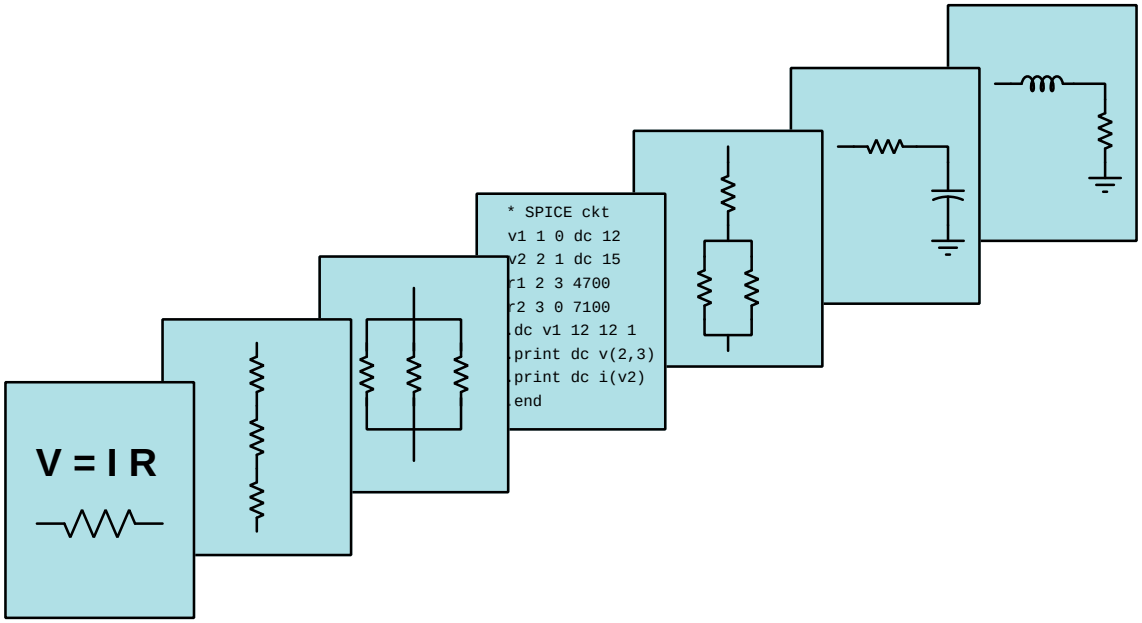
Modular Electronics Learning (ModEL) project



Electric and Magnetic Fields

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Chapter 1

Introduction

1.1 Recommendations for students

This module formally introduces the twin concepts of *electric* and *magnetic* fields, which are the foundation of a great many electrical components and practical devices explored in subsequent modules. Simply put, electric and magnetic fields are invisible "webs" of interaction extending through empty space, causing forces to be exerted on certain types of matter. Electric fields exert direct forces on electrical charges, while magnetic fields exert forces on *moving* electric charges. Both fields are somewhat similar to gravity, which is a type of field acting on any *mass*. Since electric and magnetic fields are both specific to electric charges, though, their effects depend on the charge status of that object.

Important concepts related to electric and magnetic fields include **gravity**, **voltage**, **current**, **force**, **solenoids** and **motors**, the **right-hand rule**, **saturation**, **hysteresis**, **remanence** and **coercivity**, energy **dissipation**, electromagnetic **induction**, **Lenz's Law**, **Faraday's Law** of electromagnetic induction, and **braking**.

Here are some good questions to ask of yourself while studying this subject:

- How might an experiment be designed and conducted to demonstrate the phenomenon of electric fields? What hypothesis (i.e. prediction) might you pose for that experiment, and what result(s) would either support or disprove that hypothesis?
- How might an experiment be designed and conducted to demonstrate the phenomenon of magnetic fields? What hypothesis (i.e. prediction) might you pose for that experiment, and what result(s) would either support or disprove that hypothesis?
- How might an experiment be designed and conducted to demonstrate the phenomenon of electromagnetism? What hypothesis (i.e. prediction) might you pose for that experiment, and what result(s) would either support or disprove that hypothesis?
- How might an experiment be designed and conducted to demonstrate the phenomenon of electromagnetic induction? What hypothesis (i.e. prediction) might you pose for that experiment, and what result(s) would either support or disprove that hypothesis?

- What type(s) of matter do gravitational fields interact with?
- What type(s) of matter do electric fields interact with?
- What type(s) of matter do magnetic fields interact with?
- How do field lines differ between electric and magnetic fields?
- How does the right-hand-rule apply to magnetic fields?
- How do arrow-head and arrow-tail symbols show field lines when drawn on a flat surface such as paper?
- How do air and iron differ in their responses to magnetic fields?
- What does a B-H curve tell us?
- What is "hysteresis" and how does it apply to magnetic fields?
- What special property(ies) are offered by the material ferrite?
- Why does a coil generate no voltage when a stationary magnet is positioned nearby, but generates a voltage when the magnet is moved?
- What factors determine the magnitude of an induced voltage?
- How does Lenz's Law fit with the phenomenon of electromagnetic induction?
- How could we exploit the principle of electromagnetic induction to construct a generator?

1.2 Challenging concepts related to electric and magnetic fields

The following list cites concepts related to this module's topic that are easily misunderstood, along with suggestions for properly understanding them:

- **Curled Right-Hand Rule** – relating electric current and magnetic field vectors using the right-hand rule is an exercise in three-dimensional spatial relations, and is challenging for many new students. Read the text carefully, *use your right hand* to help visualize the examples in real space, and build real circuits (at least a few Amperes DC is usually necessary to create a strong enough magnetic field to swamp the Earth's natural magnetic field) using a magnetic compass to test the principle!
- **Straight Right-Hand Rule** – relating electric current, magnetic field, and Lorentz force vectors using the "straight" version of the Right-Hand Rule is another exercise in three-dimensional spatial relations, and is also challenging for many new students. Read the text carefully, *use your right hand* to help visualize the examples in real space, and build real circuits (at least a few Amperes DC is usually necessary to create a strong enough magnetic field to swamp the Earth's natural magnetic field) using a magnetic compass to test the principle!
- **Lenz's Law** – only after the curled version of the Right-Hand Rule has been understood, may it be used as an effective tool to predict the induced current vector direction predicted by Lenz's Law. Usually students who struggle to apply Lenz's Law have not yet mastered the Right-Hand Rule!

1.3 Recommendations for instructors

This section lists realistic student learning outcomes supported by the content of the module as well as suggested means of assessing (measuring) student learning. The outcomes state what learners should be able to do, and the assessments are specific challenges to prove students have learned.

- **Outcome** – Demonstrate effective technical reading and writing

  Assessment – Students present their outlines of this module's instructional chapters (e.g. Case Tutorial, Tutorial, Historical References, etc.) ideally as an entry to a larger Journal document chronicling their learning. These outlines should exhibit good-faith effort at summarizing major concepts explained in the text.

  Assessment – Students present their "Conceptual Toolbox" Theory Journal entries listing important concepts applied in the Tutorial text(s). A "Conceptual Toolbox" serves as inspiration when solving problems, each concept suggesting a potential problem-solving action. Challenge students to collaboratively generate a Conceptual Toolbox for the day's topic during the class session and then use it when solving problems together, thus fostering the habit of reasoning from principles rather than recalling procedures!

- **Outcome** – Predict the direction of a magnetic field using the Right-Hand Rule

  Assessment – Build a circuit sending direct current through a straight conductor, predict the circular orientation of the resulting magnetic field, and use a magnetic compass to validate that predicted direction.

  Assessment – Build a circuit sending direct current through a coiled conductor, predict the linear orientation of the resulting magnetic field through the center and around the outside of that coil, and use a magnetic compass to validate those predicted directions.

- **Outcome** – Predict the Lorentz force direction(s) between two conductors

  Assessment – Build a circuit sending direct current through two parallel conductors and predict the mutual force between those conductors (either attracting or repelling) according to $F = Qv \times B$.

  Assessment – Pose problems in the form of the "Lorentz force in a rail gun" Conceptual Reasoning question.

- **Outcome** – Predict the polarity of voltage induced in a wire coil

  Assessment – First identify the magnetic polarity of a bar magnet (if unlabeled, using a magnetic compass), and then predict the polarity of voltage that will be induced in a wire coil given the magnet's approach or retreat. Then, move the magnet and use a multimeter to validate that predicted voltage polarity.

  Assessment – Pose problems in the form of the "Induced voltage polarities" Conceptual Reasoning question.

Chapter 2

Case Tutorial

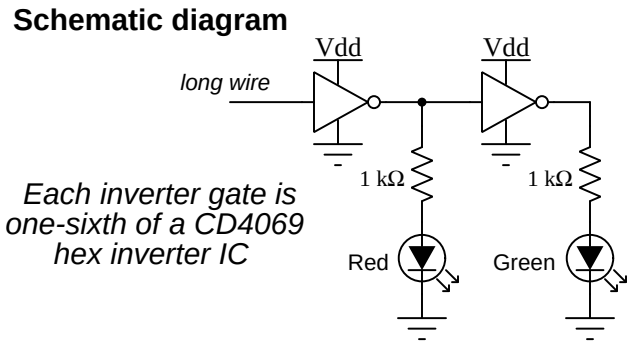
The idea behind a *Case Tutorial* is to explore new concepts by way of example. In this chapter you will read less presentation of theory compared to other Tutorial chapters, but by close observation and comparison of the given examples be able to discern patterns and principles much the same way as a scientific experimenter. Hopefully you will find these cases illuminating, and a good supplement to text-based tutorials.

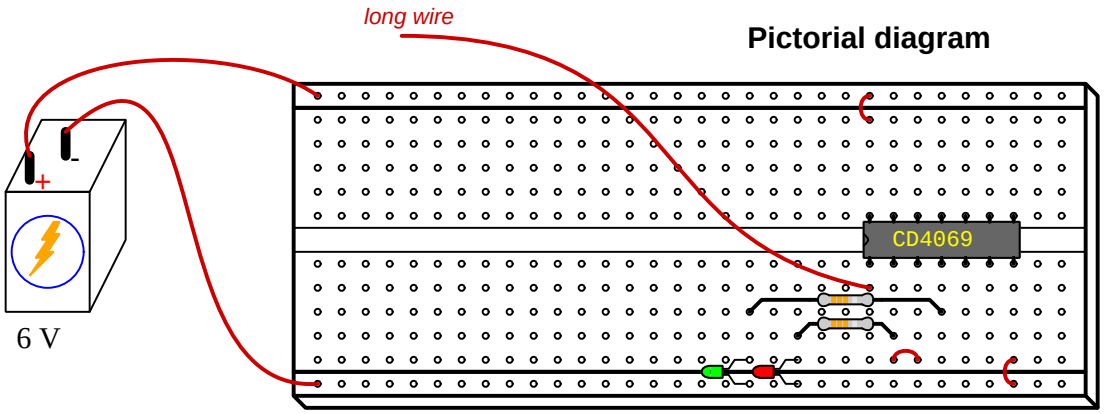
These examples also serve well as challenges following your reading of the other Tutorial(s) in this module – can you explain *why* the circuits behave as they do?

2.1 Example: CMOS inverter as electric field detector

Very strong electric fields may be detected by instruments such as *electroscopes*, but weak electric fields require some form of signal *amplification* for detection. Fortunately, devices known as *field-effect transistors* or *FETs* work exceptionally well for this purpose.

Discrete field-effect transistors are easily damaged by excessive electric field strength, and so there is always a risk when building an electric field detector from a single transistor. Digital logic circuits built using FETs, however, typically contain internal protection diodes to help guard against such damage, and therefore make more rugged electric field detectors.



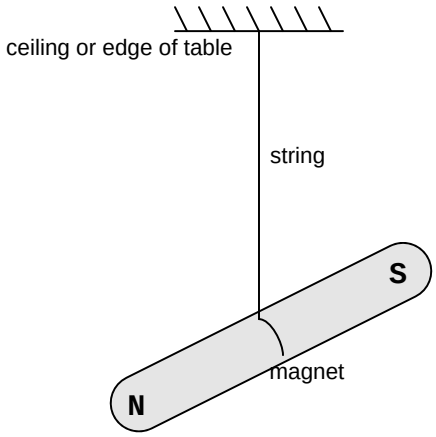


The first (left-hand) inverter functions as the electrostatic sensor: moving your hand near the long wire will cause the output of that first inverter to change state (from "low" to "high" or vice-versa). The second (right-hand) inverter takes the first inverter's signal and complements it, so that the green and red LEDs should always be in opposite states.

2.2 Example: cow magnet as magnetic field detector

Cylindrical "bar" magnets are sold to cattle ranchers for the purpose of their cattle ingesting them. The concept is that a strong magnet should attract and retain ferrous objects such as screws, nails, and fence staples near the beginning of the animal's digestive tract rather than passing completely through and possibly causing internal trauma. These magnets are quite powerful and readily available at any ranching supply store.

Hanging such a magnet from a string allows it to freely pivot and self-align with the Earth's magnetic field:



The pole pointing toward geographic north is the "magnetic north" pole of the bar magnet. Feel free to label the magnet's poles so you know which is which.

Positioning another magnet nearby this suspended magnet will cause it to shift position. Whichever end of this second magnet attracts the hanging magnet's "north" pole will be the second magnet's "south" pole.

An important caveat with using a cow magnet is that some of them have more than two magnetic poles! Some cow magnets I've tested are stainless-steel cylinders containing multiple magnet stacks inside, such that they have identical poles at each end and opposite poles near their centers (e.g. N-S-N) in order to provide more surface area to attract ferrous metal parts while inside the cow's stomach. Such a cow magnet will *not* work as a magnetic compass because there is no definite magnetic dipole from one end to the other.

2.3 Example: loudspeaker as electromagnetic demonstrator

Magnetic loudspeakers use a stationary permanent magnet positioned in close proximity to a lightweight wire coil attached to the "cone" of the speaker, so that energizing this coil causes the speaker cone to either extend or retract (depending on the direction of current through the coil) because the magnetic field generated by the current-carrying wire (i.e. *electromagnetism*) reacts with the magnetic field of the speaker's permanent magnet.

Briefly connecting a low-voltage DC battery to the speaker's terminals will cause the cone to visibly move.

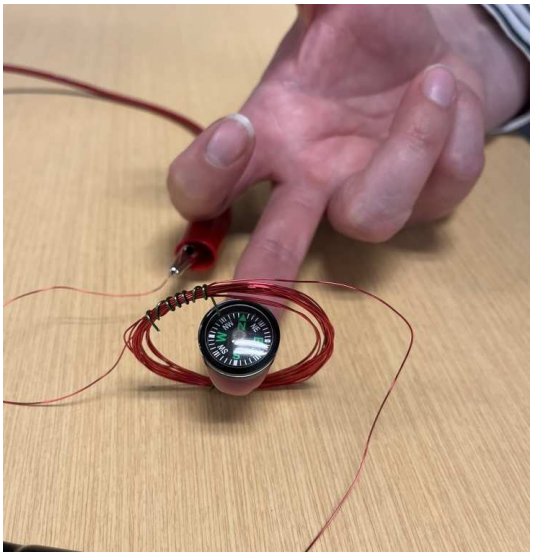
Conversely, one may use the exact same device to demonstrate electromagnetic induction. Simply connect a voltmeter to the speaker's terminals while gently pressing and releasing the speaker cone to cause the coil to move through the permanent magnet's field. Note how the voltage magnitude varies with the *speed* of the cone's motion, and not its *position*. This is in keeping with the principle of Faraday's Law, where the induced voltage is proportional to the rate at which magnetic flux varies through the coil.

2.4 Example: demonstrating the curled right-hand rule of electromagnetism

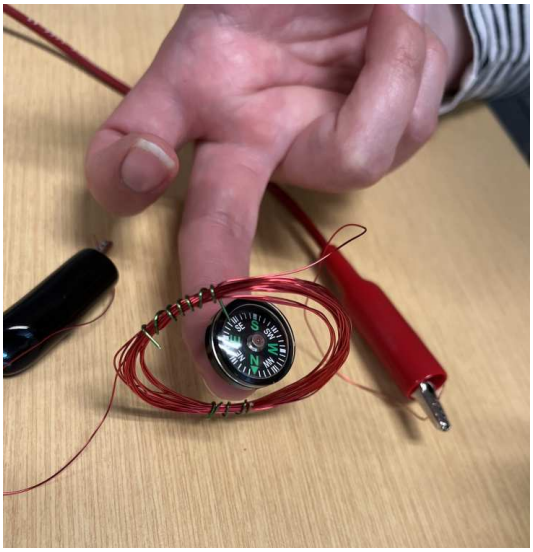
A direct current (DC) of approximately 3 Amperes passing from right to left through a straight wire with a small magnetic compass showing the magnetic field vectors below and above that wire:

To apply the Curled Right Hand Rule here, imagine curling the fingers of your right hand around this conductor with your thumb pointing toward the left. Your fingertips will show the direction of the circular magnetic field surrounding this conductor, while your thumb shows the direction of current (conventional flow notation) through it.

A direct current (DC) of approximately 1 Ampere passing clockwise through a coiled wire with a small magnetic compass showing the magnetic field vector inside of that coil:



A direct current (DC) of approximately 1 Ampere passing counter-clockwise through a coiled wire with a small magnetic compass showing the magnetic field vector inside of that coil:

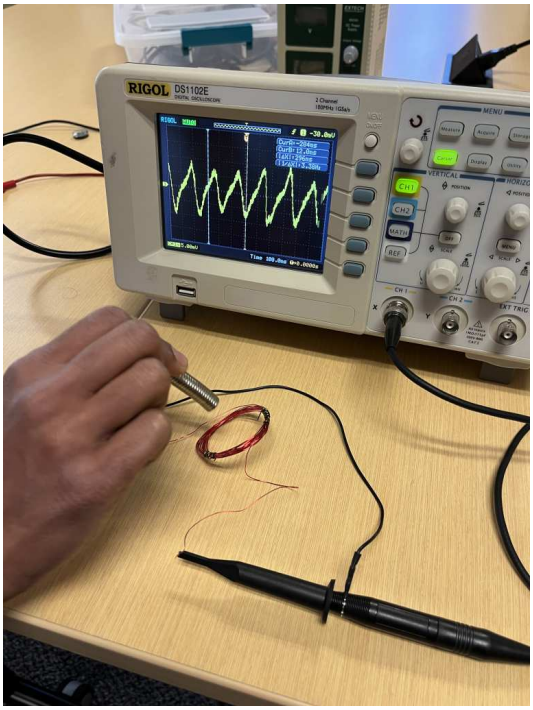


In both of these experiments the red and black wires connect to a DC power source, red positive and black negative.

2.5 Example: oscilloscope displaying induced voltage

One of the challenges of experimentally demonstrating the phenomenon of electromagnetic induction with is that the actual amount of voltage induced in any hand-wound wire coil by moving a permanent magnet near it is generally quite small, and fleeting in its duration. A sensitive analog voltmeter will register the induced voltage quite well, but digital voltmeters often struggle to do this as their digital displays often update too slowly to properly capture the peak voltage values.

A solution to this measurement problem is to use a digital *oscilloscope* rather than a voltmeter, with the horizontal (timebase) control set for a slow sweep speed of a fraction of a second per division. We see such an experiment being conducted in the photograph below:



In this experiment the student is moving the bar-style[1] magnet closer to and farther away from the center of the wire coil. The oscilloscope's vertical sensitivity is set for 5 milliVolts per division and its timebase for 100 milliseconds per division, with the triggering mode set to "Auto". The triggering voltage level is irrelevant at such slow sweep speeds.

Using an oscilloscope not only solves the interpretational problem inherent to slow-updating digits on a DMM while sensing transient voltages such as this, but it is also an excellent introduction to oscilloscope use in general because the experimenter has full control over the voltage signal itself by their hand motion.

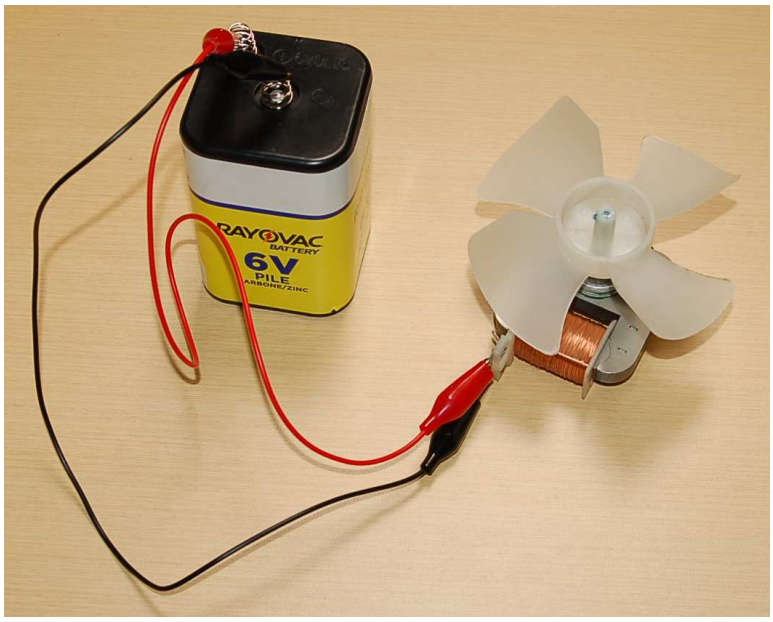
[1] Here, actually a set of rare-earth "button" style magnets stacked on end to create a longer "bar" shape. I find this approach quite convenient as it make use of button magnets which are very easy to obtain, and allows one to create as long or as short of a bar magnet as one might desire!

2.6 Example: demonstrating Lenz's Law

An easy demonstration of Lenz's Law may be performed using commonly-available components: a small AC induction motor (such as the type used in residential bathroom ventilation fans, typically sold as replacement motors at most hardware stores) and a 6 Volt dry-cell battery.

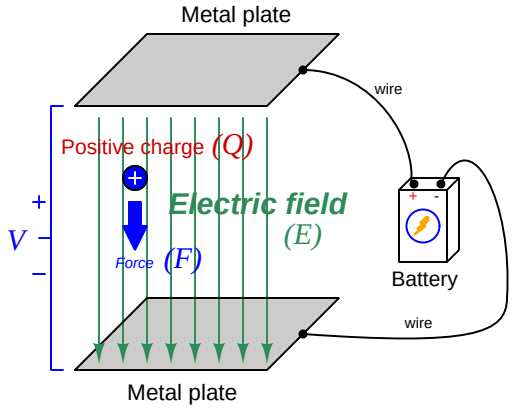
Spin the motor's shaft and feel how freely it turns. Then, connect the 6 Volt battery to the motor's terminals and try spinning the shaft again – you will notice the shaft does not spin as easily as it did before, due to the effect of Lenz's Law as the conductive rotor rotates within the stationary magnetic field produced by the stator winding energized by the (DC) battery. The currents induce in the spinning rotor produce magnetic fields that oppose its motion, making the rotor feel as though there is some sort of friction working against its motion.

Not only does this demonstration showcase Lenz's Law, but it also provides an illustrative example of a very practical AC motor "braking" technique known as *DC injection*, where a low DC voltage is applied to the motor's stator winding to produce a stationary magnetic field that the rotor must turn through.



2.7 Example: high-voltage corona

Electric fields (E) are fundamentally defined by the amount of force (F) exerted on an electric charge (Q) placed within that field:



$$\vec{F} = Q\vec{E}$$

Where,

$\vec{F}$ = Force exerted on the charge (Newtons)
Q = Charge quantity (Coulombs[2])
$\vec{E}$ = Electric field (Newtons per Coulomb)

Alternatively electric field may be defined in terms of the voltage between the end-points and the distance separating them, in which case we may express the electric field in units of *Volts per meter* as an alternative to *Newtons per Coulomb*:

$$\vec{E} = \frac{V}{\vec{d}}$$

Where,

$\vec{E}$ = Electric field (Newtons per Coulomb, or Volts per meter)
V = Voltage between two points (Volts)
d = Distance between those same two points (meters)

Relating electric fields to voltage and distance helps us compare electric potential with gravitational potential energy, the latter being the potential energy possessed by any mass (m) having the potential to fall some distance d while exposed to a gravitational field (a, the "acceleration" of gravity). The force exerted by a gravitational field on any mass is given by the formula $F = ma$, while the amount of potential energy (E_p, not to be confused with electric field $\vec{E}$) possessed by that mass is $E_p = mad$ or $E_p = Fd$. The farther a mass is lifted against the force of gravity, the more potential energy it has; the farther an electric charge is lifted against the force of an electric field, the more potential energy it has.

[2] One Coulomb of electric charge is equal to 6.2415×10^{18} electrons.

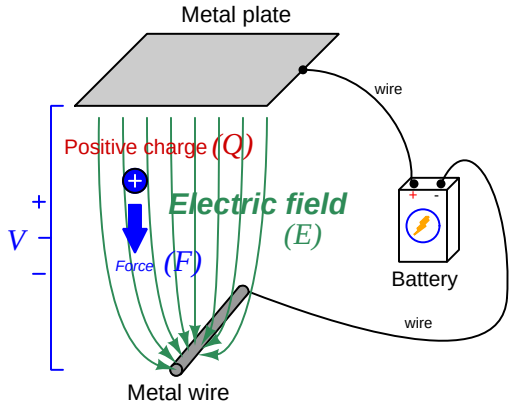
The vector arrows shown in the previous illustration representing the electric field between two metal plates actually represent electric *flux* (Φ_E). The electric field (E) is related to electric flux by area (A), the field being a measurement of how densely-packed those flux lines are per unit area:

$$\vec{E} = \frac{\Phi_E}{\vec{A}}$$

Where,

$\vec{E}$ = Electric field, or electric flux density (Newtons per Coulomb)
Φ_E = Electric flux (Newton-meter squared per Coulomb)
$\vec{A}$ = Area over which flux is distributed (square meters)

For two separated plates of infinite area[3], the electric flux lines will all be parallel to each other, and the electric field (or flux density) will be constant at any location between those plates. However, if we replace one plate with a wire we see the electric flux lines concentrating around that wire in contrast to their spreading over the plate's area:



This concentration of electric flux lines near the thin wire mean the electric flux density is strongest at the surface of that wire, and therefore that any electric charge will experience a greater force exerted on it as it approaches that wire. Stated in terms of voltage and distance, there are literally more Volts per meter of electric field strength near the wire than there are near the flat plate.

When atoms experience an electric field, that field applies opposing forces to the atoms' negative electrons and positive protons. If the field is concentrated enough, these forces will become sufficient to rip the electrons away from the atoms' positive nuclei, a process called *ionization*. When this happens to any substance that is normally an electrical insulator, the presence of these negative ions and positive ions constitute mobile charge carriers, and their resulting motion with the applied field is an electric current. In other words, when an electric field is strong enough, any insulator may *break down* and become electrically conductive. This *dielectric breakdown* limit is usually expressed in *Volts per meter*, which makes perfect sense because those are the units of electric field measurement,

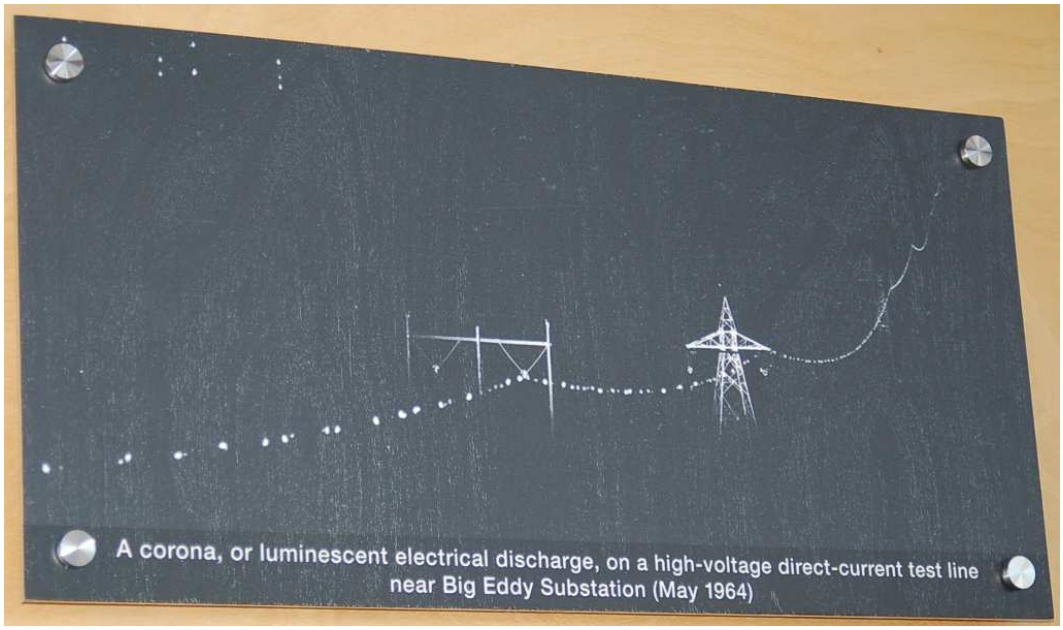
[3] For plates of finite area, electric flux lines tend to "bulge" at the outer edges of the plate, but even for realistically-sized plates the flux lines are nearly parallel to each other in the middle of the plates and therefore electric flux density is relatively constant at any location between the plates centered in the plates' areas.

and directly predict the amount of force applied to the atoms' individual electric charges (protons and electrons).

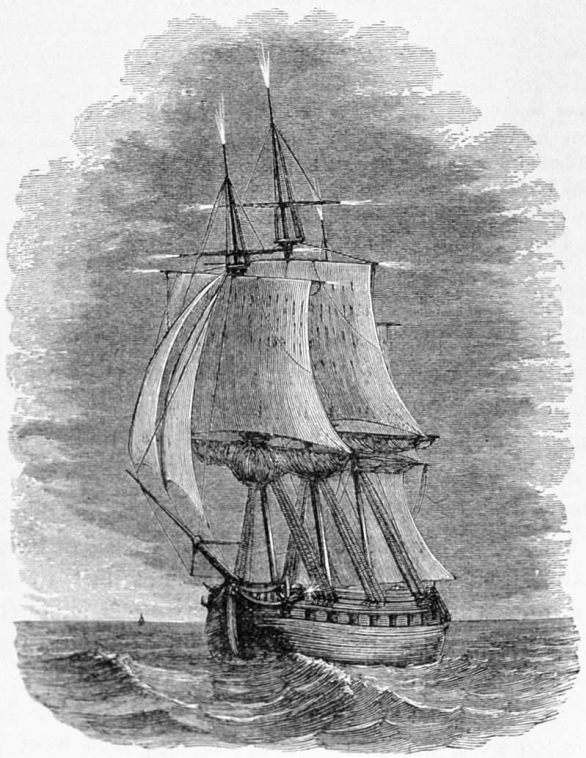
When dielectric breakdown occurs in open air, the phenomenon is called *corona discharge*. The ionization results in an emission of blue-white light, making corona discharge a spectacular sight to behold when it is intense enough. At atmospheric pressure this breakdown limit for air is approximately 30 kV/cm, or 3 MegaVolts per meter.

Corona discharge is problematic for high-voltage power lines, responsible for creating the "crackling" or "buzzing" sound audible near AC power lines with potentials in the range of hundreds of thousands of Volts. Corona discharge prompts oxygen atoms to join together in triplets to form *ozone* (O_3), as well as prompting atoms of nitrogen to join with atoms of oxygen to form various oxides of nitrogen (NO_x). Both ozone and NO_x gases are unhealthy for humans to breathe in large quantity, and they are also a causal factor in metallic corrosion which over time degrades the metal wires and fastener assemblies used in high voltage power lines.

The following image shows a picture taken in 1964 of a high-voltage DC power line undergoing corona tests at night. Glowing points of light along one of the conductors mark where corona discharge occurs:



Corona discharge sometimes occurs in nature during electrical storms, and this is historically known as *Saint Elmo's Fire*. This eerie glow occurs at the tips of tall, pointed structures such as antenna towers and sailing ship masts when the storm's electric field grows strong enough. The image below[4] is a historical representation of this phenomenon as observed by sailors:

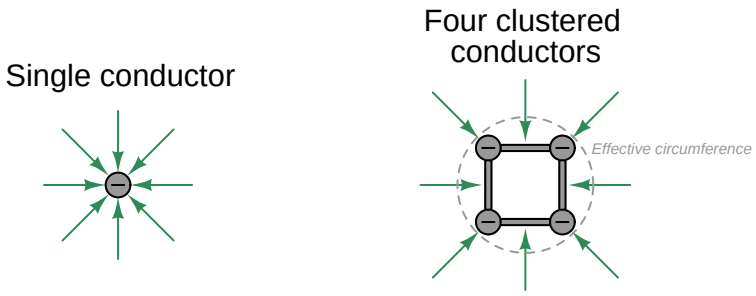


Note how the corona discharge occurs at each tip of the wooden mast structure (e.g. at the lift-beam tips as well as the mast tips on top), where the wood is wet enough (especially during a storm!) to sufficiently conduct electricity. Other pointed structures on the ship lower to the water's surface, such as the bowstrip, are not shown with any such "fire" in this image. Likely, the electric flux lines spread back out as they near the water's surface to decrease the electric flux density. Only when far away from the water's surface and sufficiently pointed (such as the mast tips and lift-beam ends) will the flux lines concentrate enough to ionize the air.

