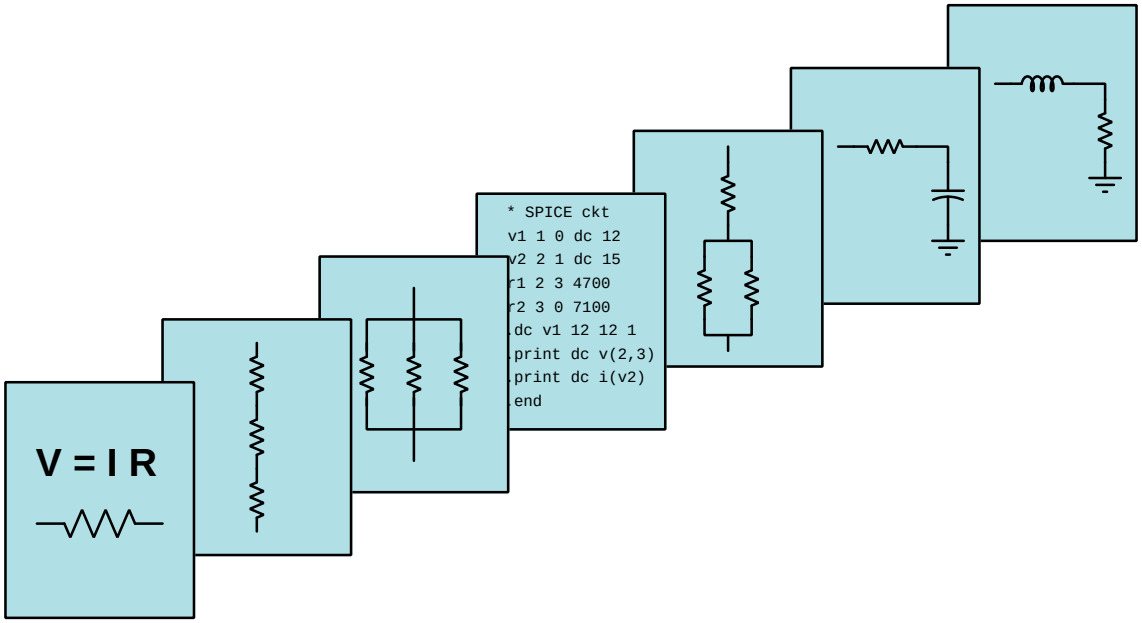
Modular Electronics Learning (ModEL) project



Feedback

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Chapter 1

Introduction

1.1 Recommendations for students

The concept of *feedback* is one of the most sweeping and powerful in all of electronics. At root, it involves "recycling" at least a portion of a signal from the output of a system back to its input. Feedback comes in two generalized forms: *positive feedback* where the recycled signal tends to reinforce itself, and *negative feedback* where the recycled signal tends to diminish itself. A great many electronic circuits utilize both forms of feedback to practical effect.

Important concepts related to feedback include **amplification**, **gain**, **passive averagers**, the **normal status of a switch**, **saturation**, **analog** versus **digital** signals, **block diagrams**, **functional inversion**, **servos**, **reference** signals, **disturbances**, and **open-** versus **closed-loop** systems.

Here are some good questions to ask of yourself while studying this subject:

- How might an experiment be designed and conducted to explore the phenomenon of negative feedback? What hypothesis (i.e. prediction) might you pose for that experiment, and what result(s) would either support or disprove that hypothesis?
- What are some practical applications of feedback in complex systems?
- What is *gain*, in your own words?
- What is the significance of a *negative* gain value for an amplifier?
- What is the general effect of positive feedback on any system?
- What is the general effect of negative feedback on any system?
- Why might we intentionally add positive feedback to an electrical or electronic circuit?
- Why might we intentionally add negative feedback to an electrical or electronic circuit?
- How does a mechanical pendulum help illustrate the phenomenon of positive feedback?

- How does a mechanical pendulum help illustrate the phenomenon of negative feedback?
- How may we intentionally design an electronic amplifier to have an extremely high voltage gain?
- Why doesn't negative feedback guarantee oscillation in an amplifier or in any other type of system employing negative feedback?
- Why might we want to design an amplifier system with an enormous gain only to reduce that enormous gain through negative feedback?
- What are some practical applications of servo systems?
- How are block diagrams useful for modeling feedback systems?
- How could you alter a block diagram for a negative feedback system, to make it have positive feedback instead?
- What is *functional inversion*, and why is it useful?

1.2 Challenging concepts related to feedback

The following list cites concepts related to this module's topic that are easily misunderstood, along with suggestions for properly understanding them:

- **Negative feedback** – this non-intuitive concept is difficult to grasp due its recursive nature: the output of an amplifier is fed back to its input, which affects its output, which affects its input, *ad infinitum*. A particular challenge with negative feedback is comprehending why this does not irresistably lead to oscillations, given the fed-back signal's tendency to counter whatever direction the output signal is changing in. One rationale for the oscillation hypothesis is that we as human beings tend to imagine changes in discrete steps rather than as continuous trends, and so it is understandable why a new student might assume negative feedback causes the amplifier to leap back and forth between the (only) two extremes of signal range they might be envisioning. In reality, as the output of an amplifier with negative feedback drifts in one direction, the fed-back signal has the effect of slowing that drift down until it settles at some value rather than oscillate: as the "error" approaches zero, so does the rate of drift.

- **Functional inversion** – one of the non-intuitive consequences of negative feedback is that whatever function is placed in the feedback loop ends up being inverted in the amplifier's overall response. For example, feeding back a divided signal results in the amplifier multiplying its input voltage. This can and will make sense if one takes the time to trace signal values in a feedback system, which is one great application for block diagrams!

- **Faults in a feedback system** – predicting the effect(s) of a given fault in a negative-feedback system can be confusing because of the functional inversion that system provides. For example, if something happens in the feedback signal path to attenuate that signal, the result will be the amplifier generating an ever *larger* output signal to compensate!

1.3 Recommendations for instructors

This section lists realistic student learning outcomes supported by the content of the module as well as suggested means of assessing (measuring) student learning. The outcomes state what learners should be able to do, and the assessments are specific challenges to prove students have learned.

- **Outcome** – Demonstrate effective technical reading and writing

  Assessment – Students present their outlines of this module's instructional chapters (e.g. Case Tutorial, Tutorial, Historical References, etc.) ideally as an entry to a larger Journal document chronicling their learning. These outlines should exhibit good-faith effort at summarizing major concepts explained in the text.

  Assessment – Students show how quantitative results were obtained by the author in the Tutorial chapter's examples.

  Assessment – Students present their "Conceptual Toolbox" Theory Journal entries listing important concepts applied in the Tutorial text(s). A "Conceptual Toolbox" serves as inspiration when solving problems, each concept suggesting a potential problem-solving action. Challenge students to collaboratively generate a Conceptual Toolbox for the day's topic during the class session and then use it when solving problems together, thus fostering the habit of reasoning from principles rather than recalling procedures!

- **Outcome** – Apply the concept of negative feedback to real systems

  Assessment – Explain how negative feedback works in a generator voltage-regulation system; e.g. pose problems in the form of the "Fooling a voltage regulator" Conceptual Reasoning question.

  Assessment – Predict the effects of changes within negative feedback systems; e.g. pose problems in the form of the "Miscalibrated speedometer" or "Miscalibrated thermostat sensor" Conceptual Reasoning questions.

- **Outcome** – Apply the concept of functional inversion to negative-feedback systems

  Assessment – Calculate signal values in a given negative-feedback system shown in block-diagram form; e.g. pose problems in the form of the "Mathematical functions in feedback path" Quantitative Reasoning question.

- **Outcome** – Independent research

  Assessment – Read and summarize in your own words Harold Black's 1937 patent on negative feedback (US Patent 2,102,671 "Wave Translation System").

  Assessment – Read and summarize in your own words reliable source documents on the subject of operational amplifiers. Recommended readings include anything written by the early opamp researcher George A. Philbrick, as well as patents on opamp design innovations.

Chapter 2

Case Tutorial

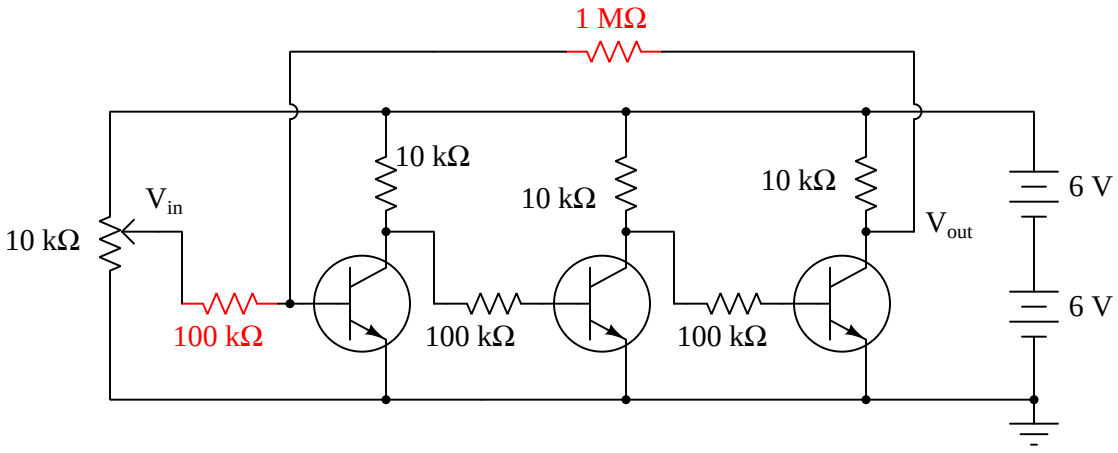
The idea behind a *Case Tutorial* is to explore new concepts by way of example. In this chapter you will read less presentation of theory compared to other Tutorial chapters, but by close observation and comparison of the given examples be able to discern patterns and principles much the same way as a scientific experimenter. Hopefully you will find these cases illuminating, and a good supplement to text-based tutorials.

These examples also serve well as challenges following your reading of the other Tutorial(s) in this module – can you explain *why* the circuits behave as they do?

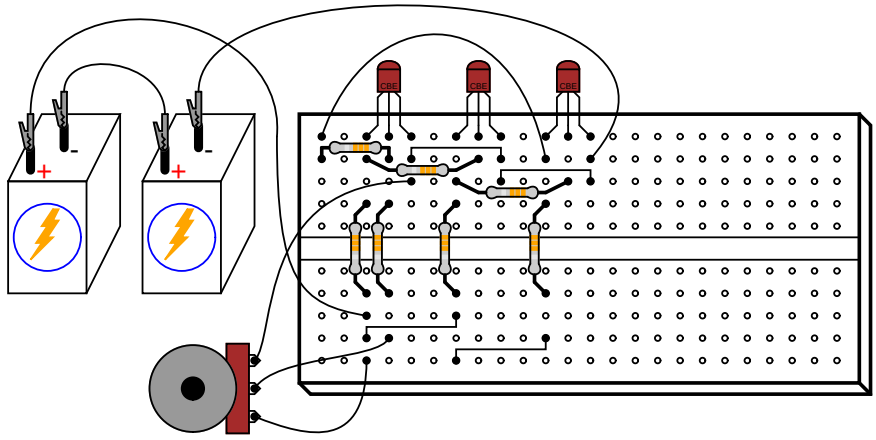
2.1 Example: multi-stage transistor amplifier with negative feedback

Single-stage transistor amplifiers are well-known for their somewhat unpredictable voltage gain behavior due to uncontrollable variations in transistor gain (beta for BJTs, and transconductance for FETs). However, if multiple stages are cascaded to produce a very high voltage gain, then negative feedback may be used to diminish that high voltage gain to a more reasonable (and much more stable!) value.

The following circuit shows how this may be done, using three grounded-emitter BJT amplifier stages to provide very high voltage gain and the inverting characteristic necessary for negative feedback:



The 1 MΩ feedback resistor in conjunction with the 100 kΩ "input" resistor yield a 10:1 ratio, translating into a voltage gain value that is nearly exactly 10.



Each stage of this amplifier circuit is *inverting*, in that a rising input signal causes the output signal to fall. Over-all, the entire amplifier is also inverting because there is an odd number of these inverting stages cascaded – we could have just as readily coupled five common-emitter stages together, or seven, or nine, and still maintained the inverting characteristic we need. This characteristic is important because we want the feedback to be *negative* in sign: the output signal's direction must fight against the causal input signal's direction in order to bring stability to the circuit.

Although the notion of building an amplifier circuit to have an extremely high gain and then intentionally reducing that gain by way of negative feedback may seem counter-productive, the result is that the over-all gain becomes much more stable and the amplification becomes much more linear than with no feedback at all. As the transistors heat and cool, or if one must be replaced with another having slightly different characteristics, the strategy of arbitrarily high voltage gain intentionally scaled down via negative feedback results in those individual transistor variations having very little effect on the whole circuit's idealized performance.

This non-intuitive concept lies at the heart of Harold Black's 1920's era innovation, developed at a time when telephone repeater amplifiers used vacuum tubes as amplifying elements. These early tube circuits were notoriously unstable, the tubes' operating characteristics changing significantly as they aged. Black's application of negative feedback to the problem of electronic amplifier stability was revolutionary, and still holds merit with today's solid-state amplifier circuitry.

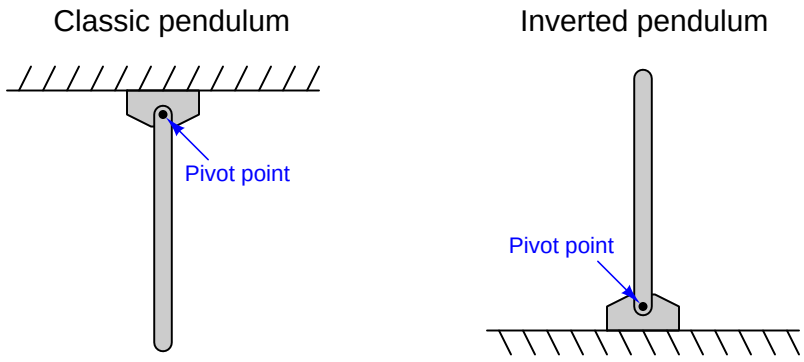
Operational amplifiers are perhaps the best and most common electronic application of this concept. Being integrate-circuit (IC) modules designed to provide extremely high voltage gain on their own, "opamps" are designed to be used in conjunction with negative feedback resistor networks like this to provide highly accurate and stable gains. This discrete transistor amplifier demonstrates the fundamental concept, but without the use of any integrated circuitry.

Chapter 3

Tutorial

3.1 Mechanical pendula

Consider two types of pendula, one suspended from an elevated pivot, and the other "inverted" with its pivot point at ground level:

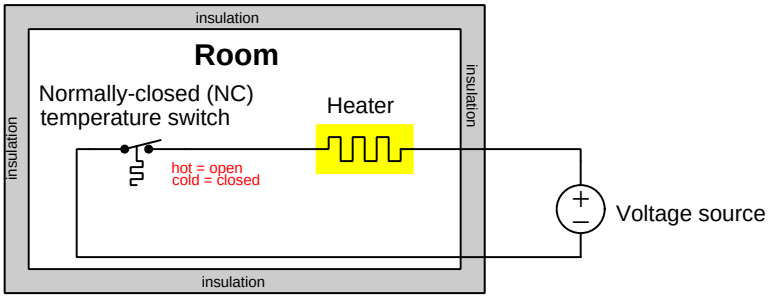


These two mechanisms are identical in construction, but have very different behaviors. The classic pendulum, when nudged by an external force, always returns to its original (downward) position; the inverted pendulum, when nudged by an external force, falls over and remains there. Described in more technical terms, the classic pendulum *self-corrects* while the inverted pendulum *saturates*.

Key to understanding why each of these pendula behave as they do is an analysis of the forces acting on each. There is, of course, the external force nudging each pendulum in a particular direction. More significant, though, is *the weight of each pendulum acting upon itself*. In the case of the classic pendulum, its own weight constantly drives it *toward* a perfectly vertical (downward) orientation; in the case of the inverted pendulum, its own weight drives it *away* from a perfectly vertical (upward) orientation. The weight of a pendulum combined with its angle from vertical creates a force independent of any external nudge, and as such we could say that a pendulum's weight *feeds back* on itself. For a classic pendulum, this feedback force always *opposes* any nudge away from vertical. For an inverted pendulum, this feedback force always *aids* any nudge away from vertical. Using the nudge's direction as a reference, we could say that the classic pendulum's feedback is *negative* (i.e. opposing) and that the inverted pendulum's feedback is *positive* (i.e. aiding).

3.2 Thermostatic control

Feedback applies to a wide range of applications. Having seen how the concept of feedback applies to a pendulum, we will now consider a room *thermostat*:



A simple temperature switch located inside this room senses air temperature, closing the circuit and energizing the electrical heater if the room becomes too cold, and opening the circuit and de-energizing the heater if the room becomes too hot. The electric circuit in this room is another example of a *feedback* system, where its action feeds back on itself: the electric heater doesn't just add thermal energy to the room – it also affects the temperature switch which in turn affects the heater, and so on in a causal loop[1]. Specifically, the thermostatic system shown above is a *negative feedback* system because it counter-acts any thermal "nudge" externally imposed on the room: if cold weather forces the room to cool down, the switch closes to warm it back up. This is analogous to a classic pendulum, whose own weight acts to restore its vertical orientation if ever some external force "nudges" it away from vertical.

An interesting thought experiment is to replace the normally-closed (NC) temperature switch with one that is normally-open (NO). Now, the system exhibits *positive feedback*: if the room cools down, the switch remains open and the heater never energizes; if the room heats up for any reason, the switch closes which energizes the heater and makes the room heat up even more. This is now the thermal equivalent of an inverted pendulum, "falling over" in the direction of the nudge.

Another interesting thought experiment is to return to the normally-closed (NC) temperature switch scenario where we have negative feedback (self-regulation) again, then place some ice near the temperature-sensing switch. The ice, of course, will decrease temperature in the vicinity of the switch, causing the switch to "think" the room is cold. This makes the switch contact close to energize the heater, which in turn adds heat to the room, the end-result being a warmer room. The reverse is true if we place a localized heat source near the temperature switch: the room will cool because the switch is "fooled" into "thinking" the room is warmer than it really is.

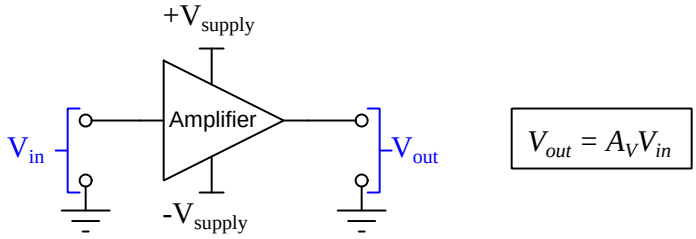
As you can see, feedback adds interesting behavioral characteristics to systems. Another lesson from these examples is the power of *thought experiments* as an analytical tool.

[1] If the relationship between a single *cause* and its *effect* may be thought of as an arrow pointing from cause to effect, the relationship between cause and effect in a feedback system may be thought of as a set of curved arrows arranged tip-to-tail, with the last tip pointed toward the first tail to form a loop. In other words, multiple elements in a feedback system play simultaneous roles as causes and effects to each other. Here, the switch causes the heater to operate, and also the heater causes the switch to operate. Causal loops are the essence of all feedback systems, and they are what makes feedback so interesting.

3.3 Amplifier feedback

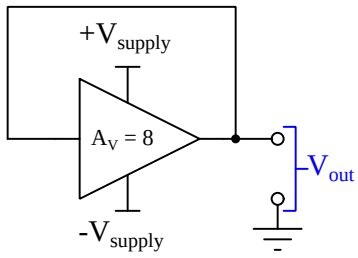
Feedback is widely used in *electronic amplifier* circuits to achieve desired behaviors. For the sake of this investigation, we will consider an "amplifier" to be an electronic circuit with one voltage input and one voltage output, energized by a suitable DC source, the output function being some direct multiple of the input voltage. For now, the internal design of the amplifier is irrelevant. All we need to know is that the output voltage is at the command of the input voltage, the input voltage signal "controlling" the output voltage's magnitude within the limits of the DC power supply.

The generic symbol for an electronic amplifier is a triangle, with input at the wide end and output at the pointed end. Each signal voltage is ground-referenced, and has a range no greater than the power supply "rail" voltages ($+V_{supply}$ and $-V_{supply}$):



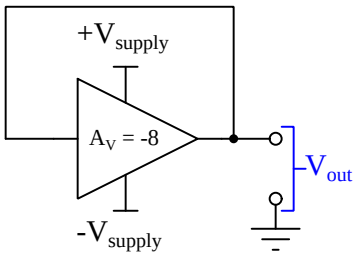
Amplification implies something being magnified, and in most cases this is true of V_{out} versus V_{in}. The ratio $\frac{V_{out}}{V_{in}}$ for a DC amplifier is called its *voltage gain* and is symbolized as A_V. Being a unitless ratio, it expresses how many Volts V_{out} will be per Volt of V_{in} (again, within the limits of $\pm V_{supply}$). For example, a DC voltage amplifier with a gain of 8 should output 5.6 Volts given an input voltage signal of 0.7 Volts.

Suppose we wired the output terminal of such an amplifier directly to its input terminal. How might such a circuit behave?



With a voltage gain of eight, $V_{out} = 8V_{in}$. If we imagine V_{in} being exactly zero Volts, then V_{out} will also be exactly zero Volts, and the system remains at zero Volts like an inverted pendulum perched in a perfectly vertical position. However, the slightest "nudge" one way or the other would force this amplifier to rapidly saturate to either $+V_{supply}$ or $-V_{supply}$, because *any* input voltage at all will become magnified by the amplifier, fed back, and amplified even more. This is pure positive feedback, yielding the same result as any other positive feedback system: it tends to "fall over" to a saturated condition.

How would *negative* feedback be implemented in such an amplifier? The solution here is to build the internals of the amplifier circuit to have a *negative voltage gain*, for example $A_V = -8$ instead of $A_V = 8$:



Once again, this amplifier is theoretically stable at zero Volts, because when V_{in} is exactly zero Volts, V_{out} (being magnified -8 times compared to the input) must also be exactly zero Volts. This is analogous to a classic pendulum hanging straight down. If we imagine some external source trying to "nudge" the signal in either a positive or a negative direction, the amplifier's natural reaction is to respond eight times as aggressively with a signal of the opposite polarity. The effect of this reaction would be to re-center[2] the signal at zero Volts, just as a classic pendulum with its negative feedback characteristic will "fight" any external force in its "attempt" to re-center itself in a vertical-down position.

