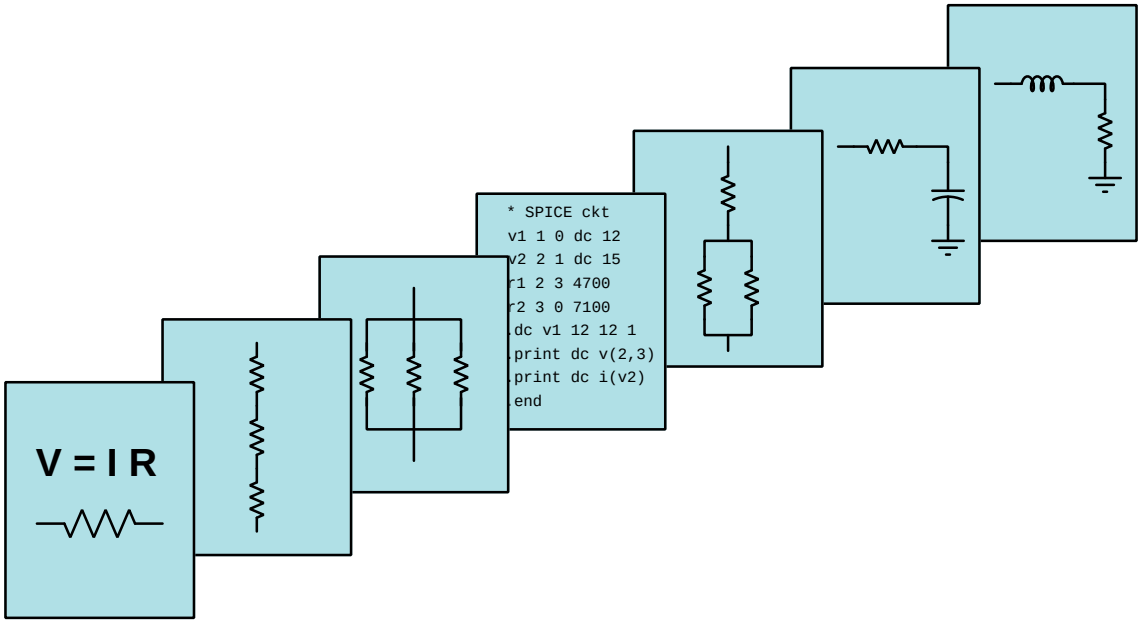
Modular Electronics Learning (ModEL) project



DC Motors

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Last update = 24 November 2025

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Chapter 1

Introduction

DC electric motors were the first practical electric motor technology, and dominated early industry until the advent of AC induction motors and AC power distribution systems. Even then, for a while DC motors were still preferred over AC for applications where variable-speed operation was essential, as AC induction motor speed is a function of power frequency.

This module explores the fundamental principles of electromagnetism and electromagnetic induction as they apply to the operation of electric motors. Multiple types of DC motor will be discussed, including the most modern type which is the brushless DC motor.

Important concepts related to DC electric motors include **electromagnetism**, **magnetic attraction/repulsion**, **magnetic fields**, **magnetic poles**, **right-hand rule**, **Lorentz force**, electromagnetic **induction**, **Faraday's Law** of electromagnetic induction, **rates of change**, **Lenz's Law**, **brushes**, **commutators**, **counter-EMF**, **Ohm's Law**, **Kirchhoff's Voltage Law**, **field windings**, **series** versus **shunt** versus **compound** motors, **transistors**, and the **Hall Effect**.

Here are some good questions to ask of yourself while studying this subject:

- How do magnetic fields and electric currents relate to one another?
- What is electromagnetic induction, and what conditions are necessary for it to occur?
- How does the Right Hand Rule apply to straight-line conductors?
- How does the Right Hand Rule apply to conductive loops?
- What is Lenz's Law, and how does it apply to electromagnetism?
- How is mechanical torque produced by an electric motor?
- How does the Right Hand Rule apply to the Tutorial's illustrations showing simple wire and motor structures?
- What practical purpose do brushes and commutator serve in a DC electric motor?
- What replaces the brushes and commutator within a "brushless" DC electric motor?

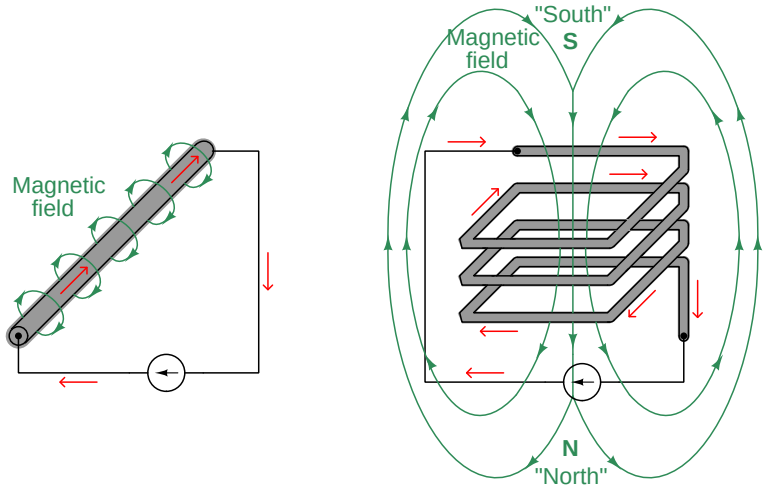
- How may we force a motor to function as a generator?
- Why does motor current surge when it is first started?
- Why do some motors have permanent magnets while others have field windings?
- What are the characteristics of a series-wound DC motor?
- What are the characteristics of a shunt-wound DC motor?
- What are the characteristics of a compound-wound DC motor?
- What must be done to a DC motor to reverse its direction of rotation?

Chapter 2

Tutorial

2.1 Magnetism and induction

The phenomenon of an electric current creating a magnetic field is called *electromagnetism*. This effect may be enhanced by forming the current-carrying conductor into a coil shape, which causes each wire turn's magnetic field to link with the magnetic fields of the other turns in the wire coil, producing a greater net magnetic field:



The terms "north" and "south" refer to the tips of a magnetic compass needle, in honor of which direction they point[1] when allowed to freely orient to the natural magnetic field of the Earth. Like poles repel, while opposite poles attract. While the magnetic field passing through and around the coiled wire is an endless loop just like the circular field surrounding the straight wire, we are able to assign "north" and "south" poles only to the coil because the coil itself has definite ends for the field to enter and exit. A straight wire provides no convenient features to mark the "ends" of the field, and so we cannot place "N" and "S" symbols anywhere along its length.

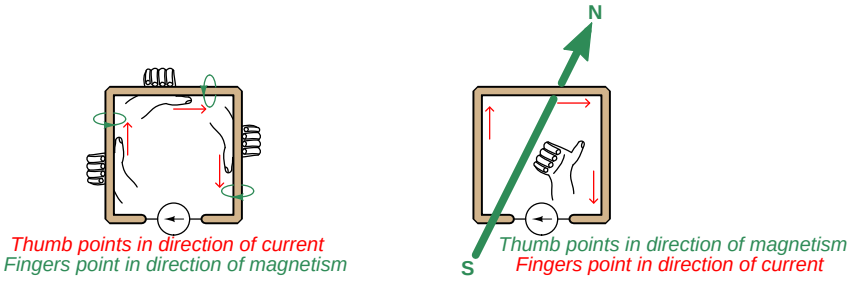
A coil's magnetic field may be further enhanced by providing a solid iron path for it to pass through, since iron offers less magnetic "resistance" than empty space or air. This is how powerful *electromagnets* are made: by wrapping coils of wire around an iron "core" material. The strength of the magnetic field is then easily controlled by varying the intensity of the electric current. Such electromagnets are the basis of *solenoid* actuators used to exert mechanical force, as well as *electric motors* used to perform mechanical work.

Note the arrowhead symbols used to denote both current (red) and magnetic field (green) directions. These directions are of course, conventions[2], but still we need a way to consistently

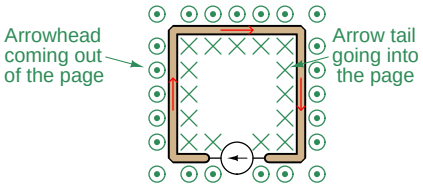
[1] This means that any bar-shaped magnet will tend to orient itself such that its "north" pole points toward the Arctic and its "south" pole toward the Antarctic. Since opposite magnetic poles attract, this actually means that the natural magnetic pole located on Earth's north (Arctic) end is actually "magnetic south" and vice-versa. Confusing, no?

[2] As we know, the direction of "conventional flow" for electric current is merely what early experimenters *assumed* was the correct direction of the mysterious "electric fluid" for which they had no reliable means to discern. Later discoveries showed that in fact electric current through metallic conductors is comprised of negatively-charged electrons

associate electromagnetic polarity with current direction, and for this purpose we have the *right-hand rule*. If you curl the fingers of your right hand as though forming a fist, and point your thumb outward (perpendicular to your wrist), your fingers and thumb will serve as a faithful guide to associating conventional-flow current with magnetic north. For a straight[3] current-carrying conductor, your thumb points in the direction of conventional current flow and your curled fingertips point in the direction of magnetic north. For a coiled conductor, your curled fingertips point in the direction of conventional flow while your thumb points in the direction of magnetic north:



The challenges of representing vectors existing in three physical dimensions accurately on a two-dimensional page are obvious. A common notation used to represent vector arrows perpendicular to the page is to show arrowheads as circles with center-dots (as though you were looking at a cone-shaped arrowhead headed toward your eyes) and arrow tails as "X" symbols (as though you were looking at the fletchings of an arrow headed away from you). Here is a view of the same circuit using this notation to show the true orientation of the magnetic field lines perpendicular to the page:



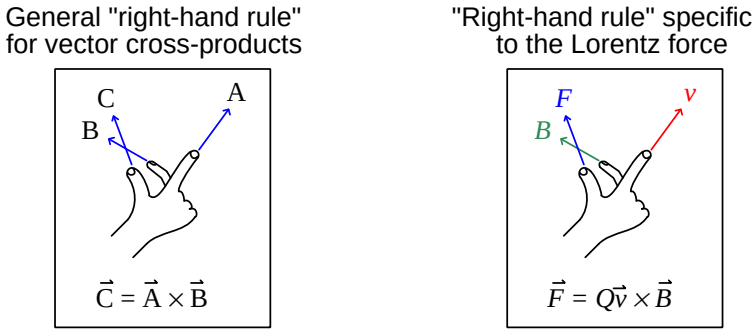
It is a good active-reading exercise to take your right hand and try applying it to this illustration, first pointing your thumb in the direction of current for each straight wire section and seeing how the curl of your fingers match the arrow head/tail symbols, then curling your fingers in the clockwise direction of the circuit's current and verifying that your thumb indeed points into the page with the arrow tails in the center of the wire loop.

which actually flow in the direction opposite to the accepted convention. This matters less than one might suppose. Magnetic fields are even more arbitrary, as there are no particles actually drifting anywhere to constitute a direction, but still it is necessary for the sake of accurate communication to define "north" and "south" polarity of these fields. Incidentally, the definition of "north" for a magnetic field is that of the north-pointing end of a magnetic compass needle, which means that the magnetic pole located near the Earth's geographic north pole (in the Arctic) is actually "magnetic south" so that the "magnetic north" of a compass' needle will be attracted in that direction!

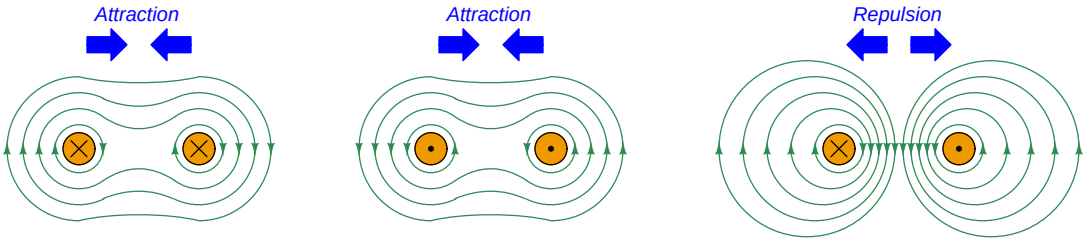
[3] Either an entirely straight wire, or a straight section of wire that is part some other shape.

Magnetic fields react with one another to produce *force*, and this is useful for converting electrical energy into mechanical work. If we know we can create magnetic fields using electric current, and we also know these fields repel (same poles) and attract (opposite poles), it is fairly simple to build electro-mechanical machinery using electricity to produce mechanical motion (e.g. electric motors).

The foundational principle of electromagnetic force generation is another variation of the *right-hand rule* where the index finger, middle finger, and thumb point in straight lines all perpendicular to each other, relating the vectors[4] of electric current motion ($Q\vec{v}$)[5], magnetic flux direction ($\vec{B}$), and Lorentz force ($\vec{F}$):



Two parallel current-carrying conductors a mutual force proportional to both their currents. If the currents travel in the same direction, the forces will attract; if the currents flow in opposite directions, the forces repel. The following illustrations show three different pairs of current-carrying conductors viewed from the end (with current either entering into or exiting out of the page, arrow tip and tail markings showing the directions of conventional flow):



Another good active-reading exercise is to map the right-hand rule onto each of these examples, aligning the index finger with the direction of electric current, the middle finger with the magnetic flux vector from the near side of the *other* conductor, and verifying the proper direction of the Lorentz force vector.

[4] A *vector* quantity is one with both a magnitude and a direction, as opposed to a *scalar* quantity which only has magnitude and no specified direction.

[5] Instead of using I to represent current, which is a scalar quantity, we express the motion of electric charges more directly in terms of charge quantity (Q, in Coulombs) multiplied by vector velocity ($\vec{v}$, in meters per second).

Complementary to electromagnetism is the phenomenon of *electromagnetic induction*. This occurs when a conductor experiences a perpendicular magnetic field varying in strength over time. This is how *generators* are constructed: by moving coils of wire past a stationary magnetic field, or vice-versa. The amount of voltage induced across the coil is proportional to the number of turns of wire in that coil and to the magnetic field's rate-of-change over time. This mathematical relationship is expressed in *Faraday's Law of Electromagnetic Induction*:

$$V = N\frac{d\Phi}{dt}$$

Where,

V = Induced voltage across the coil, in Volts

N = Number of "turns" or "wraps" in the coil

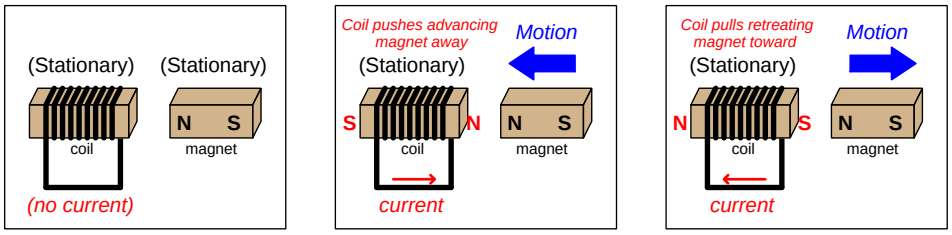
$\frac{d\Phi}{dt}$ = Rate-of-change of magnetic flux[6], in Webers per second

Electromagnetic induction is proportional to the *rate* at which a magnetic field strengthens or weakens perpendicular to a conductive coil. This means a voltmeter connected to a wire coil will register nothing when a nearby magnet is stationary, but will register voltage every time the magnet *moves* (either toward the coil or away from it):

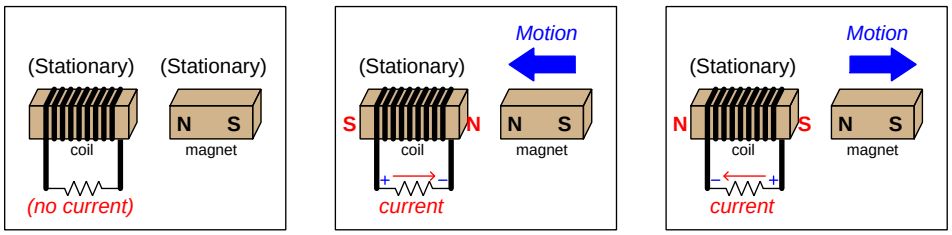
[6]Magnetic flux is related to magnetic field strength by cross-sectional area, with $B = \frac{\Phi}{A}$. Thus, it is valid to think of magnetic field strength as the degree to which magnetic flux is *concentrated* into an area. This holds true for the units of measurement used to express each: the unit of the *Tesla* used for magnetic field strength (B) is equivalent to *Webers of flux per square meter*.

If such a coil is connected to form a complete circuit allowing an electric current to form when the magnet advances or retreats, the coil will produce its own magnetic field of such a polarity as to oppose the inducing field's change. This is called *Lenz's Law*, and it stands as a universal feature of induction: *the induced effect always opposes the inducing cause.*

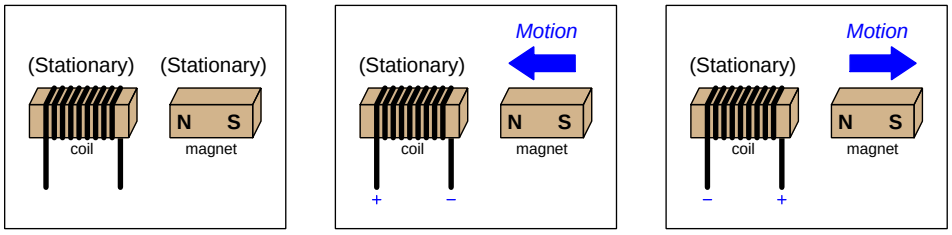
Examine the illustrations below showing a coil of wire facing a permanent magnet. In the left-hand illustration the magnet is stationary and therefore no induction occurs. In the center and right-hand illustrations the magnet moves, causing the coil to experience a varying magnetic field and thereby causing induction. The short-circuit permits current as a result of this induction, and we know any electric current through a coil produces its own magnetic field. Lenz's Law tells us this *induced* magnetic field must act to counter the motion of the permanent magnet:



Inserting an electrical resistance into the coil's circuit diminishes the strength of this reaction, but does not alter its direction. Note how the coil functions as a *source* to the resistor's *load*:



If we break the circuit, no current may pass and therefore no opposing magnetic field can form. However, the coil must still generate a voltage in accordance with Faraday's Law, and the polarity of that voltage will be the same as it would if there were a resistive load connected:

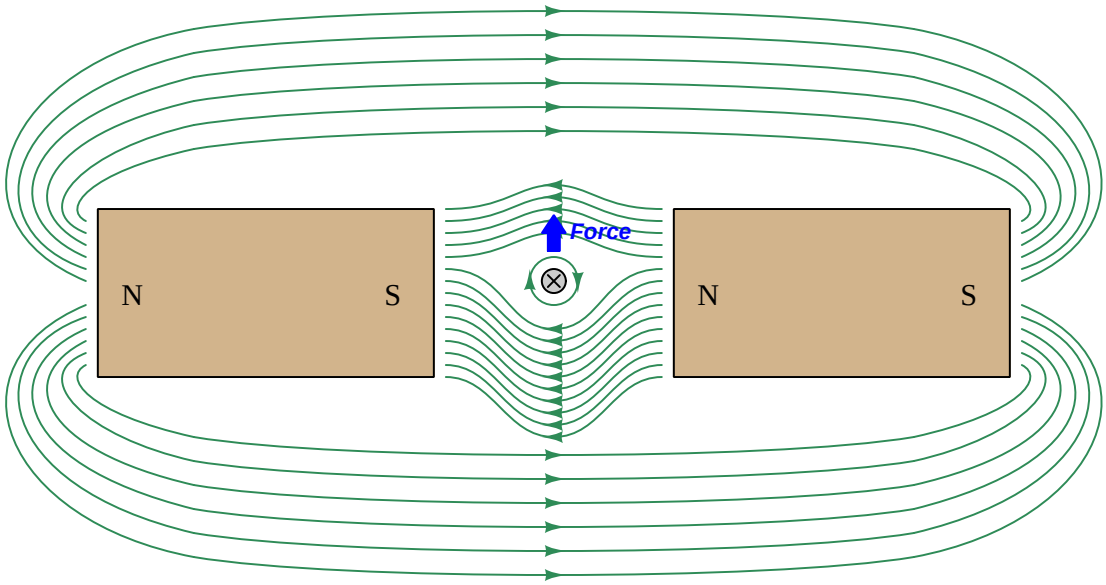


In case you were wondering, all these current directions and voltage polarities will reverse if we swap either the permanent magnet's polarity (i.e. South pole facing the coil) or the direction the coil is wrapped around its core. Reversing both of these parameters is equivalent to a double-negative, resulting in the same current directions and voltage polarities as shown above.