[4]Public domain, courtesy of Saibo.

Sharp edges and pointed structures tend to concentrate electric flux lines and therefore invite corona discharge in high-voltage electrical conductors. Therefore, care is taken to avoid such shapes on high-voltage electrical connections. Note the round metallic structures intentionally placed at the ends of these high-voltage electrical terminals found in a 500 kiloVolt electrical substation:

Another corona-mitigating design feature seen in these photographs is the clustering of multiple electrical conductors into bundles of three or four wires – all electrically common to one another – for the purpose of simulating the profile of a much larger-diameter electrical conductor. In other words, the overlapping electric field profiles of the individual metal cables spreads out the electric flux lines to keep the E field strength below the corona limit:



Chapter 3

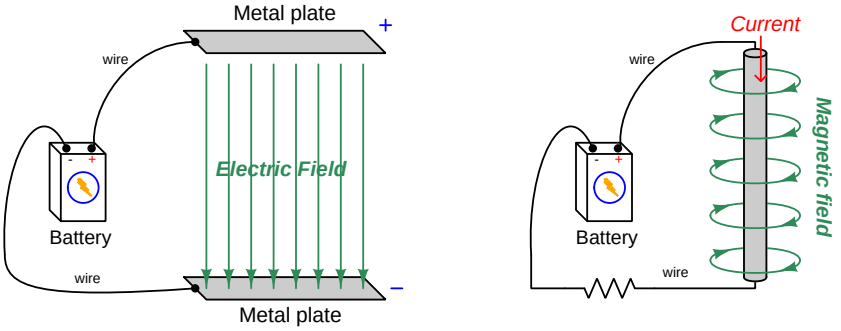
Tutorial

It has long been observed that certain objects exert forces upon each other from a distance, and that these actions-at-a-distance are not all of the same category. Rubbing electrically-insulating materials against each other in a low-humidity environment causes "static electricity" to build between them, and those objects will experience an attracting force between them. Striking an iron bar while oriented along the Earth's north-south axis causes it to "magnetize" with "north" and "south" poles able to attract other iron objects. Masses in space experience mutual attraction proportional to their mass, regardless of other circumstances.

The modern explanation for these actions-at-a-distance has been to posit the existence of *fields* filling the space between the acting objects. *Electric fields* interact with any matter possessing electric charge. *Magnetic fields* interact with any moving (but not stationary) electric charges. *Gravitational fields* interact with any object possessing mass. Each of these fields may be envisioned as invisible "webs" of interaction between their respective particles or objects, and we may even approximate the shape of these fields by sketching sets of lines or curves showing the direction of their generated forces. Our focus of study in this module will be on electric and magnetic fields, because gravitational fields are of little consequence to electric circuits.

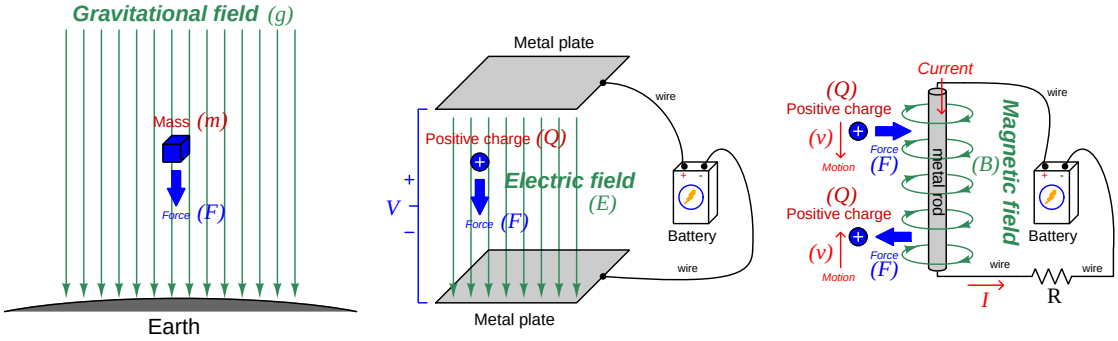
3.1 Electric versus magnetic fields

An electric field exists wherever voltage exists (i.e. wherever an imbalance exists between positive and negative electrical charges), parallel to the axis of that voltage. A magnetic field exists wherever an electric charge moves, perpendicular to the axis of that charge's motion. These fields may be represented as lines or curves in an illustration:



These two types of fields differ from each other in multiple ways. Electric fields terminate at conductors having opposite voltage polarity, while magnetic fields "loop" around current-carrying conductors and have no termination points at all. Electric field strength is proportional to the magnitude of the *voltage*, while magnetic field strength is proportional to the magnitude of the *current*.

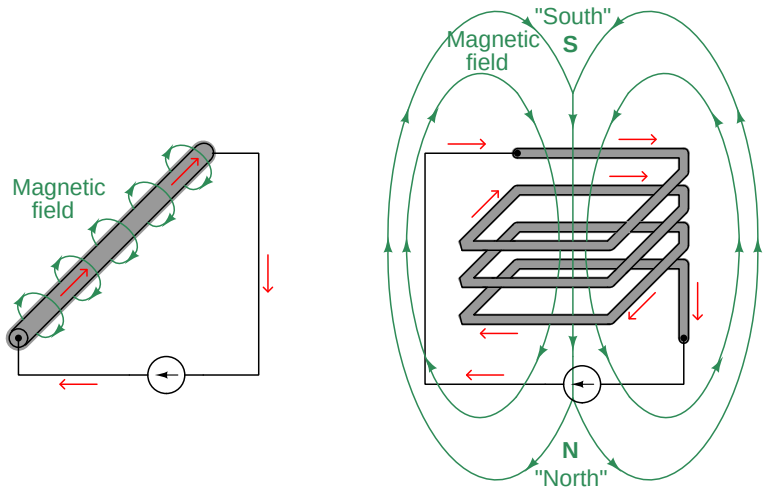
Fields are often defined and quantified in terms of the amount of *force* they exert on matter. This is true for gravitational, electric, and magnetic fields alike. Gravitational fields (g) act on mass (m) to produce force (F). Electric fields (E) act on electric charges (Q) to produce force (F). Magnetic fields (B) act on *moving* charges (charge Q with velocity v) to produce force (F):



Electric and magnetic fields alike are natural consequences of voltage and current, respectively. These fields are found *everywhere* in energized electric circuits, but are generally weak enough that we do not notice their forces.

3.2 Electromagnetism

The phenomenon of an electric current creating a magnetic field is called *electromagnetism*. This effect may be enhanced by forming the current-carrying conductor into a coil shape, which causes each wire turn's magnetic field to link with the magnetic fields of the other turns in the wire coil, producing a greater net magnetic field[1]:



The terms "north" and "south" refer to the tips of a magnetic compass needle, in honor of which direction they point[2] when allowed to freely orient to the natural magnetic field of the Earth. While the magnetic field passing through and around the coiled wire is an endless loop just like the circular field surrounding the straight wire, we are able to assign "north" and "south" poles only to the coil because the coil itself has definite ends for the field to enter and exit. A straight wire provides no convenient features to mark the "ends" of the field, and so we cannot place "N" and "S" symbols anywhere along its length.

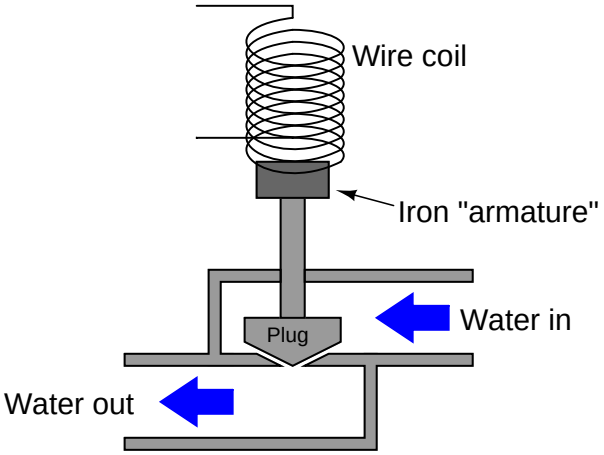
An interesting photograph appears next, showing an image of an electric arc welding cable lying on a concrete surface at a construction site for a steel-framed building. Tiny pieces of steel ("filings") scattered on the concrete floor oriented themselves perpendicular to the cable (i.e. aligning with the magnetic field encircling the cable) as electric current passed through it from the arc-welding process:

[1] It should be noted that every single current-carrying conductor has a circular magnetic field around it, including all the connecting wires. Here only the fields around the grey-colored conductors are shown, for simplicity.

[2] This means that any bar-shaped magnet will tend to orient itself such that its "north" pole points toward the Arctic and its "south" pole toward the Antarctic. Since opposite magnetic poles attract, this actually means that the natural magnetic pole located on Earth's north (Arctic) end is actually "magnetic south" and vice-versa. Confusing, no?

A coil's magnetic field may be further enhanced by providing a solid iron path for it to pass through, since iron offers less magnetic "resistance" than empty space or air. This is how powerful *electromagnets* are made: by wrapping coils of wire around an iron "core" material. The strength of the magnetic field is then easily controlled by varying the intensity of the electric current. Such electromagnets are the basis of *solenoid* actuators used to exert mechanical force, as well as *electric motors* used to perform mechanical work.

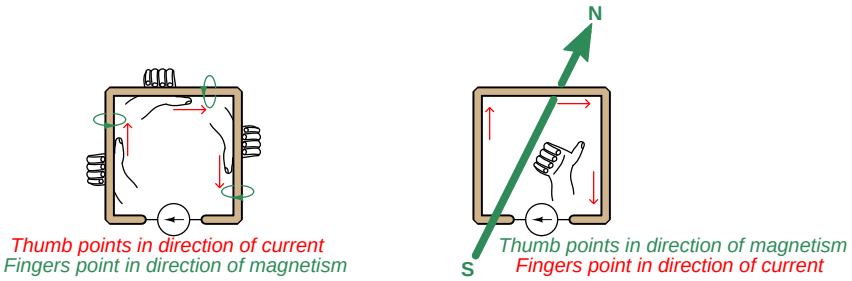
The following illustration shows an electric *solenoid valve* using an electromagnet coil to magnetically attract a movable iron "armature" and thereby operate a fluid-controlling valve mechanism:



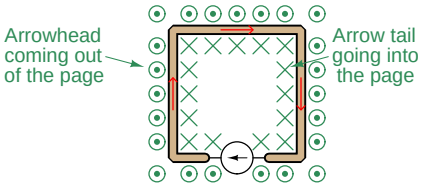
In this photograph we see an example of a solenoid valve (manufactured by ASCO) with threaded ports for the fluid connections and wires at the right-hand end for connection to the solenoid coil:

3.3 Curled right-hand rule

When we sketch electric and/or magnetic fields, as well as electric currents, we show them as arrow symbols with heads pointing in specific directions. In physics and mathematics we refer to any quantity having both magnitude and spatial direction as a *vector*, and so electric currents, electric fields, and magnetic fields are all vector quantities. The directions of these vectors and their relations to one another are merely conventions[3], and the method we use to properly associate these directions is something called the *right-hand rule*. If you curl the fingers of your right hand as though forming a fist, and point your thumb outward (perpendicular to your wrist), your fingers and thumb will serve as a faithful guide to associating conventional-flow current vectors with magnetic field vectors. For a straight[4] current-carrying conductor, your thumb points in the direction of conventional current flow and your curled fingertips point in the direction of magnetic north. For a coiled conductor, your curled fingertips point in the direction of conventional flow while your thumb points in the direction of magnetic north:



The challenges of accurately representing three-dimensional vectors on a two-dimensional page are obvious. A common notation used for vector arrows perpendicular to the page is to show arrowheads as circles with center-dots (as though you were looking at a cone-shaped arrowhead headed toward your eyes) and arrow tails as "X" symbols (as though you were looking at the fletchings of an arrow headed away from you). Here is a view of the same circuit using this notation to show the true orientation of the magnetic field lines perpendicular to the page:

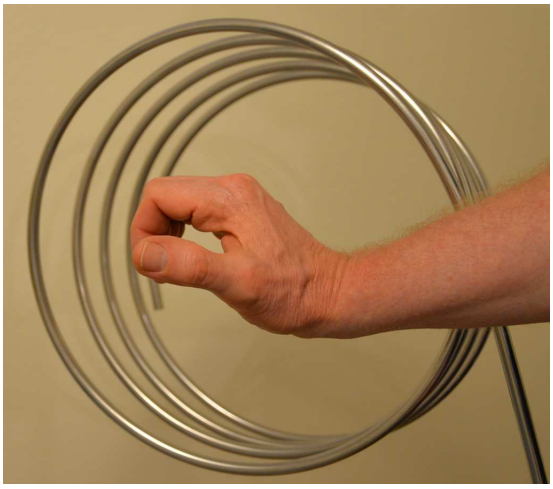


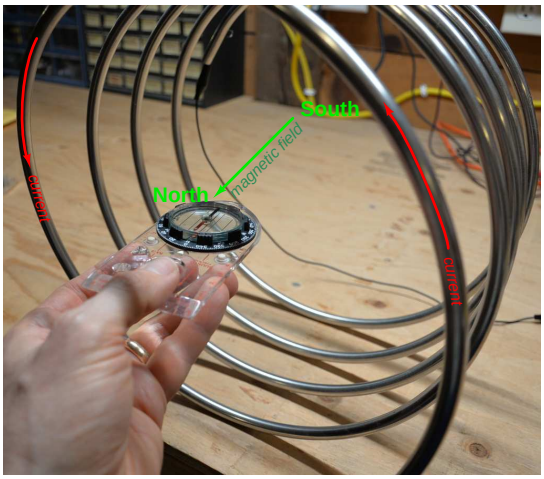
[3] As we know, the direction of "conventional flow" for electric current is merely what early experimenters *assumed* was the correct direction of the mysterious "electric fluid" for which they had no reliable means to discern. Later discoveries showed that in fact electric current through metallic conductors is comprised of negatively-charged electrons which actually flow in the direction opposite to the accepted convention. This matters less than one might suppose. Magnetic fields are even more arbitrary, as there are no particles actually drifting anywhere to constitute a direction, but still it is necessary for the sake of accurate communication to define "north" and "south" polarity of these fields. Incidentally, the definition of "north" for a magnetic field is that of the north-pointing end of a magnetic compass needle, which means that the magnetic pole located near the Earth's geographic north pole (in the Arctic) is actually "magnetic south" so that the "magnetic north" of a compass' needle will be attracted in that direction!

[4] Either an entirely straight wire, or a straight section of wire that is part some other shape.

It is a good active-reading exercise to take your right hand and try applying it to these illustrations, first pointing your thumb in the direction of current for each straight wire section and seeing how the curl of your fingers match the arrow head/tail symbols, then curling your fingers in the clockwise direction of the circuit's current and verifying that your thumb indeed points into the page with the arrow tails in the center of the wire loop.

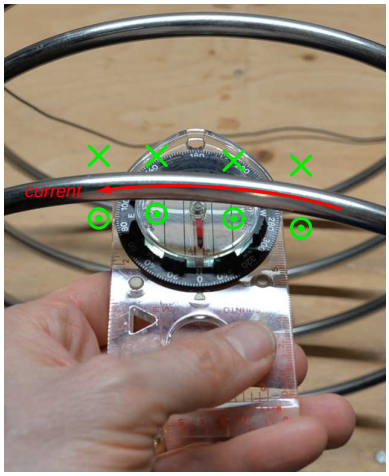
The following photographs depict the curled right-hand rule in relation to a large coiled conductor. In the left-hand image we imagine this coiled conductor connected to an electrical source, with current entering the coil's end shown in the lower-right of the foreground and exiting the other end visible in the rear, the curled fingers of the right hand tracing the counter-clockwise circular motion of this imagined current through the coil while the thumb points in the direction of magnetic north (i.e. toward you, the reader). In the right-hand image we see the same coiled conductor actually being energized in this manner, a magnetic compass clearly indicating the magnetic field vector with current moving in the same counter-clockwise direction:

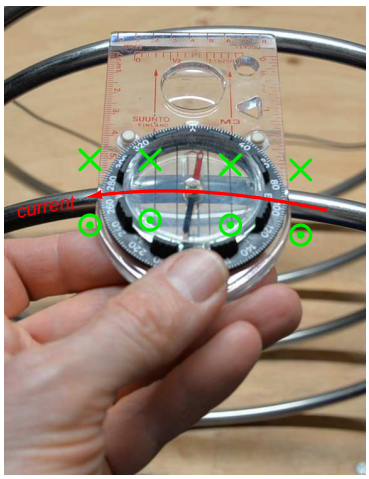




Magnetic north happens to be indicated by the red-colored end of this magnetic compass's needle.

If we use the same magnetic compass to map the magnetic field vector surrounding a section of this same conductor, holding it first below the conductor and then above the conductor as shown in the left and right photographs (respectively) following this paragraph, we may see another application of the curled right-hand rule in action. This time we apply the rule by imagining a person's right hand fingers curling around the conductor with their thumb pointed in the direction of current from right to left, the curled fingers representing magnetism and the thumb representing current:





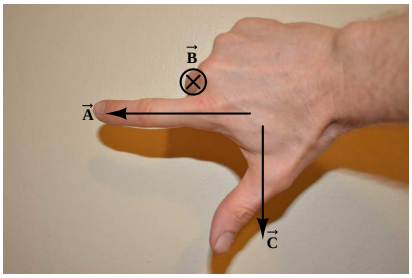
Again, it is a good active-reading exercise to take your right hand and try applying it to these photographs. Whether using your curled fingers to point in the circular direction of current through a coiled conductor and your thumb to predict the magnetic field vector in the center of that coil, or using your thumb to point in the straight direction of current through a section of conductor and your curled fingers predicting the magnetic field vector encircling that conductor, the activity of actually positioning your right hand in relation to the images will help make the concept clearer than only reading about it.

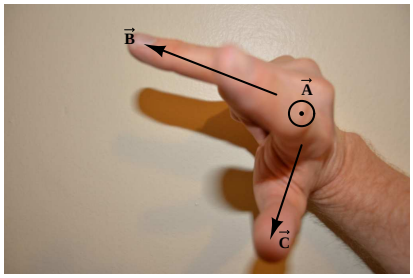
3.4 Straight right-hand rule

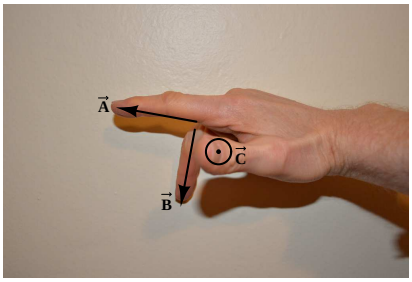
Another version of the right-hand rule uses the thumb as well as the index and middle fingers pointing in straight lines all perpendicular to each other. In vector mathematics, a certain type of multiplication called the *cross-product* ($\times$) frequently applied to electromagnetism describes such mutually-perpendicular vectors:

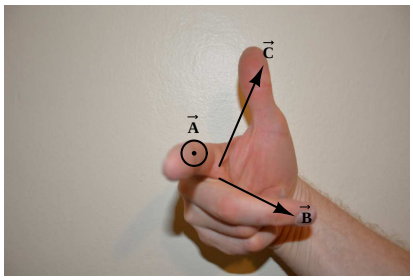
$$\vec{C} = \vec{A} \times \vec{B}$$

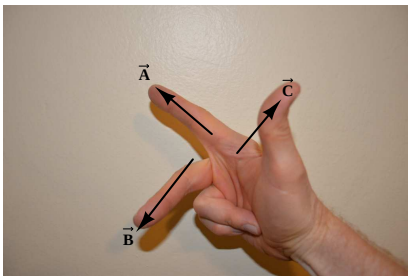
Using your right hand as a guide, the vector represented by your index finger ($\vec{A}$) and the vector represented by your middle finger ($\vec{B}$) form the cross-product vector $\vec{C}$ represented by your thumb. Photographs of a right hand with index finger, middle finger, and thumb extended perpendicularly to each other are shown here, with corresponding annotations:



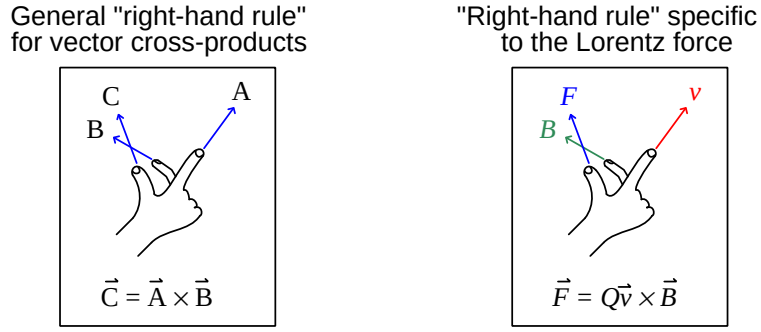






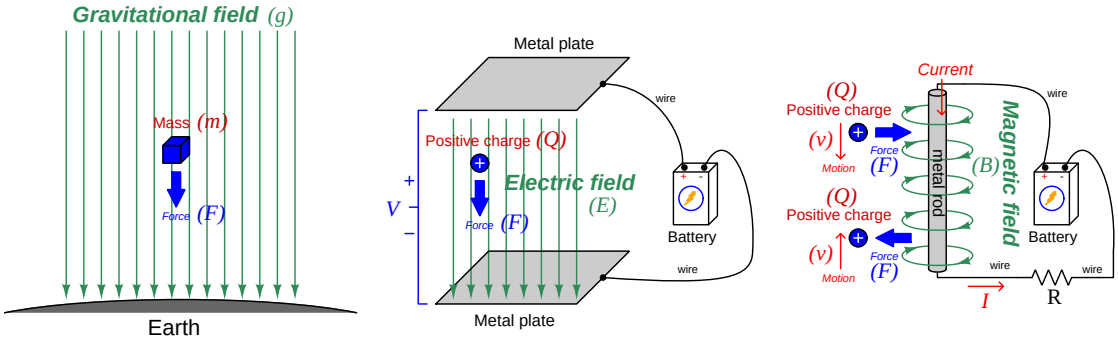


This "straight-fingered" version of the right-hand rule specifically relates to magnetism when the velocity of an electric charge ($\vec{Q}v$) moves perpendicular to a magnetic field ($\vec{B}$) and experiences a resulting force ($\vec{F}$) called the *Lorentz force* that is perpendicular to both its motion vector and the magnetic field vector:



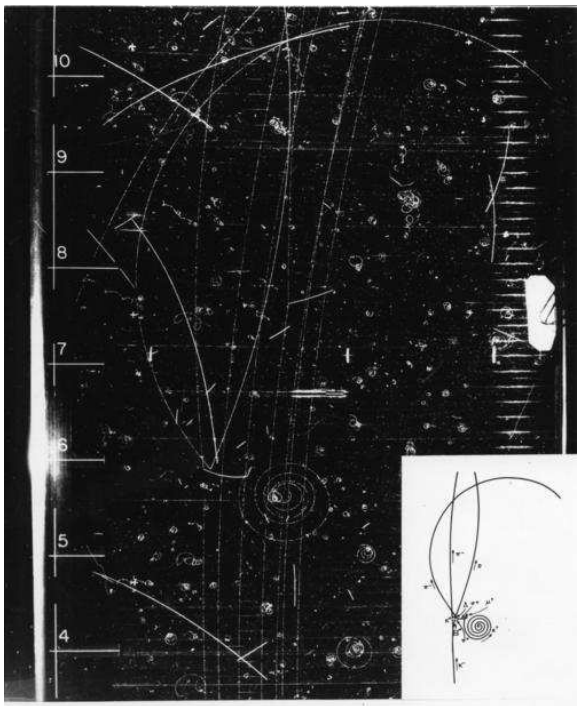
A simple mnemonic I use to remember these relationships of fingers to vectors is that the **I**ndex finger points in the direction of current ($\vec{I}$, conventional flow notation), the **M**iddle finger points in the direction of the **m**agnetic field ($\vec{B}$), and the **Th**umb points in the direction of the **th**rust (i.e. Lorentz force $\vec{F}$) acting upon the moving charge.

An important detail to note regarding the Lorentz force is that it only exists when an electric charge is *moving*, and not when the charge in question is stationary. This stands in stark contrast to electric fields which exert force directly on electric charges whether they are in motion or at rest, similar to how gravity exerts force on objects having mass whether in motion or at rest. The reason for this is based on what we learned from the curled version of the right-hand rule: that *electric charges produce their own magnetic fields as they move*. The Lorentz force is really the result of *two* magnetic fields reacting with one another. Shown below (again) is a comparison of gravitational, electric, and magnetic fields and their respective forces:



$F = mg$ $F = QE$ $F = Qv \times B$

A practical example of this phenomenon is when electrically-charged particles ejected from the Sun travel into Earth's magnetic field, and begin to spiral due to the Lorentz force. Particle physicists exploit this same effect by imposing a strong magnetic field perpendicular to the beam of charged particles exiting a *particle accelerator*[5], that magnetic field causing those charged particles to curve in their trajectories. The photograph[6] shown below depicts an antiproton shot into a *bubble chamber* from a particle accelerator, and soon thereafter decomposing into multiple charged particles each with its own electric charge sign, mass, and velocity:

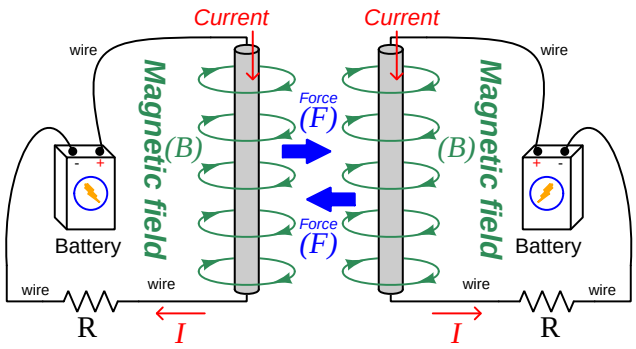


Note the particle charge signs shown in the inset annotation, depicting negatively-charged particles curving clockwise in the magnetic field while positively-charged particles curve counter-clockwise. The orientation of the magnetic field imposed on this bubble chamber accounts for these particles' differing directions of curvature: the magnetic field vector in this bubble chamber faces down (into the page) so that all positively-charged particles experience a Lorentz force to the left of their velocity vector, and all negatively-charged particles experience a Lorentz force to the right. Many other tightly-curled spiral tracks may also be seen in this bubble chamber photograph, showing random charged particles with significantly less velocity than those ejected from the particle accelerator.

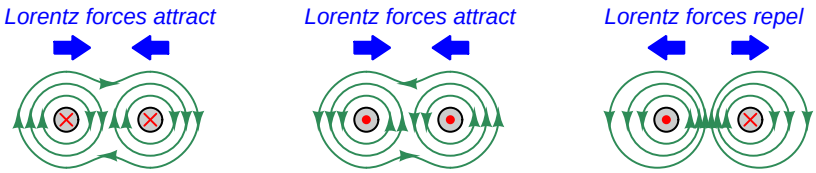
[5] A particle accelerator is a scientific research device used to accelerate subatomic particles to extremely high velocities – often significant fractions of the speed of light in a vacuum – for the purpose of studying their behavior and composition. Devices such as *bubble chambers* track the trajectories of these particles as they exit the accelerator.

[6] This free-use image provided courtesy of the American Institute of Physics, Emilio Segrè Visual Archives.

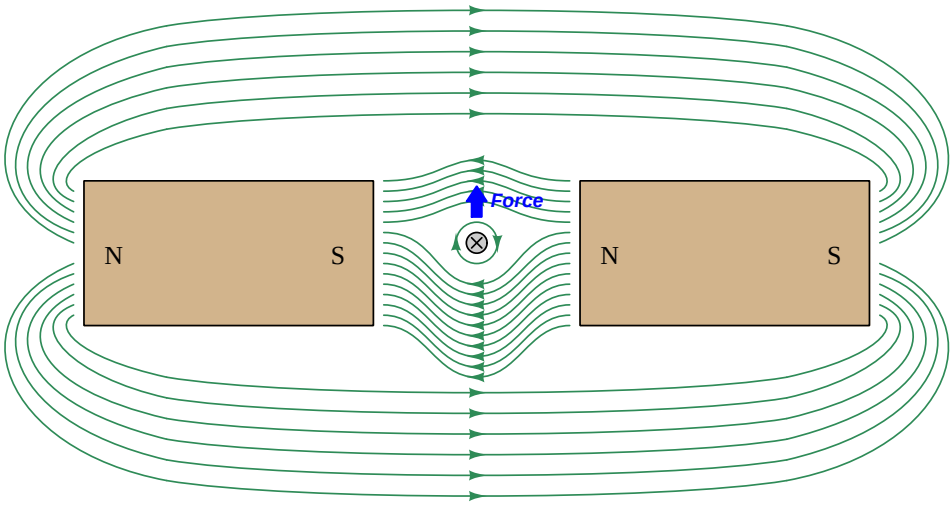
A more common example of this phenomenon is the mutual force of attraction or repulsion between two parallel current-carrying conductors. Below we see an illustration showing how two parallel conductors carrying currents in the same direction will be attracted toward one another by the Lorentz force exerted on each by the other's magnetic field:



The directions of the Lorentz force vectors will always be toward each other (i.e. pulling the conductors together) so long as the currents are in the same direction. This is just as true with both currents going up as well as going down. As you might suspect, if the two conductors' currents travel in opposite directions the Lorentz force vectors will point outward to force the conductors apart from each other. The following illustrations showing end-views of conductors with currents in various directions explains the Lorentz attraction versus repulsion in terms of the conductors' adjacent magnetic field circles:



Placing a current-carrying conductor perpendicular to the magnetic field produced by permanent magnets will also generate a Lorentz force as shown in the illustration below. Here, a straight wire (perpendicular to the page) carries an electric current (into the page) while exposed to a magnetic field pointing from right to left between two permanent magnet poles. The resulting Lorentz force will be upwards:

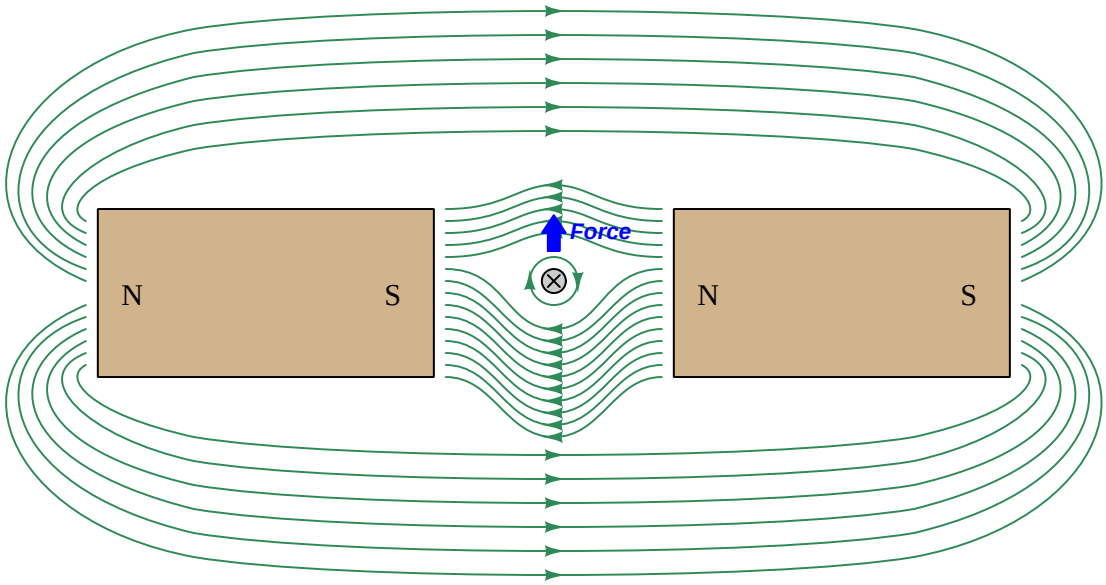


This is the basis of an elementary *electric motor*, the subject of the next section. In an electric motor, a current is passed through a conductor (usually a *coiled* conductor) in order to create a magnetic field that will react against the magnetic field produced by either a permanent magnet or another energized conductor, ultimately producing a Lorentz force causing one or more conductors to physically move.

