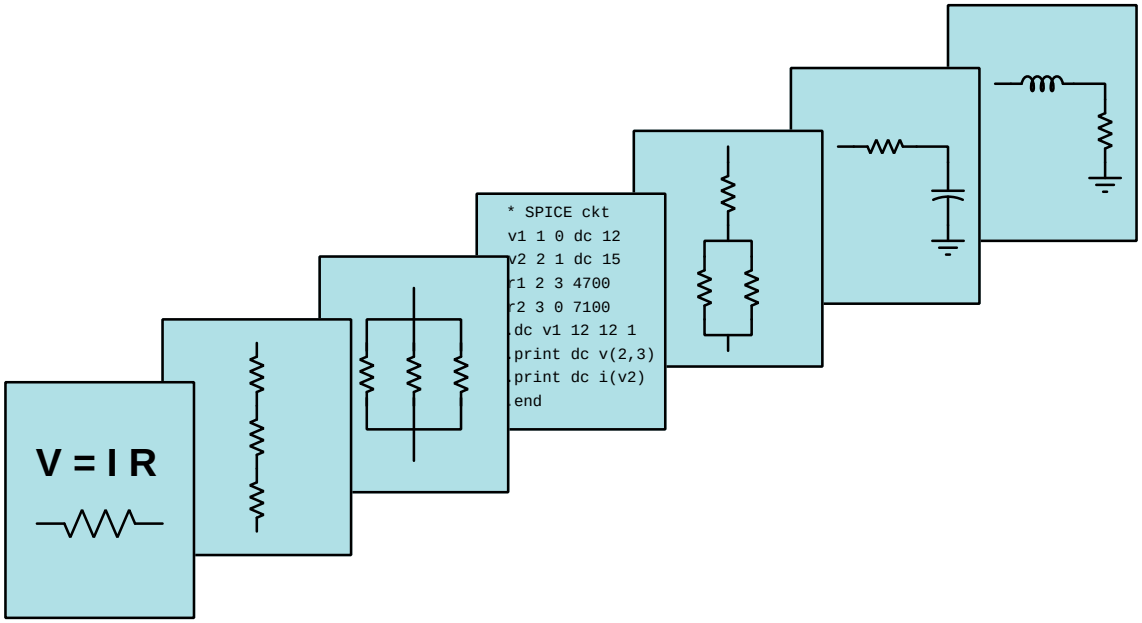
Modular Electronics Learning (ModEL) project



Ideal and Real Sources

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Chapter 1

Introduction

1.1 Recommendations for students

Idealized voltage sources maintain absolutely constant terminal voltage under all load conditions, and idealized current sources maintain absolutely constant terminal current under all load conditions, but real sources fall short of these ideals. This learning module explores to limitations of real sources, provides modeling techniques to approximate real sources by a collection of idealized components, and demonstrates the functional equivalence of non-ideal voltage and current sources.

An important analytical tool introduced with the concept of ideal versus real sources is that of *modeling* one circuit in terms of another circuit. We will apply this tool in later studies of Thévenin's and Norton's theorems.

Important concepts related to ideal and real sources include **Ohm's Law**, **internal resistance**, effects of **opens** versus **shorts**, **voltage sources** versus **current sources**, properties of **series** networks, properties of **parallel** networks, **sources** versus **loads**, voltage and current **sag**, **stiffness** of an electrical source, **equivalent networks**, and **light** versus **heavy** loads.

Here are some good questions to ask of yourself while studying this subject:

- How might an experiment be designed and conducted to measure the internal resistance of an electrical source? What hypothesis (i.e. prediction) might you pose for that experiment, and what result(s) would either support or disprove that hypothesis?
- How might an experiment be designed and conducted to test whether or not two different electrical sources were equivalent to one another? What hypothesis (i.e. prediction) might you pose for that experiment, and what result(s) would either support or disprove that hypothesis?
- In what ways do voltage sources differ from current sources?
- Why do electrical sources provide different values of voltage/current when loaded versus when unloaded?
- What are some physical origins of internal source resistance?

- What does it mean to *model* some real-life component characteristic in terms of ideal component behaviors?
- What characteristics of a battery change as it ages?
- What makes a load either "heavy" or "light" from the perspective of the source supplying energy to it?
- What makes a source "stiffer" or "stronger" than another?
- What does it mean if two networks are *electrically equivalent* to one another?
- How may a voltage source be used as an approximation of a current source?
- What voltage and internal resistance values make a voltage source more closely approximate a true current source?

Some recommended strategies for active reading include:

- Run the **thought experiment** suggested in the "Approximating ideal sources" section.
- **Limiting cases** – consider the behavior of sources given "limiting case" resistance values of *opens* versus *shorts*.
- **Annotate diagrams** – you will find your understanding of ideal and real sources enhanced if you annotate each and every schematic diagram with all voltages, currents, polarities, and directions.
- **Work "backward" from solution to given condition(s)** – a good application of this strategy is to take a realistic source circuit where you know all the voltages and currents, and then pretend you no longer know that source's internal resistance. Figure out how you could work *backward* from parameters measurable at the source's external terminals to determine the source's internal resistance.
- **Experiment using simulations** – setting up SPICE (or other) computer simulations of these circuits is another way to actively engage with the text's presentation of this subject.

1.2 Challenging concepts related to ideal versus real electrical sources

The following list cites concepts related to this module's topic that are easily misunderstood, along with suggestions for properly understanding them:

- **Equivalent electrical networks** – the possibility that completely different networks of components may nevertheless *behave* identically from the perspective of two or more connection terminals is non-intuitive for many students new to electric circuits. However, there are many analogues in life that may be used to help explain. For example, a box containing an unknown assortment of items may have precisely the same weight as a different box containing something completely different from the first: i.e. from the perspective of weight sense at the handle when lifting each box, the boxes may behave the same even though their contents may be wholly different.

- **Schematic annotation as a problem-solving tool** – as an instructor you will find most students new to the study of electronics attempt to solve circuit-analysis problems by inspection, or by plugging given values into familiar equations until something resembling an answer emerges. Such efforts are usually fruitless because it is difficult to track the real-world context of every circuit parameter doing things this way. You will need to demonstrate, encourage, and at times *require* your students to apply the alternative strategy of first sketching a schematic diagram of the circuit and then annotating it with all known values. Once the given information gains context through this annotation, appropriate principles (e.g. Ohm's Law, KVL, KCL) are much easier to identify and then properly apply for that circuit. Then, as calculated values get annotated on the same schematic diagram the next logical steps to follow become easier and easier to identify. I recommend using different colors to label different parameters (e.g. blue for voltage, red for current, black for component values, green for power). Students should have blank "scratch" paper and writing instruments with them at all times, even during discussions, so as to neatly organize information and to articulate and focus their thoughts!

The *Case Tutorial* chapter contains multiple examples showing how to infer internal source resistance values from external measurements under loaded versus unloaded conditions.

1.3 Recommendations for instructors

This section lists realistic student learning outcomes supported by the content of the module as well as suggested means of assessing (measuring) student learning. The outcomes state what learners should be able to do, and the assessments are specific challenges to prove students have learned.

- **Outcome** – Demonstrate effective technical reading and writing

  Assessment – Students present their outlines of this module's instructional chapters (e.g. Case Tutorial, Tutorial, Historical References, etc.) ideally as an entry to a larger Journal document chronicling their learning. These outlines should exhibit good-faith effort at summarizing major concepts explained in the text.

  Assessment – Students show how quantitative results were obtained by the author in the Tutorial chapter's examples.

  Assessment – Students present their "Conceptual Toolbox" Theory Journal entries listing important concepts applied in the Tutorial text(s). A "Conceptual Toolbox" serves as inspiration when solving problems, each concept suggesting a potential problem-solving action. Challenge students to collaboratively generate a Conceptual Toolbox for the day's topic during the class session and then use it when solving problems together, thus fostering the habit of reasoning from principles rather than recalling procedures!

- **Outcome** – Apply the concept of ideal sources to hypothetical loads

  Assessment – Calculate circuit conditions given an ideal voltage or current source with varying load resistance; e.g. pose problems in the form of the "Ideal current source behavior" Quantitative Reasoning question.

- **Outcome** – Model a real source using idealized components

  Assessment – Determine ideal source and resistance values for a real electrical source given loaded and unloaded measurements; e.g. pose problems in the form of the "Testing a power supply" Diagnostic Reasoning question.

Chapter 2

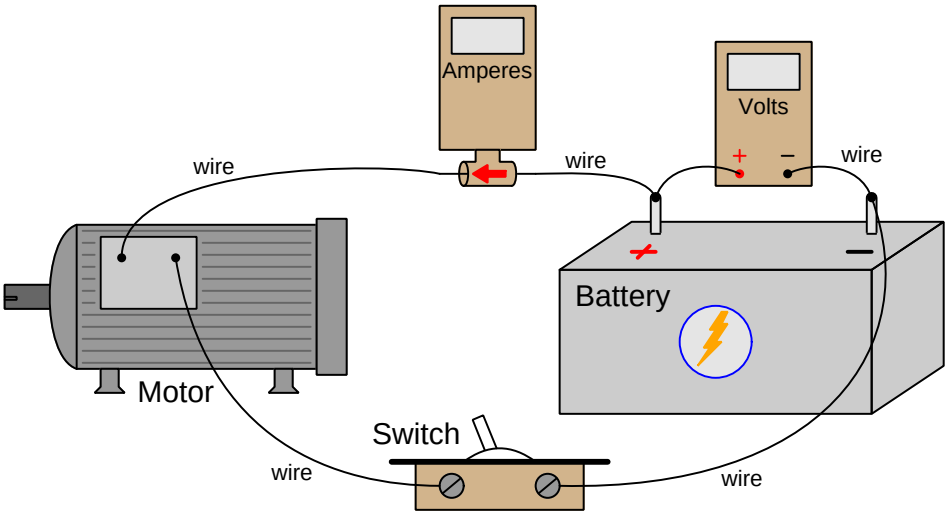
Case Tutorial

The idea behind a *Case Tutorial* is to explore new concepts by way of example. In this chapter you will read less presentation of theory compared to other Tutorial chapters, but by close observation and comparison of the given examples be able to discern patterns and principles much the same way as a scientific experimenter. Hopefully you will find these cases illuminating, and a good supplement to text-based tutorials.

These examples also serve well as challenges following your reading of the other Tutorial(s) in this module – can you explain *why* the circuits behave as they do?

2.1 Example: battery voltage under load

Electrochemical batteries, like all voltage sources, have some internal (series) resistance. This resistance causes the battery's terminal voltage to "sag" when placed under load, rather than hold to a constant voltage as we would expect for an ideal voltage source. In the following circuit, the battery experiences a "heavier" load (i.e. greater current) when the motor works harder against a "heavier" mechanical load (i.e. greater torque):



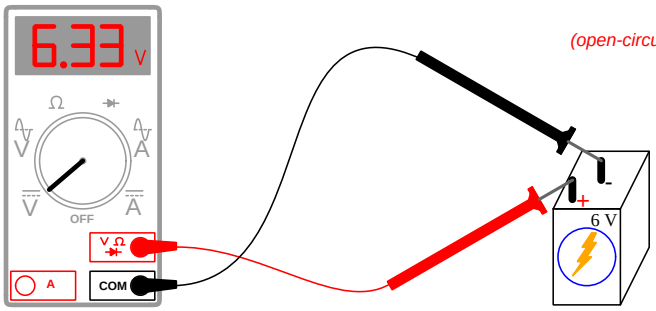
The following meter measurements are consistent with a internal battery resistance of 0.035 Ω:

Switch	Ammeter	Voltmeter
Off	0 Amperes	13.50 Volts
On	5 Amperes	13.33 Volts
On	10 Amperes	13.15 Volts
On	20 Amperes	12.80 Volts
On	30 Amperes	12.45 Volts
On	50 Amperes	11.75 Volts

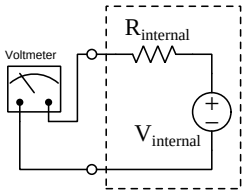
2.2 Example: dying dry-cell battery under load

The following voltage measurements were taken from a "6 Volt" dry-cell battery under different load conditions:

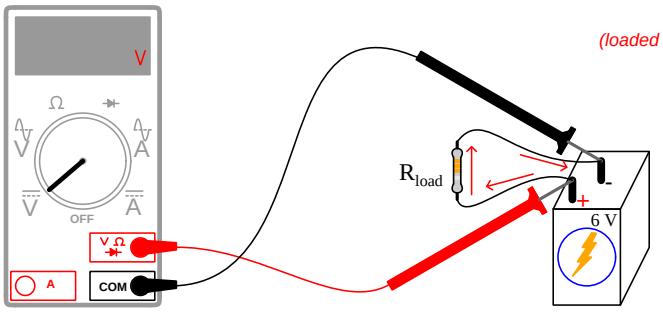
Pictorial diagram



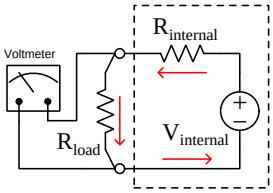
Schematic diagram



Pictorial diagram



Schematic diagram



Load resistance	Measured voltage
(open)	6.33 Volts
48.1 kΩ	5.99 Volts
15 kΩ	5.33 Volts
6.8 kΩ	4.48 Volts

If we assume the voltmeter draws negligible current when connected to the battery's terminals (i.e. the meter has nearly-infinite insertion resistance), we may conclude that the amount of voltage it registers with no load resistor connected to the battery will be the battery's true internal voltage. Therefore, $V_{internal}$ = 6.33 Volts. For each attached load resistance, current in the circuit will be the voltmeter's indicated voltage divided by that load resistance ($I = \frac{V_{meter}}{R_{load}}$). The battery's internal resistance is in series with the load, which means it must experience that same amount of current. Taking the difference between $V_{internal}$ (6.33 Volts) and the meter's measured voltage with connected load tells us how much voltage is being dropped by that internal battery resistance, and that voltage difference divided by the calculated current gives us the internal resistance value in Ohms ($R_{internal}$).