2.2 Permanent-magnet motors

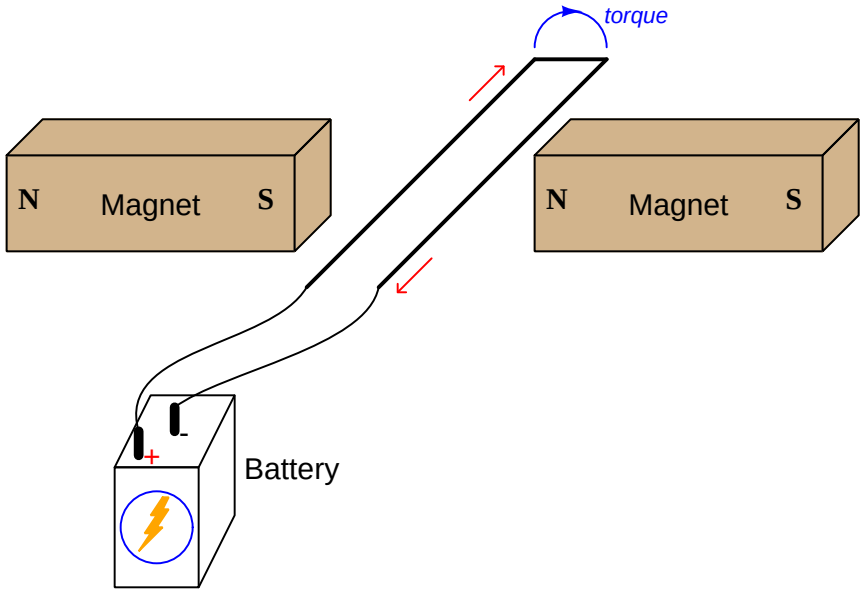
Magnetism, of course, is a direct consequence of moving electrical charge carriers, which is why every current-carrying conductor is surrounded by its own magnetic field. Certain materials[7], however, may be made "permanently" magnetic, requiring no electric current to sustain their own magnetic field. The common *bar magnet* and *horseshoe magnet* are examples of this phenomenon.

If a pair of permanent magnets are arranged to form a magnetic flux spanning a gap, and a current-carrying conductor placed within that gap perpendicular to the permanent magnetism, the conductor will experience a lateral force perpendicular to both the current and the magnetic flux vectors. This is the familiar *Lorentz force*, following the right-hand rule:



[7] All ferromagnetic materials such as iron have the ability to retain at least some magnetic flux after the magnetizing stimulus has been removed. Certain so-called *rare earth* compounds are exceptionally good in this regard.

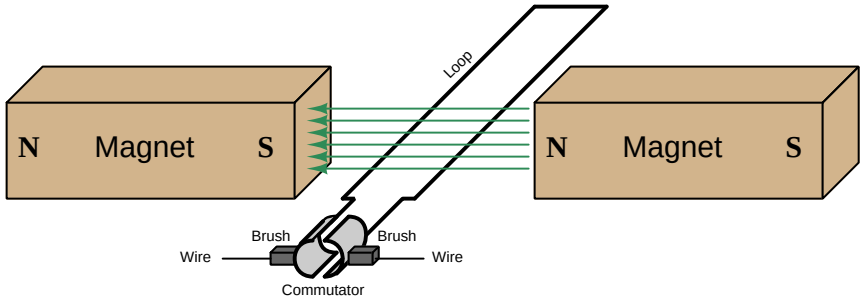
If we revise this experiment slightly and place a *loop* of wire within that same gap, the loop experiences a *torque* (i.e. twisting force) as one wire is forced upward and the other downward:



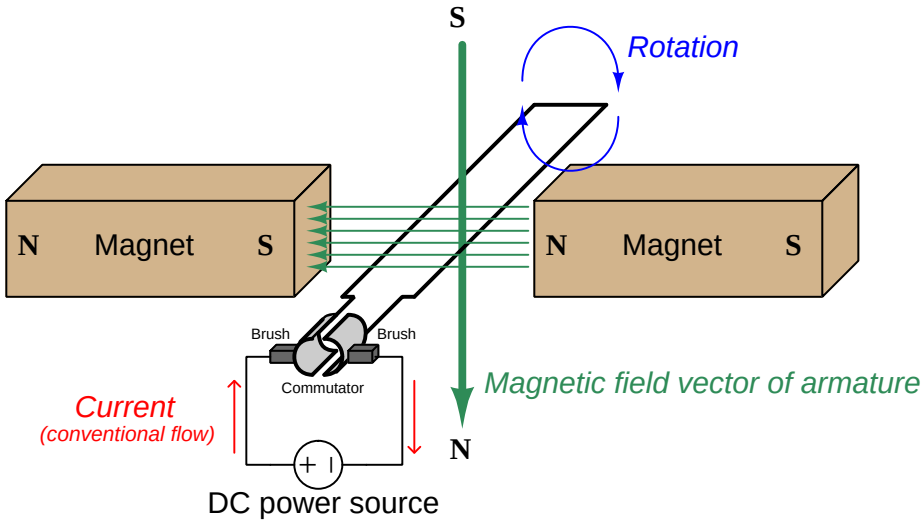
This makes sense if we view the loop as a pair of wires with each wire experiencing its own force by the perpendicular right-hand rule (left-hand wire forced up and right-hand wire forced down), or alternatively if we consider the loop as a partial *coil* with the curled-finger version of the right-hand rule informing us of the coil's magnetic polarity (north down and south up), with north repelling north, south repelling south, and north-south attracting. Either way the loop tries to rotate clockwise as viewed from the perspective of the battery.

As promising as this seems to be as the foundation for the design of an electric motor, there is a problem: the loop will not spin in a full circle, but instead will stop when its magnetic poles align with the permanent magnets, only one-quarter of a turn away from where it is shown above. Furthermore, the wires connecting the loop to the battery would become twisted together if the loop were (somehow) to turn more than 180 degrees. If only we had a way to *make* and *break* connection to the battery at just the right positions as the loop rotated, we might have a way to make this loop fully spin.

An ingenious solution to this problem is a sliding-contact assembly called a *commutator*, and a pair of graphite (carbon) *brushes* serving as contact points between the battery's wires and the commutator segments. The commutator is simply a pair of semi-circular metal pieces terminating the ends of the wire loop and arranged about the centerline of rotation. These pieces rotate along with the loop as one assembly, making and breaking contact with the brushes as it spins:

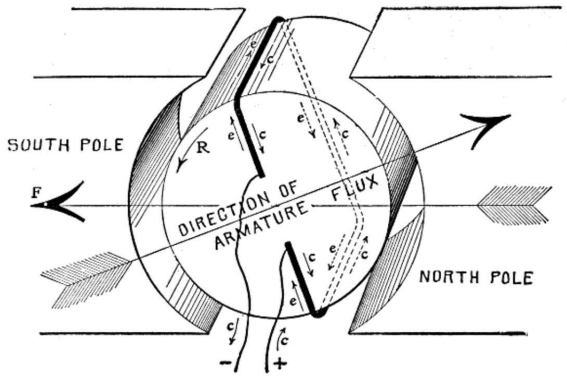


When energized by a DC electrical source, the loop continuously turns while the commutator and brushes make and break contact as necessary to maintain the same direction of rotation. Reversing the direction of a permanent-magnet (PM) DC motor is as simple as reversing the polarity of the energizing source:



In order to maintain a strong magnetic field through the rotating loop, it is generally wound around an iron core mounted on a bearing-supported shaft called an *armature*. The armature supports the rotating loop while providing a nearly continuous pathway for magnetic flux lines from one permanent magnet to the other. A circular iron structure joining the magnets' outer poles together similarly enhances the magnetism by providing a complete magnetic path.

The following illustration taken from page 394 of *Hawkins Electrical Guide Number Two* (published in 1914) shows how the wire loop may be wrapped around an iron armature, as well as directions of current (both *conventional* c and *electron* flow e):



For greater effect, we may substitute a multi-turn coil for a single loop. For even greater effect, we may position multiple coils around the armature, with a multitude of commutator bars for the coils' termination. Additional illustrations (with captions) taken from pages 400 and 401 of the same text show photographs of typical DC motor components:

Fig. 400.—Fairbanks-Morse armature for 7½ H. P., 1300 R. P. M., TR type motor. The armature core is built up of thin sheet steel laminations with notches in the circumference, which, when the discs are placed together, form grooves or slots to receive the armature coils. The armature cores for the larger machines are mounted on a cast iron spider, which also carries the commutator, making the two parts entirely self-contained, and with this construction, it is possible to remove the armature shaft, without disturbing the core, commutator or windings. Cores of all sizes are provided with ventilating spaces, running from the surface to the central opening of the core, so that air is drawn through the core and blown out over the windings by the revolution of the armature.

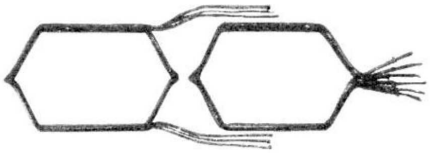
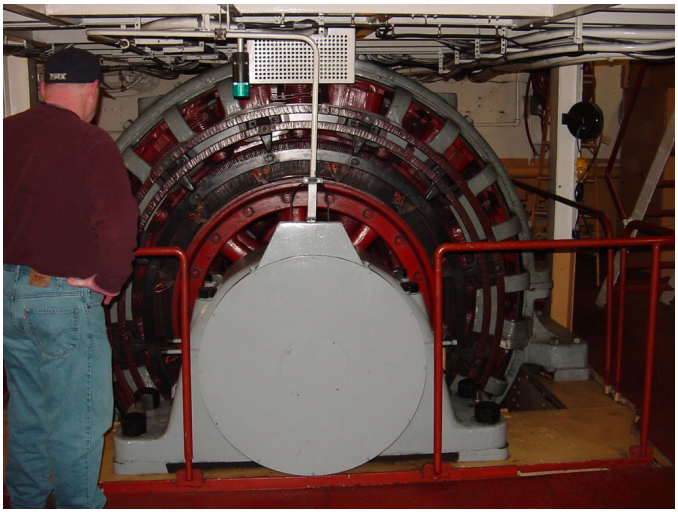


Fig. 401.—Fairbanks-Morse wire wound armature coils. These coils are form wound and are thoroughly insulated and baked before assembling in the slots. Material of great mechanical strength as well as high insulating value is used, and the coils are subjected to dippings in insulating compound and to bakings, thus driving out all moisture and making a coil which is practically waterproof and which will withstand rough handling. These coils, when completed, are placed in the slots, where they are retained by bands on the three smaller sizes and by hardwood wedges on the larger sizes.

Most industrial DC motors have multiple armature coils and commutator segments (also called *bars*), not just a single "loop" and two segments as shown in the previous model. A photograph of a large (1250 horsepower) DC motor used to propel a passenger ferry ship is shown here, with the field and armature poles clearly seen (appearing much like spokes in a wheel):



A close-up of one brush assembly on this large motor shows both the graphite brush, the brush's spring-loaded holder, and the myriad of commutator bars the brush makes contact with as the armature rotates. Each armature coil terminates at two of the commutator bars:

2.3 Counter-EMF

As the coils of a DC motor's armature rotate within the stationary magnetic field of the permanent magnets, they experience an induced voltage as described by *Faraday's Law of Electromagnetic Induction*:

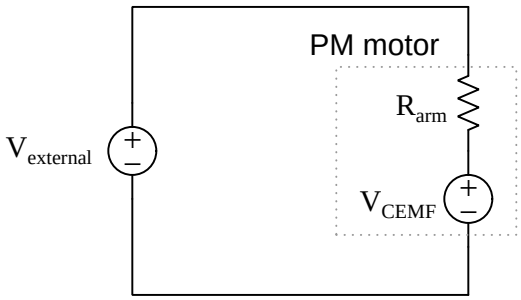
$$V = N\frac{d\Phi}{dt}$$

Where,

V = Induced voltage across the coil, in Volts
N = Number of "turns" or "wraps" in the coil
$\frac{d\Phi}{dt}$ = Rate-of-change of magnetic flux, in Webers per second

Thus, a PM motor also functions as a *generator*. This is true whether the armature is spun by an external (mechanical) torque or if it is running as a motor. When running as a motor, the induced voltage actually *opposes* that of the external source – an example of Lenz's Law in action: *the induced effect always opposing the inducing cause*.

A helpful problem-solving strategy to simplify complex systems is to model them as collections of simple elements. Here, we will visualize counter-voltage (commonly known as *counter-EMF* or *back-EMF*)[8] as an internal DC voltage source series-opposed to the external source powering the motor. For a permanent-magnet DC motor, the magnitude of this counter-EMF is a direct function of shaft speed:



Armature current may be calculated by applying Ohm's Law and Kirchhoff's Voltage Law, since armature coil resistance is fairly constant, and the net voltage dropped across that coil's resistance will be the difference between the external source voltage and the counter-EMF:

$$I_{arm} = \frac{V_{external} - V_{CEMF}}{R_{arm}}$$

Counter-EMF explains why for any given applied voltage there is a top speed for a PM motor: at this top speed, the counter-EMF is only slightly less than the external voltage source. If the motor were to somehow spin faster, its counter-EMF would equal the applied voltage and armature current would cease, resulting in zero torque (i.e. zero motor action). Spinning faster yet would result in $V_{CEMF} > V_{external}$ which would result in the motor functioning entirely as a generator and sending

[8] *EMF* stands for *electromotive force*, and it is an older (and not particularly accurate) synonym for voltage.

power to $V_{external}$ which would act as a load. Therefore, since the amount of source voltage must always be greater than the amount of counter-EMF in order for the machine to function as a motor (load), the source voltage magnitude places an upper limit on a PM motor's top speed.

Torque (twisting force) is a function of both the stationary magnetic field strength (which is constant for a PM motor) and the rotating armature's magnetic field strength (which varies with current). Therefore, current determines a PM motor's torque.

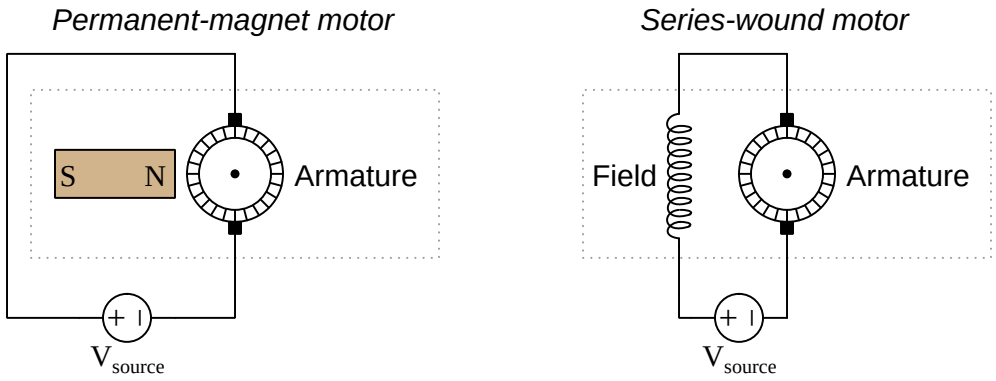
When electrical power is first applied to a stationary PM motor, the result is a high armature current and a corresponding high torque because counter-EMF is zero. As shaft speed increases, counter-EMF increases with it, resulting in less net voltage dropped across the armature coil resistance and less armature current. This initial surge of current during motor start-up is commonly known as *inrush current*.

Interestingly, this phenomenon of high starting current and relatively low running current is common to all electric motors regardless of design, because it is founded on the universal principle of Lenz's Law. Every electric motor simultaneously functions as a motor and as a generator, with the generated voltage (counter-EMF) of a running motor less than the applied voltage from the external source. This also means that every motor may be pressed into service as a generator simply by mechanically driving it to a faster speed than it would ordinarily run when powered as a motor, forcing its counter-EMF to exceed the applied voltage. In fact, this is one of the reasons why electric-drive vehicles enjoy high energy efficiency when driven in stop-and-go conditions: the vehicle's drive motor functions as a generator when decelerating, converting the vehicle's kinetic energy into electricity to re-charge the on-board battery with potential (chemical) energy. Combustion-based engines have no means to recover the vehicle's kinetic energy, and so they are only able to decelerate by *dissipating* that kinetic energy as heat (e.g. friction brakes or engine compression braking).

Permanent-magnet (PM) electric motors are the simplest form of DC motor design, but they have significant limitations. Permanent magnets are expensive and dangerous to handle at large sizes, and so PM motors tend to be small. Additionally, the range of operating speed for a PM motor is limited by the range of supply voltage. In order to build large DC motors more economically, as well as to enjoy wider operating speed range, it becomes necessary to replace the permanent magnets with electromagnet coils called *field windings*. Multiple options exist for field winding configurations, which we will explore in subsequent sections.

