorted/etc.)?
- What would have to occur to make this system unsafe?

8.1 Conceptual reasoning

These questions are designed to stimulate your analytic and synthetic thinking[3]. In a Socratic discussion with your instructor, the goal is for these questions to prompt an extended dialogue where assumptions are revealed, conclusions are tested, and understanding is sharpened. Your instructor may also pose additional questions based on those assigned, in order to further probe and refine your conceptual understanding.

Questions that follow are presented to challenge and probe your understanding of various concepts presented in the tutorial. These questions are intended to serve as a guide for the Socratic dialogue between yourself and the instructor. Your instructor's task is to ensure you have a sound grasp of these concepts, and the questions contained in this document are merely a means to this end. Your instructor may, at his or her discretion, alter or substitute questions for the benefit of tailoring the discussion to each student's needs. The only absolute requirement is that each student is challenged and assessed at a level equal to or greater than that represented by the documented questions.

It is far more important that you convey your *reasoning* than it is to simply convey a correct answer. For this reason, you should refrain from researching other information sources to answer questions. What matters here is that *you* are doing the thinking. If the answer is incorrect, your instructor will work with you to correct it through proper reasoning. A correct answer without an adequate explanation of how you derived that answer is unacceptable, as it does not aid the learning or assessment process.

You will note a conspicuous lack of answers given for these conceptual questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your conceptual answers, where applicable, is to use circuit simulation software to explore the effects of changes made to circuits. For example, if one of these conceptual questions challenges you to predict the effects of altering some component parameter in a circuit, you may check the validity of your work by simulating that same parameter change within software and seeing if the results agree.

[3] *Analytical* thinking involves the "disassembly" of an idea into its constituent parts, analogous to dissection. *Synthetic* thinking involves the "assembly" of a new idea comprised of multiple concepts, analogous to construction. Both activities are high-level cognitive skills, extremely important for effective problem-solving, necessitating frequent challenge and regular practice to fully develop.

8.1.1 Reading outline and reflections

"Reading maketh a full man; conference a ready man; and writing an exact man" – Francis Bacon

Francis Bacon's advice is a blueprint for effective education: reading provides the learner with knowledge, writing focuses the learner's thoughts, and critical dialogue equips the learner to confidently communicate and apply their learning. Independent acquisition and application of knowledge is a powerful skill, well worth the effort to cultivate. To this end, students should read these educational resources closely, journal their own reflections on the reading, and discuss in detail their findings with classmates and instructor(s). You should be able to do all of the following after reading any instructional text:

☑ Briefly SUMMARIZE THE TEXT in the form of a journal entry documenting your learning as you progress through the course of study. Share this summary in dialogue with your classmates and instructor. Journaling is an excellent self-test of thorough reading because you cannot clearly express what you have not read or did not comprehend.

☑ Demonstrate ACTIVE READING STRATEGIES, including verbalizing your impressions as you read, simplifying long passages to convey the same ideas using fewer words, annotating text and illustrations with your own interpretations, working through mathematical examples shown in the text, cross-referencing passages with relevant illustrations and/or other passages, identifying problem-solving strategies applied by the author, etc. Technical reading is a special case of problem-solving, and so these strategies work precisely because they help solve any problem: paying attention to your own thoughts (metacognition), eliminating unnecessary complexities, identifying what makes sense, paying close attention to details, drawing connections between separated facts, and noting the successful strategies of others.

☑ Identify IMPORTANT THEMES, especially GENERAL LAWS and PRINCIPLES, expounded in the text and express them in the simplest of terms as though you were teaching an intelligent child. This emphasizes connections between related topics and develops your ability to communicate complex ideas to anyone.

☑ Form YOUR OWN QUESTIONS based on the reading, and then pose them to your instructor and classmates for their consideration. Anticipate both correct and incorrect answers, the incorrect answer(s) assuming one or more plausible misconceptions. This helps you view the subject from different perspectives to grasp it more fully.

☑ Devise EXPERIMENTS to test claims presented in the reading, or to disprove misconceptions. Predict possible outcomes of these experiments, and evaluate their meanings: what result(s) would confirm, and what would constitute disproof? Running mental simulations and evaluating results is essential to scientific and diagnostic reasoning.

☑ Specifically identify any points you found CONFUSING. The reason for doing this is to help diagnose misconceptions and overcome barriers to learning.

8.1.2 Foundational concepts

Correct analysis and diagnosis of electric circuits begins with a proper understanding of some basic concepts. The following is a list of some important concepts referenced in this module's full tutorial. Define each of them in your own words, and be prepared to illustrate each of these concepts with a description of a practical example and/or a live demonstration.

Energy

Conservation of Energy

Potential energy

Kinetic energy

Isaac Newton's First Law of Motion

Electrical source

Electrical load

Gravitational field

Electric field

Magnetic field

Capacitance

Inductance

Thought experiments as a problem-solving strategy

Capacitor

Inductor

Superconductivity

AC

DC

Diode

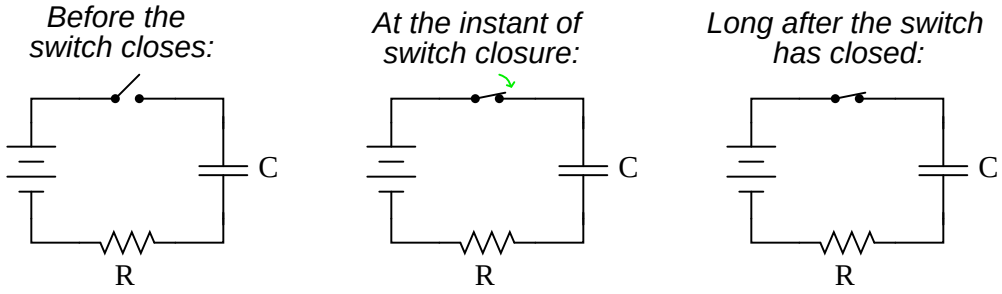
Rectification

Resonance

Frequency

8.1.3 Capacitor charging circuit

Qualitatively determine the voltages across all components as well as the current through all components in this simple resistor-capacitor circuit at three different times: (1) just before the switch closes, (2) at the instant the switch contacts touch, and (3) after the switch has been closed for a long time. Assume that the capacitor begins in a completely discharged state:



Express your answers qualitatively: "maximum," "minimum," or perhaps "zero" if you know that to be the case.

Before the switch closes:

E_C (energy stored in capacitor) =
V_C =
V_R =
V_{switch} =
I =

At the instant of switch closure:

E_C (energy stored in capacitor) =
V_C =
V_R =
V_{switch} =
I =

Long after the switch has closed:

E_C (energy stored in capacitor) =
V_C =
V_R =
V_{switch} =
I =

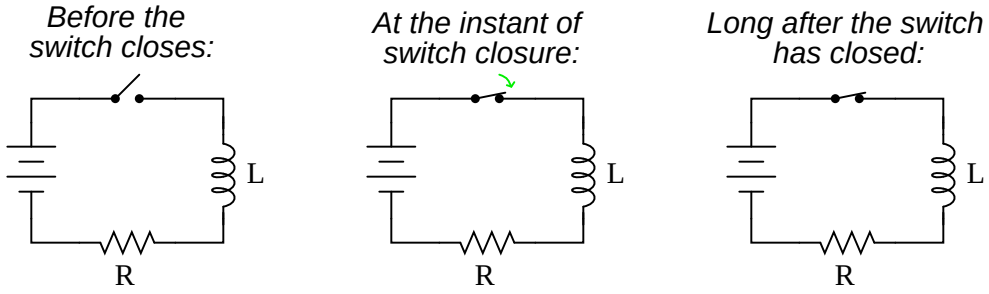
Hint: a graph may be a helpful tool for determining the answers!

Challenges

- Explain how the *Conservation of Energy* relates to changes in *voltage* and *current* for a capacitor.
- In this experiment, is the capacitor absorbing energy or releasing energy? How can we tell?
- Explain what will happen in the circuit when the switch is re-opened after the switch has been closed for a long period of time.
- Explain how drawing multiple versions of a circuit and annotating conditions before and after switch actuation is a helpful problem-solving strategy for any time-delay circuit.

8.1.4 Inductor charging circuit

Qualitatively determine the voltages across all components as well as the current through all components in this simple resistor-inductor circuit at three different times: (1) just before the switch closes, (2) at the instant the switch contacts touch, and (3) after the switch has been closed for a long time. Assume that the inductor begins in a completely discharged state:



Express your answers qualitatively: "maximum," "minimum," or perhaps "zero" if you know that to be the case.

Before the switch closes:

E_L (energy stored in inductor) =
V_L =
V_R =
V_{switch} =
I =

At the instant of switch closure:

E_L (energy stored in inductor) =
V_L =
V_R =
V_{switch} =
I =

Long after the switch has closed:

E_L (energy stored in inductor) =
V_L =
V_R =
V_{switch} =
I =

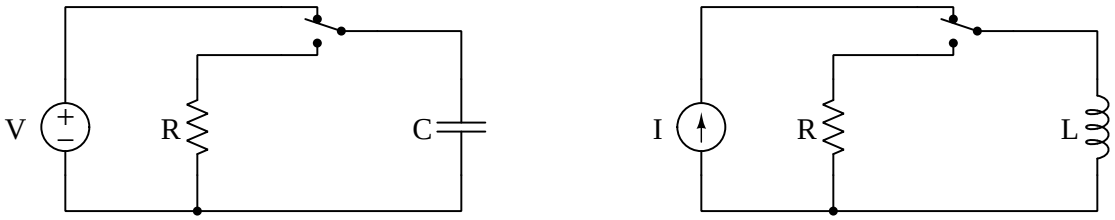
Hint: a graph may be a helpful tool for determining the answers!

Challenges

- Explain how the *Conservation of Energy* relates to changes in *voltage* and *current* for an inductor.
- In this experiment, is the inductor absorbing energy or releasing energy? How can we tell?
- Explain what will happen in the circuit when the switch is re-opened after the switch has been closed for a long period of time.
- Explain how drawing multiple versions of a circuit and annotating conditions before and after switch actuation is a helpful problem-solving strategy for any time-delay circuit.

8.1.5 Controlling the rate of de-energization

Examine the following circuits, each one designed to energize the energy-storing component and then dissipate that stored energy in a resistor when the switch is thrown to the other position:



Identify the direction of current and the polarity of voltage for each of the energy-storing components during the time their respective fields are growing in strength, and explain your answers.

Identify the direction of current and the polarity of voltage for each of the energy-storing components during the time their respective fields are weakening, and explain your answers.

Suppose we wished to de-energize each of the energy-storing components *as rapidly as possible*. Select qualitative resistor values (e.g. low-resistance, high-resistance) to achieve this goal in each of the circuits, and explain your reasoning in each case.

Challenges

- A *timing circuit* may be constructed from a resistor and capacitor, the time required to deplete the capacitor's stored energy being a function of capacitance (C) and resistance (R). For any given resistance value, will a greater capacitance result in a faster or slower depletion rate? Explain why.
- A *timing circuit* may be constructed from a resistor and capacitor, the time required to deplete the capacitor's stored energy being a function of capacitance (C) and resistance (R). For any given capacitance value, will a greater resistance result in a faster or slower depletion rate? Explain why.
- A *timing circuit* may be constructed from a resistor and inductor, the time required to deplete the inductor's stored energy being a function of inductance (L) and resistance (R). For any given resistance value, will a greater inductance result in a faster or slower depletion rate? Explain why.
- A *timing circuit* may be constructed from a resistor and inductor, the time required to deplete the inductor's stored energy being a function of inductance (L) and resistance (R). For any given inductance value, will a greater resistance result in a faster or slower depletion rate? Explain why.

8.1.6 Longer time constants

We know that the time constant (τ) for a resistor-capacitor network increases with increased capacitance because $\tau = RC$. We also know that the time constant for a resistor-inductor network increases with increased inductance because $\tau = \frac{L}{R}$. Why does τ increase with both increased C and increased L, when so many other properties of capacitance and inductance are opposite of each other? What is the conceptual commonality that makes time constant directly proportional to either C or L regardless of their other dissimilarities?

Challenges

- In a "tank circuit" where C connects directly to L, does this same principle hold true?

8.1.7 Safe de-energization

Capacitors and inductors are both energy-storing devices. As such, it is theoretically possible for these devices to store enough energy to be *lethal* if you make accidental contact with the component terminals.

Suppose you find a capacitor in a storage bin and decide to intentionally de-energize it just in case it happens to have some energy still stored in it from the last time it was used in a circuit. Explain the procedure by which you might safely de-energize this capacitor.

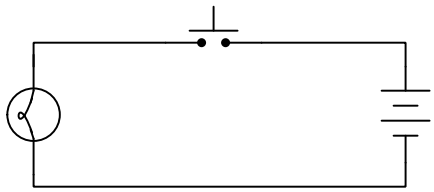
Suppose you find an inductor in a storage bin and decide to intentionally de-energize it just in case it happens to have some energy still stored in it from the last time it was used in a circuit. Explain why this step is not necessary at all the way it is with capacitors.

Challenges

- Which component is guaranteed to be in a zero-energy state when open?
- Which component is guaranteed to be in a zero-energy state when shorted?

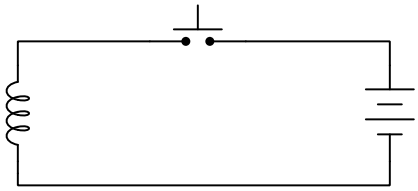
8.1.8 Switching an inductive load

Suppose a student connects a battery, switch, and incandescent lamp together to form this simple circuit:



When the pushbutton switch is pressed, the lamp energizes. Nothing could be simpler, right?

