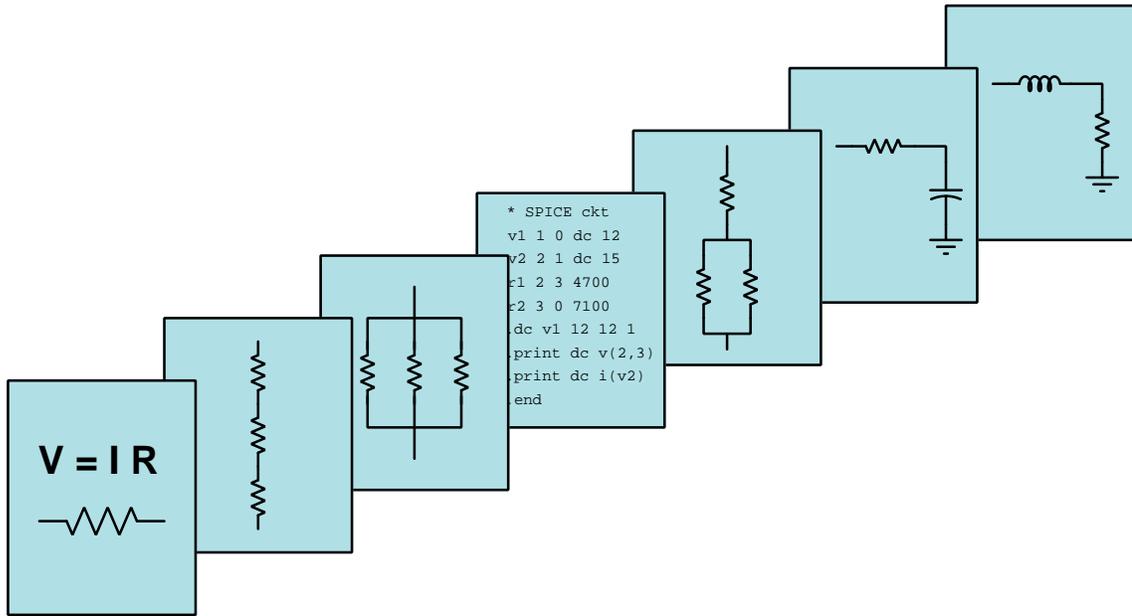


MODULAR ELECTRONICS LEARNING (MODEL) PROJECT



KIRCHHOFF'S CURRENT LAW

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Contents

1	Introduction	3
1.1	Recommendations for students	3
1.2	Challenging concepts related to Kirchhoff's Current Law	5
1.3	Recommendations for instructors	6
2	Case Tutorial	7
2.1	Example: Three-wire power system	8
3	Simplified Tutorial	13
4	Full Tutorial	17
4.1	Concept review	17
4.2	Charge conservation at nodes	24
4.3	Application to parallel circuits	26
5	Historical References	29
5.1	Gustav Robert Kirchhoff in his own words	30
6	Derivations and Technical References	31
6.1	Ground in electric and electronic circuits	32
7	Questions	37
7.1	Conceptual reasoning	41
7.1.1	Reading outline and reflections	42
7.1.2	Foundational concepts	43
7.1.3	Water pipe system	45
7.1.4	Three-wire electrical power distribution	46
7.1.5	Unusual ammeter measurements	47
7.1.6	Battery charging status	48
7.1.7	Generator failure	51
7.1.8	Series current sources	52
7.2	Quantitative reasoning	53
7.2.1	Miscellaneous physical constants	54
7.2.2	Introduction to spreadsheets	55
7.2.3	Paralleled current sources	58

CONTENTS	1
7.2.4 Currents in resistor networks	59
7.2.5 Currents in a large resistor network	60
7.2.6 Battery currents	61
7.2.7 Current divider	62
7.2.8 Solving for currents in a multi-source circuit	62
7.2.9 Interpreting a SPICE analysis	63
7.3 Diagnostic reasoning	65
7.3.1 Mysterious fuse failure	66
7.3.2 4-20 mA transmitter circuit fault	68
7.3.3 Multiple-LED circuit	70
8 Projects and Experiments	73
8.1 Recommended practices	73
8.1.1 Safety first!	74
8.1.2 Other helpful tips	76
8.1.3 Terminal blocks for circuit construction	77
8.1.4 Conducting experiments	80
8.1.5 Constructing projects	84
8.2 Experiment: Confirming Kirchhoff's Voltage Law	85
8.3 Project: (first project)	86
A Problem-Solving Strategies	87
B Instructional philosophy	89
C Tools used	95
D Creative Commons License	99
E References	107
F Version history	109
Index	111

Chapter 1

Introduction

1.1 Recommendations for students

Two extremely important principles of electric circuits are Kirchhoff's Voltage Law and Kirchhoff's Current Law, both named after Gustav Robert Kirchhoff who formulated them in the middle of the 19th century. These principles are every bit as important as Ohm's Law and Joule's Law, as they aid our analysis of all kinds of circuits. They are also pedagogically important because they clarify the essential natures of voltage and current, respectively.

All practitioners of electricity and electronics owe it to themselves to master Kirchhoff's Laws in order to build a firm conceptual foundation for later studies. This particular learning module focuses on the latter of Kirchhoff's Laws (current), beginning with a review of basic concepts as well as series and parallel circuit connections. Mastery of Kirchhoff's Current Law enables the determination of currents in any multi-path electrical circuit based on the magnitudes and directions of other currents in the same circuit.

Important concepts related to Kirchhoff's Current Law include **conservation laws**, the energy possessed by mass with certain types of **fields**, **series** versus **parallel** networks, **conductors** versus **insulators**, electrical **resistance**, **Ohm's Law**, **Joule's Law**, **sources** versus **loads**, **branch currents** in parallel networks, current **direction** and its relationship to values displayed by **ammeters**, and different types of electrical **charge carriers**.

Here are some good questions to ask of yourself while studying this subject:

- How might an experiment be designed and conducted to demonstrate the principle of Kirchhoff's Current Law? What hypothesis (i.e. prediction) might you pose for that experiment, and what result(s) would either support or disprove that hypothesis?
- How does current in a circuit relate analogously to the flow of liquid through pipes?
- Why is Kirchhoff's Current Law true – i.e. what is the fundamental principle it is based on?
- Why is current the same everywhere in a series network – what is the fundamental principle at work here?
- Why is voltage the same across all components within a parallel network – what is the fundamental principle at work here?
- Why is electrical power a function of both voltage and current?
- How is Kirchhoff's Current Law useful in ways that Ohm's Law and Joule's Law are not? In other words, how is Kirchhoff's Current Law uniquely useful?
- How might circuits behave if Kirchhoff's Current Law were not true?
- Why is it important to *annotate* a circuit's schematic diagram with symbols showing current direction when applying Kirchhoff's Current Law?

A good resource is the *Case Tutorial* chapter where you will find examples to apply and test your understanding of Kirchhoff's Current Law.

1.2 Challenging concepts related to Kirchhoff's Current Law

The following list cites concepts related to this module's topic that are easily misunderstood, along with suggestions for properly understanding them:

- **Defining a parallel network** – parallel networks are defined by all components spanning the same two sets of electrically-common points. From this definition, and from the Conservation of Energy, follows the conclusion that parallel-connected components must experience the same amount of continuous voltage. One challenge is that sometimes parallel-connected components do not physically or pictorially align in a parallel fashion, even though they are in fact parallel in the *electrical* sense. Another challenge is reasoning logically from this definition of “between the same two sets of electrically-common points” to the necessary conclusion of “same voltage”, as many students are so accustomed to memorizing facts without seeing the logical connections between those facts. A helpful way to both identify parallel connections as well as to conceptualize the commonality of voltage is to use color-highlighting to mark all electrically-common points with the same color. This way, all points in one set have one color, all points in the other set have another color, and all components connecting to points with those two colors must be connected in parallel with each other. This coloring approach is used in the Tutorial chapters where points A-B-C-D are all colored blue and points E-F-G-H are all colored green. The *Case Tutorial* section “Example: potential in open and shorted circuits” is a helpful reference for using color-highlighting to denote different amounts of electrical potential in a circuit.
- **Schematic annotation as a problem-solving tool** – as an instructor you will find most students new to the study of electronics attempt to solve circuit-analysis problems by inspection, or by plugging given values into familiar equations until something resembling an answer emerges. Such efforts are usually fruitless because it is difficult to track the real-world context of every circuit parameter doing things this way. You will need to demonstrate, encourage, and at times *require* your students to apply the alternative strategy of first sketching a schematic diagram of the circuit and then annotating it with all known values. Once the given information gains context through this annotation, appropriate principles (e.g. Ohm's Law, KVL, KCL) are much easier to identify and then properly apply for that circuit. Then, as calculated values get annotated on the same schematic diagram the next logical steps to follow become easier and easier to identify. I recommend using different colors to label different parameters (e.g. blue for voltage, red for current, black for component values, green for power). Students should have blank “scratch” paper and writing instruments with them at all times, even during discussions, so as to neatly organize information and to articulate and focus their thoughts!

1.3 Recommendations for instructors

This section lists realistic student learning outcomes supported by the content of the module as well as suggested means of assessing (measuring) student learning. The outcomes state what learners should be able to do, and the assessments are specific challenges to prove students have learned.

- **Outcome** – Demonstrate effective technical reading and writing

Assessment – Students present their outlines of this module’s instructional chapters (e.g. Case Tutorial, Tutorial, Historical References, etc.) ideally as an entry to a larger Journal document chronicling their learning. These outlines should exhibit good-faith effort at summarizing major concepts explained in the text.

Assessment – Students show how quantitative KCL results were obtained by the author in the Tutorial chapter’s examples.

- **Outcome** – Apply the concept of Kirchhoff’s Current Law to solve currents in any circuit

Assessment – Solve for an unknown current in a circuit where the schematic diagram shows other known currents; e.g. pose problems in the form of the “Currents in a large resistor network” Quantitative Reasoning question.

Assessment – Solve for currents in a circuit containing components capable of being either sources or loads; e.g. pose problems in the form of the “Battery currents” Quantitative Reasoning question.

- **Outcome** – Independent research

Assessment – Translate¹ Gustav Robert Kirchhoff’s own explanation of his current law from German into English and then summarize in your own words portions of it. A recommended source is page 513 of *Annalen der Physik und Chemie*, No. 4, Band LXIV, published in Germany in the year 1845 by Johann Christian Poggendorff.

¹If you aren’t fluent in German, feel free to use language-translation software to do this for you!

Chapter 2

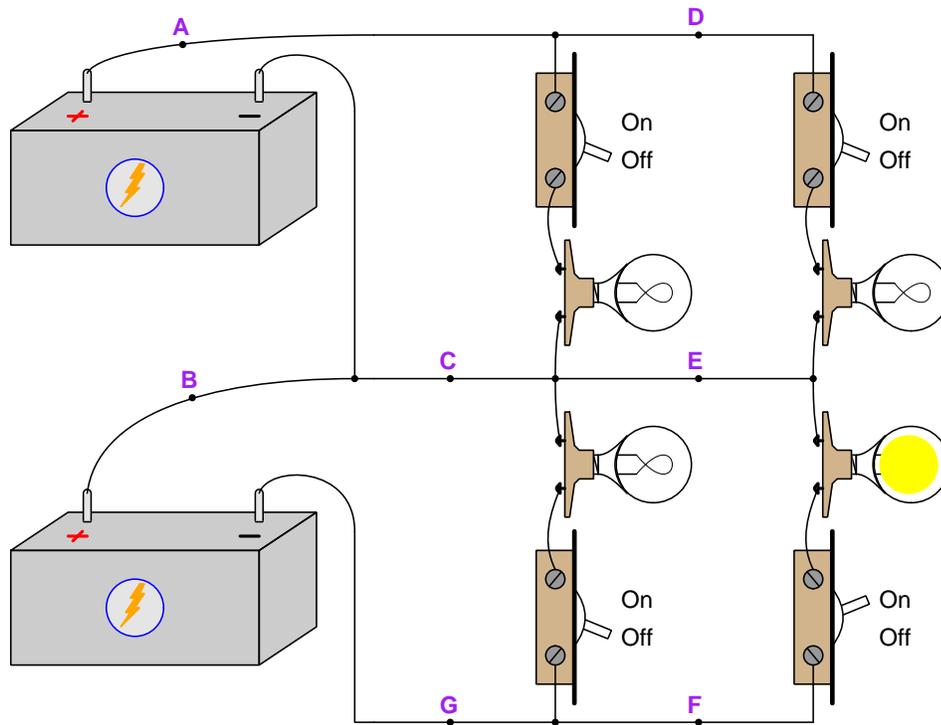
Case Tutorial

The idea behind a *Case Tutorial* is to explore new concepts by way of example. In this chapter you will read less presentation of theory compared to other Tutorial chapters, but by close observation and comparison of the given examples be able to discern patterns and principles much the same way as a scientific experimenter. Hopefully you will find these cases illuminating, and a good supplement to text-based tutorials.

These examples also serve well as challenges following your reading of the other Tutorial(s) in this module – can you explain *why* the circuits behave as they do?

2.1 Example: Three-wire power system

A common form of circuit for electric power distribution is the so-called *three-wire* system, an example of which appears here:



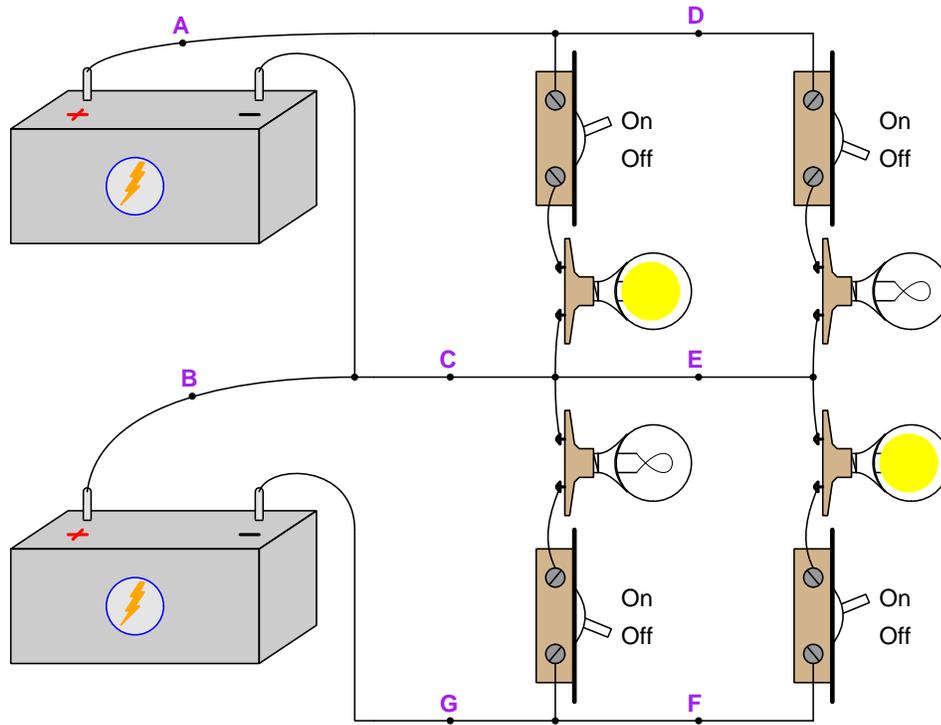
Rather than visually show where ammeters connect to circuit components, ammeter indications will be in written form as I values¹. With the circuit configured as shown in the above illustration, we obtain the following measurements:

- $I_A = 0$ Amperes
- $I_B = 2$ Amperes (left to right)
- $I_C = 2$ Amperes (left to right)
- $I_D = 0$ Amperes
- $I_E = 2$ Amperes (left to right)
- $I_F = 2$ Amperes (right to left)
- $I_G = 2$ Amperes (right to left)

¹Current measurements (I) imply measurements made at a single location, and so there will only ever be single-lettered subscripts for I . Conventional flow notation will be used to specify current direction.