Expanding our table of load resistance and measured voltage values to include these calculated $R_{internal}$ values, we get the following:

Load resistance	**Measured voltage**	**Calculated internal resistance**
(open)	6.33 Volts	– – – –
48.1 kΩ	5.99 Volts	2730.2 Ohms
15 kΩ	5.33 Volts	2814.3 Ohms
6.8 kΩ	4.48 Volts	2808.0 Ohms

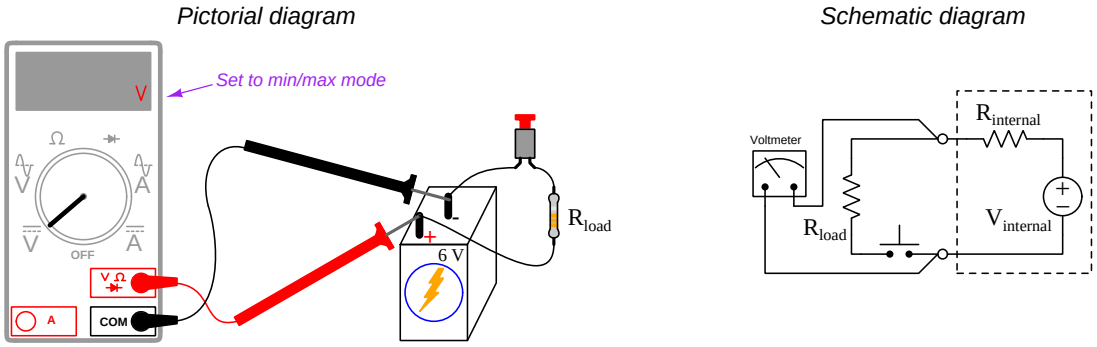
Averaging these three calculated $R_{internal}$ values yields a figure of 2784.2 Ohms, which is likely more accurate than any of the three individual calculated values. Now that we know this value, we may predict how much current we will see from this battery if we short-circuit it:

$$I_{short} = \frac{V_{internal}}{R_{internal}} = 2.2736 \text{ milliAmperes}$$

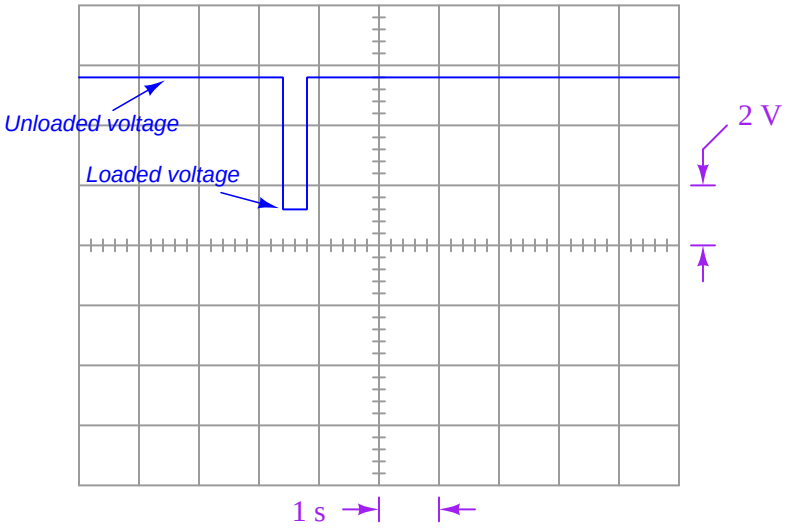
Sure enough, an actual test performed on this battery using a digital ammeter registered approximately 2.28 mA of current with just the ammeter attached. Please note that usually it is very dangerous practice to connect an ammeter directly to any voltage source because ammeters generally have very low insertion resistance. However, *in this case only* we know it is safe because the high internal resistance of this dying battery naturally limits the short-circuit current to a value well within the safe measurement range of the meter.

Also note that any "loaded" battery tests should be done for *short durations only* to minimize energy depletion. If a resistor (or a short!) is left connected to a battery for significant lengths of time, the battery will lose some of its internal store of energy. This energy loss affects $V_{internal}$ which our calculations assume is constant. If $V_{internal}$ is not actually constant over time as assumed, then our internal resistance calculations will be inaccurate.

A practical test strategy to minimize energy depletion while load-testing a battery is to use a multimeter with minimum/maximum recording or datalogging capability, and then to connect loads only for *brief* intervals. Similarly, one could use a digital oscilloscope set to a slow sweep rate (i.e. timebase set to whole seconds per division) as a graphical voltage recorder, noting how low the battery's terminal voltage dips when the load resistor is momentarily connected to the battery's terminals. A momentary pushbutton switch in series with the load resistor makes brief connection easy to perform, pressing the button only as long as required for the measuring instrument to record the loading effect at the battery's terminals:



If an oscilloscope is used instead of a recording voltmeter, the trace left on the oscilloscope's screen after performing the pushbutton test will look something like what is shown below:

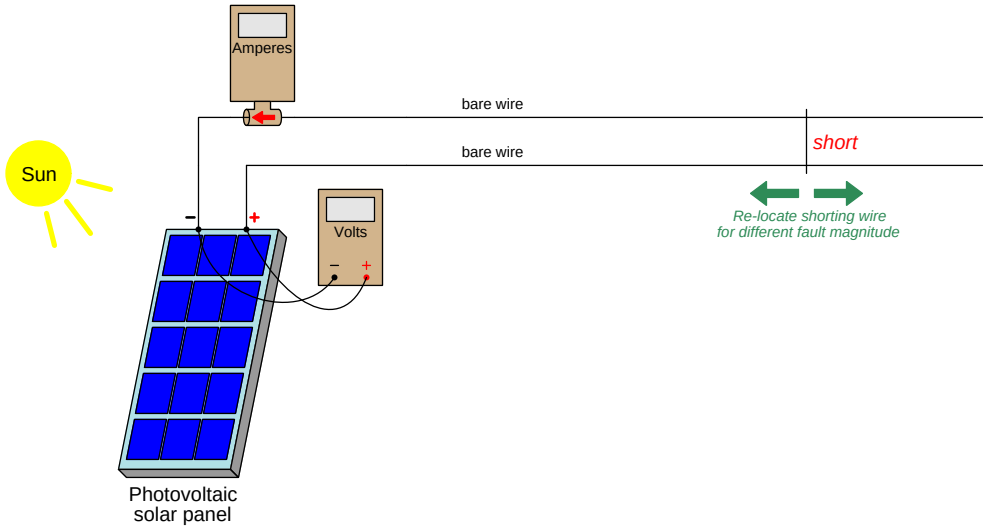


In this particular example with the oscilloscope's vertical sensitivity set to 2 Volts per division and its horizontal timebase set to 1 second per division, we see[1] the battery's unloaded voltage at 5.6 Volts but sagging to 1.2 Volts during the 0.4 seconds the pushbutton was pressed.

[1] If you are unfamiliar with interpreting graphed results on an oscilloscope display, know that each "square" on

2.3 Example: shorted solar panel

A photovoltaic "solar panel" behaves as a current source when exposed to constant sunlight intensity while being short-circuited (i.e. connected to a load of extremely small resistance value). Like all real current sources, though, a solar panel contains internal (parallel) resistance which causes its output voltage and current to vary somewhat as load conditions change. In the following circuit, two bare-metal conductors normally conveying electrical energy from the solar panel to a load are instead short-circuited by a "shorting wire" at different locations, the fault's distance from the panel causing different amount of "loop" resistances and therefore different degrees of short-circuiting:



The following meter measurements are consistent with an internal panel resistance of 0.37 Ω:

Shorting wire location	Ammeter	Voltmeter
Far	9 Amperes	0.574 Volts
	9.2 Amperes	0.500 Volts
	9.4 Amperes	0.426 Volts
	9.6 Amperes	0.352 Volts
	9.8 Amperes	0.277 Volts
Near	10 Amperes	0.204 Volts

the screen's grid is one *major division*, and since the smaller *minor divisions* subdivide each major division into five smaller pieces we may consider their value to be $\frac{1}{5}$ of a division each. So, if the vertical sensitivity is 2 Volts per (major) division, then each of the vertical minor divisions represents 0.4 Volts because one-fifth of 2 is 0.4. Similarly, if the horizontal timebase setting is 1 second per (major) division, then each of the horizontal minor divisions represents 0.2 seconds because one-fifth of 1 is 0.2.

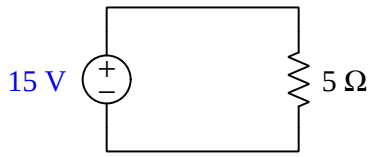
Note how for every 0.2 Amperes of current demanded from the solar panel its terminal voltage sags by another 0.074 Volts. This means the panel's equivalent internal resistance value must drop that addition 0.074 Volts for each additional 0.2 Amperes of current. The ratio of that incremental voltage sag to the corresponding current increase is the panel's equivalent internal resistance value, $\frac{0.074 \text{ V}}{0.2 \text{ A}} = 0.37$ Ohms.

Chapter 3

Tutorial

3.1 Source resistance

Consider the following circuit, consisting of nothing but a DC voltage source and a resistor:



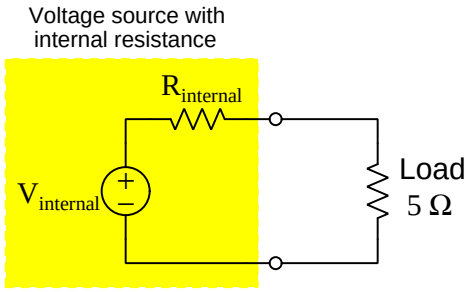
The amount of current flowing in this simple circuit is a direct function of voltage (15 Volts) and resistance (5 Ohms) as predicted by Ohm's Law:

$$I = \frac{V}{R} = \frac{15\ \text{V}}{5\ \Omega} = 3\ \text{A}$$

Assuming a source voltage of precisely 15 Volts and a resistor having precisely 5 Ohms of resistance, and no resistance elsewhere in the circuit, the current should be precisely 3 Amperes. Altering the resistor's value will result in different amounts of current as predicted by Ohm's Law. For example, replacing the 5 Ω resistor with a 1 Ω resistor should result in a current of 15 Amperes, assuming the DC source still outputs 15 Volts.

If we were to replace the 5 Ω resistor with a 1 Ω resistor in a *real* circuit, however, we would likely find the new current value to be something less than 15 Amperes. This is not a failure of Ohm's Law, but rather the result of assuming ideal behavior from non-ideal components. The voltage source in this circuit may be rated at 15 Volts, and it may happen to output precisely 15 Volts at 3 Amperes of load current, but it will likely not output precisely 15 Volts at other load current values. In other words, a real 15 Volt DC source may not be perfectly *stable* under changing load conditions.

The chief cause of instability for electrical sources is *internal resistance*, which we may model as a series-connected resistance inside the source itself:



Being in series with the load, the voltage source's internal resistance drops some voltage under loaded conditions, resulting in the source's external terminal voltage being less than what it is under no-load (open) conditions. As load current increases, the internal resistance drops more voltage than before, causing the source's terminal voltage to decrease as load current increases. This is why our hypothetical 15 Volt source will likely output less than 15 Volts if we replace the 5 Ω resistor with a 1 Ω resistor.

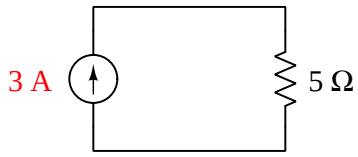
Before we move on in the Tutorial, it should be fully realized that this internal resistance almost never takes the form of an actual *resistor*. What we're modeling here as a discrete component called $R_{internal}$ is almost always the combined effect of stray resistances throughout the voltage source. For a chemical battery, the majority of this internal resistance takes the form of electrical resistance within the liquid or paste electrolyte material between the battery's cell plates. For an electromechanical generator, it will be resistance distributed throughout the length of all the wire coils. No matter the physical form taken, though, it is valid to *represent* a source's internal resistance as a separate component (i.e. resistor) connected to an idealized source. Even if this modeling does not *precisely* emulate the behavior of the real source, it will likely achieve theoretical results close enough[1] for our practical purposes.

Resistance internal to an electrical source may have a profound effect on that source's behavior under loaded conditions. Consider, for example, what will happen if we short-circuit the terminals of a real voltage source versus short-circuiting the terminals of an ideal (perfect) voltage source. A short-circuit, of course, is an electrical connection made between two points possessing negligible resistance. Placing such a "short" across an ideal voltage source's terminals results in an effectively[2] *infinite* amount of current, as the source's full voltage becomes impressed across essentially zero Ohms of resistance. However, short-circuiting the terminals of a real voltage source results in some finite (i.e. limited) amount of current rather than an infinite (unlimited) amount of current owing to the current-limiting effect of that internal resistance.

[1] You will find that a great deal of engineering consists of finding simple(r) approximations of reality that work well enough for some intended purpose.

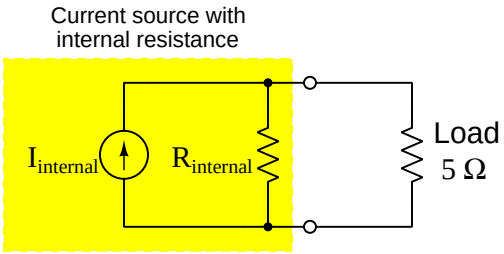
[2] Taken literally, an ideal voltage source subjected to a short-circuit leads to a logical contradiction. Recall that the effect of any short is to prevent voltage from developing between the shorted points, because if two points have absolutely no resistance between them it means charge carriers may pass from one point to the other without losing or gaining any energy. However, an ideal voltage source *must* impart energy to charge carriers passing through – a fact contradicting the short's zero-voltage drop. It is analogous to posing the scenario of an immovable object encountering an unopposable force: these two conditions *by definition* cannot coexist.

Real current sources suffer the same imperfections as real voltage sources, and we may model those same imperfections in much the same way. An ideal current source is supposed to output a definite amount of current through *any* load regardless of the amount of voltage necessary to satisfy Ohm's Law at the load. Consider the following example circuit consisting only of a 3 Ampere DC current source and a 5 Ohm load:



Ohm's Law predicts a voltage drop of 15 Volts ($V = IR$) in this circuit, and if we were to exchange a larger resistor value for the 5 Ω unit shown here, we would expect a proportionately larger voltage drop. However, a real current source, much like a real voltage source, is limited in how much power it may deliver to a load due to internal energy dissipation. A "heavier" load – defined as one demanding more power – will cause the source's internal resistance to dissipate more power too. This means increasing the load's resistance will result a greater voltage, but probably not as great as our new predicted value owing to the fact that the current source will not be able to maintain its full rated value of 3 Amperes through the heavier load. For example, swapping the 5 Ω load for a 10 Ω load should ideally result in 30 Volts instead of 15 Volts, but for a real 3 Ampere source we will likely see something less than 30 Volts because its actual current will probably fall below 3 Amperes driving this heavier load.

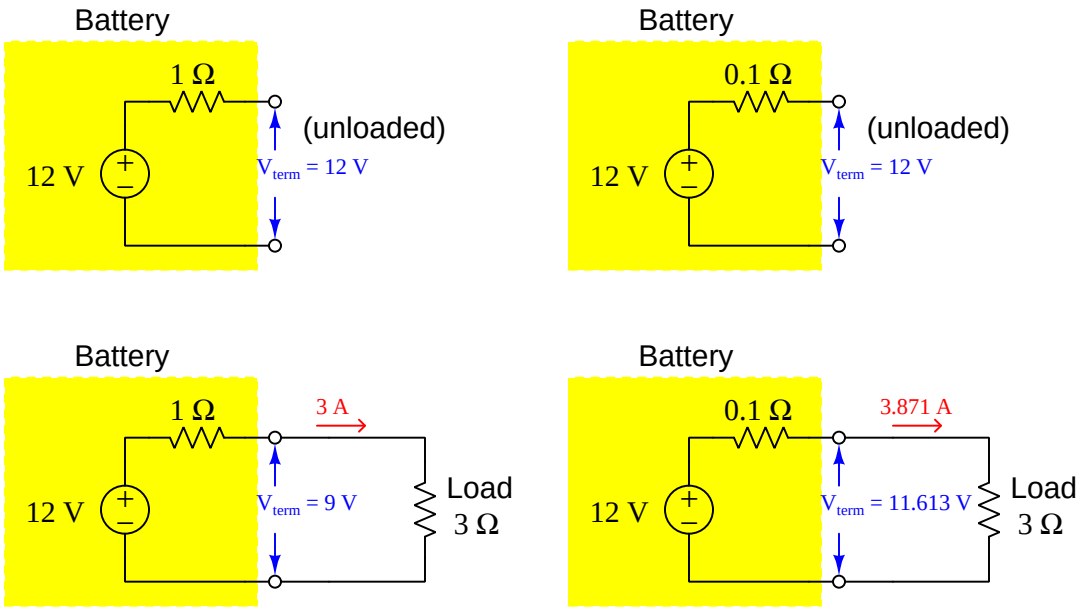
We may model the limitations of any current source by representing it as an ideal current source connected in *parallel* with an internal resistance:



A useful method for exploring the behavior of this current source model is to perform *thought experiments* on it, whereby we try connecting different load resistor value to its terminals. First, imagine the output terminals being short-circuited. This would obviously result in a voltage drop of zero between the terminals, which would mean $R_{internal}$ experiences zero voltage. At zero voltage drop, that internal resistance will pass no current, and the result will be that the full current of the source ($I_{internal}$) passes through the short-circuit load – just the same as if the current source were ideal (i.e. no internal resistance). For our next thought experiment, we will consider open-circuiting the source's terminals. Here, with no load current, the entirety of the source's current must pass through its own internal resistance, resulting in a maximum output voltage equal to the product of its undiverted source current and its internal resistance: $V = I_{internal}R_{internal}$.