Next, the student decides to replace the lamp with a *solenoid*, which creates a magnetic field when energized:

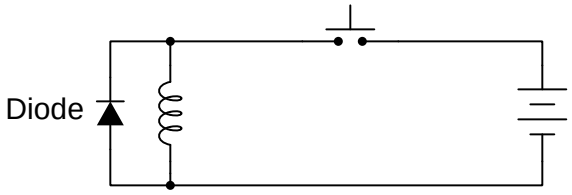


The solenoid energizes whenever the switch is pressed, but something strange now happens when the switch is released. Instead of turning off without incident, the switch contacts are seen to *arc* every time it returns to its normal (open) state! No such arcing was visible when the load was a lamp.

Thinking the problem must be one of excess current with the solenoid, the student uses an ammeter to measure current in both circuits. Surprisingly, the amount of current is nearly the same: 120 mA for the lamp and 125 mA for the solenoid.

Explain why the switch contacts arc every time they open in the solenoid circuit, but not in the lamp circuit.

Presenting this problem to a more experienced student, a solution is proposed: connect a *diode* in parallel with the solenoid coil as shown in the diagram below. "This diode," claims the experienced student, "will fix the arcing problem for sure!"



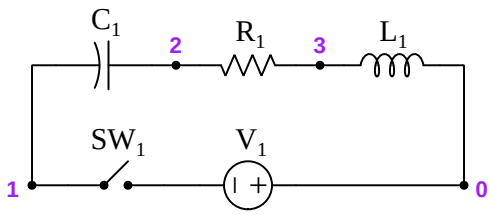
As promised, this circuit modification works to eliminate the switch arcing. Explain why!

Challenges

- Suppose the student accidently installs the diode the wrong way. What will happen in this (incorrect) circuit?

8.1.9 RLC circuit

Qualitatively determine voltages and currents at the specified node numbers for this circuit at two points in time: (1) long before the switch is closed, assuming the capacitor and inductor are fully de-energized, and (2) long after the switch has been closed:



Time	V_{2-1}	V_{3-0}	I_1
At the very instant the switch closes			
After switch closed for long time			

Challenges

- What do you suspect will happen during the time in-between initial switch closure and long after switch closure?

8.1.10 The Meissner Effect

If a strong magnet is dropped above a thick copper plate, the magnet's impact on that plate will be "cushioned" by the effect of Lenz's Law as the magnet's field intersects the plate. As the falling magnet approaches the copper plate's surface, the advancing magnetic field induces circular electric currents within the plate to create an opposing magnetic field. In accordance with Lenz's Law, this induced magnetic field repels the advancing magnet, the end-result being that the magnet slows down and comes to a gentle rest on the copper plate instead of crashing into it as one might expect.

Since we know energy is always conserved, the kinetic energy of the falling magnet must be accounted for as it slows down just prior to contacting the copper plate. Where does this kinetic energy go to as the magnet slows down in its fall?

If we exchange a flat surface made of *superconducting* material for the thick copper plate, and cool down that material until it achieves superconductivity (i.e. zero electrical resistance), we notice a dramatically different result from dropping a magnet above it. Rather than the falling magnet gently touching down on the surface as it did when the surface was made of copper metal, the magnet dropped above the superconductor will suddenly arrest its own fall and remain suspended just above the superconducting surface for as long as that surface is superconducting. Only if the material warms up too much and loses its superconducting property will the magnet touch down like it did above copper.

This is known as the *Meissner Effect*, and it is one of the more dramatic demonstrations of superconductivity. Explain why the magnet remains suspended in the air and falls no more above the superconducting surface, in contrast to its gentle touchdown toward a copper plate. Also, explain where the falling magnet's kinetic energy goes.

Challenges

- How does the Meissner Effect relate to the experimental storage of energy within superconducting inductors (i.e. rings of superconducting material) that are "charged" with a current and then monitored from time to time to demonstrate self-maintenance of that current over long periods of time?

8.2 Quantitative reasoning

These questions are designed to stimulate your computational thinking. In a Socratic discussion with your instructor, the goal is for these questions to reveal your mathematical approach(es) to problem-solving so that good technique and sound reasoning may be reinforced. Your instructor may also pose additional questions based on those assigned, in order to observe your problem-solving firsthand.

Mental arithmetic and estimations are strongly encouraged for all calculations, because without these abilities you will be unable to readily detect errors caused by calculator misuse (e.g. keystroke errors).

You will note a conspicuous lack of answers given for these quantitative questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. My advice is to use circuit simulation software such as SPICE to check the correctness of quantitative answers. Refer to those learning modules within this collection focusing on SPICE to see worked examples which you may use directly as practice problems for your own study, and/or as templates you may modify to run your own analyses and generate your own practice problems.

Completely worked example problems found in the Tutorial may also serve as "test cases[4]" for gaining proficiency in the use of circuit simulation software, and then once that proficiency is gained you will never need to rely[5] on an answer key!

[4] In other words, set up the circuit simulation software to analyze the same circuit examples found in the Tutorial. If the simulated results match the answers shown in the Tutorial, it confirms the simulation has properly run. If the simulated results disagree with the Tutorial's answers, something has been set up incorrectly in the simulation software. Using every Tutorial as practice in this way will quickly develop proficiency in the use of circuit simulation software.

[5] This approach is perfectly in keeping with the instructional philosophy of these learning modules: *teaching students to be self-sufficient thinkers*. Answer keys can be useful, but it is even more useful to your long-term success to have a set of tools on hand for checking your own work, because once you have left school and are on your own, there will no longer be "answer keys" available for the problems you will have to solve.

8.2.1 Miscellaneous physical constants

Note: constants shown in **bold** type are *exact*, not approximations. Values inside of parentheses show one standard deviation (σ) of uncertainty in the final digits: for example, the magnetic permeability of free space value given as $1.25663706212(19) \times 10^{-6}$ H/m represents a center value (i.e. the location parameter) of $1.25663706212 \times 10^{-6}$ Henrys per meter with one standard deviation of uncertainty equal to $0.0000000000019 \times 10^{-6}$ Henrys per meter.

Avogadro's number (N_A) = **6.02214076** $\times 10^{23}$ **per mole** (mol^{-1})

Boltzmann's constant (k) = **1.380649** $\times 10^{-23}$ **Joules per Kelvin** (J/K)

Electronic charge (e) = **1.602176634** $\times 10^{-19}$ **Coulomb** (C)

Faraday constant (F) = **96,485.33212...** $\times 10^{4}$ **Coulombs per mole** (C/mol)

Magnetic permeability of free space (μ_0) = $1.25663706212(19) \times 10^{-6}$ Henrys per meter (H/m)

Electric permittivity of free space (ϵ_0) = $8.8541878128(13) \times 10^{-12}$ Farads per meter (F/m)

Characteristic impedance of free space (Z_0) = 376.730313668(57) Ohms (Ω)

Gravitational constant (G) = $6.67430(15) \times 10^{-11}$ cubic meters per kilogram-seconds squared ($m^3/kg\text{-}s^2$)

Molar gas constant (R) = **8.314462618... Joules per mole-Kelvin** (J/mol-K) = 0.08205746(14) liters-atmospheres per mole-Kelvin

Planck constant (h) = **6.62607015** $\times 10^{-34}$ **joule-seconds** (J-s)

Stefan-Boltzmann constant (σ) = **5.670374419...** $\times 10^{-8}$ **Watts per square meter-Kelvin**^{4} ($W/m^2 \cdot K^4$)

Speed of light in a vacuum (c) = **299,792,458 meters per second** (m/s) = 186282.4 miles per second (mi/s)

Note: All constants taken from NIST data "Fundamental Physical Constants – Complete Listing", from `http://physics.nist.gov/constants`, National Institute of Standards and Technology (NIST), 2018 CODATA Adjustment.

8.2.2 Introduction to spreadsheets

A powerful computational tool you are encouraged to use in your work is a *spreadsheet*. Available on most personal computers (e.g. Microsoft Excel), *spreadsheet* software performs numerical calculations based on number values and formulae entered into cells of a grid. This grid is typically arranged as lettered columns and numbered rows, with each cell of the grid identified by its column/row coordinates (e.g. cell `B3`, cell `A8`). Each cell may contain a string of text, a number value, or a mathematical formula. The spreadsheet automatically updates the results of all mathematical formulae whenever the entered number values are changed. This means it is possible to set up a spreadsheet to perform a series of calculations on entered data, and those calculations will be re-done by the computer any time the data points are edited in any way.

For example, the following spreadsheet calculates average speed based on entered values of distance traveled and time elapsed:

	A	B	C	D
1	Distance traveled	46.9	Kilometers	
2	Time elapsed	1.18	Hours	
3	Average speed	= B1 / B2	km/h	
4				
5				

Text labels contained in cells `A1` through `A3` and cells `C1` through `C3` exist solely for readability and are not involved in any calculations. Cell `B1` contains a sample distance value while cell `B2` contains a sample time value. The formula for computing speed is contained in cell `B3`. Note how this formula begins with an "equals" symbol (`=`), references the values for distance and speed by lettered column and numbered row coordinates (`B1` and `B2`), and uses a forward slash symbol for division (`/`). The coordinates `B1` and `B2` function as *variables*[6] would in an algebraic formula.

When this spreadsheet is executed, the numerical value `39.74576` will appear in cell `B3` rather than the formula `= B1 / B2`, because 39.74576 is the computed speed value given 46.9 kilometers traveled over a period of 1.18 hours. If a different numerical value for distance is entered into cell `B1` or a different value for time is entered into cell `B2`, cell `B3`'s value will automatically update. All you need to do is set up the given values and any formulae into the spreadsheet, and the computer will do all the calculations for you.

Cell `B3` may be referenced by other formulae in the spreadsheet if desired, since it is a variable just like the given values contained in `B1` and `B2`. This means it is possible to set up an entire chain of calculations, one dependent on the result of another, in order to arrive at a final value. The arrangement of the given data and formulae need not follow any pattern on the grid, which means you may place them anywhere.

[6] Spreadsheets may also provide means to attach text labels to cells for use as variable names (Microsoft Excel simply calls these labels "names"), but for simple spreadsheets such as those shown here it's usually easier just to use the standard coordinate naming for each cell.

Common[7] arithmetic operations available for your use in a spreadsheet include the following:

- Addition (`+`)
- Subtraction (`-`)
- Multiplication (`*`)
- Division (`/`)
- Powers (`^`)
- Square roots (`sqrt()`)
- Logarithms (`ln()` , `log10()`)

Parentheses may be used to ensure[8] proper order of operations within a complex formula. Consider this example of a spreadsheet implementing the *quadratic formula*, used to solve for roots of a polynomial expression in the form of $ax^2 + bx + c$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

	A	B
1	x_1	= (-B4 + sqrt((B4^2) - (4*B3*B5))) / (2*B3)
2	x_2	= (-B4 - sqrt((B4^2) - (4*B3*B5))) / (2*B3)
3	a =	9
4	b =	5
5	c =	-2

This example is configured to compute roots[9] of the polynomial $9x^2 + 5x - 2$ because the values of 9, 5, and -2 have been inserted into cells `B3`, `B4`, and `B5`, respectively. Once this spreadsheet has been built, though, it may be used to calculate the roots of *any* second-degree polynomial expression simply by entering the new a, b, and c coefficients into cells `B3` through `B5`. The numerical values appearing in cells `B1` and `B2` will be automatically updated by the computer immediately following any changes made to the coefficients.

[7] Modern spreadsheet software offers a bewildering array of mathematical functions you may use in your computations. I recommend you consult the documentation for your particular spreadsheet for information on operations other than those listed here.

[8] Spreadsheet programs, like text-based programming languages, are designed to follow standard order of operations by default. However, my personal preference is to use parentheses even where strictly unnecessary just to make it clear to any other person viewing the formula what the intended order of operations is.

[9] Reviewing some algebra here, a *root* is a value for x that yields an overall value of zero for the polynomial. For this polynomial ($9x^2 + 5x - 2$) the two roots happen to be $x = 0.269381$ and $x = -0.82494$, with these values displayed in cells `B1` and `B2`, respectively upon execution of the spreadsheet.

Alternatively, one could break up the long quadratic formula into smaller pieces like this:

$$y = \sqrt{b^2 - 4ac} \qquad z = 2a$$

$$x = \frac{-b \pm y}{z}$$

	A	B	C
1	x_1	= (-B4 + C1) / C2	= sqrt((B4^2) - (4*B3*B5))
2	x_2	= (-B4 - C1) / C2	= 2*B3
3	a =	9	
4	b =	5	
5	c =	-2	

Note how the square-root term (y) is calculated in cell C1, and the denominator term (z) in cell C2. This makes the two final formulae (in cells B1 and B2) simpler to interpret. The positioning of all these cells on the grid is completely arbitrary[10] – all that matters is that they properly reference each other in the formulae.

Spreadsheets are particularly useful for situations where the same set of calculations representing a circuit or other system must be repeated for different initial conditions. The power of a spreadsheet is that it automates what would otherwise be a tedious set of calculations. One specific application of this is to simulate the effects of various components within a circuit failing with abnormal values (e.g. a shorted resistor simulated by making its value nearly zero; an open resistor simulated by making its value extremely large). Another application is analyzing the behavior of a circuit design given new components that are out of specification, and/or aging components experiencing drift over time.