To review, configuring a voltage amplifier ($A_V = 8$) for pure positive feedback resulted in an unstable circuit with a tendency to saturate its output signal to either $+V_{supply}$ or $-V_{supply}$ depending on which way it happened to be "nudged". Incorporating pure negative feedback resulted in a circuit refusing to output anything but zero Volts.

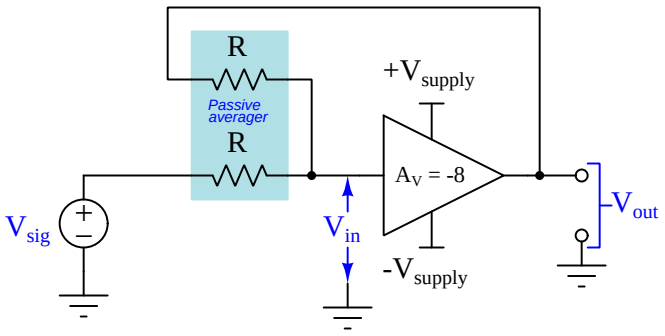
Neither of these outcomes is particularly useful for anything, but they both hint at useful properties: positive feedback tends to drive the voltage signal toward one of two extreme conditions, while negative feedback tends to stabilize the voltage signal against external influences. In *digital* electronic systems where all voltage signals are limited to "high" and "low" (on/off) states, having a circuit that conspicuously avoids "middle" states could be advantageous[3]. In *analog* electronic systems where voltage signals serve as proportional representations of other variables (e.g. temperature, light intensity, speed, sound pressure, strain), having a circuit actively work to stabilize a signal against corrupting influences could serve practical purposes[4].

[2] It is tempting to conclude that this negative feedback circuit might begin to *oscillate* back and forth between positive and negative polarities, given its gain of -8 *over*-correcting for any external nudge. Indeed, this can happen in negative feedback circuits, but only if there is a *time delay* in the feedback response. If no such time delay exists, then the amplifiers corrective action is *instantaneous*, and the "nudge" is immediately neutralized with no opportunity for oscillation.

[3] Indeed, such a circuit is called a *Schmitt trigger*.

[4] A huge array of practical analog circuits exist for this and similar purposes, as you will discover in subsequent learning modules!

Negative feedback is widely used in electronic systems, and it is this form of feedback which will occupy our attention for the remainder of this tutorial. To explore this concept in detail, we will add a resistive network to our negative-gain amplifier circuit called a *passive averager*, simply consisting of two equal-sized resistances. This passive averager network will average two ground-referenced voltages, one originating from an external signal source (V_{sig}) and the other being the "feedback" signal from the amplifier's output (V_{out}):



The output of the passive averager network is simply[5] $\frac{V_{sig}+V_{out}}{2}$ and this becomes the input signal for the amplifier (V_{in}). The amplifier still produces an output voltage that is eight times larger (and opposite in polarity) than its input ($V_{out} = -8V_{in}$). If we combine these two mathematical equations, we may derive a single equation expressing the output voltage as a function of the signal voltage and of the amplifier's internal gain of -8:

$$V_{in} = \frac{V_{sig} + V_{out}}{2} \qquad\qquad V_{out} = -8V_{in}$$

To combine these two equations, we must find a common variable shared by both which we may substitute. It is apparent that both V_{out} and V_{in} are shared by these two equations, and since our desire is to end up with one equation containing V_{sig} and V_{out}, the proper variable to substitute (i.e. eliminate) is V_{in}. First, manipulating the right-hand equation to express V_{in} as a function of V_{out}:

$$\frac{V_{out}}{-8} = V_{in}$$

Next, we will substitute this "definition" of V_{in} into the passive averager equation:

$$\frac{V_{out}}{-8} = \frac{V_{sig} + V_{out}}{2}$$

Now we have one equation containing V_{sig} and V_{out}. The only remaining step is to combine V_{out} terms into one, so we may be able to solve for V_{out} given any arbitrary value for V_{sig}.

[5] This assumes no "loading" effect of the amplifier's input on the averager network; in other words, an electronic amplifier with an infinite input resistance, like that of a perfect voltmeter.

$$\frac{V_{out}}{-8} = \frac{V_{sig} + V_{out}}{2}$$

$$\frac{V_{out}}{-4} = V_{sig} + V_{out}$$

$$\frac{V_{out}}{-4} - V_{out} = V_{sig}$$

$$\frac{V_{out}}{-4} + \frac{4V_{out}}{-4} = V_{sig}$$

$$\frac{5V_{out}}{-4} = V_{sig}$$

$$V_{out} = -\frac{4}{5}V_{sig}$$

Without negative feedback, $V_{out} = -8V_{in}$. With negative feedback applied to the passive averager network, $V_{out} = -\frac{4}{5}V_{sig}$, representing a dramatic reduction in over-all voltage gain. The output voltage is not zero as was the case with direct (wire-connected) negative feedback, but it certainly is diminished.

At this point we may be tempted to ask, *"What good is this?"* Clearly, we have taken an amplifier with a healthy voltage gain and caused that gain to be less than one by adding negative feedback – hardly a useful achievement. This criticism is valid, but only for the particular amplifier shown with its modest gain. If we imagine the internal voltage gain of the amplifier being much greater than the one shown, we will begin to notice more interesting behavior.

Consider the case where the amplifier in question has a voltage gain of $-800,000$ rather than -8. If we perform all the same algebraic steps as before, what we end up with for V_{out} as a function of V_{in} looks substantially different:

$$V_{out} = -\frac{400000}{400001}V_{sig}$$

Note the value of the fraction $-\frac{400000}{400001}$, which is approximately equal to -0.9999975. This means the over-all voltage gain of the circuit is very nearly -1. This is remarkable, starting with an amplifier having such a large voltage gain (nearly negative one *million*) and through the action of negative feedback achieving a voltage gain quite close to negative unity (-1).

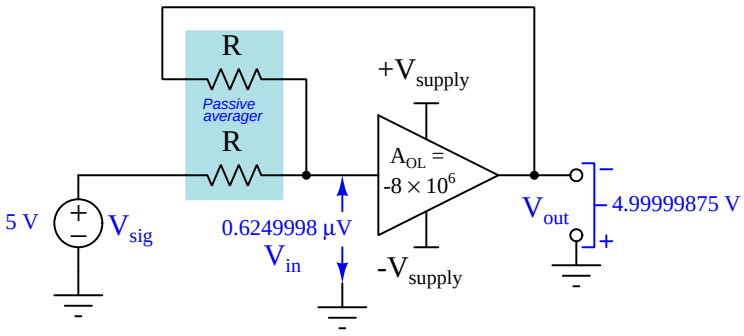
Taking this "thought experiment" further, let us see what happens to this equation if we raise the amplifier's internal voltage gain even more, all the way to negative eighty million ($-80,000,000$):

$$V_{out} = -\frac{40000000}{40000001}V_{sig}$$

Now the over-all voltage gain is -0.999999975, which is even closer yet to -1. It should be clear from this thought experiment that the amplifier's over-all gain approaches -1 as its internal voltage gain increases toward negative infinity.

Let us investigate how this amplifier circuit will function with a signal voltage of 5 Volts and an internal ("open-loop")[6] amplifier voltage gain of negative eight million. The following equations will be used to predict V_{out} from[7] V_{sig}, and V_{sig} from V_{out}:

$$V_{out} = \frac{A_{OL}}{2 - A_{OL}} V_{sig} \qquad\qquad V_{out} = A_{OL} V_{in}$$



Two details are noteworthy here. First, the output voltage is very nearly equal in absolute value to the signal voltage from the 5 Volt source. This is expected because we know the over-all voltage gain of this circuit with negative feedback in effect closely approaches negative 1. Secondly, the voltage seen at the input terminal of the high-gain amplifier with respect to ground is practically zero, being less than one microVolt. This is a consequence of the amplifier's intrinsic voltage gain being so large, that hardly any input voltage is necessary to achieve a modest output voltage.

If we imagine the amplifier's open-loop gain approaching negative infinity, we would expect the output voltage to approach $-V_{sig}$ and the input terminal voltage of the high-gain amplifier to approach zero. These are convenient approximations when the amplifier has an arbitrarily high gain value, and may be used with confidence to estimate the behavior of any negative-feedback circuit.

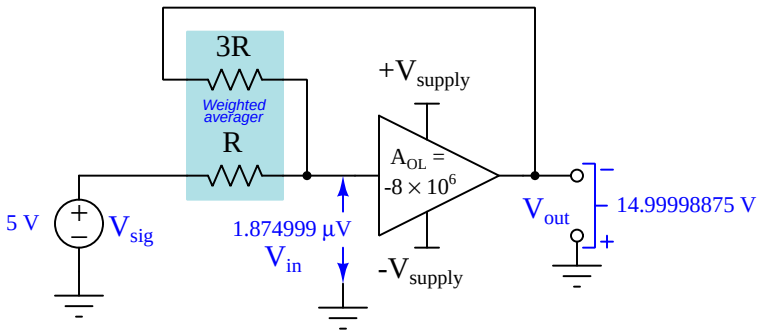
Again, it is tempting to ask the question, *"What good is this?"* If we want a voltage gain of 1, why not just build the amplifier with that amount of voltage gain rather than go to all the trouble of designing it with an extraordinarily high gain only to attenuate that gain all the way back down to one using negative feedback? This is a very reasonable question, and the answer is that electronic amplifying components (e.g. transistors) are not naturally *stable*, and because of this fact the gain of any simple amplifier circuit drifts uncontrollably. In other words, a stable open-loop gain of 1 is difficult if not impossible to achieve. *Negative feedback is remarkable in that it enables us to build nearly-perfect amplifying systems using imperfect amplifying components.* As we saw in previous negative-feedback calculations, the internal gain of our amplifier may vary widely (e.g. from 800,000 to 80,000,000 – a *hundred*-fold change in gain) while its over-all gain hardly deviates at all (e.g. from −0.9999975 to −0.999999975).

[6] A_V is, of course, the common way to symbolize an amplifier's voltage gain. However, in negative feedback circuits we are dealing with two different voltage gains: the gain of the over-all circuit (with feedback) and the intrinsic gain of the amplifier itself (without feedback). In order to more clearly distinguish between these two different voltage gains, it is common practice to use A_{OL} to refer to the "open-loop" voltage gain of the amplifier with no negative feedback in effect, while A_V refers to the over-all voltage gain of the amplifier circuit with negative feedback in effect.

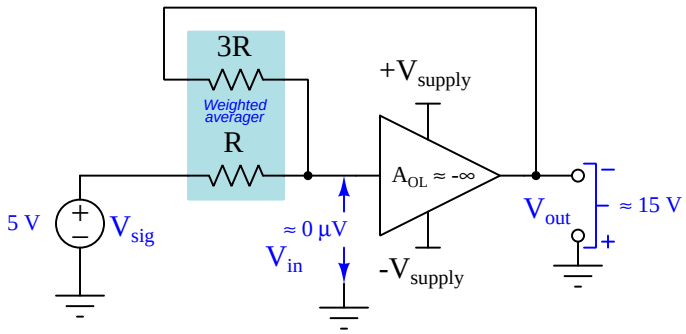
[7] $V_{out} = -\frac{A_{OL}}{2+A_{OL}} V_{sig}$ is a generalized form of the equation previously derived, where A_{OL} is the "open-loop" voltage gain of the amplifier without any negative feedback applied.

Now that we have seen how to achieve a stable gain of (negative) 1 using feedback, let's see what happens when we modify our resistive feedback network. If we replace the feedback resistor in the passive averager network with one of a different value, the result will be a *weighted* averaging function. For example, if we replace the upper resistor R with one three times larger ($3R$), the feedback signal's effect on the average (V_{in}) will be three times weaker than it was before, having just *one-third* as much "weight" in the averaging function. Instead of the resistor network's average being $\frac{V_{sig}+V_{out}}{2}$, it will be $\frac{V_{sig}+\frac{1}{3}V_{out}}{2}$. Modifying the over-all gain equation with this new (weighted) average and applying to the case of a 5 Volt signal:

$$V_{out} = \frac{3A_{OL}}{6 - A_{OL}} V_{sig} \qquad\qquad V_{out} = A_{OL} V_{in}$$



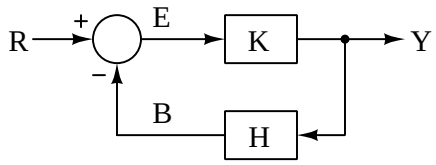
The output is very nearly -15 Volts, corresponding to an over-all voltage gain that is very nearly -3. Once again, the terminal voltage at the amplifier's input is practically zero. These results, predicted from the gain equations, could have just as well been estimated by assuming an infinite open-loop gain, a V_{in} of zero, and from those assumptions concluding that with one-third the weight at the averager network the circuit's output voltage must rise to *three times* the signal voltage in order for negative feedback to achieve a condition of equilibrium:



This is the "magic" of negative feedback: with a sufficiently sensitive amplifier, negative feedback *does whatever it must* to regulate V_{in} at zero, and in doing so achieves a precise voltage gain defined not by the internals of the amplifier itself, but by the external resistor network.

3.4 Block diagrams

Negative feedback is used for a wide range of engineering purposes, not just stable electronic amplification. A popular form of diagram to represent feedback systems in engineering is the *block diagram*, so called because each segment of the system is represented by a block with arrows going in (inputs) and arrows coming out (outputs) at each. An example of a block diagram showing our feedback-stabilized electronic amplifier appears here:



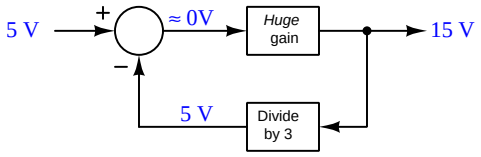
Block diagrams are mathematical abstractions of the real physical system, and so the layout of a block diagram may not exactly match the layout of the real system. Each block represents some mathematical function, and each variable represents a signal passed between functions. The letters used to symbolize each are fairly standard within engineering: R is the *Reference* or input signal to the system, K represents the *gain* of the amplifier or "compensator", Y represents the output signal from the system, H represents the feedback network (in the last case, H would represent the "weighted" value of the averaging network), B represents the *Back*-fed signal, and E represents the *Error* (or *difference*) between the reference signal R and the back-fed signal B.

The feedback polarity (either positive or negative) is shown in the block diagram by a mathematical sign where the B signal enters the *summation* point (the circle), and this stands as a good example of how a block diagram may differ from the physical system. In our feedback-stabilized amplifier system, we made the feedback negative by using an amplifier with a negative voltage gain. In the block diagram K is assumed to be a positive gain, with the inversion happening at the summation point where the back-fed signal B *subtracts* from the reference signal R.

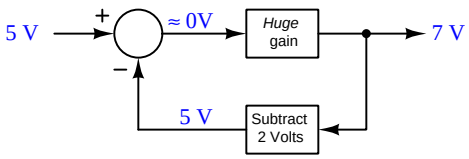
3.5 Functional inversion

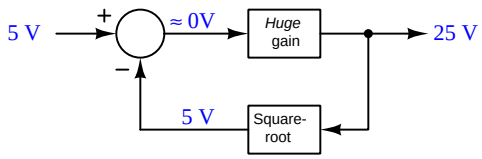
One of the very interesting properties of negative feedback systems is *functional inversion*, of which we have already seen one example. In any negative-feedback system where the compensator (i.e. amplifier) has a nearly-infinite gain, the error signal will be reduced to almost nothing ($E \approx 0$), making back-fed and reference signals practically equal to each other ($B \approx R$). Thus, H really defines the relationship between R and Y. If the back-fed signal is the output signal run through the function H, and the reference and back-fed signals are practically equal to each other, then the output signal must be equivalent to the reference signal run *backwards* through the function H. In other words, whatever mathematical function H is, the output signal (Y) becomes that inverse-function[8] of the reference signal (R).

Admittedly, this is an abstract concept to grasp, but it is made clear by example. In our last feedback-stabilized amplifier system using the weighted averager network, the feedback represented a signal division of *one-third*. The result of this divided feedback was a multiplied output: the output voltage was *three times* as great as the 5 Volt signal sent to the system as a reference:



The high-gain amplifier *did what it had to* to make the weakened back-fed signal practically equal to the reference signal, and in so doing created a strengthened output. If our feedback network had instead *subtracted* a certain amount of voltage, the result would have been an output with that same voltage offset *added*. Likewise, if the feedback were somehow made to be a *square-root* function, the output would become the *square* of the reference signal:





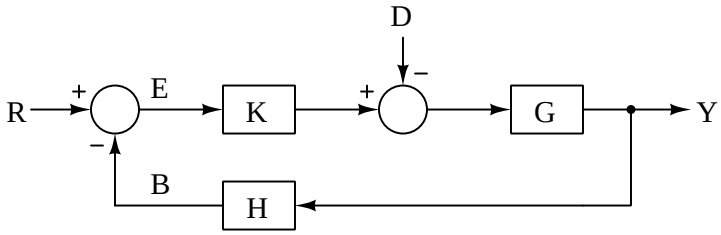
Functional inversion is extremely useful in analog signal processing, because it allows us to create twice as many mathematical functions as would otherwise be possible using simple electrical networks. For example, a collection of resistors can only *attenuate* a signal, but when used in conjunction with negative feedback we can now *boost* signals just as easily: a feat impossible with resistors alone.

[8] Using mathematical symbology, if $R = H(Y)$ then $Y = H^{-1}(R)$.

3.6 Servo systems

Another use for high-gain negative-feedback amplification is in *servo* systems. A "servo" system is one where some form of motor controls the motion of a physical object in response to a command (reference) signal, despite interference from one or more disturbances. These disturbances include such things as weight, external forces applied to the object, friction, etc. Servo systems are central to robotics, machine tool control, and vehicle guidance systems.

A basic servo system closely resembles a feedback-stabilized electronic amplifier when viewed as a block diagram. Added to our former block diagram are two new elements, a *disturbance* (D) subtracting from our amplifier's output, and the dynamics (e.g. friction, inertia) of the physical object being moved (G). The feedback network (H) is usually some form of sensor feeding accurate information to the amplifier regarding the actual state of the object:



Suppose this servo system controls the position of a robotic arm. R would be the reference, or desired, arm position; B would be the arm position as sensed by sensor H; and Y would be the true arm position. Disturbances (D) would include weight borne by the arm, as well as any other forces acting upon the arm from external sources. The arm's inertia, bearing friction, servo motor characteristics, etc. would all be a part of block G.

Just as in the stabilized amplifier system, an extraordinarily high open-loop gain in the amplifier (K) ensures the system will "fight hard" to ensure the back-fed signal and the reference signal are always nearly equal to each other. If some new disturbance (D) occurs and pushes the object away from its target (reference) position, sensor H will detect and report that change, creating an error (E) which prompts the amplifier (K) to respond. The over-all result is that the arm's true position (Y) will very nearly equal the reference position signal (R) at all times despite any external interference (D) or internal limitations of the servo mechanism (G).

Servo systems are just one type of a larger classification of negative-feedback *control systems* used extensively throughout the field of automation. Returning to our example of the temperature-controlled room, imagine replacing the on/off temperature switch with an analog temperature sensor (H in the block diagram), and using an electronic amplifier (K) to drive power to the room's electric heating element in a continual effort to maintain zero error. The reference signal (R) would then be the "set-point" for the room: the temperature we desire for it to maintain. Disturbances (D) would include opening a window or a door (letting in colder air), introducing warm-bodied people into the room, variations in ambient temperature outside the room, etc. The room's own thermal characteristics such as furniture mass and wall insulation would constitute G in the block diagram. Just like the robotic arm servo, the combination of negative feedback and high amplifier gain would "fight hard" to maintain a steady room temperature, automatically adjusting power to the heating element in an effort to maintain zero error (E).

3.7 Summary

Feedback is a highly complicated topic, and is far from intuitive. Summarizing some of the major points discussed so far:

- Adding feedback to any system creates new complexities
- Positive feedback results in a system tending toward saturation, useful in digital systems where we desire the signal to fall reliably into one of two valid states
- Negative feedback results in a system tending toward stability, useful in analog systems where we desire the signal to be regulated at some target (reference) value
- In any negative feedback with a high-gain signal amplifier, the relationship between the output and input (reference) is defined by the feedback network and nothing else; specifically, the relationship from reference to output is the mathematical inverse of the relationship between output and back-fed signals through the feedback network
- Servo systems use negative feedback and high-gain signal amplifiers to control the motion of objects according to the command of an input (reference) signal
- Negative feedback with high gain results in an error that is nearly zero at all times

Chapter 4

Historical References

This chapter is where you will find references to historical texts and technologies related to the module's topic.

Readers may wonder why historical references might be included in any modern lesson on a subject. Why dwell on old ideas and obsolete technologies? One answer to this question is that the initial discoveries and early applications of scientific principles typically present those principles in forms that are unusually easy to grasp. Anyone who first discovers a new principle must necessarily do so from a perspective of ignorance (i.e. if you truly *discover* something yourself, it means you must have come to that discovery with no prior knowledge of it and no hints from others knowledgeable in it), and in so doing the discoverer lacks any hindsight or advantage that might have otherwise come from a more advanced perspective. Thus, discoverers are forced to think and express themselves in less-advanced terms, and this often makes their explanations more readily accessible to others who, like the discoverer, comes to this idea with no prior knowledge. Furthermore, early discoverers often faced the daunting challenge of explaining their new and complex ideas to a naturally skeptical scientific community, and this pressure incentivized clear and compelling communication. As James Clerk Maxwell eloquently stated in the Preface to his book *A Treatise on Electricity and Magnetism* written in 1873,

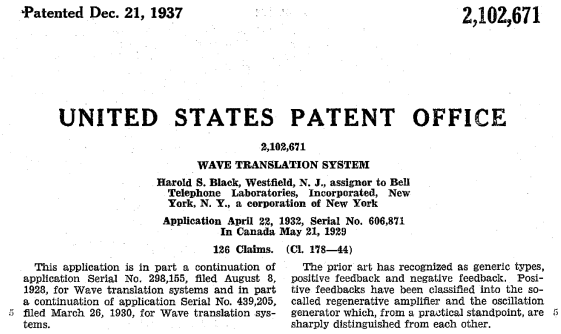
> It is of great advantage to the student of any subject to read the original memoirs on that subject, for science is always most completely assimilated when it is in its nascent state . . . `[page xi]`

Furthermore, grasping the historical context of technological discoveries is important for understanding how science intersects with culture and civilization, which is ever important because new discoveries and new applications of existing discoveries will always continue to impact our lives. One will often find themselves impressed by the ingenuity of previous generations, and by the high degree of refinement to which now-obsolete technologies were once raised. There is much to learn and much inspiration to be drawn from the technological past, and to the inquisitive mind these historical references are treasures waiting to be (re)-discovered.