3.2 Source strength

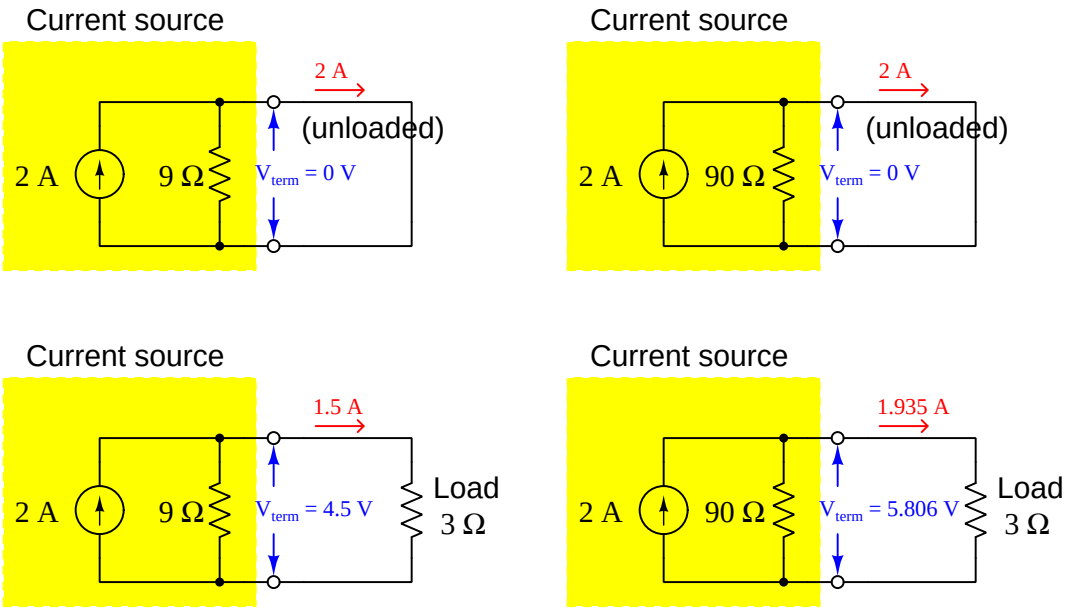
Not all electrical sources are created equal. This adage obviously applies to the voltage ratings of voltage sources just as well as to the current ratings of current sources, but it also applies to internal resistance. For example, two electrochemical batteries may have the same voltage rating but vastly different internal resistances, the result being that they will perform very differently under load even though sharing the same open-circuit (i.e. no-load) terminal voltage. To illustrate, let us consider two "12 Volt" batteries, one of the batteries having an internal resistance of 1 Ohm and the other having just one-tenth as much. These two batteries will be compared in two different loading scenarios, unloaded versus loaded by a 3 Ohm resistance:



The 12-Volt battery with 1 Ohm of internal resistance "sags" down to just 9 Volts at its terminals while powering a 3 Ohm load, while the 12-Volt battery with only 0.1 Ohms of internal resistance maintains a terminal voltage much closer to its rating while powering an identical load. A typical way to compare such sources would be to say that the voltage source with less internal resistance is *stiffer* or *stronger* than the source with more internal resistance. These terms evoke imagery of a physical support structure such as a bridge or truss, the "stiffer" or "stronger" support sagging less under a heavy (i.e. higher power demand) burden than a "softer" or "weaker" structure.

This comparison of internal resistance values helps explain why any comprehensive battery test must include a *loaded condition*, to verify the battery's ability to output an acceptable amount of voltage while bearing the burden of a load. One of the effects of battery aging is that internal resistance tends to increase, and this effect will not be apparent if the battery's terminal voltage is only measured in an unloaded state.

Now let us compare realistic current sources, modeled as ideal current sources in parallel with internal resistances. Here, we will select two current sources rated at 2 Amperes each, but one having ten times more internal resistance than the other:



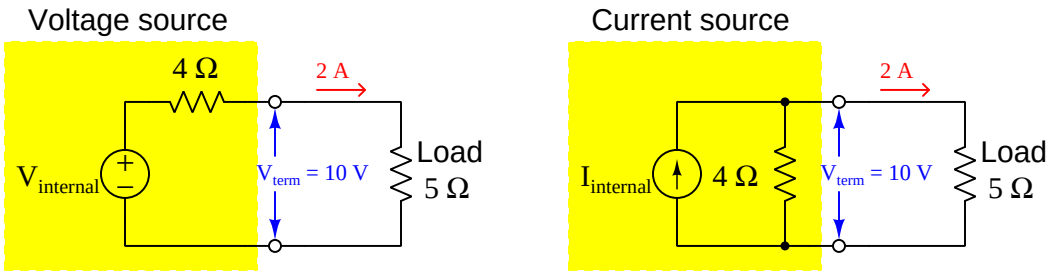
The 2-Ampere source with 9 Ohms of internal resistance "sags" down to just 1.5 Amperes at its terminals while powering a 3 Ohm load, while the 2-Ampere source with 90 Ohms of internal resistance maintains a terminal current much closer to its rating while powering an identical load. As with the voltage sources previously compared, we would say the current source able to maintain current closer to its short-circuit (unloaded) rating is "stiffer" or "stronger" than the other.

What is very different about the "stiffer" current source versus the "stiffer" voltage source are their respective values of internal resistance. A voltage source is "stiffer" when its internal resistance is low; a current source is "stiffer" when its internal resistance is high. Taking this concept to its limits, we could say that an *ideal* voltage source has zero internal resistance while an *ideal* current source has infinite internal resistance. These are important concepts to understand, as we will see in later circuit-analysis techniques that we must frequently consider the internal resistance of ideal sources as they appear in circuit models.

3.3 Source equivalence

A very interesting consequence of modeling realistic voltage and current sources as idealized sources connected to internal resistances is that it becomes possible to find voltage and current sources that are *equivalent to one another*. This is easily proven by illustration.

Consider two non-ideal sources – one voltage and one current – possessing the exact same internal resistance (4 Ohms), powering the exact same load (5 Ohms), at the exact same output values of 2 Amperes and 10 Volts:

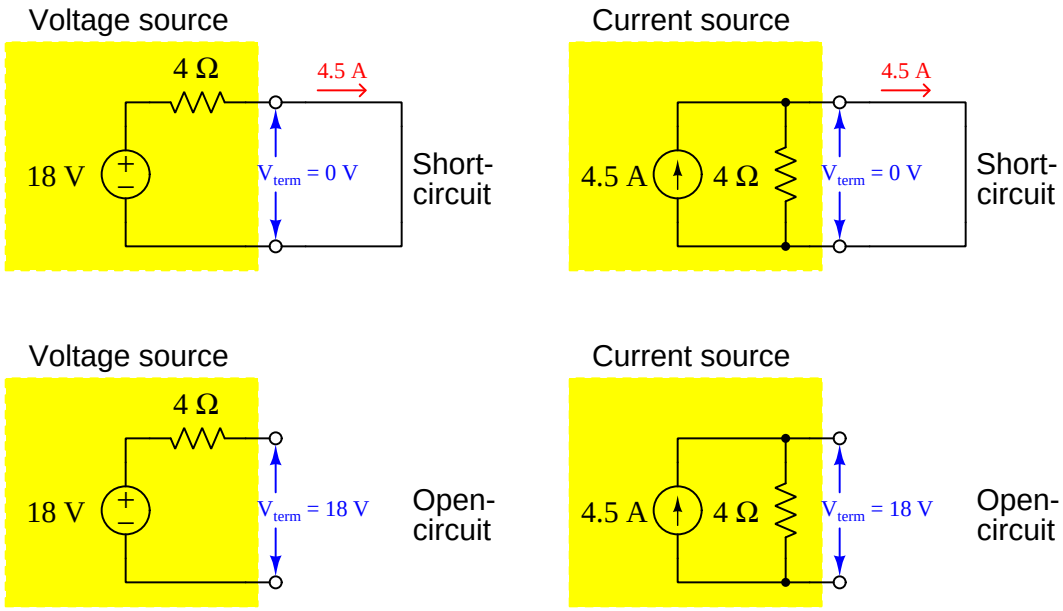


Solving for $V_{internal}$ we get 18 Volts: the sum of the two resistor voltages (10 Volts across the load resistor added to 8 Volts across the internal resistance of 4 Ohms carrying 2 Amperes). Solving for $I_{internal}$ we get 4.5 Amperes: the sum of the two resistor currents (2 Amperes through the load resistor added to 2.5 Amperes through the internal resistance of 4 Ohms dropping 10 Volts).

From the perspective of the two terminals where the load attaches in each case, the voltage source and current source networks behave identically. If the yellow-shaded areas represented opaque boxes enclosing the ideal V and I sources and their respective 4 Ω internal resistances, it would be impossible for us to distinguish the box enclosing the 18-Volt source and 4 Ohm resistor versus from the box enclosing the 4.5-Ampere source and 4 Ohm resistor. This is fundamentally what it means for two electrical networks to be *equivalent* to one another: from a certain perspective their electrical behaviors are truly identical.

Electrical equivalence is a powerful tool for simplifying and analyzing complex electrical networks, and one whose use will span multiple modules in this series. The first step to understanding how to use this principle in practical ways is to first grasp its existence, which is the purpose of this section of the Tutorial.

Returning to our two example sources – 18 Volts in series with 4 Ohms, and 4.5 Amperes in parallel with 4 Ohms – we find they still behave *identically* from the perspective of their output terminals even when the load resistance is replaced by either an open or a short. When short-circuited they both output 4.5 Amperes, and when open-circuited they both output 18 Volts:



Note that this equivalence between non-ideal voltage and current sources is not limited to the example shown. It should be clear that we could have chosen *any* internal resistance value, chosen *any* load resistance value, and chosen *any* current and voltage output values appropriate to that load, and from those arbitrarily-chosen values computed the internal (ideal) voltage and current source values necessary to make the two networks functionally equivalent to each other. The procedure we followed will work the same for any arbitrarily-chosen values, and this makes voltage-current source equivalence a universal principle.

This equivalence may seem rather strange to anyone familiar with the treatment of ideal voltage and current sources in electric circuits. An ideal voltage source is definitely *not* the same as an ideal current source: the former maintains constant voltage regardless of current, and the latter maintains constant current regardless of voltage. However, *real* voltage and current sources are limited in their ability to maintain their rated values, and it is within these limits we find one may emulate the behavior of the other.

Generalizing this behavior, we may say that non-ideal sources (voltage or current) approach ideal behavior if their loads are *very light*, and act like the other type of source if their loads are *very heavy*. This means voltage sources maintain nearly-constant output voltage when powering high-resistance loads (i.e. load power $P_L = \frac{V^2}{R_L}$ is minimized when R_L is large), and current sources maintain nearly-constant output current when powering low-resistance loads (i.e. load power $P_L = I^2 R_L$ is minimized

when R_L is small)[3]. Conversely, voltage sources become current-limited and current sources become voltage-limited under heavy-load conditions (i.e. voltage sources powering low-resistance loads and current sources powering high-resistance loads).

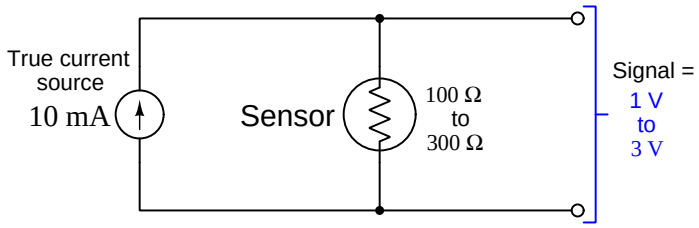
All of this discussion about equivalent sources may seem impractically theoretical, but it is actually important for multiple reasons. First, it is good to understand how real sources behave under extreme-load conditions, realizing how (and why) the source may fail to maintain the constant output assumed of an ideal source. Circuit designs may fail if this fact is not taken into consideration. Second, it is a common practice to construct electrical power systems with internal resistance in order to limit the amount of damage that might occur during a fault (e.g. an accidental short-circuit or open-circuit) when power demand surges. A voltage source's ability to deliver current to a short-circuit fault will be limited by its internal resistance; a current source's ability to produce a high voltage across an open-circuit fault will also be limited by its internal resistance. Third, you may find yourself confronted with the task of making one type of source behave like the other, in which case understanding the behavior of real versus ideal sources gives us tools to do so. This last point will be the topic of the next section.

[3] What constitutes "high/large" or "low/small" resistance values for a load is its size compared to the source's own internal resistance. In the example previously shown where sources having 4 Ω of internal resistance power a load of 5 Ω, the voltage/current regulation is rather poor: modest changes in load resistance for these two sources will result in proportionate changes in voltage and current. If we were to exchange the 5 Ω load for a 5 kΩ load, we would find *voltage* output by either source remains much more constant with the same variation in load resistance. If we were to place a 5 mΩ (5 milliOhm) load resistance across the source terminals, we would find *current* very constant with the same variation in load resistance.

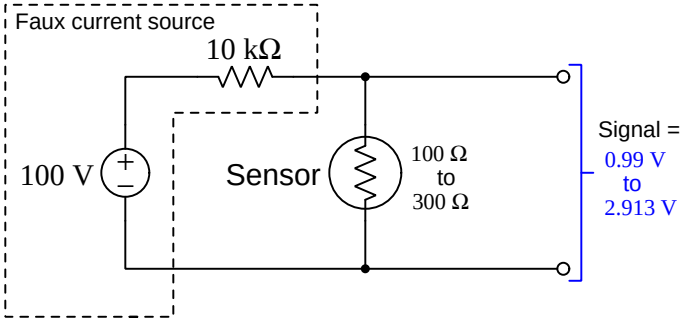
3.4 Approximating ideal sources

Imagine if we needed to pass a constant current through a resistive sensor in order to form a measurement circuit, but we lacked a true current source. We may achieve results that are good enough by connecting a voltage source (of high value) in series with a resistor (also of high value) so that the relatively low resistance of the sensor severely loads the voltage source network and results in a nearly-constant current as sensor resistance varies.