[10] My personal preference is to locate all the "given" data in the upper-left cells of the spreadsheet grid (each data point flanked by a sensible name in the cell to the left and units of measurement in the cell to the right as illustrated in the first distance/time spreadsheet example), sometimes coloring them in order to clearly distinguish which cells contain entered data versus which cells contain computed results from formulae. I like to place all formulae in cells below the given data, and try to arrange them in logical order so that anyone examining my spreadsheet will be able to figure out *how* I constructed a solution. This is a general principle I believe all computer programmers should follow: *document and arrange your code to make it easy for other people to learn from it.*

8.2.3 Capacitor energy calculations

Capacitors store energy as a function of voltage and capacitance. Calculate the missing value given the other two, for each of the following examples:

- 270 microFarad capacitor at 25 Volts ; Stored energy =
- 80 Joules of energy at 170 Volts ; Capacitance =
- 35 Volts for a 15,000 μF capacitor ; Stored energy =
- 45 milliJoules of energy in a 0.22 Farad capacitor ; Voltage =

Challenges

- What will result in a greater increase of stored energy: doubling the capacitance *or* doubling the voltage? Explain why.
- Suppose you were constructing your own capacitor for one of these applications, and needed to increase the amount of capacitance. How could you alter the construction of the capacitor to yield more C?

8.2.4 Inductor energy calculations

Inductors store energy as a function of current and inductance. Calculate the missing value given the other two, for each of the following examples:

- 100 milliHenry inductor at 3 Amperes ; Stored energy =
- 13 Joules of energy at 10 Amperes ; Inductance =
- 150 milliAmperes for a 25 mH inductor ; Stored energy =
- 300 milliJoules of energy in a 7.2 Henry inductor ; Current =

Challenges

- What will result in a greater increase of stored energy: doubling the inductance *or* doubling the current? Explain why.
- Suppose you were constructing your own inductor for one of these applications, and needed to increase the amount of inductance. How could you alter the construction of the inductor to yield more L?

8.2.5 Defibrillator capacitor

A *defibrillator* is a medical device used to deliver an electric shock to a patient suffering from *cardiac fibrillation*, which is an ineffectual heart rhythm. The amount of electrical energy necessary to shock a person's heart back to normal rhythm varies with body weight and a variety of other factors, but typically ranges from 70 to 360 Joules[11].

Suppose you are part of a team designing a new model of defibrillator, using a capacitor to store the energy delivered to the patient. Assuming the capacitor will reach a voltage of 1200 Volts when fully energized, how large of a capacitor will be necessary for this design?

Suppose the design of this new defibrillator varies the amount of delivered energy by controlling the voltage value of the energized capacitor. How many Volts would the capacitor need to be energized to in order to store the 90 Joules of energy?

Challenges

- This illustration should be enough to convince you that energized capacitors can be very dangerous. Explain how you could test a capacitor for hazardous levels of energy before handling it with your bare hands.

[11] If defibrillation is done to a patient during chest surgery, where the shock may be administered directly to the heart muscle and not through the chest, this energy figure is significantly lower!

8.2.6 Rail gun capacitor

A *rail gun* is a cannon using electrical energy rather than a chemical propellant to launch a projectile. Such weapons have virtually unlimited theoretical muzzle velocities because there is no intrinsic limitation to thrust as there is with the combustion of a chemical propellant. Rail guns typically use large capacitors to release enormous amounts of energy in very short amounts of time necessary to launch projectiles at extremely high rates of speed.

Suppose a rail gun needs enough energy to launch a 0.5 kilogram projectile at a muzzle velocity of 4000 meters per second. Calculate the amount of energy necessary to do this, in units of Joules.

Next, calculate the voltage necessary to energize a 5 Farad capacitor to deliver this much energy to the rail gun.

Lastly, assume this particular rail gun is 45% efficient. That is to say, 55% of the energy released by the capacitor upon firing the gun is "wasted" in the form of heat, with only 45% of its energy actually going into the projectile. Knowing this fact, re-calculate the voltage necessary to energize a 5 Farad capacitor to deliver enough energy to the rail gun to launch the 0.5 kg projectile at 4000 m/s.

Challenges

- Why not use an inductor to store and release the firing energy rather than a capacitor?

8.2.7 Capacitors and inductors compared against batteries

A typical 9 Volt "transistor" battery has a terminal voltage of approximately 9 Volts and an amp-hour capacity of approximately 500 milliAmpere-hours[12] (500 mAh) while supplying current continuously to a load at a value of 60 mA. This means the battery has a total energy capacity of 4.5 Watt-hours (9 Volts $\times$ 500 mAh = 4.5 Wh), which is equal to 16,200 Joules[13].

Calculate the size of capacitor necessary to store the same amount of energy at the same terminal voltage as the battery.

Calculate the size of inductor necessary to store the same amount of energy at the same continuous current as the battery.

Challenges

- How practical do you think it is to substitute capacitors or inductors for a chemical-based 9 Volt battery?
- Where might chemical-based batteries be preferred over capacitors and inductors for energy storage?
- Where might capacitors or inductors be preferred over chemical-based batteries for energy storage?

[12] An Ampere-hour is a unit of electrical charge, equal to the current in Amperes multiplied by the time in hours. 1 Ampere of current passing through a conductor for 1 hour's worth of time is equivalent to 1 Ampere-hour of total charge. Likewise, one-half of an Ampere flowing for 2 hours would also be 1 Ampere-hour of total charge. For many chemical batteries, the assumed time duration for their Ampere-hour rating is twenty hours, so 500 mAh would be a continuous current of 25 mA flowing for 20 hours.

[13] One Joule is a Watt-second, so one Watt-hour is 3600 Watt-seconds, or 3600 Joules.

8.2.8 Mixed-source circuits with capacitors and inductors

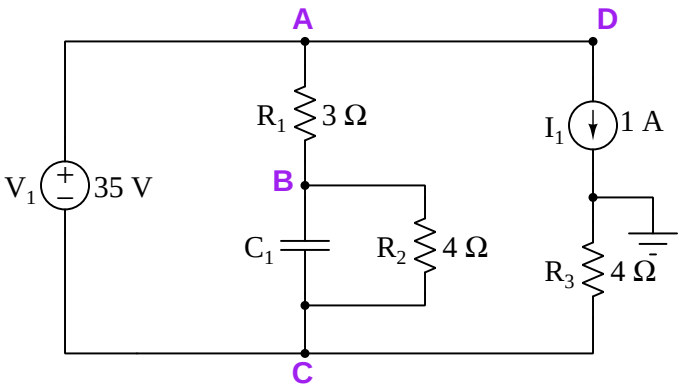
The following circuits contain a combination of constant-voltage and constant-current components[14] as well as ideal *capacitors* and ideal *inductors* which function as energy-storage devices.

Constant-voltage components, of course, maintain the same voltage for all current conditions while constant-current components maintain the same current for all voltage conditions. Capacitors store energy within electric fields and may absorb energy (i.e. act as a load) or release energy (i.e. act as a source) depending on whether their voltage is increasing or decreasing, respectively. Inductors store energy within magnetic fields and also may act as either sources or loads depending on whether their current is increasing or decreasing, respectively. Circuits containing such energy-storing devices commonly exhibit voltages and currents that *change* over time.

Based on this knowledge, as well as Ohm's Law, Kirchhoff's Laws, and other foundational concepts of electric circuits, determine all requested voltages in the following circuits, being sure to specify the mathematical sign of each requested voltage based on the convention of the first subscript letter being the voltmeter's red test lead point and the second subscript letter being the black. If only a single subscript letter appears, the assumed reference point is ground (wherever that is shown in the schematic):

[14] While these constant-parameter components are commonly referred to as *sources*, it should be realized that they may in fact behave as loads if overpowered by another source.

Circuit #1 (at a moment in time when $V_{BC} = +8$ Volts)



- $V_A =$
- $V_B =$
- $V_C =$
- $V_{CD} =$

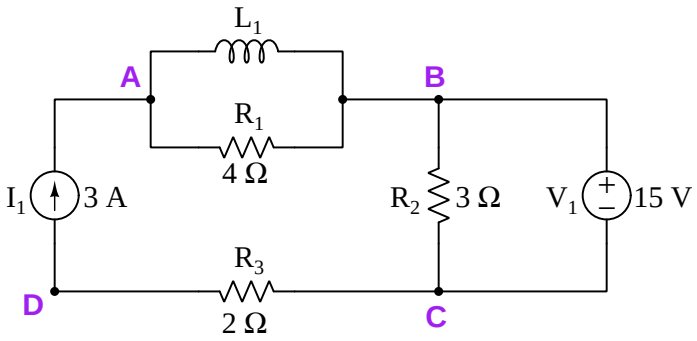
Is capacitor C_1 functioning as a source or as a load at this moment in time?

After a sufficient amount of time passes, the capacitor will reach a state of equilibrium with the rest of the circuit, its current decreasing to zero so that it acts the same as an *open*. Re-evaluate the voltages once more for this condition:

- $V_A =$
- $V_B =$
- $V_C =$
- $V_{CD} =$

How much voltage will be across C_1's terminals in this equilibrium condition?

Circuit #2 (at a moment in time when $I_{L1} = 2$ Amperes right-to-left)



- $V_{BD} =$
- $V_{AC} =$
- $V_{CB} =$
- $V_{DC} =$

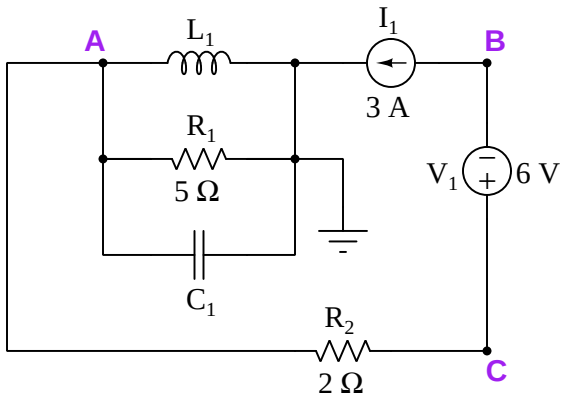
Is inductor L_1 functioning as a source or as a load at this moment in time?

After a sufficient amount of time passes, the inductor will reach a state of equilibrium with the rest of the circuit, its voltage decreasing to zero so that it acts the same as a *short*. Re-evaluate the voltages once more for this condition:

- $V_{BD} =$
- $V_{AC} =$
- $V_{CB} =$
- $V_{DC} =$

How much current will pass through L_1 in this equilibrium condition?

Circuit #3 (at a moment in time when $I_{L1} = 4$ Amperes right-to-left and $V_A = +10$ Volts)



- $V_A =$
- $V_B =$
- $V_C =$
- $V_{BA} =$
- $V_{AC} =$

Is capacitor C_1 functioning as a source or as a load at this moment in time?

Is inductor L_1 functioning as a source or as a load at this moment in time?

After a sufficient amount of time passes, both the capacitor and the inductor will reach states of equilibrium with the rest of the circuit, the capacitor's current decreasing to zero so that it acts like an *open*, and the inductor's voltage decreasing to zero so that it acts like a *short*. Re-evaluate the voltages once more for this condition:

- $V_A =$
- $V_B =$
- $V_C =$
- $V_{BA} =$
- $V_{AC} =$

How much voltage will there be across C_1 in this equilibrium condition?

How much current will pass through L_1 in this equilibrium condition?

Challenges

- Identify each constant-voltage component's and each constant-current component's functional identity as either a source or a load.
- Modify any single component value and determine the effect(s) of that change on all voltages, currents, and source/load determinations.

8.3 Diagnostic reasoning

These questions are designed to stimulate your deductive and inductive thinking, where you must apply general principles to specific scenarios (deductive) and also derive conclusions about the failed circuit from specific details (inductive). In a Socratic discussion with your instructor, the goal is for these questions to reinforce your recall and use of general circuit principles and also challenge your ability to integrate multiple symptoms into a sensible explanation of what's wrong in a circuit. Your instructor may also pose additional questions based on those assigned, in order to further challenge and sharpen your diagnostic abilities.

As always, your goal is to fully *explain* your analysis of each problem. Simply obtaining a correct answer is not good enough – you must also demonstrate sound reasoning in order to successfully complete the assignment. Your instructor's responsibility is to probe and challenge your understanding of the relevant principles and analytical processes in order to ensure you have a strong foundation upon which to build further understanding.

You will note a conspicuous lack of answers given for these diagnostic questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your diagnostic answers, where applicable, is to use circuit simulation software to explore the effects of faults placed in circuits. For example, if one of these diagnostic questions requires that you predict the effect of an open or a short in a circuit, you may check the validity of your work by simulating that same fault (substituting a very high resistance in place of that component for an open, and substituting a very low resistance for a short) within software and seeing if the results agree.

8.3.1 Capacitor and inductor fault pathology

A capacitor is fundamentally just a pair of conductive "plates" separated from each other by an insulating gap. An inductor is fundamentally a coil of conductive wire. Based on this knowledge, identify the likely fault pathology for each device (i.e. the fault mode(s) you think would be more probable).

Are capacitors more likely to fail *open* or fail *shorted*? Explain your answer.

Describe a test you could perform using nothing but a multimeter to check for this type of capacitor fault.

Are inductors more likely to fail *open* or fail *shorted*? Explain your answer.

Describe a test you could perform using nothing but a multimeter to check for this type of inductor fault.

Challenges

- Suppose a capacitor fails in the manner you determine to be more likely, while energized with significant voltage. What would be the result of this fault?
- Suppose an inductor fails in the manner you determine to be more likely, while energized with significant current. What would be the result of this fault?

8.3.2 Strange ohmmeter readings

Suppose that an electronics technician uses an ohmmeter to test a capacitor. Upon connecting the ohmmeter's test leads to the capacitor, she initially obtains a low-resistance reading which quickly increases until the meter registers "open" a few seconds later.