2.4 Series-wound motors

An alternative to using permanent magnets to establish the stationary magnetic field needed for a DC motor, we may use electromagnets formed of wire coils wrapped around iron poles. When such a "field winding" is connected in series with the motor's armature, we have a *series-wound motor*:



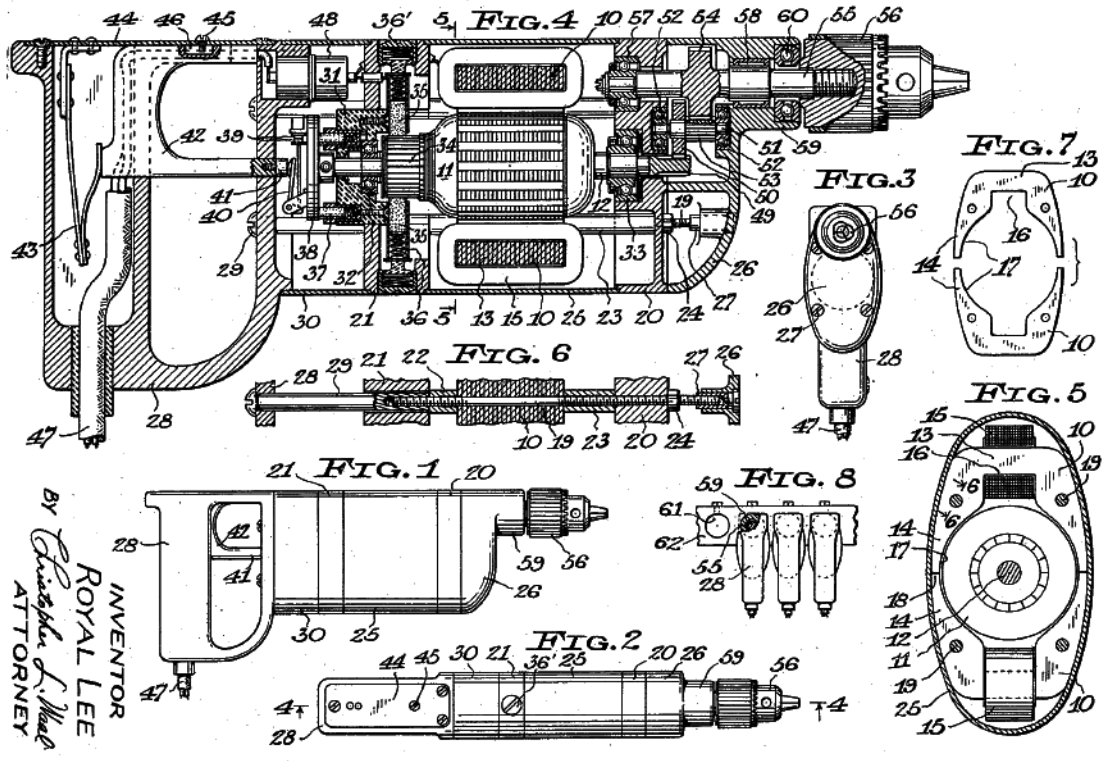
A wound-field motor does not reverse rotational direction when we reverse the polarity of the DC source, because reversing source polarity reverses the magnetic polarities of *both* field and armature. If north attracted south with the old polarity, south will still attract north with the new polarity, resulting in the same direction of rotation. In fact, polarity is so irrelevant to the operation of a wound-field motor that we may operate it on AC as well as DC!

The speed and torque characteristics of a series-wound motor differ greatly from that of a PM motor, principally because the strength of the stationary magnetic field varies with armature current for the series-wound motor, whereas it was constant for the PM motor. A "thought experiment" helps us understand the impact of this difference.

Imagine how a series-wound motor reacts when first powered from a stand-still condition. Before the shaft begins to rotate, counter-EMF is zero. This results in maximum voltage drop across the resistances of the field and armature windings, and therefore maximum current. This strong current creates strong magnetic fields for both the field and armature windings, thereby creating a very high torque (twisting force). As shaft speed increases, however, counter-EMF begins to rise. The increasing counter-EMF subtracts from the source voltage to leave less available to drop across the field and armature windings, thereby reducing current and weakening both their magnetic fields. In turn, this weakening of the field winding's current results in less counter-EMF for any given speed, which in turn results in current remaining fairly constant as speed increases. Rather than counter-EMF building linearly with speed as was the case with the PM motor, a series-wound motor's counter-EMF is self-limiting. This allows motor speed to increase almost without limit, resulting in exceptionally high top speeds for modest source voltages.

For this reason, series-wound motors are used where a very wide speed range is necessary, and where excessive speed will be constrained by the mechanical load.

Series-wound motors find popular use in power tools. An early example of this application is found in Royal Lee's 1946 patent[9], where he shows such a motor at the heart of a hand-held drill:

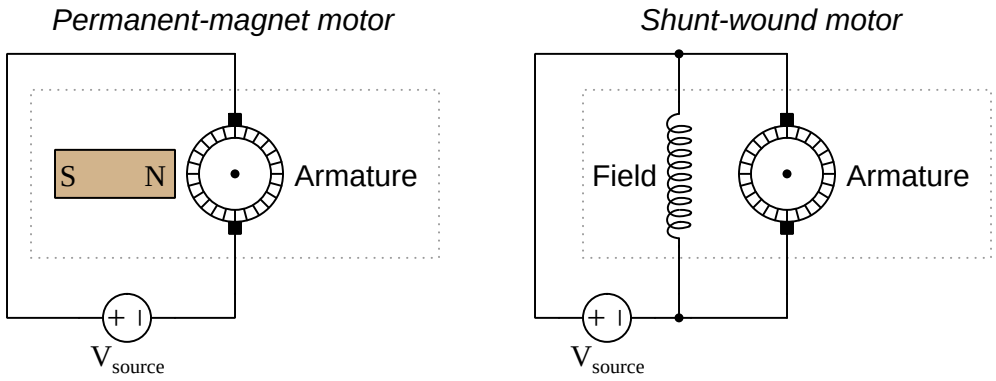


Noteworthy items shown in this diagram include the armature (11), field windings (10), commutator (34), graphite brushes (35), and springs (36) designed to maintain firm contact between the brushes and commutator bars. Threaded access caps (36') allow for easy replacement of the brushes when they wear out. Graphite is a fairly soft material – much softer than copper – and so these wear faster than the commutator bars. Like a mechanical pencil whose graphite "lead" wears down and must be replaced, so must motor brushes on a regular basis.

[9] *US Patent 2,406,389*, "Electric Motor" application filed 30 November 1942, with patent granted 27 August 1946.

2.5 Shunt-wound motors

Another variation on the wound-field concept is the *shunt-wound motor*, so-called because the field winding is wired in parallel (i.e. "shunt") with the armature. Like the series-wound DC motor, this field winding substitutes for permanent magnets:



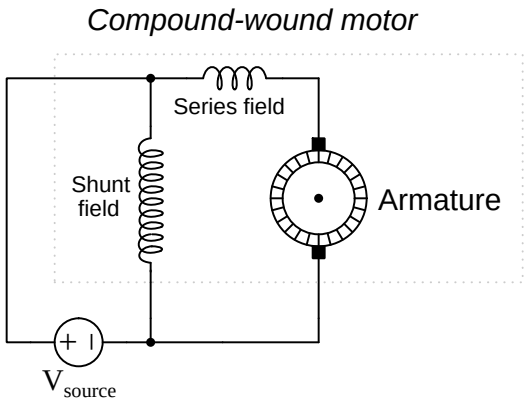
Shunt-wound field coils are made of many turns of thin-gauge wire, compared to the relatively thick-gauge wire windings of a series-wound motor. In a series-wound motor the field winding had to be able to handle the full current of the armature, but here the field winding is its own parallel branch and so we must build that winding such that it limits its own current by resistance ($I_{field} = \frac{V_{source}}{R_{field}}$).

Like the series-wound motor, a shunt-wound motor's direction of rotation does not reverse when the source polarity reverses, and for the exact same reason: reversing the source polarity swaps magnetic polarities for *both* the field and armature windings. In fact, the only way to reverse the rotation of either a series-wound or shunt-wound motor is to re-wire the connections between armature and field so that one coil's polarity reverses while the other remains the same.

The speed and torque characteristics of a series-wound motor differ greatly from that of a PM motor and that of a series-wound motor, principally because the strength of the stationary magnetic field varies with applied voltage, whereas it was constant for the PM motor and it varied with armature current for the series-wound motor. If we increase the applied voltage to a shunt-wound motor in an attempt to increase its speed, what will happen is the field winding's strength increases, which in turn increases the motor's counter-EMF by the same proportion, thereby providing additional opposition to the newly-increased source voltage value. The end-result is that speed increases only *slightly* with increased source voltage. Altering the operating speed of a shunt-wound motor requires that the excitation of one winding must be varied *independently* of the other.

2.6 Compound-wound motors

A compromise between the wide speed range of the series-wound motor and the nearly-constant speed of the shunt-wound motor is to build a wound-field motor with two field windings: one in series and one in parallel (shunt):



Not surprisingly, each of these field windings resembles the respective windings of the series and shunt motors: the shunt field winding is a long coil of thin wire for high resistance and low current, while the series winding is a short coil of thick wire designed to bear the full current drawn by the armature.

2.7 Brushless DC motors

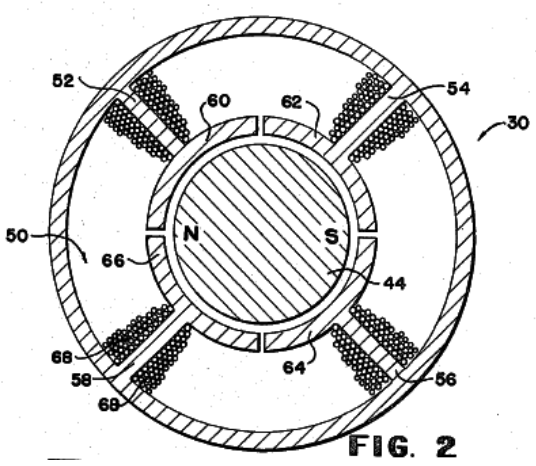
A major weakness of all DC motors described thus far are the carbon brushes and commutator bars. The need to conduct relatively high amounts of current through sliding graphite-on-copper contact represents a daunting engineering challenge. Decades of brush design refinement, armature winding techniques, and other technical innovations have permitted electric DC motors to become fairly reliable machines, but still the fundamental problem remains. Brushes and commutator bars wear out over time. Sparking is all but inevitable at the point of interface between the brush and commutator. The intrusion of dust or oil or other contaminants inside the motor will very rapidly destroy the delicate contact between brushes and commutator bars. Clearly, if there was some way to eliminate the need for brushes and commutators, it would represent a significant advance in electric motor performance and reliability.

The first motor technology to challenge DC was the *AC induction motor*, which is a topic for another module. AC induction motors rely on the alternating nature of the source voltage and current to create a "rotating magnetic field" for the rotor to follow, and therefore completely eliminate brushes, commutator, and even coil windings on the rotor. This advance, combined with the advent of an electrical device called a *transformer* established AC as the dominant mode of electric power transmission in the world. It was not until many decades later that solid-state (transistor) technology made possible a new type of DC motor that could rival the AC induction motor's legendary reliability.

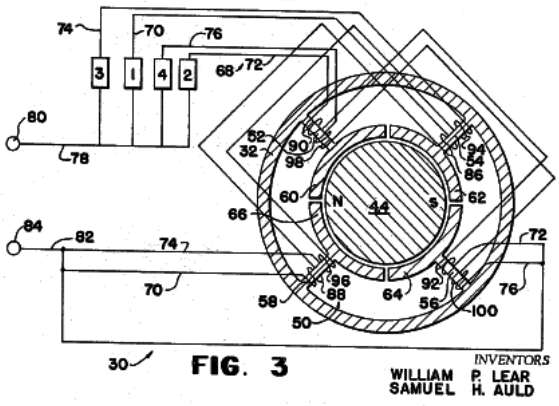
Now that we have reliable solid-state switching devices called *transistors* capable of turning on and off at incredibly high speed, we have the means to build a new type of permanent-magnet motor requiring no brushes and no commutator. Instead of letting the permanent magnets be stationary around a wound armature, we may place the permanent magnets on a rotor and use transistors to switch current on and off (and in different directions) through stationary windings located around the rotor. This is called a *brushless DC motor* or *BLDC* motor, and it may be thought of as a permanent-magnet motor turned inside-out: stationary wire coils surrounding a permanently-magnetized rotor.

For many years brushless DC motors were limited to small power levels for the same reasons as regular PM motors. However, with the advent of *rare-earth magnets* it is now possible to construct powerful brushless DC motors in compact sizes. This technology is popular for drive motors in electric vehicles, as well as countless small-motor applications such as computer processor cooling fans.

An early patent[10] on a form of brushless DC motor invented by William Lear and Samuel Auld shows one representation of a two-pole permanent-magnet rotor (item number 44) surrounded by a four-pole stationary armature (pole face item numbers 60, 62, 64, and 66). Item number 68 represents a cross-sectional illustration of the wire coil (winding) wrapped around each pole:



Lear and Auld's patent also showed a diagram of the armature windings with four switching elements numbered 1, 2, 3, and 4 for sequential energization from the DC source (terminals 80 and 84):



These numbered elements could, in principle, be any suitable mechanical or electronic switch. Typically they are transistors triggered by Hall-effect sensors magnetically detecting the position of the rotor as it spins. These transistors and rotor position sensors substitute for brushes and commutator bars, closing and opening circuits to energize and de-energize coils at the appropriate times to keep the rotor spinning in the desired direction.

[10] *US Patent 3,331,005*, "Brushless D.C. Motor", application filed 16 March 1964 and patent granted 11 July 1967. William Lear was the founder of the famous Lear Jet Corporation, a manufacturer of business-class jet aircraft.

Chapter 3

Historical References

This chapter is where you will find references to historical texts and technologies related to the module's topic.

Readers may wonder why historical references might be included in any modern lesson on a subject. Why dwell on old ideas and obsolete technologies? One answer to this question is that the initial discoveries and early applications of scientific principles typically present those principles in forms that are unusually easy to grasp. Anyone who first discovers a new principle must necessarily do so from a perspective of ignorance (i.e. if you truly *discover* something yourself, it means you must have come to that discovery with no prior knowledge of it and no hints from others knowledgeable in it), and in so doing the discoverer lacks any hindsight or advantage that might have otherwise come from a more advanced perspective. Thus, discoverers are forced to think and express themselves in less-advanced terms, and this often makes their explanations more readily accessible to others who, like the discoverer, comes to this idea with no prior knowledge. Furthermore, early discoverers often faced the daunting challenge of explaining their new and complex ideas to a naturally skeptical scientific community, and this pressure incentivized clear and compelling communication. As James Clerk Maxwell eloquently stated in the Preface to his book *A Treatise on Electricity and Magnetism* written in 1873,

> It is of great advantage to the student of any subject to read the original memoirs on that subject, for science is always most completely assimilated when it is in its nascent state . . . `[page xi]`

Furthermore, grasping the historical context of technological discoveries is important for understanding how science intersects with culture and civilization, which is ever important because new discoveries and new applications of existing discoveries will always continue to impact our lives. One will often find themselves impressed by the ingenuity of previous generations, and by the high degree of refinement to which now-obsolete technologies were once raised. There is much to learn and much inspiration to be drawn from the technological past, and to the inquisitive mind these historical references are treasures waiting to be (re)-discovered.

3.1 Transformation of industry by electric motors

Technological innovation generally occurs as an answer to practical problems. The phrase *necessity is the mother of invention* certainly explains the rise of many new technologies, as solutions to pressing needs of individuals and/or society as a whole. This is most definitely the case with electric motors. The scientific discoveries of electromagnetism and electromagnetic induction may have remained laboratory curiosities if not for the incentives of industry to employ cheaper and safer methods of transferring mechanical power to do useful work.

The following pair of photographs shows the machinery within a textile mill, in this particular case a silk-weaving factory, powered by mechanical shafts and belt drives turned by a centrally-located steam engine (left), and later powered by electric motors (right). These images were taken from the text *Electric Motors – their action, control and application* published in 1919. The contrast between these two images, in terms of mechanical complexity and employee safety alone, is stark:

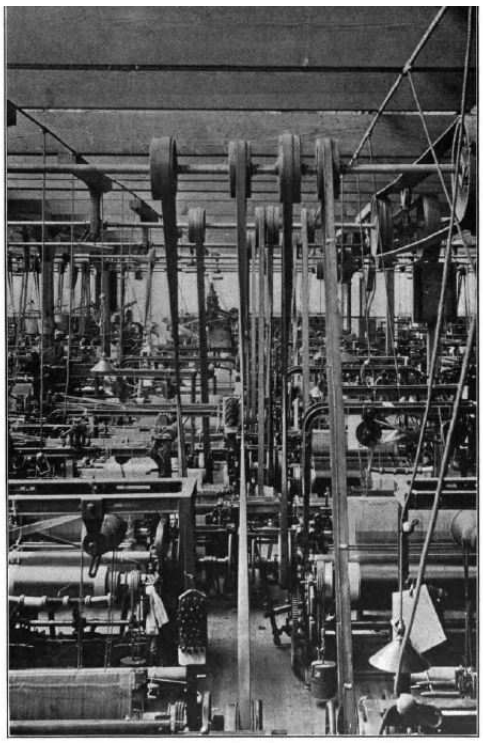


PLATE A.—SILK-WEAVING SHED WITH BELT DRIVE.

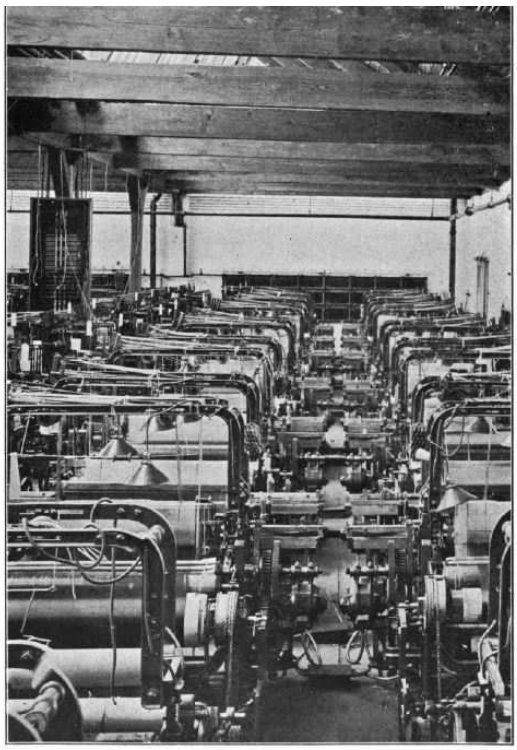


PLATE B.—THE SAME SILK-WEAVING SHED WITH MOTOR DRIVE.

Perhaps the most obvious improvement in factory conditions resulting from the replacement of mechanical drive systems with electric is that of personnel safety. All those open belts posed significant hazard to anyone working nearby. Anyone unfortunate enough to get an article of clothing or a limb caught up in one of those moving belts faced a grim fate.