Let's explore this concept with an example. Suppose the sensor's resistance varies from 100 Ω to 300 Ω, and we need a constant current of 10 mA through the sensor to produce a 1 V to 3 V voltage signal[4] over the sensor's full measurement range:



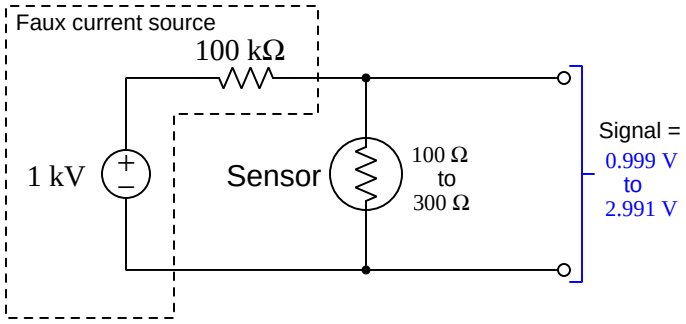
If we lack a true 10 mA current source, we could build a current-limited voltage source using a supplemental series resistor selected to be *much* larger than the sensor, and a voltage value high enough to ensure (approximately) 10 mA through the total series resistance. Let's try a 100 Volt voltage source in series with a 10 kΩ supplemental resistor, in place of the 10 mA current source:



As you can see from the results (0.99 Volts at 100 Ω and 2.913 Volts at 300 Ω), this approximation is no perfect substitute for a 10 mA current source, but it is fairly close.

[4] A good "active reading" strategy to apply here is to actually run these calculations for yourself to see that the signal voltage is indeed 1 Volt at 100 Ω and 3 Volts at 300 Ω. Although these particular values are simple enough to perform the Ohm's Law calculations mentally, later examples will be more complex and require the use of a calculator. Still, the point being made here is that you will gain a better understanding of any mathematical concept presented in text if you work through that same math on your own and then check to see if you get the same results presented in the text. Don't just accept what the text tells you – *prove to yourself that the claims are correct!* Such effort strengthens your understanding, and is well worth the effort.

To minimize the error, we could use a larger voltage value and proportionately larger current-limiting resistor value as shown here:



Now the voltage values across the sensor are significantly closer to being equal with the values when powered by an ideal 10 mA current source. In other words, boosting the internal voltage and internal resistance values of the "faux" current source results in behavior more closely approximating an ideal 10 mA current source. One way to view the behavior of this faux current source is to say the source's very high internal resistance *swamps* the sensor's low resistance, so that total resistance is dominated by the former and fairly unchanged by variations in the latter, maintaining a fairly constant sensor current as the sensor's resistance varies between 100 Ω and 300 Ω.

A reasonable prediction is that every increase in the faux current source's internal voltage-source value (and proportionate increase in current-limiting resistance) will improve this current-source approximation by swamping to a greater degree. A recommended exercise for the reader is to run "thought experiments" with progressively larger internal source values (e.g. 10 kV and 1 MΩ, 100 kV and 10 MΩ) and see for yourself how the results compare against the ideal 10 mA current source, as a way to test this hypothesis.

Chapter 4

Questions

This learning module, along with all others in the ModEL collection, is designed to be used in an inverted instructional environment where students independently read[1] the tutorials and attempt to answer questions on their own *prior* to the instructor's interaction with them. In place of lecture[2], the instructor engages with students in Socratic-style dialogue, probing and challenging their understanding of the subject matter through inquiry.

Answers are not provided for questions within this chapter, and this is by design. Solved problems may be found in the Tutorial and Derivation chapters, instead. The goal here is *independence*, and this requires students to be challenged in ways where others cannot think for them. Remember that you always have the tools of *experimentation* and *computer simulation* (e.g. SPICE) to explore concepts!

The following lists contain ideas for Socratic-style questions and challenges. Upon inspection, one will notice a strong theme of *metacognition* within these statements: they are designed to foster a regular habit of examining one's own thoughts as a means toward clearer thinking. As such these sample questions are useful both for instructor-led discussions as well as for self-study.

[1] Technical reading is an essential academic skill for any technical practitioner to possess for the simple reason that the most comprehensive, accurate, and useful information to be found for developing technical competence is in textual form. Technical careers in general are characterized by the need for continuous learning to remain current with standards and technology, and therefore any technical practitioner who cannot read well is handicapped in their professional development. An excellent resource for educators on improving students' reading prowess through intentional effort and strategy is the book textitReading For Understanding – How Reading Apprenticeship Improves Disciplinary Learning in Secondary and College Classrooms by Ruth Schoenbach, Cynthia Greenleaf, and Lynn Murphy.

[2] Lecture is popular as a teaching method because it is easy to implement: any reasonably articulate subject matter expert can talk to students, even with little preparation. However, it is also quite problematic. A good lecture always makes complicated concepts seem easier than they are, which is bad for students because it instills a false sense of confidence in their own understanding; reading and re-articulation requires more cognitive effort and serves to verify comprehension. A culture of teaching-by-lecture fosters a debilitating dependence upon direct personal instruction, whereas the challenges of modern life demand independent and critical thought made possible only by gathering information and perspectives from afar. Information presented in a lecture is ephemeral, easily lost to failures of memory and dictation; text is forever, and may be referenced at any time.

General challenges following tutorial reading

- Summarize as much of the text as you can in one paragraph of your own words. A helpful strategy is to explain ideas as you would for an intelligent child: as simple as you can without compromising too much accuracy.
- Simplify a particular section of the text, for example a paragraph or even a single sentence, so as to capture the same fundamental idea in fewer words.
- Where did the text make the most sense to you? What was it about the text's presentation that made it clear?
- Identify where it might be easy for someone to misunderstand the text, and explain why you think it could be confusing.
- Identify any new concept(s) presented in the text, and explain in your own words.
- Identify any familiar concept(s) such as physical laws or principles applied or referenced in the text.
- Devise a proof of concept experiment demonstrating an important principle, physical law, or technical innovation represented in the text.
- Devise an experiment to disprove a plausible misconception.
- Did the text reveal any misconceptions you might have harbored? If so, describe the misconception(s) and the reason(s) why you now know them to be incorrect.
- Describe any useful problem-solving strategies applied in the text.
- Devise a question of your own to challenge a reader's comprehension of the text.

General follow-up challenges for assigned problems

- Identify where any fundamental laws or principles apply to the solution of this problem, especially before applying any mathematical techniques.
- Devise a thought experiment to explore the characteristics of the problem scenario, applying known laws and principles to mentally model its behavior.
- Describe in detail your own strategy for solving this problem. How did you identify and organized the given information? Did you sketch any diagrams to help frame the problem?
- Is there more than one way to solve this problem? Which method seems best to you?
- Show the work you did in solving this problem, even if the solution is incomplete or incorrect.
- What would you say was the most challenging part of this problem, and why was it so?
- Was any important information missing from the problem which you had to research or recall?
- Was there any extraneous information presented within this problem? If so, what was it and why did it not matter?
- Examine someone else's solution to identify where they applied fundamental laws or principles.
- Simplify the problem from its given form and show how to solve this simpler version of it. Examples include eliminating certain variables or conditions, altering values to simpler (usually whole) numbers, applying a limiting case (i.e. altering a variable to some extreme or ultimate value).
- For quantitative problems, identify the real-world meaning of all intermediate calculations: their units of measurement, where they fit into the scenario at hand. Annotate any diagrams or illustrations with these calculated values.
- For quantitative problems, try approaching it qualitatively instead, thinking in terms of "increase" and "decrease" rather than definite values.
- For qualitative problems, try approaching it quantitatively instead, proposing simple numerical values for the variables.
- Were there any assumptions you made while solving this problem? Would your solution change if one of those assumptions were altered?
- Identify where it would be easy for someone to go astray in attempting to solve this problem.
- Formulate your own problem based on what you learned solving this one.

General follow-up challenges for experiments or projects

- In what way(s) was this experiment or project easy to complete?
- Identify some of the challenges you faced in completing this experiment or project.

- Show how thorough documentation assisted in the completion of this experiment or project.
- Which fundamental laws or principles are key to this system's function?
- Identify any way(s) in which one might obtain false or otherwise misleading measurements from test equipment in this system.
- What will happen if (component X) fails (open/shorted/etc.)?
- What would have to occur to make this system unsafe?

4.1 Conceptual reasoning

These questions are designed to stimulate your analytic and synthetic thinking[3]. In a Socratic discussion with your instructor, the goal is for these questions to prompt an extended dialogue where assumptions are revealed, conclusions are tested, and understanding is sharpened. Your instructor may also pose additional questions based on those assigned, in order to further probe and refine your conceptual understanding.

Questions that follow are presented to challenge and probe your understanding of various concepts presented in the tutorial. These questions are intended to serve as a guide for the Socratic dialogue between yourself and the instructor. Your instructor's task is to ensure you have a sound grasp of these concepts, and the questions contained in this document are merely a means to this end. Your instructor may, at his or her discretion, alter or substitute questions for the benefit of tailoring the discussion to each student's needs. The only absolute requirement is that each student is challenged and assessed at a level equal to or greater than that represented by the documented questions.

It is far more important that you convey your *reasoning* than it is to simply convey a correct answer. For this reason, you should refrain from researching other information sources to answer questions. What matters here is that *you* are doing the thinking. If the answer is incorrect, your instructor will work with you to correct it through proper reasoning. A correct answer without an adequate explanation of how you derived that answer is unacceptable, as it does not aid the learning or assessment process.

You will note a conspicuous lack of answers given for these conceptual questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your conceptual answers, where applicable, is to use circuit simulation software to explore the effects of changes made to circuits. For example, if one of these conceptual questions challenges you to predict the effects of altering some component parameter in a circuit, you may check the validity of your work by simulating that same parameter change within software and seeing if the results agree.

[3] *Analytical* thinking involves the "disassembly" of an idea into its constituent parts, analogous to dissection. *Synthetic* thinking involves the "assembly" of a new idea comprised of multiple concepts, analogous to construction. Both activities are high-level cognitive skills, extremely important for effective problem-solving, necessitating frequent challenge and regular practice to fully develop.

4.1.1 Reading outline and reflections

"Reading maketh a full man; conference a ready man; and writing an exact man" – Francis Bacon

Francis Bacon's advice is a blueprint for effective education: reading provides the learner with knowledge, writing focuses the learner's thoughts, and critical dialogue equips the learner to confidently communicate and apply their learning. Independent acquisition and application of knowledge is a powerful skill, well worth the effort to cultivate. To this end, students should read these educational resources closely, journal their own reflections on the reading, and discuss in detail their findings with classmates and instructor(s). You should be able to do all of the following after reading any instructional text:

☑ Briefly SUMMARIZE THE TEXT in the form of a journal entry documenting your learning as you progress through the course of study. Share this summary in dialogue with your classmates and instructor. Journaling is an excellent self-test of thorough reading because you cannot clearly express what you have not read or did not comprehend.

☑ Demonstrate ACTIVE READING STRATEGIES, including verbalizing your impressions as you read, simplifying long passages to convey the same ideas using fewer words, annotating text and illustrations with your own interpretations, working through mathematical examples shown in the text, cross-referencing passages with relevant illustrations and/or other passages, identifying problem-solving strategies applied by the author, etc. Technical reading is a special case of problem-solving, and so these strategies work precisely because they help solve any problem: paying attention to your own thoughts (metacognition), eliminating unnecessary complexities, identifying what makes sense, paying close attention to details, drawing connections between separated facts, and noting the successful strategies of others.

☑ Identify IMPORTANT THEMES, especially GENERAL LAWS and PRINCIPLES, expounded in the text and express them in the simplest of terms as though you were teaching an intelligent child. This emphasizes connections between related topics and develops your ability to communicate complex ideas to anyone.

☑ Form YOUR OWN QUESTIONS based on the reading, and then pose them to your instructor and classmates for their consideration. Anticipate both correct and incorrect answers, the incorrect answer(s) assuming one or more plausible misconceptions. This helps you view the subject from different perspectives to grasp it more fully.

☑ Devise EXPERIMENTS to test claims presented in the reading, or to disprove misconceptions. Predict possible outcomes of these experiments, and evaluate their meanings: what result(s) would confirm, and what would constitute disproof? Running mental simulations and evaluating results is essential to scientific and diagnostic reasoning.

☑ Specifically identify any points you found CONFUSING. The reason for doing this is to help diagnose misconceptions and overcome barriers to learning.

4.1.2 Foundational concepts

Correct analysis and diagnosis of electric circuits begins with a proper understanding of some basic concepts. The following is a list of some important concepts referenced in this module's full tutorial. Define each of them in your own words, and be prepared to illustrate each of these concepts with a description of a practical example and/or a live demonstration.

Thought experiments as a problem-solving strategy

Ohm's Law

Series connection

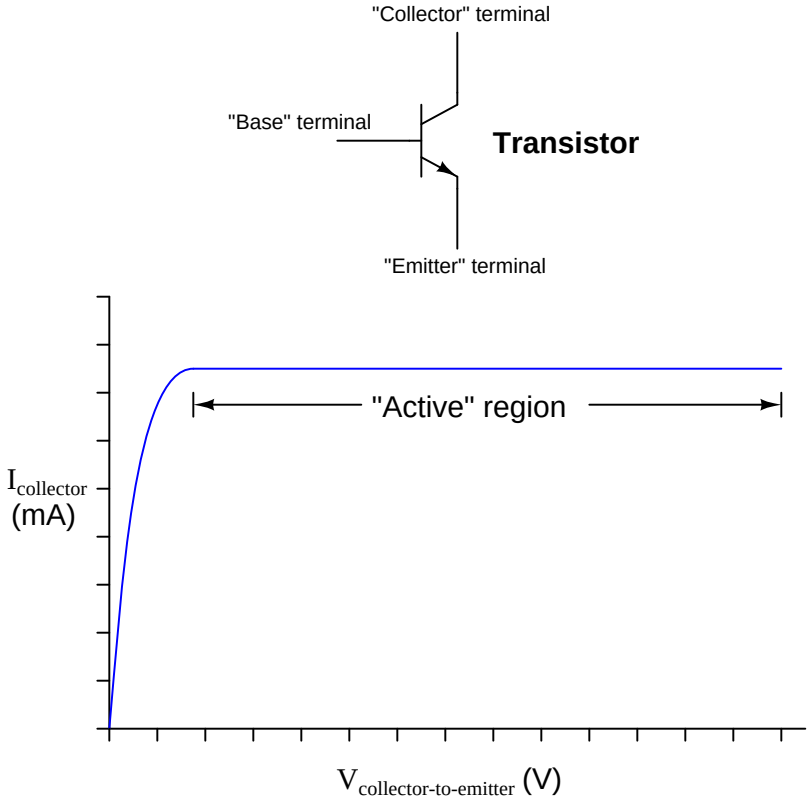
Parallel connection

Electrical source

Electrical load

4.1.3 Transistor modeled as a source

If we were to "model" a transistor with a standard passive component (resistor, voltage source, current source, capacitor, or inductor) for the sake of mathematically analyzing any circuit containing a transistor, which of these passive components would best represent the characteristics of the transistor within its "active" region?