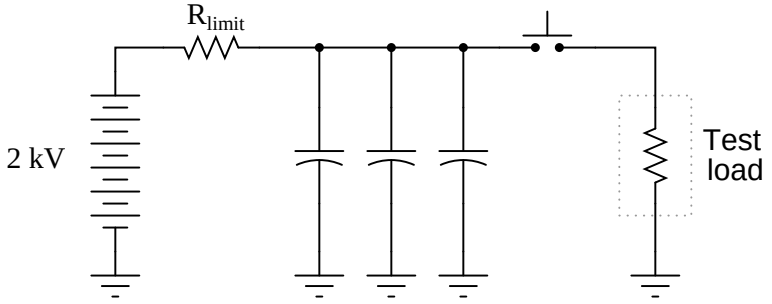
Explain why the meter registered this way, based on what you know of capacitors.

Challenges

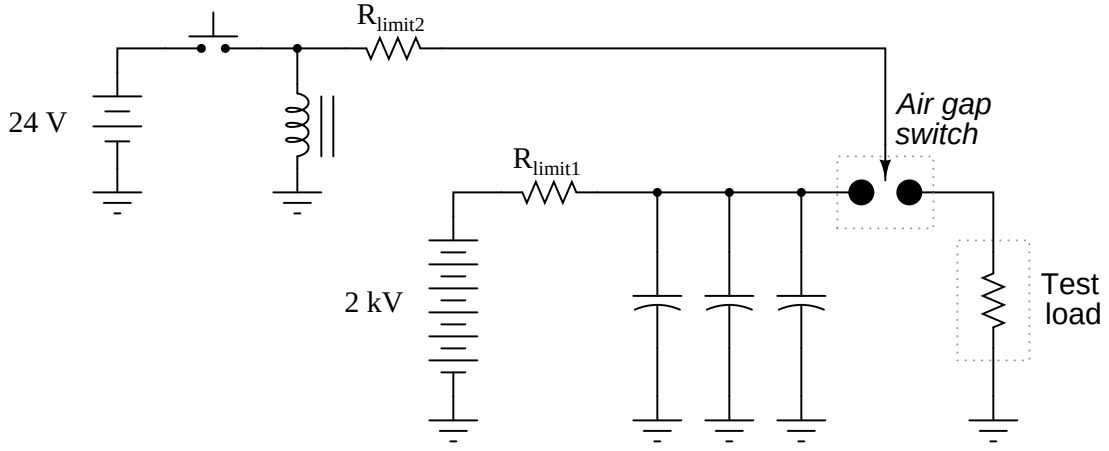
- Identify a likely fault mode for a capacitor, and explain how an ohmmeter would interpret such a faulted capacitor.

8.3.3 High-voltage surge supply

An electrical testing laboratory has designed and built a high-current "surge" power supply for testing the effects of large electrical currents on certain types of components. The basic idea is that a bank of capacitors are charged to a high voltage by a DC power source (through a current-limiting resistor to protect the power source from overcurrent damage), then quickly discharged through a switch to the test load:



Due to the magnitude of the transient currents generated by the discharging capacitors, a mechanical switch (as suggested by the pushbutton symbol) is impractical. Instead, an *air gap switch* is constructed from two metal balls with a needle-point "ionizer" electrode between them. The ionizer electrode connects to a simple inductor/switch circuit that generates a high-voltage pulse sufficient to create a spark. The ionized air created in the spark provides a low-resistance path through the air which the 2000 Volts from the capacitor bank can now traverse, thus completing the circuit for surge current between the capacitors and the test load:



Suppose this surge power supply circuit failed to work after several months of good service. Nothing happens any more when the pushbutton switch is pressed and released. No spark is heard or seen in the air gap, and the test load receives no surge of current.

A voltmeter placed (carefully!) in parallel with the capacitor bank indicates a full charge voltage of 2035 Volts, which is within normal parameters. Based on this information, identify these things:

- Two components in the circuit that you know must be in good working condition.

- Two components in the circuit that could possibly be bad, and the mode of their failure (either open or shorted).

Challenges

- Explain how one would *safely* continue diagnostic measurements in a circuit such as this where there is much potential for electric shock and other hazards.

Chapter 9

Projects and Experiments

The following project and experiment descriptions outline things you can build to help you understand circuits. With any real-world project or experiment there exists the potential for physical harm. *Electricity can be very dangerous in certain circumstances, and you should follow proper safety precautions at all times!*

9.1 Recommended practices

This section outlines some recommended practices for all circuits you design and construct.

9.1.1 Safety first!

Electricity, when passed through the human body, causes uncomfortable sensations and in large enough measures[1] will cause muscles to involuntarily contract. The overriding of your nervous system by the passage of electrical current through your body is particularly dangerous in regard to your heart, which is a vital muscle. Very large amounts of current can produce serious internal burns in addition to all the other effects.

Cardio-pulmonary resuscitation (CPR) is the standard first-aid for any victim of electrical shock. This is a very good skill to acquire if you intend to work with others on dangerous electrical circuits. You should never perform tests or work on such circuits unless someone else is present who is proficient in CPR.

As a general rule, any voltage in excess of 30 Volts poses a definitive electric shock hazard, because beyond this level human skin does not have enough resistance to safely limit current through the body. "Live" work of any kind with circuits over 30 volts should be avoided, and if unavoidable should only be done using electrically insulated tools and other protective equipment (e.g. insulating shoes and gloves). If you are unsure of the hazards, or feel unsafe at any time, stop all work and distance yourself from the circuit!

A policy I strongly recommend for students learning about electricity is to *never come into electrical contact*[2] *with an energized conductor, no matter what the circuit's voltage*[3] *level!* Enforcing this policy may seem ridiculous when the circuit in question is powered by a single battery smaller than the palm of your hand, but it is precisely this instilled habit which will save a person from bodily harm when working with more dangerous circuits. Experience has taught me that students who learn early on to be careless with safe circuits have a tendency to be careless later with dangerous circuits!

In addition to the electrical hazards of shock and burns, the construction of projects and running of experiments often poses other hazards such as working with hand and power tools, potential

[1] Professor Charles Dalziel published a research paper in 1961 called "The Deleterious Effects of Electric Shock" detailing the results of electric shock experiments with both human and animal subjects. The threshold of perception for human subjects holding a conductor in their hand was in the range of 1 milliampere of current (less than this for alternating current, and generally less for female subjects than for male). Loss of muscular control was exhibited by half of Dalziel's subjects at less than 10 milliamperes alternating current. Extreme pain, difficulty breathing, and loss of all muscular control occurred for over 99% of his subjects at direct currents less than 100 milliamperes and alternating currents less than 30 milliamperes. In summary, it doesn't require much electric current to induce painful and even life-threatening effects in the human body! Your first and best protection against electric shock is maintaining an insulating barrier between your body and the circuit in question, such that current from that circuit will be unable to flow through your body.

[2] By "electrical contact" I mean either directly touching an energized conductor with any part of your body, or indirectly touching it through a conductive tool. The only physical contact you should ever make with an energized conductor is via an electrically insulated tool, for example a screwdriver with an electrically insulated handle, or an insulated test probe for some instrument.

[3] Another reason for consistently enforcing this policy, even on low-voltage circuits, is due to the dangers that even some low-voltage circuits harbor. A single 12 Volt automobile battery, for example, can cause a surprising amount of damage if short-circuited simply due to the high current levels (i.e. very low internal resistance) it is capable of, even though the voltage level is too low to cause a shock through the skin. Mechanics wearing metal rings, for example, are at risk from severe burns if their rings happen to short-circuit such a battery! Furthermore, even when working on circuits that are simply too low-power (low voltage and low current) to cause any bodily harm, touching them while energized can pose a threat to the circuit components themselves. In summary, it's generally wise (and *always* a good habit to build) to "power down" *any* circuit before making contact between it and your body.

contact with high temperatures, potential chemical exposure, etc. You should never proceed with a project or experiment if you are unaware of proper tool use or lack basic protective measures (e.g. personal protective equipment such as safety glasses) against such hazards.

Some other safety-related practices should be followed as well:

- All power conductors extending outward from the project must be *firmly* strain-relieved (e.g. "cord grips" used on line power cords), so that an accidental tug or drop will not compromise circuit integrity.
- All electrical connections must be sound and appropriately made (e.g. soldered wire joints rather than twisted-and-taped; terminal blocks rather than solderless breadboards for high-current or high-voltage circuits). Use "touch-safe" terminal connections with recessed metal parts to minimize risk of accidental contact.
- Always provide overcurrent protection in any circuit you build. *Always.* This may be in the form of a fuse, a circuit breaker, and/or an electronically current-limited power supply.
- Always ensure circuit conductors are rated for more current than the overcurrent protection limit. *Always.* A fuse does no good if the wire or printed circuit board trace will "blow" before it does!
- Always bond metal enclosures to Earth ground for any line-powered circuit. *Always.* Ensuring an equipotential state between the enclosure and Earth by making the enclosure electrically common with Earth ground ensures no electric shock can occur simply by one's body bridging between the Earth and the enclosure.
- Avoid building a high-energy circuit when a low-energy circuit will suffice. For example, I always recommend beginning students power their first DC resistor circuits using small batteries rather than with line-powered DC power supplies. The intrinsic energy limitations of a dry-cell battery make accidents highly unlikely.
- Use line power receptacles that are GFCI (Ground Fault Current Interrupting) to help avoid electric shock from making accidental contact with a "hot" line conductor.
- Always wear eye protection when working with tools or live systems having the potential to eject material into the air. Examples of such activities include soldering, drilling, grinding, cutting, wire stripping, working on or near energized circuits, etc.
- Always use a step-stool or stepladder to reach high places. Never stand on something not designed to support a human load.
- When in doubt, *ask an expert.* If anything even seems remotely unsafe to you, do not proceed without consulting a trusted person fully knowledgeable in electrical safety.

9.1.2 Other helpful tips

Experience has shown the following practices to be very helpful, especially when students make their own component selections, to ensure the circuits will be well-behaved:

- Avoid resistor values less than 1 kΩ or greater than 100 kΩ, unless such values are definitely necessary[4]. Resistances below 1 kΩ may draw excessive current if directly connected to a voltage source of significant magnitude, and may also complicate the task of accurately measuring current since any ammeter's non-zero resistance inserted in series with a low-value circuit resistor will significantly alter the total resistance and thereby skew the measurement. Resistances above 100 kΩ may complicate the task of measuring voltage since any voltmeter's finite resistance connected in parallel with a high-value circuit resistor will significantly alter the total resistance and thereby skew the measurement. Similarly, AC circuit impedance values should be between 1 kΩ and 100 kΩ, and for all the same reasons.

- Ensure all electrical connections are low-resistance and physically rugged. For this reason, one should avoid *compression splices* (e.g. "butt" connectors), solderless breadboards[5], and wires that are simply twisted together.

- Build your circuit with **testing** in mind. For example, provide convenient connection points for test equipment (e.g. multimeters, oscilloscopes, signal generators, logic probes).

- Design permanent projects with **maintenance** in mind. The more convenient you make maintenance tasks, the more likely they will get done.

- **Always document and save your work**. Circuits lacking schematic diagrams are more difficult to troubleshoot than documented circuits. Similarly, circuit construction is simpler when a schematic diagram precedes construction. Experimental results are easier to interpret when comprehensively recorded. Consider modern videorecording technology for this purpose where appropriate.

- **Record your steps** when troubleshooting. **Talk to yourself** when solving problems. These simple steps clarify thought and simplify identification of errors.

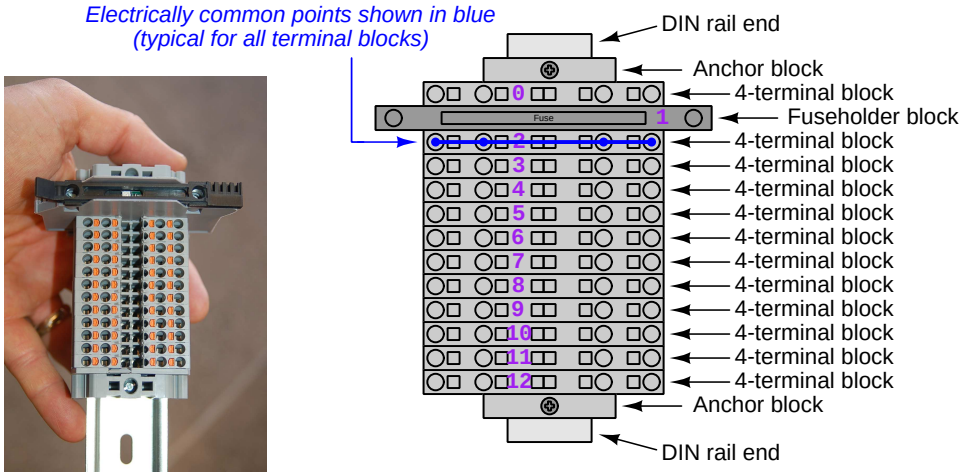
[4] An example of a necessary resistor value much less than 1 kΩ is a *shunt resistor* used to produce a small voltage drop for the purpose of sensing current in a circuit. Such shunt resistors must be low-value in order not to impose an undue load on the rest of the circuit. An example of a necessary resistor value much greater than 100 kΩ is an electrostatic *drain resistor* used to dissipate stored electric charges from body capacitance for the sake of preventing damage to sensitive semiconductor components, while also preventing a path for current that could be dangerous to the person (i.e. shock).

[5] Admittedly, solderless breadboards are very useful for constructing complex electronic circuits with many components, especially DIP-style integrated circuits (ICs), but they tend to give trouble with connection integrity after frequent use. An alternative for projects using low counts of ICs is to solder IC sockets into prototype printed circuit boards (PCBs) and run wires from the soldered pins of the IC sockets to terminal blocks where reliable temporary connections may be made.