The authors of this text presented a long list of advantages for the use of electric motor drives over mechanical shaft and belt drives, many of which are simply not apparent to anyone innocent of shaft- and belt-driven industrial environments. Given that it is highly unlikely any reader of *this* text will have that sort of personal experience, it is worthwhile to recount some of the advantages of electric motor drives cited by the authors of *Electric Motors – their action, control and application.* If nothing else, it gives a glimpse of early industrial work environments and engineering considerations:

- **Energy efficiency** – long mechanical shafts with journal bearings as well as belt drives incur substantial frictional power losses. Even considering the inefficiencies of converting a steam engine's mechanical power to electric power (via a generator) and then back to mechanical power again (via a motor), electric drive systems still used less fuel to do the same work as mechanical drive systems!

- **Lower building cost** – structures containing belt-driven machinery needed to support the weight of driveshafts from their ceiling beams, necessitating framework capable of bearing this load. Furthermore, as structures tend to settle over time, misalignments of shafts and belts would occur as the building aged. These problems disappear entirely with electric motors.

- **Building placement** – in a mechanical-drive factory, the steam engine needed to be located immediately adjacent to the factory floor where its power would be used. This is not the case with electric drives, as the generator building could be located a considerable distance from the factory floor, with wires connecting the two.

- **Expansion** – on a similar note, factory facilities are much easier to expand with electric motor drive than with mechanical drive, since building placement and distance from the power source are no longer limited.

- **Portable machinery** – small machines such as grinders may be made portable with electric motor drives, but this is impossible with shaft- and belt-drive.

- **Clear overhead space** – the elimination of shafts from the ceiling of factory buildings made lighting much easier. This advantage is plainly visible in the comparative photographs of the silk mill before and after electric motors. It also facilitated the use of overhead cranes for maintenance and/or production work.

- **Hygiene** – one of the advantages of electric motor drives most difficult to perceive by modern readers is improved air quality and general cleanliness on the factory floor. Apparently, shaft bearings constantly dripped lubricant, and belts continually generated fine dust. Not only did this pose breathing hazards to employees, but the pall of dust and oil would contaminate the very product(s) manufactured, especially in a place such as a textile mill where the last thing you want applied to new silk is *dust and oil*. As an example of improved working conditions, the 1914 text cites a 40% reduction in lost employee time due to sickness at the Government Printing Office in Washington DC following the retrofit of electric motors to printing machinery. Another working condition improvement was noise reduction, as electric motors run much quieter than shafts and belts.

- **Reliability** – electric motors, even those of circa 1914, were generally more reliable than a belt drive. Furthermore, when an electric motor failed it only affected the one machine it operated,

and repairs could be made while the rest of the machinery on the factory floor continued to operate. This was not the case with machines driven from a common shaft, which had to be stopped to replace *any* belt along the shaft's length.

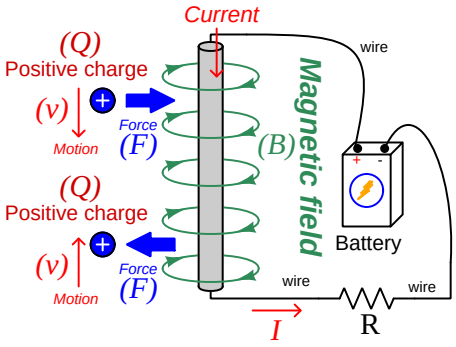
- **Speed control** – when multiple machines are powered by a common shaft, they must all run at the same speed, or be individually equipped with *transmissions* allowing speed adjustment. Electric motors, on the other hand, are relatively easy to control the speed of. An interesting behavioral note made by the authors of the 1914 text is that *what is convenient gets used*: electric motor speed controls usually take the form of a simple knob or dial that may be turned, while shifting a mechanical transmission requires manipulation of large and cumbersome levers. Human operators were found to make frequent adjustments to electric drive speed controls in order to improve efficiency and/or production, while they would be more reluctant to do the same with clumsy mechanical speed controls. This simple fact translated to improved production with electric drives.

- **Turndown** – this refers to the ability to run only *portions* of a factory's machinery after hours, during holidays, or during labor strikes. This would be inefficient in a mechanical-drive system because the (large) common steam engine would have to be run at full speed even if just one machine on the factory floor needed to operate. However, with electric drives it was possible to run a small auxiliary generator just large enough to power the reduced machinery, without having to turn the shafts of all the other machines.

As a result of these advantages, it was found that factories generally increased their production output by 20 to 30 percent following retrofit to electric motor drive.

3.2 Magnetic field quantities

A useful definition of magnetic field (B) is in terms of the force (F, called the *Lorentz force*) exerted on a moving electric charge (Q) influenced by that field:



$$\vec{F} = Q\vec{v} \times \vec{B}$$

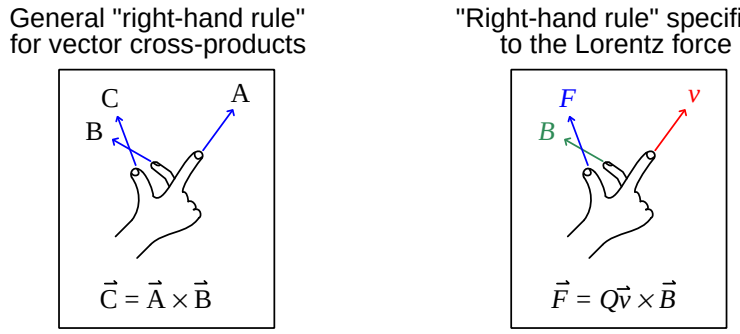
Where,

$\vec{F}$ = Force exerted on the charge (Newtons)
Q = Charge quantity (Coulombs[1])
$\vec{v}$ = Velocity of moving charge (meters per second)
$\vec{B}$ = Magnetic field (Tesla, Webers per square meter, or Newtons per Ampere-meter)

The small "arrow" symbols above the variables for force and velocity and magnetic field in the equation denote those variables as *vector quantities*, having both magnitude and direction. Charge is a *scalar quantity* having only magnitude but no direction, and as a scalar quantity when multiplied by the velocity vector it simply magnifies the magnitude but does not alter the direction. The "cross-product" ($\times$) is a specific form of vector multiplication, and it results in a product at right angles to the vector directions of both terms. Therefore, the force and velocity and electric field vectors never all point in the same direction.

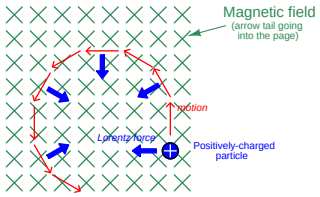
[1] One Coulomb of electric charge is equal to 6.2415×10^{18} electrons.

Vector cross-products conveniently relate to the fingers of the right hand, which is where the "right-hand rule" originates:



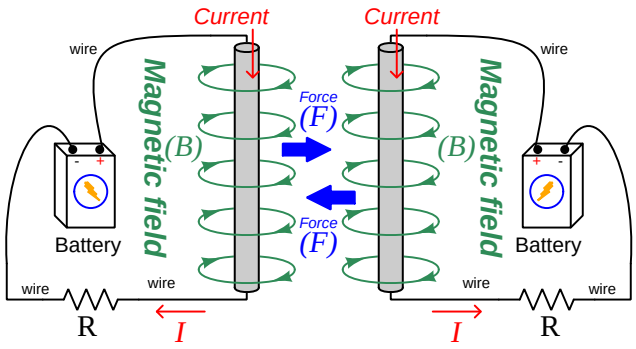
When holding the index finger, middle finger, and thumb of your right hand perpendicular to each other, your index finger points in the direction of the velocity vector ($\vec{v}$), your middle finger in the direction of the magnetic field vector ($\vec{B}$), and your thumb in the direction of the force vector ($\vec{F}$). A simple mnemonic I use to remember these relationships of fingers to vectors is that the **I**ndex finger points in the direction of current[2] (I), the **M**iddle finger points in the direction of the **m**agnetic field (B), and the **Th**umb points in the direction of the **th**rust (i.e. force) acting upon the moving charge.

The Lorentz force's effect on electrically-charged particles in motion has many applications, from redirecting the paths of charged-particle beams in particle accelerator machines, to bending the trajectory of electron beams in a cathode-ray tube (CRT), to forcing electrons to travel in spiral or circular trajectories inside of magnetron (microwave oscillator) tubes. An illustration of a positively-charged particle curving perpendicular to a magnetic field appears here:



[2]Some textbooks speak of a "left-hand rule" which is intended to make sense of electric charge motion (current) in terms of *electron flow*. As we know, electrons are the only real mobile charge carriers within metal conductors, and so technically "electron flow" notation is most physically accurate when describing the motion of electric charges in metallic circuits. However, the right-hand rule is a *mathematical definition* for vector cross products, the concept of the cross product arising in the late 18th century when electrical science was still in its infancy. Early explorers of electricity used the established mathematical tools of their time and applied it to their work with electric currents and magnetism. At that time, charge carriers in metal wires were assumed to be "positive" and this is how the motion of positively-charged carriers became associated with the first vector of the cross-product. As a result of this assumption which was later proven false, we have two different conventions for denoting the motion of electricity: electron-flow which is *physically* accurate (for metal wires, at least), and conventional flow which is *mathematically* accurate. This, perhaps more than any other reason, is why educational programs designed for mathematically rigorous fields (e.g. electrical engineering) exclusively use conventional flow notation rather than electron flow notation to denote the direction of current.

If the moving charge in question is not a single charged particle but rather part an electric *current* passing through a conductor parallel to the first, both conductors will experience a mutually-attracting force given by the following equation:



$$\vec{F} = I\vec{l} \times \vec{B}$$

Where,

$\vec{F}$ = Force exerted on both conductors (Newtons)

I = Current (Amperes)

$\vec{l}$ = Length of wire (meters)

$\vec{B}$ = Magnetic field (Tesla, or Webers per square meter, or Newtons per Ampere-meter)

The point-charge Lorentz force equation and the two-conductor Lorentz force equation are not that different from one another. Dimensional analysis validates this: the Lorentz force on a moving charge uses that charge quantity (Coulombs) multiplied by the point-charge's velocity in meters per second to give Coulomb-meters per second for the first term:

$$Q\vec{v} = [\mathrm{C}]\left[\frac{\mathrm{m}}{\mathrm{s}}\right] = \left[\frac{\mathrm{C} \cdot \mathrm{m}}{\mathrm{s}}\right]$$

The Lorentz force on a current-carrying conductor uses the current (Amperes, which is Coulombs per second) multiplied by length in meters, for the same composite units of Coulomb-meters per second:

$$I\vec{l} = \left[\frac{\mathrm{C}}{\mathrm{s}}\right][\mathrm{m}] = \left[\frac{\mathrm{C} \cdot \mathrm{m}}{\mathrm{s}}\right]$$

This dimensional equivalence makes conceptual sense as well: an electrically-charged particle moving through empty space is an electric current in its own right, and an electric current flowing through a conductor is just a collection of charged particles moving through space (just not *empty* space). In either case, the basis for the Lorentz force remains the same: the moving charge(s) create their own magnetic field, which reacts with the magnetic field of the original current-carrying wire to produce forces acting on both.

If the two currents flow in the same direction, their mutual forces *attract*. If the two currents flow in opposite directions, their mutual forces *repel*. This is the basis of electric motors: causing

mechanical motion by electro-magnetic attraction and repulsion. It also represents an interesting contrast with electric fields:

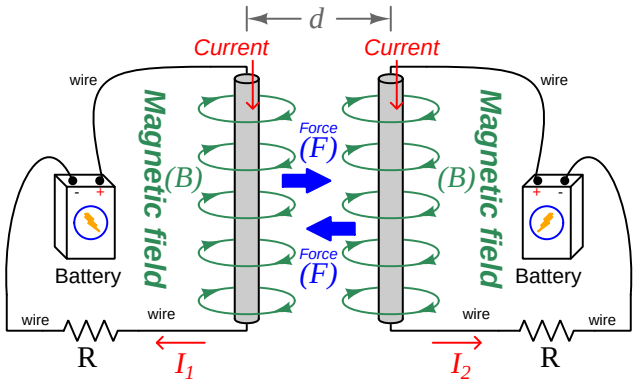
With electric fields, opposite *charges* attract and like *charges* repel.

With magnetic fields, opposite *poles* attract and like *poles* repel.

With parallel currents, opposite *directions* repel and like *directions* attract[3].

Two parallel current-carrying conductors of length l and separated by a distance d will generate a mutual force proportional to both their currents:

$$F = l\frac{\mu I_1 I_2}{2\pi d}$$



[3]That is, assuming it's *like* charges moving in these directions! If the charges in question are opposite each other – for example electrons in one circuit and holes in another – then like directions will repel and opposite directions will attract!

The circular loops surrounding the current-carrying conductors in the previous illustrations represent the magnetic lines of *flux* (Φ_B) surrounding each of those conductors. The magnetic field (B) is related to magnetic flux by area (A), the field being a measurement of how densely-packed those flux lines are per unit area. For this reason, magnetic field (B) is more properly known as *magnetic flux density*:

$$\vec{B} = \frac{\Phi_B}{\vec{A}}$$

Where,

$\vec{B}$ = Magnetic field or flux density (Tesla, Webers per square meter, or Newtons per Ampere-meter)

Φ_B = Magnetic flux (Webers)

$\vec{A}$ = Area over which flux is distributed (square meters)

An older unit of measurement for magnetic flux density B is the *Gauss* which is much smaller than a Tesla, with one Tesla equivalent to 10,000 Gauss. To put things into perspective, the Earth's natural magnetic field has a strength of approximately one-half of one Gauss[4].

Magnetic field strength is an inverse function of distance from any current-carrying wire, and also depends on the magnetic permeability of the space adjacent to the wire:

$$B = \frac{\mu I}{2\pi d}$$

Where,

B = Magnetic field or flux density (Tesla, Webers per square meter, or Newtons per Ampere-meter)

μ = Magnetic permeability of the surrounding space (Tesla-meters per Ampere, $4\pi \times 10^{-7}$ for empty space)

I = Current (Amperes)

d = Distance from conductor (meters)

[4]Using the online *Magnetic Field Calculator* application provided by NOAA (the National Oceanic and Atmospheric Administration) at `https://ngdc.noaa.gov/geomag/calculators/magcalc.shtml#igrfwmm`, applying the World Magnetic Model WMM modeling algorithm for years 2019-2024, the total magnetic field strength at my home is 53,584.4 nano-Tesla (53,584.4 nT or 0.535844 Gauss), and presently (May 2020) decaying at a rate of -104.1 nT per year.

The relation of magnetic *flux* to current through a conductor follows a similar equation:

$$\Phi = \frac{\mu A I}{2\pi d}$$

Where,

Φ = Magnetic flux (Webers)

μ = Magnetic permeability of the surrounding space (Tesla-meters per Ampere, $4\pi \times 10^{-7}$ for empty space)

A = Area over which flux is distributed (square meters)

I = Current (Amperes)

d = Distance from conductor (meters)

As this equation makes clear, the amount of magnetic flux surrounding a current-carrying conductor depends not only on the amount of current, but also on the sampled area, the distance from the wire, and also the surrounding material. Most[5] substances (gas, liquid, solid) have permeability values greater than that of empty space, and so this means magnetic flux is usually *enhanced* by the presence of matter around the current-carrying conductor.

The total magnetic flux enclosed by a circular wire loop follows a similar equation:

$$\Phi = \frac{\pi \mu I r}{2}$$

Where,

Φ = Magnetic flux (Webers)

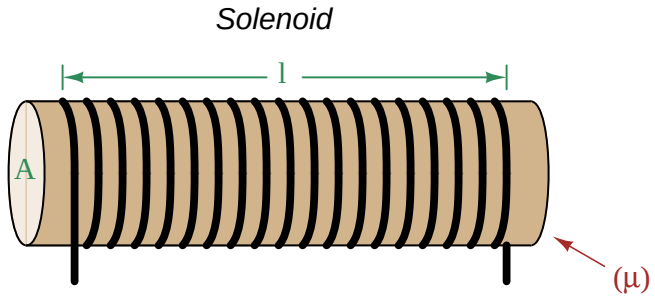
μ = Magnetic permeability of the surrounding space (Tesla-meters per Ampere, $4\pi \times 10^{-7}$ for empty space)

I = Current (Amperes)

r = Radius of circular loop (meters)

[5] Interestingly, superconducting materials *forbid* magnetic fields inside of their bulk, and so the permeability value of any superconductor must be zero!

A common form of electromagnet known as a *solenoid* takes the form of a wire coil wrapped in such a way as to form a long[6] cylinder, often wrapped around a plastic frame, and often with a ferromagnetic material such as iron in the center:



The amount of magnetic flux, and the flux density, within the interior of a current-carrying solenoid are given by the following formulae:

$$\Phi = \frac{\mu N A I}{l} \qquad\qquad B = \frac{\mu N I}{l}$$

Where,

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N = Number of turns of wire in the coil

A = Cross-sectional area of solenoid coil (square meters)

I = Current (Amperes)

l = Length of solenoid coil (meters)

These formulae have interesting implications for solenoid design. Note how a shorter (i.e. smaller length l) solenoid identical in all other respects will generate a stronger magnetic field for a given current. Note also how the flux density (B) remains constant with increasing cross-sectional area (A) if all other factors are equal, and that this necessarily means a greater amount of total magnetic flux (Φ) for a greater area A.

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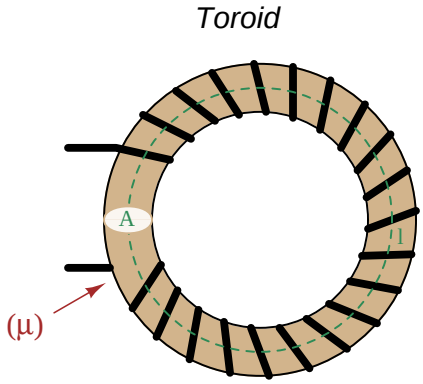

The amount of magnetic flux, and the flux density, within the interior of a current-carrying toroid are identical to that within an otherwise identical otherwise identical solenoid having a length (l) equal to the toroid's circumference:

$$\Phi = \frac{\mu N A I}{l} \qquad\qquad B = \frac{\mu N I}{l}$$

Where,

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N = Number of turns of wire in the coil

A = Cross-sectional area of toroid (square meters)

I = Current (Amperes)

l = Circumference of toroid (meters)

If we wish to substitute toroid radius (r) for circumferential length (l), the formulae become the following:

$$\Phi = \frac{\mu N A I}{2\pi r} \qquad\qquad B = \frac{\mu N I}{2\pi r}$$

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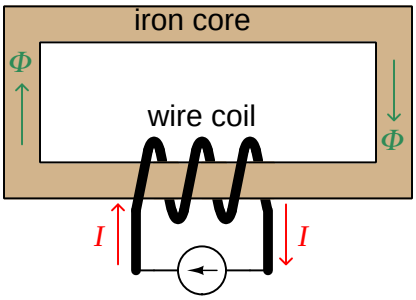
Many applications of electromagnetism involve conductive *coils* wrapped around some form of ferromagnetic core material, the purpose of that core being to provide a higher-permeability pathway for the magnetic flux than would exist otherwise through air, and the purpose of the wire coil being to intensify the amount of magnetism developed by the electric current beyond what would be possible with a straight current-carrying wire. These magnetic cores typically form a closed loop, or *magnetic circuit* for the lines of magnetic flux to naturally form a closed path. A simple example appears here:



The amount of magnetic flux (Φ) present in the magnetic "circuit" formed by the iron core depends on many factors. First and foremost is the amount of electric current (in Amperes) passing through the wire coil and the number of turns that coil makes around the iron core. The product of this current and the number of turns is called the *magnetomotive force* or *mmf* of the magnetic circuit, analogous to "electromotive force" or "emf" often used as a synonym for voltage in an electric circuit. Not surprisingly, the standard metric unit of measurement for magnetomotive force is the *Ampere-turn*.