3.5 Electric motors

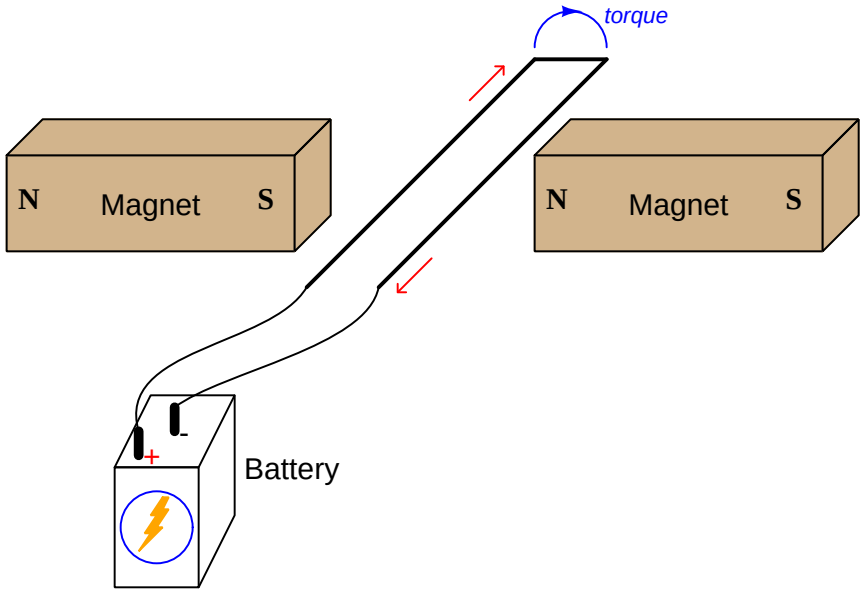
Magnetism, of course, is a direct consequence of moving electrical charge carriers, which is why every current-carrying conductor is surrounded by its own magnetic field. Certain materials[7], however, may be made "permanently" magnetic, requiring no electric current to sustain their own magnetic field. The common *bar magnet* and *horseshoe magnet* are examples of this phenomenon.

If a pair of permanent magnets are arranged to form a magnetic flux spanning a gap, and a current-carrying conductor placed within that gap perpendicular to the permanent magnetism, the conductor will experience a lateral force perpendicular to both the current and the magnetic flux vectors. This is the familiar *Lorentz force*, following the straight right-hand rule:



[7] All ferromagnetic materials such as iron have the ability to retain at least some magnetic flux after the magnetizing stimulus has been removed. Certain so-called *rare earth* compounds are exceptionally good in this regard.

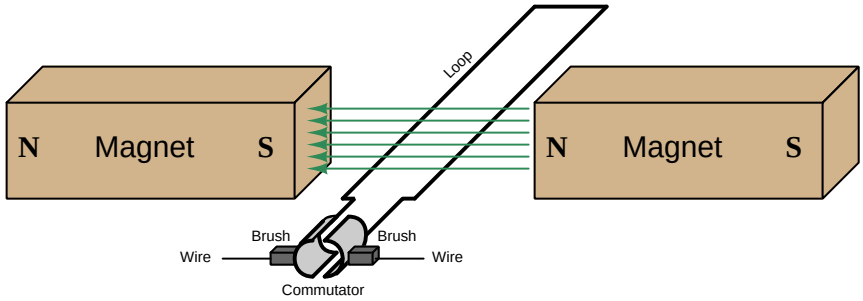
If we revise this experiment slightly and place a *loop* of wire within that same gap, the loop experiences a *torque* (i.e. twisting force) as one wire is forced upward and the other downward:



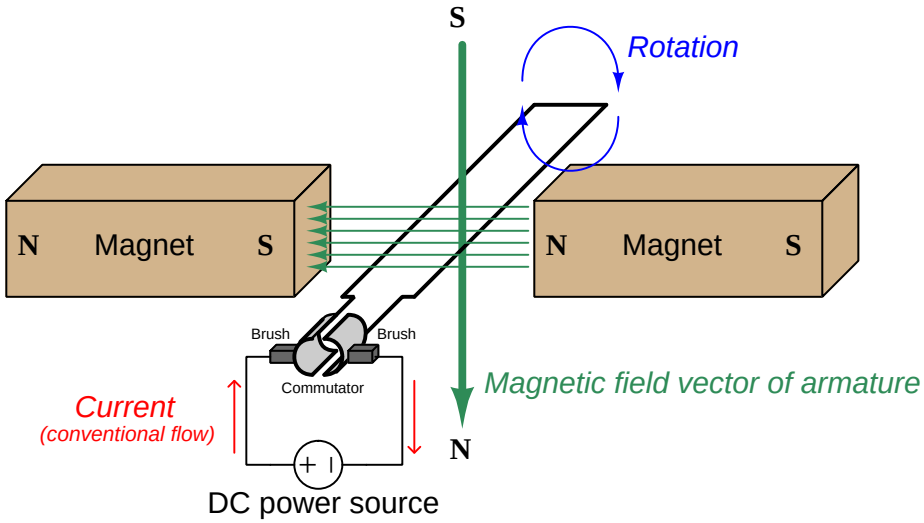
This makes sense if we view the loop as a pair of wires with each wire experiencing its own force by the straight right-hand rule (left-hand wire forced up and right-hand wire forced down), or alternatively if we consider the loop as a partial *coil* with the curled-finger version of the right-hand rule informing us of the coil's magnetic polarity (north down and south up), with north repelling north, south repelling south, and north-south attracting. Either way the loop tries to rotate clockwise as viewed from the perspective of the battery.

As promising as this seems to be as the foundation for the design of an electric motor, there is a problem: the loop will not spin in a full circle, but instead will stop when its magnetic poles align with the permanent magnets, only one-quarter of a turn away from where it is shown above. Furthermore, the wires connecting the loop to the battery would become twisted together if the loop were (somehow) to turn more than 180 degrees. If only we had a way to *make* and *break* connection to the battery at just the right positions as the loop rotated, we might have a way to make this loop fully spin.

An ingenious solution to this problem is a sliding-contact assembly called a *commutator*, and a pair of graphite (carbon) *brushes* serving as contact points between the battery's wires and the commutator segments. The commutator is simply a pair of semi-circular metal pieces terminating the ends of the wire loop and arranged about the centerline of rotation. These pieces rotate along with the loop as one assembly, making and breaking contact with the brushes as it spins:

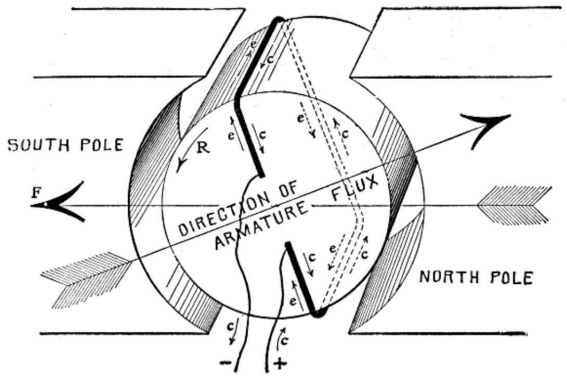


When energized by a DC electrical source, the loop continuously turns while the commutator and brushes make and break contact as necessary to maintain the same direction of rotation. Reversing the direction of a permanent-magnet (PM) DC motor is as simple as reversing the polarity of the energizing source:



In order to maintain a strong magnetic field through the rotating loop, it is generally wound around an iron core mounted on a bearing-supported shaft called an *armature*. The armature supports the rotating loop while providing a nearly continuous pathway for magnetic flux lines from one permanent magnet to the other. A circular iron structure joining the magnets' outer poles together similarly enhances the magnetism by providing a complete magnetic path.

The following illustration taken from page 394 of *Hawkins Electrical Guide Number Two* (published in 1914) shows how the wire loop may be wrapped around an iron armature, as well as directions of current (both *conventional* c and *electron* flow e):



For greater effect, we may substitute a multi-turn coil for a single loop. For even greater effect, we may position multiple coils around the armature, with a multitude of commutator bars for the coils' termination. Additional illustrations (with captions) taken from pages 400 and 401 of the same text show photographs of typical DC motor components:

Fig. 400.—Fairbanks-Morse armature for 7½ H. P., 1300 R. P. M., TR type motor. The armature core is built up of thin sheet steel laminations with notches in the circumference, which, when the discs are placed together, form grooves or slots to receive the armature coils. The armature cores for the larger machines are mounted on a cast iron spider, which also carries the commutator, making the two parts entirely self-contained, and with this construction, it is possible to remove the armature shaft, without disturbing the core, commutator or windings. Cores of all sizes are provided with ventilating spaces, running from the surface to the central opening of the core, so that air is drawn through the core and blown out over the windings by the revolution of the armature.

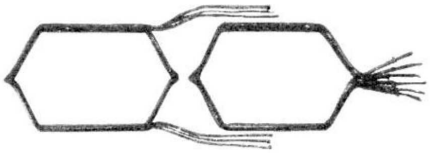
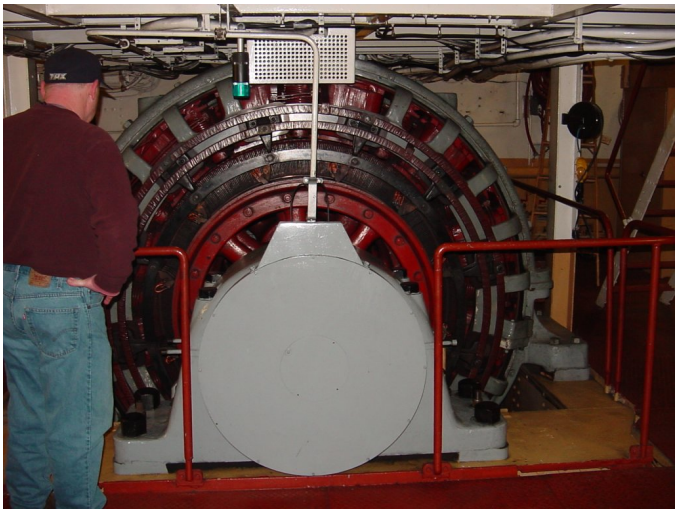


Fig. 401.—Fairbanks-Morse wire wound armature coils. These coils are form wound and are thoroughly insulated and baked before assembling in the slots. Material of great mechanical strength as well as high insulating value is used, and the coils are subjected to dippings in insulating compound and to bakings, thus driving out all moisture and making a coil which is practically waterproof and which will withstand rough handling. These coils, when completed, are placed in the slots, where they are retained by bands on the three smaller sizes and by hardwood wedges on the larger sizes.

Most industrial DC motors have multiple armature coils and commutator segments (also called *bars*), not just a single "loop" and two segments as shown in the previous model. A photograph of a large (1250 horsepower) DC motor used to propel a passenger ferry ship is shown here, with the field and armature poles clearly seen (appearing much like spokes in a wheel):



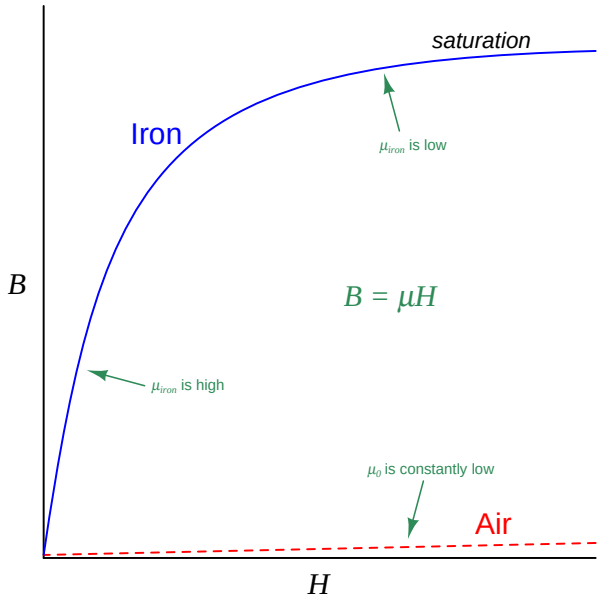
A close-up of one brush assembly on this large motor shows both the graphite brush, the brush's spring-loaded holder, and the myriad of commutator bars the brush makes contact with as the armature rotates. Each armature coil terminates at two of the commutator bars:

3.6 Field strength and field force

The relationship between electric current and magnetic field strength is a complicated one. For any current-carrying conductor in air or vacuum, or surrounded by most material types, the magnetic field strength (B, measured in the unit of the *Tesla*) is a direct and linear function of magnetizing force (H, measured in the unit of the *Ampere-turns per meter*[8]). Magnetic field strength (B) and magnetizing force (H) are related to each other by a factor known as *permeability*, symbolized by the Greek letter "mu" (μ)[9]:

$$B = \mu H$$

A perfect vacuum has a permeability (μ_0) of approximately $4\pi \times 10^{-7}$, but certain *ferromagnetic* substances such as iron have much greater magnetic permeabilities, which means their immediate presence near a current-carrying conductor help boost the magnetic field strength for any given amount of magnetizing force (i.e. more B for the same H). This is why electromagnets generally have iron cores: to maximize their magnetic field strength for any given amount of current. However, the boosting action of iron (and other ferromagnetic materials) comes with certain caveats. One is called *saturation*, which simply means its permeability is not constant but rather decreases with increasing magnetic flux. Thus, the rate of growth for magnetism (B) does not remain constant but decreases with as current continues to increase (H), as shown in the following graph:

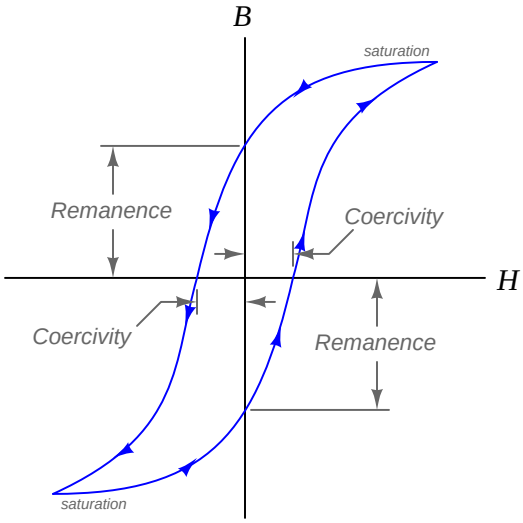


[8] This compound unit of measurement is precisely what you'd think it is: current multiplied by the number of "turns" (loops) of wire in the coil, divided by the length of the core in meters. Thus, H increases with current, with turns, and with a shorter coil. A straight conductor is considered to be equivalent to one turn.

[9] We are accustomed to seeing μ used as the metric prefix *micro* (10^{-6}) but here it is being used a symbol for a physical ratio: $\mu = \frac{B}{H}$.

For small amounts of magnetizing force (i.e. small amounts of current for a given number of "turns" in the coil), the gain in magnetic field strength with iron versus air is impressive. However, as the iron becomes more and more magnetized, we see diminishing returns until the rate of magnetic field increase versus current increase is the same as for air (i.e. the *slope* of the B-H curve for iron tapers off to eventually match the slope of the B-H curve for air).

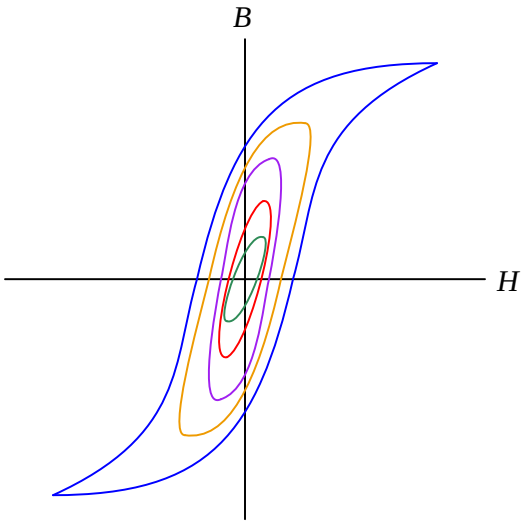
Another caveat of ferromagnetism is that ferromagnetic materials tend to *retain* some magnetism even after current through the conductor ceases, and in fact an opposing magnetizing force (H) will be necessary to restore the net magnetic flux to zero. A *hysteresis*[10] curve shows these effects, documenting the amount of magnetic field (B) versus applied magnetizing force (H) as current oscillates:



Strange-sounding as these terms may be, they actually make a lot of sense. *Remanence* is how much magnetic field *remains* in the ferromagnetic core after current returns to zero. *Coercivity* is the degree to which the ferromagnetic core must be *coerced* by a reverse magnetizing force in order to return to zero magnetism.

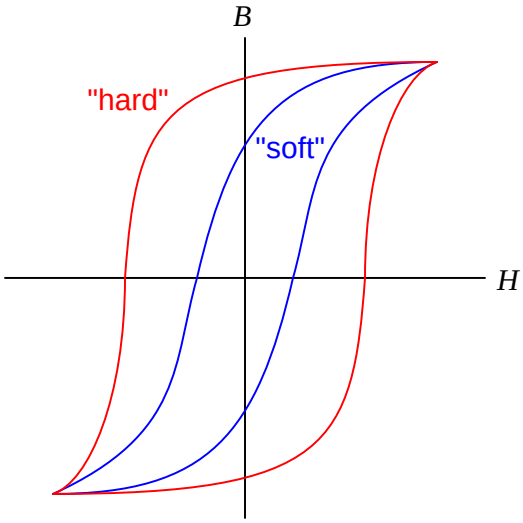
[10] Hysteresis is not limited to ferromagnetism, and in fact is applicable to many different types of systems. A good general definition for hysteresis is "That property of any system whereby it does not return exactly to its original state following the removal of a stimulus".

The example hysteresis curve shown above is just one possible trajectory among a *family* of curves for any given ferromagnetic substance. A fuller plot of multiple hysteresis curves appears next:



As you can see, the B-H curve is never linear, but it approaches linearity for small values of H. Strong applications of magnetizing force result in non-linear magnetic behavior. If we are in the business of creating *permanent magnets* (i.e. magnets designed to remain magnetic on their own without aid of a magnetizing current), then saturation is the condition we strive for. If our goal is to have magnetic field strength (B) track as proportionately as possible to the applied magnetizing force (H) then we must avoid saturation. One way to avoid saturation is to design the ferromagnetic core with a large cross-sectional area, avoiding high *concentrations* of magnetic flux, but this of course increases the size, weight, and expense of the electromagnet.

Interestingly, ferromagnetic materials will dissipate energy in the form of heat as they are magnetized and de-magnetized, and the amount of energy dissipated with each cycle is proportional to the geometric area enclosed by the material's hysteresis curve. The distinction between materials having a small enclosed area to their B-H curves versus a large enclosed area is that of "soft" versus "hard" magnetic materials, respectively:



Magnetically soft materials work well as cores for devices such as *transformers* which are energized by alternating current (AC) for the purpose of converting between different levels of voltage and current. Transformers need to transfer electrical energy from one circuit to another with maximum energy efficiency, and so "soft" magnetic materials help this goal by dissipating as little energy as possible during the magnetization and demagnetization cycle. Conversely, there are some applications where we *want* the magnetization/demagnetization cycle to dissipate maximum energy, such as when electromagnets are used to dissipate the energy of AC electrical "noise" in a circuit, or when designing an object to present the smallest possible radar signature[11], and for those specialized applications we choose classes of magnetic materials (e.g. *hard ferrite*) exhibiting very wide hysteresis curves.

[11] Radio waves consist of alternating electric and magnetic fields radiating through space, those two fields mutually supporting one another. When a radio wave encounters a magnetically "hard" material, the magnetic-field component of the wave tries to magnetize and de-magnetize the material and in so doing expends much of its energy in the form of heat, leaving far less energy available to return as an echo for the radar system to detect.

3.7 Electromagnetic induction

A phenomenon complementary to electromagnetism is *electromagnetic induction*. This occurs when a conductor experiences a perpendicular magnetic field varying in strength over time. This is how *generators* are constructed: by moving coils of wire past a stationary magnetic field, or vice-versa. The amount of voltage induced across the coil is proportional to the number of turns of wire in that coil and to the magnetic field's rate-of-change over time. This mathematical relationship is expressed in *Faraday's Law of Electromagnetic Induction*:

$$V = N\frac{d\Phi}{dt}$$

Where,

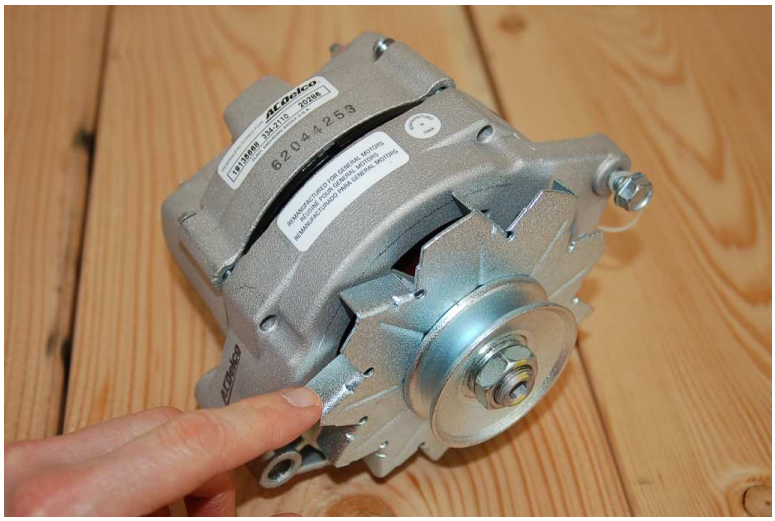
V = Induced voltage across the coil, in Volts
N = Number of "turns" or "wraps" in the coil
$\frac{d\Phi}{dt}$ = Rate-of-change of magnetic flux[12], in Webers per second

Electromagnetic induction is proportional to the *rate* at which a magnetic field strengthens or weakens perpendicular to a conductive coil. This means a voltmeter connected to a wire coil will register nothing when a nearby magnet is stationary, but will register voltage every time the magnet *moves* (either toward the coil or away from it):

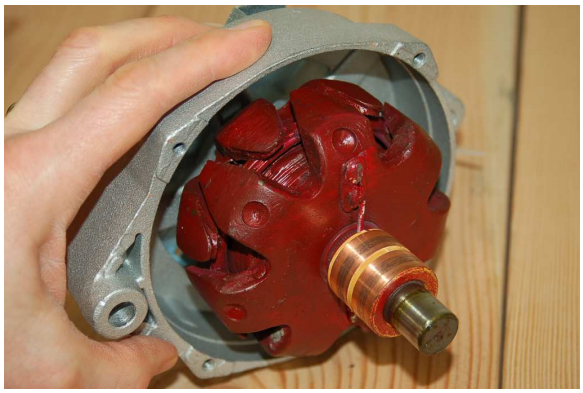
[12] Magnetic flux is related to magnetic field strength by cross-sectional area, with $B = \frac{\Phi}{A}$. Thus, it is valid to think of magnetic field strength as the degree to which magnetic flux is *concentrated* into an area. This holds true for the units of measurement used to express each: the unit of the *Tesla* used for magnetic field strength (B) is equivalent to *Webers of flux per square meter*.

As previously mentioned, electromagnetic induction is the principle by which electrical generators function. All electro-mechanical generators used for generating electrical power in the world function in basically the same way: using relative motion between a magnetic field and one or more coils of wire to induce voltage in those coils of wire.

A simple example of such a machine is the Delco-Remy model 10SI "alternator" used for years in many models of General Motors automobiles to generate electricity from the engine's mechanical power. A photograph of one in isolation is shown here, the mechanical power coupled from engine to alternator by a V-belt (not shown):



Upon disassembly of this machine's case, we see the rotor as a red-colored iron assembly containing a single coil of wire. In the Delco-Remy model 10SI, the rotor's coil is energized with DC from the automobile's battery to create a magnetic field, the North and South poles of that field existing between the "fingers" of the rotor's iron periphery:



The engine's shaft rotation spins this rotor at a high rate of speed.

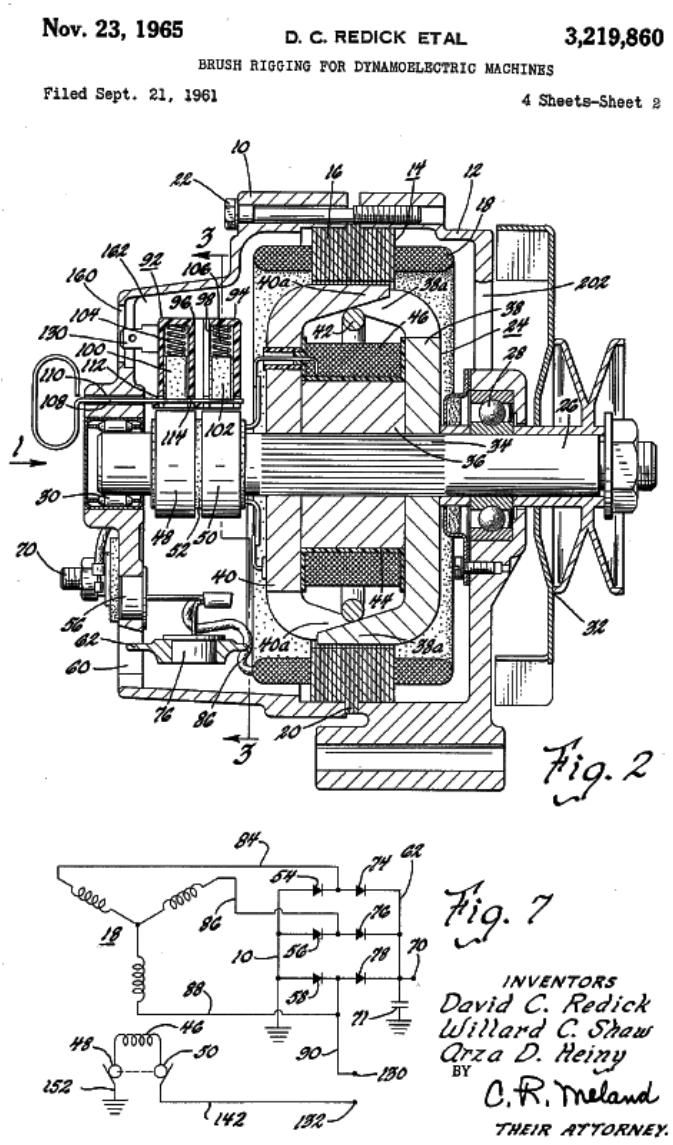
Surrounding this spinning rotor is the *stator* assembly, consisting of a set of iron poles around which enameled copper wire is wrapped into coils. These coils experience rapid growth and decay of magnetism as the rotor's North- and South-polarized "fingers" rotate past, thus inducing voltage in those stator coils. This induced voltage[13] is then tapped as a source to replenish the automobile's battery with electrical energy:

Thus, in this one machine we see both electromagnetism and electromagnetic induction at work. The phenomenon of electromagnetism finds application in the rotor, where DC from the vehicle's battery energizes the rotor coil to produce North and South poles at the rotor's tips. The phenomenon of electromagnetic induction applies to the stator, where the rapidly-rotating magnetic field from the spinning rotor induces voltage in the stator's wire coils. Together, these two phenomena along with the mechanical design of the machine work to convert mechanical energy into electrical energy, and do so in such a way as to make the electrical output easily controllable.

Controllability is why the rotor has been manufactured to be an electromagnet rather than a permanent magnet. If the rotor were permanently magnetized, the amount of voltage output by this machine would always be proportional to its shaft speed. However, if we include a control circuit to vary the amount of current passed through the rotor coils, we may intentionally strengthen or weaken the rotor's magnetic field at will, and use that adjustable field strength to keep the machine's output voltage at the desired level despite changes in shaft speed resulting from the vehicle's engine spinning at different speeds at different times.

[13] Which happens to be alternating (AC) rather than continuous (DC), hence the term "alternator" used to describe the Delco-Remy 10SI machine.

A page from a 1965 patent awarded to David Redick, Willard Shaw, and Arza Heiny of the General Motors Corporation shows a detailed cross-sectional diagram of this alternator:

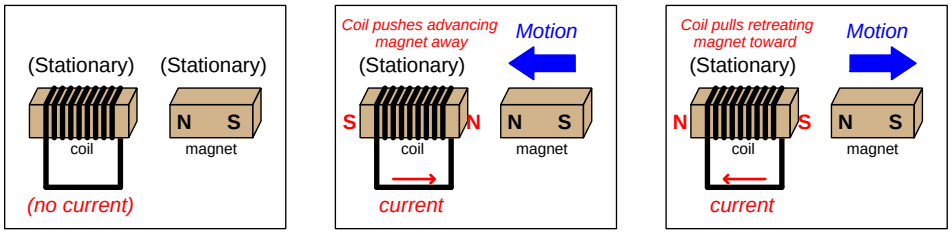


The schematic diagram at the bottom of this page shows the singular rotor coil (46) and the three-coil stator assembly (18). *Diodes* serving as one-way valves for electric current (54, 56, 58, 74, 76, 78) *rectify* the stator coils' AC voltage into DC suitable for re-charging the vehicle's battery.

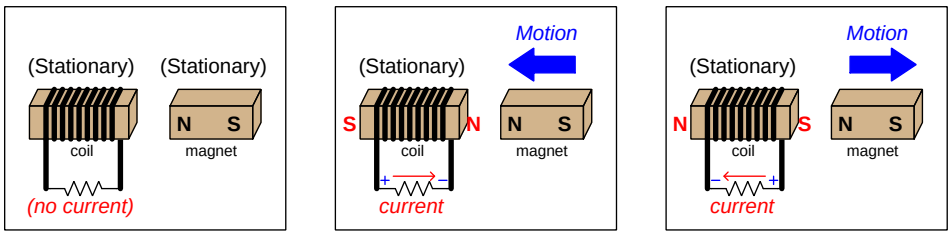
3.8 Lenz's Law

If a coil of conductive wire is connected to form a complete circuit allowing an electric current to form when a magnet advances or retreats, the coil will produce its own magnetic field of such a polarity as to oppose the inducing field's change. This is called *Lenz's Law*, and it stands as a universal feature of induction: *the induced effect always opposes the inducing cause.*

Examine the illustrations below showing a coil of wire facing a permanent magnet. In the left-hand illustration the magnet is stationary and therefore no induction occurs. In the center and right-hand illustrations the magnet moves, causing the coil to experience a varying magnetic field and thereby causing induction. The short-circuit permits current as a result of this induction, and we know any electric current through a coil produces its own magnetic field. Lenz's Law tells us this *induced* magnetic field must act to counter the motion of the permanent magnet. Closely examine the North-South magnetic field polarity induced in the coil. and you will see in each case how that induced magnetic polarity "fights against" the instigating motion of the magnet, effectively attempting to maintain a constant distance between the coil and the magnet:

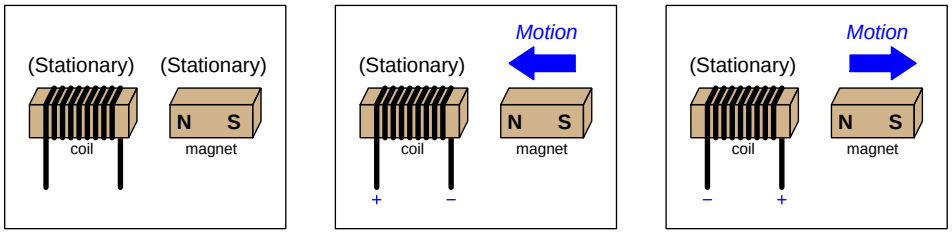


Inserting an electrical resistance into the coil's circuit diminishes the strength of this reaction, but does not alter its direction. Note how the coil acts to repel the magnet when the magnet advances toward the coil, and how the coil acts to attract the magnet when the magnet retreats from the coil:



Note also how the coil functions as a *source* to the resistor's *load* whenever the magnet moves in relation to the coil. This is the conceptual basis of electro-magnetic generators, where magnetized iron poles are moved rapidly past stationary coils of wire in order to generate electricity from the mechanical energy moving those iron poles.

If we break the circuit, no current may pass and therefore no opposing magnetic field can form. However, the coil must still generate a voltage in accordance with Faraday's Law, and the polarity of that voltage will be the same as it would if there were a resistive load connected:



Faraday's Law of Electromagnetic Induction is often (most properly) written with a negative sign in it to mathematically express Lenz's Law: the fact that the induced voltage is polarized in such a way that any resulting current will tend to oppose the direction of the magnetic flux change:

$$V = -N\frac{d\Phi}{dt}$$

Where,

V = Induced voltage across the coil, in Volts

N = Number of "turns" or "wraps" in the coil

$\frac{d\Phi}{dt}$ = Rate-of-change of magnetic flux[14], in Webers per second

In case you were wondering, all these current directions and voltage polarities will reverse if we swap either the permanent magnet's polarity (i.e. magnetic south pole facing the coil) or the direction the coil is wrapped around its core. Reversing both of these parameters is equivalent to a double-negative, resulting in the same current directions and voltage polarities as shown above.

Lenz's Law is consistent with how we know electrical generators behave under different load conditions. Generators work by moving magnets past wire coils (called *windings*) or vice-versa, employing the relative motion between magnet and coil to produce voltage. If a generator is unloaded (i.e. open-circuited terminals) there will be no current as it spins, and with no current there will be no reaction force against the spinning, and so we would expect the generator shaft to spin freely. As we load a generator (i.e. connect it to an electrical load such as a resistor), the magnetic field created by that current in the generator's own windings creates a reaction torque which makes the generator shaft more difficult to spin. A short-circuited generator is extremely difficult to turn, and in fact will act as a mechanical *brake*. This variation in shaft torque makes sense from the perspective of energy conservation: an unloaded generator conveys no electrical energy to any load, and so no energy should be required to spin its shaft other than to overcome bearing friction and aerodynamic drag; a loaded generator, by contrast, delivers electrical energy to a load and therefore must receive an input of mechanical work (i.e. rotation *and* torque) to supply that energy.

[14] Magnetic flux is related to magnetic field strength by cross-sectional area, with $B = \frac{\Phi}{A}$. Thus, it is valid to think of magnetic field strength as the degree to which magnetic flux is *concentrated* into an area. This holds true for the units of measurement used to express each: the unit of the *Tesla* used for magnetic field strength (B) is equivalent to *Webers of flux per square meter*.