4.1 Harold Black's feedback patent

In April of 1932 an electrical engineer named Harold Black applied for a United States patent on a design technique useful for making exceptionally stable electronic amplifier circuits. His job working for Bell Labs permitted wide-ranging explorations in circuit theory while providing a very practical application – telephone circuits – to apply any discoveries and innovations toward. One of the problems at that time centered around vacuum tube-based "repeating" amplifiers used to boost telephone signals at certain points along long-distance cables. The gain values of these amplifiers were known to drift beyond acceptable limits on a regular basis, partly due to the electronic characteristics of the vacuum tubes changing as the tubes aged, and this (among other factors) posed problems for reliable communication in Bell's telephone system. Black's focus was to improve the gain-stability of these amplifiers, but to do so in a way that did not require improvements to the vacuum tubes themselves. He sought to find a more comprehensive design-based solution to the problem that would work despite irregularities in the electronic components.

Black's research culminated in a patent which will be the focus of this section. His notion of a "wave translation system" applied to electronic amplifiers to be sure, but had applications to any system involving waves, even non-electrical[1]:



Patented Dec. 21, 1937 2,102,671

UNITED STATES PATENT OFFICE

2,102,671

WAVE TRANSLATION SYSTEM

Harold S. Black, Westfield, N. J., assignor to Bell Telephone Laboratories, Incorporated, New York, N. Y., a corporation of New York

Application April 22, 1932, Serial No. 606,871
In Canada May 21, 1929

126 Claims. (Cl. 178—44)

This application is in part a continuation of application Serial No. 298,155, filed August 8, 1928, for Wave translation systems and in part a continuation of application Serial No. 439,205, filed March 26, 1930, for Wave translation systems.

The prior art has recognized as generic types, positive feedback and negative feedback. Positive feedbacks have been classified into the so-called regenerative amplifier and the oscillation generator which, from a practical standpoint, are sharply distinguished from each other.

A noteworthy feature of this patent is how it patiently explains the then-common understanding of feedback as applied to electronic circuits, and goes on to explain in great detail how Black's application of negative feedback was a true innovation. This clear exposition deserves the attention of all students of electronics.

[1] Page 1 of the patent contains this notable paragraph: *"The invention is applicable to any kind of wave transmission such as electrical, mechanical or acoustical, and thus far in the description the terms used have been generic to all such systems. The invention will be disclosed herein, however, as specifically applied to electrical systems, it being understood that the principles involved are equally applicable to other types of Wave transmission and that the generic claims are intended to include electrical and other than electrical wave systems and apparatus."*

Black begins his description of how feedback might be applied to an electronic amplifier circuit. Note his use of the word *grid*, referring to the "grid" structure of a vacuum tube, the metallic screen placed between cathode and anode ("plate") to control the flow of electrons streaming between those two metal surfaces. The modern equivalent of "grid" would be *base* for a bipolar transistor amplifier and *gate* for a field-effect transistor amplifier:

> Reverting now to the elemental feedback system above, described, and taking a typical electrical case, for illustration, the amplifying element may be a grid-controlled discharge tube of ordinary type. The input circuit leads up to the input side from a suitable source of input waves to be amplified, for example, signal waves. The feedback is connected from the output circuit to the input circuit. It will be noted that the waves actually applied to the grid come from two circuit branches, identified as the incoming circuit (from the signal or other wave source) and the feedback circuit. Of course, each of these two waves, the incoming or signal wave and the fed-back wave, could be thought of as being separately applied to the grid, but it will be simpler to think of a summation wave as the voltage actually effective on the grid. Thus, there are three waves to be considered in relation to the input side of the amplifier, (1) the incoming or original wave to be amplified, (2) the fed-back wave, and (3) the voltage wave effective on the grid, this latter being the resultant produced by the algebraic addition of the other two. `[page 1]`

Having described the electrical connections involved with adding feedback to a tube-type amplifier, Black proceeds to discuss some of the historical uses (i.e. *prior art*) of feedback in electronic amplifiers as a prelude to describing his own innovation:

> The prior art has recognized as generic types, positive feedback and negative feedback. Positive feedbacks have been classified into the so-called regenerative amplifier and the oscillation generator which, from a practical standpoint, are sharply distinguished from each other. `[page 1]`

As Black notes, the principle of "feeding back" part of an amplifier's output signal back to its own input was not unknown at his time, and in fact had been practically used for multiple purposes. The two purposes he notes are *regenerative amplifiers* and *oscillators*, the fundamental difference between these two circuits being the amount of positive feedback applied.

His next paragraphs describe existing uses of positive feedback very well. Large amounts of positive feedback is used to generate self-sustaining oscillations (making the amplifier circuit an *oscillator*) while small amounts of positive feedback work to boost the gain of an amplifier:

> If we picture the fed-back wave and the incoming wave as separately adjustable, we may imagine an increasing amount of fed-back waves in a direction to augment the effect of the incoming wave with a corresponding decrease of the incoming wave. The adjustments may continue until the input wave has been entirely replaced by the fed-back wave. That is, the incoming wave has been reduced to zero and the incoming circuit can be disconnected from the amplifier. The waves in the circuit, then become self-sustained and we have the familiar case of the wave generator developing and maintaining continuous oscillations, in a closed loop consisting of the input circuit, the amplifier, the output circuit and the feedback circuit back to the input circuit.
>
> If a smaller amount of positive feedback is employed than is required to produce sustained oscillations, we have the case of the regenerative amplifier. Some of the incoming signal wave after amplification is again put back on the input circuit in such manner as to augment the original incoming signal so that reamplification occurs in the same tube, causing the signal to build up to relatively higher value than could be done by traversing the amplifying element but once. The useful limit of amplification by such a circuit is reached when a further increase in the degree of feedback would result in producing self-oscillation or when the circuit is sufficiently close to the oscillating condition to cause an intolerable amount of signal distortion.
>
> A broad practical distinction between the oscillator and the regenerative amplifier is that in the latter the output current remains under the control of the incoming signal and must follow its variations of starting, stopping, growing stronger or weaker and exhibiting the characteristic quality of the Signal, whereas in the oscillator the wave circulating around the regenerative loop is self-determined and is beyond the control of any signal or other input wave. `[page 1]`

So far, Black has only given attention to *positive* feedback. With this, he then turns to an exposition of *negative* feedback. Again, this was nothing new in his time, and so he had to make a logical case that his invention was a true innovation and not just an existing design principle in disguise:

> Turning, now, from positive feedback to negative feedback we find that the principal field of use of negative feedback has been in radio frequency amplifiers.
>
> A radio frequency amplified is a familiar instance of a circuit in which there may exist an `[page 1]`
>
> inherent tendency toward self-oscillation because of a positive feedback produced by inductive or capacitive coupling between elements of the output and input circuits even where great care in design is exercised to reduce such coupling. These effects become more pronounced as the frequencies become higher and higher. The tendency toward self-oscillation in such circuits places a limit on the amount of amplification that can be used.
>
> Negative feedback has commonly been applied in the prior art in radio frequency amplifiers to oppose the inherent positive feedback. The negative feedback in all such

> cases has had as its purpose the reduction of the positive feedback. If we imagine that the negative feedback is increased from an infinitesimal amount in any given case, it reaches its optimum value in opposing positive feedback when it just equals the positive feedback. At that point, the net or resultant total feedback is zero and the effect is that of rendering the amplifier a strictly unilateral circuit or one having no feedback, either positive or negative. [page 2]

Black's description of negative feedback as a tool to cancel a radio-frequency amplifier's intrinsic positive feedback is commonly known as *neutralization*. The problem, as Black describes in 1932, is that unintentional "coupling" between different elements within a radio-frequency circuit biases it toward instability, causing RF amplifiers to self-oscillate. Small amounts of negative feedback added to an RF amplifier – just enough to exactly cancel out or "neutralize" the undesired positive feedback – thus serves a familiar and practical purpose. Black's claim, though, was that his use of negative feedback (i.e. the *invention*) was of a different scale and for a different purpose, which he begins to describe next:

> In contrast to the negative feedback of the prior art in which, as discussed above, the resultant total feedback is substantially zero, the invention uses negative feedback for an entirely different purpose and in very much larger amounts to achieve improved results in wave translation, e.g., amplification, not envisaged in the prior art.
>
> Applicant has discovered how to use larger amounts of negative feedback than were contemplated by prior art workers with a new and important kind of improvement in tube operation. One improvement is in lowered distortion arising in the amplifier. Another improvement is greater constancy of operation, in particular a more nearly constant gain despite variable factors such as ordinarily would influence the gain. Various other operating characteristics of the circuit are likewise rendered more nearly constant. Applicant has discovered that these improvements are attained in proportion to the sacrifice that is made in amplifier gain, and that by constructing a circuit with excess gain and reducing the gain by negative feedback, any desired degree of linearity between output and input and any desired degree of constancy or stability of operating characteristics can be realized, the limiting factor being in the amount of gain that can be attained rather than any limitation in the method of improvement provided by the invention. [page 2]

In this last paragraph, Black summarizes his discoveries about the use of large amounts of negative feedback in amplifier circuits: this causes the amplifier to exhibit lower amounts of signal distortion, as well as exhibit a more stable gain. Both of these improvements were of great interest to Black's employer – Bell Labs – for use in their telephone system. Perhaps the most counter-intuitive notion Black expresses in this paragraph is that it actually makes sense to build an amplifier with an abnormally high amount of gain ("excess gain") and then use negative feedback to globally reduce this gain in order to achieve the aforementioned benefits of lower distortion and greater stability. If one were to view negative feedback merely as a way to force an amplifier to have a reduced gain, then building a high-gain amplifier only to attenuate that gain with negative feedback seems utterly pointless. No, argues Black, the real point of reducing all that "excess" gain is to enjoy lower distortion and greater stability.

Having summarized the unique benefits of large amounts of negative feedback, the author goes on to explain practical problems alleviated by negative feedback. First, the problem of amplifier distortion:

> Also, from what has been said, it is apparent that applicant uses negative feedback for a purpose quite different from that of the prior art which was to prevent self-oscillation or "singing". To make this clearer, applicant's invention is not concerned, except in a very secondary way (to be explained later on), with the singing tendency of a circuit. Its primary purpose has no relation to the phenomenon of self-oscillation. If amplifiers could be built exactly like present day amplifiers in all respects except that they were absolutely free of any tendency of self oscillation regardless of how high their gains might be pushed, it is generally true that there would still be as great need for the present invention. The "perfect" amplifier is one in which the output wave is in all respects an exact replica of the input wave multiplied by some linear factor. The singing tendency is only one of several difficulties encountered in an amplifier. Actually, amplifiers produce distortion components along with the output fundamental components. These modulation or distortion components are mingled with the fundamental wave and detract from its purity. In multiplex carrier systems the distorting effect gives rise to cross-talk and, in any high quality system, it constitutes a limiting factor of design. `[page 2]`

Next, the problem of unstable amplifier gain:

> Another difficulty in amplifier operation is instability, not used here as meaning the singing tendency but rather signifying constancy of operation as an amplifier with changes in battery voltages, temperature, apparatus changes including changes in tubes, aging and kindred causes. Such instability is a limiting factor where, for example, a large number of repeaters are operated in tandem in a line. A simultaneous tendency toward increased gain by several or all of the repeaters might be disastrous. Without a means of preventing such changes, they become a limiting factor of design. `[page 2]`

Here it is easy to understand Bell Labs' interest in Black's discovery. By using negative feedback to reduce signal distortion and to tame unstable amplification, practical problems plaguing their repeater amplifiers could be solved. This would reduce channel-to-channel interference on multiplexed[2] telephone systems. By using negative feedback to stabilize each amplifier's gain, cascaded repeater stations could provide reliable end-to-end operation despite the inevitable aging of vacuum tubes, tube replacement, environmental changes, and supply voltage variations.

[2] *Multiplexing* refers to transmitting multiple streams of data along a common communication channel. There are many different ways to do this, but in Black's time it involved *modulating* high-frequency waveforms with information from audio-frequency voice signals, much in the same way as radio transmitting stations broadcast "modulated" RF waveforms from their antennas.

As is customary with patent applications, the author provides a series of illustrations and diagrams to augment the text's explanation of the invention. Black's patent is no exception, and includes several insightful diagrams and graphs.

Figure 1, shown below, gives a generalized schematic diagram of feedback applied to a multi-tube amplifier. To anyone unfamiliar with vacuum-tube circuit schematics, the symbols used here may look very strange. The grid appears as a kind of single-leg "resistor" oriented vertically, while the plate is a small rectangle. The cathode is a triangle located between the two. Grid, plate, and cathode are analogous to base, collector, and emitter in a BJT, or gate, drain, and source in a FET. The feedback is simply a resistor labeled f connecting the output terminal (plate) of the last tube with the input terminal (grid) of the first tube, with the size of that resistor determining the amount of negative feedback imposed on the amplifier:

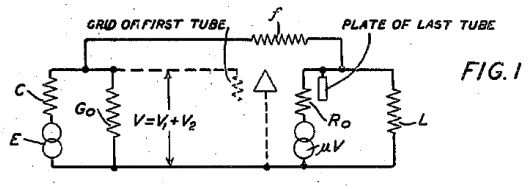
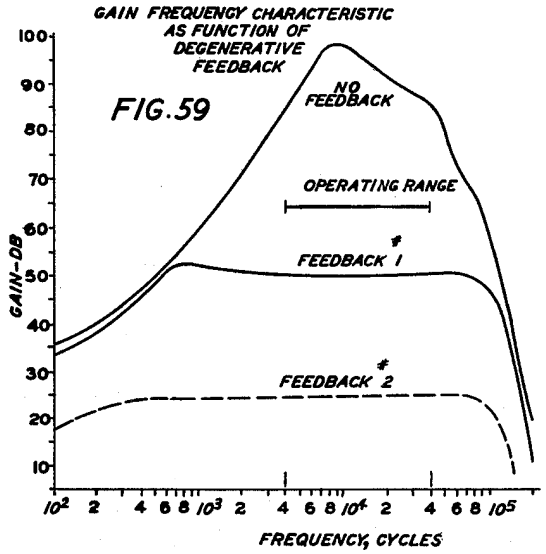
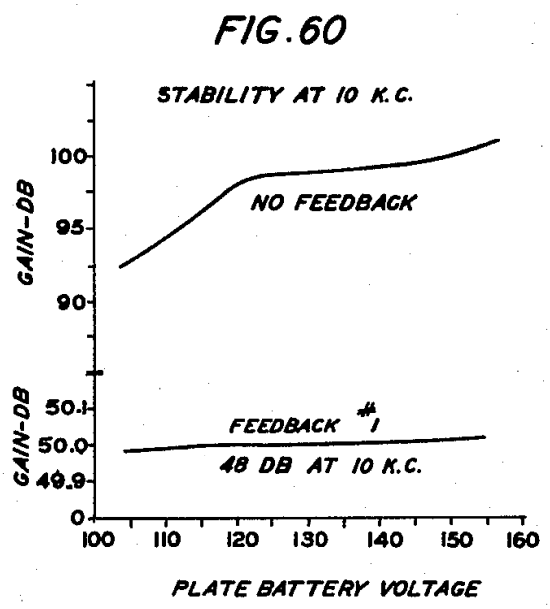


Figure 59, shown below, plots the gain of an amplifier as a function of signal frequency with varying degrees of negative (i.e. *degenerative*) feedback. As the graph shows, the amplifier's gain varies widely from 35 dB at the lowest frequency to nearly 100 dB at its peak gain without the benefit of any negative feedback. With some negative feedback added to the circuit, the gain appears much more stable, especially over the typical operating range from 4 kHz to 40 kHz. With even more feedback, the gain curve is almost perfectly "flat" at approximately 25 dB from 400 Hz all the way to 60 kHz. Bear in mind that the only modification made to the amplifier to achieve such stability was including a resistive feedback network from its output to its input:



The next graph, Figure 60, plots amplifier gain as a function of plate (DC power supply) voltage, given a constant 10 kHz signal to amplify. As in the previous graph, we see dramatic improvements in the stability of the gain due to the addition of negative feedback:



This last illustration, Figure 18, shows three different "views" of a three-stage amplifier with negative feedback. On top is a simplified schematic diagram of the amplifier circuit, using three "triode" vacuum tubes connected in common-cathode configuration (equivalent to common-emitter for BJT or common-source for FET). The odd number of common-cathode stages makes this amplifier *inverting* at the output with respect to the input, since each common-cathode stage inverts the polarity of its signal. The middle diagram represents the feedback "loop" which is even more simplified from the schematic. The bottom diagram is simplified even more, omitting the tubes entirely:

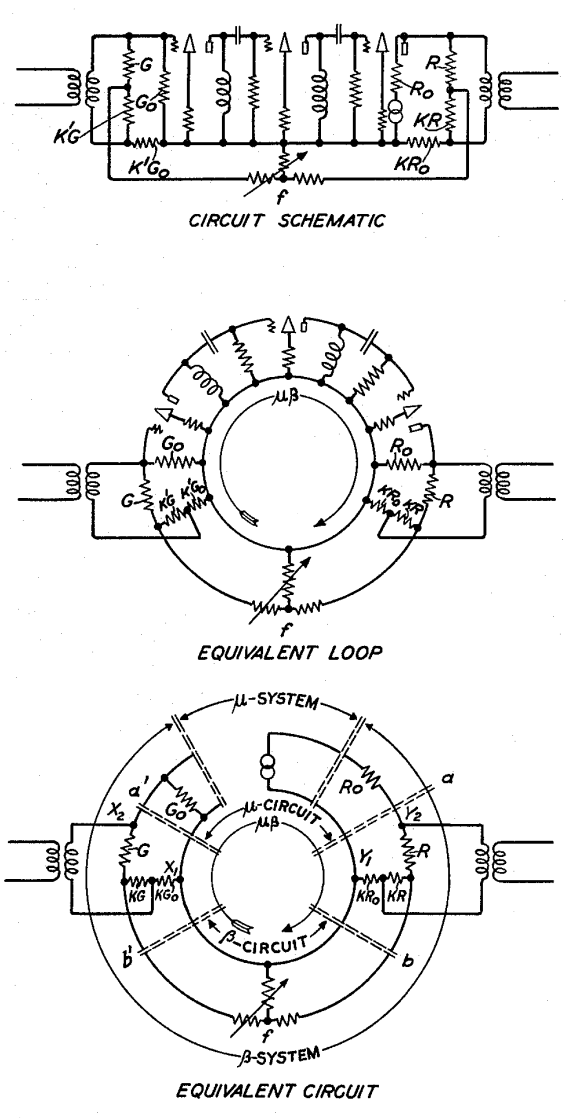


FIG. 18
ILLUSTRATIVE APPLICATION OF METHODS OF FIGS. 14, 15, 16, AND 17

Black's experimental feedback network consists of a simple "T-pad" attenuator, adjustable to

provide a convenient method for testing the amplifier with varying amounts of negative feedback. The feedback is inherently *negative* and not positive due to the inverting nature of the amplifier: with the output signal 180^o out of phase with the input signal, any feedback passed through a non-shifting (i.e. resistive) network must be negative feedback.

Throughout the patent's text, Black uses the lower-case Greek letter "mu" (μ) to symbolize the amplifier's gain without negative feedback (what we would call the *open-loop gain* of the amplifier). He also uses the Greek letter "beta" (β) to symbolize the feedback network's attenuation factor. If the amplifier's open-loop gain is arbitrarily large, such that the product $\mu\beta$ is much greater than unity, the system's gain with negative feedback in effect comes close to being equal to $-\frac{1}{\beta}$.

This conclusion is rather striking, as it tells us the system's gain may be set to a very stable value simply by adjusting a simple resistor network to the reciprocal of that desired gain. For example, if the feedback network has an attenuation of 5:1, then the amplifier's over-all (i.e. *closed-loop*) gain will approach 5 regardless of the internal (open-loop) gain of the amplifier without feedback. Modern operational amplifier circuits come very near this ideal realization, owing to the spectacular open-loop gain capabilities of modern semiconductor amplifiers.

4.2 Early telephone system repeaters

Harold Black's innovation of using negative feedback to stabilize the operating characteristics of vacuum-tube audio amplifiers found immediate application in the electronic telephone communication systems constructed and maintained by his employer, Bell Telephone Laboratories. Some interesting photographs and data on these early telephone systems were recorded in the *Bell System Technical Journal*, a regular publication of Bell Telephone Laboratories chronicling state-of-the-art research and development.

For example, contained within the first volume of the *Bell System Technical Journal*, circa 1922, is an article entitled "Philadelphia-Pittsburgh Section of the New York-Chicago Cable" by James J. Pilliod describing design and construction of a 300 mile long telephone cable connecting the cities of Pittsburgh and Philadelphia in 1921. This article described the electronic repeating amplifiers (called "repeaters") installed at approximately 100 mile intervals along this telephone network, intended to boost the degraded audio signals in order to maintain their integrity across long distances of small-gauge copper wire. A typical repeater building measured about 50 by 80 feet in size (shown in the photograph below, from page 85 of the *Journal*), and was filled with racks of electronic amplifying equipment.

Fig. 29—Test and Repeater Station at Ligonier, Pa.

These legacy vacuum-tube amplifiers required significant amounts of electrical power to operate, most of that energy needed to heat the wire filaments of the vacuum tubes to a dull glow. Each repeater building was equipped with its own electrical power plant, and had secondary-cell storage batteries for uninterruptible power service even when the power plant shut down. For a typical repeater installation the total capacity of the 24 Volt batteries was 2240 Ampere-hours, which was sufficient for one day's continuous operation without the power plant running. Assuming a 24-hour

day, this equates to 93.33 Amperes of direct current at 24 Volts to supply most of the power demand of these legacy amplifiers.

A typical rack of repeater units is shown in this next photograph, from page 80:

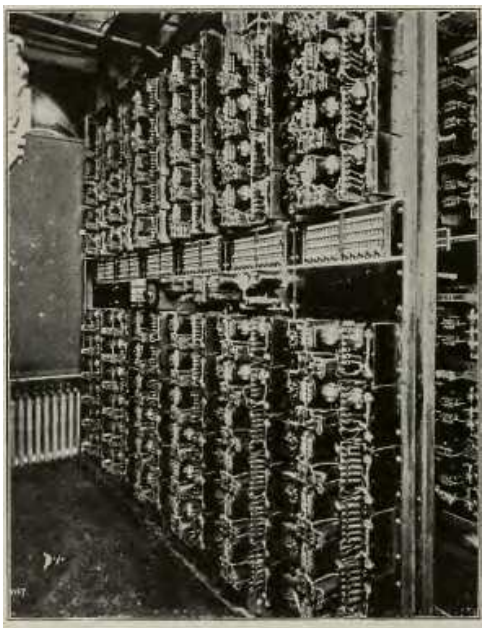


Fig. 24—Group of Repeaters at Reading, Pa.