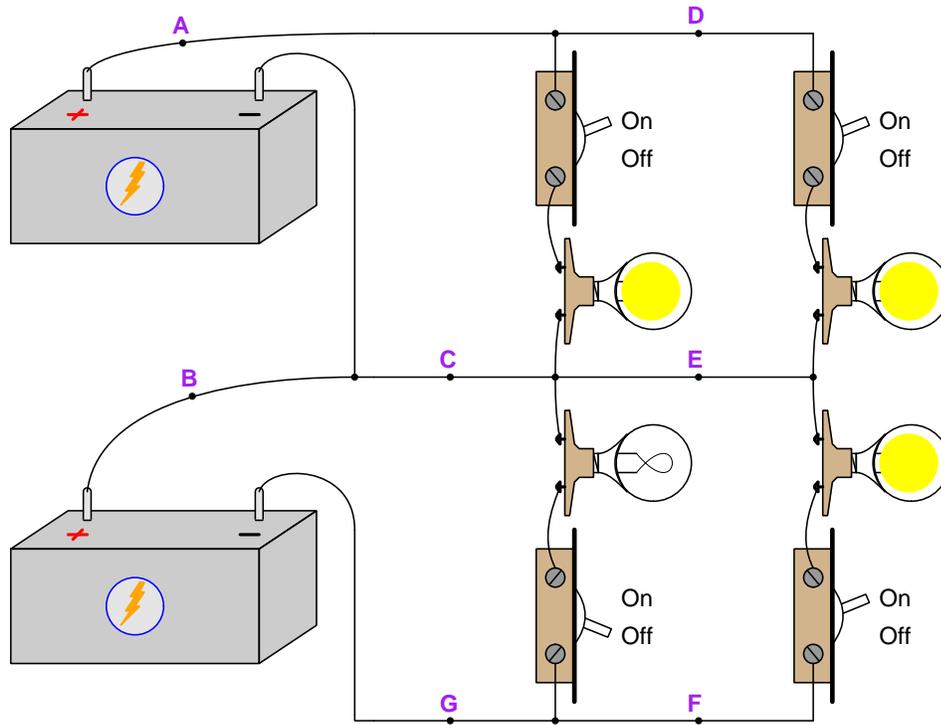
2.1. EXAMPLE: THREE-WIRE POWER SYSTEM

With a different switch configuration:



- $I_A = 2$ Amperes (left to right)
- $I_B = 0$ Amperes
- $I_C = 0$ Amperes
- $I_D = 0$ Amperes
- $I_E = 2$ Amperes (left to right)
- $I_F = 2$ Amperes (right to left)
- $I_G = 2$ Amperes (right to left)

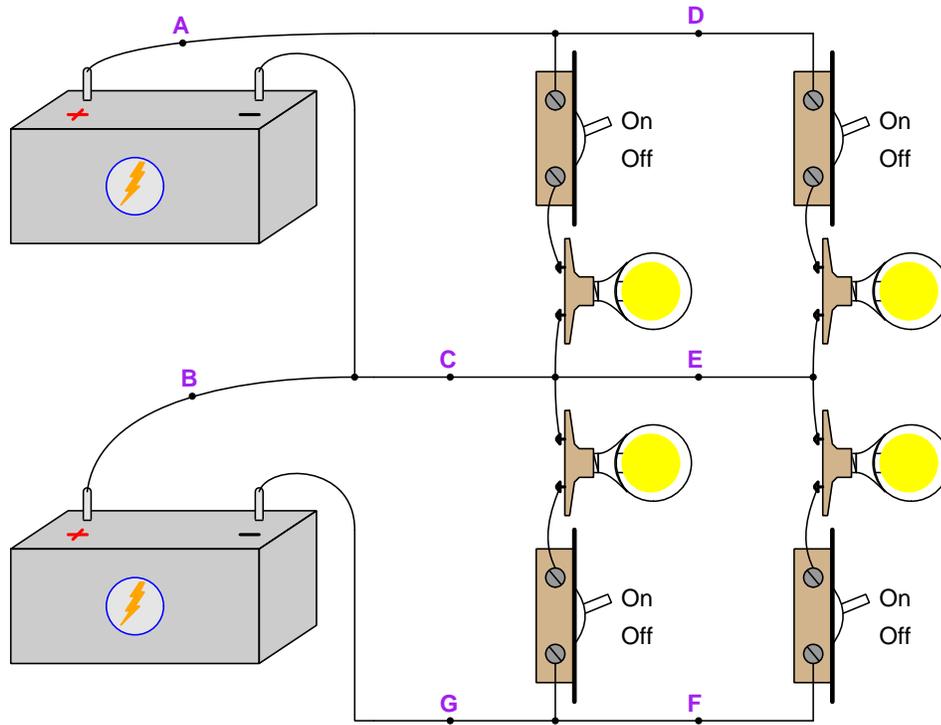
With yet another switch configuration:



- $I_A = 4$ Amperes (left to right)
- $I_B = 2$ Amperes (left to right)
- $I_C = 2$ Amperes (right to left)
- $I_D = 2$ Amperes (left to right)
- $I_E = 0$ Amperes
- $I_F = 2$ Amperes (right to left)
- $I_G = 2$ Amperes (right to left)

2.1. EXAMPLE: THREE-WIRE POWER SYSTEM

Finally, all four switches turned on:



- $I_A = 4$ Amperes (left to right)
- $I_B = 0$ Amperes
- $I_C = 0$ Amperes
- $I_D = 2$ Amperes (left to right)
- $I_E = 0$ Amperes
- $I_F = 2$ Amperes (right to left)
- $I_G = 4$ Amperes (right to left)

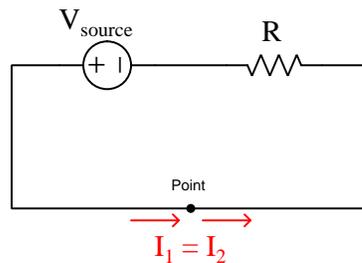
Chapter 3

Simplified Tutorial

Electricity consists of miniscule bits of matter called *charge carriers* moving through solid or ionized fluid materials. As matter, electric charge carriers possess *mass*, and mass is a *conserved quantity*. To be “conserved” means that mass is essentially a fixed quantity in the universe, being impossible to either create or destroy. Mass may alter its form, but the total quantity is fixed – this is known as the Law of Mass Conservation. Furthermore, the electric *charge* possessed by these charge carriers is also conserved quantity: its total quantity in the universe is fixed and cannot be either enhanced nor diminished – this is known as the Law of Electric Charge Conservation.

The intrinsic permanence of charge carriers leads to a simple conclusion about electricity: when charge carriers continuously drift through a conductive pathway to form an electric *current* (symbolized by the variable I), the rate of flow of these charge carriers through that pathway must be the same at all points. This is analogous to fluid moving through a length of pipe: for any continuous flow of fluid through the pipe, the rate of flow at any one point in the pipe must be the same as the rate of flow at any other point in that same pipe. Simply put, *flow in equals flow out*.

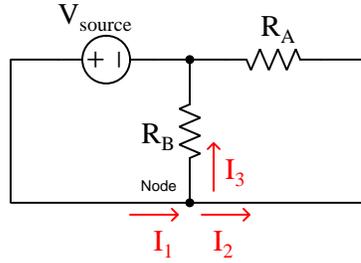
We may apply this principle to single points in a circuit by saying that the amount of current entering any *point* in a circuit must be the same as the amount of current exiting that point. A simple circuit is shown here for illustrative purposes:



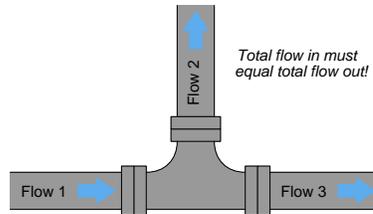
The permanence of electric charges make this equality of current fairly obvious: the only way I_1 could possibly be greater than I_2 is if charges somehow vanished after entering the point; the only way I_2 could possibly be greater than I_1 is if new charges sprang into existence within that point. The Law of Charge Conservation forbids either scenario, and so I_1 must always equal I_2 . Since a conductor is really just a series of such points, I measured at any point along a conductor must be

the same as I measured at any other point along that same conductor. This is why current is equal at any given time through *series*-connected components.

This same principle applies to junctions of multiple conductors at a single point (typically referred to as a *node*), as illustrated here:



Introducing a parallel branch for current at the labeled point means that some of the charge carriers entering the point as I_1 will exit that point as I_2 , but others will exit the point as I_3 . Adding another pathway to our circuit does not violate the Law of Electric Charge Conservation, and so it stands to reason that the charge carriers entering this point will still exit it in equal number, the only difference in this scenario being the exit route those charge carriers take. This is analogous to a “tee” junction in a piping system: the total mass flow rate into this tee must equal the total mass flow rate out of the tee regardless of the paths taken:



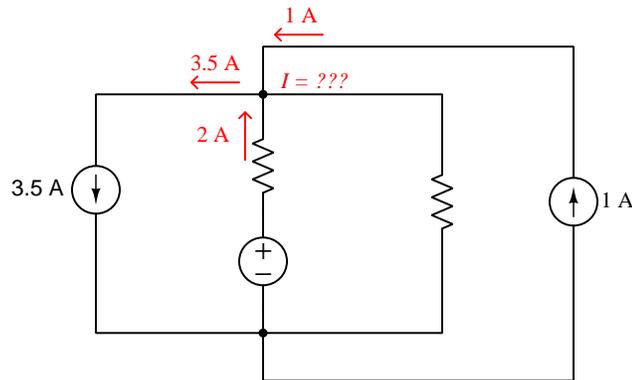
It is customary to attach mathematical signs to each current value based on its direction in relation to the point, currents entering the point being positive and currents exiting being negative. As signed quantities, their algebraic sum must be equal to zero. This is *Kirchhoff's Current Law*, or *KCL*:

$$I_1 + I_2 + I_3 = 0$$

In the example circuit shown, I_1 would be considered positive because it enters the node, while I_2 and I_3 would both be considered negative because they exit the node. This is how we may add three currents together and achieve a sum of zero: the first current is positive and the other two are negative.

This principle holds true for *any* configuration of conductors, no matter how complicated, because the underlying principle of electric charge conservation is universal.

Here is a numerical example of Kirchhoff's Current Law applied to a four-wire node in which we know three of the four currents:



The given current values are 2 Amperes entering the node from below, 1 Ampere entering the node from above, and 3.5 Amperes exiting the node to the left. If we consider the 2 Ampere and 1 Ampere currents as positive (because they are entering the node) and the 3.5 Ampere current as negative (because it is exiting the node), the algebraic sum of these currents leaves an imbalance of -0.5 Amperes:

$$2 \text{ A} + 1 \text{ A} - 3.5 \text{ A} = -0.5 \text{ A}$$

In other words, considering just these three currents we have a deficit of 0.5 Ampere at that upper node. In order to account for the Conservation of Electric Charges, this means the unknown current *must* be 0.5 Ampere in magnitude, and its direction *must* be entering the node:

$$2 \text{ A} + 1 \text{ A} - 3.5 \text{ A} + 0.5 \text{ A} = 0 \text{ A}$$

Note how Kirchhoff's Current Law allowed us to solve for this unknown current despite not knowing anything about the voltage source's value or the values of the two resistors. Since it is based on the Law of Electric Charge Conservation rather than Ohm's Laws, these other component values are not necessary so long as we happen to know all current values at a node except for one.

Chapter 4

Full Tutorial

4.1 Concept review

First, a review of some foundational principles.

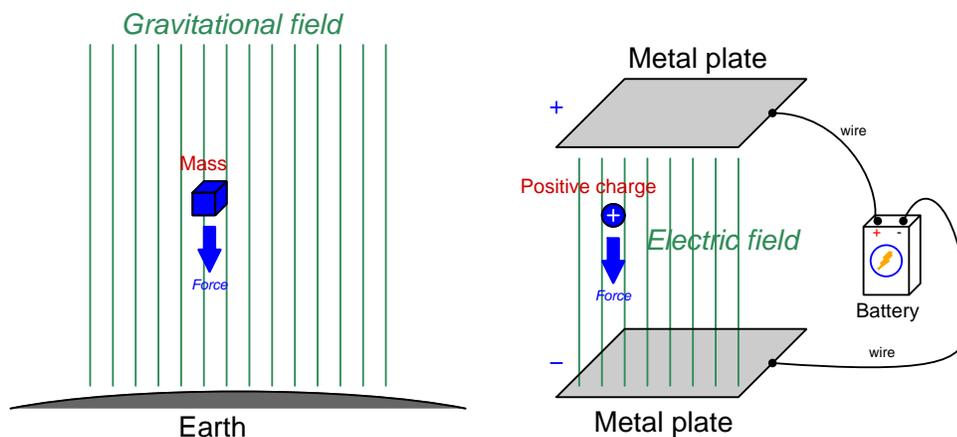
Energy is the ability to set matter into motion, and it may be found in a variety of different forms. Two major categories of energy are *kinetic energy* and *potential energy*, the former representing energy as a function of existing motion and the latter representing energy as a function of *possible* motion. For example, a moving vehicle possesses kinetic energy, while the fuel stored in that vehicle's tank possesses potential energy.

Energy is a *conserved* quantity, which means it can neither be created or destroyed, but the sum total always remains constant. This fact is codified as the *Law of Energy Conservation*, and so far as anyone knows there are no exceptions to this law.

The classic example of potential energy is a suspended mass within a gravitational field such as what we all experience living on planet Earth. The amount of potential energy possessed by any suspended mass is a function of the mass's magnitude, the strength of the gravitational field, and the height of the suspension. Mathematically, this form of potential energy is defined as $E_p = mgh$, where E_p is the amount of potential energy in Joules, m is the amount of mass in kilograms, g is the acceleration of gravity¹ in meters per second per second, and h is the amount of height in meters. Note that *height*, unlike the other variables in this formula, is a *relative* quantity between two locations. A mass only has potential energy with regard to how far it *might* fall. A mass resting at the top of a staircase has a certain amount of potential energy relative to a point half-way down the staircase, and twice that much potential energy relative to the very bottom of the staircase. It is impossible to determine exactly how much potential energy that mass resting at the top of the staircase possesses, but only how much it could potentially release if it falls to some pre-defined point. In other words, potential energy only has meaning when we compare *two* different locations or states-of-being.