9.1.3 Terminal blocks for circuit construction

Terminal blocks are the standard means for making electric circuit connections in industrial systems, and are also quite useful for student-built experiments, providing a robust alternative to solderless breadboards[6]. Terminal blocks provide highly reliable connections rated for significant voltage and current magnitudes, and they force the builder to think carefully about component placement which is an important mental practice. Terminal blocks designed to mount on standard 35 mm DIN rail[7] come in many types and capabilities.

Every student of electricity should build their own terminal block array consisting of several terminal blocks where each block has at least 4 connection points all electrically common to each other[8] and at least one terminal block that is a fuse holder for overcurrent protection. A pair of anchoring blocks hold all terminal blocks securely on the DIN rail, preventing them from sliding off the rail. Each of the terminals should bear a unique number starting from 0. An example is shown in the following photograph and illustration:



Screwless terminal blocks (using internal spring clips to clamp wire and component lead ends) are preferred over screw-based terminal blocks, as they reduce assembly and disassembly time, and also minimize repetitive wrist stress from twisting screwdrivers. Some screwless terminal blocks require the use of a special tool to release the spring clip, while others such as the Phoenix Contact model 3211019 blocks shown in the photograph provide buttons[9] for this task which may be pressed using the tip of any suitable tool such as a small screwdriver or multimeter test probe tip.

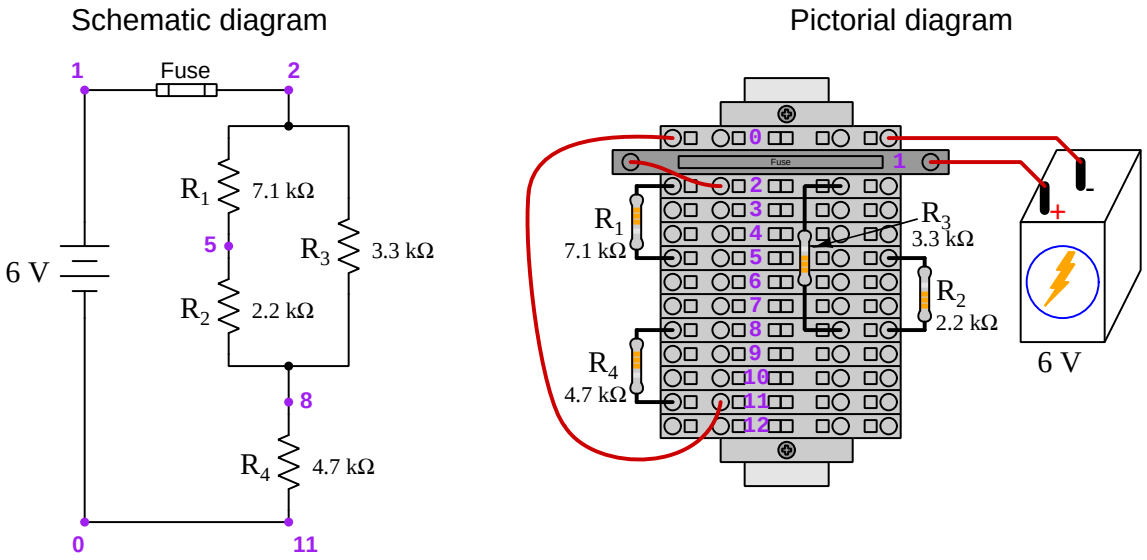
[6] Solderless breadboard are preferable for complicated electronic circuits with multiple integrated "chip" components, but for simpler circuits I find terminal blocks much more practical. An alternative to solderless breadboards for "chip" circuits is to solder chip sockets onto a PCB and then use wires to connect the socket pins to terminal blocks. This also accommodates *surface-mount* components, which solderless breadboards do not.

[7] DIN rail is a metal rail designed to serve as a mounting point for a wide range of electrical and electronic devices such as terminal blocks, fuses, circuit breakers, relay sockets, power supplies, data acquisition hardware, etc.

[8] Sometimes referred to as *equipotential*, *same-potential*, or *potential distribution* terminal blocks.

[9] The small orange-colored squares seen in the above photograph are buttons for this purpose, and may be actuated by pressing with any tool of suitable size.

The following example shows how such a terminal block array might be used to construct a series-parallel resistor circuit consisting of four resistors and a battery:



Numbering on the terminal blocks provides a very natural translation to SPICE[10] netlists, where component connections are identified by terminal number:

```
* Series-parallel resistor circuit
v1 1 0 dc 6
r1 2 5 7100
r2 5 8 2200
r3 2 8 3300
r4 8 11 4700
rfuse 1 2 0.01
rwire 0 11 0.01
.op
.end
```

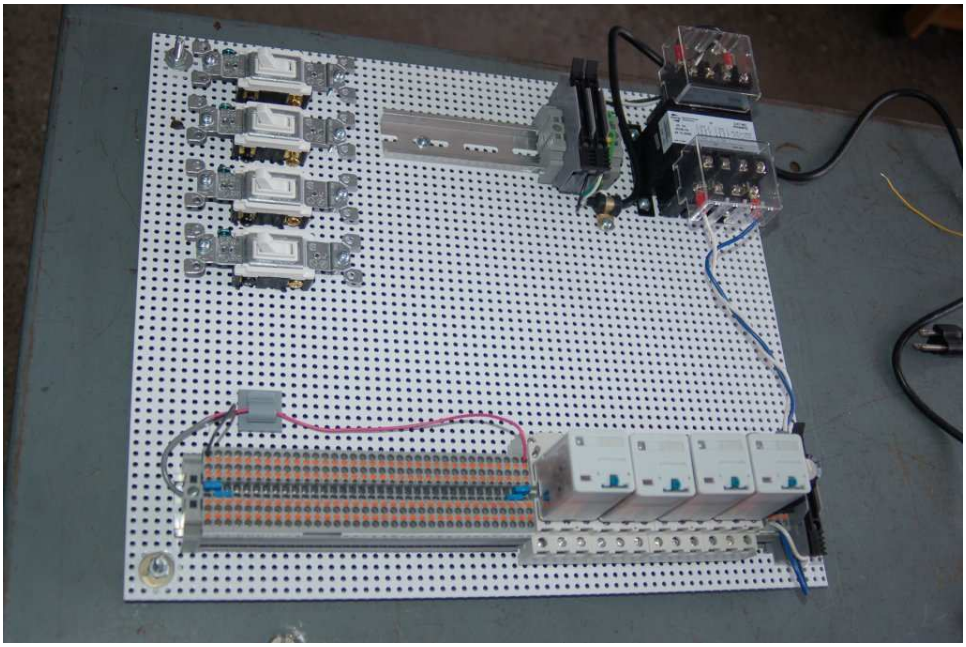
Note the use of two resistances `rfuse` and `rwire` to describe fuse resistance between terminals 1 and 2 and "jumper" wire resistance between terminals 0 and 11, respectively. Being resistances, SPICE requires a resistance value for each, and here we see they have both been set to an arbitrarily low value of 0.01 Ohm appropriate to their real-life resistance values.

Listing all components and wires along with their numbered terminals happens to be a useful documentation method for any circuit built on terminal blocks, independent of SPICE. Such a

[10] SPICE is computer software designed to analyze electrical and electronic circuits. Circuits are described for the computer in the form of *netlists* which are text files listing each component type, connection node numbers, and component values.

"wiring sequence" may be thought of as a *non-graphical description* of an electric circuit, and is exceptionally easy to follow.

An example of a more elaborate terminal block array is shown in the following photograph, with terminal blocks and "ice-cube" style electromechanical relays mounted to DIN rail, which is turn mounted to a perforated subpanel[11]. This "terminal block board" hosts an array of thirty five undedicated terminal block sections, four SPDT toggle switches, four DPDT "ice-cube" relays, a step-down control power transformer, bridge rectifier and filtering capacitor, and several fuses for overcurrent protection:



Four plastic-bottomed "feet" support the subpanel above the benchtop surface, and an unused section of DIN rail stands ready to accept other components. Safety features include electrical bonding of the AC line power cord's ground to the metal subpanel (and all metal DIN rails), mechanical strain relief for the power cord to isolate any cord tension from wire connections, clear plastic finger guards covering the transformer's screw terminals, as well as fused overcurrent protection for the 120 Volt AC line power and the transformer's 12 Volt AC output. The perforated holes happen to be on $\frac{1}{4}$ inch centers with a diameter suitable for tapping with 6-32 machine screw threads, their presence making it very easy to attach other sections of DIN rail, printed circuit boards, or specialized electrical components directly to the grounded metal subpanel. Such a "terminal block board" is an inexpensive[12] yet highly flexible means to construct physically robust circuits using industrial wiring practices.

[11] An electrical *subpanel* is a thin metal plate intended for mounting inside an electrical enclosure. Components are attached to the subpanel, and the subpanel in turn bolts inside the enclosure. Subpanels allow circuit construction outside the confines of the enclosure, which speeds assembly. In this particular usage there is no enclosure, as the subpanel is intended to be used as an open platform for the convenient construction of circuits on a benchtop by students. In essence, this is a modern version of the traditional *breadboard* which was literally a wooden board such as might be used for cutting loaves of bread, but which early electrical and electronic hobbyists used as platforms for the construction of circuits.

[12] At the time of this writing (2019) the cost to build this board is approximately $250 US dollars.

9.1.4 Conducting experiments

An *experiment* is an exploratory act, a test performed for the purpose of assessing some proposition or principle. Experiments are the foundation of the *scientific method*, a process by which careful observation helps guard against errors of speculation. All good experiments begin with an *hypothesis*, defined by the American Heritage Dictionary of the English Language as:

> An assertion subject to verification or proof, as (a) A proposition stated as a basis for argument or reasoning. (b) A premise from which a conclusion is drawn. (c) A conjecture that accounts, within a theory or ideational framework, for a set of facts and that can be used as a basis for further investigation.

Stated plainly, an hypothesis is an *educated guess* about cause and effect. The correctness of this initial guess matters little, because any well-designed experiment will reveal the truth of the matter. In fact, *incorrect* hypotheses are often the most valuable because the experiments they engender lead us to surprising discoveries. One of the beautiful aspects of science is that it is more focused on the process of *learning* than about the status of *being correct*[13]. In order for an hypothesis to be valid, it must be testable[14], which means it must be a claim possible to refute given the right data. Hypotheses impossible to critique are useless.

Once an hypothesis has been formulated, an experiment must be designed to test that hypothesis. A well-designed experiment requires careful regulation of all relevant variables, both for personal safety and for prompting the hypothesized results. If the effects of one particular variable are to be tested, the experiment must be run multiple times with different values of (only) that particular variable. The experiment set up with the "baseline" variable set is called the *control*, while the experiment set up with different value(s) is called the *test* or *experimental*.

For some hypotheses a viable alternative to a physical experiment is a *computer-simulated experiment* or even a *thought experiment*. Simulations performed on a computer test the hypothesis against the physical laws encoded within the computer simulation software, and are particularly useful for students learning new principles for which simulation software is readily available[15].

[13] Science is more about clarifying our view of the universe through a systematic process of error detection than it is about proving oneself to be right. Some scient*ists* may happen to have large egos – and this may have more to do with the ways in which large-scale scientific research is *funded* than anything else – but scientific *method* itself is devoid of ego, and if embraced as a practical philosophy is quite an effective stimulant for humility. Within the education system, scientific method is particularly valuable for helping students break free of the crippling fear of *being wrong*. So much emphasis is placed in formal education on assessing correct retention of facts that many students are fearful of saying or doing anything that might be perceived as a mistake, and of course making mistakes (i.e. having one's hypotheses disproven by experiment) is an indispensable tool for learning. Introducing science in the classroom – *real* science characterized by individuals forming actual hypotheses and testing those hypotheses by experiment – helps students become self-directed learners.

[14] This is the principle of *falsifiability*: that a scientific statement has value only insofar as it is liable to disproof given the requisite experimental evidence. Any claim that is unfalsifiable – that is, a claim which can *never* be disproven by any evidence whatsoever – could be completely wrong and we could never know it.

[15] A very pertinent example of this is learning how to analyze electric circuits using simulation software such as SPICE. A typical experimental cycle would proceed as follows: (1) Find or invent a circuit to analyze; (2) Apply your analytical knowledge to that circuit, predicting all voltages, currents, powers, etc. relevant to the concepts you are striving to master; (3) Run a simulation on that circuit, collecting "data" from the computer when complete; (4) Evaluate whether or not your hypotheses (i.e. predicted voltages, currents, etc.) agree with the computer-generated results; (5) If so, your analyses are (provisionally) correct – if not, examine your analyses and the computer simulation again to determine the source of error; (6) Repeat this process as many times as necessary until you achieve mastery.

Thought experiments are useful for detecting inconsistencies within your own understanding of some subject, rather than testing your understanding against physical reality.

Here are some general guidelines for conducting experiments:

- The clearer and more specific the hypothesis, the better. Vague or unfalsifiable hypotheses are useless because they will fit *any* experimental results, and therefore the experiment cannot teach you anything about the hypothesis.
- Collect as much data (i.e. information, measurements, sensory experiences) generated by an experiment as is practical. This includes the time and date of the experiment, too!
- *Never* discard or modify data gathered from an experiment. If you have reason to believe the data is unreliable, write notes to that effect, but never throw away data just because you think it is untrustworthy. It is quite possible that even "bad" data holds useful information, and that someone else may be able to uncover its value even if you do not.
- Prioritize *quantitative* data over *qualitative* data wherever practical. Quantitative data is more specific than qualitative, less prone to subjective interpretation on the part of the experimenter, and amenable to an arsenal of analytical methods (e.g. statistics).
- Guard against your own bias(es) by making your experimental results available to others. This allows other people to scrutinize your experimental design and collected data, for the purpose of detecting and correcting errors you may have missed. Document your experiment such that others may independently replicate it.
- Always be looking for sources of error. No physical measurement is perfect, and so it is impossible to achieve *exact* values for any variable. Quantify the amount of uncertainty (i.e. the "tolerance" of errors) whenever possible, and be sure your hypothesis does not depend on precision better than this!
- Always remember that scientific confirmation is provisional – no number of "successful" experiments will prove an hypothesis true for all time, but a single experiment can disprove it. Put into simpler terms, *truth is elusive but error is within reach.*
- Remember that scientific method is about *learning*, first and foremost. An unfortunate consequence of scientific triumph in modern society is that science is often viewed by non-practitioners as an unerring source of truth, when in fact science is an ongoing process of challenging existing ideas to probe for errors and oversights. This is why it is perfectly acceptable to have a failed hypothesis, and why the only truly failed experiment is one where nothing was learned.