However, magnetomotive force alone does not fully describe the current's effect on magnetism within the iron core. The total length of the magnetic circuit is also an important factor, since a longer path distributes that magnetomotive force over a greater distance. The quotient of magnetomotive force and magnetic circuit length is called *magnetic field intensity*, symbolized by the variable H and expressed in units of *Ampere-turns per meter*.

Magnetic permeability (μ) relates magnetic field intensity (H) to the magnetic flux density (B) within the core material, such that a greater permeability will result in higher flux density for any given amount of field intensity. Permeability is a property of the core material and not its geometry, mathematically defined as the ratio of flux density to field intensity: $\mu = \frac{B}{H}$

Magnetic reluctance ($\Re$) relates magnetomotive force (mmf) to magnetic flux (Φ), and is related not only to the core material's permeability but also its geometry. It is mathematically defined as the ratio of magnetomotive force to magnetic flux: $\Re = \frac{\text{mmf}}{\Phi}$

If all this seems confusing, you are in good company. Not only are there many magnetic variables, some related to physical geometry and others not, but there are *two different sets of metric units* appropriate for expressing each! The older units were based on the centimeter-gram-second (CGS) version of the metric system, while the newer units are based on the meter-kilogram-second or SI (Système International) version of the metric system.

Quantity	**Symbol**	**SI unit**	**CGS unit**
Magnetomotive force	mmf	Ampere-turn (A-t)	Gilbert (Gb)
Flux	Φ	Weber (Wb)	Maxwell (Mx)
Field intensity	H	Ampere-turns per meter (A-t/m)	Oersted (Oe)
Flux density	B	Tesla (T)	Gauss (G)
Permeability	μ	Tesla-meters per Ampere-turn (T-m/A-t)	Gauss per Oersted (G/Oe)
Reluctance	$\Re$	Ampere-turns per Weber (A-t/Wb)	Gilberts per Maxwell (G/Mx)

Magnetomotive force (mmf) and magnetic flux (Φ) may be thought of as the "raw" measures of magnetism, with Ampere-turns and Webers being their respective SI metric units. Reluctance ($\Re$) is the ratio of the two for any given magnetic circuit with known dimensions and core material. Simply put, reluctance tells you how many Ampere-turns of magnetomotive force will be necessary to create one Weber of magnetic flux in a given space.

Magnetic field intensity (H) and magnetic flux density (B) may be thought of as the "normalized" measures of magnetism, with Ampere-turns per meter and Tesla being their respective SI metric units. H and B relate to mmf and flux by the physical dimensions of the magnetic circuit (length and cross-sectional area, respectively). Permeability is the ratio of the two for any given magnetic core material. Simply put, permeability tells you how many Tesla of magnetic field (i.e. flux density, or Webers of flux per square meter or cross-sectional core area) you will obtain for one Ampere-turn per meter of magnetic field intensity applied to a given core material.

Conversion between the newer SI and the older CGS metric units are as follows:

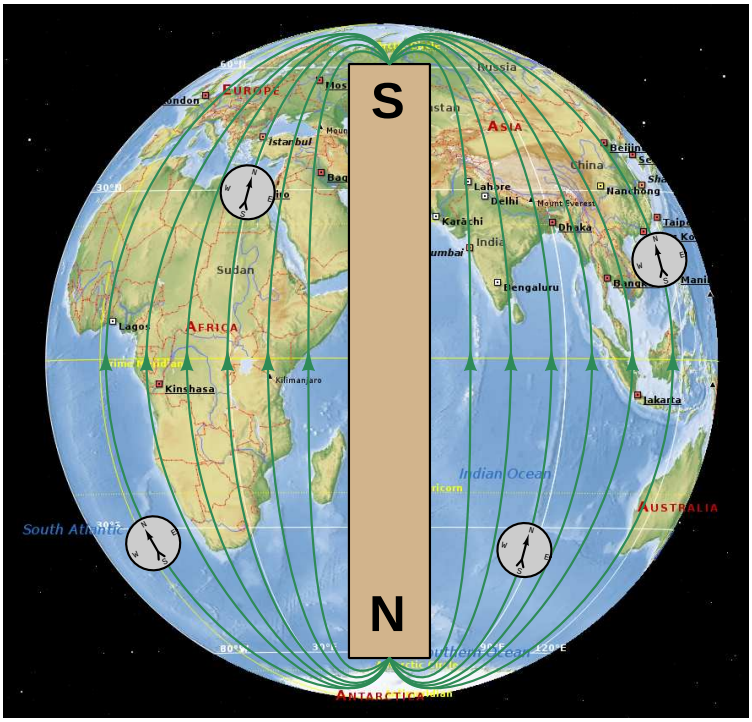
Quantity	**Conversion equivalence**
Magnetomotive force (mmf)	1 Ampere-turn = $\frac{4\pi}{10}$ Gilberts
Magnetic flux (Φ)	1 Weber = 10^8 Maxwells
Magnetic field intensity (H)	1 Ampere-turn/meter = $\frac{4\pi}{1000}$ Oersteds
Magnetic flux density (B)	1 Tesla = 10^4 Gauss
Permeability (μ)	1 Tesla-meter/Ampere-turn = $\frac{10^7}{4\pi}$ Gauss/Oersteds
Reluctance ($\Re$)	1 Ampere-turn/Weber = $\frac{4\pi}{10^9}$ Gilberts/Maxwell

3.3 Defining magnetic north and south

Magnetic pole-pairs are always called *north* and *south*, and this naming hails from the days of navigation by magnetic compass, where a magnetized iron needle would point toward the geographic north pole of the Earth (i.e. toward the Arctic region).

The compass's needle itself is, of course, nothing more than a small magnet with north and south poles. These poles naturally align with the Earth's own magnetic field. Confusion tends to arise when we consider the well-known behavior of magnetic poles to attract when they are opposite and repel when they are identical. Whichever end of a compass needle points toward the Arctic region of the Earth must have the opposite type of magnetic pole as that end of the Earth, and the same must be said of the Antarctic-pointing end of the compass needle.

By convention we have come to identify magnetic north and magnetic south in terms of a compass needle's ends: that is to say, the Arctic-pointing end of a compass needle is magnetic north while the Antarctic-pointing end of that same needle is magnetic south. This means the Earth itself must have a magnetically-south Arctic and a magnetically-north Antarctic. The following illustration shows this relationship:



Fortunately for anyone using a magnetic compass to map magnetic fields created by circuits and/or by permanent magnets, discerning the direction of the magnetic field vectors is extremely simple: *the arrow symbol shown on the compass is the vector-direction of the magnetic field.* Note in the above illustration how each of the compass needles points in the same direction as the respective magnetic flux vector, from Antarctic to Arctic.

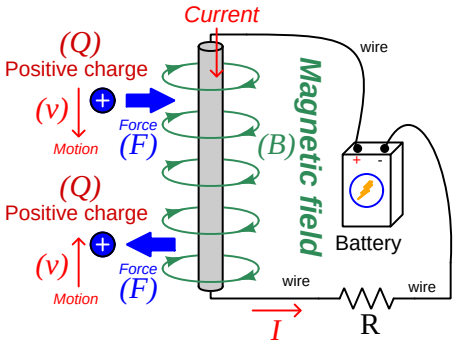
Chapter 4

Derivations and Technical References

This chapter is where you will find mathematical derivations too detailed to include in the tutorial, and/or tables and other technical reference material.

4.1 Magnetic field quantities

A useful definition of magnetic field (B) is in terms of the force (F, called the *Lorentz force*) exerted on a moving electric charge (Q) influenced by that field:



$$\vec{F} = Q\vec{v} \times \vec{B}$$

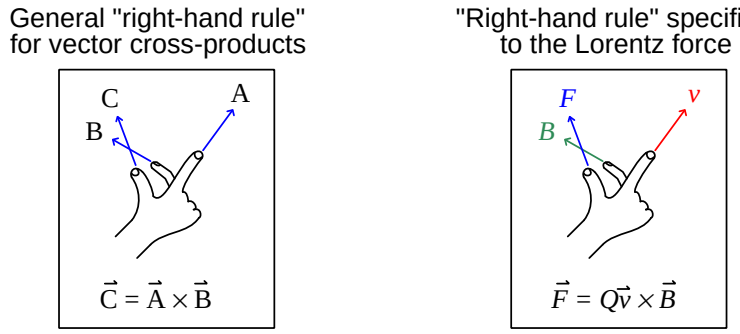
Where,

$\vec{F}$ = Force exerted on the charge (Newtons)
Q = Charge quantity (Coulombs[1])
$\vec{v}$ = Velocity of moving charge (meters per second)
$\vec{B}$ = Magnetic field (Tesla, Webers per square meter, or Newtons per Ampere-meter)

The small "arrow" symbols above the variables for force and velocity and magnetic field in the equation denote those variables as *vector quantities*, having both magnitude and direction. Charge is a *scalar quantity* having only magnitude but no direction, and as a scalar quantity when multiplied by the velocity vector it simply magnifies the magnitude but does not alter the direction. The "cross-product" ($\times$) is a specific form of vector multiplication, and it results in a product at right angles to the vector directions of both terms. Therefore, the force and velocity and electric field vectors never all point in the same direction.

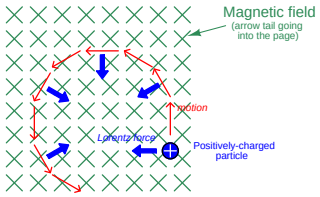
[1] One Coulomb of electric charge is equal to 6.2415×10^{18} electrons.

Vector cross-products conveniently relate to the fingers of the right hand, which is where the "right-hand rule" originates:



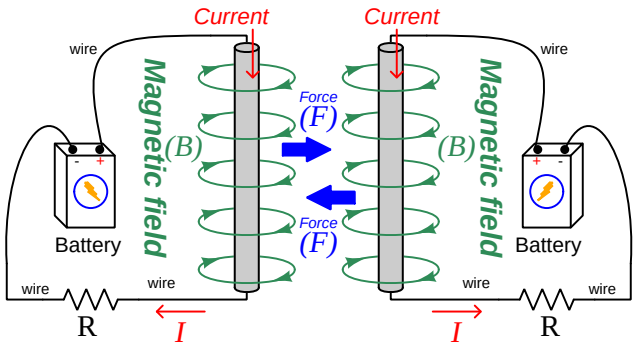
When holding the index finger, middle finger, and thumb of your right hand perpendicular to each other, your index finger points in the direction of the velocity vector ($\vec{v}$), your middle finger in the direction of the magnetic field vector ($\vec{B}$), and your thumb in the direction of the force vector ($\vec{F}$). A simple mnemonic I use to remember these relationships of fingers to vectors is that the **I**ndex finger points in the direction of current[2] (I), the **M**iddle finger points in the direction of the **m**agnetic field (B), and the **Th**umb points in the direction of the **th**rust (i.e. force) acting upon the moving charge.

The Lorentz force's effect on electrically-charged particles in motion has many applications, from redirecting the paths of charged-particle beams in particle accelerator machines, to bending the trajectory of electron beams in a cathode-ray tube (CRT), to forcing electrons to travel in spiral or circular trajectories inside of magnetron (microwave oscillator) tubes. An illustration of a positively-charged particle curving perpendicular to a magnetic field appears here:



[2]Some textbooks speak of a "left-hand rule" which is intended to make sense of electric charge motion (current) in terms of *electron flow*. As we know, electrons are the only real mobile charge carriers within metal conductors, and so technically "electron flow" notation is most physically accurate when describing the motion of electric charges in metallic circuits. However, the right-hand rule is a *mathematical definition* for vector cross products, the concept of the cross product arising in the late 18th century when electrical science was still in its infancy. Early explorers of electricity used the established mathematical tools of their time and applied it to their work with electric currents and magnetism. At that time, charge carriers in metal wires were assumed to be "positive" and this is how the motion of positively-charged carriers became associated with the first vector of the cross-product. As a result of this assumption which was later proven false, we have two different conventions for denoting the motion of electricity: electron-flow which is *physically* accurate (for metal wires, at least), and conventional flow which is *mathematically* accurate. This, perhaps more than any other reason, is why educational programs designed for mathematically rigorous fields (e.g. electrical engineering) exclusively use conventional flow notation rather than electron flow notation to denote the direction of current.

If the moving charge in question is not a single charged particle but rather part an electric *current* passing through a conductor parallel to the first, both conductors will experience a mutually-attracting force given by the following equation:



$$\vec{F} = I\vec{l} \times \vec{B}$$

Where,

$\vec{F}$ = Force exerted on both conductors (Newtons)

I = Current (Amperes)

$\vec{l}$ = Length of wire (meters)

$\vec{B}$ = Magnetic field (Tesla, or Webers per square meter, or Newtons per Ampere-meter)

The point-charge Lorentz force equation and the two-conductor Lorentz force equation are not that different from one another. Dimensional analysis validates this: the Lorentz force on a moving charge uses that charge quantity (Coulombs) multiplied by the point-charge's velocity in meters per second to give Coulomb-meters per second for the first term:

$$Q\vec{v} = [\mathrm{C}]\left[\frac{\mathrm{m}}{\mathrm{s}}\right] = \left[\frac{\mathrm{C} \cdot \mathrm{m}}{\mathrm{s}}\right]$$

The Lorentz force on a current-carrying conductor uses the current (Amperes, which is Coulombs per second) multiplied by length in meters, for the same composite units of Coulomb-meters per second:

$$I\vec{l} = \left[\frac{\mathrm{C}}{\mathrm{s}}\right][\mathrm{m}] = \left[\frac{\mathrm{C} \cdot \mathrm{m}}{\mathrm{s}}\right]$$

This dimensional equivalence makes conceptual sense as well: an electrically-charged particle moving through empty space is an electric current in its own right, and an electric current flowing through a conductor is just a collection of charged particles moving through space (just not *empty* space). In either case, the basis for the Lorentz force remains the same: the moving charge(s) create their own magnetic field, which reacts with the magnetic field of the original current-carrying wire to produce forces acting on both.

If the two currents flow in the same direction, their mutual forces *attract*. If the two currents flow in opposite directions, their mutual forces *repel*. This is the basis of electric motors: causing

mechanical motion by electro-magnetic attraction and repulsion. It also represents an interesting contrast with electric fields:

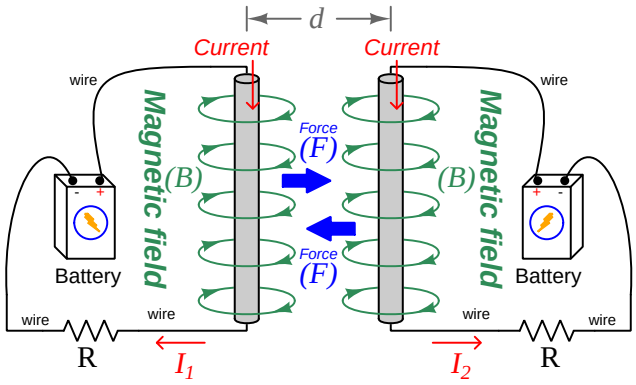
With electric fields, opposite *charges* attract and like *charges* repel.

With magnetic fields, opposite *poles* attract and like *poles* repel.

With parallel currents, opposite *directions* repel and like *directions* attract[3].

Two parallel current-carrying conductors of length l and separated by a distance d will generate a mutual force proportional to both their currents:

$$F = l\frac{\mu I_1 I_2}{2\pi d}$$



[3]That is, assuming it's *like* charges moving in these directions! If the charges in question are opposite each other – for example electrons in one circuit and holes in another – then like directions will repel and opposite directions will attract!

The circular loops surrounding the current-carrying conductors in the previous illustrations represent the magnetic lines of *flux* (Φ_B) surrounding each of those conductors. The magnetic field (B) is related to magnetic flux by area (A), the field being a measurement of how densely-packed those flux lines are per unit area. For this reason, magnetic field (B) is more properly known as *magnetic flux density*:

$$\vec{B} = \frac{\Phi_B}{\vec{A}}$$

Where,

$\vec{B}$ = Magnetic field or flux density (Tesla, Webers per square meter, or Newtons per Ampere-meter)

Φ_B = Magnetic flux (Webers)

$\vec{A}$ = Area over which flux is distributed (square meters)

An older unit of measurement for magnetic flux density B is the *Gauss* which is much smaller than a Tesla, with one Tesla equivalent to 10,000 Gauss. To put things into perspective, the Earth's natural magnetic field has a strength of approximately one-half of one Gauss[4].

Magnetic field strength is an inverse function of distance from any current-carrying wire, and also depends on the magnetic permeability of the space adjacent to the wire:

$$B = \frac{\mu I}{2\pi d}$$

Where,

B = Magnetic field or flux density (Tesla, Webers per square meter, or Newtons per Ampere-meter)

μ = Magnetic permeability of the surrounding space (Tesla-meters per Ampere, $4\pi \times 10^{-7}$ for empty space)

I = Current (Amperes)

d = Distance from conductor (meters)

[4]Using the online *Magnetic Field Calculator* application provided by NOAA (the National Oceanic and Atmospheric Administration) at `https://ngdc.noaa.gov/geomag/calculators/magcalc.shtml#igrfwmm`, applying the World Magnetic Model WMM modeling algorithm for years 2019-2024, the total magnetic field strength at my home is 53,584.4 nano-Tesla (53,584.4 nT or 0.535844 Gauss), and presently (May 2020) decaying at a rate of -104.1 nT per year.

The relation of magnetic *flux* to current through a conductor follows a similar equation:

$$\Phi = \frac{\mu A I}{2\pi d}$$

Where,

Φ = Magnetic flux (Webers)

μ = Magnetic permeability of the surrounding space (Tesla-meters per Ampere, $4\pi \times 10^{-7}$ for empty space)

A = Area over which flux is distributed (square meters)

I = Current (Amperes)

d = Distance from conductor (meters)

As this equation makes clear, the amount of magnetic flux surrounding a current-carrying conductor depends not only on the amount of current, but also on the sampled area, the distance from the wire, and also the surrounding material. Most[5] substances (gas, liquid, solid) have permeability values greater than that of empty space, and so this means magnetic flux is usually *enhanced* by the presence of matter around the current-carrying conductor.