¹Acceleration is an expression of how rapidly an object's velocity changes, and as such may seem to be a strange way to define the effect of gravity. However, it actually is quite simple to understand: g is the rate at which any object will accelerate in that gravitational field if allowed to free-fall (i.e. falling with no force to oppose its motion).

Masses suspended in gravitational fields are not the only objects possessing potential energy. Electric charges “suspended” in electric fields also possess potential energy, and just like gravitational potential energy, this *electric potential energy* is also relative between two different locations. The following illustration compares electric potential energy with gravitational potential energy:



An electric charge experiences a force when exposed to an electric field, just as a mass experiences force (called "weight") when exposed to a gravitational field

It is more convenient in the study of electric circuits to refer to the amount of electric potential energy possessed per unit of electric charge than it is to refer to the total potential energy held by any collection of charges. *Potential* is the term used to describe this “potential energy per charge”, and it is analogous to expressing gravitational potential energy in terms of energy per unit of mass². *Voltage* is the amount of potential difference between two points, and is measured in the unit of the Volt (V), a Volt defined as one Joule of electric potential energy per Coulomb of electric charge³.

Electric charge, like energy and also like mass, is a *conserved* quantity. Just as the Law of Energy Conservation states that energy is eternal and therefore cannot be created or destroyed, the Law of Charge Conservation tells us electric charges are likewise eternal and can neither appear nor disappear but must always remain.

Since energy is conserved, and electric charge is conserved, it should come as no surprise that voltage (the difference of potential energy per charge between two different locations) is also a conserved quantity. This means voltage cannot simply appear or disappear, but must always be accounted for. Electric charges may gain potential energy by moving against the pull of the electric field (like a mass being hoisted to a higher elevation) or lose potential energy by moving with the pull of the electric field (like a mass allowed to fall to a lower elevation), but in both cases this represents a *transfer* of energy either to or from that charge, respectively. If an electric charge gains potential, that energy must have come from somewhere. If an electric charge loses potential, that energy must go somewhere.

²If gravitational potential energy is defined by the formula $E_p = mgh$, then gravitational *potential* (i.e. potential energy per unit mass, or $\frac{E_p}{m}$) is simply gh .

³One Coulomb of charge is equal to 6.2415×10^{18} electrons' worth of electric charge.

A very interesting and important quality of electric charges is their ability to move through some types of matter. Metals, for example, contain a multitude of “free” electrons (a type of subatomic particle) which act as electric charge carriers which may move anywhere within a metal object’s volume when exposed to an electric field. Certain liquids contain *ions* (atoms or molecules containing an unequal number of electrons and protons, and therefore an imbalanced charge state) which likewise are free to drift within the liquid’s volume when exposed to an electric field. We classify substances possessing free charge carriers as electrical *conductors*, and the bulk motion of charge carriers in any particular direction through a conductor as an electric *current* (I). The coordinated motion of charge carriers may occur as far as the conductive material extends, which means we may use materials such as copper or aluminum (both excellent metallic conductors) to convey charge carriers across huge distances. Substances lacking free charge carriers are called *insulators*, this classification of materials including all gases⁴, most plastics, most oils, glass, dry wood, and pure water.

Not all conductors are equally conductive – that is to say, not all conductive objects share the same mobility of charge carriers. The degree to which an object opposes the free motion of electric charge carriers through it is called *resistance* (R). A perfect conductor and a perfect insulator are just limiting cases (i.e. low and high extremes) of electrical resistance, which is a continuum.

When electric charge carriers drift through a conductive substance, we may quantify their rate of motion in terms of how many Coulombs of charges move past a particular point per second of time. This is the definition of the *Ampere* (A), the basic unit of measurement for electric current. Any resistance encountered by the moving charge carriers of an electric current results in some energy lost by those charge carriers in the form of heat. This means a *voltage* will develop across any resistance as a current passes through it, that voltage being the amount of energy lost⁵ per unit charge due to resistance.

The mathematical relationship between voltage (V), current (I), and resistance (R) is called Ohm’s Law, shown here in three algebraically equivalent forms:

$$V = IR \quad I = \frac{V}{R} \quad R = \frac{V}{I}$$

Resistance is measured in the unit of the Ohm (Ω). By applying Ohm’s Law ($R = \frac{V}{I}$), we may define one Ohm as one Volt of potential difference per Ampere of current.

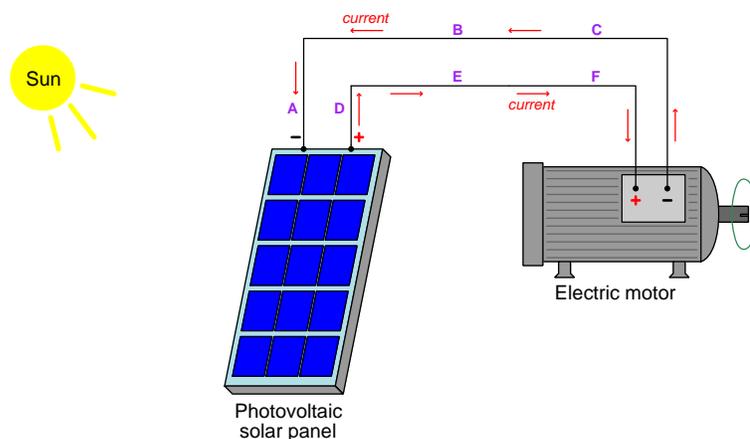
The mathematical relationship between *power* (P) which is the rate of energy transfer measured in Joules of energy per second of time (i.e. Watts, W) and the above-listed variables is called Joule’s Law, shown here in three equivalent forms:

$$P = IV \quad P = \frac{V^2}{R} \quad P = I^2R$$

⁴Gases may be made electrically conductive, but only by infusing sufficient energy into the gas molecules to cause the atoms’ electrons to be forcibly removed. Such ionization of gas molecules may be initiated by the application of highly-concentrated electric fields or by heating the gas to “glowing” temperatures. Gases outside of these extreme conditions are insulators.

⁵Since energy is always conserved, it is never really “lost” in the absolute sense of the word. What we mean here is that energy is *lost by* (i.e. given up by) each charge passing through, and that energy is then manifest as heat in the conductive substance.

Electric charges, with their ability to absorb and release potential energy, have the ability to act as *couriers of energy*. This is perhaps the single most significant reason why electricity is such a useful phenomenon in modern society: it is possible to build *circuits* made of metal wire where charge carriers are allowed to circulate, with devices infusing energy into the charge carriers at one location and other devices extracting energy from the charge carriers at another location. The electric charges moving around the circuit function as a sort of “conveyor belt” picking up energy at one location and transporting it to be used at another location. The following illustration shows such a circuit, with a photovoltaic panel transferring energy from photons⁶ of sunlight to the charge carriers in the metal wires, and an electric motor transferring energy from the charge carriers to a rotating shaft which may turn a mechanism to do useful work:



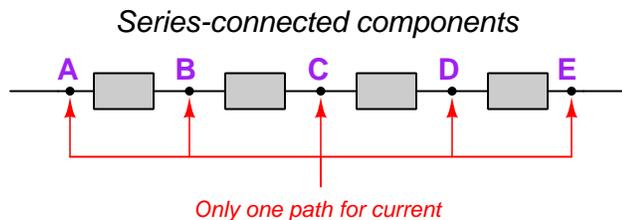
Here we see an electric current symbolized by red-colored arrows circulating counter-clockwise from the “+” terminal of the photovoltaic panel through points D, E, and F to the “+” terminal of the electric motor, through the motor to its “-” terminal, and then through points C, B, and A back to the “-” terminal of the photovoltaic panel where they pass through the panel and re-emerge at its “+” terminal. The + and - *polarity* symbols are intended to represent the relative potential energy states of charge carriers passing between two terminals on an electrical device. We see current entering the photovoltaic panel’s “-” (i.e. low energy) terminal and exiting the photovoltaic panel’s “+” (i.e. high energy) terminal because the panel transfers energy from the incident sunlight to the electric charge carriers passing through. We see current entering the electric motor’s “+” (i.e. high energy) terminal and exiting the electric motor’s “-” (i.e. low energy) terminal because the motor transfers potential energy away from the passing charge carriers and uses that energy to turn a mechanism. The orientation of the polarity symbol pairs with regard to the current direction tells us whether the device in question is a *source* of energy to the passing charges or a *load* drawing energy away from the passing charges.

Polarity marks only have meaning as *pairs*, because voltage is a function of potential energy, and potential energy is only quantifiable between *two* different points or conditions. Note that the wires themselves in the above circuit have no polarity marks shown across their ends, because we assume

⁶A “photon” is the particle form of light, possessing energy in the form of oscillating electric and magnetic fields but lacking mass.

the electrical resistance of those wires is negligible and therefore the voltage manifest between each of the wire's two ends is likewise negligible.

When electrical components are connected together such that they form a chain providing only one path for current, they are said to be connected in *series* with each other:



From this defining characteristic of only one path for current, and from the Law of Charge Conservation which states electric charges are eternal, we may conclude that any continuous current through a series network⁷ must result in exactly same rate of charge carrier motion at every point in the network; i.e. *current must be equal at all points in a series circuit*.

Incorporating the Law of Energy Conservation, we may also conclude that voltages across a set of series-connected components algebraically add to equal the series network's total voltage: as electric charge carriers move from one end of the series chain to the other, their overall gain or loss of energy must be the sum of all the individual gains and losses of energy along the way since energy can neither be created nor destroyed. It also stands to reason that the total resistance (R) of a series network is equal to the sum of all the individual component and wire resistances. Mathematically stating each of these series circuit properties:

$$I_{total} = I_1 = I_2 \cdots = I_n$$

$$V_{total} = V_1 + V_2 \cdots + V_n$$

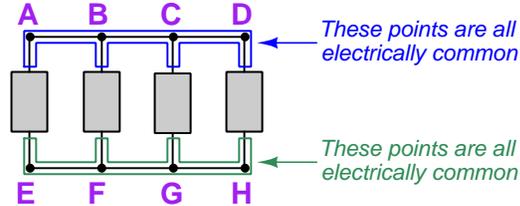
$$R_{total} = R_1 + R_2 \cdots + R_n$$

One practical application of series connections is in the measurement of electric current. Instruments designed to measure electric current are called *ammeters* and are always connected in series with the component whose current we intend to measure in order to guarantee the ammeter experiences the exact same amount of current as the component in question.

⁷A "network" defined here as any set of interconnected wires, devices, etc.

An alternative network form, called *parallel*, is where all components connect between the same two sets of conductors. Points directly connected together by a conductor of negligible resistance are said to be *electrically common* to one another, and the relative absence of resistance between those points ensures charge carriers may freely move between the connected points without losing energy. This equivalence of energy states between charge carriers is called *equipotentiality*:

Parallel-connected components



The equipotentiality of charge carriers along each of the two sets of points is important to understand, as it explains the behavior of parallel-connected components. If charge carriers at points A through D all exist at the same potential energy level, and charge carriers at points E through H likewise all exist at the same potential energy level, then the difference of potential *between* any point A-D and any point E-H (e.g. from A to E, from D to H, from B to G, etc.) must be the same. *Therefore voltage across every component in a parallel network is the same.*

Current values through each component in a parallel network, however, may differ substantially. In fact, Ohm's Law ($I = \frac{V}{R}$) guarantees this will be the case if the components in question happen to have differing resistance values (R) but all share the same voltage value (V). Given the Law of Charge Conservation, we may conclude that the total current for a parallel network must be equal to the algebraic sum of all component ("branch") currents. Another property of parallel networks is that their total resistance is less than the resistance of the lowest-resistance branch. Mathematically stating each of these parallel circuit properties:

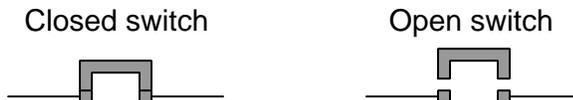
$$V_{total} = V_1 = V_2 \cdots = V_n$$

$$I_{total} = I_1 + I_2 \cdots + I_n$$

$$R_{total} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} \cdots + \frac{1}{R_n}}$$

One practical application of parallel connections is in the measurement of voltage. Instruments designed to measure electric potential difference are called *voltmeters* and are always connected in parallel with the component whose voltage we intend to measure in order to guarantee the voltmeter experiences the exact same amount of voltage as the component in question.

Switches are electrical components designed to deliberately establish (“make”) or sever (“break”) conductive pathways. Switches are usually designed such that a movable piece of metal arbitrarily contacts a stationary piece of metal, typically by the actuation of a pushbutton, lever, or some other mechanism. A switch positioned to make contact and permit the flow of electric charge carriers through it is said to be *closed* or *shorted*. A switch positioned to break contact and forbid the flow of current is said to be *open*. The following illustration shows a switch in both states:



A closed (shorted) switch guarantees negligible voltage drop across its terminals because the metallic bridge formed by the switch makes those two points electrically common, and therefore equipotential with each other. By extension, any component connected in parallel with a closed switch will also experience negligible voltage because paralleled components must share the same voltage. A closed (shorted) switch *may* conduct current, but this is not a guaranteed outcome because it depends on the presence of an external source to motivate that current.

An open switch guarantees negligible current through it, because the air gap between the switch’s metal pieces is electrically insulating. By extension, any component connected in series with an open switch will also experience negligible current, because series components must share the same current. An open switch *may* experience a voltage between its terminals, but this is not a guaranteed outcome because it depends on the presence of an external source to create that voltage.

Switches are used to insert controlled “open” and “short” conditions into electrical circuits, but sometimes these conditions appear without our consent. A metal wire that accidentally detaches from a terminal, for example, is an unintentional *open*, otherwise known as an open *fault*. A metal wire that accidentally comes into contact with another conductor, for example, is an unintentional *short*, otherwise known as a shorted *fault*. A thorough and accurate understanding of foundational electric circuit principles such as voltage, current, resistance, opens and shorts, equipotentiality, Ohm’s Law, Joule’s Law, series networks, and parallel networks is necessary to quickly identify the nature and location of faults in circuits, which is why this learning module (and others in the collection) devote so much text to the explanation and application of these concepts.

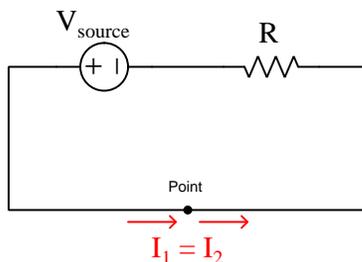
4.2 Charge conservation at nodes

Now that we have reviewed some fundamental terms and principles of electric circuits, let us move on to the topic at hand: *Kirchhoff's Current Law*, often abbreviated as *KCL*. As with so many other characteristics of electricity, KCL is really nothing more than an expression of fundamental conservation laws. In this case, the principal conservation law expressed by KCL is the *Conservation of Electric Charge*.