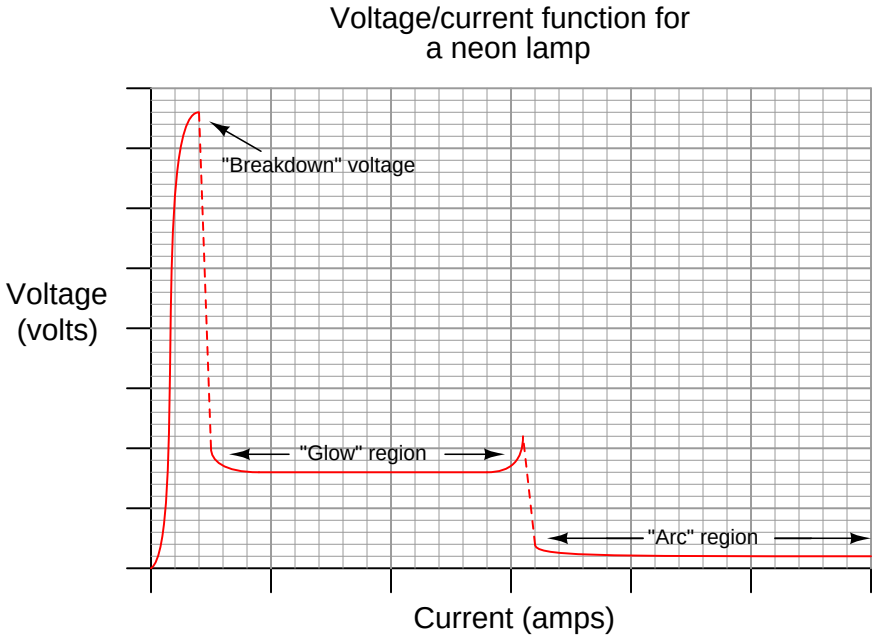


Challenges

- Transistors do not behave exactly as sources – for example, one cannot power anything using a transistor. Identify some of the limitations inherent to modeling transistors as sources. Are there any instances you can think of where such a model could be misleading?

4.1.4 Neon lamp modeled as a source

If we were to "model" a neon lamp with a standard passive component (resistor, voltage source, current source, capacitor, or inductor) for the sake of mathematically analyzing a circuit containing a neon lamp, what component would best represent the characteristics of the lamp within its "glow" region?



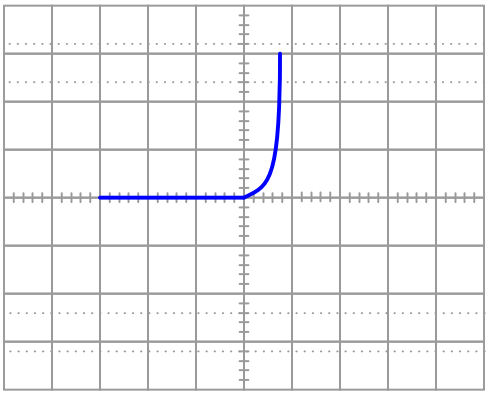
Challenges

- What type of electrical component best models a neon lamp within the vertical (dashed-line) regions on the graph?

4.1.5 Diode modeled as a source

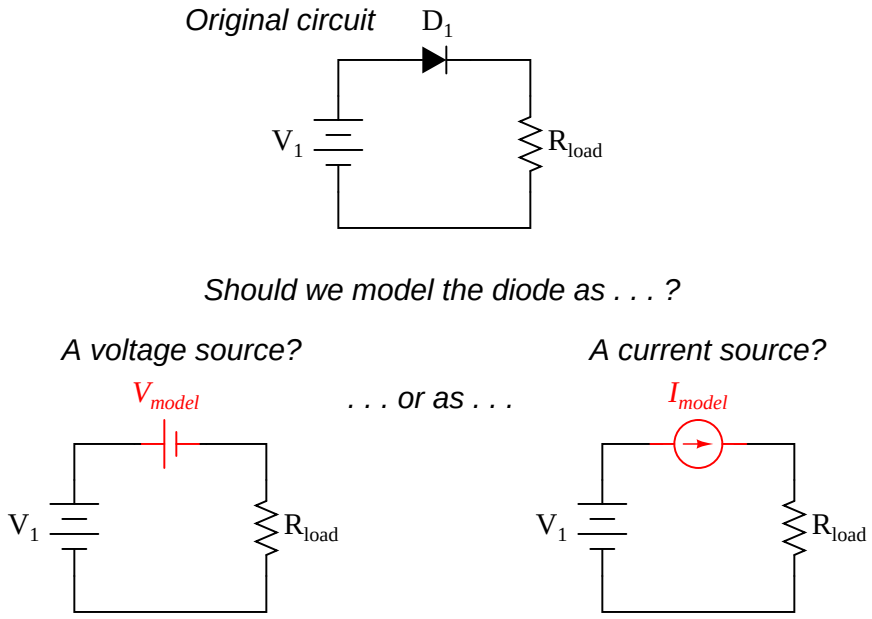
When plotted on a test instrument called a *curve tracer* designed to plot a graph showing the relationship of current to voltage for any specific device connected to it, the characteristic curve for a normal PN junction rectifying diode resembles the following graph. As conventional for curve tracer displays, the horizontal axis of this graph represents voltage applied by the curve tracer while the vertical axis represents current through the device under test:

Rectifying diode curve



Determine whether the diode behaves more like a voltage source or more like a current source (i.e. does it try to maintain constant voltage or does it try to maintain constant current?) when it is conducting current.

Models are very useful because they simplify circuit approximations. For example, we can analyze this diode circuit quite easily if we substitute an electrical source in place of the diode:



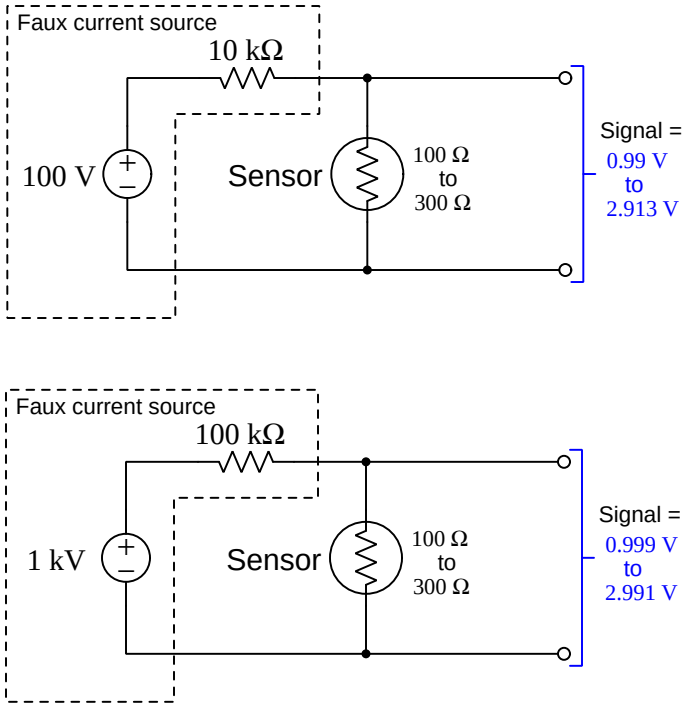
The only question here is, which substitution makes the most sense? Based on the diode's characteristic curve behavior, should we substitute a voltage source or a current source in place of it? Assuming this is a 1N4001 rectifying diode, what is the value we should use for the substituting source?

Challenges

- Identify multiple points on the voltage/current curve shown for a diode and describe what the voltage/current coordinates associated with those points mean for that diode. For example, are there points on that curve where there is voltage but no current? Are there points on the curve showing current with no voltage?
- Quite obviously, diodes do not behave exactly as sources. You cannot power anything using a diode, for instance. Identify some of the limitations inherent to modeling diodes as sources. Are there any instances you can think of where such a model could be misleading?
- Without access to a curve tracer, how might you collect data to sketch a component's characteristic curve?

4.1.6 Improving a faux current source

The following voltage/resistor sources are attempting to model a *current* source. As you can see, one of these does a better job of maintaining a constant current through the load than the other:



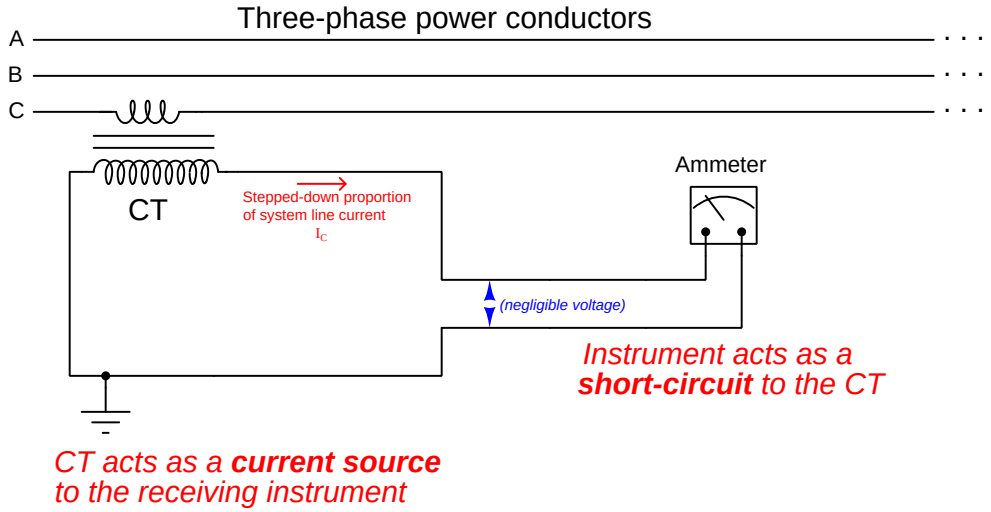
Modify this "faux" current source to be even better than it is.

Challenges

- If instead of a voltage source and internal resistance our real source consisted of an ideal current source and internal resistance, what would this source behave more or less ideally as that internal resistance increased?

4.1.7 Current transformer safety

A device widely used in the electric power industry for safely measuring current flowing through high-voltage power conductors is the *current transformer*. This device only works with AC (alternating current), and uses magnetic coupling between the power conductor and other wires to convey a "stepped-down" current to some measuring device such as an AC ammeter:



So long as alternating current flows through the power conductor sensed by the current transformer (typically abbreviated at "CT"), the CT will try as hard as it can to push current through the measuring device. For any given power line current, then, the CT essentially behaves as a *current source* to its load (e.g. an ammeter).

A photograph of a small CT having a current transformation ratio[4] of 100:5 appears below:

First, explain why it is safe to short-circuit the output of a CT as shown in the schematic diagram.

Second, explain why it would be dangerous to disconnect one of the ammeter's wires in that circuit.

Lastly, describe how you could safely disconnect the ammeter from the CT in a "live" power system where there is significant current passing through the power conductors.

Challenges

- How much current would a 100:5 CT drive through an ammeter if the associated power conductor current was 45 Amperes?

[4] A ratio of 100:5 means that for every 100 Amperes of power line current, the CT will drive 5 Amperes of current to the measuring device. This is the same as a 20:1 ratio, but is conventionally written as a ratio to 5 because five Amperes is the standard rating of most power ammeter movements.

4.2 Quantitative reasoning

These questions are designed to stimulate your computational thinking. In a Socratic discussion with your instructor, the goal is for these questions to reveal your mathematical approach(es) to problem-solving so that good technique and sound reasoning may be reinforced. Your instructor may also pose additional questions based on those assigned, in order to observe your problem-solving firsthand.

Mental arithmetic and estimations are strongly encouraged for all calculations, because without these abilities you will be unable to readily detect errors caused by calculator misuse (e.g. keystroke errors).

You will note a conspicuous lack of answers given for these quantitative questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. My advice is to use circuit simulation software such as SPICE to check the correctness of quantitative answers. Refer to those learning modules within this collection focusing on SPICE to see worked examples which you may use directly as practice problems for your own study, and/or as templates you may modify to run your own analyses and generate your own practice problems.

Completely worked example problems found in the Tutorial may also serve as "test cases[5]" for gaining proficiency in the use of circuit simulation software, and then once that proficiency is gained you will never need to rely[6] on an answer key!

[5] In other words, set up the circuit simulation software to analyze the same circuit examples found in the Tutorial. If the simulated results match the answers shown in the Tutorial, it confirms the simulation has properly run. If the simulated results disagree with the Tutorial's answers, something has been set up incorrectly in the simulation software. Using every Tutorial as practice in this way will quickly develop proficiency in the use of circuit simulation software.

[6] This approach is perfectly in keeping with the instructional philosophy of these learning modules: *teaching students to be self-sufficient thinkers*. Answer keys can be useful, but it is even more useful to your long-term success to have a set of tools on hand for checking your own work, because once you have left school and are on your own, there will no longer be "answer keys" available for the problems you will have to solve.

4.2.1 Miscellaneous physical constants

Note: constants shown in **bold** type are *exact*, not approximations. Values inside of parentheses show one standard deviation (σ) of uncertainty in the final digits: for example, the magnetic permeability of free space value given as $1.25663706212(19) \times 10^{-6}$ H/m represents a center value (i.e. the location parameter) of $1.25663706212 \times 10^{-6}$ Henrys per meter with one standard deviation of uncertainty equal to $0.0000000000019 \times 10^{-6}$ Henrys per meter.

Avogadro's number (N_A) = **6.02214076** $\times 10^{23}$ **per mole** (mol^{-1})

Boltzmann's constant (k) = **1.380649** $\times 10^{-23}$ **Joules per Kelvin** (J/K)

Electronic charge (e) = **1.602176634** $\times 10^{-19}$ **Coulomb** (C)

Faraday constant (F) = **96,485.33212...** $\times 10^{4}$ **Coulombs per mole** (C/mol)

Magnetic permeability of free space (μ_0) = $1.25663706212(19) \times 10^{-6}$ Henrys per meter (H/m)

Electric permittivity of free space (ϵ_0) = $8.8541878128(13) \times 10^{-12}$ Farads per meter (F/m)

Characteristic impedance of free space (Z_0) = 376.730313668(57) Ohms (Ω)

Gravitational constant (G) = $6.67430(15) \times 10^{-11}$ cubic meters per kilogram-seconds squared ($m^3/kg\text{-}s^2$)

Molar gas constant (R) = **8.314462618... Joules per mole-Kelvin** (J/mol-K) = 0.08205746(14) liters-atmospheres per mole-Kelvin

Planck constant (h) = **6.62607015** $\times 10^{-34}$ **joule-seconds** (J-s)

Stefan-Boltzmann constant (σ) = **5.670374419...** $\times 10^{-8}$ **Watts per square meter-Kelvin**^{4} ($W/m^2 \cdot K^4$)

Speed of light in a vacuum (c) = **299,792,458 meters per second** (m/s) = 186282.4 miles per second (mi/s)

Note: All constants taken from NIST data "Fundamental Physical Constants – Complete Listing", from `http://physics.nist.gov/constants`, National Institute of Standards and Technology (NIST), 2018 CODATA Adjustment.

4.2.2 Introduction to spreadsheets

A powerful computational tool you are encouraged to use in your work is a *spreadsheet*. Available on most personal computers (e.g. Microsoft Excel), *spreadsheet* software performs numerical calculations based on number values and formulae entered into cells of a grid. This grid is typically arranged as lettered columns and numbered rows, with each cell of the grid identified by its column/row coordinates (e.g. cell `B3`, cell `A8`). Each cell may contain a string of text, a number value, or a mathematical formula. The spreadsheet automatically updates the results of all mathematical formulae whenever the entered number values are changed. This means it is possible to set up a spreadsheet to perform a series of calculations on entered data, and those calculations will be re-done by the computer any time the data points are edited in any way.

For example, the following spreadsheet calculates average speed based on entered values of distance traveled and time elapsed:

	A	B	C	D
1	Distance traveled	46.9	Kilometers	
2	Time elapsed	1.18	Hours	
3	Average speed	= B1 / B2	km/h	
4				
5				

Text labels contained in cells `A1` through `A3` and cells `C1` through `C3` exist solely for readability and are not involved in any calculations. Cell `B1` contains a sample distance value while cell `B2` contains a sample time value. The formula for computing speed is contained in cell `B3`. Note how this formula begins with an "equals" symbol (`=`), references the values for distance and speed by lettered column and numbered row coordinates (`B1` and `B2`), and uses a forward slash symbol for division (`/`). The coordinates `B1` and `B2` function as *variables*[7] would in an algebraic formula.