The total magnetic flux enclosed by a circular wire loop follows a similar equation:

$$\Phi = \frac{\pi \mu I r}{2}$$

Where,

Φ = Magnetic flux (Webers)

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r = Radius of circular loop (meters)

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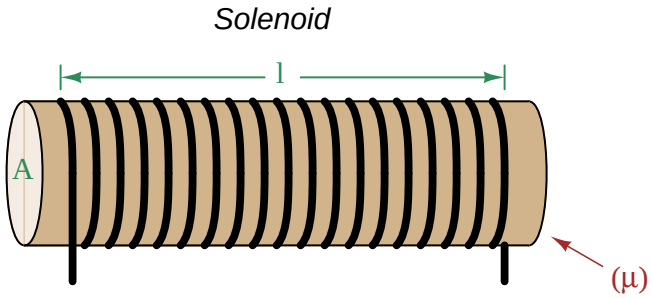

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N = Number of turns of wire in the coil

A = Cross-sectional area of solenoid coil (square meters)

I = Current (Amperes)

l = Length of solenoid coil (meters)

These formulae have interesting implications for solenoid design. Note how a shorter (i.e. smaller length l) solenoid identical in all other respects will generate a stronger magnetic field for a given current. Note also how the flux density (B) remains constant with increasing cross-sectional area (A) if all other factors are equal, and that this necessarily means a greater amount of total magnetic flux (Φ) for a greater area A.

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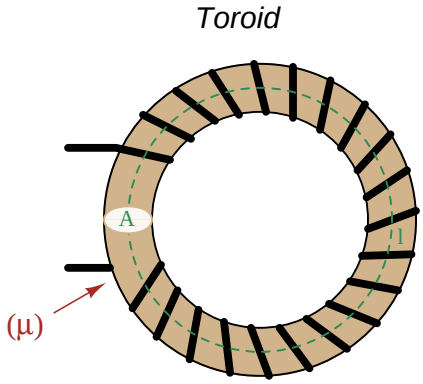

The amount of magnetic flux, and the flux density, within the interior of a current-carrying toroid are identical to that within an otherwise identical otherwise identical solenoid having a length (l) equal to the toroid's circumference:

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l = Circumference of toroid (meters)

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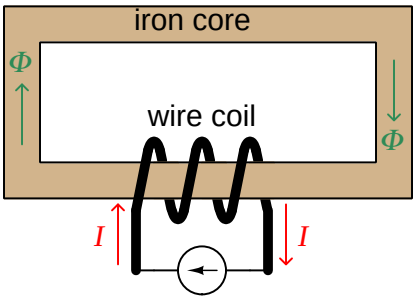

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However, magnetomotive force alone does not fully describe the current's effect on magnetism within the iron core. The total length of the magnetic circuit is also an important factor, since a longer path distributes that magnetomotive force over a greater distance. The quotient of magnetomotive force and magnetic circuit length is called *magnetic field intensity*, symbolized by the variable H and expressed in units of *Ampere-turns per meter*.

Magnetic permeability (μ) relates magnetic field intensity (H) to the magnetic flux density (B) within the core material, such that a greater permeability will result in higher flux density for any given amount of field intensity. Permeability is a property of the core material and not its geometry, mathematically defined as the ratio of flux density to field intensity: $\mu = \frac{B}{H}$

Magnetic reluctance ($\Re$) relates magnetomotive force (mmf) to magnetic flux (Φ), and is related not only to the core material's permeability but also its geometry. It is mathematically defined as the ratio of magnetomotive force to magnetic flux: $\Re = \frac{\text{mmf}}{\Phi}$

If all this seems confusing, you are in good company. Not only are there many magnetic variables, some related to physical geometry and others not, but there are *two different sets of metric units* appropriate for expressing each! The older units were based on the centimeter-gram-second (CGS) version of the metric system, while the newer units are based on the meter-kilogram-second or SI (Système International) version of the metric system.

Quantity	**Symbol**	**SI unit**	**CGS unit**
Magnetomotive force	mmf	Ampere-turn (A-t)	Gilbert (Gb)
Flux	Φ	Weber (Wb)	Maxwell (Mx)
Field intensity	H	Ampere-turns per meter (A-t/m)	Oersted (Oe)
Flux density	B	Tesla (T)	Gauss (G)
Permeability	μ	Tesla-meters per Ampere-turn (T-m/A-t)	Gauss per Oersted (G/Oe)
Reluctance	$\Re$	Ampere-turns per Weber (A-t/Wb)	Gilberts per Maxwell (G/Mx)

Magnetomotive force (mmf) and magnetic flux (Φ) may be thought of as the "raw" measures of magnetism, with Ampere-turns and Webers being their respective SI metric units. Reluctance ($\Re$) is the ratio of the two for any given magnetic circuit with known dimensions and core material. Simply put, reluctance tells you how many Ampere-turns of magnetomotive force will be necessary to create one Weber of magnetic flux in a given space.

Magnetic field intensity (H) and magnetic flux density (B) may be thought of as the "normalized" measures of magnetism, with Ampere-turns per meter and Tesla being their respective SI metric units. H and B relate to mmf and flux by the physical dimensions of the magnetic circuit (length and cross-sectional area, respectively). Permeability is the ratio of the two for any given magnetic core material. Simply put, permeability tells you how many Tesla of magnetic field (i.e. flux density, or Webers of flux per square meter or cross-sectional core area) you will obtain for one Ampere-turn per meter of magnetic field intensity applied to a given core material.

Conversion between the newer SI and the older CGS metric units are as follows:

Quantity	**Conversion equivalence**
Magnetomotive force (mmf)	1 Ampere-turn = $\frac{4\pi}{10}$ Gilberts
Magnetic flux (Φ)	1 Weber = 10^8 Maxwells
Magnetic field intensity (H)	1 Ampere-turn/meter = $\frac{4\pi}{1000}$ Oersteds
Magnetic flux density (B)	1 Tesla = 10^4 Gauss
Permeability (μ)	1 Tesla-meter/Ampere-turn = $\frac{10^7}{4\pi}$ Gauss/Oersteds
Reluctance ($\Re$)	1 Ampere-turn/Weber = $\frac{4\pi}{10^9}$ Gilberts/Maxwell

Chapter 5

Questions

This learning module, along with all others in the ModEL collection, is designed to be used in an inverted instructional environment where students independently read[1] the tutorials and attempt to answer questions on their own *prior* to the instructor's interaction with them. In place of lecture[2], the instructor engages with students in Socratic-style dialogue, probing and challenging their understanding of the subject matter through inquiry.

Answers are not provided for questions within this chapter, and this is by design. Solved problems may be found in the Tutorial and Derivation chapters, instead. The goal here is *independence*, and this requires students to be challenged in ways where others cannot think for them. Remember that you always have the tools of *experimentation* and *computer simulation* (e.g. SPICE) to explore concepts!

The following lists contain ideas for Socratic-style questions and challenges. Upon inspection, one will notice a strong theme of *metacognition* within these statements: they are designed to foster a regular habit of examining one's own thoughts as a means toward clearer thinking. As such these sample questions are useful both for instructor-led discussions as well as for self-study.

[1] Technical reading is an essential academic skill for any technical practitioner to possess for the simple reason that the most comprehensive, accurate, and useful information to be found for developing technical competence is in textual form. Technical careers in general are characterized by the need for continuous learning to remain current with standards and technology, and therefore any technical practitioner who cannot read well is handicapped in their professional development. An excellent resource for educators on improving students' reading prowess through intentional effort and strategy is the book textitReading For Understanding – How Reading Apprenticeship Improves Disciplinary Learning in Secondary and College Classrooms by Ruth Schoenbach, Cynthia Greenleaf, and Lynn Murphy.

[2] Lecture is popular as a teaching method because it is easy to implement: any reasonably articulate subject matter expert can talk to students, even with little preparation. However, it is also quite problematic. A good lecture always makes complicated concepts seem easier than they are, which is bad for students because it instills a false sense of confidence in their own understanding; reading and re-articulation requires more cognitive effort and serves to verify comprehension. A culture of teaching-by-lecture fosters a debilitating dependence upon direct personal instruction, whereas the challenges of modern life demand independent and critical thought made possible only by gathering information and perspectives from afar. Information presented in a lecture is ephemeral, easily lost to failures of memory and dictation; text is forever, and may be referenced at any time.

General challenges following tutorial reading

- Summarize as much of the text as you can in one paragraph of your own words. A helpful strategy is to explain ideas as you would for an intelligent child: as simple as you can without compromising too much accuracy.
- Simplify a particular section of the text, for example a paragraph or even a single sentence, so as to capture the same fundamental idea in fewer words.
- Where did the text make the most sense to you? What was it about the text's presentation that made it clear?
- Identify where it might be easy for someone to misunderstand the text, and explain why you think it could be confusing.
- Identify any new concept(s) presented in the text, and explain in your own words.
- Identify any familiar concept(s) such as physical laws or principles applied or referenced in the text.
- Devise a proof of concept experiment demonstrating an important principle, physical law, or technical innovation represented in the text.
- Devise an experiment to disprove a plausible misconception.
- Did the text reveal any misconceptions you might have harbored? If so, describe the misconception(s) and the reason(s) why you now know them to be incorrect.
- Describe any useful problem-solving strategies applied in the text.
- Devise a question of your own to challenge a reader's comprehension of the text.

General follow-up challenges for assigned problems

- Identify where any fundamental laws or principles apply to the solution of this problem, especially before applying any mathematical techniques.
- Devise a thought experiment to explore the characteristics of the problem scenario, applying known laws and principles to mentally model its behavior.
- Describe in detail your own strategy for solving this problem. How did you identify and organized the given information? Did you sketch any diagrams to help frame the problem?
- Is there more than one way to solve this problem? Which method seems best to you?
- Show the work you did in solving this problem, even if the solution is incomplete or incorrect.
- What would you say was the most challenging part of this problem, and why was it so?
- Was any important information missing from the problem which you had to research or recall?
- Was there any extraneous information presented within this problem? If so, what was it and why did it not matter?
- Examine someone else's solution to identify where they applied fundamental laws or principles.
- Simplify the problem from its given form and show how to solve this simpler version of it. Examples include eliminating certain variables or conditions, altering values to simpler (usually whole) numbers, applying a limiting case (i.e. altering a variable to some extreme or ultimate value).
- For quantitative problems, identify the real-world meaning of all intermediate calculations: their units of measurement, where they fit into the scenario at hand. Annotate any diagrams or illustrations with these calculated values.
- For quantitative problems, try approaching it qualitatively instead, thinking in terms of "increase" and "decrease" rather than definite values.
- For qualitative problems, try approaching it quantitatively instead, proposing simple numerical values for the variables.
- Were there any assumptions you made while solving this problem? Would your solution change if one of those assumptions were altered?
- Identify where it would be easy for someone to go astray in attempting to solve this problem.
- Formulate your own problem based on what you learned solving this one.

General follow-up challenges for experiments or projects

- In what way(s) was this experiment or project easy to complete?
- Identify some of the challenges you faced in completing this experiment or project.

- Show how thorough documentation assisted in the completion of this experiment or project.
- Which fundamental laws or principles are key to this system's function?
- Identify any way(s) in which one might obtain false or otherwise misleading measurements from test equipment in this system.
- What will happen if (component X) fails (open/shorted/etc.)?
- What would have to occur to make this system unsafe?

5.1 Conceptual reasoning

These questions are designed to stimulate your analytic and synthetic thinking[3]. In a Socratic discussion with your instructor, the goal is for these questions to prompt an extended dialogue where assumptions are revealed, conclusions are tested, and understanding is sharpened. Your instructor may also pose additional questions based on those assigned, in order to further probe and refine your conceptual understanding.

Questions that follow are presented to challenge and probe your understanding of various concepts presented in the tutorial. These questions are intended to serve as a guide for the Socratic dialogue between yourself and the instructor. Your instructor's task is to ensure you have a sound grasp of these concepts, and the questions contained in this document are merely a means to this end. Your instructor may, at his or her discretion, alter or substitute questions for the benefit of tailoring the discussion to each student's needs. The only absolute requirement is that each student is challenged and assessed at a level equal to or greater than that represented by the documented questions.

It is far more important that you convey your *reasoning* than it is to simply convey a correct answer. For this reason, you should refrain from researching other information sources to answer questions. What matters here is that *you* are doing the thinking. If the answer is incorrect, your instructor will work with you to correct it through proper reasoning. A correct answer without an adequate explanation of how you derived that answer is unacceptable, as it does not aid the learning or assessment process.

You will note a conspicuous lack of answers given for these conceptual questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your conceptual answers, where applicable, is to use circuit simulation software to explore the effects of changes made to circuits. For example, if one of these conceptual questions challenges you to predict the effects of altering some component parameter in a circuit, you may check the validity of your work by simulating that same parameter change within software and seeing if the results agree.

[3] *Analytical* thinking involves the "disassembly" of an idea into its constituent parts, analogous to dissection. *Synthetic* thinking involves the "assembly" of a new idea comprised of multiple concepts, analogous to construction. Both activities are high-level cognitive skills, extremely important for effective problem-solving, necessitating frequent challenge and regular practice to fully develop.

5.1.1 Reading outline and reflections

"Reading maketh a full man; conference a ready man; and writing an exact man" – Francis Bacon

Francis Bacon's advice is a blueprint for effective education: reading provides the learner with knowledge, writing focuses the learner's thoughts, and critical dialogue equips the learner to confidently communicate and apply their learning. Independent acquisition and application of knowledge is a powerful skill, well worth the effort to cultivate. To this end, students should read these educational resources closely, journal their own reflections on the reading, and discuss in detail their findings with classmates and instructor(s). You should be able to do all of the following after reading any instructional text:

☑ Briefly SUMMARIZE THE TEXT in the form of a journal entry documenting your learning as you progress through the course of study. Share this summary in dialogue with your classmates and instructor. Journaling is an excellent self-test of thorough reading because you cannot clearly express what you have not read or did not comprehend.

☑ Demonstrate ACTIVE READING STRATEGIES, including verbalizing your impressions as you read, simplifying long passages to convey the same ideas using fewer words, annotating text and illustrations with your own interpretations, working through mathematical examples shown in the text, cross-referencing passages with relevant illustrations and/or other passages, identifying problem-solving strategies applied by the author, etc. Technical reading is a special case of problem-solving, and so these strategies work precisely because they help solve any problem: paying attention to your own thoughts (metacognition), eliminating unnecessary complexities, identifying what makes sense, paying close attention to details, drawing connections between separated facts, and noting the successful strategies of others.

☑ Identify IMPORTANT THEMES, especially GENERAL LAWS and PRINCIPLES, expounded in the text and express them in the simplest of terms as though you were teaching an intelligent child. This emphasizes connections between related topics and develops your ability to communicate complex ideas to anyone.

☑ Form YOUR OWN QUESTIONS based on the reading, and then pose them to your instructor and classmates for their consideration. Anticipate both correct and incorrect answers, the incorrect answer(s) assuming one or more plausible misconceptions. This helps you view the subject from different perspectives to grasp it more fully.

☑ Devise EXPERIMENTS to test claims presented in the reading, or to disprove misconceptions. Predict possible outcomes of these experiments, and evaluate their meanings: what result(s) would confirm, and what would constitute disproof? Running mental simulations and evaluating results is essential to scientific and diagnostic reasoning.

☑ Specifically identify any points you found CONFUSING. The reason for doing this is to help diagnose misconceptions and overcome barriers to learning.

5.1.2 Foundational concepts

Correct analysis and diagnosis of electric circuits begins with a proper understanding of some basic concepts. The following is a list of some important concepts referenced in this module's full tutorial. Define each of them in your own words, and be prepared to illustrate each of these concepts with a description of a practical example and/or a live demonstration.

Energy

Simplification as a problem-solving strategy

Thought experiments as a problem-solving strategy

Electrical source

Electrical load

Ohm's Law

Lenz's Law

Kirchhoff's Voltage Law

Inrush current

Magnetic field

Right-hand rule

Ferromagnetism

Electromagnetism

Electromagnetic induction

Counter-EMF

Transistor

Hall effect

Lorentz force

Direct Current (DC)

Alternating Current (AC)

Commutation

Brush

Properties of series circuits

Properties of parallel circuits

5.1.3 Dynamic braking experiment

A student is experimenting with a small PM motor one day, and discovers if they short-circuit the motor's terminals with a piece of wire the shaft suddenly becomes very difficult to turn. Explain why this phenomenon – called *dynamic braking* – happens.

Challenges

- What will happen if we connect a resistor across the motor terminals rather than a direct short?
- If we couple the motor shaft to some source of mechanical energy (e.g. engine, person turning a hand crank, wind turbine, etc.), where does energy go as the motor "brakes" the prime mover's effort?

5.1.4 Coil spring solenoid

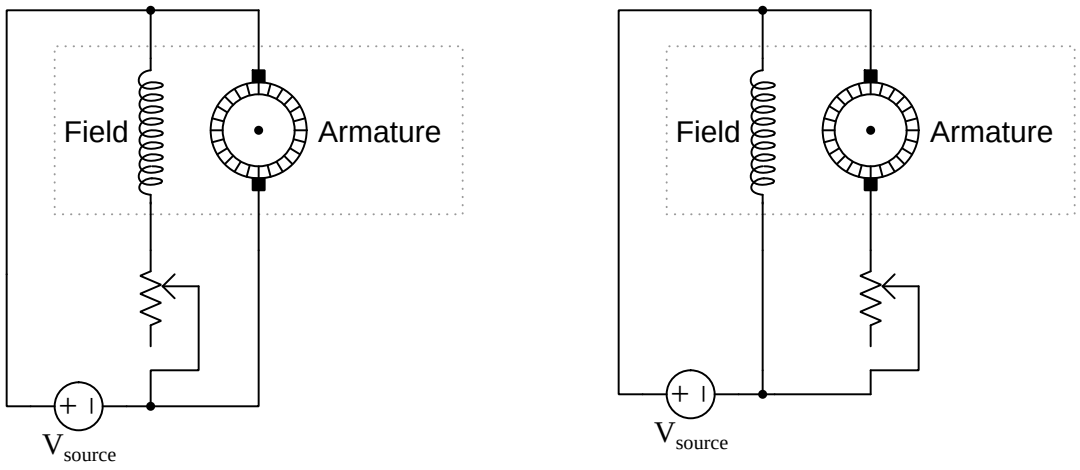
Suppose we bent a copper wire into the shape of a coil spring. Will this coil tend to expand, collapse, or experience no inter-loop forces at all if a large amount of current passes through it?

Challenges

- What will happen if current reverses direction?

5.1.5 Shunt motor speed control

Compare these two methods of controlling motor speed:



Are they equivalent? Does the motor's speed respond similarly to increases in resistance for both cases? Which might be the most practical speed-control method?

Challenges

- A more modern method of DC motor speed control relies on transistor switching elements to rapidly interrupt current to a winding, rather than inserting series resistance with a winding. The "duty cycle" of this pulsing (i.e. on-time versus total on+off time for each pulse cycle) determines the percentage of excitation for that winding. Why do you suppose this is the preferred method of speed control for a motor, rather than a variable resistance?