Like all conservation laws, the Law of Electric Charge Conservation tells us that the quantity in question (in this case, electric charge) is eternal: that is to say, it cannot be created from nothing nor can it be annihilated. At the end of every event, the sum total of all electric charges in the universe must be the same as what it was at the beginning of the event.

Electricity is generally defined as the phenomenon associated with electric charges and their motion. As previously mentioned, any substance possessing free-moving electric charges is called a *conductor*. These mobile charges may take the form of individual subatomic particles, such as the case in metallic conductors where the metal atoms' outer-most electrons are free to drift to and fro amongst and between the stationary nuclei. Alternatively, mobile electric charges may take the form of whole atoms or even molecules possessing a net charge that are able to move about, such as the case of conductive liquids and ionized gases. This is why the generic term *charge carrier* is preferred over the specific term *charge* when referring to electricity: it gives us a technically accurate way to describe any mobile electric charge regardless of form.

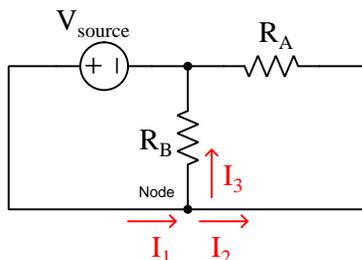
One of the basic principles of series circuits is that the amount of (continuous) current at any point in the circuit will be the same as the amount of current anywhere else in that circuit, for the simple reason that a series circuit only has one path for current to exist. Another way of stating this fact is to say that the amount of current entering a conductor must be the same as the amount of current exiting at the other end of that conductor, similar to the flow rate of a fluid entering and the exiting a length of pipe. We may apply this principle to single points in a circuit by saying that the amount of current entering any *point* in a circuit must be the same as the amount of current exiting that point. A simple circuit is shown here for illustrative purposes:



So long as we grasp the fact that electric charges cannot be created or destroyed, the equality of charge carrier flow rates in and out of this point becomes fairly obvious. The only way I_1 could possibly be greater than I_2 is if charges somehow vanished after entering the point; the only way I_2 could possibly be greater than I_1 is if new charges somehow emerged from nothing within that point. The Law of Charge Conservation forbids either scenario, and so I_1 must always⁸ equal I_2 .

⁸An important caveat to this rule is that the currents in question are *continuous*. It actually is possible for charge

At first it may appear as though this principle is so obvious that it doesn't deserve any elaboration. However, we will find that this principle becomes very useful when we apply it to points joining more than two conductors (such a point typically referred to as a *node*), such as this circuit:



Introducing a second resistor into this circuit creates a *parallel* branch for current at the labeled node. Some of the charge carriers entering the node as I_1 will exit that node as I_2 , but others will exit the node as I_3 to return to the negative pole of the voltage source. Adding another pathway to our circuit does not violate the Law of Electric Charge Conservation, and so it stands to reason that the charge carriers entering this node will still exit it in equal number, the only difference in this scenario being the exit route those charge carriers take.

So long as charges cannot be created or destroyed, it must be that the sum of currents exiting this node is equal to the one current entering the node. Mathematically expressed with all currents as positive values:

$$I_1 = I_2 + I_3$$

If we arbitrarily assign mathematical signs to each current based on its direction in relation to the node, we see that we either have one positive current (I_1 entering) balanced by two negative currents (I_2 and I_3 exiting), or alternatively two positive currents balanced by one negative current. Whatever the currents' direction and the convention we choose for attaching + or - signs to those current values, their sum total must always be zero. This is *Kirchhoff's Current Law*, or *KCL*:

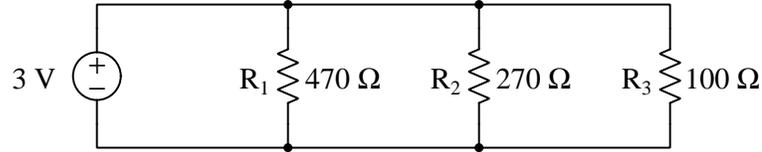
$$I_1 + I_2 + I_3 = 0$$

This principle holds true for nodes connecting any number of conductors together, and so is applicable to any point existing within any circuit.

carriers to enter an object (point) and not emerge from that point, as happens routinely in *static electric discharges* where charge carriers momentarily move to equalize what was formerly a large imbalance of electric charge. For any flow that continues unabated over time, though, current out must equal current in.

4.3 Application to parallel circuits

Parallel circuits provide an obvious application for KCL, with “branched” pathways allowing currents to split and merge. Let us consider a simple three-resistor parallel circuit to see how KCL might be useful to us:



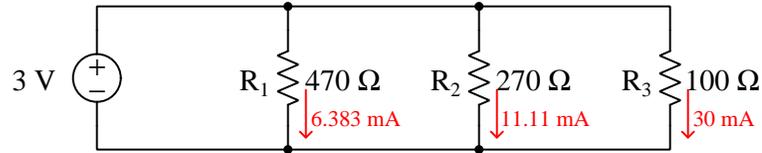
We know from our study of parallel circuits that parallel-connected components share the same voltage, since they share the same two sets of equipotential points. This means each resistor in this circuit experiences a voltage of 3 Volts, identical to the output of the voltage source. We may apply Ohm’s Law ($I = \frac{V}{R}$) to the calculation of resistor currents, since we now know the voltage across each resistor as well as the resistance of each resistor:

$$I_{R1} = \frac{V}{R_1} = \frac{3 \text{ V}}{470 \Omega} = 6.383 \text{ mA}$$

$$I_{R2} = \frac{V}{R_2} = \frac{3 \text{ V}}{270 \Omega} = 11.11 \text{ mA}$$

$$I_{R3} = \frac{V}{R_3} = \frac{3 \text{ V}}{100 \Omega} = 30.00 \text{ mA}$$

A good practice when analyzing circuits is to annotate the schematic diagram with calculated values, to remind ourselves of what those values represent. In this case, we will show the directions of each current as well as their magnitudes, using red-colored arrows:



Each node presents an opportunity to apply KCL. Since we know the algebraic sum of all currents entering and exiting any node must be zero, the ideal place to apply KCL is any node where all but one current value is known. Surveying our circuit, this rules out the nodes immediately above and below R_1 because although we know the amount of current through that branch (6.383 mA) we presently know neither of the other currents through the horizontal wires at that node.

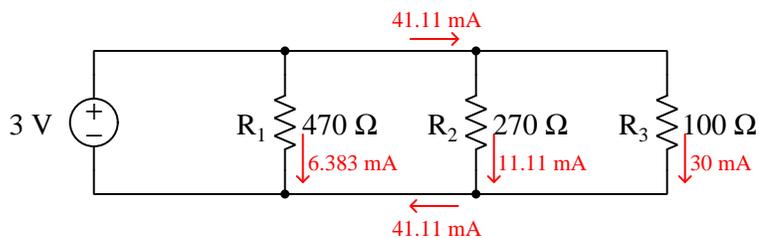
The nodes flanking R_2 are a better application for KCL right now, because for each of those nodes we happen to know two of the three current values. At the node above R_2 , we have 11.11 mA of current exiting downward and 30 mA exiting to the right. Following the convention of exiting currents being negative and entering currents being positive, we may solve for the amount of current entering that node from the left:

$$I - 11.11 \text{ mA} - 30 \text{ mA} = 0$$

$$I = 11.11 \text{ mA} + 30 \text{ mA}$$

$$I = 41.11 \text{ mA}$$

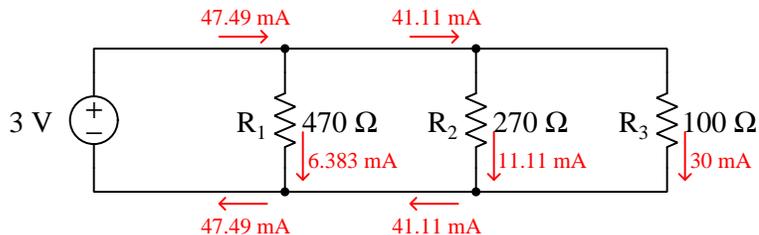
KCL applies likewise to the node immediately below R_2 . 30 mA of current enters that node from the right, and 11.11 mA enters that node from the top, which means 41.11 mA must exit it to the left. Annotating the schematic diagram once again with our calculated values:



Only now are we able to apply KCL to the nodes above and below R_1 , for now we know the current values for each of the wires to the right of each node. First applying KCL to the node above resistor R_1 , then to the node below resistor R_1 :

$$I - 6.383 \text{ mA} - 41.11 \text{ mA} = 0 \quad ; \quad I = 47.49 \text{ mA} \quad (\text{Current entering node above } R_1)$$

$$I + 6.383 \text{ mA} + 41.11 \text{ mA} = 0 \quad ; \quad I = -47.49 \text{ mA} \quad (\text{Current exiting node below } R_1)$$



This current value of 47.49 mA to the left of R_1 happens to be the total (source) current for this parallel circuit which confirms another property of parallel circuits: namely that total current is equal to the sum of all branch currents.

Chapter 5

Historical References

This chapter is where you will find references to historical texts and technologies related to the module's topic.

Readers may wonder why historical references might be included in any modern lesson on a subject. Why dwell on old ideas and obsolete technologies? One answer to this question is that the initial discoveries and early applications of scientific principles typically present those principles in forms that are unusually easy to grasp. Anyone who first discovers a new principle must necessarily do so from a perspective of ignorance (i.e. if you truly *discover* something yourself, it means you must have come to that discovery with no prior knowledge of it and no hints from others knowledgeable in it), and in so doing the discoverer lacks any hindsight or advantage that might have otherwise come from a more advanced perspective. Thus, discoverers are forced to think and express themselves in less-advanced terms, and this often makes their explanations more readily accessible to others who, like the discoverer, comes to this idea with no prior knowledge. Furthermore, early discoverers often faced the daunting challenge of explaining their new and complex ideas to a naturally skeptical scientific community, and this pressure incentivized clear and compelling communication. As James Clerk Maxwell eloquently stated in the Preface to his book *A Treatise on Electricity and Magnetism* written in 1873,

It is of great advantage to the student of any subject to read the original memoirs on that subject, for science is always most completely assimilated when it is in its nascent state . . . [page xi]

Furthermore, grasping the historical context of technological discoveries is important for understanding how science intersects with culture and civilization, which is ever important because new discoveries and new applications of existing discoveries will always continue to impact our lives. One will often find themselves impressed by the ingenuity of previous generations, and by the high degree of refinement to which now-obsolete technologies were once raised. There is much to learn and much inspiration to be drawn from the technological past, and to the inquisitive mind these historical references are treasures waiting to be (re)-discovered.

5.1 Gustav Robert Kirchhoff in his own words

Gustav Robert Kirchhoff formulated his voltage and current laws while still a university student. A reference to his current law is found on page 513 of the German scientific journal *Annals of Physics and Chemistry* published in 1845:

1) wenn die Drähte 1, 2, .. μ in einem Punkte zusammenstoßen,

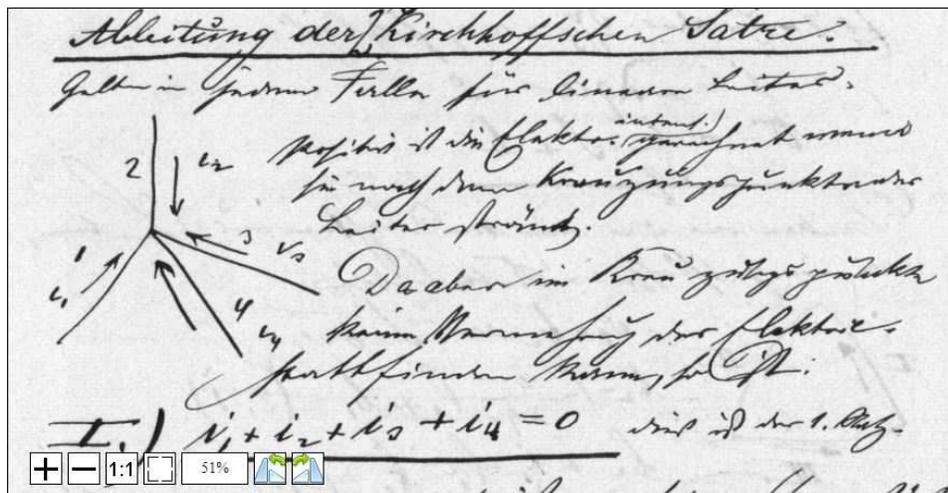
$$I_1 + I_2 + \dots + I_\mu = 0,$$

wo I_1, I_2, \dots die Intensitäten der Ströme bezeichnen, die jene Drähte durchfließen, alle nach dem Berührungspunkte zu als positiv gerechnet;

[page 513]

Here we see his statement that the algebraic sum of currents at a node equals zero ($I_1 + I_2 + \dots + I_\mu = 0$).

Kirchhoff also wrote a set of lecture notes for teaching purposes in 1869-1870, and in those notes we find the following reference to his current law written on page 76:



Note how his diagram shows four currents all entering a node, which is possible only if one uses negative values for at least one of those four currents.

Chapter 6

Derivations and Technical References

This chapter is where you will find mathematical derivations too detailed to include in the tutorial, and/or tables and other technical reference material.

6.1 Ground in electric and electronic circuits

Ground and *grounding* within both electric power and electronic circuits are commonly misunderstood topics, largely because these terms can often have very different meaning depending on context. Here we will explore some of these different meanings and hopefully eliminate some of the confusion surrounding them.

The very first concept we must understand in order to comprehend what “ground” is in any circuit is that *voltage* is fundamentally a *relative* measurement of potential energy different *between two points*. This is really true of all forms of potential energy, because “potential” means energy that has not yet been put to work setting matter into motion, implying a final state yet to be realized. When we say an electrical charge carrier possesses potential energy (i.e. electrical potential), this means it *could* (potentially) release energy to do work if it were to fall into a lower state of energy. The difference in energy held by that electric charge carrier from its original (high-potential) state to some other lower-potential state will be the amount of energy released by that charge carrier to do work, with that amount of work done dependent on both the charge carrier’s initial energy level and its final energy level. *Voltage* is the term we use to describe such a difference in electrical potential.

We say that voltage always exists “between two points” or “between two locations” in a circuit because voltage is the amount of potential energy either lost or gained by electric charge carriers from one position in a circuit to another. It is impossible to quantify how much energy will be gained or lost by charge carriers unless we know both the starting point and the ending point for those charge carriers. In order to quantify the absolute amount of potential energy possessed by anything, we would have to know where or how it could lose every bit of its stored energy to be left with zero capacity to do work¹. This is really unknowable from a practical perspective, and so the best we can do is quantify gains or losses in potential energy between different states.