A close-up photograph of a single repeating amplifier appears next, from page 81:

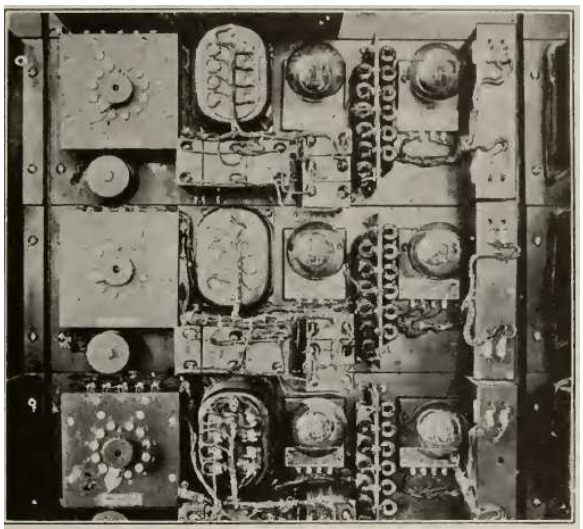


Fig. 25—Assembly of Four-Wire Repeater Apparatus

A simplified schematic diagram[3] of one vacuum tube amplifier (left) appears from page 78, along with a photograph of a typical vacuum tube of that era (right) from page 77:

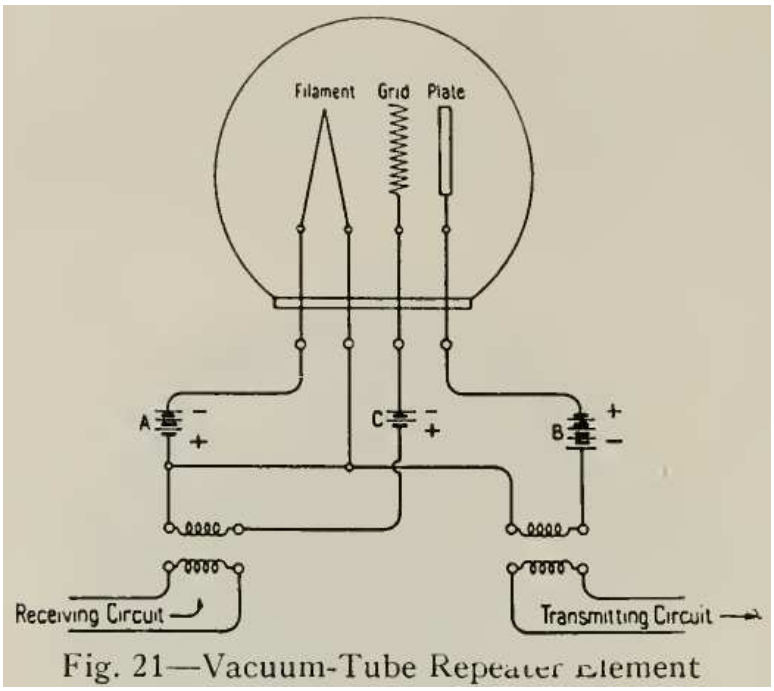


Fig. 21—Vacuum-Tube Repeater Element

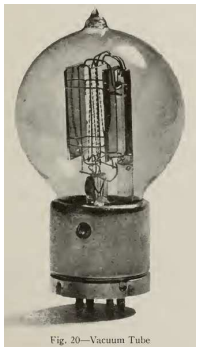


Fig. 20—Vacuum Tube

The audio-frequency signals are coupled into and out of the amplifier circuit through signal transformers. Three DC power sources are necessary for the amplifier's function: source **A** energizes the metal-wire filament to a state of incandescence, boosting the energy of individual electrons within the metal to the point where they may leave the metal surface and enter the vacuum space of the tube. A higher-voltage source **B** attracts these emitted electrons to a metal plate, the current passing through the "transmitting circuit" signal transformer. The flow of electrons through the vacuum from filament to plate is controlled by voltage impressed between a wire-screen grid and the filament, that voltage being a composite of the AC audio signal (from the "receiving circuit" signal transformer) and grid-bias battery voltage **C**. The electric field between the grid and filament modulated the flow of electrons through the tube's vacuum space much like a valve restricts a flow of fluid. In fact, for this reason vacuum tubes were first referred to as electronic *valves*. The relatively weak AC audio signal from the receiving transformer caused the current through the vacuum tube to vary at a much greater amplitude, which then generated a relatively strong AC audio signal at the transmitting transformer. Thus, this simple device *amplified* the telephone signal electronically.

[3]This diagram contains some typographical errors regarding the battery symbols. All the + and − polarity marks are correctly oriented, but the battery symbols are all drawn backwards. Interestingly, the more detailed schematic diagram from page 78 (Figure 22) shows all of them correctly.

A more complete schematic diagram of a two-way repeater is shown also on page 78, complete with provisions for signal filtering and adjustable gain:

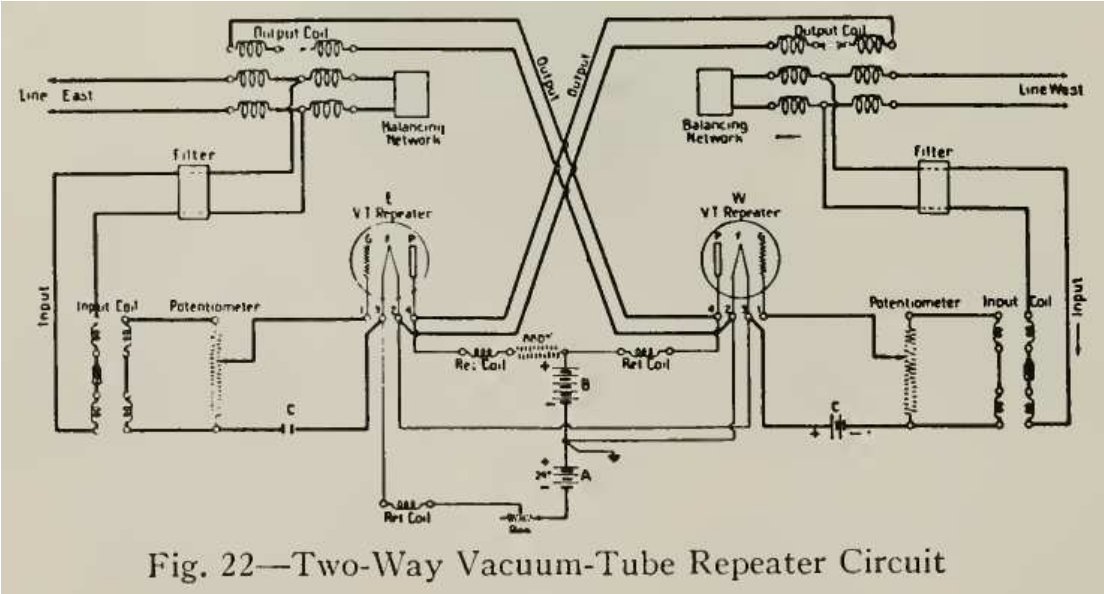


Fig. 22—Two-Way Vacuum-Tube Repeater Circuit

Dual potentiometers attenuated the AC signal from each input transformer ("input coil") before being impressed between each tube's grid and filament. Another adjustable resistance provided manual adjustment of filament current. These adjustments allowed technicians to make periodic re-alignments of the repeater circuits so they would exhibit the proper amount of gain to overcome the signal losses of the long cables. Vacuum tubes, especially of that era, wore out over time, and this wear constituted a gradual decrease in gain (amplification factor) over time as the tubes aged. Boosting filament current and input signal strength by re-adjustment could help overcome that wear, but at some point the tubes would require replacement.

Harold Black's negative feedback innovation did nothing to improve the operating life or characteristics of vacuum tubes, but it allowed these imperfect amplifying components to yield a nearly perfect gain despite aging. This stability translated into fewer necessary re-adjustments by technicians, and improved service quality in general.

4.3 George Philbrick's work on gun tracking systems

The *National Defense Research Committee* published a series of reports following the end of World War Two on their developmental work into automatic tracking systems for guns. One of these reports, "Summary Technical Report of Division 7, NDRC, Volume 3, Airborne Fire Control" contains writing of a certain George Philbrick who was to become a very important figure in the development of operational amplifiers and analog feedback systems in general. Philbrick was the founder and namesake of George Philbrick A. Researches, Inc., and his contributions to early fire-control system development were highly influential.

What follows are excerpts from this now-declassified military report.

Chapter 2
ON CERTAIN ASPECTS OF TRACKING

2.1 TRACKING AND OUR CONCERN THEREWITH

By "tracking" is here meant a continuous following or alignment procedure which is pursued through cognizance of its error or unbalance, as in the basic feedback or following operation described in Section 1.10. A good example, although somewhat abstractly disclosed, is given in Section 1.8, where an arbitrarily varying position vector is to be followed by an artificially generated position vector which is influenced by manipulation of an integration constant and a velocity.

Tracking pervades all branches of fire control, but it arises principally in aiming operations as such, whether these be manual or automatic. With regard to the latter, automatic tracking is essentially indistinguishable from other types of automatic control and will here be considered in that light. However, our major interest in the present chapter is in tracking by manual means, a human operator being involved. While the removal of human limitations gives the greater promise to automatic tracking devices and techniques, still there is a good bit to be learned, even for such unlimited future trends, from tracking of the human variety. Besides this, the human element can hardly be altogether eliminated, even ultimately, and until we give to our machines the final powers of judgment and choice we shall find it necessary personally to direct their efforts. It may also be doubted that the adaptability or educability of the brain, whereby it adjusts to altered circumstances, can soon be imparted to the automaton in any but the most trivial of cases. Or rather, if this is to be done, it will probably be through imitation of the processes by which such adaptations take place in the animal mechanism.

It is typical of the writer that he considers the entire subject of tracking to come within the broader boundaries of regulatory controls, which to him include all corrective apparatus having a closed causal loop. In this connection, and although the region has not been very far explored, there will be traced out briefly in what follows an analogy between the past development of automatic regulators and that of tracking aids. From this analogy a deduction will be made as to the future of the latter field of endeavor. Aside from this conjectural activity, we have been intimately concerned with tracking in a number of development programs where fire-control apparatus was being devised. Tracking in range, and we speak now of manual tracking, whether by stadiametric means or otherwise, has not entered particularly into our immediate sphere, and it is further

evident that this brand of tracking has been dealt with very thoroughly by other groups. Thus it may be said that tracking in angle, or more precisely in direction, is under discussion. The problems and technology of such tracking have appeared in gunnery systems, and in those for bombing, guided bombing, and for airborne rockets.

For flexible gunnery, an elaborate instrumental, statistical, and psychological study of the man-machine interactions which tracking comprises was undertaken and pursued by Section 7.2 at The Franklin Institute. This project (NO-268) dealt with standard types of computing dynamics and with known kinds of aiding controls. Many significant results were obtained; some unexpected. Other members of the section have been more directly involved, however, and their writings should be referred to rather than the present material. (See Part III, for example.) While allusions may be made here to this work, it will not be further discussed or described.

2.2 THE HUMAN FOLLOWER

People "track" during every conscious moment, unless their eyes be closed, their hands tightly tied, and their tongues clamped solidly between their teeth. Alignment processes, in which the alignment error serves as datum for its own annihilation, are forever being carried out in the familiar operations of living. We may thus expect that there is a rudimentary sort of tracking circumstance in which the human operator will be at home, and in which he will be found to be greatly and innately skilled. Pointing at a moving object with a pencil, or with a rifle under beneficial conditions of support and inertia, is a tracking operation which may be carried out relatively well. Provision of a reflecting sight, or of a nonmagnifying telescope with cross hairs, does not improve the operation extensively and has even been seen to impair it. Of course it is a different matter with magnification, since then visibility and visual resolution may be vastly increased. This is not to say that this "natural" type of tracking cannot be improved upon, for in tracking targets which are changing very slowly and uniformly an adjustable-rate device is superior, this being only one example, but on the whole for typical cases, and where scale factors in the presentation are reasonably adjusted, such natural tracking would suffice admirably. It is assumed here that closeness of tracking as such is desired, or simply smallness of tracking error. It is evident that for the larger purposes of aiming contol this is not the only index of excellence. The needs and nature of the interpretive and computing equipment cannot finally be separated from those of the tracking controls. We are thus discussing a subsidiary problem, but more on this again.

We repeat, those tracking arrangements would suffice in which the dynamic relationship between the immediate manual manipulation and the direction index were of the same character as in the arrangements referred to as natural. If this is an overstatement of the case, then we should like to assume that there might be discovered some even more ideal dynamic connection between the manual manipulation and, say, the visual index, and that the dynamic nature hereof might be expressible in quantitative form, even though the ideal differed from one individual to the next.

2.3 THE CAUSAL LOOP

It is frequently pointed out that the elements in a tracking system form a chain, and that consequently the whole assemblage perform as perfectly as each element must is intended

> to perform. But it has not been sufficiently reiterated that these elements form a *closed* chain, or a *complete loop*, together with the human operator, where one is present. This fact brings to the tracking process all the special characteristics, including the unique conditions for stability and periodicity, which are peculiar to that type of system. The formation of such a loop, in the causal sense, is particularly significant when there is included the dynamic characteristic of a "disturbed" or lead-computing sight, so-called. For in spite of the fact that the little understood dynamics of the human operator are included, many of the properties of those loops which are entirely automatic regulatory devices and in many automatic controls, are seen to be present in the tracking sequence. It seems evident that an approach to the tracking problem, thinking of this problem as requiring the provision of improved aiding equipment, would best be made a basis of the recognition that the operations involved are of this cyclic character. Operations in a closed loop are fundamentally different from those in an "open" or "straight-through" system. Both advantages and disadvantages follow from such an arrangement, and this will define our principal topic. We wish first to map out the causal circuit involved in a tracking system in operation.

Philbrick then goes on to diagram such feedback systems, beginning with a block diagram of a human-directed aiming "system" for a gun:

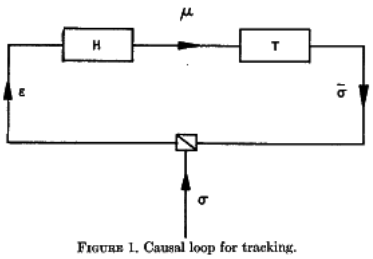


FIGURE 1. Causal loop for tracking.

In this system, variable μ is the position of the gun's handle, function H represents the nervous system of the human gunner, function T represents the physical gun, variable $\overline{\sigma}$ is the *sight index* which represents the position of the gunsight's crosshair center in the field of vision, variable σ is the "true sight direction" which represents the target's position in the field of vision, and variable ϵ is the *error* showing how far off the gun's aim is from the gunner's intended target.

Philbrick gives the feedback loop's error variable particular attention:

> Now the sight index $\overline{\sigma}$ is to be compared with the true sight direction or direction to the target, which may be given the symbol σ. Thus the difference between these two, or the error, which is visible to the human operator, is an important variable. We define the error as
>
> $$\epsilon = \sigma - \overline{\sigma}$$
>
> While under ideal tracking this difference will remain identically zero, still in real operation its value must be continually observed and interpreted to approximate this ideal.

This error between the gun's ideal and actual direction is an example of the alignment error serving as *datum for its own annihilation*, to borrow Philbrick's phrasing. The control system (in this case, the human gunner) constantly monitors this error and tries as best as possible to minimize it so that the gun points directly at its target.

Later in this same chapter, Philbrick notes the similiarities between this and standard "regulatory" control loops as applied in the process industries, no doubt informed by Philbrick's employment with The Foxboro Company which specialized in the design and manufacture of industrial instrumentation and control systems.

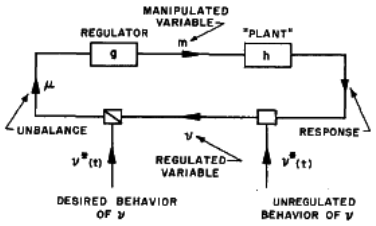


FIGURE 3. Symbolic representations for regulatory control loop.

Here, some of the labels have changed from the gun-aiming system, but the general concept of a feedback loop remains intact. A "regulator" device (g) replaces human judgment and action to influence (m) the physical process being controlled (the "plant" h). The process's response, in turn, is influenced by factors outside anyone's control to become the regulated variable (ν). This variable (known as the *process variable* or PV in modern control terms) gets compared against the desired behavior (known as the *setpoint* or SP in modern control terms), with the difference between those two variables known as the *unbalance* (or "error") μ. In an ideally-functioning control system this error has a value of zero.

Chapter 5

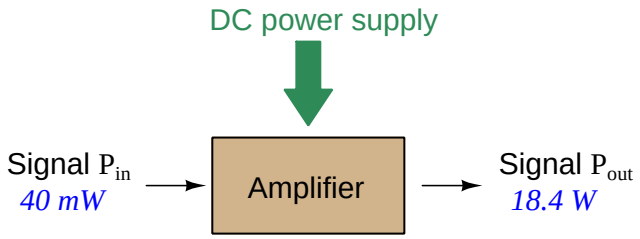
Derivations and Technical References

This chapter is where you will find mathematical derivations too detailed to include in the tutorial, and/or tables and other technical reference material.

5.1 Decibels

One of the mathematical tools popularly used to gauge increases and decreases of electrical power is the *common logarithm*, expressed as a measurement unit called the *decibel*. The basic idea of decibels is to express a ratio of two electrical power quantities in logarithmic terms. Every time you see the unit of "decibel" you can think: *this is an expression of how much greater (or how much smaller) one power is to another*. The only question is which two powers are being compared.

Electronic amplifiers are a type of electrical system where comparisons of power are useful. Students of electronics learn to compare the output power of an amplifier against the input power as a unitless ratio, called a *gain*. Take for example an electronic amplifier with a signal input of 40 milliWatts and a signal output of 18.4 Watts:



$$\text{Gain} = \frac{P_{out}}{P_{in}} = \frac{18.4\ W}{40\ mW} = 460$$

An alternative way to express the gain of this amplifier is to do so using the unit of the *Bel*, defined as the common logarithm of the gain ratio:

$$\log\left(\frac{P_{out}}{P_{in}}\right) = \log\left(\frac{18.4\ \text{W}}{40\ \text{mW}}\right) = 2.66276\ \text{B}$$

When you see an amplifier gain expressed in the unit of "Bel", it's really just a way of saying "The output signal coming from this amplifier is x powers of ten greater than the input signal." An amplifier exhibiting a gain of 1 Bel outputs 10 times as much power as the input signal. An amplifier with a gain of 2 Bels boosts the input signal by a factor of 100. The amplifier shown above, with a gain of 2.66276 Bels, boosts the input signal 460-fold.

At some point in technological history it was decided that the "Bel" (B) was too large and cumbersome, and so it became common to express powers in fractions of a Bel instead: the *deci*Bel (1 dB = $\frac{1}{10}$ of a Bel). Therefore, this is the form of formula you will commonly see for expressing electrical signal power gains or losses:

$$\text{dB} = 10 \log\left(\frac{P_{out}}{P_{in}}\right)$$

The gain of our hypothetical electronic amplifier, therefore, would be more commonly expressed as 26.6276 dB rather than 2.66276 B, although either expression is technically valid[1].

[1] It is interesting to note that although the "Bel" is a metric unit, it is seldom if ever used without the metric prefix

An operation students often struggle with is converting a decibel figure back into a ratio, since the concept of logarithms seems to be universally perplexing. Here I will demonstrate how to algebraically manipulate the decibel formula to solve for the power ratio given a dB figure.

First, we will begin with the decibel formula as given, solving for a value in decibels given a power ratio:

$$\text{dB} = 10 \log(\text{Ratio})$$

If we wish to solve for the ratio, we must "undo" all the mathematical operations surrounding that variable. One way to determine how to do this is to reverse the order of operations we would follow if we knew the ratio and were solving for the dB value. After calculating the ratio, we would then take the logarithm of that value, and then multiply that logarithm by 10: start with the ratio, then take the logarithm, then multiply last. To un-do these operations and solve for the ratio, we must un-do each of these operations in reverse order. First, we must un-do the multiplication (by dividing by 10):

$$\frac{\text{dB}}{10} = \frac{10 \log(\text{Ratio})}{10}$$

$$\frac{\text{dB}}{10} = \log(\text{Ratio})$$

Next, we must un-do the logarithm function by applying its mathematical inverse to both sides of the formula – making each expression a power of 10:

$$10^{\frac{\text{dB}}{10}} = 10^{\log(\text{Ratio})}$$

$$10^{\frac{\text{dB}}{10}} = \text{Ratio}$$

To test our algebra, we can take the previous decibel value for our hypothetical amplifier and see if this new formula yields the original gain ratio:

$$\text{Ratio} = 10^{\frac{26.6276 \text{ dB}}{10}}$$

$$\text{Ratio} = 10^{2.66276 \text{ B}}$$

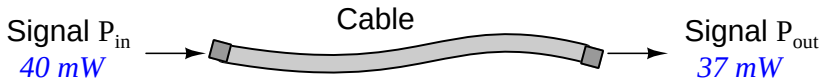
$$\text{Ratio} = 460$$

Sure enough, we arrive at the correct gain ratio of 460, starting with the decibel gain figure of 26.6276 dB.

"deci" ($\frac{1}{10}$). One could express powers in microbels, megabels, or any other metric prefix desired, but it is never done in industry: only the decibel is used.

We may also use decibels to express power *losses* in addition to power *gains*. There are many practical applications of this in signaling systems, both electronic and optical. One such application is *filtering*, where a "filter" circuit screens out certain components of the signal while letting others pass through (e.g. the bass or treble control for an audio system). Another application is *attenuation*, where the entirety of a signal is reduced in magnitude (e.g. the volume control for an audio system).