5.2 Quantitative reasoning

These questions are designed to stimulate your computational thinking. In a Socratic discussion with your instructor, the goal is for these questions to reveal your mathematical approach(es) to problem-solving so that good technique and sound reasoning may be reinforced. Your instructor may also pose additional questions based on those assigned, in order to observe your problem-solving firsthand.

Mental arithmetic and estimations are strongly encouraged for all calculations, because without these abilities you will be unable to readily detect errors caused by calculator misuse (e.g. keystroke errors).

You will note a conspicuous lack of answers given for these quantitative questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. My advice is to use circuit simulation software such as SPICE to check the correctness of quantitative answers. Refer to those learning modules within this collection focusing on SPICE to see worked examples which you may use directly as practice problems for your own study, and/or as templates you may modify to run your own analyses and generate your own practice problems.

Completely worked example problems found in the Tutorial may also serve as "test cases[4]" for gaining proficiency in the use of circuit simulation software, and then once that proficiency is gained you will never need to rely[5] on an answer key!

[4] In other words, set up the circuit simulation software to analyze the same circuit examples found in the Tutorial. If the simulated results match the answers shown in the Tutorial, it confirms the simulation has properly run. If the simulated results disagree with the Tutorial's answers, something has been set up incorrectly in the simulation software. Using every Tutorial as practice in this way will quickly develop proficiency in the use of circuit simulation software.

[5] This approach is perfectly in keeping with the instructional philosophy of these learning modules: *teaching students to be self-sufficient thinkers*. Answer keys can be useful, but it is even more useful to your long-term success to have a set of tools on hand for checking your own work, because once you have left school and are on your own, there will no longer be "answer keys" available for the problems you will have to solve.

5.2.1 Miscellaneous physical constants

Note: constants shown in **bold** type are *exact*, not approximations. Values inside of parentheses show one standard deviation (σ) of uncertainty in the final digits: for example, the magnetic permeability of free space value given as $1.25663706212(19) \times 10^{-6}$ H/m represents a center value (i.e. the location parameter) of $1.25663706212 \times 10^{-6}$ Henrys per meter with one standard deviation of uncertainty equal to $0.0000000000019 \times 10^{-6}$ Henrys per meter.

Avogadro's number (N_A) = **6.02214076** $\times 10^{23}$ **per mole** (mol^{-1})

Boltzmann's constant (k) = **1.380649** $\times 10^{-23}$ **Joules per Kelvin** (J/K)

Electronic charge (e) = **1.602176634** $\times 10^{-19}$ **Coulomb** (C)

Faraday constant (F) = **96,485.33212...** $\times 10^{4}$ **Coulombs per mole** (C/mol)

Magnetic permeability of free space (μ_0) = $1.25663706212(19) \times 10^{-6}$ Henrys per meter (H/m)

Electric permittivity of free space (ϵ_0) = $8.8541878128(13) \times 10^{-12}$ Farads per meter (F/m)

Characteristic impedance of free space (Z_0) = 376.730313668(57) Ohms (Ω)

Gravitational constant (G) = $6.67430(15) \times 10^{-11}$ cubic meters per kilogram-seconds squared ($m^3/kg\text{-}s^2$)

Molar gas constant (R) = **8.314462618... Joules per mole-Kelvin** (J/mol-K) = 0.08205746(14) liters-atmospheres per mole-Kelvin

Planck constant (h) = **6.62607015** $\times 10^{-34}$ **joule-seconds** (J-s)

Stefan-Boltzmann constant (σ) = **5.670374419...** $\times 10^{-8}$ **Watts per square meter-Kelvin**^{4} ($W/m^2 \cdot K^4$)

Speed of light in a vacuum (c) = **299,792,458 meters per second** (m/s) = 186282.4 miles per second (mi/s)

Note: All constants taken from NIST data "Fundamental Physical Constants – Complete Listing", from `http://physics.nist.gov/constants`, National Institute of Standards and Technology (NIST), 2018 CODATA Adjustment.

5.2.2 Introduction to spreadsheets

A powerful computational tool you are encouraged to use in your work is a *spreadsheet*. Available on most personal computers (e.g. Microsoft Excel), *spreadsheet* software performs numerical calculations based on number values and formulae entered into cells of a grid. This grid is typically arranged as lettered columns and numbered rows, with each cell of the grid identified by its column/row coordinates (e.g. cell `B3`, cell `A8`). Each cell may contain a string of text, a number value, or a mathematical formula. The spreadsheet automatically updates the results of all mathematical formulae whenever the entered number values are changed. This means it is possible to set up a spreadsheet to perform a series of calculations on entered data, and those calculations will be re-done by the computer any time the data points are edited in any way.

For example, the following spreadsheet calculates average speed based on entered values of distance traveled and time elapsed:

	A	B	C	D
1	Distance traveled	46.9	Kilometers	
2	Time elapsed	1.18	Hours	
3	Average speed	= B1 / B2	km/h	
4				
5				

Text labels contained in cells `A1` through `A3` and cells `C1` through `C3` exist solely for readability and are not involved in any calculations. Cell `B1` contains a sample distance value while cell `B2` contains a sample time value. The formula for computing speed is contained in cell `B3`. Note how this formula begins with an "equals" symbol (`=`), references the values for distance and speed by lettered column and numbered row coordinates (`B1` and `B2`), and uses a forward slash symbol for division (`/`). The coordinates `B1` and `B2` function as *variables*[6] would in an algebraic formula.

When this spreadsheet is executed, the numerical value `39.74576` will appear in cell `B3` rather than the formula `= B1 / B2`, because 39.74576 is the computed speed value given 46.9 kilometers traveled over a period of 1.18 hours. If a different numerical value for distance is entered into cell `B1` or a different value for time is entered into cell `B2`, cell `B3`'s value will automatically update. All you need to do is set up the given values and any formulae into the spreadsheet, and the computer will do all the calculations for you.

Cell `B3` may be referenced by other formulae in the spreadsheet if desired, since it is a variable just like the given values contained in `B1` and `B2`. This means it is possible to set up an entire chain of calculations, one dependent on the result of another, in order to arrive at a final value. The arrangement of the given data and formulae need not follow any pattern on the grid, which means you may place them anywhere.

[6] Spreadsheets may also provide means to attach text labels to cells for use as variable names (Microsoft Excel simply calls these labels "names"), but for simple spreadsheets such as those shown here it's usually easier just to use the standard coordinate naming for each cell.

Common[7] arithmetic operations available for your use in a spreadsheet include the following:

- Addition (`+`)
- Subtraction (`-`)
- Multiplication (`*`)
- Division (`/`)
- Powers (`^`)
- Square roots (`sqrt()`)
- Logarithms (`ln()` , `log10()`)

Parentheses may be used to ensure[8] proper order of operations within a complex formula. Consider this example of a spreadsheet implementing the *quadratic formula*, used to solve for roots of a polynomial expression in the form of $ax^2 + bx + c$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

	A	B
1	x_1	= (-B4 + sqrt((B4^2) - (4*B3*B5))) / (2*B3)
2	x_2	= (-B4 - sqrt((B4^2) - (4*B3*B5))) / (2*B3)
3	a =	9
4	b =	5
5	c =	-2

This example is configured to compute roots[9] of the polynomial $9x^2 + 5x - 2$ because the values of 9, 5, and -2 have been inserted into cells `B3`, `B4`, and `B5`, respectively. Once this spreadsheet has been built, though, it may be used to calculate the roots of *any* second-degree polynomial expression simply by entering the new a, b, and c coefficients into cells `B3` through `B5`. The numerical values appearing in cells `B1` and `B2` will be automatically updated by the computer immediately following any changes made to the coefficients.

[7] Modern spreadsheet software offers a bewildering array of mathematical functions you may use in your computations. I recommend you consult the documentation for your particular spreadsheet for information on operations other than those listed here.

[8] Spreadsheet programs, like text-based programming languages, are designed to follow standard order of operations by default. However, my personal preference is to use parentheses even where strictly unnecessary just to make it clear to any other person viewing the formula what the intended order of operations is.

[9] Reviewing some algebra here, a *root* is a value for x that yields an overall value of zero for the polynomial. For this polynomial ($9x^2 + 5x - 2$) the two roots happen to be $x = 0.269381$ and $x = -0.82494$, with these values displayed in cells `B1` and `B2`, respectively upon execution of the spreadsheet.

Alternatively, one could break up the long quadratic formula into smaller pieces like this:

$$y = \sqrt{b^2 - 4ac} \qquad z = 2a$$

$$x = \frac{-b \pm y}{z}$$

	A	B	C
1	x_1	= (-B4 + C1) / C2	= sqrt((B4^2) - (4*B3*B5))
2	x_2	= (-B4 - C1) / C2	= 2*B3
3	a =	9	
4	b =	5	
5	c =	-2	

Note how the square-root term (y) is calculated in cell C1, and the denominator term (z) in cell C2. This makes the two final formulae (in cells B1 and B2) simpler to interpret. The positioning of all these cells on the grid is completely arbitrary[10] – all that matters is that they properly reference each other in the formulae.

Spreadsheets are particularly useful for situations where the same set of calculations representing a circuit or other system must be repeated for different initial conditions. The power of a spreadsheet is that it automates what would otherwise be a tedious set of calculations. One specific application of this is to simulate the effects of various components within a circuit failing with abnormal values (e.g. a shorted resistor simulated by making its value nearly zero; an open resistor simulated by making its value extremely large). Another application is analyzing the behavior of a circuit design given new components that are out of specification, and/or aging components experiencing drift over time.

[10] My personal preference is to locate all the "given" data in the upper-left cells of the spreadsheet grid (each data point flanked by a sensible name in the cell to the left and units of measurement in the cell to the right as illustrated in the first distance/time spreadsheet example), sometimes coloring them in order to clearly distinguish which cells contain entered data versus which cells contain computed results from formulae. I like to place all formulae in cells below the given data, and try to arrange them in logical order so that anyone examining my spreadsheet will be able to figure out *how* I constructed a solution. This is a general principle I believe all computer programmers should follow: *document and arrange your code to make it easy for other people to learn from it.*

5.2.3 Calculating inrush current and counter-EMF

Suppose a permanent-magnet DC motor has a measured internal resistance of 2.8 Ω, including the resistance of its armature winding, brushes and brush contact with the armature, etc. When initially connected to a source it will draw a sudden "inrush" current that decreases as motor speed increases. Calculate the magnitude of this inrush current, assuming the source voltage is 18 Volts.

Next, determine the amount of counter-EMF generated by this motor as it spins at 5700 RPM (revolutions per minute) while drawing 230 milliAmperes from that same 18 Volt DC source.

Challenges

- Calculate the speed at which this motor will draw zero current.

5.3 Diagnostic reasoning

These questions are designed to stimulate your deductive and inductive thinking, where you must apply general principles to specific scenarios (deductive) and also derive conclusions about the failed circuit from specific details (inductive). In a Socratic discussion with your instructor, the goal is for these questions to reinforce your recall and use of general circuit principles and also challenge your ability to integrate multiple symptoms into a sensible explanation of what's wrong in a circuit. Your instructor may also pose additional questions based on those assigned, in order to further challenge and sharpen your diagnostic abilities.

As always, your goal is to fully *explain* your analysis of each problem. Simply obtaining a correct answer is not good enough – you must also demonstrate sound reasoning in order to successfully complete the assignment. Your instructor's responsibility is to probe and challenge your understanding of the relevant principles and analytical processes in order to ensure you have a strong foundation upon which to build further understanding.

You will note a conspicuous lack of answers given for these diagnostic questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your diagnostic answers, where applicable, is to use circuit simulation software to explore the effects of faults placed in circuits. For example, if one of these diagnostic questions requires that you predict the effect of an open or a short in a circuit, you may check the validity of your work by simulating that same fault (substituting a very high resistance in place of that component for an open, and substituting a very low resistance for a short) within software and seeing if the results agree.

5.3.1 Failed-open field winding

Compare and contrast the effects of the field winding failing open for each of these three DC motor types:

- Series-wound
- Shunt-wound
- Compound-wound

Explain *why* in each case.

Challenges

- If you could attach a speed switch the motor's shaft that actuates with excessive speed, how would you connect that switch's contacts to the motor to serve as a safety device for the motor?
- If you could attach a speed switch the motor's shaft that actuates with insufficient speed, how would you connect that switch's contacts to the motor to serve as a safety device for the motor?

Appendix A

Problem-Solving Strategies

The ability to solve complex problems is arguably one of the most valuable skills one can possess, and this skill is particularly important in any science-based discipline.

- Study principles, not procedures. Don't be satisfied with merely knowing how to compute solutions – learn *why* those solutions work.
- Identify what it is you need to solve, identify all relevant data, identify all units of measurement, identify any general principles or formulae linking the given information to the solution, and then identify any "missing pieces" to a solution. Annotate all diagrams with this data.
- Sketch a diagram to help visualize the problem. When building a real system, always devise a plan for that system and analyze its function *before* constructing it.
- Follow the units of measurement and meaning of every calculation. If you are ever performing mathematical calculations as part of a problem-solving procedure, and you find yourself unable to apply each and every intermediate result to some aspect of the problem, it means you don't understand what you are doing. Properly done, every mathematical result should have practical meaning for the problem, and not just be an abstract number. You should be able to identify the proper units of measurement for each and every calculated result, and show where that result fits into the problem.
- Perform "thought experiments" to explore the effects of different conditions for theoretical problems. When troubleshooting real systems, perform *diagnostic tests* rather than visually inspecting for faults, the best diagnostic test being the one giving you the most information about the nature and/or location of the fault with the fewest steps.
- Simplify the problem until the solution becomes obvious, and then use that obvious case as a model to follow in solving the more complex version of the problem.
- Check for exceptions to see if your solution is incorrect or incomplete. A good solution will work for *all* known conditions and criteria. A good example of this is the process of testing scientific hypotheses: the task of a scientist is not to find support for a new idea, but rather to *challenge* that new idea to see if it holds up under a battery of tests. The philosophical

principle of *reductio ad absurdum* (i.e. disproving a general idea by finding a specific case where it fails) is useful here.

- Work "backward" from a hypothetical solution to a new set of given conditions.
- Add quantities to problems that are qualitative in nature, because sometimes a little math helps illuminate the scenario.
- Sketch graphs illustrating how variables relate to each other. These may be quantitative (i.e. with realistic number values) or qualitative (i.e. simply showing increases and decreases).
- Treat quantitative problems as qualitative in order to discern the relative magnitudes and/or directions of change of the relevant variables. For example, try determining what happens if a certain variable were to increase or decrease before attempting to precisely calculate quantities: how will each of the dependent variables respond, by increasing, decreasing, or remaining the same as before?
- Consider limiting cases. This works especially well for qualitative problems where you need to determine which direction a variable will change. Take the given condition and magnify that condition to an extreme degree as a way of simplifying the direction of the system's response.
- Check your work. This means regularly testing your conclusions to see if they make sense. This does *not* mean repeating the same steps originally used to obtain the conclusion(s), but rather to use some other means to check validity. Simply repeating procedures often leads to *repeating the same errors* if any were made, which is why alternative paths are better.

Appendix B

Instructional philosophy

B.1 First principles of learning

- **Anyone can learn anything** given appropriate time, effort, resources, challenges, encouragement, and expectations. Dedicating time and investing effort are the student's responsibility; providing resources, challenges, and encouragement are the teacher's responsibility; high expectations are a responsibility shared by both student and teacher.

- **Transfer is not automatic.** The human mind has a natural tendency to compartmentalize information, which means the process of taking knowledge learned in one context and applying it to another usually does not come easy and therefore should never be taken for granted.

- **Learning is iterative.** The human mind rarely learns anything perfectly on the first attempt. Anticipate mistakes and plan for multiple tries to achieve full understanding, using the lessons of those mistakes as feedback to guide future attempts.

- **Information is absorbed, but understanding is created.** Facts and procedures may be memorized easily enough by repeated exposure, but the ability to reliably apply principles to novel scenarios only comes through intense personal effort. This effort is fundamentally creative in nature: casting new concepts in one's own words, running mental experiments to perceive patterns and trends, etc. These acts of making knowledge "one's own" need not be perfect in order to be effective, as the value lies in the activity and not necessarily the finished product. Depth of understanding and permanence of memory are both directly related to mental effort during the learning process – i.e. *easy learning is shallow and fleeting.*

- **Education trumps training.** There is no such thing as an entirely isolated subject, as all fields of knowledge are connected. Training is narrowly-focused and task-oriented. Education is broad-based and principle-oriented. When preparing for a life-long technical career, education beats training every time.

- **Character matters.** Poor habits are more destructive than deficits of knowledge or skill. This is especially true in collective endeavors, where a team's ability to function depends on trust between its members. Simply put, no one wants an untrustworthy person on their team. An essential component of education then, is character development.

- **People learn to be responsible by bearing responsibility.** An irresponsible person is someone who has never *had* to be responsible for anything that mattered enough to them. Just as anyone can learn anything, anyone can become responsible if the personal cost of irresponsibility becomes high enough.

- **What gets measured, gets done.** Accurate and relevant assessment of learning is key to ensuring all students learn. Therefore, it is imperative to measure what matters.

- **Failure is nothing to fear.** Every human being fails, and fails in multiple ways at multiple times. Eventual success only happens when we don't stop trying.

B.2 Proven strategies for instructors

- Assume every student is capable of learning anything they desire given the proper conditions. Treat them as capable adults by granting real responsibility and avoiding artificial incentives such as merit or demerit points.
- Create a consistent culture of high expectations across the entire program of study. Demonstrate and encourage patience, persistence, and a healthy sense of self-skepticism. Anticipate and de-stigmatize error. Teach respect for the capabilities of others as well as respect for one's own fallibility.
- Replace lecture with "inverted" instruction, where students first encounter new concepts through reading and then spend class time in Socratic dialogue with the instructor exploring those concepts and solving problems individually. There is a world of difference between observing someone solve a problem versus actually solving a problem yourself, and so the point of this form of instruction is to place students in a position where they *cannot* passively observe.
- Require students to read extensively, write about what they learn, and dialogue with you and their peers to sharpen their understanding. Apply Francis Bacon's advice that "reading maketh a full man; conference a ready man; and writing an exact man". These are complementary activities helping students expand their confidence and abilities.
- Use artificial intelligence (AI) to challenge student understanding rather than merely provide information. Find productive ways for AI to critique students' clarity of thought and of expression, for example by employing AI as a Socratic-style interlocutor or as a reviewer of students' journals. Properly applied, AI has the ability to expand student access to critical review well outside the bounds of their instructor's reach.
- Build frequent and rapid feedback into the learning process so that students know at all times how well they are learning, to identify problems early and fix them before they grow. Model the intellectual habit of self-assessing and self-correcting your own understanding (i.e. a cognitive *feedback loop*), encouraging students to do the same.
- Use "mastery" as the standard for every assessment, which means the exam or experiment or project must be done with 100% competence in order to pass. Provide students with multiple opportunity for re-tries (different versions of the assessment every time).
- Require students to devise their own hypotheses and procedures on all experiments, so that the process is truly a scientific one. Have students assess their proposed experimental procedures for risk and devise mitigations for those risks. Let nothing be pre-designed about students' experiments other than a stated task (i.e. what principle the experiment shall test) at the start and a set of demonstrable knowledge and skill objectives at the end.
- Have students build as much of their lab equipment as possible: building power sources, building test assemblies[1], and building complete working systems (no kits!). In order to provide

[1] In the program I teach, every student builds their own "Development Board" consisting of a metal chassis with DIN rail, terminal blocks, and an AC-DC power supply of their own making which functions as a portable lab environment they can use at school as well as take home.

this same "ground-up" experience for every new student, this means either previous students take their creations with them, or the systems get disassembled in preparation for the new students, or the systems grow and evolve with each new student group.