Chapter 4

Historical References

This chapter is where you will find references to historical texts and technologies related to the module's topic.

Readers may wonder why historical references might be included in any modern lesson on a subject. Why dwell on old ideas and obsolete technologies? One answer to this question is that the initial discoveries and early applications of scientific principles typically present those principles in forms that are unusually easy to grasp. Anyone who first discovers a new principle must necessarily do so from a perspective of ignorance (i.e. if you truly *discover* something yourself, it means you must have come to that discovery with no prior knowledge of it and no hints from others knowledgeable in it), and in so doing the discoverer lacks any hindsight or advantage that might have otherwise come from a more advanced perspective. Thus, discoverers are forced to think and express themselves in less-advanced terms, and this often makes their explanations more readily accessible to others who, like the discoverer, comes to this idea with no prior knowledge. Furthermore, early discoverers often faced the daunting challenge of explaining their new and complex ideas to a naturally skeptical scientific community, and this pressure incentivized clear and compelling communication. As James Clerk Maxwell eloquently stated in the Preface to his book *A Treatise on Electricity and Magnetism* written in 1873,

> It is of great advantage to the student of any subject to read the original memoirs on that subject, for science is always most completely assimilated when it is in its nascent state . . . `[page xi]`

Furthermore, grasping the historical context of technological discoveries is important for understanding how science intersects with culture and civilization, which is ever important because new discoveries and new applications of existing discoveries will always continue to impact our lives. One will often find themselves impressed by the ingenuity of previous generations, and by the high degree of refinement to which now-obsolete technologies were once raised. There is much to learn and much inspiration to be drawn from the technological past, and to the inquisitive mind these historical references are treasures waiting to be (re)-discovered.

4.1 Electric-field instruments

The attractive force generated by an electric field stretching between two oppositely-charged bodies is sufficient to employ as a means for measuring voltage. Some early voltmeters utilized this principle, since a voltage of greater intensity results in a stronger electric field and therefore a stronger attractive force. If any mechanism is built to respond to this force, it will function as a voltage-measuring instrument.

Perhaps the first "electrostatic voltmeter" was a device called an *electroscope*. An electroscope consists of a two delicately thin strips of hammered gold (called *gold leaf*) hung from the bottom of a metal rod situated at the mouth of an upright glass jar. The jar is flanked by metal rods to either side of the gold leaf strips. When a sufficiently high voltage is impressed between the leaf-rod and the rods outside the jar, the gold leaf strips spread apart in an inverted-V shape. The greater the voltage, the further apart these thin metal strips spread. An illustration from page 63 of *Electrical Installations*, Volume 1 by Rankin Kennedy (published in 1902) shows a sketch of an electroscope in action:

volts only, as the heating at the low pressure becomes very feeble on a wire long enough to stand 120 volts. Properly made and used hot-wire instruments have no serious errors; they are dead beat, and are equally correct on continuous or alternating circuits for measuring current and pressure.

Electrostatic instruments were the earliest of all electrical measuring apparatus. The old gold-leaf electroscope was a high-pressure voltmeter, modifications of which are now the most successful high-pressure voltmeters of to-day. In Fig. 60 we have the simple electroscope, containing two gold leaves and two earth-connected upright rods, to which the leaves are attracted when electrified. The lines of force are shown: these lines are like stretched rubber filaments, tending to contract and draw the leaves to the nearest objects, the rods, in the induction circuit; but the pull due to the force is resisted by the weight of the gold leaves, and the angle of deflection is roughly an indication of the electric pressure

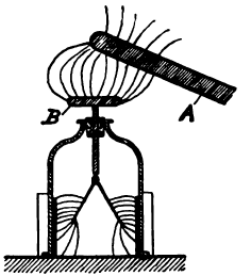


FIG. 60

63

Rod "A" in this illustration is some insulating material which has been statically charged, probably by friction with some other insulating object. The resulting electric charge imbalance on the surface of this object "induces" an opposite charge on the top plate (B) of the electroscope, which causes the gold-leaf foil strips to spread apart.

Electroscopes tend to be useful only for high voltages (in the range of *thousands* of volts), suitable for experiments with static electricity but little else. The basic principle of electrostatic attraction, however, may be employed in more sensitive mechanisms to greater effect.

A photograph of an actual electroscope is shown in the following photograph, this particular electroscope being an historical relic:

Note the graduated scale etched on a glass plate situated below the movable metal leaf, in no particular units of measurement.

The following illustrations, both from page 65 of Kennedy's book, show more sophisticated electrostatic voltmeter designs. The instrument in figure 67 relies on electrostatic attraction to pull a metal rod (a) farther into a metal tube (b) as applied voltage (from source c) increases. Figure 68 shows an instrument employing the attractive force of the electric field between a movable metal vane and a nearby metal shell to draw a pointer across a dial face against the restraining force of a small weight, causing the vane to tilt counter-clockwise with increasing voltage:

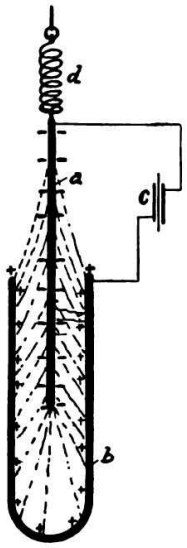


FIG. 67

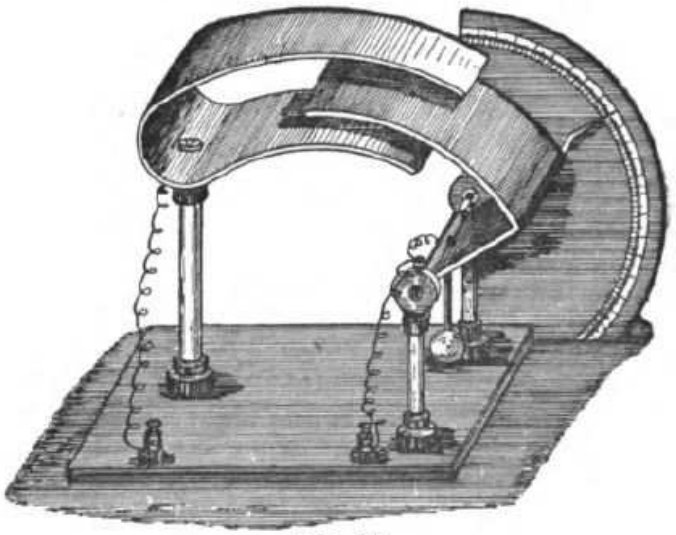


FIG. 68

It is interesting to note that these voltmeters respond equally well to DC of either polarity, and even to AC voltage, because the two conductors will always have opposite polarity and therefore will always attract one another.

4.2 Michael Faraday's experiments on electromagnetic induction

Michael Faraday describes in some detail his discovery of electromagnetic induction, by which a varying magnetic field produced by an intermittently-energized conductor is able to induce an electric current through another conductor. What follows is Faraday's account written on 24 November 1831, as recorded in the pages of his first volume of *Experimental Researches In Electricity* published in 1839. Faraday begins by defining "induction" in the electrostatic sense, describing how an electrically charged object ("electricity of tension") is able to *induce* an oppositely-charged state in a nearby object, and then proceeds to apply this term as a more general principle to the generation of an electric current ("electricity in motion") through a wire coil resulting from a pulsed current through an adjacent wire coil:

> 1. The power which electricity of tension possesses of causing an opposite electrical state in its vicinity has been expressed by the general term Induction; which, as it has been received into the scientific language, may also, with propriety, be used in the same general sense to express the power which electric currents may possess of inducing any particular state upon matter in their immediate neighborhood, otherwise indifferent. It is with this meaning that I purpose using it in the present paper.
>
> 2. Certain effects of the induction of electrical currents have already been recognized and described: as those of magnetization; Ampère's experiments of bringing a copper disk near to a flat spiral; his repetition with electro-magnets of Arago's extraordinary experiments, and perhaps a few others. Still it appeared unlikely that these could be all the effects which induction by currents could produce; especially as, upon dispensing with iron, almost the whole of them disappear, whilst yet an infinity of bodies, exhibiting definite phenomena of in- `[page 1]`
>
> duction with electricity of tension, still remain to be acted upon by the induction of electricity in motion.
>
> 3. Further: Whether Ampère's beautiful theory were adopted, or any other, or whatever reservation were mentally made, still it appeared very extraordinary, that as every electric current was accompanied by a corresponding intensity of magnetic action at right angles to the current, good conductors of electricity, when placed within the sphere of this action, should not have any current induced through them, or some sensible effect produced equivalent in force to such a current.
>
> 4. These considerations, with their consequence, the hope of obtaining electricity from ordinary magnetism, have stimulated me at various times to investigate experimentally the inductive effect of electric currents. I lately arrived at positive results; and not only had my hopes fulfilled, but obtained a key which appeared to me to open out a full explanation of Arago's magnetic phenomena, and also to discover a new state, which may probably have great influence in some of the most important effects of electric currents.
>
> 5. These results I purpose describing, not as they were obtained, but in such a manner as to give the most concise view of the whole. `[page 2]`

1. *Induction of Electric Currents*

6. About twenty-six feet of copper wire one twentieth of an inch in diameter were wound round a cylinder of wood as a helix, the different spires of which were prevented from touching by a thin interposed twine. This helix was covered with calico, and then a second wire applied in the same manner. In this way twelve helices were superposed, each containing an average length of wire of twenty-seven feet, and all in the same direction. The first, third, fifth, seventh, ninth, and eleventh of these helices were connected at their extremities end to end, so as to form one helix; the other were connected in a similar manner; and this two principal helices were produced, closely interposed, having the same direction, not touching anywhere, and each containing one hundred and fifty-five feet in length of wire.

7. One of these helices was connected with a galvanometer, the other with a voltaic battery of ten pair of plates four inches [page 2]

square with double coppers and well charged; yet not the slightest sensible deflection of the galvanometer needle could be observed.

8. A similar compound helix, consisting of six lengths of copper and six of soft iron wire, was constructed. The resulting iron helix contained two hundred and fourteen feet of wire, the resulting copper helix two hundred and eight feet; but whether the current from the trough[1] was passed through the copper or the iron helix, no effect upon the other could be perceived at the galvanometer.

9. In these and many similar experiments no difference in action of any kind appeared between iron and other metals.

10. Two hundred and three feet of copper wire in one length were coiled round a large block of wood; other two hundred and three feet of similar wire were interposed as a spiral between the turns of the first coil, and metallic contact everywhere prevented by twine. One of these helices was connected with a galvanometer, and the other with a battery of one hundred pairs of plates four inches square, with double coppers, and well charged. When the contact was made, there was a sudden and very slight effect at the galvanometer, and there was also a similar slight effect when the contact with the battery was broken. But whilst the voltaic current was continuing to pass through the one helix, no galvanometrical appearances nor any effect like induction upon the other helix could be perceived, although the active power of the battery was proved to be great, but its heating the whole of its own helix, and by the brilliancy of the discharge when made through charcoal.

11. Repetition of the experiments with a battery of one hundred and twenty pairs of plates produced no other effects; but it was ascertained, both at this time and the former time, that the slight deflection of the needle occurring at the moment of completing the [sic] connexion, was always in one direction, and that the equally slight deflection produced when the contact was broken, was in the other direction; and also, that these effects occurred with the first helices were used (6. 8.).

[1] A "trough" in Faraday's time referred to an electrochemical battery with an open-top glass vessel containing the liquid electrolyte.

> 12. The results which I had by this time obtained with magnets led me to believe that the battery current through one wire, did, in reality, induce a similar current through the other [page 3]
>
> wire, but that is continued for an instant only, and partook more of the nature of the electrical wave passed through from the shock of a common Leyden jar than of the current from a voltaic battery, and therefore might magnetize a steel needle, although it scarcely affected the galvanometer. [page 4]

Faraday then goes on to describe how this apparatus might be used to magnetize a steel needle. After that, he continues with a somewhat different apparatus consisting of two wires mounted to different surfaces, allowing one wire to be moved with respect to the other as a galvanometer registers any induced effects by its needle indicator:

> 18. In the preceding experiments the wires were placed near to each other, and the contact of the inducing one with the battery made with the inductive effect was required; but as the particular action might be supposed to be exerted only at the moments of making and breaking contact, the induction was produced in another way. Several feet of copper wire were stretched in wide zigzag forms, representing the letter W, on one surface of a broad board; a second wire was stretched in precisely similar forms on a second board, so that when brought near the first, the wires should everywhere touch, except that a sheet of thick paper was interposed. One of these wires was connected with the galvanometer, and the other with a voltaic battery. The first wire was then moved towards the second, and as it approached, the needle was deflected. Being then removed, the needle was deflected in the opposite direction. By first making the wires approach and then recede, simultaneously with the vibrations of the needle, the latter soon became very extensive; but when the wires ceased to move from or towards each other, the galvanometer needle soon came to its usual position.
>
> 19. As the wires approximated, the induced current was in the *contrary* direction to the inducing current. As the wires receded, the induced current was in the *same* direction as the inducing current. When the wires remained stationary, there was no induced current.
>
> 20. When a small voltaic arrangement was introduced into the circuit between the galvanometer and its helix or wire, so as to cause a permanent deflection of 30^o or 40^o, and then the battery of one hundred pairs of plates connected with the inducing wire, there was an instantaneous action as before; but the galvanometer-needle immediately resumed and retained its place unaltered, notwithstanding the continued contact of the inducing wire with the trough; such was the case in whichever way the contacts were made. [page 5]
>
> 21. Hence it would appear that collateral currents, either in the same or in opposite directions, exert no permanent inducing power on each other, affecting their quantity or tension. [page 6]

In order to avoid confusion with *static induction* whereby a statically-charged object "induces" an opposite static charge on any nearby object, Faraday then begins to refer to this new current-based form of induction as *volta-electric induction* (page 7). He proceeds to explain variations on this experimental theme, where the wire coils were wrapped around an iron ring rather than around

blocks of wood (pages 7-9), and after that an experiment where wire coils were wrapped around a paper tube through which various materials (iron, copper) could be inserted as magnetic "cores" (pages 9-10). In all cases, the presence of iron greatly enhanced the inductive effects.

After this, Faraday proceeds to describe an experiment demonstrating how permanent magnets could be used to induce electric currents:

> 36. Similar effects were then produced by *ordinary magnets:* thus the hollow helix just described has all its elementary helices connected with the galvanometer by two copper wires, each five feet in length; the soft iron cylinder was introduced into its axis; a couple of bar magnets, each twenty-four inches long, were arranged with their opposite poles at one end in contact, so as to resemble a horse-shoe magnet, and then contact made between the other poles and the ends of the iron cylinder, so as to convert it for the time into a magnet: by breaking the magnetic contacts, or reversing them, the magnetism of the iron cylinder could be destroyed or reversed at pleasure.
>
> 37. Upon making magnetic contact, the needle was deflected; continuing the contact, the needle became indifferent, and resumed its first position; on breaking the contact, it was again deflected, but in the opposite direction to the first effect, and then it again became indifferent. When the magnetic contacts were reverse the deflections were reversed.
>
> 38. When the magnetic contact was made, the deflection was such as to indicate an induced current of electricity in the opposite direction to that fitted to form a magnet, having the same polarity as that really produced by contact with the bar magnets. Thus when the marked and unmarked poles were placed as in fig. 3, the current in the helix was in the direction represented, P being supposed to be the end of the wire going to the positive pole of the battery, or that end towards which the zinc plates face, and N the negative wire. Such a current would have converted the cylinder into a magnet of the opposite kind to that formed by contact with the poles A and B; and such a current moves in the opposite direction to the currents which in M. Ampère's beautiful theory are considered constituting a magnet in the position figured. `[page 10]`
>
> 39. But as it might be supposed that in all the preceding experiments of this section, it was by some peculiar effect taking place during the formation of the magnet, and not by its mere virtual approximation, that the momentary induced current was excited, the following experiment was made. All the similar ends of the compound hollow helix (34.) were bound together by copper wire, forming two general terminations, and these were connected with the galvanometer. The soft iron cylinder (34.) was removed, and a cylindrical magnet, three quarters of an inch in diameter and eight inches and a half in length, used instead. One end of this magnet was introduced into the axis of the helix (fig. 4.), and then, the galvanometer-needle being stationary, the magnet was suddenly thrust in; immediately the needle was deflected in the same direction as if the magnet had been formed by either of the two preceding processes (34. 36.). Being left in, the needle resumed its first position, and then the magnet being with drawn the needle was deflected in the opposite direction. These effects were not great; but by introducing and with drawing the magnet, so that the impulse each time should be added to those previously communicated to the needle, the latter could be made to vibrate through an arc of 180^o or more.

> 40. In this experiment the magnet must not be passed entirely through the helix, for then a second action occurs. When the magnet is introduced, the needle at the galvanometer is deflected in a certain direction; but being in, whether it be pushed quite through or withdrawn, the needle is deflected in a direction the reverse of that previously produced. When the magnet is passed in and through at one continuous motion, [page 11]
>
> the needle moves one way, is then suddenly stopped, and finally moves the other way.
>
> 41. If such a hollow helix as that described (34.) be laid east and west (or in any other constant position), and a magnet be retained east and west, its marked pole always being one way; then whichever end of the helix the magnet goes in at, and consequently whichever pole of the magnet enters first, still the needle is deflected the same way: on the other hand, whichever direction is followed in withdrawing the magnet, the deflection is constant, but contrary to that due to its entrance. [page 12]

Summarizing the similarities between the battery-powered experiments and the permanent magnet experiments, Faraday says the following:

> 58. The similarity of action, almost amounting to identity, between common magnets and either electro-magnets or volta-electric currents, is strikingly in accordance with and confirmatory of M. Ampère's theory, and furnishes powerful reasons for believing that the action is the same in both cases; but, as a distinction in language is still necessary, I propose to call the agency thus exerted by ordinary magnets, *magneto-electric* or *magnelectric* induction.
>
> 59. The only difference which powerfully strikes the attention as existing between volta-electric and magneto-electric induction, is the suddenness of the former, and the sensible time required by the latter; but even in this early state of investigation there are circumstances which seem to indicate, that upon further inquiry this difference will, as a philosophical distinction, disappear. [page 16]

Indeed, with time and further research this distinction has disappeared: Faraday's observation that volta-electric induction was quicker than magneto-electric induction was purely a function of how rapidly the magnetic field could vary. When making and breaking a contact between an electromagnet coil and a battery, the transition from no magnetism to full magnetism is very rapid; when making and breaking "contact" between a permanent magnet and a soft iron bar, a much more gradual rise and fall of magnetism occurs because there is still some magnetism passing from the magnet to the iron bar even with a slight air gap between them.

4.3 Mapping magnetic fields

John Ambrose Fleming, in his book *Magnets and Electric Currents*, shows on page 48 an image of a bar magnet's field lines made by sprinkling steel filings[2] near the magnet:

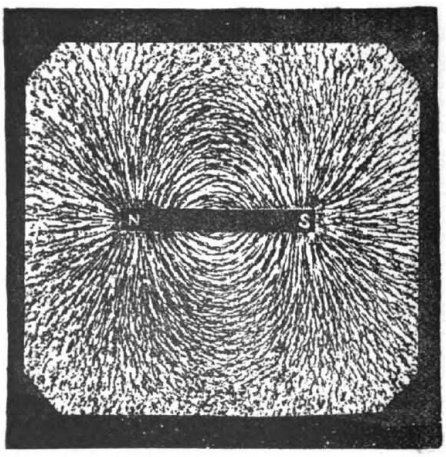


Fig. 11.—Lines of Magnetic Flux round a Bar Magnet delineated by Steel Filings.

In another of his books on elementary electricity, *Electric Lamps and Electric Lighting*, Fleming shows a series of illustrations demonstrating the shapes of magnetic fields around current-carrying wires. These two illustrations were found on pages 195 and 196, respectively:

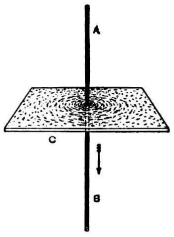


FIG. 66.—Curves formed by sprinkling Iron Filings on a Card C, perforated by a Wire conveying an Electric Current, thus showing the lines of Circular Magnetism round the Conductor A B.

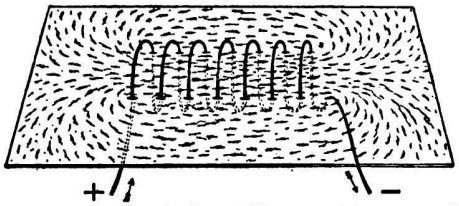


FIG. 67.—Curves formed by Iron Filings sprinkled on a Card, through holes in which a spiral wire is laced, showing the lines of Magnetic Flux or Induction linked with a Spiral Electric Current.

[2] Anyone who has ever used a grinding machine to shape iron or steel parts knows what these "filings" look like: tiny slivers of metal.

Later in *Electric Lamps and Electric Lighting* Fleming shows the magnetic fields produced by three different magnetized iron rings, one gapless (page 205) and others with increasingly large gaps (pages 206-207). Dashed circles drawn inside each ring represent the intensity of the magnetic field within, while the orientation of filings outside the ring show magnetic field lines outside:

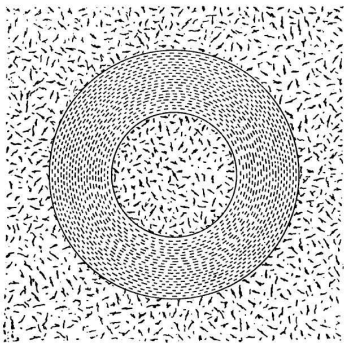


FIG. 71.—Iron Ring magnetised in lines round the ring, but producing no external magnetic field or influence.

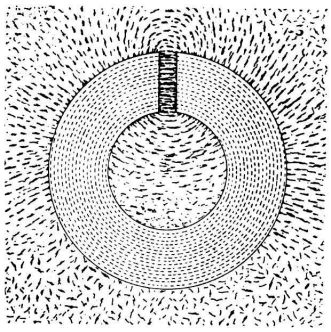


FIG. 72.—Magnetised Iron Ring with cut or air-gap. The ring is magnetised along the dotted lines. The external magnetic field or induction is delineated by iron filings.

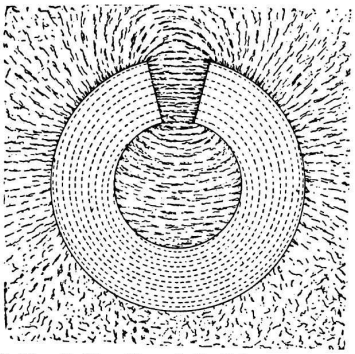


FIG. 73.—Magnetised Iron Ring, as in Fig. 72, but with wider air-gap.

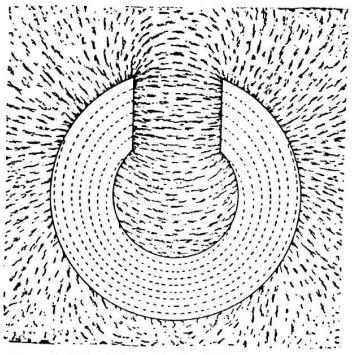


FIG. 74.—Magnetised Iron Ring, as in Fig. 72, but with still wider air-gap.

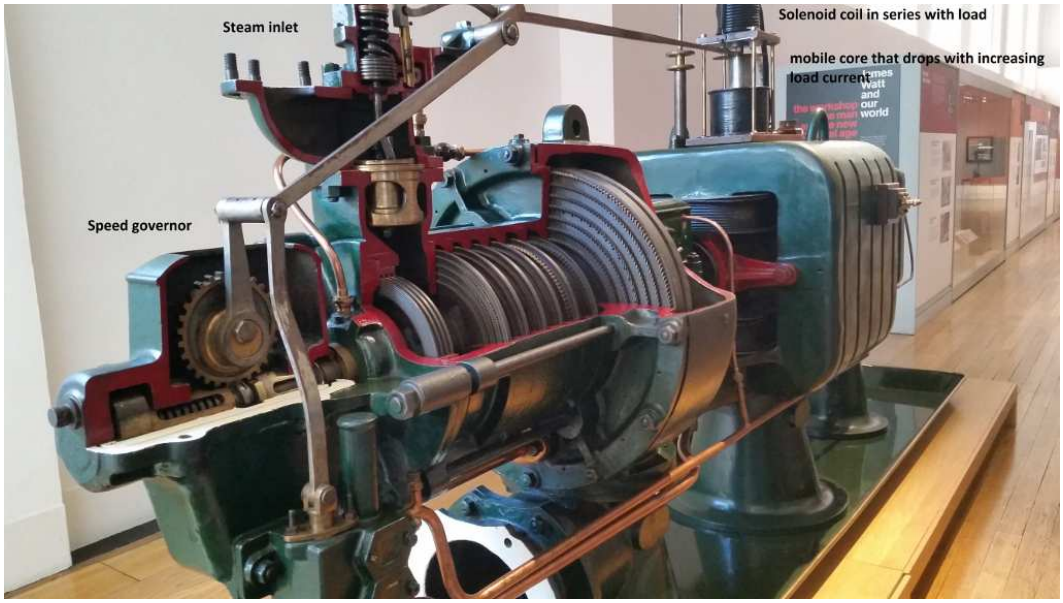
As the gap widens, more magnetism "leaks" from the iron toroid, and the field within the toroid weakens because the air gap presents greater opposition to the establishment of magnetic flux.

The upper-left illustration from page 205 shows just how effective continuous annular iron rings (also called *toroids*) are in confining their magnetic fields. *Inductors* and *transformers* using toroidal iron cores exhibit very little "leakage" flux escaping the core, and for this same reason they pose negligible magnetic interference to nearby components.

4.4 Solenoid-driven turbine control

Electromagnetism found widespread application soon after its discovery. Perhaps the most obvious of these 19th century applications include electric motors and electric generators, machines designed to convert electrical into mechanical energy, and mechanical into electrical energy, respectively.

In the following photograph[3] we see another, more subtle, application of electromagnetism. This photograph was taken within a museum, at an exhibit showcasing early electrical technology. In it we see a cutaway steam turbine engine coupled to a generator. Steam (most likely from a coal-fired boiler) drove the blades of the turbine, converting thermal energy into mechanical energy, which then spun the generator's shaft to convert into electrical energy:



The subtle application of electromagnetism suggested in the previous paragraph is the *solenoid coil* visible on the right-hand side of the image, linked to a lever arm which in turn influenced the speed governor mechanism for the turbine. A "solenoid" is nothing more than a coil of wire, typically wrapped around a cylindrical ferromagnetic core to produce strong magnetic fields. These magnetic fields, in turn, could be employed to attract other coils or ferromagnetic objects by way of magnetic attraction. This particular solenoid coil was wired in series with the output of the generator such that any changes in load current would immediately produce force on the lever arm to preload the governor mechanism and thereby alter the amount of steam fed to the turbine by the throttling valve mechanism near the steam inlet pipe. Any increase in load current, for example, would automatically result in more steam being fed to the turbine to provide more mechanical power to the generator.

[3] I am indebted to Mario Sotomaior for taking this photograph for me at a museum in London, England.

Speed regulation of such a generator is intended to stabilize the generator's output voltage. If the electrical load placed on this generator's output increases, the generator will become a "heavier" load for the turbine to drive, which in turn has a natural effect of slowing down the turbine. A slower-spinning generator outputs less voltage, all other factors being equal, and so stabilizing generator shaft speed helps the generator to maintain a more constant voltage. The governor assembly detects changes in turbine shaft speed and moves the throttling valve accordingly to correct for those disturbances. However, the governor alone is not the most effective way to stabilize generator output, as it cannot act until *after* the generator's shaft speed noticeably decreases. This is where the solenoid coil helps: by using load current to produce a mechanical force on the lever, any changes in load current *preemptively* adjust the throttling valve's position to increase or decrease steam flow as necessary to counteract the effects of electrical load changes. This means the task of restoring shaft speed does not rest solely with the mechanical governor, resulting in better dynamic stability for the whole generator system.

In control theory, this arrangement is known as *feedforward control*, in contrast to the governor's response being *feedback control*. Feedback control works to stabilize some measured quantity (called the *process variable*) by taking action *after* that measured quantity deviates from the desired level (called a *setpoint*). Feedforward control, however, takes action *before* that process variable has opportunity to deviate from its setpoint, based on the measurement of some other variable having a "loading" effect on the process variable. As you might expect, the combination of feedforward control offered by the load current solenoid and feedback control offered by the turbine's mechanical speed governor work well together to stabilize generator output when faced with rapid changes in load current.

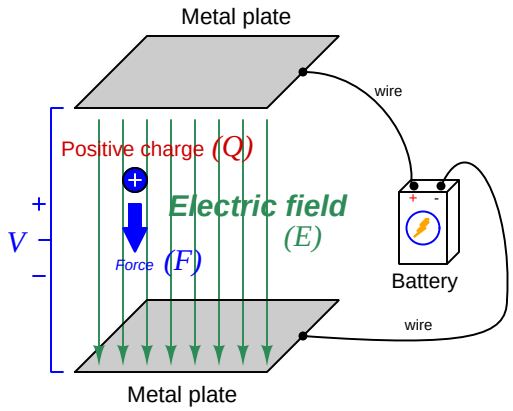
Chapter 5

Derivations and Technical References

This chapter is where you will find mathematical derivations too detailed to include in the tutorial, and/or tables and other technical reference material.

5.1 Electric field quantities

A useful definition of electric field (E) is in terms of the force (F) exerted on an electric charge (Q) influenced by that field:



$$\vec{F} = Q\vec{E}$$

Where,

$\vec{F}$ = Force exerted on the charge (Newtons)
Q = Charge quantity (Coulombs[1])
$\vec{E}$ = Electric field (Newtons per Coulomb)

The small "arrow" symbols above the variables for force and electric field in the equation denote those variables as *vector quantities*, having both magnitude and direction. Charge is a *scalar quantity* having only magnitude but no direction, and as a scalar quantity when multiplied by the electric field vector it simply magnifies the magnitude but does not alter the direction. Therefore, the force and electric field vectors always point in the same direction.

Alternatively electric field may be defined in terms of the voltage between the end-points and the distance separating them, in which case we may express the electric field in units of *Volts per meter* as an alternative to *Newtons per Coulomb*:

$$\vec{E} = \frac{V}{\vec{d}}$$

This measurement of electric field strength is very important for quantifying the *breakdown* of electrical insulators: the point at which the electric field becomes so powerful that otherwise immobile charges within the insulating substance are torn free to constitute a current and that substance is no longer an insulator. For rating the dielectric strength of insulating materials, we often see electric fields expressed in units of *kiloVolts per millimeter* rather than Volts per meter just to make the numerical quantities easier to manage (1 kV/mm = 1 *million* V/m).

[1] One Coulomb of electric charge is equal to 6.2415×10^{18} electrons.

The vector arrows shown in the previous illustration representing the electric field between two metal plates actually represent electric *flux* (Φ_E). The electric field (E) is related to electric flux by area (A), the field being a measurement of how densely-packed those flux lines are per unit area:

$$\vec{E} = \frac{\Phi_E}{\vec{A}}$$

Where,
$\vec{E}$ = Electric field, or electric flux density (Newtons per Coulomb)
Φ_E = Electric flux (Newton-meter squared per Coulomb)
$\vec{A}$ = Area over which flux is distributed (square meters)

The mere presence of an unbalanced electric charge at any point in space is sufficient to generate lines of electric flux, the total magnitude of that flux predicted by the following equation:

$$\Phi_E = \frac{Q}{\epsilon}$$

Where,
Φ_E = Electric flux (Newton-meter squared per Coulomb)
Q = Charge quantity (Coulombs)
ϵ = Electric permittivity of the surrounding space (Coulombs squared per Newton-meter squared, approximately 8.85×10^{-12} for empty space)

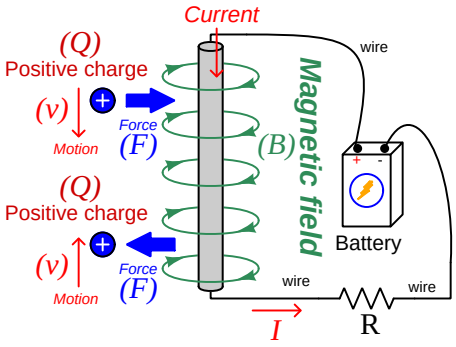
By convention, these flux vectors point *away* from positive charges and point *toward* negative charges, their direction indicating force exerted on any positively-charged particle influenced by that field[2]. As the equation states, the amount of flux depends on how much charge exists at each particle as well as the permittivity of the surrounding space:

For example, identical charges suspended in a vacuum versus in a substance such as oil will have different amounts of flux associated with them as a result of oil and vacuum having different permittivity values. Perfectly empty space has the least amount of permittivity, which means anything else (gas, liquid, or solid matter) has greater ϵ which acts to diminish the amount of electric flux surrounding any charged particle. Superconducting materials have infinite permittivity, which means they forbid the existence of any electric field inside their bulk.

[2]Conversely, the flux vectors point exactly opposite the direction of force applied to any *negatively*-charged particle within that field. This makes sense of the aphorism that *like charges repel and opposite charges attract*. If you consider the two charges shown in this illustration, the positive charge will be pulled in the direction of the flux vectors pointing toward the negative charge, as the negative charge will also be pulled opposite the direction of the flux vectors pointing away from the positive charge (i.e. the negative charge will be pulled *toward* the positive charge) – thus the positive and negative charges feel mutual attraction.