We will explore yet another application of signal power reduction as a case study for decibels: *cable loss*. Cables designed to convey signals over long distances are not perfect conduits of energy, as some of the signal's energy is inevitably lost along the way. This is true for different types of signals, electrical and optical being two popular examples. In the following illustration we see a signal cable losing power along its length[2], such that the power out is less than the power in:



$$\text{Gain} = \frac{P_{out}}{P_{in}} = \frac{37\ mW}{40\ mW} = 0.925$$

$$10 \log \left(\frac{P_{out}}{P_{in}} \right) = 10 \log \left(\frac{37 \text{ mW}}{40 \text{ mW}} \right) = -0.3386 \text{ dB}$$

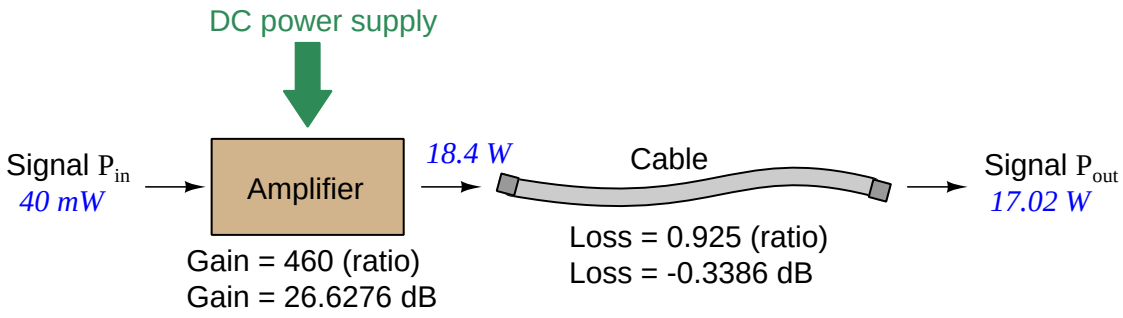
Contrasting this result against the previous result (with the amplifier) we see a very important property of decibel figures: any power *gain* is expressed as a *positive* decibel value, while any power *loss* is expressed as a *negative* decibel value. Any component outputting the exact same power as it takes in will exhibit a "gain" value of 0 dB (equivalent to a gain *ratio* of 1).

Remember that Bels and decibels are nothing more than logarithmic expressions of "greater than" and "less than". Positive values represent powers that are *greater* while negative values represent powers that are *lesser*. Zero Bel or decibel values represent *no change* (neither gain nor loss) in power.

A couple of simple decibel values are useful to remember for approximations, where you need to quickly estimate decibel values from power ratios (or vice-versa). Each addition or subtraction of 10 dB exactly represents a 10-fold multiplication or division of power ratio: e.g. +20 dB represents a power ratio gain of $10 \times 10 = 100$, whereas −30 dB represents a power ratio reduction of $\frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} = \frac{1}{1000}$. Each addition or subtraction of 3 dB approximately represents a 2-fold multiplication or division or power ratio: e.g. +6 dB is approximately equal to a power ratio gain of $2 \times 2 = 4$, whereas −12 dB is approximately equal to a power ratio reduction of $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16}$. We may combine ± 10 dB and ± 3 dB increments to come up with ratios that are products of 10 and 2: e.g. +26 dB is approximately equal to a power ratio gain of $10 \times 10 \times 2 \times 2 = 400$.

[2] For high-frequency signals such as those used in radio communications, the dominant mode of energy dissipation is *dielectric heating*, where the AC electric field between the cable conductors excites the molecules of the conductor insulation. This energy loss manifests as heat, which explains why there is less signal energy present at the load end of the cable than is input at the source end of the cable. For DC and low-frequency AC circuits the dominant mode of energy dissipation is cable conductor resistance, which is typically very small.

Observe what happens if we combine a "gain" component with a "loss" component and calculate the overall power out versus power in:



The overall gain of this amplifier and cable system expressed as a ratio is equal to the *product* of the individual component gain/loss ratios. That is, the gain ratio of the amplifier *multiplied* by the loss ratio of the cable yields the overall power ratio for the system:

$$\text{Overall gain} = \frac{17.02 \text{ W}}{40 \text{ mW}} = (460)(0.925) = 425.5$$

The overall gain may be alternatively expressed as a decibel figure, in which case it is equal to the *sum* of the individual component decibel values. That is, the decibel gain of the amplifier *added* to the decibel loss of the cable yields the overall decibel figure for the system:

$$\text{Overall gain} = 10 \log \left(\frac{17.02 \text{ W}}{40 \text{ mW}} \right) = 26.6276 \text{ dB} + (-0.3386 \text{ dB}) = 26.2890 \text{ dB}$$

It is often useful to be able to estimate decibel values from power ratios and vice-versa. If we take the gain ratio of this amplifier and cable system (425.5) and round it down to 400, we may easily express this gain ratio as an expanded product of 10 and 2:

$$425.5 \approx 400 = (10) \times (10) \times (2) \times (2)$$

Knowing that every 10-fold multiplication of power ratio is an addition of +10 dB, and that every 2-fold multiplication of power is an addition of +3 dB, we may express the expanded product as a sum of decibel values:

$$(10) \times (10) \times (2) \times (2) = (10 \text{ dB}) + (10 \text{ dB}) + (3 \text{ dB}) + (3 \text{ dB}) = 26 \text{ dB}$$

Therefore, our power ratio of 425.5 is approximately equal to +26 decibels.

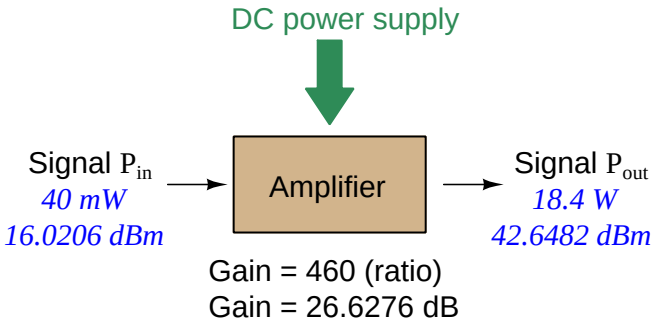
Decibels always represent comparisons of power, but that comparison need not always be P_{out}/P_{in} for a system component. We may also use decibels to express an amount of power compared to some standard reference. If, for example, we wished to express the input power to our hypothetical amplifier (40 milliWatts) using decibels, we could do so by comparing 40 mW against a standard "reference" power of exactly 1 milliWatt. The resulting decibel figure would be written as "dBm" in honor of the 1 *m*illiWatt reference:

$$P_{in} = 10 \log \left(\frac{40 \text{ mW}}{1 \text{ mW}} \right) = 16.0206 \text{ dBm}$$

The unit of "dBm" literally means the amount of dB "greater than" 1 milliWatt. In this case, our input signal of 40 milliWatts is 16.0206 dB greater than a standard reference power of exactly 1 milliWatt. The output power of that amplifier (18.4 Watts) may be expressed in dBm as well:

$$P_{out} = 10 \log \left(\frac{18.4 \text{ W}}{1 \text{ mW}} \right) = 42.6482 \text{ dBm}$$

A signal power of 18.4 Watts is 42.6482 dB greater than a standard reference power of exactly 1 milliWatt, and so it has a decibel value of 42.6482 dBm.



Notice how the output and input powers expressed in dBm relate to the power gain of the amplifier. Taking the input power and simply *adding* the amplifier's gain factor yields the amplifier's output power in dBm:

$$P_{in}(\text{dB}) + P_{gain}(\text{dB}) = P_{out}(\text{dB})$$

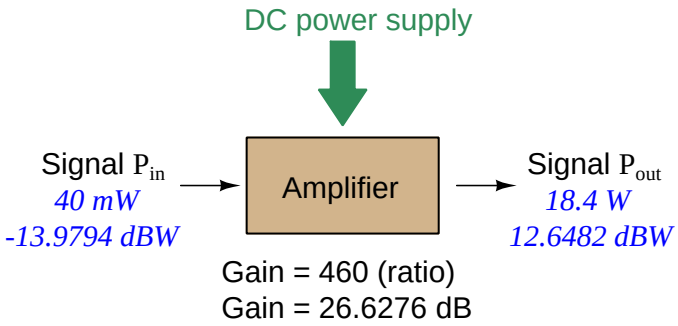
$$16.0206 \text{ dBm} + 26.6276 \text{ dB} = 42.6482 \text{ dBm}$$

An electronic signal that begins 16.0206 dB greater than 1 milliWatt, when boosted by an amplifier gain of 26.6276 dB, will become 42.6482 dB greater than the original reference power of 1 milliWatt.

We may alternatively express all powers in this hypothetical amplifier in reference to a 1-Watt standard power, with the resulting power expressed in units of "dBW" (decibels greater than 1 Watt):

$$P_{in} = 10 \log \left(\frac{40 \text{ mW}}{1 \text{ W}} \right) = -13.9794 \text{ dBW}$$

$$P_{out} = 10 \log \left(\frac{18.4 \text{ W}}{1 \text{ W}} \right) = 12.6482 \text{ dBW}$$



Note how the input power of 40 milliWatts equates to a negative dBW figure because 40 milliWatts is *less* than the 1 Watt reference, and how the output power of 18.4 Watts equates to a positive dBW figure because 18.4 Watts is *more* than the 1 Watt reference. A positive dB figure means "more than" while a negative dB figure means "less than."

Note also how the output and input powers expressed in dBW still relate to the power gain of the amplifier by simple addition, just as they did when previously expressed in units of dBm. Taking the input power in units of dBW and simply *adding* the amplifier's gain factor yields the amplifier's output power in dBW:

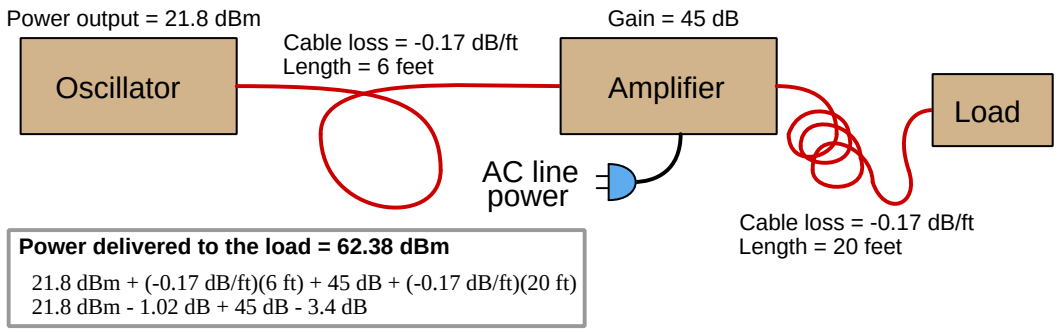
$$P_{in}(\text{dB}) + P_{gain}(\text{dB}) = P_{out}(\text{dB})$$

$$-13.9794 \text{ dBW} + 26.6276 \text{ dB} = 12.6482 \text{ dBW}$$

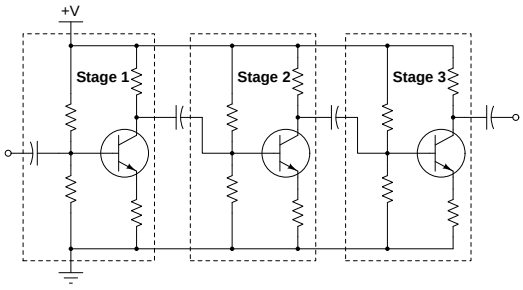
An electronic signal that begins 13.9794 dB less than 1 Watt, when boosted by an amplifier gain of 26.6276 dB, will become 12.6482 dB greater than the original reference power of 1 Watt.

This is one of the major benefits of using decibels to express powers: we may very easily calculate power gains and losses by summing a string of dB figures, each dB figure representing the power gain or power loss of a different system component. Normally, any compounding of *ratios* involves multiplication and/or division of those ratios, but with decibels we may simply add and subtract. One of the interesting mathematical properties of logarithms is that they "transform[3]" one type of problem into a simpler type: in this case, a problem of multiplying ratios into a (simpler) problem of adding decibel figures.

For example, we may express the power dissipated along a cable in terms of decibels per foot; the longer the cable, of course, the more power will be lost this way, all other factors being equal. For example, a radio-frequency signal cable having a loss figure of −0.15 decibels per foot at a signal frequency of 2.4 GHz will suffer −15 dB over 100 feet, and −150 dB over 1000 feet. To illustrate how decibels may be used to calculate power delivered to a load in such a system, accounting for various gains and losses along the way using decibel figures:



A similar application of decibels is found in multi-stage amplifier circuits, where one stage amplifies a signal to be fed into a successive stage to be amplified more. The power gains of these stages, each expressed as a ratio, *multiply* to make the over-all amplifier's power gain (ratio). The power gains of those same stages, each expressed as a decibel figure, *add* to make the over-all amplifier's power gain (dB):



[3] In fact, logarithms are one of the simplest examples of a *transform function*, converting one type of mathematical problem into another type. Other examples of mathematical transform functions used in engineering include the *Fourier transform* (converting a time-domain function into a frequency-domain function) and the *Laplace transform* (converting a differential equation into an algebraic equation).

Another common application of decibels is to express ratios of voltage and/or current rather than power. However, since the unit of the Bel has been defined as an expression of a *power* ratio, we cannot simply substitute V or I for P in any of the formulae we've seen so far.

Suppose an amplifier has a voltage gain of 2 (i.e. V_{out} is twice as large as V_{in}), and we would like to express this gain in decibels. Since decibels are intended to express power gain and not voltage gain, we must figure out how much power gain is equivalent to a voltage gain of two. Obviously, voltage and power are fundamentally different quantities, but if we imagine ourselves connecting a fixed load resistance to the input signal, and then to the output signal, we will realize that load's power dissipation will be more than double when energized by a voltage twice as large. Joule's Law is helpful to determine the exact ratio of power dissipation:

$$P = \frac{V^2}{R}$$

Doubling voltage for any given load resistance results in power *quadrupling* because power is proportional to the square of the voltage applied to a fixed resistance. Using this as the basis for applying decibels to a voltage ratio. Knowing that Joule's Law also declares power is proportional to the square of the current applied to a fixed resistance ($P = I^2R$) means this same mathematical relationship will apply to current gains and reductions as well as voltage gains and reductions:

$$\text{dB} = 10 \log \left(\frac{P_{out}}{P_{in}} \right) = 10 \log \left(\frac{V_{out}}{V_{in}} \right)^2 = 10 \log \left(\frac{I_{out}}{I_{in}} \right)^2$$

An algebraic identity of logarithms is that the logarithm of any quantity raised to a power is equal to that power multiplied by the logarithm of the quantity. Expressed in general terms:

$$\log x^y = y \log x$$

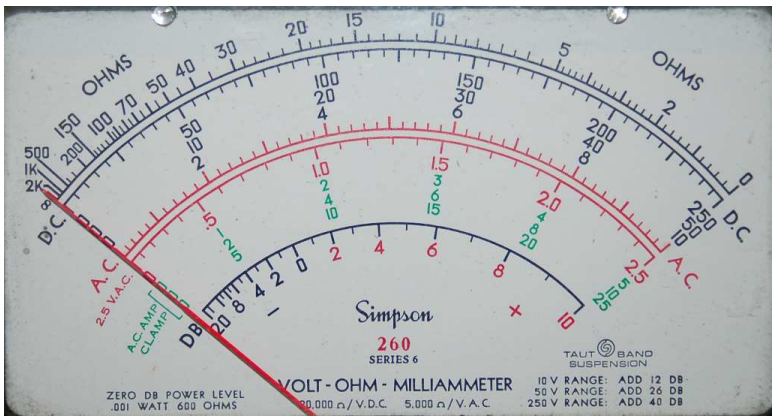
Therefore, we may simplify the decibel formula for voltage gain by removing the "2" power and making it a multiplier:

$$10 \log \left(\frac{V_{out}}{V_{in}} \right)^2 = (2)(10) \log \left(\frac{V_{out}}{V_{in}} \right) = 20 \log \left(\frac{V_{out}}{V_{in}} \right)$$

$$10 \log \left(\frac{I_{out}}{I_{in}} \right)^2 = (2)(10) \log \left(\frac{I_{out}}{I_{in}} \right) = 20 \log \left(\frac{I_{out}}{I_{in}} \right)$$

Thus, we may use decibels to express voltage or current ratios if we simply substitute 20 instead of 10 as the multiplier.

We can see the practicality of using decibels to represent something other than electrical *power* by examining this analog meter face, belonging to a Simpson model 260 VOM (Volt-Ohm-Milliammeter). Note the bottom scale on this meter's face, calibrated in decibels (DB):



Pay attention to the note on decibels written in the lower-left corner of the meter face, where 0 dB is defined as 0.001 Watt dissipated by 600 Ohms. The fact that 0 dB is defined as 1 milliWatt means it should (properly) be labeled dBm rather than dB[4]. A load resistance value is necessary as part of this definition for dB because this meter cannot measure power directly but must infer signal power from measurements of AC *voltage*. Without a specific load resistance, there is no clear relation between voltage and power. 600 Ohms is an old telecommunications standard for audio-frequency AC signals, and continues to be used today for voltage-based decibel measurements of audio-frequency AC signals.

The meter as shown is connected to nothing at all, and so registers 0 Volts AC. This, of course, corresponds to zero power, and it has no corresponding decibel value because the logarithm of zero is mathematically *undefined*[5]. Practically, it means $-\infty$ dB, which is why the needle at the 0 Volt position "falls off" the left-hand end of the dB scale.

Close inspection of the dB scale on this meter face reveals another interesting property of decibels, and that is the *nonlinear* nature of the dB scale. This contrasts starkly against all the voltage and current scales on this meter face which are linear. This nonlinearity is a fundamental property of decibels because it is based on the *logarithm* function which is nonlinear.

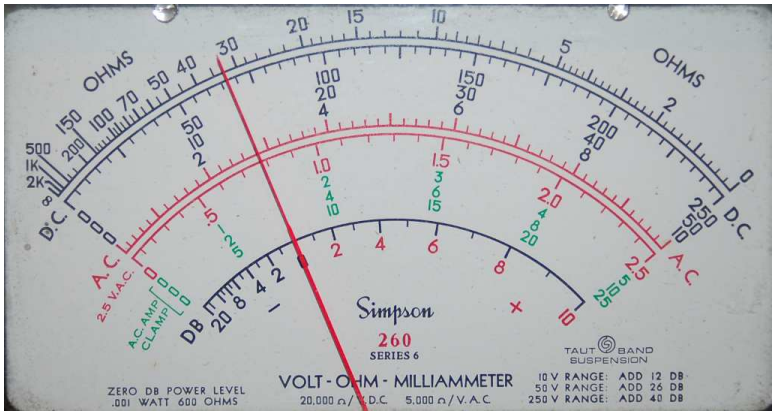
[4] Such mis-labeling is not that uncommon in the profession, the expectation being that the technician or engineer working with the instrument ought to be familiar enough with the concept of decibels to know when dB really means dBm, or dBW, etc.

[5] Your electronic calculator will complain if you attempt to take the logarithm of zero!

Now, we will explore what is necessary to make this meter register 0 dBm (i.e. 1 milliWatt) with an applied AC voltage. 1 milliWatt of power dissipated by 600 Ohms is equivalent to:

$$V = \sqrt{PR} = \sqrt{(0.001)(600)} = 0.7746 \text{ Volts}$$

Setting the VOM to the 2.5 VAC range and applying just enough AC voltage to bring the needle to the 0 dB mark allows us to verify that this is indeed equivalent to just under 0.8 Volts (read on the 2.5 VAC scale):



In the lower-right corner of the meter face we see some notes regarding correction values for decibel measurements when using different AC voltage ranges. The dB scale is read directly when the meter is set on the 2.5 VAC range. When set on the 10 VAC range (i.e. a range *four times* as great), the meter's needle will experience a deflection one-fourth as much as when set to the 2.5 VAC range, and therefore it will point to a lesser (or even negative) value on the dB scale. Converting a voltage ratio of 0.25 into a decibel figure shows us how much less the needle will register on the dB scale when the voltage range is quadrupled:

$$20 \log \left(\frac{2.5}{10} \right) = -12.04 \text{ dB}$$

Therefore, when using the 10 VAC range instead of the 2.5 VAC range, one must add 12 dB to the reading. Likewise, we may prove each of the printed correction factors for the alternative voltage-measurement ranges listed (50 Volt AC range and 250 Volt AC range):

$$20 \log \left(\frac{2.5}{50} \right) = -26.02 \text{ dB}$$

$$20 \log \left(\frac{2.5}{250} \right) = -40.0 \text{ dB}$$

Chapter 6

Questions

This learning module, along with all others in the ModEL collection, is designed to be used in an inverted instructional environment where students independently read[1] the tutorials and attempt to answer questions on their own *prior* to the instructor's interaction with them. In place of lecture[2], the instructor engages with students in Socratic-style dialogue, probing and challenging their understanding of the subject matter through inquiry.

Answers are not provided for questions within this chapter, and this is by design. Solved problems may be found in the Tutorial and Derivation chapters, instead. The goal here is *independence*, and this requires students to be challenged in ways where others cannot think for them. Remember that you always have the tools of *experimentation* and *computer simulation* (e.g. SPICE) to explore concepts!

The following lists contain ideas for Socratic-style questions and challenges. Upon inspection, one will notice a strong theme of *metacognition* within these statements: they are designed to foster a regular habit of examining one's own thoughts as a means toward clearer thinking. As such these sample questions are useful both for instructor-led discussions as well as for self-study.

[1] Technical reading is an essential academic skill for any technical practitioner to possess for the simple reason that the most comprehensive, accurate, and useful information to be found for developing technical competence is in textual form. Technical careers in general are characterized by the need for continuous learning to remain current with standards and technology, and therefore any technical practitioner who cannot read well is handicapped in their professional development. An excellent resource for educators on improving students' reading prowess through intentional effort and strategy is the book textitReading For Understanding – How Reading Apprenticeship Improves Disciplinary Learning in Secondary and College Classrooms by Ruth Schoenbach, Cynthia Greenleaf, and Lynn Murphy.

[2] Lecture is popular as a teaching method because it is easy to implement: any reasonably articulate subject matter expert can talk to students, even with little preparation. However, it is also quite problematic. A good lecture always makes complicated concepts seem easier than they are, which is bad for students because it instills a false sense of confidence in their own understanding; reading and re-articulation requires more cognitive effort and serves to verify comprehension. A culture of teaching-by-lecture fosters a debilitating dependence upon direct personal instruction, whereas the challenges of modern life demand independent and critical thought made possible only by gathering information and perspectives from afar. Information presented in a lecture is ephemeral, easily lost to failures of memory and dictation; text is forever, and may be referenced at any time.