The following is an example of a well-planned and executed experiment, in this case a physical experiment demonstrating Ohm's Law.

```
Planning Time/Date = 09:30 on 12 February 2019

HYPOTHESIS: the current through any resistor should be exactly proportional
to the voltage impressed across it.

PROCEDURE: connect a resistor rated 1 k Ohm and 1/4 Watt to a variable-voltage
DC power supply.  Use an ammeter in series to measure resistor current and
a voltmeter in parallel to measure resistor voltage.

RISKS AND MITIGATION: excessive power dissipation may harm the resistor and/
or pose a burn hazard, while excessive voltage poses an electric shock hazard.
30 Volts is a safe maximum voltage for laboratory practices, and according to
Joule's Law a 1000 Ohm resistor will dissipate 0.25 Watts at 15.81 Volts
(P = V^2 / R), so I will remain below 15 Volts just to be safe.

Experiment Time/Date = 10:15 on 12 February 2019

DATA COLLECTED:
  (Voltage)    (Current)         (Voltage)   (Current)
   0.000 V   =  0.000 mA          8.100    =  7.812 mA
   2.700 V   =  2.603 mA          10.00 V  =  9.643 mA
   5.400 V   =  5.206 mA          14.00 V  =  13.49 mA

Analysis Time/Date = 10:57 on 12 February 2019

ANALYSIS: current definitely increases with voltage, and although I expected
exactly one milliAmpere per Volt the actual current was usually less than
that.  The voltage/current ratios ranged from a low of 1036.87 (at 8.1 Volts)
to a high of 1037.81 (at 14 Volts), but this represents a variance of only
-0.0365% to +0.0541% from the average, indicating a very consistent
proportionality -- results consistent with Ohm's Law.

ERROR SOURCES: one major source of error is the resistor's value itself.  I
did not measure it, but simply assumed color bands of brown-black-red meant
exactly 1000 Ohms.  Based on the data I think the true resistance is closer
to 1037 Ohms.  Another possible explanation is multimeter calibration error.
However, neither explains the small positive and negative variances from the
average.  This might be due to electrical noise, a good test being to repeat
the same experiment to see if the variances are the same or different.  Noise
should generate slightly different results every time.
```

The following is an example of a well-planned and executed *virtual* experiment, in this case demonstrating Ohm's Law using a computer (SPICE) simulation.

```
Planning Time/Date = 12:32 on 14 February 2019

HYPOTHESIS: for any given resistor, the current through that resistor should be
exactly proportional to the voltage impressed across it.

PROCEDURE: write a SPICE netlist with a single DC voltage source and single
1000 Ohm resistor, then use NGSPICE version 26 to perform a "sweep" analysis
from 0 Volts to 25 Volts in 5 Volt increments.

    * SPICE circuit
    v1 1 0 dc
    r1 1 0 1000
    .dc v1 0 25 5
    .print dc v(1) i(v1)
    .end

RISKS AND MITIGATION: none.

DATA COLLECTED:
       DC transfer characteristic  Thu Feb 14 13:05:08  2019
    ------------------------------------------------------------
    Index   v-sweep         v(1)            v1#branch
    ------------------------------------------------------------
    0       0.000000e+00    0.000000e+00    0.000000e+00
    1       5.000000e+00    5.000000e+00    -5.00000e-03
    2       1.000000e+01    1.000000e+01    -1.00000e-02
    3       1.500000e+01    1.500000e+01    -1.50000e-02
    4       2.000000e+01    2.000000e+01    -2.00000e-02
    5       2.500000e+01    2.500000e+01    -2.50000e-02

Analysis Time/Date = 13:06 on 14 February 2019

ANALYSIS: perfect agreement between data and hypothesis -- current is precisely
1/1000 of the applied voltage for all values.  Anything other than perfect
agreement would have probably meant my netlist was incorrect.  The negative
current values surprised me, but it seems this is just how SPICE interprets
normal current through a DC voltage source.

ERROR SOURCES: none.
```

As gratuitous as it may seem to perform experiments on a physical law as well-established as Ohm's Law, even the examples listed previously demonstrate opportunity for real learning. In the physical experiment example, the student should identify and explain why their data does not perfectly agree with the hypothesis, and this leads them naturally to consider sources of error. In the computer-simulated experiment, the student is struck by SPICE's convention of denoting regular current through a DC voltage source as being *negative* in sign, and this is also useful knowledge for future simulations. Scientific experiments are most interesting when things *do not* go as planned!

Aside from verifying well-established physical laws, simple experiments are extremely useful as educational tools for a wide range of purposes, including:

- Component familiarization (e.g. *Which terminals of this switch connect to the NO versus NC contacts?*)
- System testing (e.g. *How heavy of a load can my AC-DC power supply source before the semiconductor components reach their thermal limits?*)
- Learning programming languages (e.g. *Let's try to set up an "up" counter function in this PLC!*)

Above all, the priority here is to inculcate the habit of hypothesizing, running experiments, and analyzing the results. This experimental cycle not only serves as an excellent method for self-directed learning, but it also works exceptionally well for troubleshooting faults in complex systems, and for these reasons should be a part of every technician's and every engineer's education.

9.1.5 Constructing projects

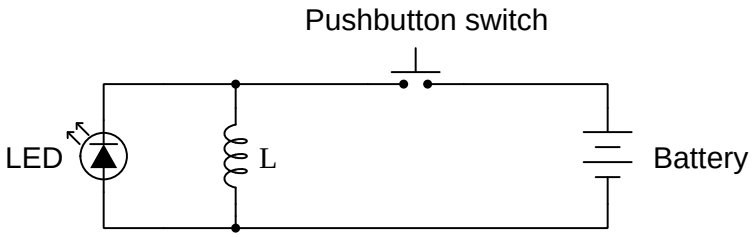
Designing, constructing, and testing projects is a very effective means of practical education. Within a formal educational setting, projects are generally chosen (or at least vetted) by an instructor to ensure they may be reasonably completed within the allotted time of a course or program of study, and that they sufficiently challenge the student to learn certain important principles. In a self-directed environment, projects are just as useful as a learning tool but there is some risk of unwittingly choosing a project beyond one's abilities, which can lead to frustration.

Here are some general guidelines for managing projects:

- Define your goal(s) before beginning a project: what do you wish to achieve in building it? What, exactly, should the completed project *do*?
- Analyze your project prior to construction. Document it in appropriate forms (e.g. schematic diagrams), predict its functionality, anticipate all associated risks. In other words, *plan ahead*.
- Set a reasonable budget for your project, and stay within it.
- Identify any deadlines, and set reasonable goals to meet those deadlines.
- Beware of *scope creep*: the tendency to modify the project's goals before it is complete.
- Document your progress! An easy way to do this is to use photography or videography: take photos and/or videos of your project as it progresses. Document failures as well as successes, because both are equally valuable from the perspective of learning.

9.2 Experiment: Powering an LED using an inductor

Inductors store energy as a function of current, and so we may prove this using the following circuit:



You will need an inductor that is fairly large in value. The high-voltage winding of a microwave oven transformer[16] works quite well for this purpose. The light-emitting diode (LED) may be of any color and any size. A 6 Volt or 9 Volt battery works very well for this experiment.

First and foremost, identify any potential hazards posed by this experiment. If any exist, identify how to mitigate each of those risks to ensure personal safety as well as ensure no components or equipment will be damaged.

Formulate a hypothesis (i.e. an educated guess) as to what will happen when the switch is pressed, and when the switch is released.

Be sure to document all data in a neat and well-organized format, easily understood by anyone viewing it. Be prepared to explain your reasoning at every step, and also to demonstrate the safe and proper use of all materials, components, and equipment. If a live demonstration is not practical, record your actions on video.

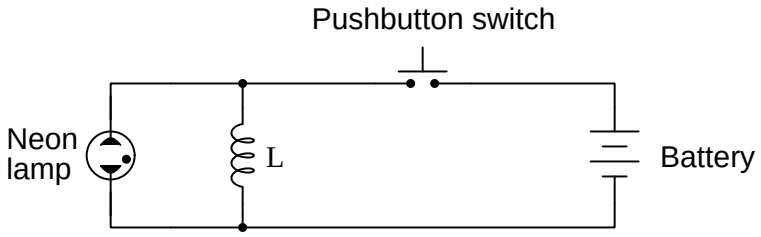
Challenges

- Explain why LED energizes when it does. Does it energize when the switch is closed, when the switch is opened, or both?
- If we were to replace the inductor with one having a smaller value (i.e. less L), what would be the result?

[16] This is the step-up power transformer used to boost AC line voltage into the kiloVolt range for powering the magnetron tube of a standard microwave oven. The high-voltage winding will be the one made of thin-gauge wire, while the low-voltage winding is made of much thicker-gauge wire. Incidentally, old microwave ovens are excellent sources of components for a variety of experiments: the large power transformer, door switches, line power cords, fan motors, etc.

9.3 Experiment: Powering a neon lamp using an inductor

When an inductor is energized, it has the ability to generate tremendous voltages if its energizing current is suddenly interrupted. We may prove this using the following circuit:



You will need an inductor that is fairly large in value. The high-voltage winding of a microwave oven transformer[17] works quite well for this purpose. The neon lamp requires substantial voltage (typically 70 Volts or so) before it is able to output any light. A 6 Volt or 9 Volt battery works very well for this experiment, outputting much less voltage than required by the neon lamp.

First and foremost, identify any potential hazards posed by this experiment. If any exist, identify how to mitigate each of those risks to ensure personal safety as well as ensure no components or equipment will be damaged.

Formulate a hypothesis (i.e. an educated guess) as to what will happen when the switch is pressed, and when the switch is released.

Identify how a digital multimeter (DMM) with "maximum" voltage capture might be useful in this experiment.

Be sure to document all data in a neat and well-organized format, easily understood by anyone viewing it. Be prepared to explain your reasoning at every step, and also to demonstrate the safe and proper use of all materials, components, and equipment. If a live demonstration is not practical, record your actions on video.

Challenges

- Explain why the neon lamp energizes when it does. Does it energize when the switch is closed, when the switch is opened, or both?
- If we were to replace the inductor with one having a smaller value (i.e. less L), what would be the result?

[17] This is the step-up power transformer used to boost AC line voltage into the kiloVolt range for powering the magnetron tube of a standard microwave oven. The high-voltage winding will be the one made of thin-gauge wire, while the low-voltage winding is made of much thicker-gauge wire. Incidentally, old microwave ovens are excellent sources of components for a variety of experiments: the large power transformer, door switches, line power cords, fan motors, etc.

- If an inductor is capable of generating hundreds or even thousands of Volts when its energizing current is suddenly interrupted, how come there is no threat posed to a digital multimeter measuring lamp voltage in this circuit?

Appendix A

Problem-Solving Strategies

The ability to solve complex problems is arguably one of the most valuable skills one can possess, and this skill is particularly important in any science-based discipline.

- Study principles, not procedures. Don't be satisfied with merely knowing how to compute solutions – learn *why* those solutions work.
- Identify what it is you need to solve, identify all relevant data, identify all units of measurement, identify any general principles or formulae linking the given information to the solution, and then identify any "missing pieces" to a solution. Annotate all diagrams with this data.
- Sketch a diagram to help visualize the problem. When building a real system, always devise a plan for that system and analyze its function *before* constructing it.
- Follow the units of measurement and meaning of every calculation. If you are ever performing mathematical calculations as part of a problem-solving procedure, and you find yourself unable to apply each and every intermediate result to some aspect of the problem, it means you don't understand what you are doing. Properly done, every mathematical result should have practical meaning for the problem, and not just be an abstract number. You should be able to identify the proper units of measurement for each and every calculated result, and show where that result fits into the problem.
- Perform "thought experiments" to explore the effects of different conditions for theoretical problems. When troubleshooting real systems, perform *diagnostic tests* rather than visually inspecting for faults, the best diagnostic test being the one giving you the most information about the nature and/or location of the fault with the fewest steps.
- Simplify the problem until the solution becomes obvious, and then use that obvious case as a model to follow in solving the more complex version of the problem.
- Check for exceptions to see if your solution is incorrect or incomplete. A good solution will work for *all* known conditions and criteria. A good example of this is the process of testing scientific hypotheses: the task of a scientist is not to find support for a new idea, but rather to *challenge* that new idea to see if it holds up under a battery of tests. The philosophical

principle of *reductio ad absurdum* (i.e. disproving a general idea by finding a specific case where it fails) is useful here.