- Incorporate external accountability for you and for your students, continuously improving the curriculum and your instructional methods based on proven results. Have students regularly network with active professionals through participation in advisory committee meetings, service projects, tours, jobshadows, internships, etc. Practical suggestions include requiring students to design and build projects for external clients (e.g. community groups, businesses, different departments within the institution), and also requiring students attend all technical advisory committee meetings and to dialogue with the industry representatives at those meetings.
- Repeatedly explore difficult-to-learn concepts across multiple courses, so that students have multiple opportunities to build their understanding.
- Relate all new concepts, whenever possible, to previous concepts and to relevant physical laws. Challenge each and every student, every day, to *reason* from concept to concept and to explain the logical connections between. Challenge students to verify their conclusions by multiple approaches (e.g. double-checking their work using different methods). Ask *"Why?"* often.
- Maintain detailed records on each student's performance and share these records privately with them. These records should include academic performance as well as professionally relevant behavioral tendencies.
- Hold mandatory "check-in" meetings between all program faculty and each new student during their first term. Offer these to all other students as an option, except for any students continuing to manifest unprofessional behaviors, poor academic performance, or who have some other need for a face-to-face meeting with faculty.
- Address problems while they are small, before they grow larger. This is equally true for tutoring technical concepts as it is for helping students build professional habits.
- Build rigorous quality control into the curriculum to ensure every student masters every important concept, and that the mastery is retained over time. This includes (1) review questions added to every exam to re-assess knowledge taught in previous terms, (2) cumulative exams at the end of every term to re-assess all important concepts back to the very beginning of the program, and (3) review assessments in practical (hands-on) coursework to ensure critically-important skills were indeed taught and are still retained. What you will find by doing this is that it actually boosts retention of students by ensuring that important knowledge gets taught and is retained over long spans of time. In the absence of such quality control, student learning and retention tends to be spotty and this contributes to drop-out and failure rates later in their education.
- Finally, *never rush learning.* Education is not a race. Give your students ample time to digest complex ideas, as you continually remind yourself of just how long it took you to achieve mastery! Long-term retention and the consistently correct application of concepts are always the result of *focused effort over long periods of time* which means there are no shortcuts to learning.

B.3 Proven strategies for students

The single most important piece of advice I have for any student of any subject is to take responsibility for your own development in all areas of life including mental development. Expecting others in your life to entirely guide your own development is a recipe for disappointment. This is just as true for students enrolled in formal learning institutions as it is for auto-didacts pursuing learning entirely on their own. Learning to think in new ways is key to being able to gainfully use information, to make informed decisions about your life, and to best serve those you care about. With this in mind, I offer the following advice to students:

- **Approach all learning as valuable**. No matter what course you take, no matter who you learn from, no matter the subject, there is something useful in every learning experience. If you don't see the value of every new experience, you are not looking closely enough!
- **Continually challenge yourself**. Let other people take shortcuts and find easy answers to easy problems. The purpose of education is to stretch your mind, in order to shape it into a more powerful tool. This doesn't come by taking the path of least resistance. An excellent analogy for an empowering education is productive physical exercise: becoming stronger, more flexible, and more persistent only comes through intense personal effort.
- **Master the use of language**. This includes reading extensively, writing every day, listening closely, and speaking articulately. To a great extent language channels and empowers thought, so the better you are at wielding language the better you will be at grasping abstract concepts and articulating them not only for your benefit but for others as well.
- **Do not limit yourself to the resources given to you**. Read books that are not on the reading list. Run experiments that aren't assigned to you. Form study groups outside of class. Take an entrepreneurial approach to your own education, as though it were a business you were building for your future benefit.
- **Express and share what you learn**. Take every opportunity to teach what you have learned to others, as this will not only help them but will also strengthen your own understanding[2].
- Realize that **no one can give you understanding**, just as no one can give you physical fitness. These both must be *built*.
- **Above all, recognize that learning is hard work, and that a certain level of frustration is unavoidable**. There are times when you will struggle to grasp some of these concepts, and that struggle is a natural thing. Take heart that it will yield with persistent and varied[3] effort, and never give up! That concepts don't immediately come to you is not a sign of something wrong, but rather of something right: that you have found a worthy challenge!

[2] On a personal note, I was surprised to learn just how much my own understanding of electronics and related subjects was strengthened by becoming a teacher. When you are tasked every day with helping other people grasp complex topics, it catalyzes your own learning by giving you powerful incentives to study, to articulate your thoughts, and to reflect deeply on the process of learning.

[3] As the old saying goes, "Insanity is trying the same thing over and over again, expecting different results." If you find yourself stumped by something in the text, you should attempt a different approach. Alter the thought experiment, change the mathematical parameters, do whatever you can to see the problem in a slightly different light, and then the solution will often present itself more readily.

B.4 Design of these learning modules

"The unexamined circuit is not worth energizing" – Socrates (if he had taught electricity)

These learning modules, although useful for self-study, were designed to be used in a formal learning environment where a subject-matter expert challenges students to digest the content and exercise their critical thinking abilities in the answering of questions and in the construction and testing of working circuits. Every effort has been made to embed the following instructional and assessment philosophies within:

- The first goal of education is to enhance clear and independent thought, in order that every student reach their fullest potential in a highly complex and inter-dependent world. Robust reasoning is *always* more important than particulars of any subject matter, because its application is universal.
- Literacy is fundamental to independent learning and thought because text continues to be the most efficient way to communicate complex ideas over space and time. Those who cannot read with ease are limited in their ability to acquire knowledge and perspective.
- Articulate communication is fundamental to work that is complex and interdisciplinary.
- Faulty assumptions and poor reasoning are best corrected through challenge, not presentation. The rhetorical technique of *reductio ad absurdum* (disproving an assertion by exposing an absurdity) works well to discipline student's minds, not only to correct the problem at hand but also to learn how to detect and correct future errors.
- Important principles should be repeatedly explored and widely applied throughout a course of study, not only to reinforce their importance and help ensure their mastery, but also to showcase the interconnectedness and utility of knowledge.

These learning modules were expressly designed to be used in an "inverted" teaching environment[4] where students first read the introductory and tutorial chapters on their own, then individually attempt to answer the questions and construct working circuits according to the experiment and project guidelines. The instructor never lectures, but instead meets regularly with each individual student to review their progress, answer questions, identify misconceptions, and challenge the student to new depths of understanding through further questioning. Regular meetings between instructor and student should resemble a Socratic[5] dialogue, where questions serve as scalpels to dissect topics and expose assumptions. The student passes each module only after consistently demonstrating their ability to logically analyze and correctly apply all major concepts in each question or project/experiment. The instructor must be vigilant in probing each student's understanding to ensure they are truly *reasoning* and not just *memorizing*. This is why "Challenge" points appear throughout, as prompts for students to think deeper about topics and as starting points for instructor queries. Sometimes these challenge points require additional knowledge that hasn't been covered in the series to answer in full. This is okay, as the major purpose of the Challenges is to stimulate analysis and synthesis on the part of each student.

The instructor must possess enough mastery of the subject matter and awareness of students' reasoning to generate their own follow-up questions to practically any student response. Even completely correct answers given by the student should be challenged by the instructor for the purpose of having students practice articulating their thoughts and defending their reasoning. Conceptual errors committed by the student should be exposed and corrected not by direct instruction, but rather by reducing the errors to an absurdity[6] through well-chosen questions and thought experiments posed by the instructor. Becoming proficient at this style of instruction requires time and dedication, but the positive effects on critical thinking for both student and instructor are spectacular.

An inspection of these learning modules reveals certain unique characteristics. One of these is a bias toward thorough explanations in the tutorial chapters. Without a live instructor to explain concepts and applications to students, the text itself must fulfill this role. This philosophy results in lengthier explanations than what you might typically find in a textbook, each step of the reasoning process fully explained, including footnotes addressing common questions and concerns students raise while learning these concepts. Each tutorial seeks to not only explain each major concept in sufficient detail, but also to explain the logic of each concept and how each may be developed

[4] In a traditional teaching environment, students first encounter new information via *lecture* from an expert, and then independently apply that information via *homework*. In an "inverted" course of study, students first encounter new information via *homework*, and then independently apply that information under the scrutiny of an expert. The expert's role in lecture is to simply *explain*, but the expert's role in an inverted session is to *challenge*, *critique*, and if necessary *explain* where gaps in understanding still exist.

[5] Socrates is a figure in ancient Greek philosophy famous for his unflinching style of questioning. Although he authored no texts, he appears as a character in Plato's many writings. The essence of Socratic philosophy is to leave no question unexamined and no point of view unchallenged. While purists may argue a topic such as electric circuits is too narrow for a true Socratic-style dialogue, I would argue that the essential thought processes involved with scientific reasoning on *any* topic are not far removed from the Socratic ideal, and that students of electricity and electronics would do very well to challenge assumptions, pose thought experiments, identify fallacies, and otherwise employ the arsenal of critical thinking skills modeled by Socrates.

[6] This rhetorical technique is known by the Latin phrase *reductio ad absurdum*. The concept is to expose errors by counter-example, since only one solid counter-example is necessary to disprove a universal claim. As an example of this, consider the common misconception among beginning students of electricity that voltage cannot exist without current. One way to apply *reductio ad absurdum* to this statement is to ask how much current passes through a fully-charged battery connected to nothing (i.e. a clear example of voltage existing without current).

from "first principles". Again, this reflects the goal of developing clear and independent thought in students' minds, by showing how clear and logical thought was used to forge each concept. Students benefit from witnessing a model of clear thinking in action, and these tutorials strive to be just that.

Another feature of these learning modules is that they do not treat topics in isolation. Rather, important concepts are introduced early in the series, and appear repeatedly as stepping-stones toward other concepts in subsequent modules. This helps to avoid the "compartmentalization" of knowledge, demonstrating the inter-connectedness of concepts and simultaneously reinforcing them. Each module is fairly complete in itself, reserving the beginning of its tutorial to a review of foundational concepts.

To high standards of education,

Tony R. Kuphaldt

Appendix C

Tools used

I am indebted to the developers of many open-source software applications in the creation of these learning modules. The following is a list of these applications with some commentary on each.

You will notice a theme common to many of these applications: a bias toward *code*. Although I am by no means an expert programmer in any computer language, I understand and appreciate the flexibility offered by code-based applications where the user (you) enters commands into a plain ASCII text file, which the software then reads and processes to create the final output. Code-based computer applications are by their very nature *extensible*, while WYSIWYG (What You See Is What You Get) applications are generally limited to whatever user interface the developer makes for you.

The `GNU/Linux` computer operating system

> There is so much to be said about Linus Torvalds' `Linux` and Richard Stallman's `GNU` project. First, to credit just these two individuals is to fail to do justice to the *mob* of passionate volunteers who contributed to make this amazing software a reality. I first learned of `Linux` back in 1996, and have been using this operating system on my personal computers almost exclusively since then. It is *free*, it is completely *configurable*, and it permits the continued use of highly efficient `Unix` applications and scripting languages (e.g. shell scripts, Makefiles, `sed`, `awk`) developed over many decades. `Linux` not only provided me with a powerful computing platform, but its open design served to inspire my life's work of creating open-source educational resources.

Bram Moolenaar's `Vim` text editor

> Writing code for any code-based computer application requires a *text editor*, which may be thought of as a word processor strictly limited to outputting plain-ASCII text files. Many good text editors exist, and one's choice of text editor seems to be a deeply personal matter within the programming world. I prefer `Vim` because it operates very similarly to `vi` which is ubiquitous on `Unix`/`Linux` operating systems, and because it may be entirely operated via keyboard (i.e. no mouse required) which makes it fast to use.

Donald Knuth's TEX typesetting system

Developed in the late 1970's and early 1980's by computer scientist extraordinaire Donald Knuth to typeset his multi-volume magnum opus *The Art of Computer Programming*, this software allows the production of formatted text for screen-viewing or paper printing, all by writing plain-text code to describe how the formatted text is supposed to appear. TEX is not just a markup language for documents, but it is also a Turing-complete programming language in and of itself, allowing useful algorithms to be created to control the production of documents. Simply put, TEX *is a programmer's approach to word processing*. Since TEX is controlled by code written in a plain-text file, this means anyone may read that plain-text file to see exactly how the document was created. This openness afforded by the code-based nature of TEX makes it relatively easy to learn how other people have created their own TEX documents. By contrast, examining a beautiful document created in a conventional WYSIWYG word processor such as Microsoft `Word` suggests nothing to the reader about *how* that document was created, or what the user might do to create something similar. As Mr. Knuth himself once quipped, conventional word processing applications should be called WYSIAYG (What You See Is *All* You Get).

Leslie Lamport's LATEX extensions to TEX

Like all true programming languages, TEX is inherently extensible. So, years after the release of TEX to the public, Leslie Lamport decided to create a massive extension allowing easier compilation of book-length documents. The result was LATEX, which is the markup language used to create all ModEL module documents. You could say that TEX is to LATEX as `C` is to `C++`. This means it is permissible to use any and all TEX commands within LATEX source code, and it all still works. Some of the features offered by LATEX that would be challenging to implement in TEX include automatic index and table-of-content creation.

Tim Edwards' `Xcircuit` drafting program

This wonderful program is what I use to create all the schematic diagrams and illustrations (but not photographic images or mathematical plots) throughout the ModEL project. It natively outputs PostScript format which is a true vector graphic format (this is why the images do not pixellate when you zoom in for a closer view), and it is so simple to use that I have never had to read the manual! Object libraries are easy to create for `Xcircuit`, being plain-text files using PostScript programming conventions. Over the years I have collected a large set of object libraries useful for drawing electrical and electronic schematics, pictorial diagrams, and other technical illustrations.

`Gimp` graphic image manipulation program

Essentially an open-source clone of Adobe's `PhotoShop`, I use `Gimp` to resize, crop, and convert file formats for all of the photographic images appearing in the ModEL modules. Although `Gimp` does offer its own scripting language (called `Script-Fu`), I have never had occasion to use it. Thus, my utilization of `Gimp` to merely crop, resize, and convert graphic images is akin to using a sword to slice bread.

`SPICE` circuit simulation program

`SPICE` is to circuit analysis as TEX is to document creation: it is a form of markup language designed to describe a certain object to be processed in plain-ASCII text. When the plain-text "source file" is compiled by the software, it outputs the final result. More modern circuit analysis tools certainly exist, but I prefer `SPICE` for the following reasons: it is *free*, it is *fast*, it is *reliable*, and it is a fantastic tool for *teaching* students of electricity and electronics how to write simple code. I happen to use rather old versions of `SPICE`, version 2g6 being my "go to" application when I only require text-based output. `NGSPICE` (version 26), which is based on Berkeley `SPICE` version 3f5, is used when I require graphical output for such things as time-domain waveforms and Bode plots. In all `SPICE` example netlists I strive to use coding conventions compatible with all `SPICE` versions.

Andrew D. Hwang's `ePiX` mathematical visualization programming library

This amazing project is a `C++` library you may link to any `C/C++` code for the purpose of generating PostScript graphic images of mathematical functions. As a completely free and open-source project, it does all the plotting I would otherwise use a Computer Algebra System (CAS) such as `Mathematica` or `Maple` to do. It should be said that `ePiX` is *not* a Computer Algebra System like `Mathematica` or `Maple`, but merely a mathematical *visualization* tool. In other words, it won't determine integrals for you (you'll have to implement that in your own `C/C++` code!), but it can graph the results, and it does so beautifully. What I really admire about `ePiX` is that it is a `C++` programming library, which means it builds on the existing power and toolset available with that programming language. Mr. Hwang could have probably developed his own stand-alone application for mathematical plotting, but by creating a `C++` library to do the same thing he accomplished something much greater.

`gnuplot` mathematical visualization software

Another open-source tool for mathematical visualization is `gnuplot`. Interestingly, this tool is *not* part of Richard Stallman's GNU project, its name being a coincidence. For this reason the authors prefer "gnu" *not* be capitalized at all to avoid confusion. This is a much "lighter-weight" alternative to a spreadsheet for plotting tabular data, and the fact that it easily outputs directly to an X11 console or a file in a number of different graphical formats (including PostScript) is very helpful. I typically set my `gnuplot` output format to default (X11 on my Linux PC) for quick viewing while I'm developing a visualization, then switch to PostScript file export once the visual is ready to include in the document(s) I'm writing. As with my use of `Gimp` to do rudimentary image editing, my use of `gnuplot` only scratches the surface of its capabilities, but the important points are that it's *free* and that it *works well*.

`Python` programming language

Both Python and C++ find extensive use in these modules as instructional aids and exercises, but I'm listing Python here as a *tool* for myself because I use it almost daily as a *calculator*. If you open a Python interpreter console and type `from math import *` you can type mathematical expressions and have it return results just as you would on a hand calculator. Complex-number (i.e. *phasor*) arithmetic is similarly supported if you include the complex-math library (`from cmath import *`). Examples of this are shown in the Programming References chapter (if included) in each module. Of course, being a fully-featured programming language, Python also supports conditionals, loops, and other structures useful for calculation of quantities. Also, running in a console environment where all entries and returned values show as text in a chronologically-ordered list makes it easy to copy-and-paste those calculations to document exactly how they were performed.

Appendix D

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Appendix E

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Appendix F

Version history

This is a list showing all significant additions, corrections, and other edits made to this learning module. Each entry is referenced by calendar date in reverse chronological order (newest version first), which appears on the front cover of every learning module for easy reference. Any contributors to this open-source document are listed here as well.

24 November 2025 – added index entries for William Lear and Samuel Auld.

5 July 2023 – added Technical Reference sections on magnetic units and also definitions of magnetic north and south.

27 October 2021 – explicitly added a reference to inrush current in the Tutorial.

25 October 2021 – reversed the magnetic polarity on the first two graphical examples in the Tutorial (image_4202 and image_4203) to make them all consistent with each other.

8 May 2021 – commented out or deleted empty chapters.

21 October 2020 – minor additions to the Introduction chapter, and made a small typo correction in the Tutorial.

5 October 2020 – significantly edited the Introduction chapter to make it more suitable as a pre-study guide and to provide cues useful to instructors leading "inverted" teaching sessions.

23-24 July 2020 – wrote more content for the Tutorial chapter.

22 July 2020 – document first created.

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