When this spreadsheet is executed, the numerical value `39.74576` will appear in cell `B3` rather than the formula `= B1 / B2`, because 39.74576 is the computed speed value given 46.9 kilometers traveled over a period of 1.18 hours. If a different numerical value for distance is entered into cell `B1` or a different value for time is entered into cell `B2`, cell `B3`'s value will automatically update. All you need to do is set up the given values and any formulae into the spreadsheet, and the computer will do all the calculations for you.

Cell `B3` may be referenced by other formulae in the spreadsheet if desired, since it is a variable just like the given values contained in `B1` and `B2`. This means it is possible to set up an entire chain of calculations, one dependent on the result of another, in order to arrive at a final value. The arrangement of the given data and formulae need not follow any pattern on the grid, which means you may place them anywhere.

[7] Spreadsheets may also provide means to attach text labels to cells for use as variable names (Microsoft Excel simply calls these labels "names"), but for simple spreadsheets such as those shown here it's usually easier just to use the standard coordinate naming for each cell.

Common[8] arithmetic operations available for your use in a spreadsheet include the following:

- Addition (`+`)
- Subtraction (`-`)
- Multiplication (`*`)
- Division (`/`)
- Powers (`^`)
- Square roots (`sqrt()`)
- Logarithms (`ln()` , `log10()`)

Parentheses may be used to ensure[9] proper order of operations within a complex formula. Consider this example of a spreadsheet implementing the *quadratic formula*, used to solve for roots of a polynomial expression in the form of $ax^2 + bx + c$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

	A	B
1	x_1	= (-B4 + sqrt((B4^2) - (4*B3*B5))) / (2*B3)
2	x_2	= (-B4 - sqrt((B4^2) - (4*B3*B5))) / (2*B3)
3	a =	9
4	b =	5
5	c =	-2

This example is configured to compute roots[10] of the polynomial $9x^2 + 5x - 2$ because the values of 9, 5, and -2 have been inserted into cells `B3`, `B4`, and `B5`, respectively. Once this spreadsheet has been built, though, it may be used to calculate the roots of *any* second-degree polynomial expression simply by entering the new a, b, and c coefficients into cells `B3` through `B5`. The numerical values appearing in cells `B1` and `B2` will be automatically updated by the computer immediately following any changes made to the coefficients.

[8] Modern spreadsheet software offers a bewildering array of mathematical functions you may use in your computations. I recommend you consult the documentation for your particular spreadsheet for information on operations other than those listed here.

[9] Spreadsheet programs, like text-based programming languages, are designed to follow standard order of operations by default. However, my personal preference is to use parentheses even where strictly unnecessary just to make it clear to any other person viewing the formula what the intended order of operations is.

[10] Reviewing some algebra here, a *root* is a value for x that yields an overall value of zero for the polynomial. For this polynomial ($9x^2 + 5x - 2$) the two roots happen to be $x = 0.269381$ and $x = -0.82494$, with these values displayed in cells `B1` and `B2`, respectively upon execution of the spreadsheet.

Alternatively, one could break up the long quadratic formula into smaller pieces like this:

$$y = \sqrt{b^2 - 4ac} \qquad z = 2a$$

$$x = \frac{-b \pm y}{z}$$

	A	B	C
1	x_1	= (-B4 + C1) / C2	= sqrt((B4^2) - (4*B3*B5))
2	x_2	= (-B4 - C1) / C2	= 2*B3
3	a =	9	
4	b =	5	
5	c =	-2	

Note how the square-root term (y) is calculated in cell C1, and the denominator term (z) in cell C2. This makes the two final formulae (in cells B1 and B2) simpler to interpret. The positioning of all these cells on the grid is completely arbitrary[11] – all that matters is that they properly reference each other in the formulae.

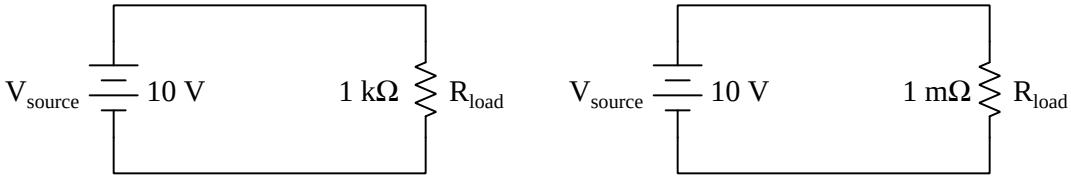
Spreadsheets are particularly useful for situations where the same set of calculations representing a circuit or other system must be repeated for different initial conditions. The power of a spreadsheet is that it automates what would otherwise be a tedious set of calculations. One specific application of this is to simulate the effects of various components within a circuit failing with abnormal values (e.g. a shorted resistor simulated by making its value nearly zero; an open resistor simulated by making its value extremely large). Another application is analyzing the behavior of a circuit design given new components that are out of specification, and/or aging components experiencing drift over time.

[11] My personal preference is to locate all the "given" data in the upper-left cells of the spreadsheet grid (each data point flanked by a sensible name in the cell to the left and units of measurement in the cell to the right as illustrated in the first distance/time spreadsheet example), sometimes coloring them in order to clearly distinguish which cells contain entered data versus which cells contain computed results from formulae. I like to place all formulae in cells below the given data, and try to arrange them in logical order so that anyone examining my spreadsheet will be able to figure out *how* I constructed a solution. This is a general principle I believe all computer programmers should follow: *document and arrange your code to make it easy for other people to learn from it.*

4.2.3 Ideal current source behavior

A *voltage source* is a source of electricity that (ideally) outputs a constant voltage. That is, a perfect voltage source will hold its output voltage constant regardless of the load imposed upon it:

Ideal voltage sources assumed



V = 10 V
I = 10 mA

V = 10 V
I = 10 kA

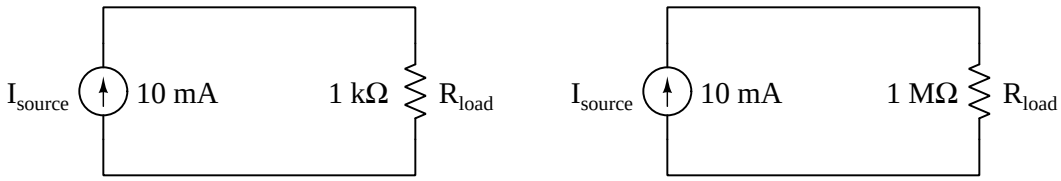
In real life, there is no such thing as a perfect voltage source, but sources having extremely low internal resistance come close.

Another type of electricity source is the *current source*, which (ideally) outputs a constant current regardless of the load imposed upon it. A common symbol for a current source is a circle with an arrow inside[12]. Another symbol is two intersecting circles, with an arrow nearby pointing in the direction of conventional flow:

Current sources

Predict how an ideal current source would behave for the following two load scenarios:

Ideal current sources assumed



V = ???
I = 10 mA

V = ???
I = 10 mA

[12] The arrow always points in the direction of conventional flow, never electron flow!

The degree to which a load is considered "heavy" is the rate at which energy is extracted from a source by that load. In other words, a "heavy" load dissipates more power than a "light" load. What type of load (high resistance or low resistance) is considered "heavier" when energized by a voltage source, and why?

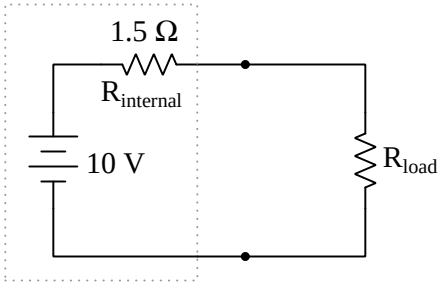
What type of load (high resistance or low resistance) is considered "heavier" when energized by a current source, and why?

Challenges

- Identify the polarity of the voltage drops across the resistors in the circuits shown above.
- For both the ideal voltage source and ideal current source circuits, identify the "heaviest" load (i.e. the load resistance scenario where the resistor dissipates the greatest amount of power).

4.2.4 Analyzing a non-ideal source

Calculate the voltage dropped across the load resistor, and the current through the load resistor, for load resistance values of 1 kΩ, 2 kΩ, 5 kΩ, 8 kΩ, and 10 kΩ:



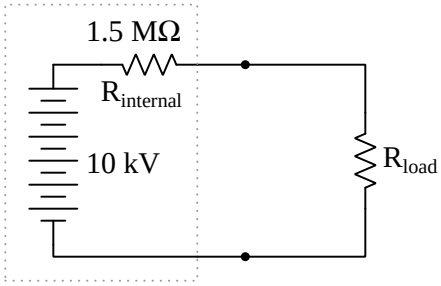
Do the "boxed" components in this circuit behave more like a constant voltage source, or a constant current source? Explain your answer.

Challenges

- Modify the "boxed" source so that it behaves more like the *other* type of source.

4.2.5 Analyzing another non-ideal source

Calculate the voltage dropped across the load resistor, and the current through the load resistor, for load resistance values of 1 kΩ, 2 kΩ, 5 kΩ, 8 kΩ, and 10 kΩ:



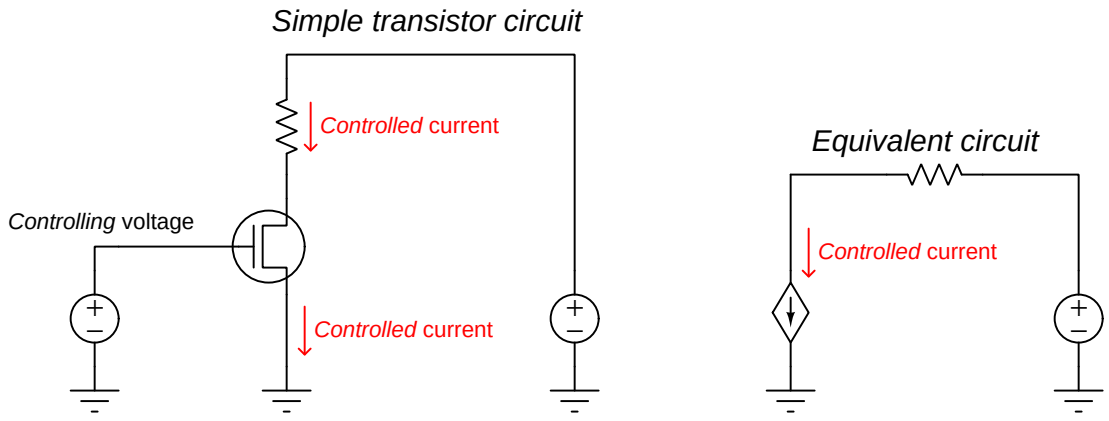
Do the "boxed" components in this circuit behave more like a constant voltage source, or a constant current source? Explain your answer.

Challenges

- Modify the "boxed" source so that it behaves more like the *other* type of source.

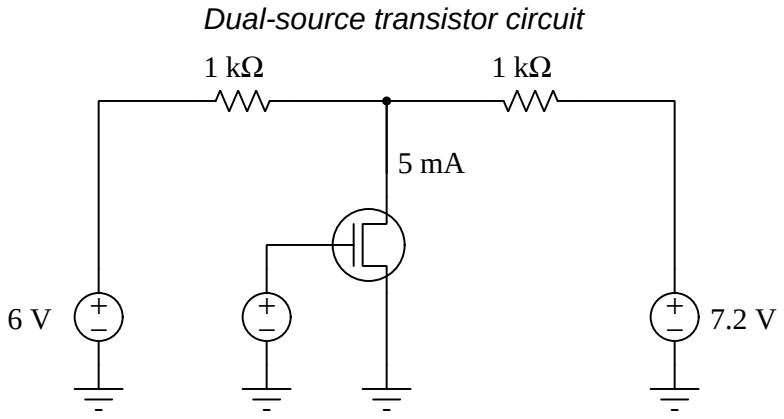
4.2.6 FET modeled as a current source

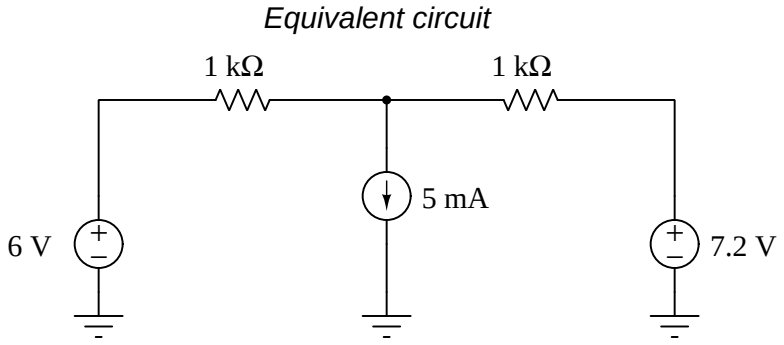
An electronic component called a *Field-Effect Transistor*, or *FET*, behaves as a constant-current regulator when activated by a controlling voltage signal applied between its gate and drain terminals:



Although transistors are not true sources, but are actually *loads*, it is useful to represent them as a current *source* if only to represent their tendency to maintain a regulated current when powered by an external source.

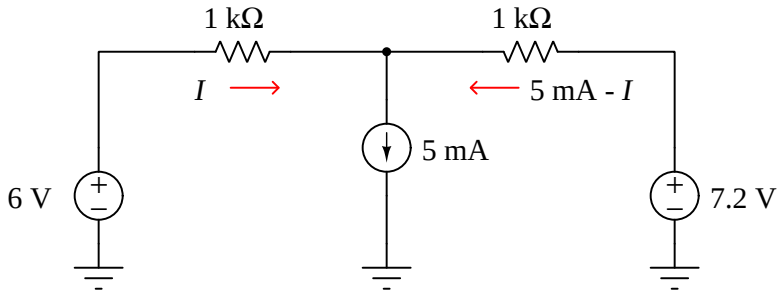
Suppose we needed to calculate the amount of current drawn from the 6-volt source in this dual-source[13] transistor circuit:



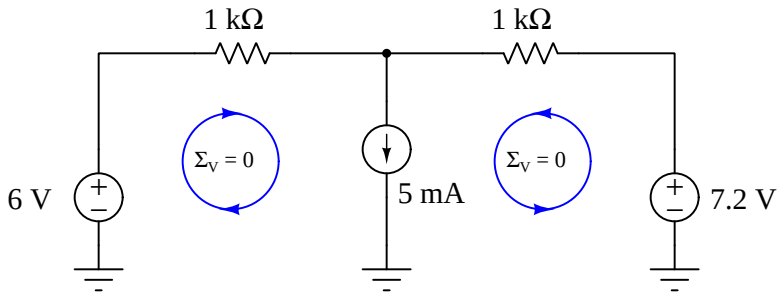


[13]The voltage source connected between the transistor's "gate" terminal and ground doesn't count because the gate terminal of an FET happens to be electrically isolated from the rest of the transistor structure, and so no gate current is possible. This third voltage source therefore merely serves to establish the transistor's controlled current value and does not actually contribute any energy to the circuit.