General challenges following tutorial reading

- Summarize as much of the text as you can in one paragraph of your own words. A helpful strategy is to explain ideas as you would for an intelligent child: as simple as you can without compromising too much accuracy.
- Simplify a particular section of the text, for example a paragraph or even a single sentence, so as to capture the same fundamental idea in fewer words.
- Where did the text make the most sense to you? What was it about the text's presentation that made it clear?
- Identify where it might be easy for someone to misunderstand the text, and explain why you think it could be confusing.
- Identify any new concept(s) presented in the text, and explain in your own words.
- Identify any familiar concept(s) such as physical laws or principles applied or referenced in the text.
- Devise a proof of concept experiment demonstrating an important principle, physical law, or technical innovation represented in the text.
- Devise an experiment to disprove a plausible misconception.
- Did the text reveal any misconceptions you might have harbored? If so, describe the misconception(s) and the reason(s) why you now know them to be incorrect.
- Describe any useful problem-solving strategies applied in the text.
- Devise a question of your own to challenge a reader's comprehension of the text.

General follow-up challenges for assigned problems

- Identify where any fundamental laws or principles apply to the solution of this problem, especially before applying any mathematical techniques.
- Devise a thought experiment to explore the characteristics of the problem scenario, applying known laws and principles to mentally model its behavior.
- Describe in detail your own strategy for solving this problem. How did you identify and organized the given information? Did you sketch any diagrams to help frame the problem?
- Is there more than one way to solve this problem? Which method seems best to you?
- Show the work you did in solving this problem, even if the solution is incomplete or incorrect.
- What would you say was the most challenging part of this problem, and why was it so?
- Was any important information missing from the problem which you had to research or recall?
- Was there any extraneous information presented within this problem? If so, what was it and why did it not matter?
- Examine someone else's solution to identify where they applied fundamental laws or principles.
- Simplify the problem from its given form and show how to solve this simpler version of it. Examples include eliminating certain variables or conditions, altering values to simpler (usually whole) numbers, applying a limiting case (i.e. altering a variable to some extreme or ultimate value).
- For quantitative problems, identify the real-world meaning of all intermediate calculations: their units of measurement, where they fit into the scenario at hand. Annotate any diagrams or illustrations with these calculated values.
- For quantitative problems, try approaching it qualitatively instead, thinking in terms of "increase" and "decrease" rather than definite values.
- For qualitative problems, try approaching it quantitatively instead, proposing simple numerical values for the variables.
- Were there any assumptions you made while solving this problem? Would your solution change if one of those assumptions were altered?
- Identify where it would be easy for someone to go astray in attempting to solve this problem.
- Formulate your own problem based on what you learned solving this one.

General follow-up challenges for experiments or projects

- In what way(s) was this experiment or project easy to complete?
- Identify some of the challenges you faced in completing this experiment or project.

- Show how thorough documentation assisted in the completion of this experiment or project.
- Which fundamental laws or principles are key to this system's function?
- Identify any way(s) in which one might obtain false or otherwise misleading measurements from test equipment in this system.
- What will happen if (component X) fails (open/shorted/etc.)?
- What would have to occur to make this system unsafe?

6.1 Conceptual reasoning

These questions are designed to stimulate your analytic and synthetic thinking[3]. In a Socratic discussion with your instructor, the goal is for these questions to prompt an extended dialogue where assumptions are revealed, conclusions are tested, and understanding is sharpened. Your instructor may also pose additional questions based on those assigned, in order to further probe and refine your conceptual understanding.

Questions that follow are presented to challenge and probe your understanding of various concepts presented in the tutorial. These questions are intended to serve as a guide for the Socratic dialogue between yourself and the instructor. Your instructor's task is to ensure you have a sound grasp of these concepts, and the questions contained in this document are merely a means to this end. Your instructor may, at his or her discretion, alter or substitute questions for the benefit of tailoring the discussion to each student's needs. The only absolute requirement is that each student is challenged and assessed at a level equal to or greater than that represented by the documented questions.

It is far more important that you convey your *reasoning* than it is to simply convey a correct answer. For this reason, you should refrain from researching other information sources to answer questions. What matters here is that *you* are doing the thinking. If the answer is incorrect, your instructor will work with you to correct it through proper reasoning. A correct answer without an adequate explanation of how you derived that answer is unacceptable, as it does not aid the learning or assessment process.

You will note a conspicuous lack of answers given for these conceptual questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your conceptual answers, where applicable, is to use circuit simulation software to explore the effects of changes made to circuits. For example, if one of these conceptual questions challenges you to predict the effects of altering some component parameter in a circuit, you may check the validity of your work by simulating that same parameter change within software and seeing if the results agree.

[3] *Analytical* thinking involves the "disassembly" of an idea into its constituent parts, analogous to dissection. *Synthetic* thinking involves the "assembly" of a new idea comprised of multiple concepts, analogous to construction. Both activities are high-level cognitive skills, extremely important for effective problem-solving, necessitating frequent challenge and regular practice to fully develop.

6.1.1 Reading outline and reflections

"Reading maketh a full man; conference a ready man; and writing an exact man" – Francis Bacon

Francis Bacon's advice is a blueprint for effective education: reading provides the learner with knowledge, writing focuses the learner's thoughts, and critical dialogue equips the learner to confidently communicate and apply their learning. Independent acquisition and application of knowledge is a powerful skill, well worth the effort to cultivate. To this end, students should read these educational resources closely, journal their own reflections on the reading, and discuss in detail their findings with classmates and instructor(s). You should be able to do all of the following after reading any instructional text:

☑ Briefly SUMMARIZE THE TEXT in the form of a journal entry documenting your learning as you progress through the course of study. Share this summary in dialogue with your classmates and instructor. Journaling is an excellent self-test of thorough reading because you cannot clearly express what you have not read or did not comprehend.

☑ Demonstrate ACTIVE READING STRATEGIES, including verbalizing your impressions as you read, simplifying long passages to convey the same ideas using fewer words, annotating text and illustrations with your own interpretations, working through mathematical examples shown in the text, cross-referencing passages with relevant illustrations and/or other passages, identifying problem-solving strategies applied by the author, etc. Technical reading is a special case of problem-solving, and so these strategies work precisely because they help solve any problem: paying attention to your own thoughts (metacognition), eliminating unnecessary complexities, identifying what makes sense, paying close attention to details, drawing connections between separated facts, and noting the successful strategies of others.

☑ Identify IMPORTANT THEMES, especially GENERAL LAWS and PRINCIPLES, expounded in the text and express them in the simplest of terms as though you were teaching an intelligent child. This emphasizes connections between related topics and develops your ability to communicate complex ideas to anyone.

☑ Form YOUR OWN QUESTIONS based on the reading, and then pose them to your instructor and classmates for their consideration. Anticipate both correct and incorrect answers, the incorrect answer(s) assuming one or more plausible misconceptions. This helps you view the subject from different perspectives to grasp it more fully.

☑ Devise EXPERIMENTS to test claims presented in the reading, or to disprove misconceptions. Predict possible outcomes of these experiments, and evaluate their meanings: what result(s) would confirm, and what would constitute disproof? Running mental simulations and evaluating results is essential to scientific and diagnostic reasoning.

☑ Specifically identify any points you found CONFUSING. The reason for doing this is to help diagnose misconceptions and overcome barriers to learning.

6.1.2 Foundational concepts

Correct analysis and diagnosis of electric circuits begins with a proper understanding of some basic concepts. The following is a list of some important concepts referenced in this module's full tutorial. Define each of them in your own words, and be prepared to illustrate each of these concepts with a description of a practical example and/or a live demonstration.

Thought experiments as a problem-solving strategy

Negative feedback

Positive feedback

Amplification

Saturation

Gain

Passive averager

Stability

Block diagram

Functional inversion

Error

Servo

Disturbance

6.1.3 Negative feedback examples

A very important concept in electronics is that of *negative feedback*. This is an extremely important concept to grasp, as a great many electronic systems exploit this principle for their operation and cannot be properly understood without a comprehension of it.

However important negative feedback might be, it is not the easiest concept to understand. In fact, it is quite a conceptual leap for some. The following is a list of examples – some electronic, some not – exhibiting negative feedback:

- A voltage regulating circuit
- An auto-pilot system for an aircraft or boat
- A thermostatic temperature control system ("thermostat")
- Emitter resistor in a BJT amplifier circuit
- Lenz's Law demonstration (magnetic damping of a moving object)
- Body temperature of a mammal
- Natural regulation of prices in a free market economy (Adam Smith's "invisible hand")
- A scientist learning about the behavior of a natural system through experimentation.

For each case, answer the following questions:

- What variable is being stabilized by negative feedback?
- How is the feedback taking place (step by step)?
- What would the system's response be like if negative feedback were not present?

Challenges

- How would these systems act if the feedback were *positive* instead of *negative*?

6.1.4 Perspiration as a feedback mechanism

What purpose does *perspiration* serve for a person or an animal, and how does this represent a form of feedback? Would this be negative feedback, or positive feedback?

Challenges

- Identify some other biological feedback loops.

6.1.5 Miscalibrated speedometer

Suppose you borrowed a friend's automobile without realizing the speedometer was miscalibrated such that it always read -5% too low. For example, when the actual road speed is 50 km/h, this speedometer reads 47.5 km/h.

What effect, if any, would this miscalibration have on your driving, if you did your best to drive at the posted speed limit on all roads?

Challenges

- If the speedometer were perfectly calibrated, but the engine in your friend's car was more powerful than what you were used to, how would this affect your ability to maintain the speed limit?
- Sketch a block diagram of you as part of a speed-control system for this automobile, then identify where a speedometer miscalibration would appear in the diagram, as well as an engine "recalibration" (i.e. more powerful engine).

6.1.6 Miscalibrated thermostat sensor

Sketch a block diagram for a home thermostat control system, and then explain what would happen if the thermostat's air temperature sensor were miscalibrated so that it "thought" the home was ten degrees warmer than it actually was.

Challenges

- If a window were opened on a cold day, how would this system respond (referring to the block diagram)?

6.1.7 Party stereo

Suppose a group of people are enjoying a party where music emanates from a stereo system. Some time during the party, a new guest arrives who happens to be hard of hearing. This new person walks over to the stereo system and re-adjusts the volume according to his liking. What do you suppose the other guests will think of this volume re-adjustment?

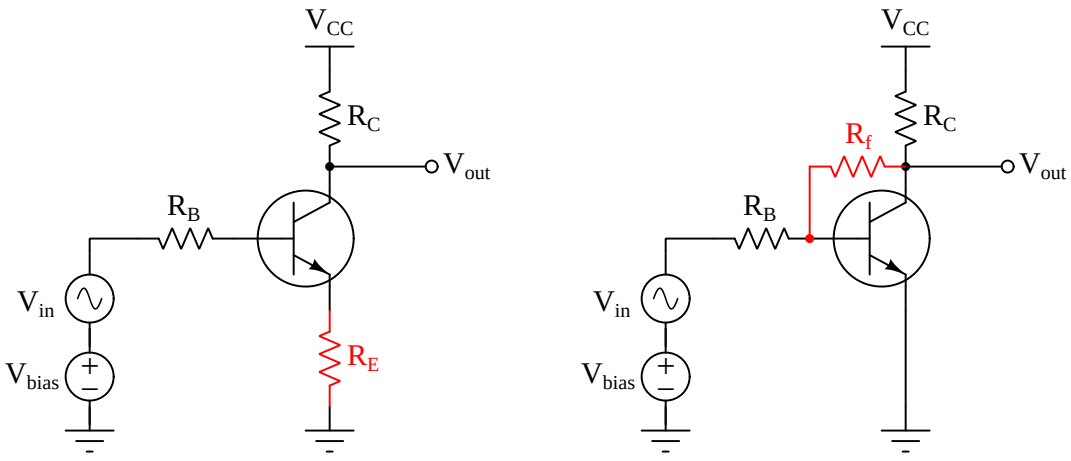
Describe this scenario in terms of *feedback*. Is this an example of negative feedback, positive feedback, or no feedback?

Challenges

- Explain how this "system" could be modified to allow this new guest to set the volume, but to *everyone else's* liking.

6.1.8 Negative feedback for common-emitter amplifiers

Two simple methods of incorporating negative (i.e. "degenerative") feedback into common-emitter amplifier circuits are shown below:



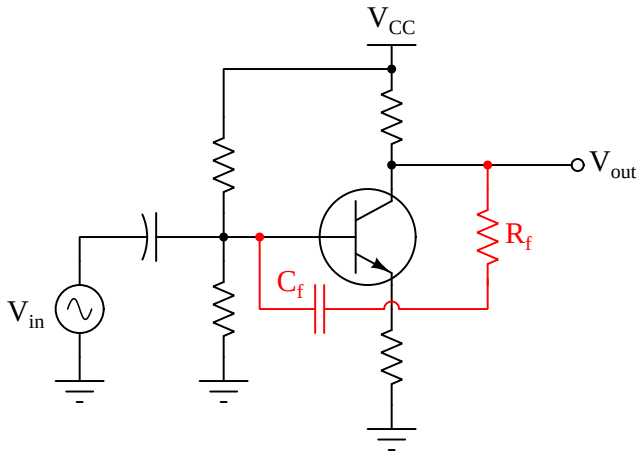
Explain what the term *negative* (or *degenerative*) means with reference to feedback, and explain how each of these techniques works to produce this type of feedback. Also, explain one *disadvantage* of applying negative feedback to a common-emitter amplifier circuit.

Challenges

- Suppose each of the colored resistors were exchanged for *inductors* – how would each amplifier's performance be altered?
- Identify where a "bypass" capacitor would be connected in the left-hand amplifier circuit, and explain how the addition of that capacitor would affect the circuit's voltage gain.

6.1.9 RC feedback network

Sometimes a *feedback network* is purposely placed in an amplifier circuit, like the R_f-C_f combination shown in the following schematic:



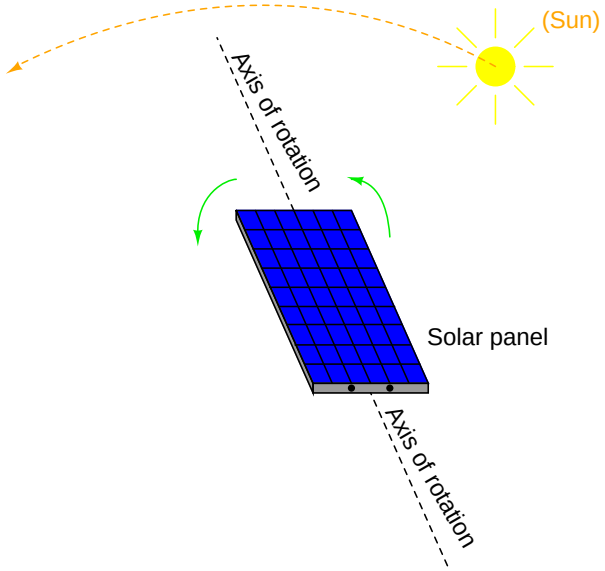
Contrast the performance of this feedback network against that of a plain resistor-only feedback network in the same type of amplifier circuit.

Challenges

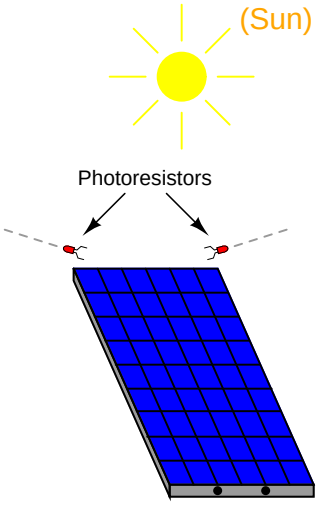
- Which way should the capacitor be installed if it were polarized (e.g. electrolytic)?

6.1.10 Solar panel tracker

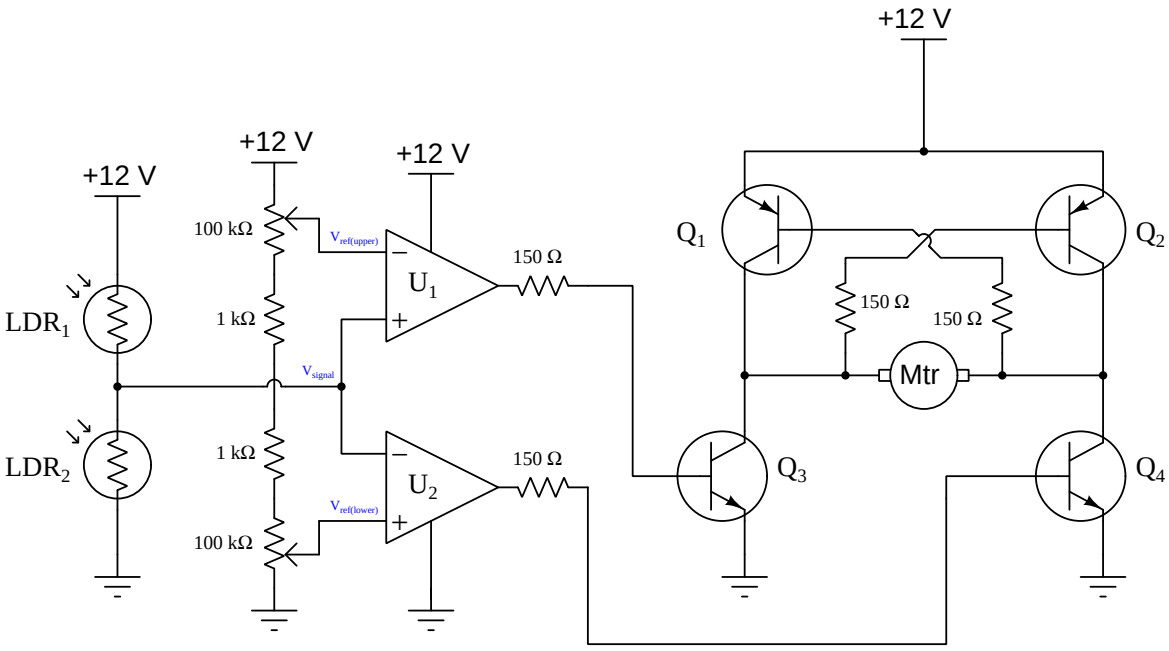
Photovoltaic solar panels produce the most output power when facing directly into sunlight. To maintain proper positioning, "tracker" systems may be used to orient the panels' direction as the sun "moves" from east to west across the sky:



One way to detect the sun's position relative to the panel is to attach a pair of Light-Dependent Resistors (LDR's) to the solar panel in such a way that each LDR will receive an equal amount of light only if the panel is pointed directly at the sun:



Two comparators are used to sense the differential resistance produced by these two LDR's, and activate a tracking motor to tilt the solar panel on its axis when the differential resistance becomes too great. An "H-drive" transistor switching circuit takes the comparators' output signals and amplifies them to drive a permanent-magnet DC motor one way or the other:



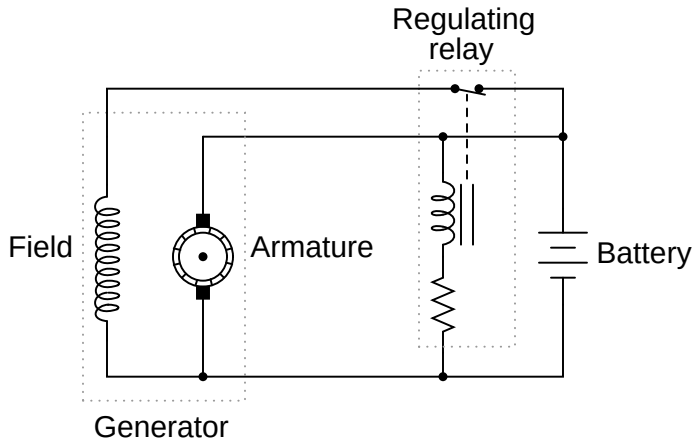
Would you classify this system as having *positive feedback*, *negative feedback*, or *no feedback*? Explain in detail.

Challenges

- Identify a *disturbance* in this control system and predict its effects on the system.
- What would happen if one of the 150 Ω resistors happened to fail open?

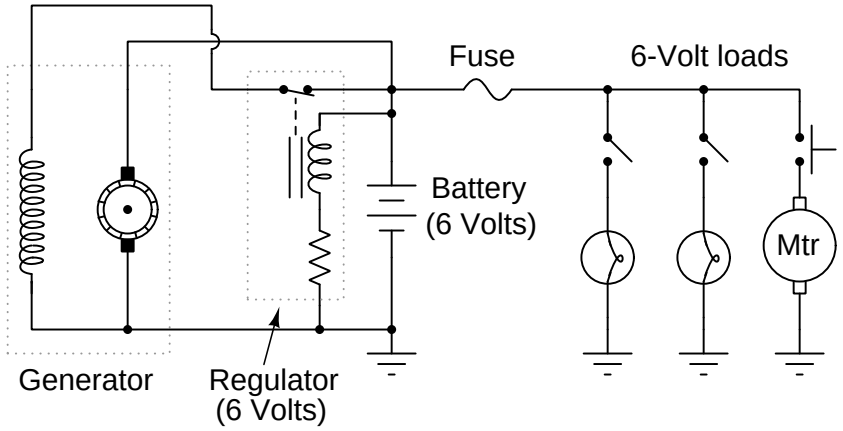
6.1.11 Fooling a voltage regulator

Generators used in battery-charging systems must be *regulated* so as to not overcharge the battery(ies) they are connected to. Here is a crude, relay-based voltage regulator for a DC generator:

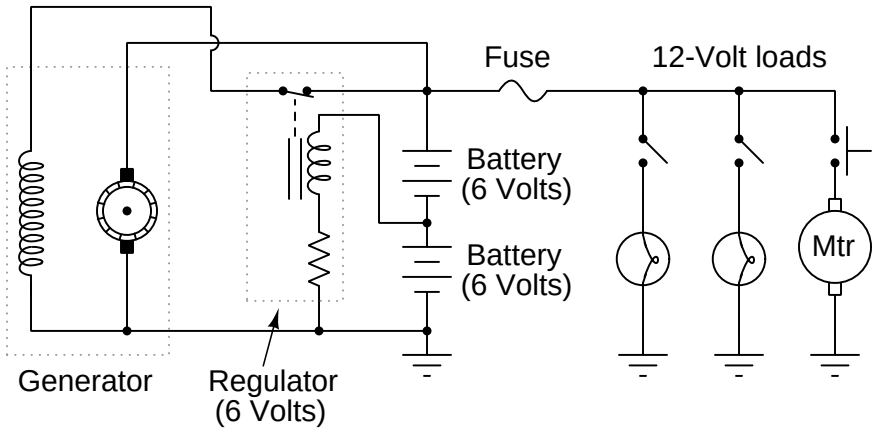


Simple electromechanical relay circuits such as this one were very common in automotive electrical systems during the 1950's, 1960's, and 1970's. The fundamental principle upon which their operation is based is called *negative feedback*: where a system takes action to *oppose* any change in a certain variable. In this case, the variable is generator output voltage. Explain how the relay works to prevent the generator from overcharging the battery with excessive voltage.