5.2 Magnetic field quantities

A useful definition of magnetic field (B) is in terms of the force (F, called the *Lorentz force*) exerted on a moving electric charge (Q) influenced by that field:



$$\vec{F} = Q\vec{v} \times \vec{B}$$

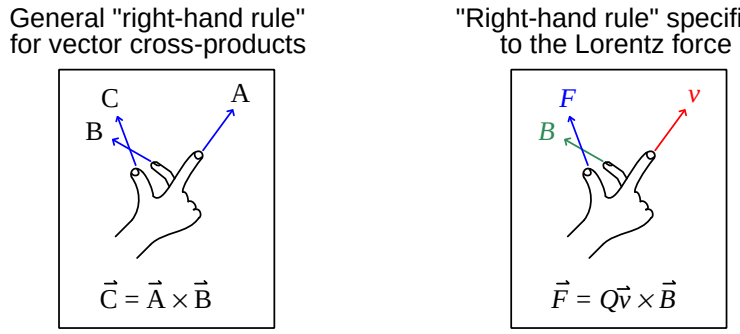
Where,

$\vec{F}$ = Force exerted on the charge (Newtons)
Q = Charge quantity (Coulombs[3])
$\vec{v}$ = Velocity of moving charge (meters per second)
$\vec{B}$ = Magnetic field (Tesla, Webers per square meter, or Newtons per Ampere-meter)

The small "arrow" symbols above the variables for force and velocity and magnetic field in the equation denote those variables as *vector quantities*, having both magnitude and direction. Charge is a *scalar quantity* having only magnitude but no direction, and as a scalar quantity when multiplied by the velocity vector it simply magnifies the magnitude but does not alter the direction. The "cross-product" ($\times$) is a specific form of vector multiplication, and it results in a product at right angles to the vector directions of both terms. Therefore, the force and velocity and electric field vectors never all point in the same direction.

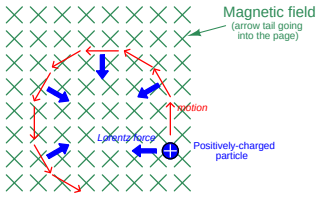
[3] One Coulomb of electric charge is equal to 6.2415×10^{18} electrons.

Vector cross-products conveniently relate to the fingers of the right hand, which is where the "right-hand rule" originates:



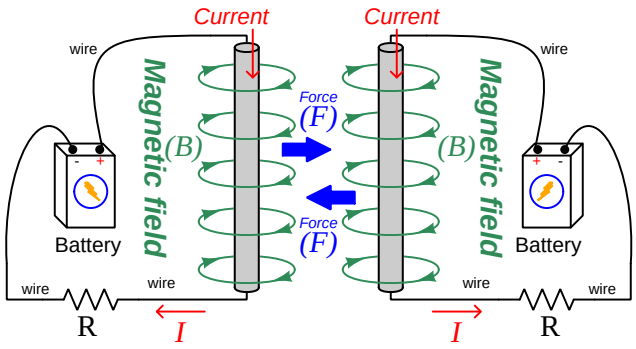
When holding the index finger, middle finger, and thumb of your right hand perpendicular to each other, your index finger points in the direction of the velocity vector ($\vec{v}$), your middle finger in the direction of the magnetic field vector ($\vec{B}$), and your thumb in the direction of the force vector ($\vec{F}$). A simple mnemonic I use to remember these relationships of fingers to vectors is that the **I**ndex finger points in the direction of current[4] (I), the **M**iddle finger points in the direction of the **m**agnetic field (B), and the **Th**umb points in the direction of the **th**rust (i.e. force) acting upon the moving charge.

The Lorentz force's effect on electrically-charged particles in motion has many applications, from redirecting the paths of charged-particle beams in particle accelerator machines, to bending the trajectory of electron beams in a cathode-ray tube (CRT), to forcing electrons to travel in spiral or circular trajectories inside of magnetron (microwave oscillator) tubes. An illustration of a positively-charged particle curving perpendicular to a magnetic field appears here:



[4]Some textbooks speak of a "left-hand rule" which is intended to make sense of electric charge motion (current) in terms of *electron flow*. As we know, electrons are the only real mobile charge carriers within metal conductors, and so technically "electron flow" notation is most physically accurate when describing the motion of electric charges in metallic circuits. However, the right-hand rule is a *mathematical definition* for vector cross products, the concept of the cross product arising in the late 18th century when electrical science was still in its infancy. Early explorers of electricity used the established mathematical tools of their time and applied it to their work with electric currents and magnetism. At that time, charge carriers in metal wires were assumed to be "positive" and this is how the motion of positively-charged carriers became associated with the first vector of the cross-product. As a result of this assumption which was later proven false, we have two different conventions for denoting the motion of electricity: electron-flow which is *physically* accurate (for metal wires, at least), and conventional flow which is *mathematically* accurate. This, perhaps more than any other reason, is why educational programs designed for mathematically rigorous fields (e.g. electrical engineering) exclusively use conventional flow notation rather than electron flow notation to denote the direction of current.

If the moving charge in question is not a single charged particle but rather part an electric *current* passing through a conductor parallel to the first, both conductors will experience a mutually-attracting force given by the following equation:



$$\vec{F} = I\vec{l} \times \vec{B}$$

Where,

$\vec{F}$ = Force exerted on both conductors (Newtons)

I = Current (Amperes)

$\vec{l}$ = Length of wire (meters)

$\vec{B}$ = Magnetic field (Tesla, or Webers per square meter, or Newtons per Ampere-meter)

The point-charge Lorentz force equation and the two-conductor Lorentz force equation are not that different from one another. Dimensional analysis validates this: the Lorentz force on a moving charge uses that charge quantity (Coulombs) multiplied by the point-charge's velocity in meters per second to give Coulomb-meters per second for the first term:

$$Q\vec{v} = [\mathrm{C}]\left[\frac{\mathrm{m}}{\mathrm{s}}\right] = \left[\frac{\mathrm{C} \cdot \mathrm{m}}{\mathrm{s}}\right]$$

The Lorentz force on a current-carrying conductor uses the current (Amperes, which is Coulombs per second) multiplied by length in meters, for the same composite units of Coulomb-meters per second:

$$I\vec{l} = \left[\frac{\mathrm{C}}{\mathrm{s}}\right][\mathrm{m}] = \left[\frac{\mathrm{C} \cdot \mathrm{m}}{\mathrm{s}}\right]$$

This dimensional equivalence makes conceptual sense as well: an electrically-charged particle moving through empty space is an electric current in its own right, and an electric current flowing through a conductor is just a collection of charged particles moving through space (just not *empty* space). In either case, the basis for the Lorentz force remains the same: the moving charge(s) create their own magnetic field, which reacts with the magnetic field of the original current-carrying wire to produce forces acting on both.

If the two currents flow in the same direction, their mutual forces *attract*. If the two currents flow in opposite directions, their mutual forces *repel*. This is the basis of electric motors: causing

mechanical motion by electro-magnetic attraction and repulsion. It also represents an interesting contrast with electric fields:

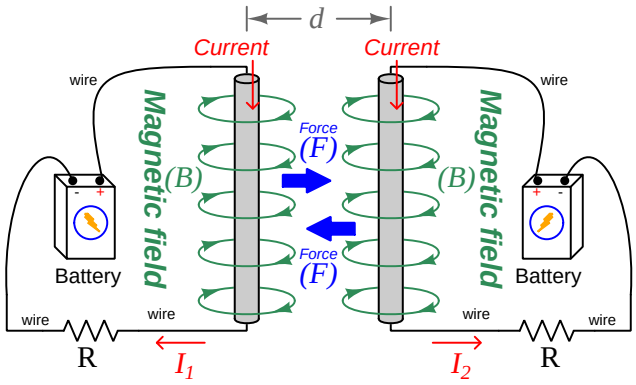
With electric fields, opposite *charges* attract and like *charges* repel.

With magnetic fields, opposite *poles* attract and like *poles* repel.

With parallel currents, opposite *directions* repel and like *directions* attract[5].

Two parallel current-carrying conductors of length l and separated by a distance d will generate a mutual force proportional to both their currents:

$$F = l\frac{\mu I_1 I_2}{2\pi d}$$



[5]That is, assuming it's *like* charges moving in these directions! If the charges in question are opposite each other – for example electrons in one circuit and holes in another – then like directions will repel and opposite directions will attract!

The circular loops surrounding the current-carrying conductors in the previous illustrations represent the magnetic lines of *flux* (Φ_B) surrounding each of those conductors. The magnetic field (B) is related to magnetic flux by area (A), the field being a measurement of how densely-packed those flux lines are per unit area. For this reason, magnetic field (B) is more properly known as *magnetic flux density*:

$$\vec{B} = \frac{\Phi_B}{\vec{A}}$$

Where,

$\vec{B}$ = Magnetic field or flux density (Tesla, Webers per square meter, or Newtons per Ampere-meter)

Φ_B = Magnetic flux (Webers)

$\vec{A}$ = Area over which flux is distributed (square meters)

An older unit of measurement for magnetic flux density B is the *Gauss* which is much smaller than a Tesla, with one Tesla equivalent to 10,000 Gauss. To put things into perspective, the Earth's natural magnetic field has a strength of approximately one-half of one Gauss[6].

Magnetic field strength is an inverse function of distance from any current-carrying wire, and also depends on the magnetic permeability of the space adjacent to the wire:

$$B = \frac{\mu I}{2\pi d}$$

Where,

B = Magnetic field or flux density (Tesla, Webers per square meter, or Newtons per Ampere-meter)

μ = Magnetic permeability of the surrounding space (Tesla-meters per Ampere, $4\pi \times 10^{-7}$ for empty space)

I = Current (Amperes)

d = Distance from conductor (meters)

[6] Using the online *Magnetic Field Calculator* application provided by NOAA (the National Oceanic and Atmospheric Administration) at `https://ngdc.noaa.gov/geomag/calculators/magcalc.shtml#igrfwmm`, applying the World Magnetic Model WMM modeling algorithm for years 2019-2024, the total magnetic field strength at my home is 53,584.4 nano-Tesla (53,584.4 nT or 0.535844 Gauss), and presently (May 2020) decaying at a rate of -104.1 nT per year.

The relation of magnetic *flux* to current through a conductor follows a similar equation:

$$\Phi = \frac{\mu A I}{2\pi d}$$

Where,

Φ = Magnetic flux (Webers)

μ = Magnetic permeability of the surrounding space (Tesla-meters per Ampere, $4\pi \times 10^{-7}$ for empty space)

A = Area over which flux is distributed (square meters)

I = Current (Amperes)

d = Distance from conductor (meters)

As this equation makes clear, the amount of magnetic flux surrounding a current-carrying conductor depends not only on the amount of current, but also on the sampled area, the distance from the wire, and also the surrounding material. Most[7] substances (gas, liquid, solid) have permeability values greater than that of empty space, and so this means magnetic flux is usually *enhanced* by the presence of matter around the current-carrying conductor.

The total magnetic flux enclosed by a circular wire loop follows a similar equation:

$$\Phi = \frac{\pi \mu I r}{2}$$

Where,

Φ = Magnetic flux (Webers)

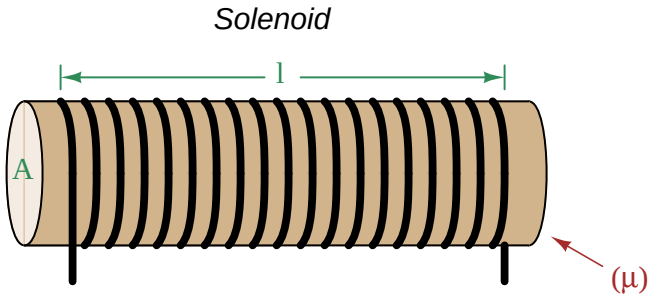
μ = Magnetic permeability of the surrounding space (Tesla-meters per Ampere, $4\pi \times 10^{-7}$ for empty space)

I = Current (Amperes)

r = Radius of circular loop (meters)

[7] Interestingly, superconducting materials *forbid* magnetic fields inside of their bulk, and so the permeability value of any superconductor must be zero!

A common form of electromagnet known as a *solenoid* takes the form of a wire coil wrapped in such a way as to form a long[8] cylinder, often wrapped around a plastic frame, and often with a ferromagnetic material such as iron in the center:



The amount of magnetic flux, and the flux density, within the interior of a current-carrying solenoid are given by the following formulae:

$$\Phi = \frac{\mu N A I}{l} \qquad\qquad B = \frac{\mu N I}{l}$$

Where,

Φ = Magnetic flux (Webers)

B = Magnetic field or flux density (Tesla, Webers per square meter, or Newtons per Ampere-meter)

μ = Magnetic permeability of the surrounding space (Tesla-meters per Ampere, $4\pi \times 10^{-7}$ for empty space)

N = Number of turns of wire in the coil

A = Cross-sectional area of solenoid coil (square meters)

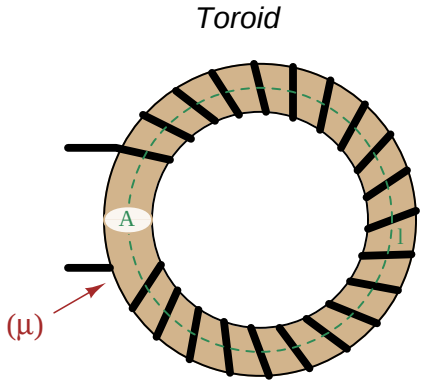
I = Current (Amperes)

l = Length of solenoid coil (meters)

These formulae have interesting implications for solenoid design. Note how a shorter (i.e. smaller length l) solenoid identical in all other respects will generate a stronger magnetic field for a given current. Note also how the flux density (B) remains constant with increasing cross-sectional area (A) if all other factors are equal, and that this necessarily means a greater amount of total magnetic flux (Φ) for a greater area A.

[8] These magnetic field formulae apply perfectly to a solenoid coil that is closely-packed (i.e. each turn adjacent to the next) and infinitely long. Therefore, they only approximate real solenoid behavior. This fact may be understood by performing a thought experiment where we decrease the solenoid coil's length to zero, in which case the formulae predict an *infinite* amount of magnetism for any amount of current at all, which of course cannot be true.

Another common form of electromagnet known as a *toroid* is really just a solenoid bent in a circle so that its two ends meet[9] cylinder, often wrapped around a plastic frame, and often with a ferromagnetic material such as iron in the center. Toroids have the unusual property of *containing* their magnetic flux lines extremely well, unlike solenoids, wires, and simple coils which all radiate magnetic fields. They find application as energy-storage devices, or as electromagnets suitable for applying magnetic fields to specimens placed *inside* the toroid's cross-section:



The amount of magnetic flux, and the flux density, within the interior of a current-carrying toroid are identical to that within an otherwise identical otherwise identical solenoid having a length (l) equal to the toroid's circumference:

$$\Phi = \frac{\mu N A I}{l} \qquad\qquad B = \frac{\mu N I}{l}$$

Where,

Φ = Magnetic flux (Webers)

B = Magnetic field or flux density (Tesla, Webers per square meter, or Newtons per Ampere-meter)

μ = Magnetic permeability of the surrounding space (Tesla-meters per Ampere, $4\pi \times 10^{-7}$ for empty space)

N = Number of turns of wire in the coil

A = Cross-sectional area of toroid (square meters)

I = Current (Amperes)

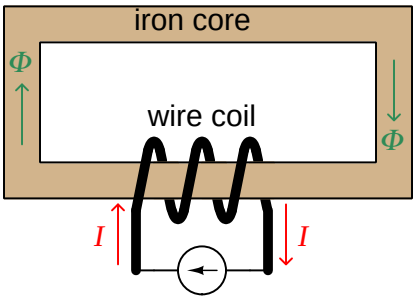
l = Circumference of toroid (meters)

If we wish to substitute toroid radius (r) for circumferential length (l), the formulae become the following:

$$\Phi = \frac{\mu N A I}{2\pi r} \qquad\qquad B = \frac{\mu N I}{2\pi r}$$

[9] Again, the magnetic field formulae are only accurate for a toroidal coil that is closely-packed (i.e. each turn adjacent to the next) and infinitely long, and therefore only approximate real toroid behavior. This fact may be understood by performing an equivalent thought experiment as before where we decrease the toroid's circumference to zero and absurdly end up with *infinite* magnetism for a finite current.

Many applications of electromagnetism involve conductive *coils* wrapped around some form of ferromagnetic core material, the purpose of that core being to provide a higher-permeability pathway for the magnetic flux than would exist otherwise through air, and the purpose of the wire coil being to intensify the amount of magnetism developed by the electric current beyond what would be possible with a straight current-carrying wire. These magnetic cores typically form a closed loop, or *magnetic circuit* for the lines of magnetic flux to naturally form a closed path. A simple example appears here:



The amount of magnetic flux (Φ) present in the magnetic "circuit" formed by the iron core depends on many factors. First and foremost is the amount of electric current (in Amperes) passing through the wire coil and the number of turns that coil makes around the iron core. The product of this current and the number of turns is called the *magnetomotive force* or *mmf* of the magnetic circuit, analogous to "electromotive force" or "emf" often used as a synonym for voltage in an electric circuit. Not surprisingly, the standard metric unit of measurement for magnetomotive force is the *Ampere-turn*.

However, magnetomotive force alone does not fully describe the current's effect on magnetism within the iron core. The total length of the magnetic circuit is also an important factor, since a longer path distributes that magnetomotive force over a greater distance. The quotient of magnetomotive force and magnetic circuit length is called *magnetic field intensity*, symbolized by the variable H and expressed in units of *Ampere-turns per meter*.

Magnetic permeability (μ) relates magnetic field intensity (H) to the magnetic flux density (B) within the core material, such that a greater permeability will result in higher flux density for any given amount of field intensity. Permeability is a property of the core material and not its geometry, mathematically defined as the ratio of flux density to field intensity: $\mu = \frac{B}{H}$

Magnetic reluctance ($\Re$) relates magnetomotive force (mmf) to magnetic flux (Φ), and is related not only to the core material's permeability but also its geometry. It is mathematically defined as the ratio of magnetomotive force to magnetic flux: $\Re = \frac{\text{mmf}}{\Phi}$

If all this seems confusing, you are in good company. Not only are there many magnetic variables, some related to physical geometry and others not, but there are *two different sets of metric units* appropriate for expressing each! The older units were based on the centimeter-gram-second (CGS) version of the metric system, while the newer units are based on the meter-kilogram-second or SI (Système International) version of the metric system.

Quantity	**Symbol**	**SI unit**	**CGS unit**
Magnetomotive force	mmf	Ampere-turn (A-t)	Gilbert (Gb)
Flux	Φ	Weber (Wb)	Maxwell (Mx)
Field intensity	H	Ampere-turns per meter (A-t/m)	Oersted (Oe)
Flux density	B	Tesla (T)	Gauss (G)
Permeability	μ	Tesla-meters per Ampere-turn (T-m/A-t)	Gauss per Oersted (G/Oe)
Reluctance	$\Re$	Ampere-turns per Weber (A-t/Wb)	Gilberts per Maxwell (G/Mx)

Magnetomotive force (mmf) and magnetic flux (Φ) may be thought of as the "raw" measures of magnetism, with Ampere-turns and Webers being their respective SI metric units. Reluctance ($\Re$) is the ratio of the two for any given magnetic circuit with known dimensions and core material. Simply put, reluctance tells you how many Ampere-turns of magnetomotive force will be necessary to create one Weber of magnetic flux in a given space.

Magnetic field intensity (H) and magnetic flux density (B) may be thought of as the "normalized" measures of magnetism, with Ampere-turns per meter and Tesla being their respective SI metric units. H and B relate to mmf and flux by the physical dimensions of the magnetic circuit (length and cross-sectional area, respectively). Permeability is the ratio of the two for any given magnetic core material. Simply put, permeability tells you how many Tesla of magnetic field (i.e. flux density, or Webers of flux per square meter or cross-sectional core area) you will obtain for one Ampere-turn per meter of magnetic field intensity applied to a given core material.

Conversion between the newer SI and the older CGS metric units are as follows:

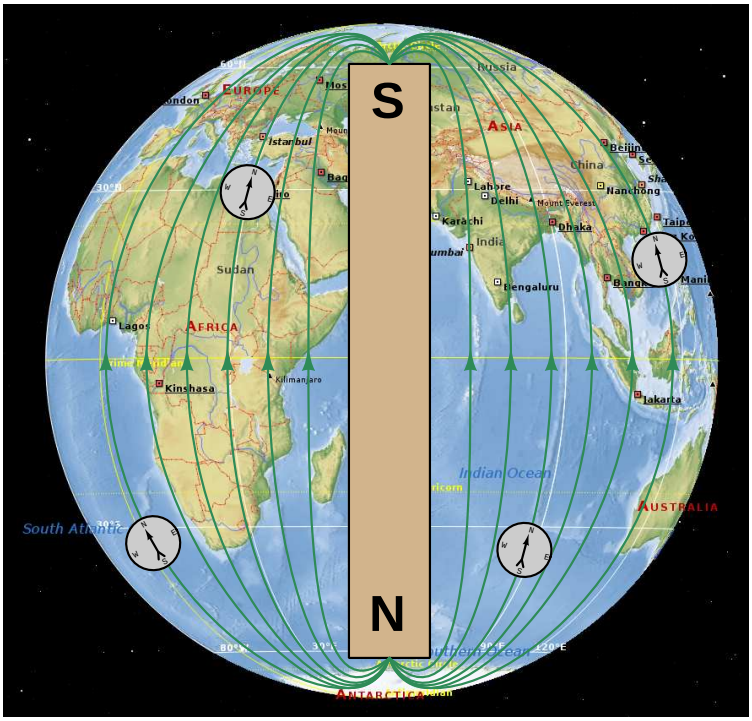
Quantity	**Conversion equivalence**
Magnetomotive force (mmf)	1 Ampere-turn = $\frac{4\pi}{10}$ Gilberts
Magnetic flux (Φ)	1 Weber = 10^8 Maxwells
Magnetic field intensity (H)	1 Ampere-turn/meter = $\frac{4\pi}{1000}$ Oersteds
Magnetic flux density (B)	1 Tesla = 10^4 Gauss
Permeability (μ)	1 Tesla-meter/Ampere-turn = $\frac{10^7}{4\pi}$ Gauss/Oersteds
Reluctance ($\Re$)	1 Ampere-turn/Weber = $\frac{4\pi}{10^9}$ Gilberts/Maxwell

5.3 Defining magnetic north and south

Magnetic pole-pairs are always called *north* and *south*, and this naming hails from the days of navigation by magnetic compass, where a magnetized iron needle would point toward the geographic north pole of the Earth (i.e. toward the Arctic region).

The compass's needle itself is, of course, nothing more than a small magnet with north and south poles. These poles naturally align with the Earth's own magnetic field. Confusion tends to arise when we consider the well-known behavior of magnetic poles to attract when they are opposite and repel when they are identical. Whichever end of a compass needle points toward the Arctic region of the Earth must have the opposite type of magnetic pole as that end of the Earth, and the same must be said of the Antarctic-pointing end of the compass needle.

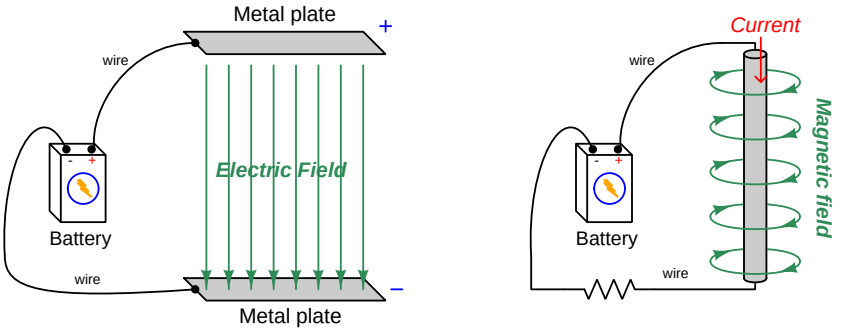
By convention we have come to identify magnetic north and magnetic south in terms of a compass needle's ends: that is to say, the Arctic-pointing end of a compass needle is magnetic north while the Antarctic-pointing end of that same needle is magnetic south. This means the Earth itself must have a magnetically-south Arctic and a magnetically-north Antarctic. The following illustration shows this relationship:



Fortunately for anyone using a magnetic compass to map magnetic fields created by circuits and/or by permanent magnets, discerning the direction of the magnetic field vectors is extremely simple: *the arrow symbol shown on the compass is the vector-direction of the magnetic field.* Note in the above illustration how each of the compass needles points in the same direction as the respective magnetic flux vector, from Antarctic to Arctic.

5.4 Near-field versus far-field regions

An electric field exists wherever voltage exists (i.e. wherever an imbalance exists between positive and negative electrical charges), parallel to the axis of that voltage. A magnetic field exists wherever an electric charge moves, perpendicular to the axis of that charge's motion. These phenomena exist for DC (direct) as well as AC (alternating) electricity, and are shown in the two following illustrations:



Electric and magnetic fields, however, are not simply manifestations of voltage and current, respectively. Mathematical discoveries made by the Scottish physicist James Clerk Maxwell (and later simplified by Oliver Heaviside) also relate electric fields and magnetic fields *directly to each other*. Consider two[10] of Maxwell's equations shown below:

$$\oint \vec{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{B} \cdot d\mathbf{s} = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

The first equation describes how an electric field ($\vec{E}$) is created by a varying magnetic flux ($\frac{d\Phi_B}{dt}$), which is otherwise known as *Faraday's Law of Electromagnetic Induction*. This is the law exploited in the function of electro-mechanical generators whereby coils of wire are subjected to changing magnetic fields, creating voltage between the coils' end-points. The negative sign in this equation is an embodiment of *Lenz's Law*, which states that any current resulting from the induced voltage will produce its own magnetic field opposing the first magnetic field's direction of change.

The second equation shown here describes *two* different ways to produce a magnetic field ($\vec{B}$). One way is to use a moving stream of electric charges known as a *current* (I), a fact also known as *Ampère's Law*. This is the law exploited in the function of electromagnets, where we produce a magnetic field by connecting a coil of wire to an electrical source. The second way is to vary an electric flux ($\frac{d\Phi_E}{dt}$) in empty space with surrounding magnetic permeability μ_0 and electric permittivity ϵ_0.

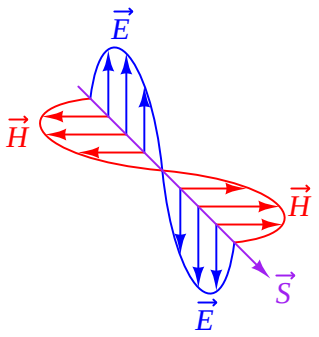
[10] Maxwell's equations of electromagnetism are typically presented as a set of *four*. Here we only list two of these, as the other two are not relevant to electromagnetic waves in particular.

Maxwell's equations suggest that in pulsed DC and AC circuits, where the resulting electric and/or magnetic fields vary over time ($\frac{d}{dt}$) rather than holding steady, those time-varying fields will mutually support one another over space and time. That is, a varying electric field will eventually produce a varying magnetic field, and vice-versa, those fields propagating one another as an *electromagnetic wave* radiating away from the circuit through space at the speed of light[11].

At close range, immediately adjacent to the circuit conductors, electric and magnetic fields are strictly functions of those conductors' voltages and currents, respectively. These fields' shapes follow the basic forms shown in the previous illustrations: magnetic field lines circling current-carrying conductors and electric field lines stretching between conductive surfaces. We generally refer to this region of space around an energized circuit as the *near-field region*, where electric and magnetic fields are distinct from one another in form and in relative magnitude.

At farther distances from the energized circuit, the electric and magnetic fields are found in pairs oscillating at right angles to one another in space, forming a single electromagnetic wave radiating away from the source circuit spherically in all directions. This form of wave is a direct consequence of Maxwell's equations describing how the rate-of-change of one field creates the other field, and vice-versa. Below we see a crude representation of an electromagnetic wave, with an electric $\vec{E}$ field, a magnetic $\vec{H}$ field, and the vector of propagation $\vec{S}$ called the *Poynting vector*:

Electromagnetic wave



This region where the electric and magnetic fields exist as coupled pairs comprising a well-formed wave is referred to as the *far-field region*. In this region the electric and magnetic fields are always perpendicular to one another and their relative magnitudes are in a fixed proportion governed by the permeability and permittivity of free space.

The demarcation between near- and far-field regions depends on the physical dimensions of the radiating circuit as well as the *wavelength* of the signal (λ), wavelength being calculated by dividing the speed of light (2.9979×10^8 meters per second in empty space) by the signal frequency in Hertz. There is no simple rule for predicting where the near-field region ends and the far-field region begins, and in fact there is a gradual morphing of one to the other, but one may reliably consider any distance from the circuit *in excess of multiple wavelengths* to be far-field.

[11] In fact, this is precisely what light is: an electromagnetic wave of exceptionally high frequency, well beyond the frequency range of contemporary electric circuits.

The following list compares and contrasts several near-field versus far-field characteristics:

- In the near-field region electric and magnetic field strengths depend greatly on the geometry of the energized circuit's conductors as well as on the specific voltage and current levels existing at each location. In the far-field region, however, the electric and magnetic field strengths are always in fixed proportion to one another.

- Far-field effects may be ignored for low-frequency AC circuits because these circuits' wavelengths are so long. At 60 Hz, for example, the wavelength is nearly 5 *million* meters. If we consider the far-field region of a circuit to begin at least multiple wavelengths from the source, this puts the far-field region of a 60 Hz circuit at least one-quarter of the Earth's circumference away from the circuit in question!

- In the near-field region the proportionality between electric and magnetic field strengths is a function of circuit impedance: high-impedance circuits will have stronger electric fields than magnetic fields, and low-impedance circuits will have stronger magnetic fields than electric fields, all other factors being equal. This is due to Ohm's Law establishing the relationship between voltage and current ($Z = \frac{V}{I}$), and how electric fields originate in a circuit from potential differences while magnetic fields originate in a circuit from currents. In the far-field region, however, where the electric and magnetic fields exist only through mutual support, their strengths are always in the same (fixed) proportion defined as the *characteristic impedance of free space* which is approximately 377 Ohms. This value is equal to the square root of the ratio of magnetic permeability to electric permittivity for free space ($Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$) because these parameters of space dictate just how strongly a varying electric field creates a magnetic field.

- The relationship between field strength and physical distance from the circuit is a very complex one within the near-field region, as this depends greatly on the geometry of the circuit conductors. In the far-field region, however, we find the *Inverse-Square Law* always holds true: as distance from the circuit increases, electromagnetic wave strength simply and reliably diminishes with the *square* of that distance as if the wave were radiating away in a spherical fashion from a point-source[12]. For example, if we double the distance from the radiating circuit (from a point in the far-field region to another point in the far-field region twice as far away in empty space), the power conveyed by the electromagnetic radiation will always be *four times* less. Tripling the far-field distance always weakens the wave's power by a factor of *nine*. This is simply because the area over which the radiated energy spreads increases with the square of the distance from any point-source, viewing that point-source as the center of a sphere.

- In the far-field region every radiating circuit has a definite *radiation pattern* where the strength of the radiated electromagnetic wave as a function of the *angle* from the geometric centerline follows a consistent pattern regardless of distance. This is a very important feature of *antennas*, where different antenna designs feature unique far-field radiation patterns. By contrast, in the near-field region of an antenna the radiation pattern varies significantly with distance.

[12] A "point source" is a hypothetical point in space having no height nor width nor depth (i.e. it has *zero* physical dimensions) that emits radiation. Point-sources are a theoretical concept only. Real sources have spatial dimensions which makes their near-field strength/distance relationships complex, but the farther away you get from them the more their radiative behavior approaches that of a theoretical point-source.

- The spatial *polarization* of the electromagnetic wave (i.e. the orientation of its perpendicular electric and magnetic fields) is always well-defined in the far-field region but often ill-defined in the near-field region. This means, among other things, that two or more antennas will exchange electromagnetic energy efficiently only if they are appropriately oriented to one another over far-field distances, but may exchange energy fairly well regardless of orientation over near-field distances.

- Any "gain" specifications for an antenna structure apply only to apparent gains in signal power over the far-field range, because they refer to the degree to which an antenna focuses its energy in one direction more than another (i.e. its directionality), implying a distance over which the electromagnetic wave has become well-formed and the antenna's radiation pattern is reliably established. At near-field distances these "gain" figures are meaningless.