This fact about voltage is also why we label it with *pairs* of “+” and “-” symbols, the “+” symbol representing the point of *higher potential* and the “-” symbol representing the point of *lower potential*. We cannot place an absolute value on electrical potential at any single location because no one can say where the location of zero energy is in the universe for electric charges. The best we can do is to rate one location as having higher or lower electrical potential than another, quantifying only the difference in potential between those two locations.

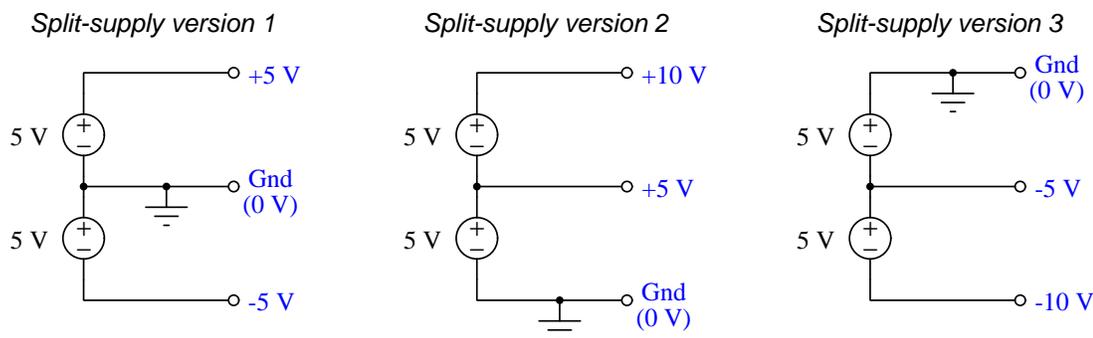
Now that we have reviewed this property of voltage and of potential energy in general, we are ready to explore what is meant by *ground* and *grounding*.

¹Using gravitational potential energy as an example, a weight hoisted above the Earth’s surface certainly possesses potential energy, and we might be tempted to quantify this potential energy by simply measuring its weight and its height above ground level and then multiplying those two quantities (i.e. potential energy equals force times distance). However, there are more places this weight could *potentially* fall than just ground level. That same weight, hoisted above the Earth’s surface to the same height, will be assessed to have a far greater amount of potential energy if we consider it potentially falling down a deep hole rather than just falling to ground level. For that matter, one could even argue the weight has a *negative* quantity of potential energy with respect to what it would possess at the summit of the highest mountain! Potential energy, defined as the amount of work that *could* be done, necessarily depends on a system’s initial condition *and* any number of different final conditions of which none are known to be truly zero-energy.

How do we rate the height of mountain peaks? Certainly, we may rate one mountain peak as being some number of meters higher or lower than another, but generally what people do instead is measure a mountain's peak height compared to *Mean Sea Level*. Since water is a liquid, and liquid tend to settle at equal heights under the influence of gravity when they are free to move about, the level of oceans throughout the world provide a relatively stable reference point for measurements of altitude. If we imagine a world without oceans, perhaps one with nothing but dry land and no open bodies of water anywhere, people living at different locations in this world would likely choose some flat-land reference points local (to them) for their measurements of nearby mountains, but these localized altitude measurements would be confusing if not meaningless for world-wide comparisons of mountain peaks.

Much the same is true of electrical and electronic circuits, where voltage is an important measure of electrical potential energy as well as a measure of signal strength where electricity is used as an analogue for representing other things². One way to conveniently rate electrical potentials at different points in a circuit is by arbitrarily choosing one of those points to be the “zero-energy” reference point, and calling it *ground*. Ground in an electrical or electronic circuit is analogous to sea level for rating mountain peak heights or a *datum* or *benchmark* used for surveying land: simply a point of reference agreed-upon to be “zero” for the sake of ranking all other potentials to it.

A good example of this is an electrical network popularly known as a *split DC power supply*. Three different versions of this power supply network are shown with the same pair of 5-Volt DC sources, the only difference being the arbitrary location of the “Ground” (Gnd) point:



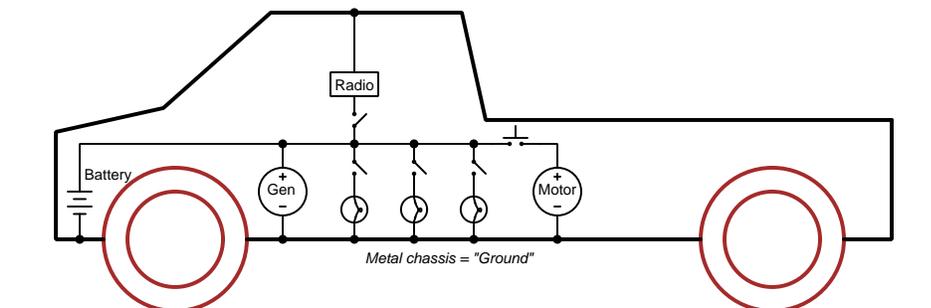
Each terminal's voltage label represents the amount of electrical potential there *relative to the “Ground” terminal*; i.e. “Ground” is simply a “sea-level” reference point against which all other potentials may be measured. The three different versions shown here illustrate just how arbitrary the location of “Ground” really is. Any load connected between the same pairs of terminals on each power supply would experience the same amount of voltage regardless of the “Ground” terminal assignment.

This is one meaning of the word “ground” in electric and electronic circuits: an arbitrary point of reference for the measurement and comparison of electrical power and signal potentials at different locations in the circuit.

²Many, many examples exist of voltage being used to represent non-electrical quantities. In digital computer systems, DC voltage levels represent binary “1” and “0” number states. In analog electronic sensor systems, voltage may represent temperature, speed, force, weight, sound pressure, or any number of other physical quantities.

Another meaning of the word “ground” in circuits is a metallic surface used as a common conductor throughout some device or system for reasons of wiring economy. Automobiles with metal frames and chassis typically use their metallic bodies as an electrical conductor for nearly all circuits contained within. Doing so eliminates the need to install separate copper-wire conductors for every source and every load, thereby saving expense and weight:

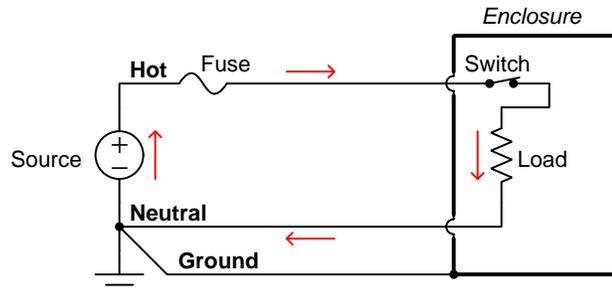
“Negative-ground” vehicle electrical system



Modern automotive electrical systems are “negative-ground” which means the vehicle chassis is always the lowest-potential point in any portion of the circuit. Interestingly, some early automobiles used “positive-ground” wiring which simply had the reverse polarity on all sources and loads: the vehicle chassis being the highest-potential point and all other points being either equal to it or at lower (–) potentials. This choice of polarity is really arbitrary, as electric circuits may be designed and built to function just as well either way. The “negative-ground” standardization we see today is really nothing more than automobile manufacturers agreeing to wiring their different vehicles the same way in order to enjoy interoperability of components.

This usage of the word *ground* – describing the metal chassis of some complex system exploited as a common electrical conductor for multiple circuits – is not incompatible with the previous usage of *ground*: a point of reference in a circuit arbitrarily defined to be “zero” potential. An automotive technician troubleshooting circuit faults in a vehicle often connects the black test lead of their voltmeter to the metal chassis, expecting to measure positive or zero voltage values on that meter when probing circuit test points with the red test lead. In a negative-ground electrical system, the chassis’ electrical potential is *defined* as zero for the purpose of specifying voltages at other points in the circuit(s).

Yet another electrical meaning of the word “ground” is associated with *safety*, in which one metallic conductor of a power circuit is made electrically common to the Earth through a *grounding rod* buried in the soil. In the following circuit, the ground symbol in the lower-left corner is not merely a reference marker intended for anyone viewing the schematic diagram, but actually represents a metal rod driven deep into the soil and connected to one terminal of the voltage source with intent to achieve equipotentiality with the Earth:



Here the purpose of “grounding” is more important than merely establishing a reference point for voltage measurements. Instead, designating the conductor attached to this metal rod as “Ground” and attaching it to any conductive surface on or around an otherwise energized electrical appliance forces that surface to be equipotential to the same Earth that any humans and animals are standing on. This greatly mitigates the risk of electric shock should there be an internal fault within that appliance by making it impossible to have a difference of potential (i.e. voltage) between the grounded surface and the soil.

The following photograph shows the grounding rod for a commercial building, a large-gauge bare copper conductor clamped to the top of the rod which leads into the building’s electrical room:



Ground wires in electric power systems should never carry current during normal operation. In fact, the presence of current through a ground conductor indicates a problem in the system! Note the two current-bearing power conductors in the previous schematic, specifically how the current-carrying conductor made common to Earth ground is called the *neutral*, while the other (non-grounded) power conductors is called *hot* by comparison. The Hot/Neutral designations always refer to which is grounded and which is not, rather than polarity. For example, if we were to reverse

the polarity of the DC voltage source shown in that schematic, the upper conductor (now $-$) would still be considered “hot” and the lower conductor (now $+$) would still be considered “neutral”.

Chapter 7

Questions

This learning module, along with all others in the ModEL collection, is designed to be used in an inverted instructional environment where students independently read¹ the tutorials and attempt to answer questions on their own *prior* to the instructor's interaction with them. In place of lecture², the instructor engages with students in Socratic-style dialogue, probing and challenging their understanding of the subject matter through inquiry.

Answers are not provided for questions within this chapter, and this is by design. Solved problems may be found in the Tutorial and Derivation chapters, instead. The goal here is *independence*, and this requires students to be challenged in ways where others cannot think for them. Remember that you always have the tools of *experimentation* and *computer simulation* (e.g. SPICE) to explore concepts!

The following lists contain ideas for Socratic-style questions and challenges. Upon inspection, one will notice a strong theme of *metacognition* within these statements: they are designed to foster a regular habit of examining one's own thoughts as a means toward clearer thinking. As such these sample questions are useful both for instructor-led discussions as well as for self-study.

¹Technical reading is an essential academic skill for any technical practitioner to possess for the simple reason that the most comprehensive, accurate, and useful information to be found for developing technical competence is in textual form. Technical careers in general are characterized by the need for continuous learning to remain current with standards and technology, and therefore any technical practitioner who cannot read well is handicapped in their professional development. An excellent resource for educators on improving students' reading prowess through intentional effort and strategy is the book *Reading For Understanding – How Reading Apprenticeship Improves Disciplinary Learning in Secondary and College Classrooms* by Ruth Schoenbach, Cynthia Greenleaf, and Lynn Murphy.

²Lecture is popular as a teaching method because it is easy to implement: any reasonably articulate subject matter expert can talk to students, even with little preparation. However, it is also quite problematic. A good lecture always makes complicated concepts seem easier than they are, which is bad for students because it instills a false sense of confidence in their own understanding; reading and re-articulation requires more cognitive effort and serves to verify comprehension. A culture of teaching-by-lecture fosters a debilitating dependence upon direct personal instruction, whereas the challenges of modern life demand independent and critical thought made possible only by gathering information and perspectives from afar. Information presented in a lecture is ephemeral, easily lost to failures of memory and dictation; text is forever, and may be referenced at any time.

GENERAL CHALLENGES FOLLOWING TUTORIAL READING

- Summarize as much of the text as you can in one paragraph of your own words. A helpful strategy is to explain ideas as you would for an intelligent child: as simple as you can without compromising too much accuracy.
- Simplify a particular section of the text, for example a paragraph or even a single sentence, so as to capture the same fundamental idea in fewer words.
- Where did the text make the most sense to you? What was it about the text's presentation that made it clear?
- Identify where it might be easy for someone to misunderstand the text, and explain why you think it could be confusing.
- Identify any new concept(s) presented in the text, and explain in your own words.
- Identify any familiar concept(s) such as physical laws or principles applied or referenced in the text.
- Devise a proof of concept experiment demonstrating an important principle, physical law, or technical innovation represented in the text.
- Devise an experiment to disprove a plausible misconception.
- Did the text reveal any misconceptions you might have harbored? If so, describe the misconception(s) and the reason(s) why you now know them to be incorrect.
- Describe any useful problem-solving strategies applied in the text.
- Devise a question of your own to challenge a reader's comprehension of the text.

GENERAL FOLLOW-UP CHALLENGES FOR ASSIGNED PROBLEMS

- Identify where any fundamental laws or principles apply to the solution of this problem, especially before applying any mathematical techniques.
- Devise a thought experiment to explore the characteristics of the problem scenario, applying known laws and principles to mentally model its behavior.
- Describe in detail your own strategy for solving this problem. How did you identify and organized the given information? Did you sketch any diagrams to help frame the problem?
- Is there more than one way to solve this problem? Which method seems best to you?
- Show the work you did in solving this problem, even if the solution is incomplete or incorrect.
- What would you say was the most challenging part of this problem, and why was it so?
- Was any important information missing from the problem which you had to research or recall?
- Was there any extraneous information presented within this problem? If so, what was it and why did it not matter?
- Examine someone else's solution to identify where they applied fundamental laws or principles.
- Simplify the problem from its given form and show how to solve this simpler version of it. Examples include eliminating certain variables or conditions, altering values to simpler (usually whole) numbers, applying a limiting case (i.e. altering a variable to some extreme or ultimate value).
- For quantitative problems, identify the real-world meaning of all intermediate calculations: their units of measurement, where they fit into the scenario at hand. Annotate any diagrams or illustrations with these calculated values.
- For quantitative problems, try approaching it qualitatively instead, thinking in terms of “increase” and “decrease” rather than definite values.
- For qualitative problems, try approaching it quantitatively instead, proposing simple numerical values for the variables.
- Were there any assumptions you made while solving this problem? Would your solution change if one of those assumptions were altered?
- Identify where it would be easy for someone to go astray in attempting to solve this problem.
- Formulate your own problem based on what you learned solving this one.

GENERAL FOLLOW-UP CHALLENGES FOR EXPERIMENTS OR PROJECTS

- In what way(s) was this experiment or project easy to complete?
- Identify some of the challenges you faced in completing this experiment or project.

- Show how thorough documentation assisted in the completion of this experiment or project.
- Which fundamental laws or principles are key to this system's function?
- Identify any way(s) in which one might obtain false or otherwise misleading measurements from test equipment in this system.
- What will happen if (component X) fails (open/shorted/etc.)?
- What would have to occur to make this system unsafe?

7.1 Conceptual reasoning

These questions are designed to stimulate your analytic and synthetic thinking³. In a Socratic discussion with your instructor, the goal is for these questions to prompt an extended dialogue where assumptions are revealed, conclusions are tested, and understanding is sharpened. Your instructor may also pose additional questions based on those assigned, in order to further probe and refine your conceptual understanding.