We know the combined currents from the two voltage sources must add up to 5 mA, because Kirchhoff's Current Law tells us that currents add algebraically at any node. Based on this knowledge, we may label the current through the 6-volt battery as "I", and the current through the 7.2 volt battery as "5 mA $-I$":



Kirchhoff's Voltage Law tells us that the algebraic sum of voltage drops around any "loop" in a circuit must equal zero. Based on all this data, calculate the value of I:



Challenges

- Explain why we cannot analyze this as a series-parallel network, but must apply Kirchhoff's Laws as we did.

4.3 Diagnostic reasoning

These questions are designed to stimulate your deductive and inductive thinking, where you must apply general principles to specific scenarios (deductive) and also derive conclusions about the failed circuit from specific details (inductive). In a Socratic discussion with your instructor, the goal is for these questions to reinforce your recall and use of general circuit principles and also challenge your ability to integrate multiple symptoms into a sensible explanation of what's wrong in a circuit. Your instructor may also pose additional questions based on those assigned, in order to further challenge and sharpen your diagnostic abilities.

As always, your goal is to fully *explain* your analysis of each problem. Simply obtaining a correct answer is not good enough – you must also demonstrate sound reasoning in order to successfully complete the assignment. Your instructor's responsibility is to probe and challenge your understanding of the relevant principles and analytical processes in order to ensure you have a strong foundation upon which to build further understanding.

You will note a conspicuous lack of answers given for these diagnostic questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your diagnostic answers, where applicable, is to use circuit simulation software to explore the effects of faults placed in circuits. For example, if one of these diagnostic questions requires that you predict the effect of an open or a short in a circuit, you may check the validity of your work by simulating that same fault (substituting a very high resistance in place of that component for an open, and substituting a very low resistance for a short) within software and seeing if the results agree.

4.3.1 Battery testing

As an electro-chemical battery ages, it degrades in two different ways. First, the internal (ideal) voltage decreases. Second, its internal resistance increases. These two mechanisms of degradation do not necessarily occur at the same pace!

Based on these facts, explain why a simple voltmeter measurement of a battery may be insufficient to determine if a battery is nearing the end of its useful life.

Also, devise a comprehensive test procedure for a battery to independently determine its internal voltage and internal resistance.

Challenges

- Explain how you could build a "battery test set" to integrate these testing functions into a single special-purpose instrument so that an operator unfamiliar with ideal and real sources could reliably test batteries for their health.

4.3.2 Testing a power supply

Suppose a "constant-voltage" AC-to-DC power supply built by a student outputs 14.8 Volts when unloaded, but its output voltage sags to 13.5 Volts when powering a 50 Ω resistor. How much internal resistance does this power supply have, and what is its internal voltage value?

Next, predict how far the output voltage of this power supply will sag when connected to a 20 Ω resistor.

Finally, describe how you could build a test circuit where it is easy for you to connect different load resistors for extremely brief periods of time, to avoid overheating these loads if energized for too long.

Challenges

- Identify some helpful problem-solving strategies you could apply to this.
- What are some possible sources of this internal resistance?

4.3.3 Linear and non-linear sources

The real sources shown in the Tutorial are all *linear*, because they contain purely linear components (ideal sources plus a resistance). Explain how we can tell that a source is linear just by conducting tests at its terminals, and conversely how we could tell a source is nonlinear.

Challenges

- Identify any electrical/electronic components you know of that have non-linear characteristics.

Appendix A

Problem-Solving Strategies

The ability to solve complex problems is arguably one of the most valuable skills one can possess, and this skill is particularly important in any science-based discipline.

- Study principles, not procedures. Don't be satisfied with merely knowing how to compute solutions – learn *why* those solutions work.
- Identify what it is you need to solve, identify all relevant data, identify all units of measurement, identify any general principles or formulae linking the given information to the solution, and then identify any "missing pieces" to a solution. Annotate all diagrams with this data.
- Sketch a diagram to help visualize the problem. When building a real system, always devise a plan for that system and analyze its function *before* constructing it.
- Follow the units of measurement and meaning of every calculation. If you are ever performing mathematical calculations as part of a problem-solving procedure, and you find yourself unable to apply each and every intermediate result to some aspect of the problem, it means you don't understand what you are doing. Properly done, every mathematical result should have practical meaning for the problem, and not just be an abstract number. You should be able to identify the proper units of measurement for each and every calculated result, and show where that result fits into the problem.
- Perform "thought experiments" to explore the effects of different conditions for theoretical problems. When troubleshooting real systems, perform *diagnostic tests* rather than visually inspecting for faults, the best diagnostic test being the one giving you the most information about the nature and/or location of the fault with the fewest steps.
- Simplify the problem until the solution becomes obvious, and then use that obvious case as a model to follow in solving the more complex version of the problem.
- Check for exceptions to see if your solution is incorrect or incomplete. A good solution will work for *all* known conditions and criteria. A good example of this is the process of testing scientific hypotheses: the task of a scientist is not to find support for a new idea, but rather to *challenge* that new idea to see if it holds up under a battery of tests. The philosophical

principle of *reductio ad absurdum* (i.e. disproving a general idea by finding a specific case where it fails) is useful here.

- Work "backward" from a hypothetical solution to a new set of given conditions.
- Add quantities to problems that are qualitative in nature, because sometimes a little math helps illuminate the scenario.
- Sketch graphs illustrating how variables relate to each other. These may be quantitative (i.e. with realistic number values) or qualitative (i.e. simply showing increases and decreases).
- Treat quantitative problems as qualitative in order to discern the relative magnitudes and/or directions of change of the relevant variables. For example, try determining what happens if a certain variable were to increase or decrease before attempting to precisely calculate quantities: how will each of the dependent variables respond, by increasing, decreasing, or remaining the same as before?
- Consider limiting cases. This works especially well for qualitative problems where you need to determine which direction a variable will change. Take the given condition and magnify that condition to an extreme degree as a way of simplifying the direction of the system's response.
- Check your work. This means regularly testing your conclusions to see if they make sense. This does *not* mean repeating the same steps originally used to obtain the conclusion(s), but rather to use some other means to check validity. Simply repeating procedures often leads to *repeating the same errors* if any were made, which is why alternative paths are better.

Appendix B

Instructional philosophy

B.1 First principles of learning

- **Anyone can learn anything** given appropriate time, effort, resources, challenges, encouragement, and expectations. Dedicating time and investing effort are the student's responsibility; providing resources, challenges, and encouragement are the teacher's responsibility; high expectations are a responsibility shared by both student and teacher.

- **Transfer is not automatic.** The human mind has a natural tendency to compartmentalize information, which means the process of taking knowledge learned in one context and applying it to another usually does not come easy and therefore should never be taken for granted.

- **Learning is iterative.** The human mind rarely learns anything perfectly on the first attempt. Anticipate mistakes and plan for multiple tries to achieve full understanding, using the lessons of those mistakes as feedback to guide future attempts.

- **Information is absorbed, but understanding is created.** Facts and procedures may be memorized easily enough by repeated exposure, but the ability to reliably apply principles to novel scenarios only comes through intense personal effort. This effort is fundamentally creative in nature: casting new concepts in one's own words, running mental experiments to perceive patterns and trends, etc. These acts of making knowledge "one's own" need not be perfect in order to be effective, as the value lies in the activity and not necessarily the finished product. Depth of understanding and permanence of memory are both directly related to mental effort during the learning process – i.e. *easy learning is shallow and fleeting.*

- **Education trumps training.** There is no such thing as an entirely isolated subject, as all fields of knowledge are connected. Training is narrowly-focused and task-oriented. Education is broad-based and principle-oriented. When preparing for a life-long technical career, education beats training every time.

- **Character matters.** Poor habits are more destructive than deficits of knowledge or skill. This is especially true in collective endeavors, where a team's ability to function depends on trust between its members. Simply put, no one wants an untrustworthy person on their team. An essential component of education then, is character development.

- **People learn to be responsible by bearing responsibility.** An irresponsible person is someone who has never *had* to be responsible for anything that mattered enough to them. Just as anyone can learn anything, anyone can become responsible if the personal cost of irresponsibility becomes high enough.

- **What gets measured, gets done.** Accurate and relevant assessment of learning is key to ensuring all students learn. Therefore, it is imperative to measure what matters.

- **Failure is nothing to fear.** Every human being fails, and fails in multiple ways at multiple times. Eventual success only happens when we don't stop trying.

B.2 Proven strategies for instructors

- Assume every student is capable of learning anything they desire given the proper conditions. Treat them as capable adults by granting real responsibility and avoiding artificial incentives such as merit or demerit points.
- Create a consistent culture of high expectations across the entire program of study. Demonstrate and encourage patience, persistence, and a healthy sense of self-skepticism. Anticipate and de-stigmatize error. Teach respect for the capabilities of others as well as respect for one's own fallibility.
- Replace lecture with "inverted" instruction, where students first encounter new concepts through reading and then spend class time in Socratic dialogue with the instructor exploring those concepts and solving problems individually. There is a world of difference between observing someone solve a problem versus actually solving a problem yourself, and so the point of this form of instruction is to place students in a position where they *cannot* passively observe.
- Require students to read extensively, write about what they learn, and dialogue with you and their peers to sharpen their understanding. Apply Francis Bacon's advice that "reading maketh a full man; conference a ready man; and writing an exact man". These are complementary activities helping students expand their confidence and abilities.
- Use artificial intelligence (AI) to challenge student understanding rather than merely provide information. Find productive ways for AI to critique students' clarity of thought and of expression, for example by employing AI as a Socratic-style interlocutor or as a reviewer of students' journals. Properly applied, AI has the ability to expand student access to critical review well outside the bounds of their instructor's reach.
- Build frequent and rapid feedback into the learning process so that students know at all times how well they are learning, to identify problems early and fix them before they grow. Model the intellectual habit of self-assessing and self-correcting your own understanding (i.e. a cognitive *feedback loop*), encouraging students to do the same.
- Use "mastery" as the standard for every assessment, which means the exam or experiment or project must be done with 100% competence in order to pass. Provide students with multiple opportunity for re-tries (different versions of the assessment every time).
- Require students to devise their own hypotheses and procedures on all experiments, so that the process is truly a scientific one. Have students assess their proposed experimental procedures for risk and devise mitigations for those risks. Let nothing be pre-designed about students' experiments other than a stated task (i.e. what principle the experiment shall test) at the start and a set of demonstrable knowledge and skill objectives at the end.
- Have students build as much of their lab equipment as possible: building power sources, building test assemblies[1], and building complete working systems (no kits!). In order to provide

[1] In the program I teach, every student builds their own "Development Board" consisting of a metal chassis with DIN rail, terminal blocks, and an AC-DC power supply of their own making which functions as a portable lab environment they can use at school as well as take home.

this same "ground-up" experience for every new student, this means either previous students take their creations with them, or the systems get disassembled in preparation for the new students, or the systems grow and evolve with each new student group.

- Incorporate external accountability for you and for your students, continuously improving the curriculum and your instructional methods based on proven results. Have students regularly network with active professionals through participation in advisory committee meetings, service projects, tours, jobshadows, internships, etc. Practical suggestions include requiring students to design and build projects for external clients (e.g. community groups, businesses, different departments within the institution), and also requiring students attend all technical advisory committee meetings and to dialogue with the industry representatives at those meetings.
- Repeatedly explore difficult-to-learn concepts across multiple courses, so that students have multiple opportunities to build their understanding.
- Relate all new concepts, whenever possible, to previous concepts and to relevant physical laws. Challenge each and every student, every day, to *reason* from concept to concept and to explain the logical connections between. Challenge students to verify their conclusions by multiple approaches (e.g. double-checking their work using different methods). Ask *"Why?"* often.
- Maintain detailed records on each student's performance and share these records privately with them. These records should include academic performance as well as professionally relevant behavioral tendencies.
- Hold mandatory "check-in" meetings between all program faculty and each new student during their first term. Offer these to all other students as an option, except for any students continuing to manifest unprofessional behaviors, poor academic performance, or who have some other need for a face-to-face meeting with faculty.
- Address problems while they are small, before they grow larger. This is equally true for tutoring technical concepts as it is for helping students build professional habits.
- Build rigorous quality control into the curriculum to ensure every student masters every important concept, and that the mastery is retained over time. This includes (1) review questions added to every exam to re-assess knowledge taught in previous terms, (2) cumulative exams at the end of every term to re-assess all important concepts back to the very beginning of the program, and (3) review assessments in practical (hands-on) coursework to ensure critically-important skills were indeed taught and are still retained. What you will find by doing this is that it actually boosts retention of students by ensuring that important knowledge gets taught and is retained over long spans of time. In the absence of such quality control, student learning and retention tends to be spotty and this contributes to drop-out and failure rates later in their education.
- Finally, *never rush learning.* Education is not a race. Give your students ample time to digest complex ideas, as you continually remind yourself of just how long it took you to achieve mastery! Long-term retention and the consistently correct application of concepts are always the result of *focused effort over long periods of time* which means there are no shortcuts to learning.

B.3 Proven strategies for students

The single most important piece of advice I have for any student of any subject is to take responsibility for your own development in all areas of life including mental development. Expecting others in your life to entirely guide your own development is a recipe for disappointment. This is just as true for students enrolled in formal learning institutions as it is for auto-didacts pursuing learning entirely on their own. Learning to think in new ways is key to being able to gainfully use information, to make informed decisions about your life, and to best serve those you care about. With this in mind, I offer the following advice to students:

- **Approach all learning as valuable**. No matter what course you take, no matter who you learn from, no matter the subject, there is something useful in every learning experience. If you don't see the value of every new experience, you are not looking closely enough!
- **Continually challenge yourself**. Let other people take shortcuts and find easy answers to easy problems. The purpose of education is to stretch your mind, in order to shape it into a more powerful tool. This doesn't come by taking the path of least resistance. An excellent analogy for an empowering education is productive physical exercise: becoming stronger, more flexible, and more persistent only comes through intense personal effort.
- **Master the use of language**. This includes reading extensively, writing every day, listening closely, and speaking articulately. To a great extent language channels and empowers thought, so the better you are at wielding language the better you will be at grasping abstract concepts and articulating them not only for your benefit but for others as well.
- **Do not limit yourself to the resources given to you**. Read books that are not on the reading list. Run experiments that aren't assigned to you. Form study groups outside of class. Take an entrepreneurial approach to your own education, as though it were a business you were building for your future benefit.
- **Express and share what you learn**. Take every opportunity to teach what you have learned to others, as this will not only help them but will also strengthen your own understanding[2].
- Realize that **no one can give you understanding**, just as no one can give you physical fitness. These both must be *built*.
- **Above all, recognize that learning is hard work, and that a certain level of frustration is unavoidable**. There are times when you will struggle to grasp some of these concepts, and that struggle is a natural thing. Take heart that it will yield with persistent and varied[3] effort, and never give up! That concepts don't immediately come to you is not a sign of something wrong, but rather of something right: that you have found a worthy challenge!

[2] On a personal note, I was surprised to learn just how much my own understanding of electronics and related subjects was strengthened by becoming a teacher. When you are tasked every day with helping other people grasp complex topics, it catalyzes your own learning by giving you powerful incentives to study, to articulate your thoughts, and to reflect deeply on the process of learning.

[3] As the old saying goes, "Insanity is trying the same thing over and over again, expecting different results." If you find yourself stumped by something in the text, you should attempt a different approach. Alter the thought experiment, change the mathematical parameters, do whatever you can to see the problem in a slightly different light, and then the solution will often present itself more readily.