5.5 The Poynting vector

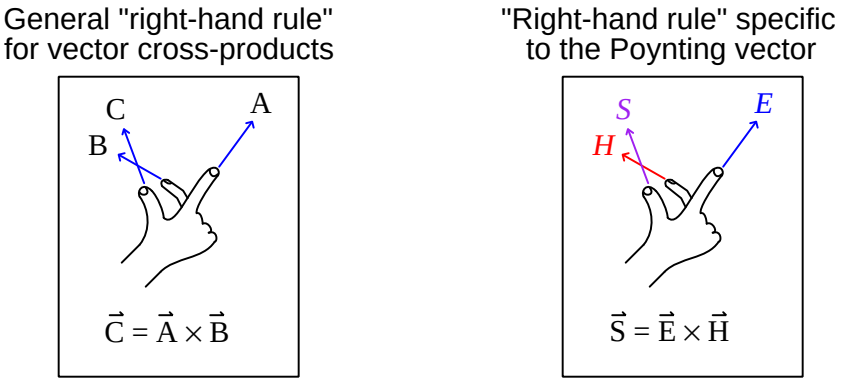
We know that electric fields (E) are always associated with a voltage (V), and magnetic fields (field strength B and field force H) with a current (I). Specifically, electric field strength is the quotient of voltage and distance between two points which is why electric fields are measured in units of *Volts per meter*. Similarly, magnetic field force around a straight current-carrying conductor is the quotient of current and circumference of the circular magnetic path (equal to 2π times the distance from the conductor) which is why magnetic field force is measured in units of *Amperes per meter*.

From Joule's Law we know that the rate of energy transfer (called power) whenever there is simultaneous voltage and current ($P = VI$). It stands to reason, then, that just as voltage multiplied by current yields power, an electric field multiplied by a magnetic field must similarly express power transferred by those two fields through space. That is, if $P = VI$, then the product of E and H should yield a power *intensity* which we shall label S:

$$P = VI \qquad \text{[Watts]} = \text{[Volts][Amperes]}$$

$$S = EH \qquad \text{[Watts/meter}^2\text{]} = \text{[Volts/meter][Amperes/meter]}$$

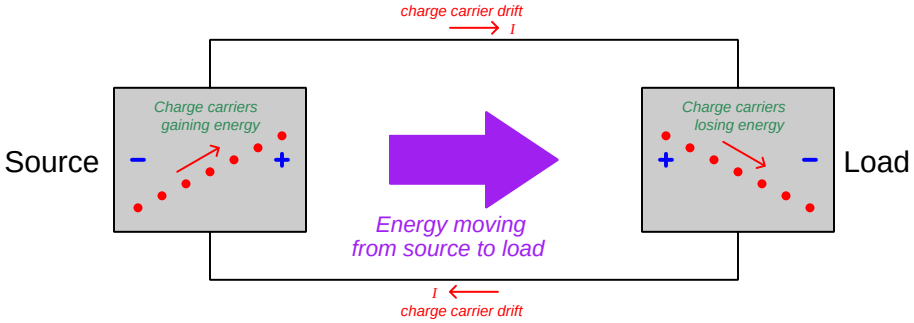
Notice how the Joule's Law equation $P = VI$ involves scalar quantities, while the $S = EH$ equation necessarily involves spatial dimensions of distance (meters) and area (square meters). This is because both electric and magnetic fields are *vector* quantities having both magnitude and spatial direction. Power intensity (S) is the measure of how much power is flowing per square meter of area in open space. As it so happens, power intensity in Watts per square meter relates to electric and magnetic fields by the right-hand rule of vector cross-products, the vector expression of that power intensity being called the *Poynting vector* ($\vec{S}$) after its formulator, the English physicist John H. Poynting:



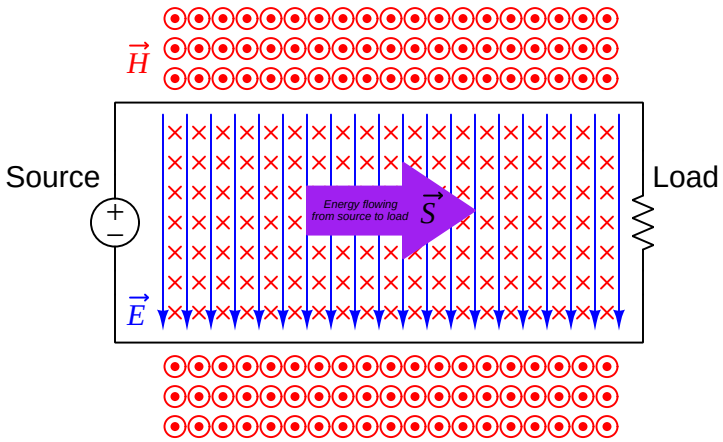
Poynting's contribution to the science of electromagnetism was expressing energy conservation and energy transfer in terms of electric and magnetic fields. That is, the electric and magnetic fields associated with any electrical *source* result in a flow of energy away from that source *through space*, and the electric and magnetic fields associated with any *load* result in a flow of energy toward that load *through space*. The direction of the Poynting vector shows the direction of that energy flow,

and the Poynting vector magnitude shows how many Watts of power flow per square meter of space.

This should be a rather startling conclusion to anyone familiar with voltage, current, and power in simple DC circuits. Beginning students of electricity regard energy transfer in simple circuits as the gain and loss of potential energy in mobile electric charges, those charges gaining energy as they pass through a source and losing energy as they pass through a load:

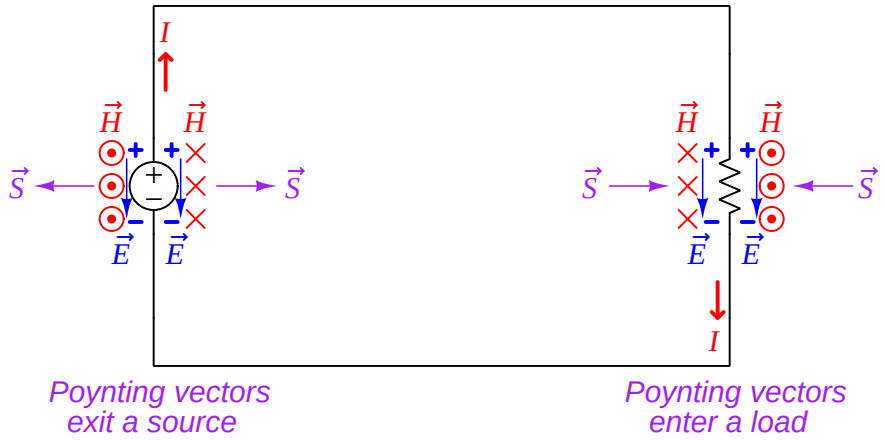


Though this model is conceptually correct, *so is Poynting's model of energy flowing through space*. In fact, if we re-draw this simple circuit illustration showing the electric and magnetic field vectors between the two wires, we can see the Poynting vector expressing that flow of energy from source to load[13]:



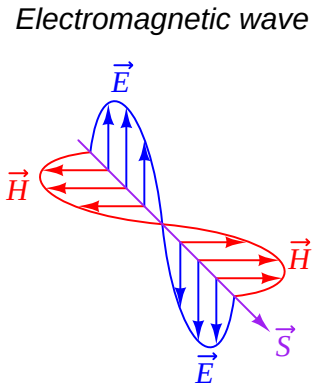
[13] Holding your right hand such that your index finger points downward ($\vec{E}$) and your middle finger into the page ($\vec{H}$), your thumb will point to the right ($\vec{S}$) from source to load.

If we examine the source and load separately using the right-hand rule, we see how the Poynting vector in each case shows the proper direction of energy transfer:



It is well-worth your time to apply the right-hand rule to each of the four Poynting vectors shown above, to familiarize yourself with the use of this tool as a means to determine vector cross-product directions. For example, to the left of the source we see the electric field vector pointing down which means your index finger should point downward as well. The magnetic field vector on that side is coming out of the page (middle finger pointing toward you), and holding your right hand in that manner points your thumb to the left.

For simple DC circuits and circuit elements, Poynting's theory of power transfer may seem like a mere curiosity. It fails to explain anything we could not already explain using the model of potential energy gained and lost by mobile charge carriers, and it requires contortions of your right hand to properly align all the vectors. However, in some applications such as *radio* where energy radiates away from structures called *antennas* in the form of electromagnetic waves (oscillating electric and magnetic fields), Poynting's theory is the *only* suitable explanation for power transfer because there are no wires to convey mobile electric charges from source (transmitting antenna) to load (receiving antenna). Below we see a simplified representation of an electromagnetic wave consisting of two oscillating fields (shown here as an electric field oscillating vertically and a magnetic field oscillating horizontally), and if you use the right-hand rule in each half-cycle you will find that the Poynting vector consistently points in the same direction at the speed of light:



Based on the direction of the Poynting vector we see in this illustration, the source (transmitting antenna) would be located somewhere above and to the left of the drawn wave. There is no definite location for a load, as electromagnetic waves spread out in *all directions* away from the source. Our simple illustration merely shows a "cross-section" of a wave radiating *cylindrically* away from the transmitting antenna. The real electromagnetic wave (of which this illustration shows just a portion) would have an electric field resembling ripples on a pond oscillating vertically while radiating away in all horizontal directions from a single point, and have a magnetic field resembling a set of concentric circles oscillating rotationally in the horizontal plane while also radiating in all directions from that same point.

5.6 Maxwell's equations of electromagnetism

The Scottish physicist James Clerk Maxwell mathematically proved the interrelationships of electric and magnetic fields, publishing his results in the book *A Treatise on Electricity and Magnetism* in 1873. His equations were later simplified and consolidated by the work of Oliver Heaviside, commonly expressed in the following forms describing electric fields (**E**) and magnetic fields (**B**) within a vacuum:

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\epsilon_0}$$

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0$$

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

We will now explore each of these four equations, relating them to laws codified by other early electrical researchers, as well as describe how they relate to the phenomenon of *electromagnetic waves*.

5.6.1 Gauss's Law of electric fields

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\epsilon_0}$$

The first of these equations simply describes electric fields as spreading out from a quantity of electric charge (Q). If we imagine a three-dimensional surface completely enclosing an electric charge (e.g. a soap bubble surrounding a charge), the total quantity of electric flux passing through that surface will be the product of the electric field (also known as the electric flux density, **E**) and the enclosing area (**A**). This is also known as *Gauss's Law of electric fields*.

5.6.2 Gauss's Law of magnetic fields

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0$$

The second of these equations describes magnetic fields passing through a three-dimensional enclosing surface. If we imagine a magnet completely enclosed within a soap bubble, the sum total of magnetic flux passing out through that soap bubble's surface must equal the sum total of magnetic flux re-entering it somewhere else. Literally, this equation says the product of magnetic field (also known as magnetic flux density **B**) and the enclosing area (**A**) must be algebraically equal to zero. This is to say, magnetic fields must always exist as *closed loops*, or alternatively we may say there is no such thing as a magnetic monopole (i.e. a solitary North or South pole without its complement, or a *magnetic charge*). In this way magnetic fields are quite different from electric fields which radiate away from discrete electric charges without ever returning. This is also known as *Gauss's Law of magnetic fields*.

5.6.3 Faraday's Law of electromagnetic induction, extended

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

The third equation states that an electric field ($\mathbf{E}$) will be produced in open space by a varying magnetic flux $\left(\frac{d\Phi_B}{dt}\right)$. Michael Faraday expressed this phenomenon in terms of voltage induced along the length of a conductor by a time-varying magnetic field, but Maxwell demonstrated mathematically that this same thing happens even in empty space!

5.6.4 Ampère's Circuital Law, extended

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

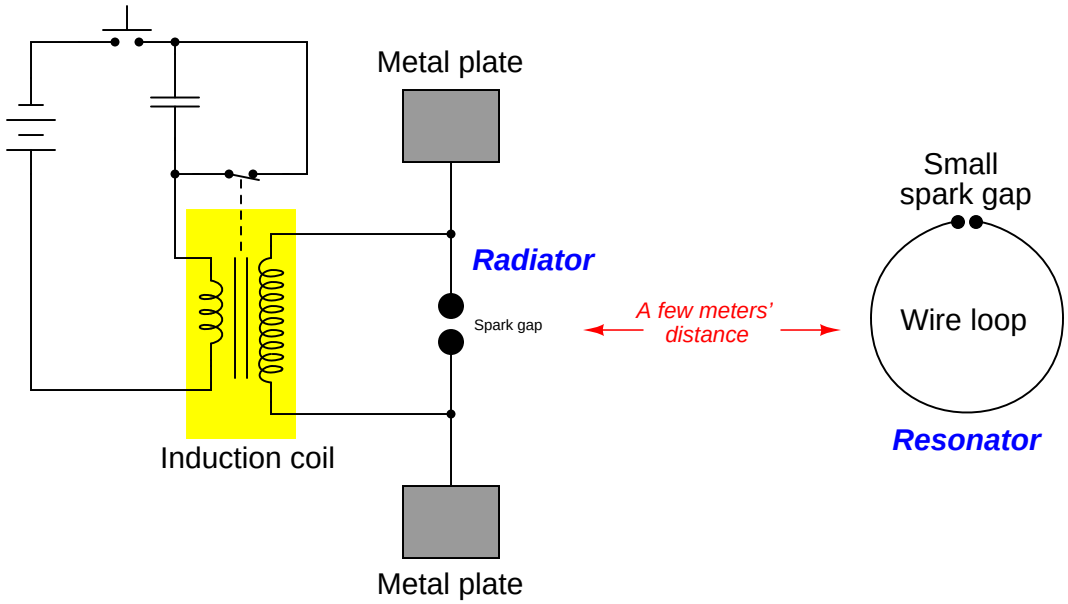
The fourth equation states than a magnetic field ($\mathbf{B}$) will be produced in open space either by an electric current (I) and/or by a varying electric flux $\left(\frac{d\Phi_E}{dt}\right)$. The French scientist André-Marie Ampère described the relationship between current passing through a conductor and the magnetic field created around that conductor by the current, but once again Maxwell extended this concept to include magnetic fields generated in empty space. In empty space, where there are no moving electric charges (i.e. current, I) to generate a magnetic field, only the second term of the equation applies with a time-varying electric field substituting for actual electric current.

5.6.5 Proof of electromagnetic waves

Given the complementary relationship between time-varying electric and magnetic fields, Maxwell reasoned, it was possible for a varying electric field to create a varying magnetic field which would then create another varying electric field, and so on. This cause-and-effect cycle could continue, ad infinitum, with fast-changing electric and magnetic fields radiating off into open space without needing wires to carry or guide them, or a tank circuit to sustain the oscillations. In other words, *these complementary fields self-sustain as they travel*, although they do become weaker due to their spreading out of a finite amount of energy over larger and larger volumes of space.

The Prussian Academy of Science offered a reward to anyone who could experimentally validate Maxwell's prediction, and this challenge was answered by Professor Heinrich Hertz at the Engineering College in Karlsruhe, Germany in 1887, eight years after Maxwell's death. Hertz constructed and tested a pair of devices: a "radiator" to produce the electromagnetic waves, and a "resonator" to receive them.

A simplified diagram showing Hertz's experimental device is shown here:



An "induction coil" (a buzzing device constructed of a self-interrupting relay and step-up transformer winding to generate a continuous pulsing waveform of high voltage) provided an extremely noisy (i.e. harmonically rich)[14] AC signal to the radiator, while a spark gap at the

[14] A fundamental property of waves is that any wave-shape, no matter how complex, is mathematically equivalent to a series of perfectly sinusoidal waves of differing frequencies added together. Waveforms with fast rise/fall times, such as the pulse waveforms produced by switched induction coils, contain many such high-frequency "harmonic" signals, and it is these high-frequency signals that created the radiated electromagnetic waves in Hertz's experiments. The induction coil's secondary winding inductance combined with the parasitic capacitance of the radiator's wires and plates provided the means to momentarily sustain some of these high-frequency oscillations long enough to sustain visible sparks at the resonator's gap.

resonator provided indication that the electromagnetic waves were captured and converted into voltage by the resonator wire.

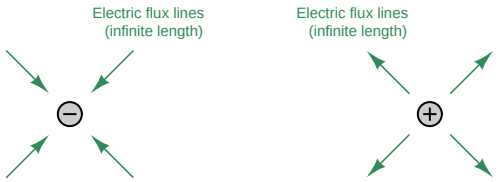
Both the radiator and the resonator are antennas. The purpose of the transmitting antenna (radiator) is to take high-frequency AC power and radiate that power in the form of electromagnetic waves: self-sustaining electric and magnetic fields propagating outward into open space. The purpose of the receiving antenna is to capture those electromagnetic waves and convert them into an electrical signal (i.e. AC voltage and current). All antennas – from historical to the most modern – behave in fundamentally the same way: energize them with high-frequency AC electrical power, and they will radiate electromagnetic waves at that frequency; expose them to electromagnetic waves, and they will produce a very small AC electrical signal at the same frequency as the incident radiation.

5.7 Gauss' Law of Electric Fields

One of fundamental principles relating to electric fields is *Gauss' Law of Electric Fields*. A common mathematical expression for this law is as follows:

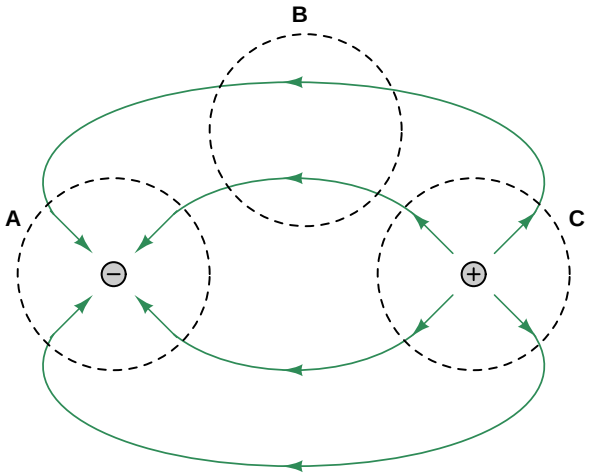
$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\epsilon_0}$$

This equation simply describes electric fields (E) as lines of flux spreading out from a quantity of electric charge (Q). By convention we draw these electric flux lines as radiating away from positive charges and heading toward negative charges as shown below:



If we imagine a three-dimensional surface completely enclosing any electric charge, like a soap bubble surrounding that charge, the total quantity of electric flux lines passing through that surface will be the product of the electric field (also known as the electric flux density, **E**) and the enclosing area (**A**). This "soap bubble" is formally known as a *Gaussian surface*, and we may arbitrarily place that surface anywhere within space for the sake of solving an electric field problem. The major point to understand here is that *any* Gaussian surface drawn will experience an amount of electric flux penetrating its surface proportional to the amount of electric charge enclosed within that surface regardless of the size or shape.

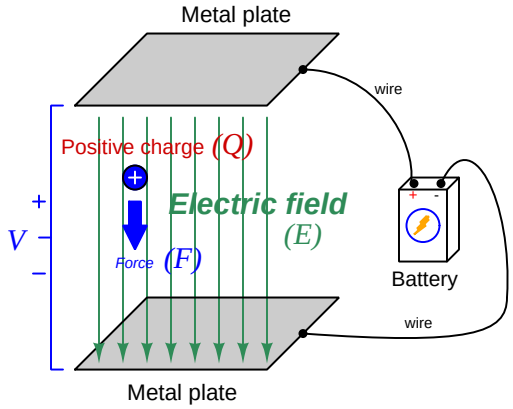
Observe what happens if we connect the electric flux lines together from two opposite charges, and then draw Gaussian surfaces in different regions of space labeled **A**, **B**, and **C**:



Gaussian surface **A** encloses a single negative charge and we see inward-pointing electric flux lines penetrating that surface. Gaussian surface **C** encloses a single positive charge and we see outward-

pointing electric flux lines penetrating that surface. Gaussian surface **B** contains no electric charges at all because it encloses empty space, and we see the number of electric flux lines entering the enclosed volume exactly equaling the number of flux lines leaving it. In other words, for Gaussian surface **B** there is zero net electric flux enclosed within it and also zero *net* penetrations since the number going in balances the number going out.

If we apply an electric field across open space containing a single positive charge, that charge will experience a force proportional to its charge-quantity and the strength of the electric field. The direction of this force will be downward to the negative plate and away from the positive as shown in the following illustration:



$$\vec{F} = Q\vec{E}$$

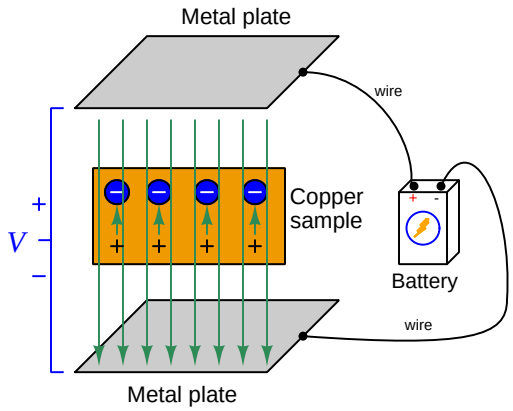
Where,

$\vec{F}$ = Force exerted on the charge (Newtons)
Q = Charge quantity (Coulombs[15])
$\vec{E}$ = Electric field (Newtons per Coulomb)

Unsurprisingly, a negative charge in that same space will experience a force in the opposite direction, upward to the positive plate and away from the negative. With nothing to impede the motion of these charges, they will accelerate as a result of the applied force from the electric field, their acceleration predicted by the familiar equation $F = ma$.

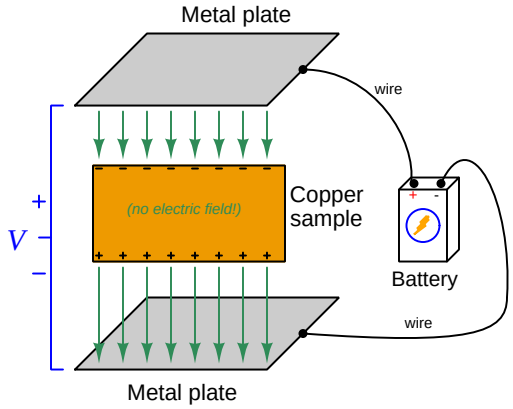
[15] One Coulomb of electric charge is equal to 6.2415×10^{18} electrons.

Now imagine a solid piece of electrically conductive material such as copper metal suspended between the same two metal plates. The outer-most electrons of the copper atoms are free to drift throughout the metal, which is the fact that makes copper metal a conductive material. The effect of the two plates' electric field on the free electrons within the copper sample will be much the same as upon free electrons in a vacuum space between two metal plates: those free electrons will accelerate upward in the direction of the positive plate and away from the negative. This migration of free electrons toward the upper surface of the copper sample leaves positively-charged regions near the lower surface of the copper:



The electrons will migrate as far as they are able, which in this case is the upper surface of the conductive copper sample. The separation of negative and positive charges within the copper sample – the negative charges being mobile electrons and the positive charges being immobile protons in the copper atom nuclei – itself forms an electric field within the copper, pointed in the direction opposite that of the battery's external field. These short vector-arrows pointed upward in the above illustration represent the flux lines from the positive stationary nuclei to the displaced negative electrons.

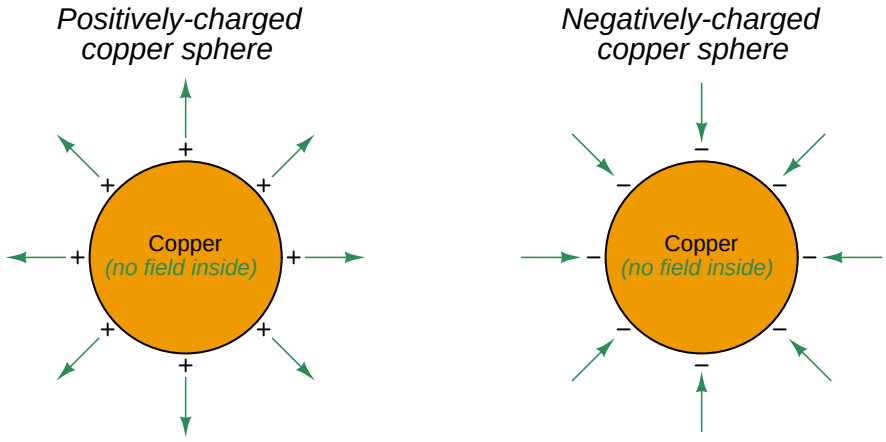
Since these two sets of electric flux lines point in opposite directions, their net effect is to cancel each other out. As more electrons drift upward this cancellation continues until there is zero net electric field within the copper sample. When the net electric field reaches zero, the electrons no longer experience any driving force upward. In this *equilibrium* condition of zero net force on all electric charges, the interior of the copper sample contains no net electric field:



It should not come as a surprise to anyone familiar with basic concepts of electric circuits that this sample of copper naturally excludes any electric field imposed externally upon it, since the mass of copper may be thought of as a large collection of electrically common points, and we know one of the characteristics of electrically common points is equipotentiality. *And, where there is no difference of electrical potential between two points (i.e. no voltage), there can be no electric field between those points.* A conductive object in a state of electrical equilibrium, therefore, contains no internal electric field(s).

This phenomenon of electrical conductors naturally cancelling out internal electric fields finds wide application in electrical and electronic systems. One such application is the *shielding* of sensitive components by surrounding them with a continuous conductive surface. Another application is in insulated-gate field-effect transistors (also known as MOSFETs) where an externally-applied electric field causes electron and hole charge carriers to reposition themselves and in so doing cause a conductive channel transverse to that field to either grow wider or narrower.

An interesting consequence of Gauss' Law of Electric Fields is that any net gain or loss of electric charge by a conductive object results in the unbalanced charges moving to the very exterior surface of the object. For example, if we inject extra electrons into a copper block, those electrons quickly settle into positions on the very outer skin of that block where they remain. Likewise, if electrons are extracted from a copper block, the remaining electrons will migrate within the block so as to leave the resulting positive charges (i.e. electron deficiencies) at the outer surface of that block:



All unbalanced charges migrate to the outside of the object

We may understand this by appealing to the proven fact that a conductive object in a state of electrical equilibrium must have no electric field within its interior. If we imagine drawing a Gaussian surface just beneath the skin of that conductive object, we may conclude from Gauss' Law that the interior of that Gaussian surface must contain no net electric charge (i.e. all positive and negative charges within it are perfectly balanced) because zero net electric field penetrating that Gaussian surface implies zero net electric charge within. This must mean that any and all unbalanced electric charges in that conductive object must reside *outside* the area of no electric field; i.e. outside the Gaussian surface on the conductive object's outer surface. Thus, a Gaussian surface drawn anywhere inside a charged conductive object experiences no electric flux lines but a Gaussian surface drawn outside that same object does.

Chapter 6

Questions

This learning module, along with all others in the ModEL collection, is designed to be used in an inverted instructional environment where students independently read[1] the tutorials and attempt to answer questions on their own *prior* to the instructor's interaction with them. In place of lecture[2], the instructor engages with students in Socratic-style dialogue, probing and challenging their understanding of the subject matter through inquiry.

Answers are not provided for questions within this chapter, and this is by design. Solved problems may be found in the Tutorial and Derivation chapters, instead. The goal here is *independence*, and this requires students to be challenged in ways where others cannot think for them. Remember that you always have the tools of *experimentation* and *computer simulation* (e.g. SPICE) to explore concepts!

The following lists contain ideas for Socratic-style questions and challenges. Upon inspection, one will notice a strong theme of *metacognition* within these statements: they are designed to foster a regular habit of examining one's own thoughts as a means toward clearer thinking. As such these sample questions are useful both for instructor-led discussions as well as for self-study.

[1] Technical reading is an essential academic skill for any technical practitioner to possess for the simple reason that the most comprehensive, accurate, and useful information to be found for developing technical competence is in textual form. Technical careers in general are characterized by the need for continuous learning to remain current with standards and technology, and therefore any technical practitioner who cannot read well is handicapped in their professional development. An excellent resource for educators on improving students' reading prowess through intentional effort and strategy is the book textitReading For Understanding – How Reading Apprenticeship Improves Disciplinary Learning in Secondary and College Classrooms by Ruth Schoenbach, Cynthia Greenleaf, and Lynn Murphy.

[2] Lecture is popular as a teaching method because it is easy to implement: any reasonably articulate subject matter expert can talk to students, even with little preparation. However, it is also quite problematic. A good lecture always makes complicated concepts seem easier than they are, which is bad for students because it instills a false sense of confidence in their own understanding; reading and re-articulation requires more cognitive effort and serves to verify comprehension. A culture of teaching-by-lecture fosters a debilitating dependence upon direct personal instruction, whereas the challenges of modern life demand independent and critical thought made possible only by gathering information and perspectives from afar. Information presented in a lecture is ephemeral, easily lost to failures of memory and dictation; text is forever, and may be referenced at any time.

General challenges following tutorial reading

- Summarize as much of the text as you can in one paragraph of your own words. A helpful strategy is to explain ideas as you would for an intelligent child: as simple as you can without compromising too much accuracy.
- Simplify a particular section of the text, for example a paragraph or even a single sentence, so as to capture the same fundamental idea in fewer words.
- Where did the text make the most sense to you? What was it about the text's presentation that made it clear?
- Identify where it might be easy for someone to misunderstand the text, and explain why you think it could be confusing.
- Identify any new concept(s) presented in the text, and explain in your own words.
- Identify any familiar concept(s) such as physical laws or principles applied or referenced in the text.
- Devise a proof of concept experiment demonstrating an important principle, physical law, or technical innovation represented in the text.
- Devise an experiment to disprove a plausible misconception.
- Did the text reveal any misconceptions you might have harbored? If so, describe the misconception(s) and the reason(s) why you now know them to be incorrect.
- Describe any useful problem-solving strategies applied in the text.
- Devise a question of your own to challenge a reader's comprehension of the text.

General follow-up challenges for assigned problems

- Identify where any fundamental laws or principles apply to the solution of this problem, especially before applying any mathematical techniques.
- Devise a thought experiment to explore the characteristics of the problem scenario, applying known laws and principles to mentally model its behavior.
- Describe in detail your own strategy for solving this problem. How did you identify and organized the given information? Did you sketch any diagrams to help frame the problem?
- Is there more than one way to solve this problem? Which method seems best to you?
- Show the work you did in solving this problem, even if the solution is incomplete or incorrect.
- What would you say was the most challenging part of this problem, and why was it so?
- Was any important information missing from the problem which you had to research or recall?
- Was there any extraneous information presented within this problem? If so, what was it and why did it not matter?
- Examine someone else's solution to identify where they applied fundamental laws or principles.
- Simplify the problem from its given form and show how to solve this simpler version of it. Examples include eliminating certain variables or conditions, altering values to simpler (usually whole) numbers, applying a limiting case (i.e. altering a variable to some extreme or ultimate value).
- For quantitative problems, identify the real-world meaning of all intermediate calculations: their units of measurement, where they fit into the scenario at hand. Annotate any diagrams or illustrations with these calculated values.
- For quantitative problems, try approaching it qualitatively instead, thinking in terms of "increase" and "decrease" rather than definite values.
- For qualitative problems, try approaching it quantitatively instead, proposing simple numerical values for the variables.
- Were there any assumptions you made while solving this problem? Would your solution change if one of those assumptions were altered?
- Identify where it would be easy for someone to go astray in attempting to solve this problem.
- Formulate your own problem based on what you learned solving this one.

General follow-up challenges for experiments or projects

- In what way(s) was this experiment or project easy to complete?
- Identify some of the challenges you faced in completing this experiment or project.

- Show how thorough documentation assisted in the completion of this experiment or project.
- Which fundamental laws or principles are key to this system's function?
- Identify any way(s) in which one might obtain false or otherwise misleading measurements from test equipment in this system.
- What will happen if (component X) fails (open/shorted/etc.)?
- What would have to occur to make this system unsafe?

6.1 Conceptual reasoning

These questions are designed to stimulate your analytic and synthetic thinking[3]. In a Socratic discussion with your instructor, the goal is for these questions to prompt an extended dialogue where assumptions are revealed, conclusions are tested, and understanding is sharpened. Your instructor may also pose additional questions based on those assigned, in order to further probe and refine your conceptual understanding.

Questions that follow are presented to challenge and probe your understanding of various concepts presented in the tutorial. These questions are intended to serve as a guide for the Socratic dialogue between yourself and the instructor. Your instructor's task is to ensure you have a sound grasp of these concepts, and the questions contained in this document are merely a means to this end. Your instructor may, at his or her discretion, alter or substitute questions for the benefit of tailoring the discussion to each student's needs. The only absolute requirement is that each student is challenged and assessed at a level equal to or greater than that represented by the documented questions.

It is far more important that you convey your *reasoning* than it is to simply convey a correct answer. For this reason, you should refrain from researching other information sources to answer questions. What matters here is that *you* are doing the thinking. If the answer is incorrect, your instructor will work with you to correct it through proper reasoning. A correct answer without an adequate explanation of how you derived that answer is unacceptable, as it does not aid the learning or assessment process.

You will note a conspicuous lack of answers given for these conceptual questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your conceptual answers, where applicable, is to use circuit simulation software to explore the effects of changes made to circuits. For example, if one of these conceptual questions challenges you to predict the effects of altering some component parameter in a circuit, you may check the validity of your work by simulating that same parameter change within software and seeing if the results agree.

[3] *Analytical* thinking involves the "disassembly" of an idea into its constituent parts, analogous to dissection. *Synthetic* thinking involves the "assembly" of a new idea comprised of multiple concepts, analogous to construction. Both activities are high-level cognitive skills, extremely important for effective problem-solving, necessitating frequent challenge and regular practice to fully develop.