Questions that follow are presented to challenge and probe your understanding of various concepts presented in the tutorial. These questions are intended to serve as a guide for the Socratic dialogue between yourself and the instructor. Your instructor’s task is to ensure you have a sound grasp of these concepts, and the questions contained in this document are merely a means to this end. Your instructor may, at his or her discretion, alter or substitute questions for the benefit of tailoring the discussion to each student’s needs. The only absolute requirement is that each student is challenged and assessed at a level equal to or greater than that represented by the documented questions.

It is far more important that you convey your *reasoning* than it is to simply convey a correct answer. For this reason, you should refrain from researching other information sources to answer questions. What matters here is that *you* are doing the thinking. If the answer is incorrect, your instructor will work with you to correct it through proper reasoning. A correct answer without an adequate explanation of how you derived that answer is unacceptable, as it does not aid the learning or assessment process.

You will note a conspicuous lack of answers given for these conceptual questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your conceptual answers, where applicable, is to use circuit simulation software to explore the effects of changes made to circuits. For example, if one of these conceptual questions challenges you to predict the effects of altering some component parameter in a circuit, you may check the validity of your work by simulating that same parameter change within software and seeing if the results agree.

³*Analytical* thinking involves the “disassembly” of an idea into its constituent parts, analogous to dissection. *Synthetic* thinking involves the “assembly” of a new idea comprised of multiple concepts, analogous to construction. Both activities are high-level cognitive skills, extremely important for effective problem-solving, necessitating frequent challenge and regular practice to fully develop.

7.1.1 Reading outline and reflections

“Reading maketh a full man; conference a ready man; and writing an exact man” – Francis Bacon

Francis Bacon’s advice is a blueprint for effective education: reading provides the learner with knowledge, writing focuses the learner’s thoughts, and critical dialogue equips the learner to confidently communicate and apply their learning. Independent acquisition and application of knowledge is a powerful skill, well worth the effort to cultivate. To this end, students should read these educational resources closely, journal their own reflections on the reading, and discuss in detail their findings with classmates and instructor(s). You should be able to do all of the following after reading any instructional text:

Briefly **SUMMARIZE THE TEXT** in the form of a journal entry documenting your learning as you progress through the course of study. Share this summary in dialogue with your classmates and instructor. Journaling is an excellent self-test of thorough reading because you cannot clearly express what you have not read or did not comprehend.

Demonstrate **ACTIVE READING STRATEGIES**, including verbalizing your impressions as you read, simplifying long passages to convey the same ideas using fewer words, annotating text and illustrations with your own interpretations, working through mathematical examples shown in the text, cross-referencing passages with relevant illustrations and/or other passages, identifying problem-solving strategies applied by the author, etc. Technical reading is a special case of problem-solving, and so these strategies work precisely because they help solve any problem: paying attention to your own thoughts (metacognition), eliminating unnecessary complexities, identifying what makes sense, paying close attention to details, drawing connections between separated facts, and noting the successful strategies of others.

Identify **IMPORTANT THEMES**, especially **GENERAL LAWS** and **PRINCIPLES**, expounded in the text and express them in the simplest of terms as though you were teaching an intelligent child. This emphasizes connections between related topics and develops your ability to communicate complex ideas to anyone.

Form **YOUR OWN QUESTIONS** based on the reading, and then pose them to your instructor and classmates for their consideration. Anticipate both correct and incorrect answers, the incorrect answer(s) assuming one or more plausible misconceptions. This helps you view the subject from different perspectives to grasp it more fully.

Devise **EXPERIMENTS** to test claims presented in the reading, or to disprove misconceptions. Predict possible outcomes of these experiments, and evaluate their meanings: what result(s) would confirm, and what would constitute disproof? Running mental simulations and evaluating results is essential to scientific and diagnostic reasoning.

Specifically identify any points you found **CONFUSING**. The reason for doing this is to help diagnose misconceptions and overcome barriers to learning.

7.1.2 Foundational concepts

Correct analysis and diagnosis of electric circuits begins with a proper understanding of some basic concepts. The following is a list of some important concepts referenced in this module's full tutorial. Define each of them in your own words, and be prepared to illustrate each of these concepts with a description of a practical example and/or a live demonstration.

Energy

Conservation of Energy

Conservation of Electric Charge

Voltage

Conductors versus Insulators

Resistance

Current

Ohm's Law

Joule's Law

Electrical source

Electrical load

Series

Parallel

Equipotential points

Electrically common points

Switch

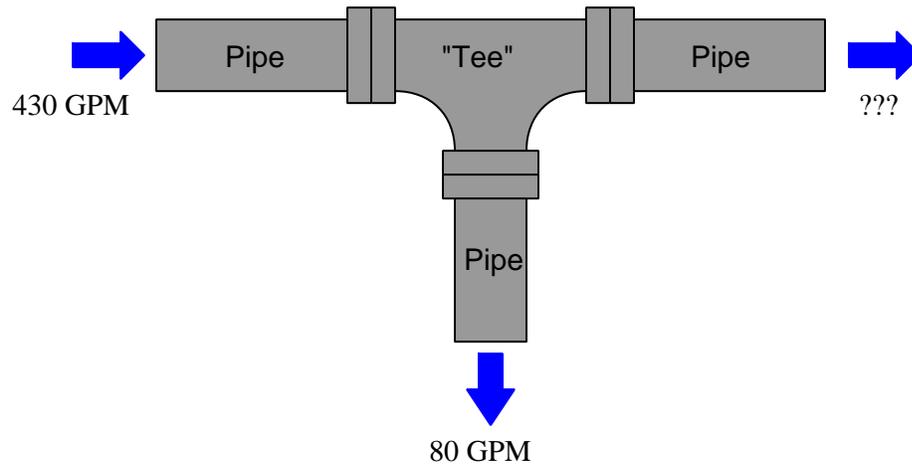
Open

Short

Kirchhoff's Current Law

7.1.3 Water pipe system

How much water must flow out of the pipe with the question-mark symbols next to it?

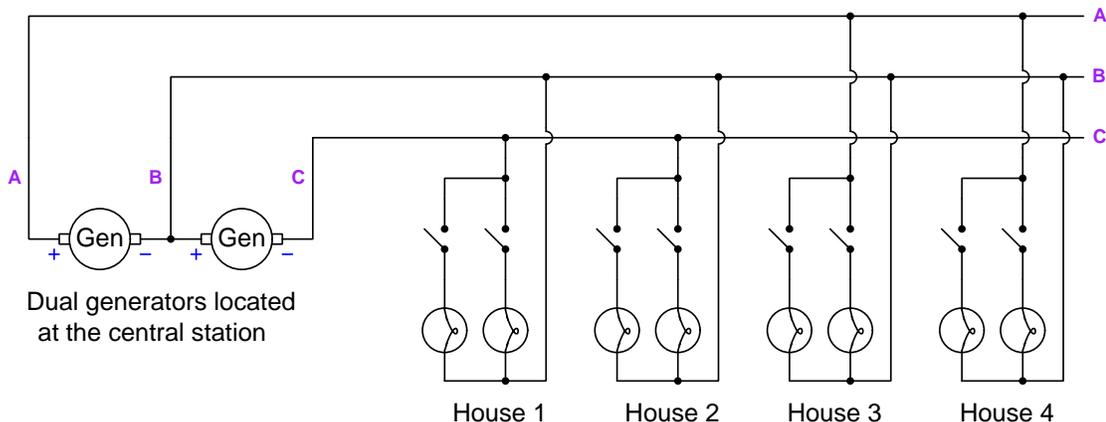


Challenges

- Reverse the direction of either the 430 GPM flow or the 80 GPM flow and re-calculate for the unknown flow.
- Identify any relevant conservation law(s) applicable to this scenario.

7.1.4 Three-wire electrical power distribution

A patent dated July 27, 1882 by the British engineer John Hopkinson describes a system for distributing electric power from central DC generators (“dynamo machines”) to residential loads using *three* wires instead of the more conventional *two* wires. An example of this kind of power distribution circuitry is shown below, as well as a quotation from Hopkinson’s patent:



“For the purpose of economizing the cost of main conductors I place two dynamo machines in series and place two systems of lamps or other appliances consuming electricity of approximately equal capacity also in a series. A main conductor is taken from each extreme pole of the two dynamos to points between the two systems of lamps, the intermediate conductor serving to bring back to the central station any electricity required for one system of lamps in excess of that required by the other system of lamps.”

Identify the “main” conductors in the diagram referred to by Hopkinson’s patent, as well as the “intermediate” conductor.

Explain how Kirchhoff’s Current Law applies to this design.

Identify at least two different conditions where the intermediate conductor will carry zero current at the central station.

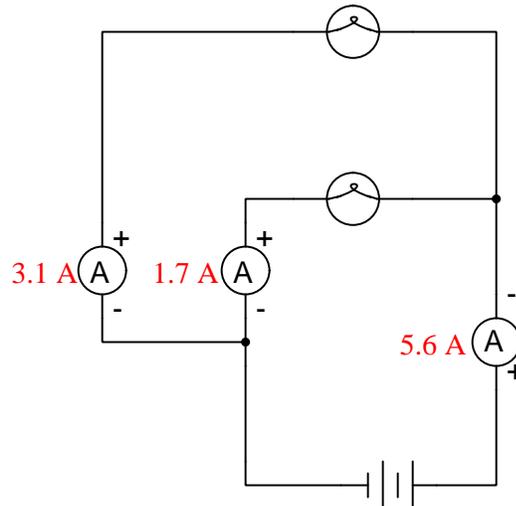
Challenges

- A useful problem-solving technique for this circuit is to apply some numerical quantities and then treat it as a quantitative (calculation) problem instead of a conceptual problem. Show how you can do this, and explain your results.
- Suppose all houses have all their lamps energized. Are there *any* portions of the intermediate conductor that will be carrying current in this scenario? If so, identify where those portions lie.

- The purpose of Hopkinson’s design was to “economize the cost of main conductors” compared to two-conductor power systems of equal power capacity. Explain how this goal is achieved in the three-wire system.
- Identify a scenario maximizing current through conductor B (at the generators).
- Predict the direction of current through conductor B (at the generators) if all lamps are on except for the lamps in house 2.

7.1.5 Unusual ammeter measurements

Something is strange about these ammeter measurements:



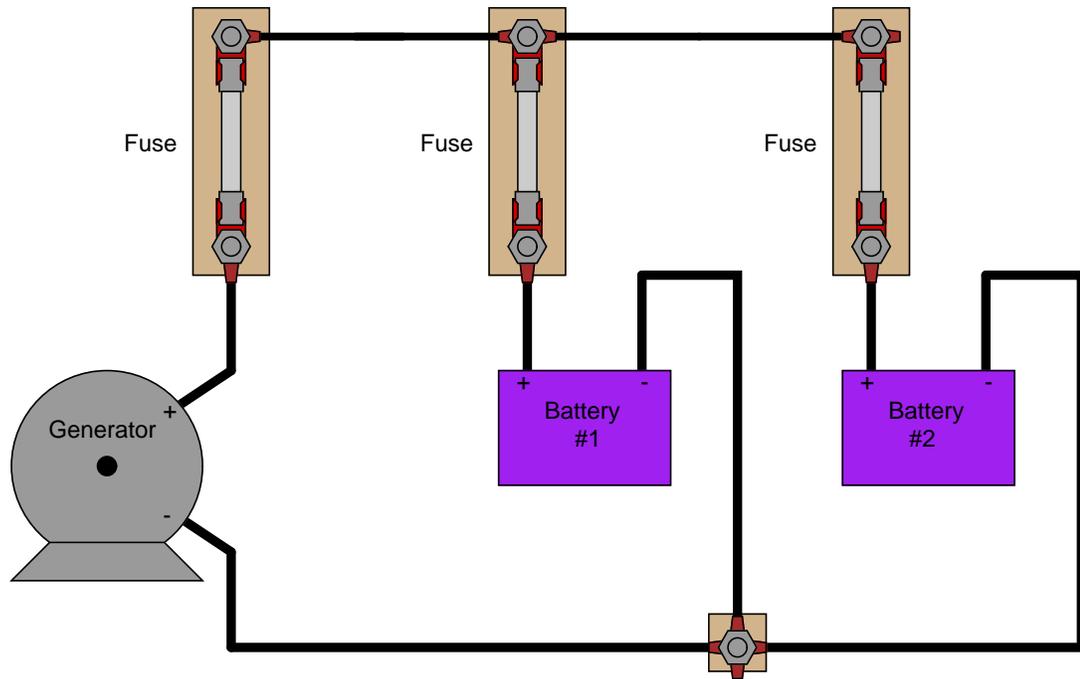
Identify what is non-sensical about these measurements, being sure to explain your reasoning.

Challenges

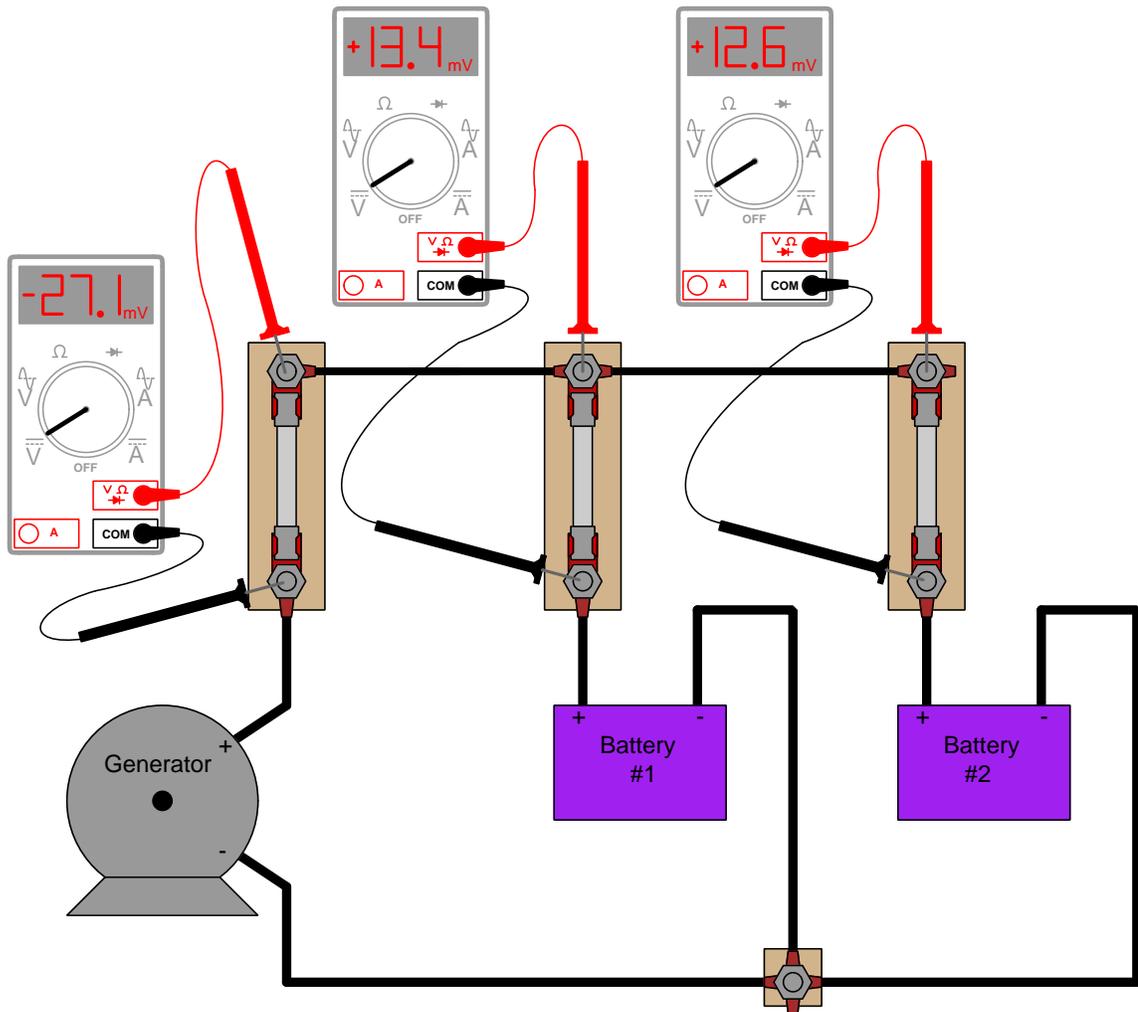
- Suppose that a fellow technician comes to you with an idea for diagnosing the problem here: swap ammeters within this circuit and compare their old readings versus their new readings. Explain how this diagnostic technique might be useful.
- Suppose that a fellow technician comes to you with an idea for diagnosing the problem here: remove the lamps from their sockets. Explain how this diagnostic technique might be useful.
- Which lamp has the greatest amount of electrical resistance?