- Work "backward" from a hypothetical solution to a new set of given conditions.
- Add quantities to problems that are qualitative in nature, because sometimes a little math helps illuminate the scenario.
- Sketch graphs illustrating how variables relate to each other. These may be quantitative (i.e. with realistic number values) or qualitative (i.e. simply showing increases and decreases).
- Treat quantitative problems as qualitative in order to discern the relative magnitudes and/or directions of change of the relevant variables. For example, try determining what happens if a certain variable were to increase or decrease before attempting to precisely calculate quantities: how will each of the dependent variables respond, by increasing, decreasing, or remaining the same as before?
- Consider limiting cases. This works especially well for qualitative problems where you need to determine which direction a variable will change. Take the given condition and magnify that condition to an extreme degree as a way of simplifying the direction of the system's response.
- Check your work. This means regularly testing your conclusions to see if they make sense. This does *not* mean repeating the same steps originally used to obtain the conclusion(s), but rather to use some other means to check validity. Simply repeating procedures often leads to *repeating the same errors* if any were made, which is why alternative paths are better.

Appendix B

Instructional philosophy

B.1 First principles of learning

- **Anyone can learn anything** given appropriate time, effort, resources, challenges, encouragement, and expectations. Dedicating time and investing effort are the student's responsibility; providing resources, challenges, and encouragement are the teacher's responsibility; high expectations are a responsibility shared by both student and teacher.

- **Transfer is not automatic.** The human mind has a natural tendency to compartmentalize information, which means the process of taking knowledge learned in one context and applying it to another usually does not come easy and therefore should never be taken for granted.

- **Learning is iterative.** The human mind rarely learns anything perfectly on the first attempt. Anticipate mistakes and plan for multiple tries to achieve full understanding, using the lessons of those mistakes as feedback to guide future attempts.

- **Information is absorbed, but understanding is created.** Facts and procedures may be memorized easily enough by repeated exposure, but the ability to reliably apply principles to novel scenarios only comes through intense personal effort. This effort is fundamentally creative in nature: casting new concepts in one's own words, running mental experiments to perceive patterns and trends, etc. These acts of making knowledge "one's own" need not be perfect in order to be effective, as the value lies in the activity and not necessarily the finished product. Depth of understanding and permanence of memory are both directly related to mental effort during the learning process – i.e. *easy learning is shallow and fleeting.*

- **Education trumps training.** There is no such thing as an entirely isolated subject, as all fields of knowledge are connected. Training is narrowly-focused and task-oriented. Education is broad-based and principle-oriented. When preparing for a life-long technical career, education beats training every time.

- **Character matters.** Poor habits are more destructive than deficits of knowledge or skill. This is especially true in collective endeavors, where a team's ability to function depends on trust between its members. Simply put, no one wants an untrustworthy person on their team. An essential component of education then, is character development.

- **People learn to be responsible by bearing responsibility.** An irresponsible person is someone who has never *had* to be responsible for anything that mattered enough to them. Just as anyone can learn anything, anyone can become responsible if the personal cost of irresponsibility becomes high enough.

- **What gets measured, gets done.** Accurate and relevant assessment of learning is key to ensuring all students learn. Therefore, it is imperative to measure what matters.

- **Failure is nothing to fear.** Every human being fails, and fails in multiple ways at multiple times. Eventual success only happens when we don't stop trying.

B.2 Proven strategies for instructors

- Assume every student is capable of learning anything they desire given the proper conditions. Treat them as capable adults by granting real responsibility and avoiding artificial incentives such as merit or demerit points.
- Create a consistent culture of high expectations across the entire program of study. Demonstrate and encourage patience, persistence, and a healthy sense of self-skepticism. Anticipate and de-stigmatize error. Teach respect for the capabilities of others as well as respect for one's own fallibility.
- Replace lecture with "inverted" instruction, where students first encounter new concepts through reading and then spend class time in Socratic dialogue with the instructor exploring those concepts and solving problems individually. There is a world of difference between observing someone solve a problem versus actually solving a problem yourself, and so the point of this form of instruction is to place students in a position where they *cannot* passively observe.
- Require students to read extensively, write about what they learn, and dialogue with you and their peers to sharpen their understanding. Apply Francis Bacon's advice that "reading maketh a full man; conference a ready man; and writing an exact man". These are complementary activities helping students expand their confidence and abilities.
- Use artificial intelligence (AI) to challenge student understanding rather than merely provide information. Find productive ways for AI to critique students' clarity of thought and of expression, for example by employing AI as a Socratic-style interlocutor or as a reviewer of students' journals. Properly applied, AI has the ability to expand student access to critical review well outside the bounds of their instructor's reach.
- Build frequent and rapid feedback into the learning process so that students know at all times how well they are learning, to identify problems early and fix them before they grow. Model the intellectual habit of self-assessing and self-correcting your own understanding (i.e. a cognitive *feedback loop*), encouraging students to do the same.
- Use "mastery" as the standard for every assessment, which means the exam or experiment or project must be done with 100% competence in order to pass. Provide students with multiple opportunity for re-tries (different versions of the assessment every time).
- Require students to devise their own hypotheses and procedures on all experiments, so that the process is truly a scientific one. Have students assess their proposed experimental procedures for risk and devise mitigations for those risks. Let nothing be pre-designed about students' experiments other than a stated task (i.e. what principle the experiment shall test) at the start and a set of demonstrable knowledge and skill objectives at the end.
- Have students build as much of their lab equipment as possible: building power sources, building test assemblies[1], and building complete working systems (no kits!). In order to provide

[1] In the program I teach, every student builds their own "Development Board" consisting of a metal chassis with DIN rail, terminal blocks, and an AC-DC power supply of their own making which functions as a portable lab environment they can use at school as well as take home.

this same "ground-up" experience for every new student, this means either previous students take their creations with them, or the systems get disassembled in preparation for the new students, or the systems grow and evolve with each new student group.

- Incorporate external accountability for you and for your students, continuously improving the curriculum and your instructional methods based on proven results. Have students regularly network with active professionals through participation in advisory committee meetings, service projects, tours, jobshadows, internships, etc. Practical suggestions include requiring students to design and build projects for external clients (e.g. community groups, businesses, different departments within the institution), and also requiring students attend all technical advisory committee meetings and to dialogue with the industry representatives at those meetings.
- Repeatedly explore difficult-to-learn concepts across multiple courses, so that students have multiple opportunities to build their understanding.
- Relate all new concepts, whenever possible, to previous concepts and to relevant physical laws. Challenge each and every student, every day, to *reason* from concept to concept and to explain the logical connections between. Challenge students to verify their conclusions by multiple approaches (e.g. double-checking their work using different methods). Ask *"Why?"* often.
- Maintain detailed records on each student's performance and share these records privately with them. These records should include academic performance as well as professionally relevant behavioral tendencies.
- Hold mandatory "check-in" meetings between all program faculty and each new student during their first term. Offer these to all other students as an option, except for any students continuing to manifest unprofessional behaviors, poor academic performance, or who have some other need for a face-to-face meeting with faculty.
- Address problems while they are small, before they grow larger. This is equally true for tutoring technical concepts as it is for helping students build professional habits.
- Build rigorous quality control into the curriculum to ensure every student masters every important concept, and that the mastery is retained over time. This includes (1) review questions added to every exam to re-assess knowledge taught in previous terms, (2) cumulative exams at the end of every term to re-assess all important concepts back to the very beginning of the program, and (3) review assessments in practical (hands-on) coursework to ensure critically-important skills were indeed taught and are still retained. What you will find by doing this is that it actually boosts retention of students by ensuring that important knowledge gets taught and is retained over long spans of time. In the absence of such quality control, student learning and retention tends to be spotty and this contributes to drop-out and failure rates later in their education.
- Finally, *never rush learning.* Education is not a race. Give your students ample time to digest complex ideas, as you continually remind yourself of just how long it took you to achieve mastery! Long-term retention and the consistently correct application of concepts are always the result of *focused effort over long periods of time* which means there are no shortcuts to learning.

B.3 Proven strategies for students

The single most important piece of advice I have for any student of any subject is to take responsibility for your own development in all areas of life including mental development. Expecting others in your life to entirely guide your own development is a recipe for disappointment. This is just as true for students enrolled in formal learning institutions as it is for auto-didacts pursuing learning entirely on their own. Learning to think in new ways is key to being able to gainfully use information, to make informed decisions about your life, and to best serve those you care about. With this in mind, I offer the following advice to students:

- **Approach all learning as valuable**. No matter what course you take, no matter who you learn from, no matter the subject, there is something useful in every learning experience. If you don't see the value of every new experience, you are not looking closely enough!
- **Continually challenge yourself**. Let other people take shortcuts and find easy answers to easy problems. The purpose of education is to stretch your mind, in order to shape it into a more powerful tool. This doesn't come by taking the path of least resistance. An excellent analogy for an empowering education is productive physical exercise: becoming stronger, more flexible, and more persistent only comes through intense personal effort.
- **Master the use of language**. This includes reading extensively, writing every day, listening closely, and speaking articulately. To a great extent language channels and empowers thought, so the better you are at wielding language the better you will be at grasping abstract concepts and articulating them not only for your benefit but for others as well.
- **Do not limit yourself to the resources given to you**. Read books that are not on the reading list. Run experiments that aren't assigned to you. Form study groups outside of class. Take an entrepreneurial approach to your own education, as though it were a business you were building for your future benefit.
- **Express and share what you learn**. Take every opportunity to teach what you have learned to others, as this will not only help them but will also strengthen your own understanding[2].
- Realize that **no one can give you understanding**, just as no one can give you physical fitness. These both must be *built*.
- **Above all, recognize that learning is hard work, and that a certain level of frustration is unavoidable**. There are times when you will struggle to grasp some of these concepts, and that struggle is a natural thing. Take heart that it will yield with persistent and varied[3] effort, and never give up! That concepts don't immediately come to you is not a sign of something wrong, but rather of something right: that you have found a worthy challenge!

[2] On a personal note, I was surprised to learn just how much my own understanding of electronics and related subjects was strengthened by becoming a teacher. When you are tasked every day with helping other people grasp complex topics, it catalyzes your own learning by giving you powerful incentives to study, to articulate your thoughts, and to reflect deeply on the process of learning.

[3] As the old saying goes, "Insanity is trying the same thing over and over again, expecting different results." If you find yourself stumped by something in the text, you should attempt a different approach. Alter the thought experiment, change the mathematical parameters, do whatever you can to see the problem in a slightly different light, and then the solution will often present itself more readily.

B.4 Design of these learning modules

"The unexamined circuit is not worth energizing" – Socrates (if he had taught electricity)

These learning modules, although useful for self-study, were designed to be used in a formal learning environment where a subject-matter expert challenges students to digest the content and exercise their critical thinking abilities in the answering of questions and in the construction and testing of working circuits. Every effort has been made to embed the following instructional and assessment philosophies within:

- The first goal of education is to enhance clear and independent thought, in order that every student reach their fullest potential in a highly complex and inter-dependent world. Robust reasoning is *always* more important than particulars of any subject matter, because its application is universal.
- Literacy is fundamental to independent learning and thought because text continues to be the most efficient way to communicate complex ideas over space and time. Those who cannot read with ease are limited in their ability to acquire knowledge and perspective.
- Articulate communication is fundamental to work that is complex and interdisciplinary.
- Faulty assumptions and poor reasoning are best corrected through challenge, not presentation. The rhetorical technique of *reductio ad absurdum* (disproving an assertion by exposing an absurdity) works well to discipline student's minds, not only to correct the problem at hand but also to learn how to detect and correct future errors.
- Important principles should be repeatedly explored and widely applied throughout a course of study, not only to reinforce their importance and help ensure their mastery, but also to showcase the interconnectedness and utility of knowledge.

These learning modules were expressly designed to be used in an "inverted" teaching environment[4] where students first read the introductory and tutorial chapters on their own, then individually attempt to answer the questions and construct working circuits according to the experiment and project guidelines. The instructor never lectures, but instead meets regularly with each individual student to review their progress, answer questions, identify misconceptions, and challenge the student to new depths of understanding through further questioning. Regular meetings between instructor and student should resemble a Socratic[5] dialogue, where questions serve as scalpels to dissect topics and expose assumptions. The student passes each module only after consistently demonstrating their ability to logically analyze and correctly apply all major concepts in each question or project/experiment. The instructor must be vigilant in probing each student's understanding to ensure they are truly *reasoning* and not just *memorizing*. This is why "Challenge" points appear throughout, as prompts for students to think deeper about topics and as starting points for instructor queries. Sometimes these challenge points require additional knowledge that hasn't been covered in the series to answer in full. This is okay, as the major purpose of the Challenges is to stimulate analysis and synthesis on the part of each student.

The instructor must possess enough mastery of the subject matter and awareness of students' reasoning to generate their own follow-up questions to practically any student response. Even completely correct answers given by the student should be challenged by the instructor for the purpose of having students practice articulating their thoughts and defending their reasoning. Conceptual errors committed by the student should be exposed and corrected not by direct instruction, but rather by reducing the errors to an absurdity[6] through well-chosen questions and thought experiments posed by the instructor. Becoming proficient at this style of instruction requires time and dedication, but the positive effects on critical thinking for both student and instructor are spectacular.

An inspection of these learning modules reveals certain unique characteristics. One of these is a bias toward thorough explanations in the tutorial chapters. Without a live instructor to explain concepts and applications to students, the text itself must fulfill this role. This philosophy results in lengthier explanations than what you might typically find in a textbook, each step of the reasoning process fully explained, including footnotes addressing common questions and concerns students raise while learning these concepts. Each tutorial seeks to not only explain each major concept in sufficient detail, but also to explain the logic of each concept and how each may be developed

[4] In a traditional teaching environment, students first encounter new information via *lecture* from an expert, and then independently apply that information via *homework*. In an "inverted" course of study, students first encounter new information via *homework*, and then independently apply that information under the scrutiny of an expert. The expert's role in lecture is to simply *explain*, but the expert's role in an inverted session is to *challenge*, *critique*, and if necessary *explain* where gaps in understanding still exist.