B.4 Design of these learning modules

"The unexamined circuit is not worth energizing" – Socrates (if he had taught electricity)

These learning modules, although useful for self-study, were designed to be used in a formal learning environment where a subject-matter expert challenges students to digest the content and exercise their critical thinking abilities in the answering of questions and in the construction and testing of working circuits. Every effort has been made to embed the following instructional and assessment philosophies within:

- The first goal of education is to enhance clear and independent thought, in order that every student reach their fullest potential in a highly complex and inter-dependent world. Robust reasoning is *always* more important than particulars of any subject matter, because its application is universal.
- Literacy is fundamental to independent learning and thought because text continues to be the most efficient way to communicate complex ideas over space and time. Those who cannot read with ease are limited in their ability to acquire knowledge and perspective.
- Articulate communication is fundamental to work that is complex and interdisciplinary.
- Faulty assumptions and poor reasoning are best corrected through challenge, not presentation. The rhetorical technique of *reductio ad absurdum* (disproving an assertion by exposing an absurdity) works well to discipline student's minds, not only to correct the problem at hand but also to learn how to detect and correct future errors.
- Important principles should be repeatedly explored and widely applied throughout a course of study, not only to reinforce their importance and help ensure their mastery, but also to showcase the interconnectedness and utility of knowledge.

These learning modules were expressly designed to be used in an "inverted" teaching environment[4] where students first read the introductory and tutorial chapters on their own, then individually attempt to answer the questions and construct working circuits according to the experiment and project guidelines. The instructor never lectures, but instead meets regularly with each individual student to review their progress, answer questions, identify misconceptions, and challenge the student to new depths of understanding through further questioning. Regular meetings between instructor and student should resemble a Socratic[5] dialogue, where questions serve as scalpels to dissect topics and expose assumptions. The student passes each module only after consistently demonstrating their ability to logically analyze and correctly apply all major concepts in each question or project/experiment. The instructor must be vigilant in probing each student's understanding to ensure they are truly *reasoning* and not just *memorizing*. This is why "Challenge" points appear throughout, as prompts for students to think deeper about topics and as starting points for instructor queries. Sometimes these challenge points require additional knowledge that hasn't been covered in the series to answer in full. This is okay, as the major purpose of the Challenges is to stimulate analysis and synthesis on the part of each student.

The instructor must possess enough mastery of the subject matter and awareness of students' reasoning to generate their own follow-up questions to practically any student response. Even completely correct answers given by the student should be challenged by the instructor for the purpose of having students practice articulating their thoughts and defending their reasoning. Conceptual errors committed by the student should be exposed and corrected not by direct instruction, but rather by reducing the errors to an absurdity[6] through well-chosen questions and thought experiments posed by the instructor. Becoming proficient at this style of instruction requires time and dedication, but the positive effects on critical thinking for both student and instructor are spectacular.

An inspection of these learning modules reveals certain unique characteristics. One of these is a bias toward thorough explanations in the tutorial chapters. Without a live instructor to explain concepts and applications to students, the text itself must fulfill this role. This philosophy results in lengthier explanations than what you might typically find in a textbook, each step of the reasoning process fully explained, including footnotes addressing common questions and concerns students raise while learning these concepts. Each tutorial seeks to not only explain each major concept in sufficient detail, but also to explain the logic of each concept and how each may be developed

[4] In a traditional teaching environment, students first encounter new information via *lecture* from an expert, and then independently apply that information via *homework*. In an "inverted" course of study, students first encounter new information via *homework*, and then independently apply that information under the scrutiny of an expert. The expert's role in lecture is to simply *explain*, but the expert's role in an inverted session is to *challenge*, *critique*, and if necessary *explain* where gaps in understanding still exist.

[5] Socrates is a figure in ancient Greek philosophy famous for his unflinching style of questioning. Although he authored no texts, he appears as a character in Plato's many writings. The essence of Socratic philosophy is to leave no question unexamined and no point of view unchallenged. While purists may argue a topic such as electric circuits is too narrow for a true Socratic-style dialogue, I would argue that the essential thought processes involved with scientific reasoning on *any* topic are not far removed from the Socratic ideal, and that students of electricity and electronics would do very well to challenge assumptions, pose thought experiments, identify fallacies, and otherwise employ the arsenal of critical thinking skills modeled by Socrates.

[6] This rhetorical technique is known by the Latin phrase *reductio ad absurdum*. The concept is to expose errors by counter-example, since only one solid counter-example is necessary to disprove a universal claim. As an example of this, consider the common misconception among beginning students of electricity that voltage cannot exist without current. One way to apply *reductio ad absurdum* to this statement is to ask how much current passes through a fully-charged battery connected to nothing (i.e. a clear example of voltage existing without current).

from "first principles". Again, this reflects the goal of developing clear and independent thought in students' minds, by showing how clear and logical thought was used to forge each concept. Students benefit from witnessing a model of clear thinking in action, and these tutorials strive to be just that.

Another feature of these learning modules is that they do not treat topics in isolation. Rather, important concepts are introduced early in the series, and appear repeatedly as stepping-stones toward other concepts in subsequent modules. This helps to avoid the "compartmentalization" of knowledge, demonstrating the inter-connectedness of concepts and simultaneously reinforcing them. Each module is fairly complete in itself, reserving the beginning of its tutorial to a review of foundational concepts.

To high standards of education,

Tony R. Kuphaldt

Appendix C

Tools used

I am indebted to the developers of many open-source software applications in the creation of these learning modules. The following is a list of these applications with some commentary on each.

You will notice a theme common to many of these applications: a bias toward *code*. Although I am by no means an expert programmer in any computer language, I understand and appreciate the flexibility offered by code-based applications where the user (you) enters commands into a plain ASCII text file, which the software then reads and processes to create the final output. Code-based computer applications are by their very nature *extensible*, while WYSIWYG (What You See Is What You Get) applications are generally limited to whatever user interface the developer makes for you.

The `GNU/Linux` computer operating system

> There is so much to be said about Linus Torvalds' `Linux` and Richard Stallman's `GNU` project. First, to credit just these two individuals is to fail to do justice to the *mob* of passionate volunteers who contributed to make this amazing software a reality. I first learned of `Linux` back in 1996, and have been using this operating system on my personal computers almost exclusively since then. It is *free*, it is completely *configurable*, and it permits the continued use of highly efficient `Unix` applications and scripting languages (e.g. shell scripts, Makefiles, `sed`, `awk`) developed over many decades. `Linux` not only provided me with a powerful computing platform, but its open design served to inspire my life's work of creating open-source educational resources.

Bram Moolenaar's `Vim` text editor

> Writing code for any code-based computer application requires a *text editor*, which may be thought of as a word processor strictly limited to outputting plain-ASCII text files. Many good text editors exist, and one's choice of text editor seems to be a deeply personal matter within the programming world. I prefer `Vim` because it operates very similarly to `vi` which is ubiquitous on `Unix`/`Linux` operating systems, and because it may be entirely operated via keyboard (i.e. no mouse required) which makes it fast to use.

Donald Knuth's TEX typesetting system

Developed in the late 1970's and early 1980's by computer scientist extraordinaire Donald Knuth to typeset his multi-volume magnum opus *The Art of Computer Programming*, this software allows the production of formatted text for screen-viewing or paper printing, all by writing plain-text code to describe how the formatted text is supposed to appear. TEX is not just a markup language for documents, but it is also a Turing-complete programming language in and of itself, allowing useful algorithms to be created to control the production of documents. Simply put, TEX *is a programmer's approach to word processing*. Since TEX is controlled by code written in a plain-text file, this means anyone may read that plain-text file to see exactly how the document was created. This openness afforded by the code-based nature of TEX makes it relatively easy to learn how other people have created their own TEX documents. By contrast, examining a beautiful document created in a conventional WYSIWYG word processor such as Microsoft `Word` suggests nothing to the reader about *how* that document was created, or what the user might do to create something similar. As Mr. Knuth himself once quipped, conventional word processing applications should be called WYSIAYG (What You See Is *All* You Get).

Leslie Lamport's LATEX extensions to TEX

Like all true programming languages, TEX is inherently extensible. So, years after the release of TEX to the public, Leslie Lamport decided to create a massive extension allowing easier compilation of book-length documents. The result was LATEX, which is the markup language used to create all ModEL module documents. You could say that TEX is to LATEX as `C` is to `C++`. This means it is permissible to use any and all TEX commands within LATEX source code, and it all still works. Some of the features offered by LATEX that would be challenging to implement in TEX include automatic index and table-of-content creation.

Tim Edwards' `Xcircuit` drafting program

This wonderful program is what I use to create all the schematic diagrams and illustrations (but not photographic images or mathematical plots) throughout the ModEL project. It natively outputs PostScript format which is a true vector graphic format (this is why the images do not pixellate when you zoom in for a closer view), and it is so simple to use that I have never had to read the manual! Object libraries are easy to create for `Xcircuit`, being plain-text files using PostScript programming conventions. Over the years I have collected a large set of object libraries useful for drawing electrical and electronic schematics, pictorial diagrams, and other technical illustrations.

`Gimp` graphic image manipulation program

Essentially an open-source clone of Adobe's `PhotoShop`, I use `Gimp` to resize, crop, and convert file formats for all of the photographic images appearing in the ModEL modules. Although `Gimp` does offer its own scripting language (called `Script-Fu`), I have never had occasion to use it. Thus, my utilization of `Gimp` to merely crop, resize, and convert graphic images is akin to using a sword to slice bread.

`SPICE` circuit simulation program

`SPICE` is to circuit analysis as TEX is to document creation: it is a form of markup language designed to describe a certain object to be processed in plain-ASCII text. When the plain-text "source file" is compiled by the software, it outputs the final result. More modern circuit analysis tools certainly exist, but I prefer `SPICE` for the following reasons: it is *free*, it is *fast*, it is *reliable*, and it is a fantastic tool for *teaching* students of electricity and electronics how to write simple code. I happen to use rather old versions of `SPICE`, version 2g6 being my "go to" application when I only require text-based output. `NGSPICE` (version 26), which is based on Berkeley `SPICE` version 3f5, is used when I require graphical output for such things as time-domain waveforms and Bode plots. In all `SPICE` example netlists I strive to use coding conventions compatible with all `SPICE` versions.

Andrew D. Hwang's `ePiX` mathematical visualization programming library

This amazing project is a `C++` library you may link to any `C/C++` code for the purpose of generating PostScript graphic images of mathematical functions. As a completely free and open-source project, it does all the plotting I would otherwise use a Computer Algebra System (CAS) such as `Mathematica` or `Maple` to do. It should be said that `ePiX` is *not* a Computer Algebra System like `Mathematica` or `Maple`, but merely a mathematical *visualization* tool. In other words, it won't determine integrals for you (you'll have to implement that in your own `C/C++` code!), but it can graph the results, and it does so beautifully. What I really admire about `ePiX` is that it is a `C++` programming library, which means it builds on the existing power and toolset available with that programming language. Mr. Hwang could have probably developed his own stand-alone application for mathematical plotting, but by creating a `C++` library to do the same thing he accomplished something much greater.

`gnuplot` mathematical visualization software

Another open-source tool for mathematical visualization is `gnuplot`. Interestingly, this tool is *not* part of Richard Stallman's GNU project, its name being a coincidence. For this reason the authors prefer "gnu" *not* be capitalized at all to avoid confusion. This is a much "lighter-weight" alternative to a spreadsheet for plotting tabular data, and the fact that it easily outputs directly to an X11 console or a file in a number of different graphical formats (including PostScript) is very helpful. I typically set my `gnuplot` output format to default (X11 on my Linux PC) for quick viewing while I'm developing a visualization, then switch to PostScript file export once the visual is ready to include in the document(s) I'm writing. As with my use of `Gimp` to do rudimentary image editing, my use of `gnuplot` only scratches the surface of its capabilities, but the important points are that it's *free* and that it *works well*.

`Python` programming language

Both Python and C++ find extensive use in these modules as instructional aids and exercises, but I'm listing Python here as a *tool* for myself because I use it almost daily as a *calculator*. If you open a Python interpreter console and type `from math import *` you can type mathematical expressions and have it return results just as you would on a hand calculator. Complex-number (i.e. *phasor*) arithmetic is similarly supported if you include the complex-math library (`from cmath import *`). Examples of this are shown in the Programming References chapter (if included) in each module. Of course, being a fully-featured programming language, Python also supports conditionals, loops, and other structures useful for calculation of quantities. Also, running in a console environment where all entries and returned values show as text in a chronologically-ordered list makes it easy to copy-and-paste those calculations to document exactly how they were performed.

Appendix D

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Appendix E

References

Appendix F

Version history

This is a list showing all significant additions, corrections, and other edits made to this learning module. Each entry is referenced by calendar date in reverse chronological order (newest version first), which appears on the front cover of every learning module for easy reference. Any contributors to this open-source document are listed here as well.

4 October 2026 – added another Challenge question to the "Diode modeled as a source" Conceptual Reasoning question.

27 September 2025 – edited the "Example: dying dry-cell battery under load" Case Tutorial section to actually show a schematic diagram of how a battery could be briefly load-tested by including a pushbutton switch into the circuit. Also added definitions of heavy versus light loads to the "Ideal current source behavior" Quantitative Reasoning question. Also added more instructor notes.

26-27 February 2025 – edited the "Example: shorted solar panel" Case Tutorial section to discuss more about how the internal resistance was obtained. Also added clearer definitions within the Tutorial for "heavy" versus "light" loads.

2 October 2024 – added another "Challenging concepts" bullet-item to the list in the Introduction chapter.

1 October 2024 – minor edits to the Tutorial, as well as to the "Example: dying dry-cell battery under load" section of the Case Tutorial chapter. Also added a sub-question to the "Testing a power supply" Diagnostic Reasoning question.

23 September 2024 – divided the Introduction chapter into sections, one with recommendations for students, one with a listing of challenging concepts, and one with recommendations for instructors.

29 February 2024 – fixed a typographical error courtesy of Logan Farrell.

4 October 2023 – clarified details about how real current sources handle increasing load resistances.

1 March 2023 – minor edits to the Tutorial for better readability.

27 November 2022 – placed questions at the top of the itemized list in the Introduction chapter prompting students to devise experiments related to the tutorial content.

5 October 2022 – added Challenge questions, and also made some additions to the Case Tutorial example of battery testing where it was suggested these loaded tests be done quickly. Also added a Conceptual Reasoning question regarding current transformers in AC power systems, which behave as current sources.

2 March 2022 – minor additions to the Introduction chapter, as well as additions to the Tutorial to elaborate on the meaning of electrically-equivalent networks and approximating ideal sources.

6 October 2021 – divided Tutorial into separate sections, and also added a Case Tutorial section showing real data from a dying dry-cell battery with high internal resistance, and added a Diagnostic Reasoning question ("Testing a power supply").

2 October 2021 – minor changes made to the "Diode modeled as a source" Conceptual Reasoning question.

8 May 2021 – commented out or deleted empty chapters.

16 March 2021 – added some content to the Introduction.

2 March 2021 – added brief explanation of a curve tracer to the "Diode modeled as a source" question.

6 October 2020 – minor edits to Tutorial.

29 September 2020 – significantly edited the Introduction chapter to make it more suitable as a pre-study guide and to provide cues useful to instructors leading "inverted" teaching sessions.

27 September 2020 – added a Case Tutorial chapter.

17 April 2020 – added questions and Challenges.

27 August 2019 – continued writing Tutorial.

26 August 2019 – document first created.

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