7.1.6 Battery charging status

Suppose you are asked to determine whether or not each battery in this circuit is charging or discharging *without the benefit of an ammeter*:



You are about to give up when a master technician comes along and uses her multimeter (set to measure *milliVolts*) to take these three voltage measurements:



“There,” she says, “it’s easy!” With that, she walks away, leaving you to figure out what those measurements mean. Determine the following:

- How these DC millivoltage measurements indicate direction of current.
- Whether or not each battery is charging or discharging.
- Whether or not these millivoltage measurements can tell you *how much* current is going through each battery.
- How this relates to Kirchhoff’s Current Law.

Challenges

- How this general principle of measurement may be extended to other practical applications.

7.1.7 Generator failure

In the book *Protective Relays* written in 1922, a description is given of a DC electric power system with an engine-driven generator, a storage battery, and load. The following diagram shows the typical operation of this system:

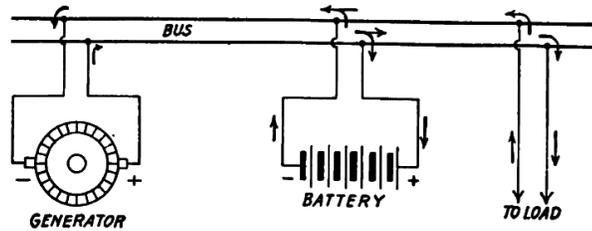


FIG. 65.—Diagram of connections of battery, load and generator. Arrows show normal direction of current flow.

In the next figure the same system is shown in a condition where the mechanical engine driving the generator has failed, and is no longer providing mechanical energy to turn the generator's shaft:

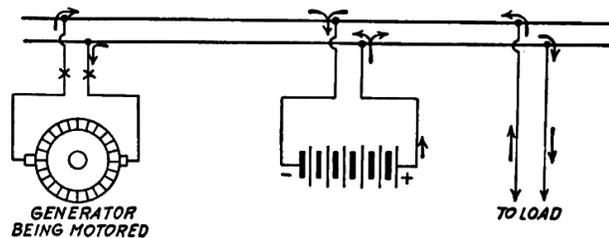


FIG. 66.—Arrows show direction of current flow upon failure of generator.

For both illustrations, describe how Kirchhoff's Current Law applies.

Label the two "bus" conductors with their proper voltage polarity symbols (i.e. "+" and "-").

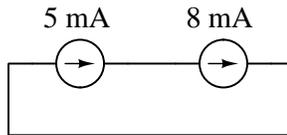
Also, explain what the author means by the phrase "Generator being motored".

Challenges

- What do you suppose the term "bus" refers to in this context?
- Is the author of this book and of the schematic diagram assuming the use of electron flow notation or conventional flow notation?
- Identify adverse effects of the generator losing motive power from the engine.

7.1.8 Series current sources

Does the following circuit violate Kirchhoff's Current Law? Explain why or why not.

**Challenges**

- Modify the circuit so that it is no longer problematic, without altering either source's current.

7.2 Quantitative reasoning

These questions are designed to stimulate your computational thinking. In a Socratic discussion with your instructor, the goal is for these questions to reveal your mathematical approach(es) to problem-solving so that good technique and sound reasoning may be reinforced. Your instructor may also pose additional questions based on those assigned, in order to observe your problem-solving firsthand.

Mental arithmetic and estimations are strongly encouraged for all calculations, because without these abilities you will be unable to readily detect errors caused by calculator misuse (e.g. keystroke errors).

You will note a conspicuous lack of answers given for these quantitative questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. My advice is to use circuit simulation software such as SPICE to check the correctness of quantitative answers. Refer to those learning modules within this collection focusing on SPICE to see worked examples which you may use directly as practice problems for your own study, and/or as templates you may modify to run your own analyses and generate your own practice problems.

Completely worked example problems found in the Tutorial may also serve as “test cases⁴” for gaining proficiency in the use of circuit simulation software, and then once that proficiency is gained you will never need to rely⁵ on an answer key!

⁴In other words, set up the circuit simulation software to analyze the same circuit examples found in the Tutorial. If the simulated results match the answers shown in the Tutorial, it confirms the simulation has properly run. If the simulated results disagree with the Tutorial’s answers, something has been set up incorrectly in the simulation software. Using every Tutorial as practice in this way will quickly develop proficiency in the use of circuit simulation software.

⁵This approach is perfectly in keeping with the instructional philosophy of these learning modules: *teaching students to be self-sufficient thinkers*. Answer keys can be useful, but it is even more useful to your long-term success to have a set of tools on hand for checking your own work, because once you have left school and are on your own, there will no longer be “answer keys” available for the problems you will have to solve.

7.2.1 Miscellaneous physical constants

Note: constants shown in **bold** type are *exact*, not approximations. Values inside of parentheses show one standard deviation (σ) of uncertainty in the final digits: for example, the magnetic permeability of free space value given as $1.25663706212(19) \times 10^{-6}$ H/m represents a center value (i.e. the location parameter) of $1.25663706212 \times 10^{-6}$ Henrys per meter with one standard deviation of uncertainty equal to $0.0000000000019 \times 10^{-6}$ Henrys per meter.

Avogadro's number (N_A) = **6.02214076** $\times 10^{23}$ **per mole** (mol⁻¹)

Boltzmann's constant (k) = **1.380649** $\times 10^{-23}$ **Joules per Kelvin** (J/K)

Electronic charge (e) = **1.602176634** $\times 10^{-19}$ **Coulomb** (C)

Faraday constant (F) = **96,485.33212...** $\times 10^4$ **Coulombs per mole** (C/mol)

Magnetic permeability of free space (μ_0) = **1.25663706212(19)** $\times 10^{-6}$ Henrys per meter (H/m)

Electric permittivity of free space (ϵ_0) = **8.8541878128(13)** $\times 10^{-12}$ Farads per meter (F/m)

Characteristic impedance of free space (Z_0) = **376.730313668(57)** Ohms (Ω)

Gravitational constant (G) = **6.67430(15)** $\times 10^{-11}$ cubic meters per kilogram-seconds squared (m³/kg-s²)

Molar gas constant (R) = **8.314462618...** **Joules per mole-Kelvin** (J/mol-K) = 0.08205746(14) liters-atmospheres per mole-Kelvin

Planck constant (h) = **6.62607015** $\times 10^{-34}$ **joule-seconds** (J-s)

Stefan-Boltzmann constant (σ) = **5.670374419...** $\times 10^{-8}$ **Watts per square meter-Kelvin⁴** (W/m²·K⁴)

Speed of light in a vacuum (c) = **299,792,458 meters per second** (m/s) = 186282.4 miles per second (mi/s)

Note: All constants taken from NIST data "Fundamental Physical Constants – Complete Listing", from <http://physics.nist.gov/constants>, National Institute of Standards and Technology (NIST), 2018 CODATA Adjustment.

7.2.2 Introduction to spreadsheets

A powerful computational tool you are encouraged to use in your work is a *spreadsheet*. Available on most personal computers (e.g. Microsoft Excel), *spreadsheet* software performs numerical calculations based on number values and formulae entered into cells of a grid. This grid is typically arranged as lettered columns and numbered rows, with each cell of the grid identified by its column/row coordinates (e.g. cell B3, cell A8). Each cell may contain a string of text, a number value, or a mathematical formula. The spreadsheet automatically updates the results of all mathematical formulae whenever the entered number values are changed. This means it is possible to set up a spreadsheet to perform a series of calculations on entered data, and those calculations will be re-done by the computer any time the data points are edited in any way.

For example, the following spreadsheet calculates average speed based on entered values of distance traveled and time elapsed:

	A	B	C	D
1	Distance traveled	46.9	Kilometers	
2	Time elapsed	1.18	Hours	
3	Average speed	= B1 / B2	km/h	
4				
5				

Text labels contained in cells A1 through A3 and cells C1 through C3 exist solely for readability and are not involved in any calculations. Cell B1 contains a sample distance value while cell B2 contains a sample time value. The formula for computing speed is contained in cell B3. Note how this formula begins with an “equals” symbol (=), references the values for distance and speed by lettered column and numbered row coordinates (B1 and B2), and uses a forward slash symbol for division (/). The coordinates B1 and B2 function as *variables*⁶ would in an algebraic formula.

When this spreadsheet is executed, the numerical value 39.74576 will appear in cell B3 rather than the formula = B1 / B2, because 39.74576 is the computed speed value given 46.9 kilometers traveled over a period of 1.18 hours. If a different numerical value for distance is entered into cell B1 or a different value for time is entered into cell B2, cell B3’s value will automatically update. All you need to do is set up the given values and any formulae into the spreadsheet, and the computer will do all the calculations for you.

Cell B3 may be referenced by other formulae in the spreadsheet if desired, since it is a variable just like the given values contained in B1 and B2. This means it is possible to set up an entire chain of calculations, one dependent on the result of another, in order to arrive at a final value. The arrangement of the given data and formulae need not follow any pattern on the grid, which means you may place them anywhere.

⁶Spreadsheets may also provide means to attach text labels to cells for use as variable names (Microsoft Excel simply calls these labels “names”), but for simple spreadsheets such as those shown here it’s usually easier just to use the standard coordinate naming for each cell.

Common⁷ arithmetic operations available for your use in a spreadsheet include the following:

- Addition (+)
- Subtraction (-)
- Multiplication (*)
- Division (/)
- Powers (^)
- Square roots (sqrt())
- Logarithms (ln() , log10())

Parentheses may be used to ensure⁸ proper order of operations within a complex formula. Consider this example of a spreadsheet implementing the *quadratic formula*, used to solve for roots of a polynomial expression in the form of $ax^2 + bx + c$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

	A	B
1	x_1	= (-B4 + sqrt((B4^2) - (4*B3*B5))) / (2*B3)
2	x_2	= (-B4 - sqrt((B4^2) - (4*B3*B5))) / (2*B3)
3	a =	9
4	b =	5
5	c =	-2

This example is configured to compute roots⁹ of the polynomial $9x^2 + 5x - 2$ because the values of 9, 5, and -2 have been inserted into cells B3, B4, and B5, respectively. Once this spreadsheet has been built, though, it may be used to calculate the roots of *any* second-degree polynomial expression simply by entering the new a , b , and c coefficients into cells B3 through B5. The numerical values appearing in cells B1 and B2 will be automatically updated by the computer immediately following any changes made to the coefficients.

⁷Modern spreadsheet software offers a bewildering array of mathematical functions you may use in your computations. I recommend you consult the documentation for your particular spreadsheet for information on operations other than those listed here.

⁸Spreadsheet programs, like text-based programming languages, are designed to follow standard order of operations by default. However, my personal preference is to use parentheses even where strictly unnecessary just to make it clear to any other person viewing the formula what the intended order of operations is.

⁹Reviewing some algebra here, a *root* is a value for x that yields an overall value of zero for the polynomial. For this polynomial ($9x^2 + 5x - 2$) the two roots happen to be $x = 0.269381$ and $x = -0.82494$, with these values displayed in cells B1 and B2, respectively upon execution of the spreadsheet.

Alternatively, one could break up the long quadratic formula into smaller pieces like this:

$$y = \sqrt{b^2 - 4ac} \quad z = 2a$$

$$x = \frac{-b \pm y}{z}$$

	A	B	C
1	x_1	= (-B4 + C1) / C2	= sqrt((B4^2) - (4*B3*B5))
2	x_2	= (-B4 - C1) / C2	= 2*B3
3	a =	9	
4	b =	5	
5	c =	-2	

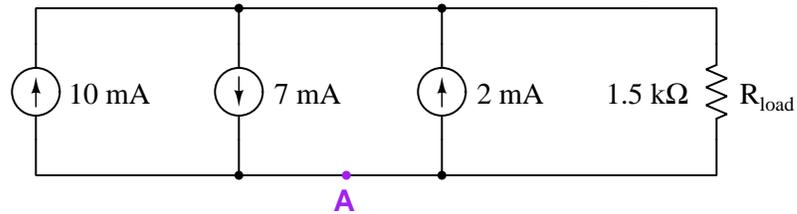
Note how the square-root term (y) is calculated in cell C1, and the denominator term (z) in cell C2. This makes the two final formulae (in cells B1 and B2) simpler to interpret. The positioning of all these cells on the grid is completely arbitrary¹⁰ – all that matters is that they properly reference each other in the formulae.

Spreadsheets are particularly useful for situations where the same set of calculations representing a circuit or other system must be repeated for different initial conditions. The power of a spreadsheet is that it automates what would otherwise be a tedious set of calculations. One specific application of this is to simulate the effects of various components within a circuit failing with abnormal values (e.g. a shorted resistor simulated by making its value nearly zero; an open resistor simulated by making its value extremely large). Another application is analyzing the behavior of a circuit design given new components that are out of specification, and/or aging components experiencing drift over time.