[5] Socrates is a figure in ancient Greek philosophy famous for his unflinching style of questioning. Although he authored no texts, he appears as a character in Plato's many writings. The essence of Socratic philosophy is to leave no question unexamined and no point of view unchallenged. While purists may argue a topic such as electric circuits is too narrow for a true Socratic-style dialogue, I would argue that the essential thought processes involved with scientific reasoning on *any* topic are not far removed from the Socratic ideal, and that students of electricity and electronics would do very well to challenge assumptions, pose thought experiments, identify fallacies, and otherwise employ the arsenal of critical thinking skills modeled by Socrates.

[6] This rhetorical technique is known by the Latin phrase *reductio ad absurdum*. The concept is to expose errors by counter-example, since only one solid counter-example is necessary to disprove a universal claim. As an example of this, consider the common misconception among beginning students of electricity that voltage cannot exist without current. One way to apply *reductio ad absurdum* to this statement is to ask how much current passes through a fully-charged battery connected to nothing (i.e. a clear example of voltage existing without current).

from "first principles". Again, this reflects the goal of developing clear and independent thought in students' minds, by showing how clear and logical thought was used to forge each concept. Students benefit from witnessing a model of clear thinking in action, and these tutorials strive to be just that.

Another feature of these learning modules is that they do not treat topics in isolation. Rather, important concepts are introduced early in the series, and appear repeatedly as stepping-stones toward other concepts in subsequent modules. This helps to avoid the "compartmentalization" of knowledge, demonstrating the inter-connectedness of concepts and simultaneously reinforcing them. Each module is fairly complete in itself, reserving the beginning of its tutorial to a review of foundational concepts.

To high standards of education,

Tony R. Kuphaldt

Appendix C

Tools used

I am indebted to the developers of many open-source software applications in the creation of these learning modules. The following is a list of these applications with some commentary on each.

You will notice a theme common to many of these applications: a bias toward *code*. Although I am by no means an expert programmer in any computer language, I understand and appreciate the flexibility offered by code-based applications where the user (you) enters commands into a plain ASCII text file, which the software then reads and processes to create the final output. Code-based computer applications are by their very nature *extensible*, while WYSIWYG (What You See Is What You Get) applications are generally limited to whatever user interface the developer makes for you.

The `GNU/Linux` computer operating system

> There is so much to be said about Linus Torvalds' `Linux` and Richard Stallman's `GNU` project. First, to credit just these two individuals is to fail to do justice to the *mob* of passionate volunteers who contributed to make this amazing software a reality. I first learned of `Linux` back in 1996, and have been using this operating system on my personal computers almost exclusively since then. It is *free*, it is completely *configurable*, and it permits the continued use of highly efficient `Unix` applications and scripting languages (e.g. shell scripts, Makefiles, `sed`, `awk`) developed over many decades. `Linux` not only provided me with a powerful computing platform, but its open design served to inspire my life's work of creating open-source educational resources.

Bram Moolenaar's `Vim` text editor

> Writing code for any code-based computer application requires a *text editor*, which may be thought of as a word processor strictly limited to outputting plain-ASCII text files. Many good text editors exist, and one's choice of text editor seems to be a deeply personal matter within the programming world. I prefer `Vim` because it operates very similarly to `vi` which is ubiquitous on `Unix`/`Linux` operating systems, and because it may be entirely operated via keyboard (i.e. no mouse required) which makes it fast to use.

Donald Knuth's TEX typesetting system

Developed in the late 1970's and early 1980's by computer scientist extraordinaire Donald Knuth to typeset his multi-volume magnum opus *The Art of Computer Programming*, this software allows the production of formatted text for screen-viewing or paper printing, all by writing plain-text code to describe how the formatted text is supposed to appear. TEX is not just a markup language for documents, but it is also a Turing-complete programming language in and of itself, allowing useful algorithms to be created to control the production of documents. Simply put, TEX *is a programmer's approach to word processing*. Since TEX is controlled by code written in a plain-text file, this means anyone may read that plain-text file to see exactly how the document was created. This openness afforded by the code-based nature of TEX makes it relatively easy to learn how other people have created their own TEX documents. By contrast, examining a beautiful document created in a conventional WYSIWYG word processor such as Microsoft `Word` suggests nothing to the reader about *how* that document was created, or what the user might do to create something similar. As Mr. Knuth himself once quipped, conventional word processing applications should be called WYSIAYG (What You See Is *All* You Get).

Leslie Lamport's LATEX extensions to TEX

Like all true programming languages, TEX is inherently extensible. So, years after the release of TEX to the public, Leslie Lamport decided to create a massive extension allowing easier compilation of book-length documents. The result was LATEX, which is the markup language used to create all ModEL module documents. You could say that TEX is to LATEX as `C` is to `C++`. This means it is permissible to use any and all TEX commands within LATEX source code, and it all still works. Some of the features offered by LATEX that would be challenging to implement in TEX include automatic index and table-of-content creation.

Tim Edwards' `Xcircuit` drafting program

This wonderful program is what I use to create all the schematic diagrams and illustrations (but not photographic images or mathematical plots) throughout the ModEL project. It natively outputs PostScript format which is a true vector graphic format (this is why the images do not pixellate when you zoom in for a closer view), and it is so simple to use that I have never had to read the manual! Object libraries are easy to create for `Xcircuit`, being plain-text files using PostScript programming conventions. Over the years I have collected a large set of object libraries useful for drawing electrical and electronic schematics, pictorial diagrams, and other technical illustrations.

`Gimp` graphic image manipulation program

Essentially an open-source clone of Adobe's `PhotoShop`, I use `Gimp` to resize, crop, and convert file formats for all of the photographic images appearing in the ModEL modules. Although `Gimp` does offer its own scripting language (called `Script-Fu`), I have never had occasion to use it. Thus, my utilization of `Gimp` to merely crop, resize, and convert graphic images is akin to using a sword to slice bread.

`SPICE` circuit simulation program

`SPICE` is to circuit analysis as TEX is to document creation: it is a form of markup language designed to describe a certain object to be processed in plain-ASCII text. When the plain-text "source file" is compiled by the software, it outputs the final result. More modern circuit analysis tools certainly exist, but I prefer `SPICE` for the following reasons: it is *free*, it is *fast*, it is *reliable*, and it is a fantastic tool for *teaching* students of electricity and electronics how to write simple code. I happen to use rather old versions of `SPICE`, version 2g6 being my "go to" application when I only require text-based output. `NGSPICE` (version 26), which is based on Berkeley `SPICE` version 3f5, is used when I require graphical output for such things as time-domain waveforms and Bode plots. In all `SPICE` example netlists I strive to use coding conventions compatible with all `SPICE` versions.

Andrew D. Hwang's `ePiX` mathematical visualization programming library

This amazing project is a `C++` library you may link to any `C/C++` code for the purpose of generating PostScript graphic images of mathematical functions. As a completely free and open-source project, it does all the plotting I would otherwise use a Computer Algebra System (CAS) such as `Mathematica` or `Maple` to do. It should be said that `ePiX` is *not* a Computer Algebra System like `Mathematica` or `Maple`, but merely a mathematical *visualization* tool. In other words, it won't determine integrals for you (you'll have to implement that in your own `C/C++` code!), but it can graph the results, and it does so beautifully. What I really admire about `ePiX` is that it is a `C++` programming library, which means it builds on the existing power and toolset available with that programming language. Mr. Hwang could have probably developed his own stand-alone application for mathematical plotting, but by creating a `C++` library to do the same thing he accomplished something much greater.

`gnuplot` mathematical visualization software

Another open-source tool for mathematical visualization is `gnuplot`. Interestingly, this tool is *not* part of Richard Stallman's GNU project, its name being a coincidence. For this reason the authors prefer "gnu" *not* be capitalized at all to avoid confusion. This is a much "lighter-weight" alternative to a spreadsheet for plotting tabular data, and the fact that it easily outputs directly to an X11 console or a file in a number of different graphical formats (including PostScript) is very helpful. I typically set my `gnuplot` output format to default (X11 on my Linux PC) for quick viewing while I'm developing a visualization, then switch to PostScript file export once the visual is ready to include in the document(s) I'm writing. As with my use of `Gimp` to do rudimentary image editing, my use of `gnuplot` only scratches the surface of its capabilities, but the important points are that it's *free* and that it *works well*.

`Python` programming language

Both Python and C++ find extensive use in these modules as instructional aids and exercises, but I'm listing Python here as a *tool* for myself because I use it almost daily as a *calculator*. If you open a Python interpreter console and type `from math import *` you can type mathematical expressions and have it return results just as you would on a hand calculator. Complex-number (i.e. *phasor*) arithmetic is similarly supported if you include the complex-math library (`from cmath import *`). Examples of this are shown in the Programming References chapter (if included) in each module. Of course, being a fully-featured programming language, Python also supports conditionals, loops, and other structures useful for calculation of quantities. Also, running in a console environment where all entries and returned values show as text in a chronologically-ordered list makes it easy to copy-and-paste those calculations to document exactly how they were performed.

Appendix D

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Appendix E

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Appendix F

Version history

This is a list showing all significant additions, corrections, and other edits made to this learning module. Each entry is referenced by calendar date in reverse chronological order (newest version first), which appears on the front cover of every learning module for easy reference. Any contributors to this open-source document are listed here as well.

26 January 2026 – made some clarifications in a footnote about leakage resistance in a real capacitor, at the suggestion of Wyatt Gleason. Wyatt also recommended I alter my description of a rail gun to be grammatically less ambiguous. Also made some minor edits in other parts of the Tutorial. Also corrected a spelling error in the title of "The Meissner Effect"

12-13 October 2025 – added some instructor notesm, and also added/re-ordered several Technical Reference sections.

10 March 2025 – added a Conceptual Reasoning question on the Meissner Effect.

25 February 2025 – added a new Historical References section on early examples of inductors.

8 November 2024 – added a new Programming Reference section on solving differential equations using C++.

15 October 2024 – divided the Full Tutorial section "Energy storage in fields" into two sections, one on electric fields and the other on magnetic fields.

18-21 August 2024 – added another section to the Introduction chapter specifically for challenges related to this module's topic.

21 July 2024 – created a new Historical References section on old capacitor technology.

24 June 2024 – divided the Introduction chapter into two sections, one for students and one for instructors, and added content to the instructor section recommending learning outcomes and measures.

14 November 2023 – added a new Case Tutorial section on empirically determining signal rates of change.

24 October 2023 – added a Conceptual Reasoning question regarding qualititative analysis of an RLC circuit. Also added Challenge questions to the "Capacitor charging circuit" and "Inductor charging circuit" Conceptual Reasoning questions.

24 August 2023 – minor edits to some instructor comments.

17 August 2023 – added $Q = CV$ to the Full Tutorial chapter. Also, created a new Technical Reference section on capacitive displacement current from two separate Technical References, and shared this new reference among multiple modules.

3 July 2023 – added new Quantitative Reasoning question with mixed-source circuits containing capacitors and inductors.

27 November 2022 – placed questions at the top of the itemized list in the Introduction chapter prompting students to devise experiments related to the tutorial content.

19 July 2022 – significant change made to image_6312 which is in the Case Tutorial section on inductive energy storage.

13 June 2022 – added two new Case Tutorial sections showing energy storage calculations for series versus parallel capacitor networks and for series versus parallel inductor networks.

10 June 2022 – minor edits to Full Tutorial, and also added experimental suggestions to the two Case Tutorial sections on capacitor and inductor energy storage/release.

18 October 2021 – divided Full Tutorial into sections.

9 July 2021 – added a Case Tutorial section showing how to demonstrate resonance using an inductor, a capacitor, a battery, and an oscilloscope.

2 June 2021 – added a Case Tutorial chapter with sections showing how to store and release energy from inductors and capacitors.

1-2 April 2021 – minor edits to Full Tutorial as well as wording of some questions.

9 February 2021 – added a Technical Reference section on inductor labeling.

25 January 2021 – corrected a typographical error identified by Jacob Stormes in the "Rail gun capacitor" question. I had 5 kg as a mass when it should have been 0.5 kg. Also capitalized "Volts" in a couple of cases, and also fixed a couple of incorrect quotation marks.

2 January 2021 – added a Technical Reference section on capacitor labeling.

27 September 2020 – significantly edited the Introduction chapter to make it more suitable as a pre-study guide and to provide cues useful to instructors leading "inverted" teaching sessions.

15 September 2020 – added row to tables in the Full Tutorial commenting on work being "done on" versus work being "done by" capacitors and inductors.

6 April 2020 – minor typographical error correction in the *Current "through" a capacitor* section of the *Derivations and Technical References* chapter, courtesy of Ron Felix.

11 March 2020 – edited "rate-of-change" to read "speed" for clarity to readers unfamiliar with calculus.

10 December 2019 – added more questions. Also added units of measurement to the explanation of the inductance formula.

7 November 2019 – added Technical Reference about displacement current, and made minor edits to Full Tutorial.

26 August 2019 – added mention of difficult concepts to the Introduction.

8 April 2019 – minor edit to wording within two illustrations (replaced "creates" with "means" to avoid the implication of a unidirectional causality).

October 2018 – clarified a number of details, with constructive criticism from Ron Felix. Added a new experiment to test an inductor's ability to energize a neon lamp.

August 2018 – added date of the Tacoma Narrows bridge collapse (7 November 1940) to a footnote in the Simplified Tutorial.

July 2018 – added a Simplified Tutorial and made minor edits to the Full Tutorial.

September 2017 – added rate-of-change equations describing the relationships between voltage and current for both capacitance and inductance.

August 2017 – elaborated more on the conservation of voltage for secondary-cell batteries and voltage for capacitors and current for inductors based on how each component stores energy.

July 2017 – added some more Conceptual questions, and also added LaTeX comments describing table formatting.

June 2017 – document first published.

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