6.1.1 Reading outline and reflections

"Reading maketh a full man; conference a ready man; and writing an exact man" – Francis Bacon

Francis Bacon's advice is a blueprint for effective education: reading provides the learner with knowledge, writing focuses the learner's thoughts, and critical dialogue equips the learner to confidently communicate and apply their learning. Independent acquisition and application of knowledge is a powerful skill, well worth the effort to cultivate. To this end, students should read these educational resources closely, journal their own reflections on the reading, and discuss in detail their findings with classmates and instructor(s). You should be able to do all of the following after reading any instructional text:

☑ Briefly SUMMARIZE THE TEXT in the form of a journal entry documenting your learning as you progress through the course of study. Share this summary in dialogue with your classmates and instructor. Journaling is an excellent self-test of thorough reading because you cannot clearly express what you have not read or did not comprehend.

☑ Demonstrate ACTIVE READING STRATEGIES, including verbalizing your impressions as you read, simplifying long passages to convey the same ideas using fewer words, annotating text and illustrations with your own interpretations, working through mathematical examples shown in the text, cross-referencing passages with relevant illustrations and/or other passages, identifying problem-solving strategies applied by the author, etc. Technical reading is a special case of problem-solving, and so these strategies work precisely because they help solve any problem: paying attention to your own thoughts (metacognition), eliminating unnecessary complexities, identifying what makes sense, paying close attention to details, drawing connections between separated facts, and noting the successful strategies of others.

☑ Identify IMPORTANT THEMES, especially GENERAL LAWS and PRINCIPLES, expounded in the text and express them in the simplest of terms as though you were teaching an intelligent child. This emphasizes connections between related topics and develops your ability to communicate complex ideas to anyone.

☑ Form YOUR OWN QUESTIONS based on the reading, and then pose them to your instructor and classmates for their consideration. Anticipate both correct and incorrect answers, the incorrect answer(s) assuming one or more plausible misconceptions. This helps you view the subject from different perspectives to grasp it more fully.

☑ Devise EXPERIMENTS to test claims presented in the reading, or to disprove misconceptions. Predict possible outcomes of these experiments, and evaluate their meanings: what result(s) would confirm, and what would constitute disproof? Running mental simulations and evaluating results is essential to scientific and diagnostic reasoning.

☑ Specifically identify any points you found CONFUSING. The reason for doing this is to help diagnose misconceptions and overcome barriers to learning.

6.1.2 Foundational concepts

Correct analysis and diagnosis of electric circuits begins with a proper understanding of some basic concepts. The following is a list of some important concepts referenced in this module's full tutorial. Define each of them in your own words, and be prepared to illustrate each of these concepts with a description of a practical example and/or a live demonstration.

Electric field

Magnetic field

Gravitational field

Electric field strength

Electric flux

Electromagnetism

Electromagnetic induction

Right-hand rule

Ferromagnetism

Magnetic field strength

Magnetizing force

Magnetic flux

Saturation

Hysteresis

Remanence

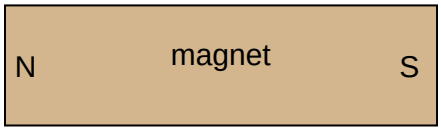
Coercivity

Faraday's Law of electromagnetic induction

Lenz's Law

6.1.3 Sketching magnetic flux lines

If we were to trace the magnetic lines of flux extending from this bar magnet, what would they appear like?



Challenges

- Design a physical experiment to reveal the patterns of magnetic flux lines in visual form.

6.1.4 Defining "north" and "south"

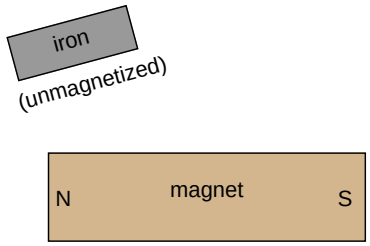
Magnetic poles are designated by two labels: "North" and "South". How are these labels defined? Explain how we can experimentally determine which ends of a magnet are "North" and "South", respectively.

Challenges

- If two magnetic compasses are held close enough to each other to interact, which way will their needles point?

6.1.5 Effect of an iron piece on magnetic flux

What happens to the magnetic lines of flux emanating from a magnet, when an unmagnetized piece of iron is placed near it?



Challenges

- Identify a practical application of this effect.

6.1.6 Compass in a lightning storm

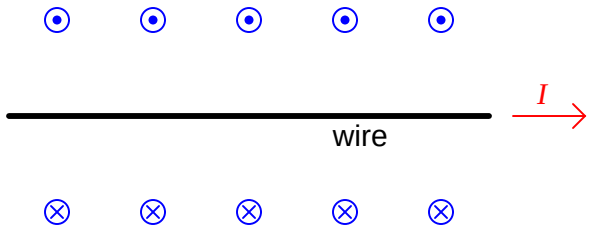
When lightning strikes, nearby magnetic compass needles may be seen to jerk in response to the electrical discharge. No compass needle deflection results during the accumulation of electrostatic charge preceding the lightning bolt, but only when the bolt actually strikes. What does this phenomenon indicate about voltage, current, and magnetism?

Challenges

- What would a sensitive *electroscope* (an instrument designed to detect electric fields) register alongside the compass during a lightning storm.

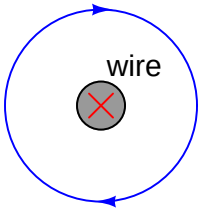
6.1.7 Circle-and-dot notation

When engineers and physicists draw pictures illustrating the magnetic field produced by a straight current-carrying wire drawn parallel to the printed page, they usually do so using this notation:



Explain what the circle-and-dot and circle-and-cross symbols mean, with reference to the *right-hand rule*.

Likewise, in illustrations depicting a current-carrying wire drawn perpendicular to the printed page, we use similar notation:



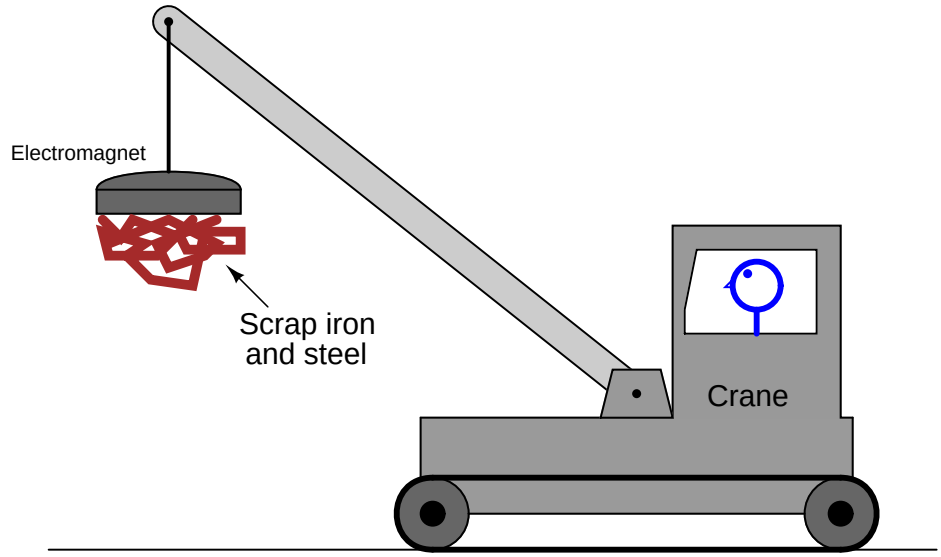
Explain what the circle-and-cross symbol means, with reference to the *right-hand rule*.

Challenges

- What would change, if anything, if the direction of electric current were to reverse?

6.1.8 Magnetic crane circuit

Cranes used to move scrap iron and steel use electrically powered magnets to hold the metal pieces, rather than a scoop or some other mechanical grasping device:



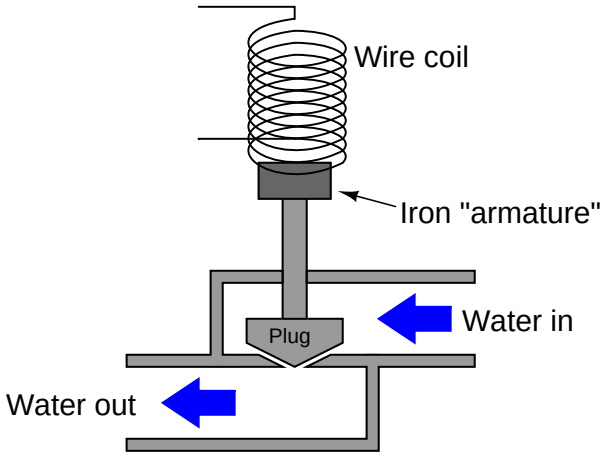
In this illustration of a crane, superimpose a drawing showing the electromagnet, electrical power supply and wiring necessary for this to work. Also include a switch so the crane operator can turn the magnet on and off. Also, draw an electrical schematic diagram of the same circuit, showing all components in the crane's magnet circuit.

Challenges

- Suppose the electromagnetic proved too weak to be practical. What could be changed in the circuit to strengthen the magnetic field?

6.1.9 Solenoid valve

A *solenoid valve* uses magnetism from an electromagnet coil to actuate a valve mechanism:



Essentially, this is an electrically-controlled on/off water valve. In the development of this valve, though, the design engineers discover that the magnetic force produced by the electromagnet coil is not strong enough to achieve reliable valve actuation every time. What can be changed in this solenoid valve design to produce a greater actuating force?

Challenges

- Suppose this valve did not open like it was supposed to when the solenoid coil was energized. Identify some possible reasons for this type of failure.

6.1.10 Ampére's experiment

In 1820, the French physicist André Marie Ampère discovered that two parallel wires carrying electrical current would either be attracted to one another, or repelled by one another, depending on what directions the two currents were going. Devise an experiment to reproduce Ampère's results, and determine which directions current must go to produce an attractive versus a repulsive force.

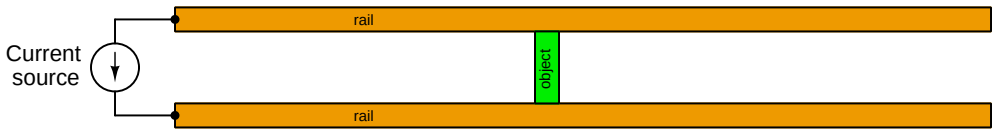
Challenges

- When electric current is passed through a coil of wire, does the inter-loop force tend to compress the coil or extend it? What if the electric current were *alternating* and not *direct*?

6.1.11 Lorentz force in a rail gun

A *rail gun* is a kinetic energy weapon using electricity to launch a projectile. One of the advantages of a rail gun over conventional gun-type weapons is that the velocity of the projectile is not limited by the speed of chemical reactions as is the case when using gunpowder or some other combustible substance to thermally propel a projectile.

Rail guns work by placing an electrically-conductive object between two metal "rails" connected to the poles of a high-energy DC source. In the following diagram, the rails are colored orange while the conductive object is colored green. The green-colored object is free to slide along the rails while maintaining electrical contact with both:



Based on the above diagram, which way will the movable object be pushed by the resulting Lorentz force? *Hint: annotate the diagram with all directions of current and directions of magnetic flux vectors.*

Will the object's motion reverse if the current source's direction is reverse?

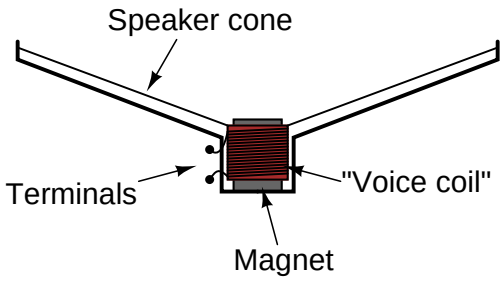
Will the object's motion reverse if the current source is connected to the right-hand end of the rails rather than the left-hand end as shown?

Challenges

- What difference does the resistor's value make, if any?
- Which ways will the two rails be forced by their own currents? In other words, will the two rails be attracted toward each other, repulsed by each other, or experience no mutual force at all?

6.1.12 Audio speaker

A large audio speaker may serve to demonstrate both the principles of *electromagnetism* and of *electromagnetic induction*. Explain how this may be done, based on the following cut-away illustration of such a device:

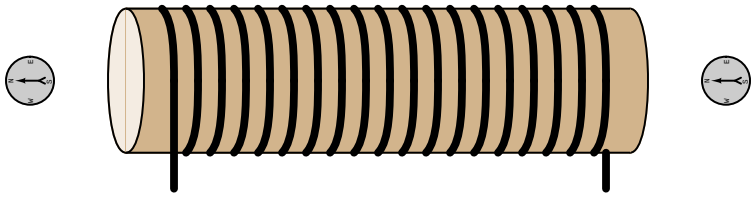


Challenges

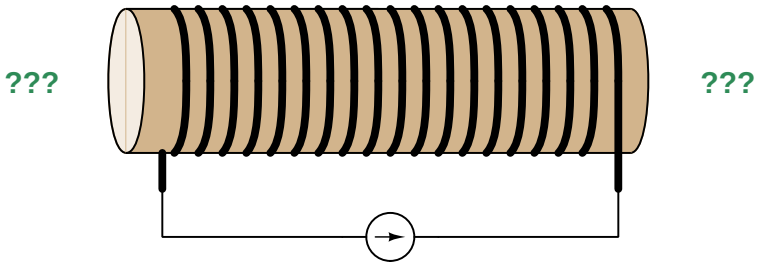
- Identify some possible points of failure in a speaker which would prevent it from operating properly.

6.1.13 An energized solenoid

Sketch an appropriate electrical source connected to this wire coil (called a "solenoid") in order to make the two magnetic compasses point in the directions shown:



Next, determine how those same compasses would point if the solenoid's coil were wound in the opposite direction and energized by the following current source:

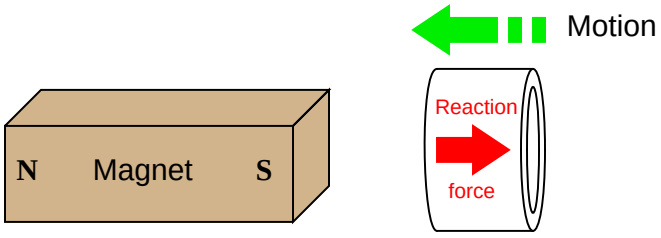


Challenges

- What would happen if the source's polarity were reversed while powering the same solenoid coil?

6.1.14 Magnet and copper ring

If a copper ring is brought closer to the end of a permanent magnet, a repulsive force will develop between the magnet and the ring. This force will cease, however, when the ring stops moving. What is this effect called?



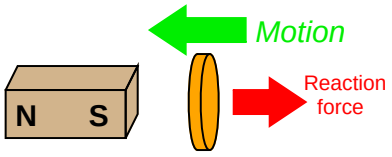
Also, describe what will happen if the copper ring is moved *away* from the end of the permanent magnet.

Challenges

- Trace the direction of rotation for the induced electric current in the ring necessary to produce both the repulsive and the attractive force.
- What would happen if the magnet's orientation were reversed (south pole on left and north pole on right)?

6.1.15 Magnet and non-magnetic metal disk

Lenz's Law describes the opposition to changes in magnetic flux resulting from electromagnetic induction between a magnetic field and an electrical conductor. One apparatus capable of demonstrating Lenz's Law is a copper or aluminum disk (electrically conductive, but non-magnetic) positioned close to the end of a powerful permanent magnet. There is no attraction or repulsion between the disk and magnet when there is no motion, but a force will develop between the two objects if either is suddenly moved. This force will be in such a direction that it tries to resist the motion (i.e. the force tries to maintain the gap constant between the two objects):



We know this force is magnetic in nature. That is, the induced current causes the disk itself to *become* a magnet in order to react against the permanent magnet's field and produce the opposing force. For each of the following scenarios, label the disk's induced magnetic poles (North and South) as it reacts to the motion imposed by an outside force:

Figure 1

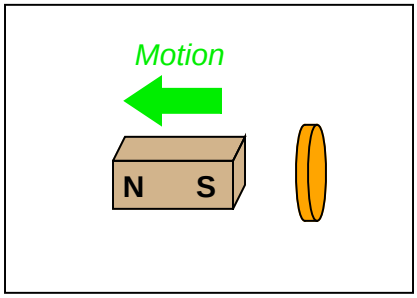


Figure 2

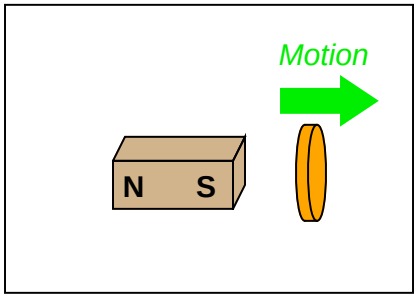


Figure 3

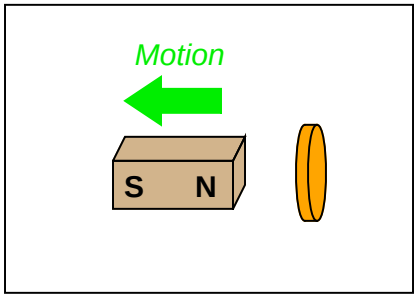
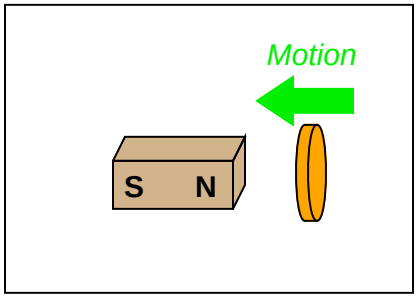


Figure 4



Challenges

- Trace the direction of rotation for the induced electric current in the disk necessary to produce both the repulsive and the attractive force.
- What would happen if the magnet's orientation were reversed (south pole on left and north pole on right)?

6.1.16 Induced current directions

In the following examples, trace the direction of current through the load resistor:

Figure 1

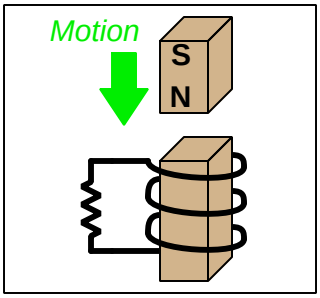


Figure 2

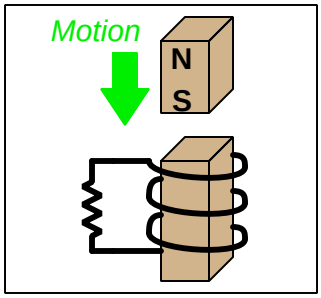


Figure 3

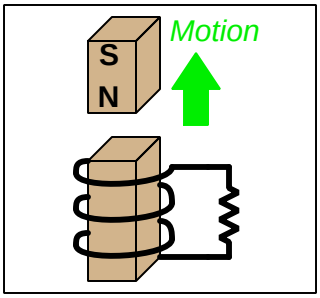


Figure 4

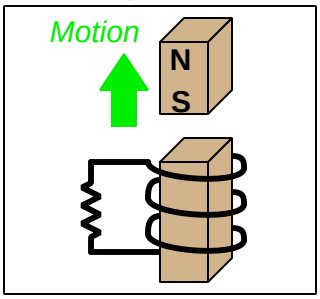


Figure 5

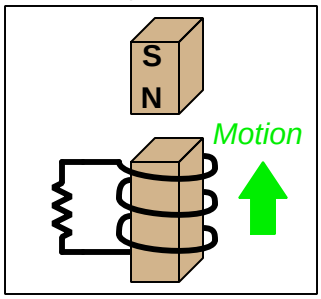
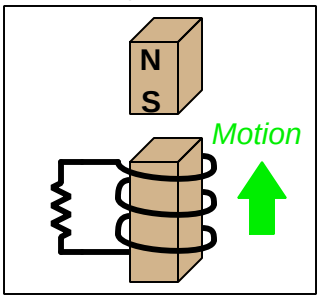


Figure 6



Challenges

- What difference does the resistor's value make, if any?

6.1.17 Induced voltage polarities

Determine the polarity of the coil's induced voltage for each of the following examples. Be careful to note the direction each coil is wrapped around its core – the coils are not all identical!

Figure 1

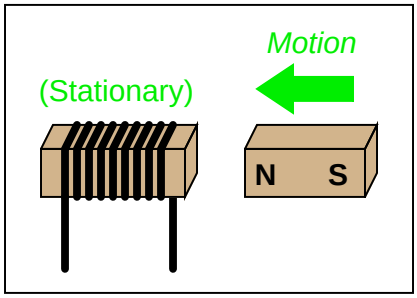


Figure 2

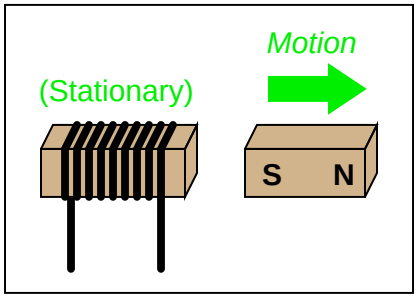


Figure 3

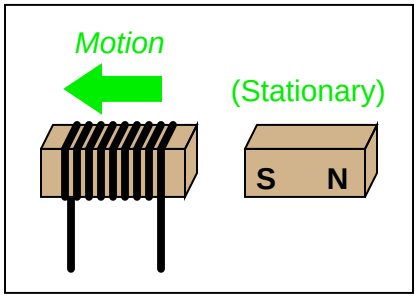
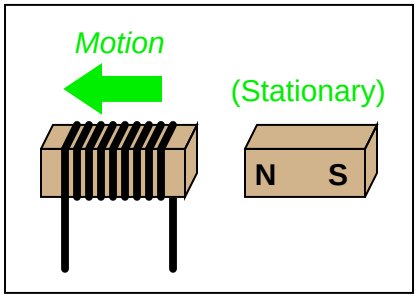


Figure 4

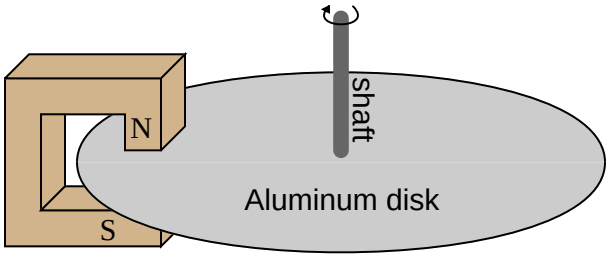


Challenges

- Explain why imagining the presence of a load between the two terminals is helpful.
- What difference does the number of turns in the coil make, if any?

6.1.18 Magnetic drag disk

Electromechanical watt-hour meters use an aluminum disk that is spun by an electric motor. To generate a constant "drag" on the disk necessary to limit its rotational speed, a strong magnet is placed in such a way that its lines of magnetic flux pass perpendicularly through the disk's thickness:



Explain the phenomenon behind this magnetic "drag" mechanism, and also explain how the permanent magnet assembly should be re-positioned so that it provides *less* drag on the disk for the same rotational speed.

Challenges

- Suppose you *move* a strong magnet past the surface of an aluminum disk. What will happen to the disk, if anything?
- If you were working an an electrical power meter technician and calibrating a watt-hour meter that was turning slower than it should, which way would you have to reposition the drag magnet to correct the meter's calibration error?

6.1.19 Fluid induction

If the motion of a conductor through a magnetic field induces a voltage in that conductor, it stands to reason that a conductive fluid moving through a pipe can also generate a voltage, if properly exposed to a magnetic field. Draw a picture showing the necessary orientation of the pipe, the magnetic field, and the electrodes intercepting the induced voltage.

Challenges

- What effect does a change in the liquid's electrical conductivity have, if any, on the induced voltage?

6.2 Quantitative reasoning

These questions are designed to stimulate your computational thinking. In a Socratic discussion with your instructor, the goal is for these questions to reveal your mathematical approach(es) to problem-solving so that good technique and sound reasoning may be reinforced. Your instructor may also pose additional questions based on those assigned, in order to observe your problem-solving firsthand.

Mental arithmetic and estimations are strongly encouraged for all calculations, because without these abilities you will be unable to readily detect errors caused by calculator misuse (e.g. keystroke errors).

You will note a conspicuous lack of answers given for these quantitative questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. My advice is to use circuit simulation software such as SPICE to check the correctness of quantitative answers. Refer to those learning modules within this collection focusing on SPICE to see worked examples which you may use directly as practice problems for your own study, and/or as templates you may modify to run your own analyses and generate your own practice problems.

Completely worked example problems found in the Tutorial may also serve as "test cases[4]" for gaining proficiency in the use of circuit simulation software, and then once that proficiency is gained you will never need to rely[5] on an answer key!

[4] In other words, set up the circuit simulation software to analyze the same circuit examples found in the Tutorial. If the simulated results match the answers shown in the Tutorial, it confirms the simulation has properly run. If the simulated results disagree with the Tutorial's answers, something has been set up incorrectly in the simulation software. Using every Tutorial as practice in this way will quickly develop proficiency in the use of circuit simulation software.

[5] This approach is perfectly in keeping with the instructional philosophy of these learning modules: *teaching students to be self-sufficient thinkers*. Answer keys can be useful, but it is even more useful to your long-term success to have a set of tools on hand for checking your own work, because once you have left school and are on your own, there will no longer be "answer keys" available for the problems you will have to solve.

6.2.1 Miscellaneous physical constants

Note: constants shown in **bold** type are *exact*, not approximations. Values inside of parentheses show one standard deviation (σ) of uncertainty in the final digits: for example, the magnetic permeability of free space value given as $1.25663706212(19) \times 10^{-6}$ H/m represents a center value (i.e. the location parameter) of $1.25663706212 \times 10^{-6}$ Henrys per meter with one standard deviation of uncertainty equal to $0.0000000000019 \times 10^{-6}$ Henrys per meter.

Avogadro's number (N_A) = **6.02214076** $\times 10^{23}$ **per mole** (mol^{-1})

Boltzmann's constant (k) = **1.380649** $\times 10^{-23}$ **Joules per Kelvin** (J/K)

Electronic charge (e) = **1.602176634** $\times 10^{-19}$ **Coulomb** (C)

Faraday constant (F) = **96,485.33212...** $\times 10^{4}$ **Coulombs per mole** (C/mol)

Magnetic permeability of free space (μ_0) = $1.25663706212(19) \times 10^{-6}$ Henrys per meter (H/m)

Electric permittivity of free space (ϵ_0) = $8.8541878128(13) \times 10^{-12}$ Farads per meter (F/m)

Characteristic impedance of free space (Z_0) = 376.730313668(57) Ohms (Ω)

Gravitational constant (G) = $6.67430(15) \times 10^{-11}$ cubic meters per kilogram-seconds squared ($m^3/kg\text{-}s^2$)

Molar gas constant (R) = **8.314462618... Joules per mole-Kelvin** (J/mol-K) = 0.08205746(14) liters-atmospheres per mole-Kelvin

Planck constant (h) = **6.62607015** $\times 10^{-34}$ **joule-seconds** (J-s)

Stefan-Boltzmann constant (σ) = **5.670374419...** $\times 10^{-8}$ **Watts per square meter-Kelvin**^{4} ($W/m^2 \cdot K^4$)

Speed of light in a vacuum (c) = **299,792,458 meters per second** (m/s) = 186282.4 miles per second (mi/s)

Note: All constants taken from NIST data "Fundamental Physical Constants – Complete Listing", from `http://physics.nist.gov/constants`, National Institute of Standards and Technology (NIST), 2018 CODATA Adjustment.

6.2.2 Introduction to spreadsheets

A powerful computational tool you are encouraged to use in your work is a *spreadsheet*. Available on most personal computers (e.g. Microsoft Excel), *spreadsheet* software performs numerical calculations based on number values and formulae entered into cells of a grid. This grid is typically arranged as lettered columns and numbered rows, with each cell of the grid identified by its column/row coordinates (e.g. cell `B3`, cell `A8`). Each cell may contain a string of text, a number value, or a mathematical formula. The spreadsheet automatically updates the results of all mathematical formulae whenever the entered number values are changed. This means it is possible to set up a spreadsheet to perform a series of calculations on entered data, and those calculations will be re-done by the computer any time the data points are edited in any way.

For example, the following spreadsheet calculates average speed based on entered values of distance traveled and time elapsed:

	A	B	C	D
1	Distance traveled	46.9	Kilometers	
2	Time elapsed	1.18	Hours	
3	Average speed	= B1 / B2	km/h	
4				
5				

Text labels contained in cells `A1` through `A3` and cells `C1` through `C3` exist solely for readability and are not involved in any calculations. Cell `B1` contains a sample distance value while cell `B2` contains a sample time value. The formula for computing speed is contained in cell `B3`. Note how this formula begins with an "equals" symbol (`=`), references the values for distance and speed by lettered column and numbered row coordinates (`B1` and `B2`), and uses a forward slash symbol for division (`/`). The coordinates `B1` and `B2` function as *variables*[6] would in an algebraic formula.

When this spreadsheet is executed, the numerical value `39.74576` will appear in cell `B3` rather than the formula `= B1 / B2`, because 39.74576 is the computed speed value given 46.9 kilometers traveled over a period of 1.18 hours. If a different numerical value for distance is entered into cell `B1` or a different value for time is entered into cell `B2`, cell `B3`'s value will automatically update. All you need to do is set up the given values and any formulae into the spreadsheet, and the computer will do all the calculations for you.

Cell `B3` may be referenced by other formulae in the spreadsheet if desired, since it is a variable just like the given values contained in `B1` and `B2`. This means it is possible to set up an entire chain of calculations, one dependent on the result of another, in order to arrive at a final value. The arrangement of the given data and formulae need not follow any pattern on the grid, which means you may place them anywhere.

[6] Spreadsheets may also provide means to attach text labels to cells for use as variable names (Microsoft Excel simply calls these labels "names"), but for simple spreadsheets such as those shown here it's usually easier just to use the standard coordinate naming for each cell.

Common[7] arithmetic operations available for your use in a spreadsheet include the following:

- Addition (`+`)
- Subtraction (`-`)
- Multiplication (`*`)
- Division (`/`)
- Powers (`^`)
- Square roots (`sqrt()`)
- Logarithms (`ln()` , `log10()`)

Parentheses may be used to ensure[8] proper order of operations within a complex formula. Consider this example of a spreadsheet implementing the *quadratic formula*, used to solve for roots of a polynomial expression in the form of $ax^2 + bx + c$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

	A	B
1	x_1	= (-B4 + sqrt((B4^2) - (4*B3*B5))) / (2*B3)
2	x_2	= (-B4 - sqrt((B4^2) - (4*B3*B5))) / (2*B3)
3	a =	9
4	b =	5
5	c =	-2

This example is configured to compute roots[9] of the polynomial $9x^2 + 5x - 2$ because the values of 9, 5, and -2 have been inserted into cells `B3`, `B4`, and `B5`, respectively. Once this spreadsheet has been built, though, it may be used to calculate the roots of *any* second-degree polynomial expression simply by entering the new a, b, and c coefficients into cells `B3` through `B5`. The numerical values appearing in cells `B1` and `B2` will be automatically updated by the computer immediately following any changes made to the coefficients.

[7] Modern spreadsheet software offers a bewildering array of mathematical functions you may use in your computations. I recommend you consult the documentation for your particular spreadsheet for information on operations other than those listed here.

[8] Spreadsheet programs, like text-based programming languages, are designed to follow standard order of operations by default. However, my personal preference is to use parentheses even where strictly unnecessary just to make it clear to any other person viewing the formula what the intended order of operations is.

[9] Reviewing some algebra here, a *root* is a value for x that yields an overall value of zero for the polynomial. For this polynomial ($9x^2 + 5x - 2$) the two roots happen to be $x = 0.269381$ and $x = -0.82494$, with these values displayed in cells `B1` and `B2`, respectively upon execution of the spreadsheet.

Alternatively, one could break up the long quadratic formula into smaller pieces like this:

$$y = \sqrt{b^2 - 4ac} \qquad z = 2a$$

$$x = \frac{-b \pm y}{z}$$

	A	B	C
1	x_1	= (-B4 + C1) / C2	= sqrt((B4^2) - (4*B3*B5))
2	x_2	= (-B4 - C1) / C2	= 2*B3
3	a =	9	
4	b =	5	
5	c =	-2	

Note how the square-root term (y) is calculated in cell C1, and the denominator term (z) in cell C2. This makes the two final formulae (in cells B1 and B2) simpler to interpret. The positioning of all these cells on the grid is completely arbitrary[10] – all that matters is that they properly reference each other in the formulae.

Spreadsheets are particularly useful for situations where the same set of calculations representing a circuit or other system must be repeated for different initial conditions. The power of a spreadsheet is that it automates what would otherwise be a tedious set of calculations. One specific application of this is to simulate the effects of various components within a circuit failing with abnormal values (e.g. a shorted resistor simulated by making its value nearly zero; an open resistor simulated by making its value extremely large). Another application is analyzing the behavior of a circuit design given new components that are out of specification, and/or aging components experiencing drift over time.

[10] My personal preference is to locate all the "given" data in the upper-left cells of the spreadsheet grid (each data point flanked by a sensible name in the cell to the left and units of measurement in the cell to the right as illustrated in the first distance/time spreadsheet example), sometimes coloring them in order to clearly distinguish which cells contain entered data versus which cells contain computed results from formulae. I like to place all formulae in cells below the given data, and try to arrange them in logical order so that anyone examining my spreadsheet will be able to figure out *how* I constructed a solution. This is a general principle I believe all computer programmers should follow: *document and arrange your code to make it easy for other people to learn from it.*

6.2.3 Magnetic field detector coil

Suppose you were designing a wire coil to be used as part of a magnetic field detection apparatus. It will detect the presence of oscillating (AC) magnetic fields by producing a voltage that will be sensed by a voltmeter.