A mechanic has an idea for upgrading the electrical system in an automobile originally designed for 6 Volt operation. He wants to upgrade the 6 Volt headlights, starter motor, battery, etc, to 12 Volts, but wishes to retain the original 6-Volt generator and regulator. Shown here is the original 6-Volt electrical system:



The mechanic's plan is to replace all the 6-Volt loads with 12-Volt loads, and use two 6-Volt batteries connected in series, with the original (6-Volt) regulator sensing voltage across only one of those batteries:



Explain how this system is supposed to work. Do you think the mechanic's plan is practical, or are there any problems with it?

Challenges

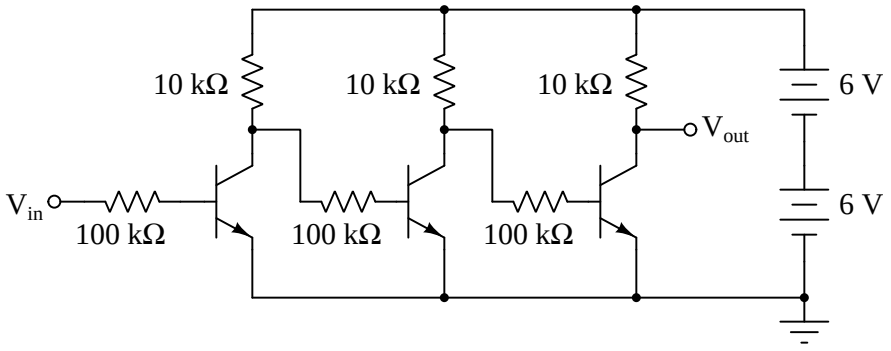
- What would we have to change in the original circuit to alter the generator's voltage regulation set-point (the "target" voltage at which the generator's output is supposed to be regulated).

- Identify factors that may prevent the generator from outputting enough voltage with the regulator connected as shown in the last diagram.

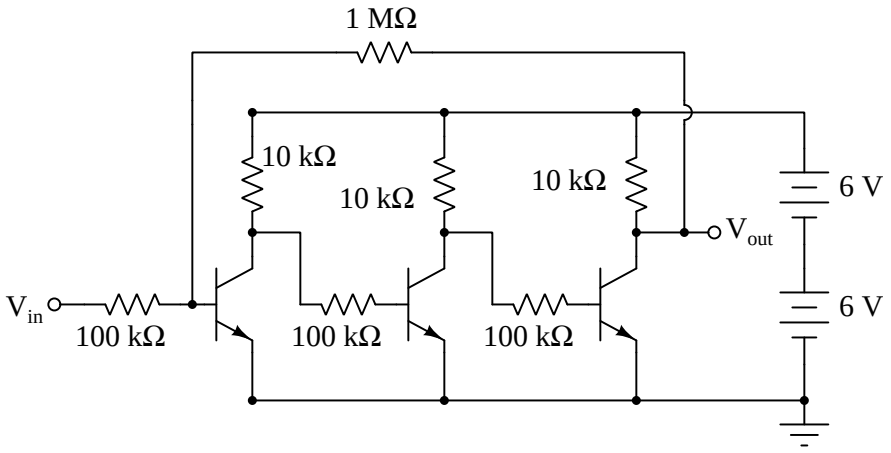
6.1.12 Taming an unruly amplifier

Unswamped common-emitter amplifiers are notoriously unstable. Both their biasing requirements and their voltage gains drift significantly as transistor temperature varies, but they do have the advantage of exhibiting very high voltage gain.

Multiple stages' worth of unswamped common-emitter amplifiers coupled by resistors is even worse. When one stage drifts, it skews the bias of the next stage, etc.



However, such an unruly beast as a three-stage unswamped common-emitter amplifier may be "tamed" simply by adding a single resistor to the circuit:



Explain why the addition of that resistor stabilized the amplifier.

Challenges

- Identify how to increase the over-all voltage gain of the stabilized amplifier.

6.2 Quantitative reasoning

These questions are designed to stimulate your computational thinking. In a Socratic discussion with your instructor, the goal is for these questions to reveal your mathematical approach(es) to problem-solving so that good technique and sound reasoning may be reinforced. Your instructor may also pose additional questions based on those assigned, in order to observe your problem-solving firsthand.

Mental arithmetic and estimations are strongly encouraged for all calculations, because without these abilities you will be unable to readily detect errors caused by calculator misuse (e.g. keystroke errors).

You will note a conspicuous lack of answers given for these quantitative questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. My advice is to use circuit simulation software such as SPICE to check the correctness of quantitative answers. Refer to those learning modules within this collection focusing on SPICE to see worked examples which you may use directly as practice problems for your own study, and/or as templates you may modify to run your own analyses and generate your own practice problems.

Completely worked example problems found in the Tutorial may also serve as "test cases[4]" for gaining proficiency in the use of circuit simulation software, and then once that proficiency is gained you will never need to rely[5] on an answer key!

[4] In other words, set up the circuit simulation software to analyze the same circuit examples found in the Tutorial. If the simulated results match the answers shown in the Tutorial, it confirms the simulation has properly run. If the simulated results disagree with the Tutorial's answers, something has been set up incorrectly in the simulation software. Using every Tutorial as practice in this way will quickly develop proficiency in the use of circuit simulation software.

[5] This approach is perfectly in keeping with the instructional philosophy of these learning modules: *teaching students to be self-sufficient thinkers*. Answer keys can be useful, but it is even more useful to your long-term success to have a set of tools on hand for checking your own work, because once you have left school and are on your own, there will no longer be "answer keys" available for the problems you will have to solve.

6.2.1 Miscellaneous physical constants

Note: constants shown in **bold** type are *exact*, not approximations. Values inside of parentheses show one standard deviation (σ) of uncertainty in the final digits: for example, the magnetic permeability of free space value given as $1.25663706212(19) \times 10^{-6}$ H/m represents a center value (i.e. the location parameter) of $1.25663706212 \times 10^{-6}$ Henrys per meter with one standard deviation of uncertainty equal to $0.0000000000019 \times 10^{-6}$ Henrys per meter.

Avogadro's number (N_A) = **6.02214076** $\times 10^{23}$ **per mole** (mol^{-1})

Boltzmann's constant (k) = **1.380649** $\times 10^{-23}$ **Joules per Kelvin** (J/K)

Electronic charge (e) = **1.602176634** $\times 10^{-19}$ **Coulomb** (C)

Faraday constant (F) = **96,485.33212...** $\times 10^{4}$ **Coulombs per mole** (C/mol)

Magnetic permeability of free space (μ_0) = $1.25663706212(19) \times 10^{-6}$ Henrys per meter (H/m)

Electric permittivity of free space (ϵ_0) = $8.8541878128(13) \times 10^{-12}$ Farads per meter (F/m)

Characteristic impedance of free space (Z_0) = 376.730313668(57) Ohms (Ω)

Gravitational constant (G) = $6.67430(15) \times 10^{-11}$ cubic meters per kilogram-seconds squared ($m^3/kg\text{-}s^2$)

Molar gas constant (R) = **8.314462618... Joules per mole-Kelvin** (J/mol-K) = 0.08205746(14) liters-atmospheres per mole-Kelvin

Planck constant (h) = **6.62607015** $\times 10^{-34}$ **joule-seconds** (J-s)

Stefan-Boltzmann constant (σ) = **5.670374419...** $\times 10^{-8}$ **Watts per square meter-Kelvin**^{4} ($W/m^2 \cdot K^4$)

Speed of light in a vacuum (c) = **299,792,458 meters per second** (m/s) = 186282.4 miles per second (mi/s)

Note: All constants taken from NIST data "Fundamental Physical Constants – Complete Listing", from `http://physics.nist.gov/constants`, National Institute of Standards and Technology (NIST), 2018 CODATA Adjustment.

6.2.2 Introduction to spreadsheets

A powerful computational tool you are encouraged to use in your work is a *spreadsheet*. Available on most personal computers (e.g. Microsoft Excel), *spreadsheet* software performs numerical calculations based on number values and formulae entered into cells of a grid. This grid is typically arranged as lettered columns and numbered rows, with each cell of the grid identified by its column/row coordinates (e.g. cell `B3`, cell `A8`). Each cell may contain a string of text, a number value, or a mathematical formula. The spreadsheet automatically updates the results of all mathematical formulae whenever the entered number values are changed. This means it is possible to set up a spreadsheet to perform a series of calculations on entered data, and those calculations will be re-done by the computer any time the data points are edited in any way.

For example, the following spreadsheet calculates average speed based on entered values of distance traveled and time elapsed:

	A	B	C	D
1	Distance traveled	46.9	Kilometers	
2	Time elapsed	1.18	Hours	
3	Average speed	= B1 / B2	km/h	
4				
5				

Text labels contained in cells `A1` through `A3` and cells `C1` through `C3` exist solely for readability and are not involved in any calculations. Cell `B1` contains a sample distance value while cell `B2` contains a sample time value. The formula for computing speed is contained in cell `B3`. Note how this formula begins with an "equals" symbol (`=`), references the values for distance and speed by lettered column and numbered row coordinates (`B1` and `B2`), and uses a forward slash symbol for division (`/`). The coordinates `B1` and `B2` function as *variables*[6] would in an algebraic formula.

When this spreadsheet is executed, the numerical value `39.74576` will appear in cell `B3` rather than the formula `= B1 / B2`, because 39.74576 is the computed speed value given 46.9 kilometers traveled over a period of 1.18 hours. If a different numerical value for distance is entered into cell `B1` or a different value for time is entered into cell `B2`, cell `B3`'s value will automatically update. All you need to do is set up the given values and any formulae into the spreadsheet, and the computer will do all the calculations for you.

Cell `B3` may be referenced by other formulae in the spreadsheet if desired, since it is a variable just like the given values contained in `B1` and `B2`. This means it is possible to set up an entire chain of calculations, one dependent on the result of another, in order to arrive at a final value. The arrangement of the given data and formulae need not follow any pattern on the grid, which means you may place them anywhere.

[6] Spreadsheets may also provide means to attach text labels to cells for use as variable names (Microsoft Excel simply calls these labels "names"), but for simple spreadsheets such as those shown here it's usually easier just to use the standard coordinate naming for each cell.

Common[7] arithmetic operations available for your use in a spreadsheet include the following:

- Addition (`+`)
- Subtraction (`-`)
- Multiplication (`*`)
- Division (`/`)
- Powers (`^`)
- Square roots (`sqrt()`)
- Logarithms (`ln()` , `log10()`)

Parentheses may be used to ensure[8] proper order of operations within a complex formula. Consider this example of a spreadsheet implementing the *quadratic formula*, used to solve for roots of a polynomial expression in the form of $ax^2 + bx + c$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

	A	B
1	x_1	= (-B4 + sqrt((B4^2) - (4*B3*B5))) / (2*B3)
2	x_2	= (-B4 - sqrt((B4^2) - (4*B3*B5))) / (2*B3)
3	a =	9
4	b =	5
5	c =	-2

This example is configured to compute roots[9] of the polynomial $9x^2 + 5x - 2$ because the values of 9, 5, and -2 have been inserted into cells `B3`, `B4`, and `B5`, respectively. Once this spreadsheet has been built, though, it may be used to calculate the roots of *any* second-degree polynomial expression simply by entering the new a, b, and c coefficients into cells `B3` through `B5`. The numerical values appearing in cells `B1` and `B2` will be automatically updated by the computer immediately following any changes made to the coefficients.

[7] Modern spreadsheet software offers a bewildering array of mathematical functions you may use in your computations. I recommend you consult the documentation for your particular spreadsheet for information on operations other than those listed here.

[8] Spreadsheet programs, like text-based programming languages, are designed to follow standard order of operations by default. However, my personal preference is to use parentheses even where strictly unnecessary just to make it clear to any other person viewing the formula what the intended order of operations is.

[9] Reviewing some algebra here, a *root* is a value for x that yields an overall value of zero for the polynomial. For this polynomial ($9x^2 + 5x - 2$) the two roots happen to be $x = 0.269381$ and $x = -0.82494$, with these values displayed in cells `B1` and `B2`, respectively upon execution of the spreadsheet.

Alternatively, one could break up the long quadratic formula into smaller pieces like this:

$$y = \sqrt{b^2 - 4ac} \qquad z = 2a$$

$$x = \frac{-b \pm y}{z}$$

	A	B	C
1	x_1	= (-B4 + C1) / C2	= sqrt((B4^2) - (4*B3*B5))
2	x_2	= (-B4 - C1) / C2	= 2*B3
3	a =	9	
4	b =	5	
5	c =	-2	

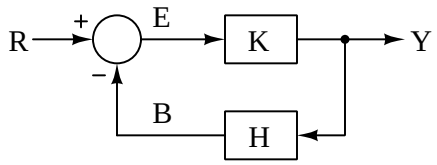
Note how the square-root term (y) is calculated in cell C1, and the denominator term (z) in cell C2. This makes the two final formulae (in cells B1 and B2) simpler to interpret. The positioning of all these cells on the grid is completely arbitrary[10] – all that matters is that they properly reference each other in the formulae.

Spreadsheets are particularly useful for situations where the same set of calculations representing a circuit or other system must be repeated for different initial conditions. The power of a spreadsheet is that it automates what would otherwise be a tedious set of calculations. One specific application of this is to simulate the effects of various components within a circuit failing with abnormal values (e.g. a shorted resistor simulated by making its value nearly zero; an open resistor simulated by making its value extremely large). Another application is analyzing the behavior of a circuit design given new components that are out of specification, and/or aging components experiencing drift over time.

[10] My personal preference is to locate all the "given" data in the upper-left cells of the spreadsheet grid (each data point flanked by a sensible name in the cell to the left and units of measurement in the cell to the right as illustrated in the first distance/time spreadsheet example), sometimes coloring them in order to clearly distinguish which cells contain entered data versus which cells contain computed results from formulae. I like to place all formulae in cells below the given data, and try to arrange them in logical order so that anyone examining my spreadsheet will be able to figure out *how* I constructed a solution. This is a general principle I believe all computer programmers should follow: *document and arrange your code to make it easy for other people to learn from it.*

6.2.3 Mathematical functions in feedback path

Suppose we have a high-gain electronic amplifier connected as shown in the following block diagram, where K is the amplifier itself and H is the feedback network:



Determine the values for E, B, and Y for the following feedback functions and reference signal (R) values, assuming K has an infinitely large positive gain:

- H = square-root function ($B = +\sqrt{Y}$); $R = 3$
- H = negative offset function ($B = Y - 4$); $R = 2$
- H = multiplying function ($B = 2 \times Y$); $R = 10$
- H = exponential function ($B = 5^Y$); $R = 2$

In general, what can we say about Y as a function of R?

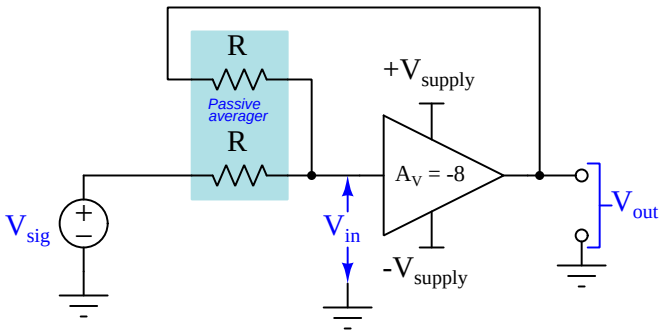
Challenges

- Modify this system to output the cube-root of R.

6.2.4 Proving gain formula with feedback

For a negative feedback amplifier circuit using a passive averager network, derive the following equation for over-all voltage gain from the fundamental equations showing the amplifier's open-loop gain and the passive averager's operation:

$$V_{out} = \frac{A_{OL}}{2 - A_{OL}} V_{sig}$$



$$V_{in} = \frac{V_{sig} + V_{out}}{2} \qquad V_{out} = A_{OL} V_{in}$$

Challenges

- Prove the equation for a passive averager network.

6.3 Diagnostic reasoning

These questions are designed to stimulate your deductive and inductive thinking, where you must apply general principles to specific scenarios (deductive) and also derive conclusions about the failed circuit from specific details (inductive). In a Socratic discussion with your instructor, the goal is for these questions to reinforce your recall and use of general circuit principles and also challenge your ability to integrate multiple symptoms into a sensible explanation of what's wrong in a circuit. Your instructor may also pose additional questions based on those assigned, in order to further challenge and sharpen your diagnostic abilities.

As always, your goal is to fully *explain* your analysis of each problem. Simply obtaining a correct answer is not good enough – you must also demonstrate sound reasoning in order to successfully complete the assignment. Your instructor's responsibility is to probe and challenge your understanding of the relevant principles and analytical processes in order to ensure you have a strong foundation upon which to build further understanding.

You will note a conspicuous lack of answers given for these diagnostic questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your diagnostic answers, where applicable, is to use circuit simulation software to explore the effects of faults placed in circuits. For example, if one of these diagnostic questions requires that you predict the effect of an open or a short in a circuit, you may check the validity of your work by simulating that same fault (substituting a very high resistance in place of that component for an open, and substituting a very low resistance for a short) within software and seeing if the results agree.

6.3.1 Loss of servo feedback

Imagine a robotic crane, designed to hoist a load to whatever height you command. This robotic crane uses negative feedback to compare a reference height signal (R) against the actual height of the hoisted load (Y), adjusting a winch accordingly to make the two equal.

Suppose the height sensor for a robotic crane were to fail, such that it always output a sensed height signal of 0 meters (ground level) regardless of the actual load height. How would this crane respond in that event?

Challenges

- Explain why the scenario as described could be very dangerous.
- How would the crane respond if the height sensor were working properly, but the cable suspending the load snapped in two?

6.3.2 Reversal of height feedback

Imagine a robotic crane, designed to hoist a load to whatever height you command. This robotic crane uses negative feedback to compare a reference height signal (R) against the actual height of the hoisted load (Y), adjusting a winch accordingly to make the two equal.

Suppose someone accidently reversed the height feedback sensor so that it registered inversely: reporting a low height when the load was high, and a tall height when the load was low. How would this affect the crane's ability to maintain a steady load position as commanded?

Challenges

- Suppose the height sensor was properly installed, but the crane's servo motor was reversed so that the load moved up when it should be moving down. Would this have a similar effect on the crane's behavior?

Appendix A

Problem-Solving Strategies

The ability to solve complex problems is arguably one of the most valuable skills one can possess, and this skill is particularly important in any science-based discipline.

- Study principles, not procedures. Don't be satisfied with merely knowing how to compute solutions – learn *why* those solutions work.
- Identify what it is you need to solve, identify all relevant data, identify all units of measurement, identify any general principles or formulae linking the given information to the solution, and then identify any "missing pieces" to a solution. Annotate all diagrams with this data.
- Sketch a diagram to help visualize the problem. When building a real system, always devise a plan for that system and analyze its function *before* constructing it.
- Follow the units of measurement and meaning of every calculation. If you are ever performing mathematical calculations as part of a problem-solving procedure, and you find yourself unable to apply each and every intermediate result to some aspect of the problem, it means you don't understand what you are doing. Properly done, every mathematical result should have practical meaning for the problem, and not just be an abstract number. You should be able to identify the proper units of measurement for each and every calculated result, and show where that result fits into the problem.
- Perform "thought experiments" to explore the effects of different conditions for theoretical problems. When troubleshooting real systems, perform *diagnostic tests* rather than visually inspecting for faults, the best diagnostic test being the one giving you the most information about the nature and/or location of the fault with the fewest steps.
- Simplify the problem until the solution becomes obvious, and then use that obvious case as a model to follow in solving the more complex version of the problem.
- Check for exceptions to see if your solution is incorrect or incomplete. A good solution will work for *all* known conditions and criteria. A good example of this is the process of testing scientific hypotheses: the task of a scientist is not to find support for a new idea, but rather to *challenge* that new idea to see if it holds up under a battery of tests. The philosophical

principle of *reductio ad absurdum* (i.e. disproving a general idea by finding a specific case where it fails) is useful here.

- Work "backward" from a hypothetical solution to a new set of given conditions.
- Add quantities to problems that are qualitative in nature, because sometimes a little math helps illuminate the scenario.
- Sketch graphs illustrating how variables relate to each other. These may be quantitative (i.e. with realistic number values) or qualitative (i.e. simply showing increases and decreases).
- Treat quantitative problems as qualitative in order to discern the relative magnitudes and/or directions of change of the relevant variables. For example, try determining what happens if a certain variable were to increase or decrease before attempting to precisely calculate quantities: how will each of the dependent variables respond, by increasing, decreasing, or remaining the same as before?
- Consider limiting cases. This works especially well for qualitative problems where you need to determine which direction a variable will change. Take the given condition and magnify that condition to an extreme degree as a way of simplifying the direction of the system's response.
- Check your work. This means regularly testing your conclusions to see if they make sense. This does *not* mean repeating the same steps originally used to obtain the conclusion(s), but rather to use some other means to check validity. Simply repeating procedures often leads to *repeating the same errors* if any were made, which is why alternative paths are better.

Appendix B

Instructional philosophy

B.1 First principles of learning

- **Anyone can learn anything** given appropriate time, effort, resources, challenges, encouragement, and expectations. Dedicating time and investing effort are the student's responsibility; providing resources, challenges, and encouragement are the teacher's responsibility; high expectations are a responsibility shared by both student and teacher.

- **Transfer is not automatic.** The human mind has a natural tendency to compartmentalize information, which means the process of taking knowledge learned in one context and applying it to another usually does not come easy and therefore should never be taken for granted.

- **Learning is iterative.** The human mind rarely learns anything perfectly on the first attempt. Anticipate mistakes and plan for multiple tries to achieve full understanding, using the lessons of those mistakes as feedback to guide future attempts.

- **Information is absorbed, but understanding is created.** Facts and procedures may be memorized easily enough by repeated exposure, but the ability to reliably apply principles to novel scenarios only comes through intense personal effort. This effort is fundamentally creative in nature: casting new concepts in one's own words, running mental experiments to perceive patterns and trends, etc. These acts of making knowledge "one's own" need not be perfect in order to be effective, as the value lies in the activity and not necessarily the finished product. Depth of understanding and permanence of memory are both directly related to mental effort during the learning process – i.e. *easy learning is shallow and fleeting.*

- **Education trumps training.** There is no such thing as an entirely isolated subject, as all fields of knowledge are connected. Training is narrowly-focused and task-oriented. Education is broad-based and principle-oriented. When preparing for a life-long technical career, education beats training every time.