If this coil is to be placed at a distance from an oscillating magnetic field where the coil covers an area large enough to experience a peak rate-of-change of flux ($\frac{d\Phi}{dt}$) equal to 0.2 Webers per second. How many "turns" of wire would this coil have to possess in order to generate a peak voltage of 5 Volts?

Challenges

- Identify one way for the coil to generate this much voltage, given the same magnetic field source, using fewer turns of wire.

6.2.4 Speaker coil voltage

It is possible to measure a small voltage by connecting a voltmeter to the terminals of a loudspeaker and gently tapping on the cone of that speaker. Suppose you do this, and measure a peak voltage of 45 milliVolts when doing so.

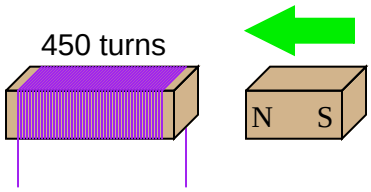
If the speaker's coil happens to contain 180 turns of wire, calculate the maximum rate-of-change of magnetic flux experienced by the coil during this experiment.

Challenges

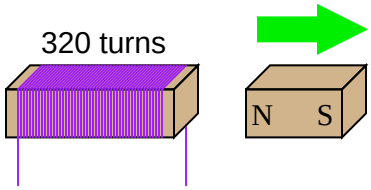
- Identify different ways to have this coil generate a larger peak voltage.

6.2.5 Induced voltage

If a wire coil with 450 turns is exposed to a magnetic flux increasing at a rate of 0.008 Webers per second, how much voltage will be induced across the coil?



If a wire coil with 320 turns is exposed to a magnetic flux decreasing at a rate of 0.03 Webers per second (as shown in the illustration), how much voltage will be induced across the coil, and what will its polarity be?



Challenges

- Are the motions shown in the illustrations the only valid way to physically expose these coils to changing magnetic flux?

6.2.6 Force between wires

Imagine a pair of wires 2 meters long (each) laid parallel to each other, separated by a distance of 10 cm. If 3 Amperes of current passes through one, and 5 Amperes of current passes through the other, both currents in the same direction, how much force will be produced between them?

Challenges

- What would happen if the direction of one of these currents was reversed?
- What would happen if the direction of *both* currents was reversed?
- Identify how to increase this force while not changing either of the current's values.

6.3 Diagnostic reasoning

These questions are designed to stimulate your deductive and inductive thinking, where you must apply general principles to specific scenarios (deductive) and also derive conclusions about the failed circuit from specific details (inductive). In a Socratic discussion with your instructor, the goal is for these questions to reinforce your recall and use of general circuit principles and also challenge your ability to integrate multiple symptoms into a sensible explanation of what's wrong in a circuit. Your instructor may also pose additional questions based on those assigned, in order to further challenge and sharpen your diagnostic abilities.

As always, your goal is to fully *explain* your analysis of each problem. Simply obtaining a correct answer is not good enough – you must also demonstrate sound reasoning in order to successfully complete the assignment. Your instructor's responsibility is to probe and challenge your understanding of the relevant principles and analytical processes in order to ensure you have a strong foundation upon which to build further understanding.

You will note a conspicuous lack of answers given for these diagnostic questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your diagnostic answers, where applicable, is to use circuit simulation software to explore the effects of faults placed in circuits. For example, if one of these diagnostic questions requires that you predict the effect of an open or a short in a circuit, you may check the validity of your work by simulating that same fault (substituting a very high resistance in place of that component for an open, and substituting a very low resistance for a short) within software and seeing if the results agree.

6.3.1 Identifying magnet poles

A simply way to identify the poles of a bar magnet is to suspend that magnet so it is free to turn, and let it orient itself to the Earth's magnetic field. Devise a method to identify the poles of a bar magnet that is too large and heavy to move.

Challenges

- Similarly, explain how we could identify the "North" and "South" poles of a horseshoe magnet, which will not orient itself quite as readily as a bar magnet.

6.3.2 Generator with low output

Suppose a generator used to output 14.6 Volts when its shaft was turned at a speed of 1800 RPM (revolutions per minute), but now only outputs 12.9 Volts at that same speed. Identify some possible causes of this reduced voltage.

Challenges

- What would you expect to see inside of a generator if you disassembled one?

Appendix A

Problem-Solving Strategies

The ability to solve complex problems is arguably one of the most valuable skills one can possess, and this skill is particularly important in any science-based discipline.

- Study principles, not procedures. Don't be satisfied with merely knowing how to compute solutions – learn *why* those solutions work.
- Identify what it is you need to solve, identify all relevant data, identify all units of measurement, identify any general principles or formulae linking the given information to the solution, and then identify any "missing pieces" to a solution. Annotate all diagrams with this data.
- Sketch a diagram to help visualize the problem. When building a real system, always devise a plan for that system and analyze its function *before* constructing it.
- Follow the units of measurement and meaning of every calculation. If you are ever performing mathematical calculations as part of a problem-solving procedure, and you find yourself unable to apply each and every intermediate result to some aspect of the problem, it means you don't understand what you are doing. Properly done, every mathematical result should have practical meaning for the problem, and not just be an abstract number. You should be able to identify the proper units of measurement for each and every calculated result, and show where that result fits into the problem.
- Perform "thought experiments" to explore the effects of different conditions for theoretical problems. When troubleshooting real systems, perform *diagnostic tests* rather than visually inspecting for faults, the best diagnostic test being the one giving you the most information about the nature and/or location of the fault with the fewest steps.
- Simplify the problem until the solution becomes obvious, and then use that obvious case as a model to follow in solving the more complex version of the problem.
- Check for exceptions to see if your solution is incorrect or incomplete. A good solution will work for *all* known conditions and criteria. A good example of this is the process of testing scientific hypotheses: the task of a scientist is not to find support for a new idea, but rather to *challenge* that new idea to see if it holds up under a battery of tests. The philosophical

principle of *reductio ad absurdum* (i.e. disproving a general idea by finding a specific case where it fails) is useful here.

- Work "backward" from a hypothetical solution to a new set of given conditions.
- Add quantities to problems that are qualitative in nature, because sometimes a little math helps illuminate the scenario.
- Sketch graphs illustrating how variables relate to each other. These may be quantitative (i.e. with realistic number values) or qualitative (i.e. simply showing increases and decreases).
- Treat quantitative problems as qualitative in order to discern the relative magnitudes and/or directions of change of the relevant variables. For example, try determining what happens if a certain variable were to increase or decrease before attempting to precisely calculate quantities: how will each of the dependent variables respond, by increasing, decreasing, or remaining the same as before?
- Consider limiting cases. This works especially well for qualitative problems where you need to determine which direction a variable will change. Take the given condition and magnify that condition to an extreme degree as a way of simplifying the direction of the system's response.
- Check your work. This means regularly testing your conclusions to see if they make sense. This does *not* mean repeating the same steps originally used to obtain the conclusion(s), but rather to use some other means to check validity. Simply repeating procedures often leads to *repeating the same errors* if any were made, which is why alternative paths are better.

Appendix B

Instructional philosophy

B.1 First principles of learning

- **Anyone can learn anything** given appropriate time, effort, resources, challenges, encouragement, and expectations. Dedicating time and investing effort are the student's responsibility; providing resources, challenges, and encouragement are the teacher's responsibility; high expectations are a responsibility shared by both student and teacher.

- **Transfer is not automatic.** The human mind has a natural tendency to compartmentalize information, which means the process of taking knowledge learned in one context and applying it to another usually does not come easy and therefore should never be taken for granted.

- **Learning is iterative.** The human mind rarely learns anything perfectly on the first attempt. Anticipate mistakes and plan for multiple tries to achieve full understanding, using the lessons of those mistakes as feedback to guide future attempts.

- **Information is absorbed, but understanding is created.** Facts and procedures may be memorized easily enough by repeated exposure, but the ability to reliably apply principles to novel scenarios only comes through intense personal effort. This effort is fundamentally creative in nature: casting new concepts in one's own words, running mental experiments to perceive patterns and trends, etc. These acts of making knowledge "one's own" need not be perfect in order to be effective, as the value lies in the activity and not necessarily the finished product. Depth of understanding and permanence of memory are both directly related to mental effort during the learning process – i.e. *easy learning is shallow and fleeting.*

- **Education trumps training.** There is no such thing as an entirely isolated subject, as all fields of knowledge are connected. Training is narrowly-focused and task-oriented. Education is broad-based and principle-oriented. When preparing for a life-long technical career, education beats training every time.

- **Character matters.** Poor habits are more destructive than deficits of knowledge or skill. This is especially true in collective endeavors, where a team's ability to function depends on trust between its members. Simply put, no one wants an untrustworthy person on their team. An essential component of education then, is character development.

- **People learn to be responsible by bearing responsibility.** An irresponsible person is someone who has never *had* to be responsible for anything that mattered enough to them. Just as anyone can learn anything, anyone can become responsible if the personal cost of irresponsibility becomes high enough.

- **What gets measured, gets done.** Accurate and relevant assessment of learning is key to ensuring all students learn. Therefore, it is imperative to measure what matters.

- **Failure is nothing to fear.** Every human being fails, and fails in multiple ways at multiple times. Eventual success only happens when we don't stop trying.

B.2 Proven strategies for instructors

- Assume every student is capable of learning anything they desire given the proper conditions. Treat them as capable adults by granting real responsibility and avoiding artificial incentives such as merit or demerit points.
- Create a consistent culture of high expectations across the entire program of study. Demonstrate and encourage patience, persistence, and a healthy sense of self-skepticism. Anticipate and de-stigmatize error. Teach respect for the capabilities of others as well as respect for one's own fallibility.
- Replace lecture with "inverted" instruction, where students first encounter new concepts through reading and then spend class time in Socratic dialogue with the instructor exploring those concepts and solving problems individually. There is a world of difference between observing someone solve a problem versus actually solving a problem yourself, and so the point of this form of instruction is to place students in a position where they *cannot* passively observe.
- Require students to read extensively, write about what they learn, and dialogue with you and their peers to sharpen their understanding. Apply Francis Bacon's advice that "reading maketh a full man; conference a ready man; and writing an exact man". These are complementary activities helping students expand their confidence and abilities.
- Use artificial intelligence (AI) to challenge student understanding rather than merely provide information. Find productive ways for AI to critique students' clarity of thought and of expression, for example by employing AI as a Socratic-style interlocutor or as a reviewer of students' journals. Properly applied, AI has the ability to expand student access to critical review well outside the bounds of their instructor's reach.
- Build frequent and rapid feedback into the learning process so that students know at all times how well they are learning, to identify problems early and fix them before they grow. Model the intellectual habit of self-assessing and self-correcting your own understanding (i.e. a cognitive *feedback loop*), encouraging students to do the same.
- Use "mastery" as the standard for every assessment, which means the exam or experiment or project must be done with 100% competence in order to pass. Provide students with multiple opportunity for re-tries (different versions of the assessment every time).
- Require students to devise their own hypotheses and procedures on all experiments, so that the process is truly a scientific one. Have students assess their proposed experimental procedures for risk and devise mitigations for those risks. Let nothing be pre-designed about students' experiments other than a stated task (i.e. what principle the experiment shall test) at the start and a set of demonstrable knowledge and skill objectives at the end.
- Have students build as much of their lab equipment as possible: building power sources, building test assemblies[1], and building complete working systems (no kits!). In order to provide

[1] In the program I teach, every student builds their own "Development Board" consisting of a metal chassis with DIN rail, terminal blocks, and an AC-DC power supply of their own making which functions as a portable lab environment they can use at school as well as take home.

this same "ground-up" experience for every new student, this means either previous students take their creations with them, or the systems get disassembled in preparation for the new students, or the systems grow and evolve with each new student group.

- Incorporate external accountability for you and for your students, continuously improving the curriculum and your instructional methods based on proven results. Have students regularly network with active professionals through participation in advisory committee meetings, service projects, tours, jobshadows, internships, etc. Practical suggestions include requiring students to design and build projects for external clients (e.g. community groups, businesses, different departments within the institution), and also requiring students attend all technical advisory committee meetings and to dialogue with the industry representatives at those meetings.
- Repeatedly explore difficult-to-learn concepts across multiple courses, so that students have multiple opportunities to build their understanding.
- Relate all new concepts, whenever possible, to previous concepts and to relevant physical laws. Challenge each and every student, every day, to *reason* from concept to concept and to explain the logical connections between. Challenge students to verify their conclusions by multiple approaches (e.g. double-checking their work using different methods). Ask *"Why?"* often.
- Maintain detailed records on each student's performance and share these records privately with them. These records should include academic performance as well as professionally relevant behavioral tendencies.
- Hold mandatory "check-in" meetings between all program faculty and each new student during their first term. Offer these to all other students as an option, except for any students continuing to manifest unprofessional behaviors, poor academic performance, or who have some other need for a face-to-face meeting with faculty.
- Address problems while they are small, before they grow larger. This is equally true for tutoring technical concepts as it is for helping students build professional habits.
- Build rigorous quality control into the curriculum to ensure every student masters every important concept, and that the mastery is retained over time. This includes (1) review questions added to every exam to re-assess knowledge taught in previous terms, (2) cumulative exams at the end of every term to re-assess all important concepts back to the very beginning of the program, and (3) review assessments in practical (hands-on) coursework to ensure critically-important skills were indeed taught and are still retained. What you will find by doing this is that it actually boosts retention of students by ensuring that important knowledge gets taught and is retained over long spans of time. In the absence of such quality control, student learning and retention tends to be spotty and this contributes to drop-out and failure rates later in their education.
- Finally, *never rush learning.* Education is not a race. Give your students ample time to digest complex ideas, as you continually remind yourself of just how long it took you to achieve mastery! Long-term retention and the consistently correct application of concepts are always the result of *focused effort over long periods of time* which means there are no shortcuts to learning.

B.3 Proven strategies for students

The single most important piece of advice I have for any student of any subject is to take responsibility for your own development in all areas of life including mental development. Expecting others in your life to entirely guide your own development is a recipe for disappointment. This is just as true for students enrolled in formal learning institutions as it is for auto-didacts pursuing learning entirely on their own. Learning to think in new ways is key to being able to gainfully use information, to make informed decisions about your life, and to best serve those you care about. With this in mind, I offer the following advice to students:

- **Approach all learning as valuable**. No matter what course you take, no matter who you learn from, no matter the subject, there is something useful in every learning experience. If you don't see the value of every new experience, you are not looking closely enough!
- **Continually challenge yourself**. Let other people take shortcuts and find easy answers to easy problems. The purpose of education is to stretch your mind, in order to shape it into a more powerful tool. This doesn't come by taking the path of least resistance. An excellent analogy for an empowering education is productive physical exercise: becoming stronger, more flexible, and more persistent only comes through intense personal effort.
- **Master the use of language**. This includes reading extensively, writing every day, listening closely, and speaking articulately. To a great extent language channels and empowers thought, so the better you are at wielding language the better you will be at grasping abstract concepts and articulating them not only for your benefit but for others as well.
- **Do not limit yourself to the resources given to you**. Read books that are not on the reading list. Run experiments that aren't assigned to you. Form study groups outside of class. Take an entrepreneurial approach to your own education, as though it were a business you were building for your future benefit.
- **Express and share what you learn**. Take every opportunity to teach what you have learned to others, as this will not only help them but will also strengthen your own understanding[2].
- Realize that **no one can give you understanding**, just as no one can give you physical fitness. These both must be *built*.
- **Above all, recognize that learning is hard work, and that a certain level of frustration is unavoidable**. There are times when you will struggle to grasp some of these concepts, and that struggle is a natural thing. Take heart that it will yield with persistent and varied[3] effort, and never give up! That concepts don't immediately come to you is not a sign of something wrong, but rather of something right: that you have found a worthy challenge!

[2] On a personal note, I was surprised to learn just how much my own understanding of electronics and related subjects was strengthened by becoming a teacher. When you are tasked every day with helping other people grasp complex topics, it catalyzes your own learning by giving you powerful incentives to study, to articulate your thoughts, and to reflect deeply on the process of learning.

[3] As the old saying goes, "Insanity is trying the same thing over and over again, expecting different results." If you find yourself stumped by something in the text, you should attempt a different approach. Alter the thought experiment, change the mathematical parameters, do whatever you can to see the problem in a slightly different light, and then the solution will often present itself more readily.

B.4 Design of these learning modules

"The unexamined circuit is not worth energizing" – Socrates (if he had taught electricity)

These learning modules, although useful for self-study, were designed to be used in a formal learning environment where a subject-matter expert challenges students to digest the content and exercise their critical thinking abilities in the answering of questions and in the construction and testing of working circuits. Every effort has been made to embed the following instructional and assessment philosophies within:

- The first goal of education is to enhance clear and independent thought, in order that every student reach their fullest potential in a highly complex and inter-dependent world. Robust reasoning is *always* more important than particulars of any subject matter, because its application is universal.
- Literacy is fundamental to independent learning and thought because text continues to be the most efficient way to communicate complex ideas over space and time. Those who cannot read with ease are limited in their ability to acquire knowledge and perspective.
- Articulate communication is fundamental to work that is complex and interdisciplinary.
- Faulty assumptions and poor reasoning are best corrected through challenge, not presentation. The rhetorical technique of *reductio ad absurdum* (disproving an assertion by exposing an absurdity) works well to discipline student's minds, not only to correct the problem at hand but also to learn how to detect and correct future errors.
- Important principles should be repeatedly explored and widely applied throughout a course of study, not only to reinforce their importance and help ensure their mastery, but also to showcase the interconnectedness and utility of knowledge.

These learning modules were expressly designed to be used in an "inverted" teaching environment[4] where students first read the introductory and tutorial chapters on their own, then individually attempt to answer the questions and construct working circuits according to the experiment and project guidelines. The instructor never lectures, but instead meets regularly with each individual student to review their progress, answer questions, identify misconceptions, and challenge the student to new depths of understanding through further questioning. Regular meetings between instructor and student should resemble a Socratic[5] dialogue, where questions serve as scalpels to dissect topics and expose assumptions. The student passes each module only after consistently demonstrating their ability to logically analyze and correctly apply all major concepts in each question or project/experiment. The instructor must be vigilant in probing each student's understanding to ensure they are truly *reasoning* and not just *memorizing*. This is why "Challenge" points appear throughout, as prompts for students to think deeper about topics and as starting points for instructor queries. Sometimes these challenge points require additional knowledge that hasn't been covered in the series to answer in full. This is okay, as the major purpose of the Challenges is to stimulate analysis and synthesis on the part of each student.

The instructor must possess enough mastery of the subject matter and awareness of students' reasoning to generate their own follow-up questions to practically any student response. Even completely correct answers given by the student should be challenged by the instructor for the purpose of having students practice articulating their thoughts and defending their reasoning. Conceptual errors committed by the student should be exposed and corrected not by direct instruction, but rather by reducing the errors to an absurdity[6] through well-chosen questions and thought experiments posed by the instructor. Becoming proficient at this style of instruction requires time and dedication, but the positive effects on critical thinking for both student and instructor are spectacular.

An inspection of these learning modules reveals certain unique characteristics. One of these is a bias toward thorough explanations in the tutorial chapters. Without a live instructor to explain concepts and applications to students, the text itself must fulfill this role. This philosophy results in lengthier explanations than what you might typically find in a textbook, each step of the reasoning process fully explained, including footnotes addressing common questions and concerns students raise while learning these concepts. Each tutorial seeks to not only explain each major concept in sufficient detail, but also to explain the logic of each concept and how each may be developed

[4] In a traditional teaching environment, students first encounter new information via *lecture* from an expert, and then independently apply that information via *homework*. In an "inverted" course of study, students first encounter new information via *homework*, and then independently apply that information under the scrutiny of an expert. The expert's role in lecture is to simply *explain*, but the expert's role in an inverted session is to *challenge*, *critique*, and if necessary *explain* where gaps in understanding still exist.

[5] Socrates is a figure in ancient Greek philosophy famous for his unflinching style of questioning. Although he authored no texts, he appears as a character in Plato's many writings. The essence of Socratic philosophy is to leave no question unexamined and no point of view unchallenged. While purists may argue a topic such as electric circuits is too narrow for a true Socratic-style dialogue, I would argue that the essential thought processes involved with scientific reasoning on *any* topic are not far removed from the Socratic ideal, and that students of electricity and electronics would do very well to challenge assumptions, pose thought experiments, identify fallacies, and otherwise employ the arsenal of critical thinking skills modeled by Socrates.

[6] This rhetorical technique is known by the Latin phrase *reductio ad absurdum*. The concept is to expose errors by counter-example, since only one solid counter-example is necessary to disprove a universal claim. As an example of this, consider the common misconception among beginning students of electricity that voltage cannot exist without current. One way to apply *reductio ad absurdum* to this statement is to ask how much current passes through a fully-charged battery connected to nothing (i.e. a clear example of voltage existing without current).

from "first principles". Again, this reflects the goal of developing clear and independent thought in students' minds, by showing how clear and logical thought was used to forge each concept. Students benefit from witnessing a model of clear thinking in action, and these tutorials strive to be just that.

Another feature of these learning modules is that they do not treat topics in isolation. Rather, important concepts are introduced early in the series, and appear repeatedly as stepping-stones toward other concepts in subsequent modules. This helps to avoid the "compartmentalization" of knowledge, demonstrating the inter-connectedness of concepts and simultaneously reinforcing them. Each module is fairly complete in itself, reserving the beginning of its tutorial to a review of foundational concepts.

To high standards of education,

Tony R. Kuphaldt

Appendix C

Tools used

I am indebted to the developers of many open-source software applications in the creation of these learning modules. The following is a list of these applications with some commentary on each.

You will notice a theme common to many of these applications: a bias toward *code*. Although I am by no means an expert programmer in any computer language, I understand and appreciate the flexibility offered by code-based applications where the user (you) enters commands into a plain ASCII text file, which the software then reads and processes to create the final output. Code-based computer applications are by their very nature *extensible*, while WYSIWYG (What You See Is What You Get) applications are generally limited to whatever user interface the developer makes for you.

The `GNU/Linux` computer operating system

> There is so much to be said about Linus Torvalds' `Linux` and Richard Stallman's `GNU` project. First, to credit just these two individuals is to fail to do justice to the *mob* of passionate volunteers who contributed to make this amazing software a reality. I first learned of `Linux` back in 1996, and have been using this operating system on my personal computers almost exclusively since then. It is *free*, it is completely *configurable*, and it permits the continued use of highly efficient `Unix` applications and scripting languages (e.g. shell scripts, Makefiles, `sed`, `awk`) developed over many decades. `Linux` not only provided me with a powerful computing platform, but its open design served to inspire my life's work of creating open-source educational resources.

Bram Moolenaar's `Vim` text editor

> Writing code for any code-based computer application requires a *text editor*, which may be thought of as a word processor strictly limited to outputting plain-ASCII text files. Many good text editors exist, and one's choice of text editor seems to be a deeply personal matter within the programming world. I prefer `Vim` because it operates very similarly to `vi` which is ubiquitous on `Unix`/`Linux` operating systems, and because it may be entirely operated via keyboard (i.e. no mouse required) which makes it fast to use.

Donald Knuth's TEX typesetting system

Developed in the late 1970's and early 1980's by computer scientist extraordinaire Donald Knuth to typeset his multi-volume magnum opus *The Art of Computer Programming*, this software allows the production of formatted text for screen-viewing or paper printing, all by writing plain-text code to describe how the formatted text is supposed to appear. TEX is not just a markup language for documents, but it is also a Turing-complete programming language in and of itself, allowing useful algorithms to be created to control the production of documents. Simply put, TEX *is a programmer's approach to word processing*. Since TEX is controlled by code written in a plain-text file, this means anyone may read that plain-text file to see exactly how the document was created. This openness afforded by the code-based nature of TEX makes it relatively easy to learn how other people have created their own TEX documents. By contrast, examining a beautiful document created in a conventional WYSIWYG word processor such as Microsoft `Word` suggests nothing to the reader about *how* that document was created, or what the user might do to create something similar. As Mr. Knuth himself once quipped, conventional word processing applications should be called WYSIAYG (What You See Is *All* You Get).

Leslie Lamport's LATEX extensions to TEX

Like all true programming languages, TEX is inherently extensible. So, years after the release of TEX to the public, Leslie Lamport decided to create a massive extension allowing easier compilation of book-length documents. The result was LATEX, which is the markup language used to create all ModEL module documents. You could say that TEX is to LATEX as `C` is to `C++`. This means it is permissible to use any and all TEX commands within LATEX source code, and it all still works. Some of the features offered by LATEX that would be challenging to implement in TEX include automatic index and table-of-content creation.

Tim Edwards' `Xcircuit` drafting program

This wonderful program is what I use to create all the schematic diagrams and illustrations (but not photographic images or mathematical plots) throughout the ModEL project. It natively outputs PostScript format which is a true vector graphic format (this is why the images do not pixellate when you zoom in for a closer view), and it is so simple to use that I have never had to read the manual! Object libraries are easy to create for `Xcircuit`, being plain-text files using PostScript programming conventions. Over the years I have collected a large set of object libraries useful for drawing electrical and electronic schematics, pictorial diagrams, and other technical illustrations.

`Gimp` graphic image manipulation program

Essentially an open-source clone of Adobe's `PhotoShop`, I use `Gimp` to resize, crop, and convert file formats for all of the photographic images appearing in the ModEL modules. Although `Gimp` does offer its own scripting language (called `Script-Fu`), I have never had occasion to use it. Thus, my utilization of `Gimp` to merely crop, resize, and convert graphic images is akin to using a sword to slice bread.

`SPICE` circuit simulation program

`SPICE` is to circuit analysis as TEX is to document creation: it is a form of markup language designed to describe a certain object to be processed in plain-ASCII text. When the plain-text "source file" is compiled by the software, it outputs the final result. More modern circuit analysis tools certainly exist, but I prefer `SPICE` for the following reasons: it is *free*, it is *fast*, it is *reliable*, and it is a fantastic tool for *teaching* students of electricity and electronics how to write simple code. I happen to use rather old versions of `SPICE`, version 2g6 being my "go to" application when I only require text-based output. `NGSPICE` (version 26), which is based on Berkeley `SPICE` version 3f5, is used when I require graphical output for such things as time-domain waveforms and Bode plots. In all `SPICE` example netlists I strive to use coding conventions compatible with all `SPICE` versions.

Andrew D. Hwang's `ePiX` mathematical visualization programming library

This amazing project is a `C++` library you may link to any `C/C++` code for the purpose of generating PostScript graphic images of mathematical functions. As a completely free and open-source project, it does all the plotting I would otherwise use a Computer Algebra System (CAS) such as `Mathematica` or `Maple` to do. It should be said that `ePiX` is *not* a Computer Algebra System like `Mathematica` or `Maple`, but merely a mathematical *visualization* tool. In other words, it won't determine integrals for you (you'll have to implement that in your own `C/C++` code!), but it can graph the results, and it does so beautifully. What I really admire about `ePiX` is that it is a `C++` programming library, which means it builds on the existing power and toolset available with that programming language. Mr. Hwang could have probably developed his own stand-alone application for mathematical plotting, but by creating a `C++` library to do the same thing he accomplished something much greater.

`gnuplot` mathematical visualization software

Another open-source tool for mathematical visualization is `gnuplot`. Interestingly, this tool is *not* part of Richard Stallman's GNU project, its name being a coincidence. For this reason the authors prefer "gnu" *not* be capitalized at all to avoid confusion. This is a much "lighter-weight" alternative to a spreadsheet for plotting tabular data, and the fact that it easily outputs directly to an X11 console or a file in a number of different graphical formats (including PostScript) is very helpful. I typically set my `gnuplot` output format to default (X11 on my Linux PC) for quick viewing while I'm developing a visualization, then switch to PostScript file export once the visual is ready to include in the document(s) I'm writing. As with my use of `Gimp` to do rudimentary image editing, my use of `gnuplot` only scratches the surface of its capabilities, but the important points are that it's *free* and that it *works well*.

`Python` programming language

Both Python and C++ find extensive use in these modules as instructional aids and exercises, but I'm listing Python here as a *tool* for myself because I use it almost daily as a *calculator*. If you open a Python interpreter console and type `from math import *` you can type mathematical expressions and have it return results just as you would on a hand calculator. Complex-number (i.e. *phasor*) arithmetic is similarly supported if you include the complex-math library (`from cmath import *`). Examples of this are shown in the Programming References chapter (if included) in each module. Of course, being a fully-featured programming language, Python also supports conditionals, loops, and other structures useful for calculation of quantities. Also, running in a console environment where all entries and returned values show as text in a chronologically-ordered list makes it easy to copy-and-paste those calculations to document exactly how they were performed.

Appendix D

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Appendix E

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Appendix F

Version history

This is a list showing all significant additions, corrections, and other edits made to this learning module. Each entry is referenced by calendar date in reverse chronological order (newest version first), which appears on the front cover of every learning module for easy reference. Any contributors to this open-source document are listed here as well.

13 October 2025 – typographical error correction courtesy of Drew Kelly.

6-7 October 2025 – minor edits to the Tutorial chapter, including multiple grammatical corrections suggested by Wyatt Gleason.

2 April 2025 – added another Technical References section, this one on Gauss' Law of Electric Fields.

14 March 2025 – added a personal photograph of an historical electroscope to the Historical References section on electric-field instruments.

9 October 2024 – added some instructor notes.

24 September 2024 – added a new Introduction section on challenging concepts. Also added a new Technical References section on Maxwell's equations.

5 July 2024 – replaced some non-ASCII "degree" symbols that got copied and pasted from historical books into the Historical References section on Michael Faraday's experiments.

21-24 June 2024 – divided the Introduction chapter into two sections, one for students and one for instructors, and added content to the instructor section recommending learning outcomes and measures. Also divided the right-hand rule into two versions: curled versus straight. Also added a new Case Tutorial section showing photographs of the curled right-hand rule in action, as well as another Case Tutorial showing the use of an oscilloscope to measure induced voltage from a coil and a moving magnet. Also added a photograph depicting the curled right-hand rule in action.

3 March 2024 – minor edits to instructor comments.

19 February 2024 – added a new section to the Technical References chapter on near-field versus far-field effects.

12 October 2023 – typo correction in the Introduction chapter ("to" instead of "do") courtesy of Erik Brown.

5-6 July 2023 – added a new Technical Reference section on defining magnetic north and magnetic south. Also edited the Case Tutorial section on cow magnets to describe the odd configuration of certain cow magnets that make them useless as magnetic compasses.

13 May 2023 – added a new Case Tutorial section on high-voltage power line corona.

21 March 2023 – minor edits to the "Lenz's Law" section of the Tutorial.

27 November 2022 – placed questions at the top of the itemized list in the Introduction chapter prompting students to devise experiments related to the tutorial content.

16 November 2022 – added a new Historical References section (historical_solenoid_turbine.latex) showing a solenoid used as part of a feedforward control system for an antique steam turbine-driven electrical generator.

17 October 2022 – added comments to the Tutorial describing a way to easily remember the finger-vector relations in the right-hand rule. Also added a Conceptual Reasoning question on Lorentz forces in a rail gun.

13 July 2022 – clarified some of the images showing magnetic fields created by electric current.

27 November 2021 – added a new Tutorial section on basic motor theory.

26 November 2021 – added a new Technical Reference section on the Poynting vector, and added many details to the Tutorial including an expanded the right-hand rule section to include the vector cross-product form of the right-hand rule and photographs of my right hand annotated with vector symbols.

2 August 2021 – added a photograph to the Case Tutorial section demonstrating Lenz's Law using a battery and a small AC motor.

9 July 2021 – added a Case Tutorial section with a demonstration of Lenz's Law using a battery and a small AC motor.

28 May 2021 – added a Case Tutorial chapter with a section showing how to use a CMOS inverter IC as a sensitive electric field detector. Also, divided the main Tutorial into sections.

8 May 2021 – commented out or deleted empty chapters.

30 March 2021 – qualified that only some types of ferrites are high-loss, and added distinction between magnetically "soft" versus "hard" materials.

13 October 2020 – minor additions to the Introduction chapter, and to the Tutorial. Also, changed the direction of coil "wrap" in one of the examples in the "Induced current directions" question.

2 October 2020 – significantly edited the Introduction chapter to make it more suitable as a pre-study guide and to provide cues useful to instructors leading "inverted" teaching sessions.

4 May 2020 – added questions, as well as Tutorial content on magnetic saturation.

1 May 2020 - 30 April 2020 – added questions, as well as Technical Reference sections on electric and magnetic field measurements.

27 January 2020 – added Foundational Concepts to the list in the Conceptual Reasoning section.

1 October 2019 – added more text from Michael Faraday's book to the Historical Reference section on induction.

13 January 2019 – renamed Simplified Tutorial to Tutorial, since I haven't written a Full Tutorial yet.

19 December 2018 – added photograph of steel filings oriented around an arc welding cable.

6 November 2018 – added historical references showing electrostatic-field instruments.

5 November 2018 – added historical references showing magnetic fields mapped by steel filings. Also edited Simplified Tutorial to elaborate more on the distinctions between electric and magnetic fields.

4 November 2018 – document first created.

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