- **Character matters.** Poor habits are more destructive than deficits of knowledge or skill. This is especially true in collective endeavors, where a team's ability to function depends on trust between its members. Simply put, no one wants an untrustworthy person on their team. An essential component of education then, is character development.

- **People learn to be responsible by bearing responsibility.** An irresponsible person is someone who has never *had* to be responsible for anything that mattered enough to them. Just as anyone can learn anything, anyone can become responsible if the personal cost of irresponsibility becomes high enough.

- **What gets measured, gets done.** Accurate and relevant assessment of learning is key to ensuring all students learn. Therefore, it is imperative to measure what matters.

- **Failure is nothing to fear.** Every human being fails, and fails in multiple ways at multiple times. Eventual success only happens when we don't stop trying.

B.2 Proven strategies for instructors

- Assume every student is capable of learning anything they desire given the proper conditions. Treat them as capable adults by granting real responsibility and avoiding artificial incentives such as merit or demerit points.
- Create a consistent culture of high expectations across the entire program of study. Demonstrate and encourage patience, persistence, and a healthy sense of self-skepticism. Anticipate and de-stigmatize error. Teach respect for the capabilities of others as well as respect for one's own fallibility.
- Replace lecture with "inverted" instruction, where students first encounter new concepts through reading and then spend class time in Socratic dialogue with the instructor exploring those concepts and solving problems individually. There is a world of difference between observing someone solve a problem versus actually solving a problem yourself, and so the point of this form of instruction is to place students in a position where they *cannot* passively observe.
- Require students to read extensively, write about what they learn, and dialogue with you and their peers to sharpen their understanding. Apply Francis Bacon's advice that "reading maketh a full man; conference a ready man; and writing an exact man". These are complementary activities helping students expand their confidence and abilities.
- Use artificial intelligence (AI) to challenge student understanding rather than merely provide information. Find productive ways for AI to critique students' clarity of thought and of expression, for example by employing AI as a Socratic-style interlocutor or as a reviewer of students' journals. Properly applied, AI has the ability to expand student access to critical review well outside the bounds of their instructor's reach.
- Build frequent and rapid feedback into the learning process so that students know at all times how well they are learning, to identify problems early and fix them before they grow. Model the intellectual habit of self-assessing and self-correcting your own understanding (i.e. a cognitive *feedback loop*), encouraging students to do the same.
- Use "mastery" as the standard for every assessment, which means the exam or experiment or project must be done with 100% competence in order to pass. Provide students with multiple opportunity for re-tries (different versions of the assessment every time).
- Require students to devise their own hypotheses and procedures on all experiments, so that the process is truly a scientific one. Have students assess their proposed experimental procedures for risk and devise mitigations for those risks. Let nothing be pre-designed about students' experiments other than a stated task (i.e. what principle the experiment shall test) at the start and a set of demonstrable knowledge and skill objectives at the end.
- Have students build as much of their lab equipment as possible: building power sources, building test assemblies[1], and building complete working systems (no kits!). In order to provide

[1] In the program I teach, every student builds their own "Development Board" consisting of a metal chassis with DIN rail, terminal blocks, and an AC-DC power supply of their own making which functions as a portable lab environment they can use at school as well as take home.

this same "ground-up" experience for every new student, this means either previous students take their creations with them, or the systems get disassembled in preparation for the new students, or the systems grow and evolve with each new student group.

- Incorporate external accountability for you and for your students, continuously improving the curriculum and your instructional methods based on proven results. Have students regularly network with active professionals through participation in advisory committee meetings, service projects, tours, jobshadows, internships, etc. Practical suggestions include requiring students to design and build projects for external clients (e.g. community groups, businesses, different departments within the institution), and also requiring students attend all technical advisory committee meetings and to dialogue with the industry representatives at those meetings.
- Repeatedly explore difficult-to-learn concepts across multiple courses, so that students have multiple opportunities to build their understanding.
- Relate all new concepts, whenever possible, to previous concepts and to relevant physical laws. Challenge each and every student, every day, to *reason* from concept to concept and to explain the logical connections between. Challenge students to verify their conclusions by multiple approaches (e.g. double-checking their work using different methods). Ask *"Why?"* often.
- Maintain detailed records on each student's performance and share these records privately with them. These records should include academic performance as well as professionally relevant behavioral tendencies.
- Hold mandatory "check-in" meetings between all program faculty and each new student during their first term. Offer these to all other students as an option, except for any students continuing to manifest unprofessional behaviors, poor academic performance, or who have some other need for a face-to-face meeting with faculty.
- Address problems while they are small, before they grow larger. This is equally true for tutoring technical concepts as it is for helping students build professional habits.
- Build rigorous quality control into the curriculum to ensure every student masters every important concept, and that the mastery is retained over time. This includes (1) review questions added to every exam to re-assess knowledge taught in previous terms, (2) cumulative exams at the end of every term to re-assess all important concepts back to the very beginning of the program, and (3) review assessments in practical (hands-on) coursework to ensure critically-important skills were indeed taught and are still retained. What you will find by doing this is that it actually boosts retention of students by ensuring that important knowledge gets taught and is retained over long spans of time. In the absence of such quality control, student learning and retention tends to be spotty and this contributes to drop-out and failure rates later in their education.
- Finally, *never rush learning.* Education is not a race. Give your students ample time to digest complex ideas, as you continually remind yourself of just how long it took you to achieve mastery! Long-term retention and the consistently correct application of concepts are always the result of *focused effort over long periods of time* which means there are no shortcuts to learning.

B.3 Proven strategies for students

The single most important piece of advice I have for any student of any subject is to take responsibility for your own development in all areas of life including mental development. Expecting others in your life to entirely guide your own development is a recipe for disappointment. This is just as true for students enrolled in formal learning institutions as it is for auto-didacts pursuing learning entirely on their own. Learning to think in new ways is key to being able to gainfully use information, to make informed decisions about your life, and to best serve those you care about. With this in mind, I offer the following advice to students:

- **Approach all learning as valuable**. No matter what course you take, no matter who you learn from, no matter the subject, there is something useful in every learning experience. If you don't see the value of every new experience, you are not looking closely enough!
- **Continually challenge yourself**. Let other people take shortcuts and find easy answers to easy problems. The purpose of education is to stretch your mind, in order to shape it into a more powerful tool. This doesn't come by taking the path of least resistance. An excellent analogy for an empowering education is productive physical exercise: becoming stronger, more flexible, and more persistent only comes through intense personal effort.
- **Master the use of language**. This includes reading extensively, writing every day, listening closely, and speaking articulately. To a great extent language channels and empowers thought, so the better you are at wielding language the better you will be at grasping abstract concepts and articulating them not only for your benefit but for others as well.
- **Do not limit yourself to the resources given to you**. Read books that are not on the reading list. Run experiments that aren't assigned to you. Form study groups outside of class. Take an entrepreneurial approach to your own education, as though it were a business you were building for your future benefit.
- **Express and share what you learn**. Take every opportunity to teach what you have learned to others, as this will not only help them but will also strengthen your own understanding[2].
- Realize that **no one can give you understanding**, just as no one can give you physical fitness. These both must be *built*.
- **Above all, recognize that learning is hard work, and that a certain level of frustration is unavoidable**. There are times when you will struggle to grasp some of these concepts, and that struggle is a natural thing. Take heart that it will yield with persistent and varied[3] effort, and never give up! That concepts don't immediately come to you is not a sign of something wrong, but rather of something right: that you have found a worthy challenge!

[2] On a personal note, I was surprised to learn just how much my own understanding of electronics and related subjects was strengthened by becoming a teacher. When you are tasked every day with helping other people grasp complex topics, it catalyzes your own learning by giving you powerful incentives to study, to articulate your thoughts, and to reflect deeply on the process of learning.

[3] As the old saying goes, "Insanity is trying the same thing over and over again, expecting different results." If you find yourself stumped by something in the text, you should attempt a different approach. Alter the thought experiment, change the mathematical parameters, do whatever you can to see the problem in a slightly different light, and then the solution will often present itself more readily.

B.4 Design of these learning modules

"The unexamined circuit is not worth energizing" – Socrates (if he had taught electricity)

These learning modules, although useful for self-study, were designed to be used in a formal learning environment where a subject-matter expert challenges students to digest the content and exercise their critical thinking abilities in the answering of questions and in the construction and testing of working circuits. Every effort has been made to embed the following instructional and assessment philosophies within:

- The first goal of education is to enhance clear and independent thought, in order that every student reach their fullest potential in a highly complex and inter-dependent world. Robust reasoning is *always* more important than particulars of any subject matter, because its application is universal.
- Literacy is fundamental to independent learning and thought because text continues to be the most efficient way to communicate complex ideas over space and time. Those who cannot read with ease are limited in their ability to acquire knowledge and perspective.
- Articulate communication is fundamental to work that is complex and interdisciplinary.
- Faulty assumptions and poor reasoning are best corrected through challenge, not presentation. The rhetorical technique of *reductio ad absurdum* (disproving an assertion by exposing an absurdity) works well to discipline student's minds, not only to correct the problem at hand but also to learn how to detect and correct future errors.
- Important principles should be repeatedly explored and widely applied throughout a course of study, not only to reinforce their importance and help ensure their mastery, but also to showcase the interconnectedness and utility of knowledge.

These learning modules were expressly designed to be used in an "inverted" teaching environment[4] where students first read the introductory and tutorial chapters on their own, then individually attempt to answer the questions and construct working circuits according to the experiment and project guidelines. The instructor never lectures, but instead meets regularly with each individual student to review their progress, answer questions, identify misconceptions, and challenge the student to new depths of understanding through further questioning. Regular meetings between instructor and student should resemble a Socratic[5] dialogue, where questions serve as scalpels to dissect topics and expose assumptions. The student passes each module only after consistently demonstrating their ability to logically analyze and correctly apply all major concepts in each question or project/experiment. The instructor must be vigilant in probing each student's understanding to ensure they are truly *reasoning* and not just *memorizing*. This is why "Challenge" points appear throughout, as prompts for students to think deeper about topics and as starting points for instructor queries. Sometimes these challenge points require additional knowledge that hasn't been covered in the series to answer in full. This is okay, as the major purpose of the Challenges is to stimulate analysis and synthesis on the part of each student.

The instructor must possess enough mastery of the subject matter and awareness of students' reasoning to generate their own follow-up questions to practically any student response. Even completely correct answers given by the student should be challenged by the instructor for the purpose of having students practice articulating their thoughts and defending their reasoning. Conceptual errors committed by the student should be exposed and corrected not by direct instruction, but rather by reducing the errors to an absurdity[6] through well-chosen questions and thought experiments posed by the instructor. Becoming proficient at this style of instruction requires time and dedication, but the positive effects on critical thinking for both student and instructor are spectacular.

An inspection of these learning modules reveals certain unique characteristics. One of these is a bias toward thorough explanations in the tutorial chapters. Without a live instructor to explain concepts and applications to students, the text itself must fulfill this role. This philosophy results in lengthier explanations than what you might typically find in a textbook, each step of the reasoning process fully explained, including footnotes addressing common questions and concerns students raise while learning these concepts. Each tutorial seeks to not only explain each major concept in sufficient detail, but also to explain the logic of each concept and how each may be developed

[4] In a traditional teaching environment, students first encounter new information via *lecture* from an expert, and then independently apply that information via *homework*. In an "inverted" course of study, students first encounter new information via *homework*, and then independently apply that information under the scrutiny of an expert. The expert's role in lecture is to simply *explain*, but the expert's role in an inverted session is to *challenge*, *critique*, and if necessary *explain* where gaps in understanding still exist.

[5] Socrates is a figure in ancient Greek philosophy famous for his unflinching style of questioning. Although he authored no texts, he appears as a character in Plato's many writings. The essence of Socratic philosophy is to leave no question unexamined and no point of view unchallenged. While purists may argue a topic such as electric circuits is too narrow for a true Socratic-style dialogue, I would argue that the essential thought processes involved with scientific reasoning on *any* topic are not far removed from the Socratic ideal, and that students of electricity and electronics would do very well to challenge assumptions, pose thought experiments, identify fallacies, and otherwise employ the arsenal of critical thinking skills modeled by Socrates.

[6] This rhetorical technique is known by the Latin phrase *reductio ad absurdum*. The concept is to expose errors by counter-example, since only one solid counter-example is necessary to disprove a universal claim. As an example of this, consider the common misconception among beginning students of electricity that voltage cannot exist without current. One way to apply *reductio ad absurdum* to this statement is to ask how much current passes through a fully-charged battery connected to nothing (i.e. a clear example of voltage existing without current).

from "first principles". Again, this reflects the goal of developing clear and independent thought in students' minds, by showing how clear and logical thought was used to forge each concept. Students benefit from witnessing a model of clear thinking in action, and these tutorials strive to be just that.

Another feature of these learning modules is that they do not treat topics in isolation. Rather, important concepts are introduced early in the series, and appear repeatedly as stepping-stones toward other concepts in subsequent modules. This helps to avoid the "compartmentalization" of knowledge, demonstrating the inter-connectedness of concepts and simultaneously reinforcing them. Each module is fairly complete in itself, reserving the beginning of its tutorial to a review of foundational concepts.

To high standards of education,

Tony R. Kuphaldt

Appendix C

Tools used

I am indebted to the developers of many open-source software applications in the creation of these learning modules. The following is a list of these applications with some commentary on each.

You will notice a theme common to many of these applications: a bias toward *code*. Although I am by no means an expert programmer in any computer language, I understand and appreciate the flexibility offered by code-based applications where the user (you) enters commands into a plain ASCII text file, which the software then reads and processes to create the final output. Code-based computer applications are by their very nature *extensible*, while WYSIWYG (What You See Is What You Get) applications are generally limited to whatever user interface the developer makes for you.

The `GNU/Linux` computer operating system

> There is so much to be said about Linus Torvalds' `Linux` and Richard Stallman's `GNU` project. First, to credit just these two individuals is to fail to do justice to the *mob* of passionate volunteers who contributed to make this amazing software a reality. I first learned of `Linux` back in 1996, and have been using this operating system on my personal computers almost exclusively since then. It is *free*, it is completely *configurable*, and it permits the continued use of highly efficient `Unix` applications and scripting languages (e.g. shell scripts, Makefiles, `sed`, `awk`) developed over many decades. `Linux` not only provided me with a powerful computing platform, but its open design served to inspire my life's work of creating open-source educational resources.

Bram Moolenaar's `Vim` text editor

> Writing code for any code-based computer application requires a *text editor*, which may be thought of as a word processor strictly limited to outputting plain-ASCII text files. Many good text editors exist, and one's choice of text editor seems to be a deeply personal matter within the programming world. I prefer `Vim` because it operates very similarly to `vi` which is ubiquitous on `Unix`/`Linux` operating systems, and because it may be entirely operated via keyboard (i.e. no mouse required) which makes it fast to use.

Donald Knuth's TEX typesetting system

Developed in the late 1970's and early 1980's by computer scientist extraordinaire Donald Knuth to typeset his multi-volume magnum opus *The Art of Computer Programming*, this software allows the production of formatted text for screen-viewing or paper printing, all by writing plain-text code to describe how the formatted text is supposed to appear. TEX is not just a markup language for documents, but it is also a Turing-complete programming language in and of itself, allowing useful algorithms to be created to control the production of documents. Simply put, TEX *is a programmer's approach to word processing*. Since TEX is controlled by code written in a plain-text file, this means anyone may read that plain-text file to see exactly how the document was created. This openness afforded by the code-based nature of TEX makes it relatively easy to learn how other people have created their own TEX documents. By contrast, examining a beautiful document created in a conventional WYSIWYG word processor such as Microsoft `Word` suggests nothing to the reader about *how* that document was created, or what the user might do to create something similar. As Mr. Knuth himself once quipped, conventional word processing applications should be called WYSIAYG (What You See Is *All* You Get).

Leslie Lamport's LATEX extensions to TEX

Like all true programming languages, TEX is inherently extensible. So, years after the release of TEX to the public, Leslie Lamport decided to create a massive extension allowing easier compilation of book-length documents. The result was LATEX, which is the markup language used to create all ModEL module documents. You could say that TEX is to LATEX as `C` is to `C++`. This means it is permissible to use any and all TEX commands within LATEX source code, and it all still works. Some of the features offered by LATEX that would be challenging to implement in TEX include automatic index and table-of-content creation.

Tim Edwards' `Xcircuit` drafting program

This wonderful program is what I use to create all the schematic diagrams and illustrations (but not photographic images or mathematical plots) throughout the ModEL project. It natively outputs PostScript format which is a true vector graphic format (this is why the images do not pixellate when you zoom in for a closer view), and it is so simple to use that I have never had to read the manual! Object libraries are easy to create for `Xcircuit`, being plain-text files using PostScript programming conventions. Over the years I have collected a large set of object libraries useful for drawing electrical and electronic schematics, pictorial diagrams, and other technical illustrations.

`Gimp` graphic image manipulation program

Essentially an open-source clone of Adobe's `PhotoShop`, I use `Gimp` to resize, crop, and convert file formats for all of the photographic images appearing in the ModEL modules. Although `Gimp` does offer its own scripting language (called `Script-Fu`), I have never had occasion to use it. Thus, my utilization of `Gimp` to merely crop, resize, and convert graphic images is akin to using a sword to slice bread.

`SPICE` circuit simulation program

`SPICE` is to circuit analysis as TEX is to document creation: it is a form of markup language designed to describe a certain object to be processed in plain-ASCII text. When the plain-text "source file" is compiled by the software, it outputs the final result. More modern circuit analysis tools certainly exist, but I prefer `SPICE` for the following reasons: it is *free*, it is *fast*, it is *reliable*, and it is a fantastic tool for *teaching* students of electricity and electronics how to write simple code. I happen to use rather old versions of `SPICE`, version 2g6 being my "go to" application when I only require text-based output. `NGSPICE` (version 26), which is based on Berkeley `SPICE` version 3f5, is used when I require graphical output for such things as time-domain waveforms and Bode plots. In all `SPICE` example netlists I strive to use coding conventions compatible with all `SPICE` versions.

Andrew D. Hwang's `ePiX` mathematical visualization programming library

This amazing project is a `C++` library you may link to any `C/C++` code for the purpose of generating PostScript graphic images of mathematical functions. As a completely free and open-source project, it does all the plotting I would otherwise use a Computer Algebra System (CAS) such as `Mathematica` or `Maple` to do. It should be said that `ePiX` is *not* a Computer Algebra System like `Mathematica` or `Maple`, but merely a mathematical *visualization* tool. In other words, it won't determine integrals for you (you'll have to implement that in your own `C/C++` code!), but it can graph the results, and it does so beautifully. What I really admire about `ePiX` is that it is a `C++` programming library, which means it builds on the existing power and toolset available with that programming language. Mr. Hwang could have probably developed his own stand-alone application for mathematical plotting, but by creating a `C++` library to do the same thing he accomplished something much greater.

`gnuplot` mathematical visualization software

Another open-source tool for mathematical visualization is `gnuplot`. Interestingly, this tool is *not* part of Richard Stallman's GNU project, its name being a coincidence. For this reason the authors prefer "gnu" *not* be capitalized at all to avoid confusion. This is a much "lighter-weight" alternative to a spreadsheet for plotting tabular data, and the fact that it easily outputs directly to an X11 console or a file in a number of different graphical formats (including PostScript) is very helpful. I typically set my `gnuplot` output format to default (X11 on my Linux PC) for quick viewing while I'm developing a visualization, then switch to PostScript file export once the visual is ready to include in the document(s) I'm writing. As with my use of `Gimp` to do rudimentary image editing, my use of `gnuplot` only scratches the surface of its capabilities, but the important points are that it's *free* and that it *works well*.

`Python` programming language

Both Python and C++ find extensive use in these modules as instructional aids and exercises, but I'm listing Python here as a *tool* for myself because I use it almost daily as a *calculator*. If you open a Python interpreter console and type `from math import *` you can type mathematical expressions and have it return results just as you would on a hand calculator. Complex-number (i.e. *phasor*) arithmetic is similarly supported if you include the complex-math library (`from cmath import *`). Examples of this are shown in the Programming References chapter (if included) in each module. Of course, being a fully-featured programming language, Python also supports conditionals, loops, and other structures useful for calculation of quantities. Also, running in a console environment where all entries and returned values show as text in a chronologically-ordered list makes it easy to copy-and-paste those calculations to document exactly how they were performed.

Appendix D

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Appendix E

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Appendix F

Version history

This is a list showing all significant additions, corrections, and other edits made to this learning module. Each entry is referenced by calendar date in reverse chronological order (newest version first), which appears on the front cover of every learning module for easy reference. Any contributors to this open-source document are listed here as well.

22 October 2025 – minor edits to the Tutorial.

19 March 2025 – edited some instructor comments.

3 March 2025 – corrected a grammatical error ("we will may").

16 October 2024 – divided the Introduction chapter into sections, one with recommendations for students, one with a listing of challenging concepts, and one with recommendations for instructors.

30 October 2023 – minor edits to the Tutorial, and new questions added to the Introduction chapter.

28 November 2022 – placed questions at the top of the itemized list in the Introduction chapter prompting students to devise experiments related to the tutorial content.

24 March 2022 – significantly edited the Introduction chapter to make it more suitable as a pre-study guide and to provide cues useful to instructors leading "inverted" teaching sessions. Also made minor edits to the Tutorial.

11 December 2021 – added a Case Tutorial chapter with a section showing a three-stage BJT amplifier using negative feedback for precise gain.

6 September 2021 – divided Tutorial into sections, and also added explanatory content to clarify certain hard-to-grasp concepts.

10 May 2021 – commented out or deleted empty chapters.

3-5 May 2020 – minor edits to Tutorial. Also added Historical Reference on George Philbrick's

work for the US military on automatic gun tracking systems, and more questions.

15 February 2020 – added some problems involving single-transistor amplifier circuits.

27 January 2020 – added Foundational Concepts to the list in the Conceptual Reasoning section.

27 December 2019 – added a Diagnostic Reasoning question.

18 December 2019 – added a Conceptual Reasoning question.

17 December 2019 – added Technical Reference on the topic of decibels.

1 October 2019 – commented out the Derivations chapter because there's no content for it (yet).

17 June 2019 – corrected a typographical error, where I used an example of a thermostatically controlled room, then later referred to that example as being an *oven*.

9 June 2019 – added more Tutorial content, as well as some questions.

5 June 2019 – added Historical Reference to Harold Black's patent on negative feedback.

2 June 2019 – document first created.

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