¹⁰My personal preference is to locate all the “given” data in the upper-left cells of the spreadsheet grid (each data point flanked by a sensible name in the cell to the left and units of measurement in the cell to the right as illustrated in the first distance/time spreadsheet example), sometimes coloring them in order to clearly distinguish which cells contain entered data versus which cells contain computed results from formulae. I like to place all formulae in cells below the given data, and try to arrange them in logical order so that anyone examining my spreadsheet will be able to figure out *how* I constructed a solution. This is a general principle I believe all computer programmers should follow: *document and arrange your code to make it easy for other people to learn from it.*

7.2.3 Paralleled current sources

Calculate the voltage dropped by the load resistor by this set of parallel-connected current sources, as well as the amount of current passing through point “A” and the direction it is flowing:

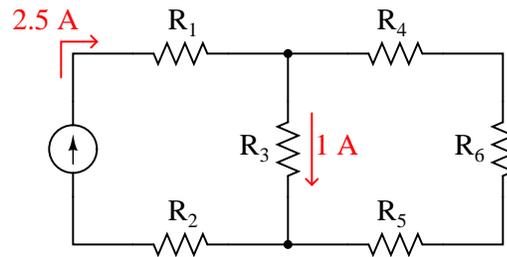


Challenges

- Predict the effects of the 10 mA source failing open.
- Predict the effects of the 2 mA source failing shorted.

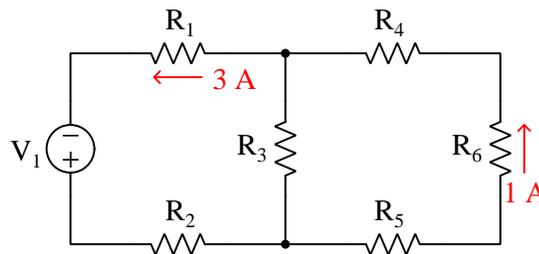
7.2.4 Currents in resistor networks

Calculate the magnitude and direction of the current through resistor R_4 in this resistor network:



Note: all current arrows point in the direction of conventional flow!

Calculate the magnitude and direction of current through resistors R_2 , R_3 , and R_4 in this network:

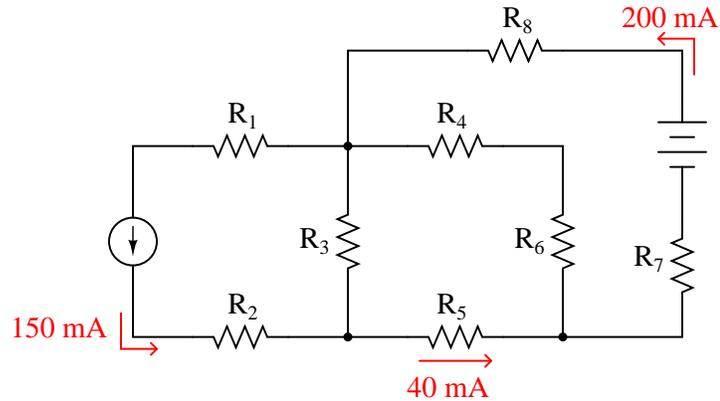


Challenges

- Qualitatively predict (i.e. *increase*, *decrease*, or *no change*) the changes to these three resistor currents if R_2 happens to fail open.
- Qualitatively predict (i.e. *increase*, *decrease*, or *no change*) the changes to these three resistor currents if R_3 happens to fail shorted.
- Qualitatively predict (i.e. *increase*, *decrease*, or *no change*) the changes to these three resistor currents if R_5 happens to fail shorted.

7.2.5 Currents in a large resistor network

Use Kirchhoff's Current Law to calculate the magnitudes and directions of currents through *all* resistors in this circuit:



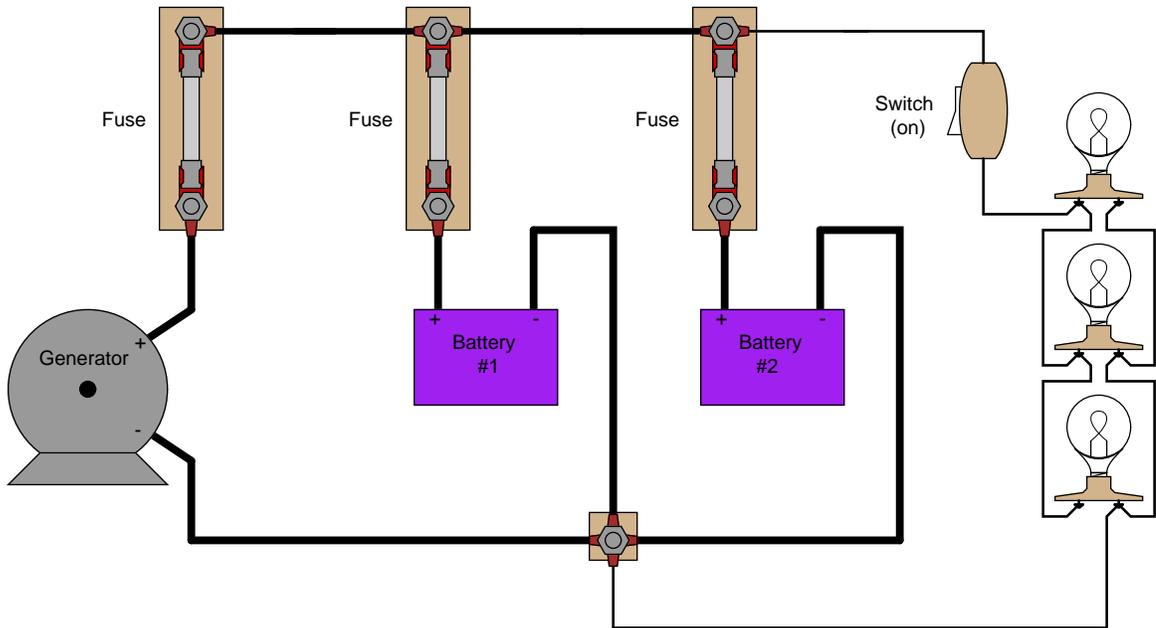
Note: all current arrows point in the direction of conventional flow!

Challenges

- Qualitatively predict (i.e. *increase*, *decrease*, or *no change*) the changes to these three resistor currents if R_3 happens to fail open.

7.2.6 Battery currents

Determine the current through battery #2 in this power system, if the generator is outputting 40 Amperes, battery #1 is charging at a rate of 18 Amperes, and each lamp draws 8 Amperes of current on its own. Be sure to indicate whether battery #2 is charging or discharging:



Also, identify ways you can double-check your work to ensure all your answers are correct.

Challenges

- Where would you install an ammeter to sense the generator's current in this circuit?
- What would change if one of the lamps failed open?
- What would change if battery #1 developed an internal short?

7.2.7 Current divider

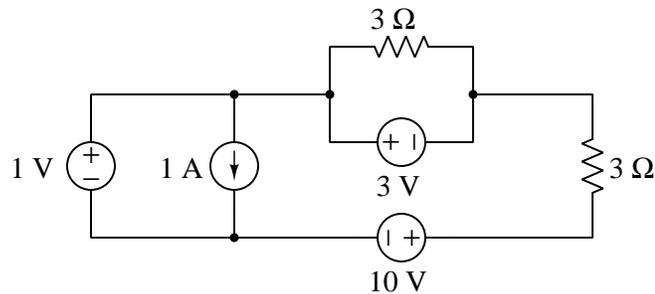
Choose two resistors to be connected in parallel such that one of them will carry 80% of the total current and the other will carry 20%. Make sure the total resistance of the two paralleled resistors will not exceed 100 Ohms.

Challenges

- Identify the correct color codes for your two resistors, assuming a four-band code.
- Explain how the solution to this problem does not require use of the “current divider formula”.

7.2.8 Solving for currents in a multi-source circuit

Determine the amount of current passing through every component and every wire in this circuit:



Challenges

- Note every “source” in this circuit is actually operating as an energy source. Some of them function as loads. Identify which are sources and which are loads.

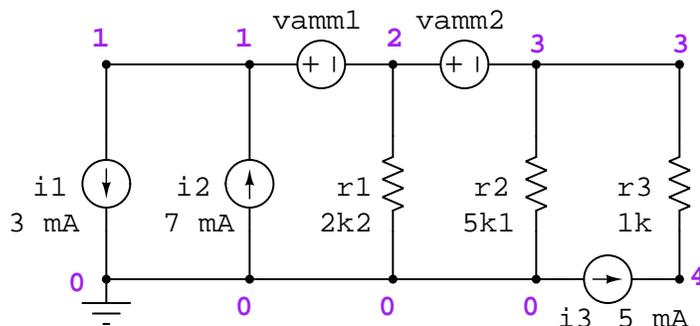
7.2.9 Interpreting a SPICE analysis

A computer program called SPICE was developed in the early 1970's, whereby a text-based description of an electric circuit (called a *netlist*) could be entered into a computer, and then computer would be directed to apply fundamental laws of electric circuits to the netlist circuit description according to algorithms coded in SPICE.

Here is a netlist text for a multi-source, multi-resistor parallel network complete with “dummy” voltage sources set to 0 Volts each functioning as ammeters¹¹:

```
* Multi-source, multi-resistor network
i1 1 0 dc 3e-3
i2 0 1 dc 7e-3
i3 0 4 dc 5e-3
r1 2 0 2200
r2 3 0 5100
r3 3 4 1000
vamm1 1 2
vamm2 2 3
.dc i1 3e-3 3e-3 1
.print dc i(vamm1) i(vamm2)
.end
```

The corresponding schematic diagram appears here:



¹¹A limitation of legacy SPICE versions is that they could only display the current through a voltage source, and not through any other type of component. Therefore, if one desired to measure current somewhere in a circuit where there was not already a voltage source, it was necessary to place a zero-voltage source in that location simply to serve as an ammeter.

When SPICE (software version 2G6) processes the contents of this “netlist” file, it outputs a text description of the analysis. The following text has been edited for clarity (e.g. blank lines, extraneous characters, and statistical data removed):

Ground connected to node 3:

```
i1          i(vamm1)    i(vamm2)
3.000E-03   4.000E-03   -2.288E-03
```

Interpret the output of SPICE to the best of your ability. Identify all parameters that you can, and see if the results of the computer’s analysis agree with your own calculations.

Once you become familiar with the analysis data format of SPICE, you will be able to use the “Gallery” of SPICE simulations found in the “*SPICE Modeling of Resistor Circuits*” module as practice problems for developing your own circuit analysis skills.

Challenges

- Explain the meaning of the negative value SPICE prints for `i(vamm2)`.
- Modify this SPICE netlist so that current source `i3` outputs 2 mA instead of 5 mA.
- Modify this SPICE netlist to specify resistor R_2 ’s resistance using scientific (power-of-ten) notation.
- Modify this SPICE netlist to specify resistor R_3 ’s resistance using metric prefix (“kilo”) notation.

7.3 Diagnostic reasoning

These questions are designed to stimulate your deductive and inductive thinking, where you must apply general principles to specific scenarios (deductive) and also derive conclusions about the failed circuit from specific details (inductive). In a Socratic discussion with your instructor, the goal is for these questions to reinforce your recall and use of general circuit principles and also challenge your ability to integrate multiple symptoms into a sensible explanation of what's wrong in a circuit. Your instructor may also pose additional questions based on those assigned, in order to further challenge and sharpen your diagnostic abilities.

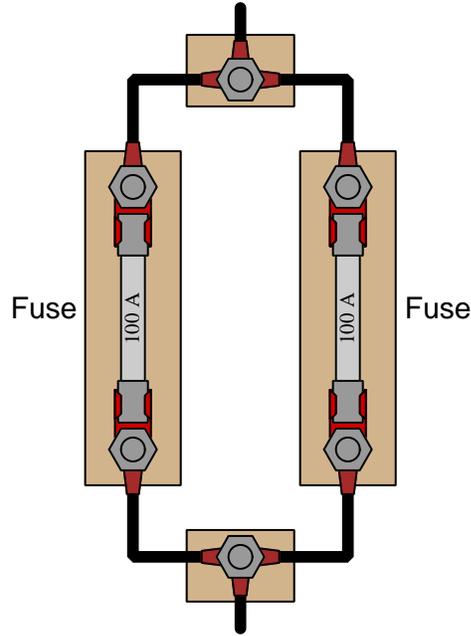
As always, your goal is to fully *explain* your analysis of each problem. Simply obtaining a correct answer is not good enough – you must also demonstrate sound reasoning in order to successfully complete the assignment. Your instructor's responsibility is to probe and challenge your understanding of the relevant principles and analytical processes in order to ensure you have a strong foundation upon which to build further understanding.

You will note a conspicuous lack of answers given for these diagnostic questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your diagnostic answers, where applicable, is to use circuit simulation software to explore the effects of faults placed in circuits. For example, if one of these diagnostic questions requires that you predict the effect of an open or a short in a circuit, you may check the validity of your work by simulating that same fault (substituting a very high resistance in place of that component for an open, and substituting a very low resistance for a short) within software and seeing if the results agree.

7.3.1 Mysterious fuse failure

In an effort to obtain greater overcurrent ratings than a single fuse can provide, an electrician decides to wire two fuses in parallel. Each fuse has a full-load current rating of 100 Amperes, and so the electrician's reasoning is that together the two fuses will provide a total rating of 200 Amperes:



After a few years of operation, this dual-fuse system begins to fail: both fuses blow even when the load is known to be drawing significantly less than 200 Amperes of current. Upon investigation, it is found that one of the fuse holders has developed some corrosion on a terminal lug where one of the wires connect.

Explain how a small accumulation of corrosion led to this condition of fuses blowing when there was no overcurrent condition (load current less than 200 Amperes), and also why connecting fuses in parallel like this is generally not a good idea.

Challenges

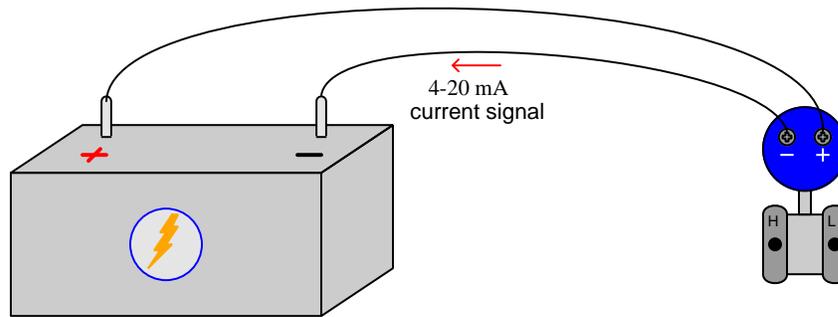
- If the corrosion formed on the left-hand fuse's holder, which fuse (left or right) will be the first to blow?
- How much corrosion do you suppose would need to accumulate before this becomes a serious problem? Do you think a fair amount of corrosion would need to build up, or would even a small amount be problematic?
- Suppose the dual-fuse arrangement shown does not suffer from any corrosion at all. If an overcurrent event occurred, do you think the two fuses would “blow” simultaneously or would

one blow before the other?

- Sketch wires connecting to the parallel fuses shown, in order to make a complete circuit with a source and a load.

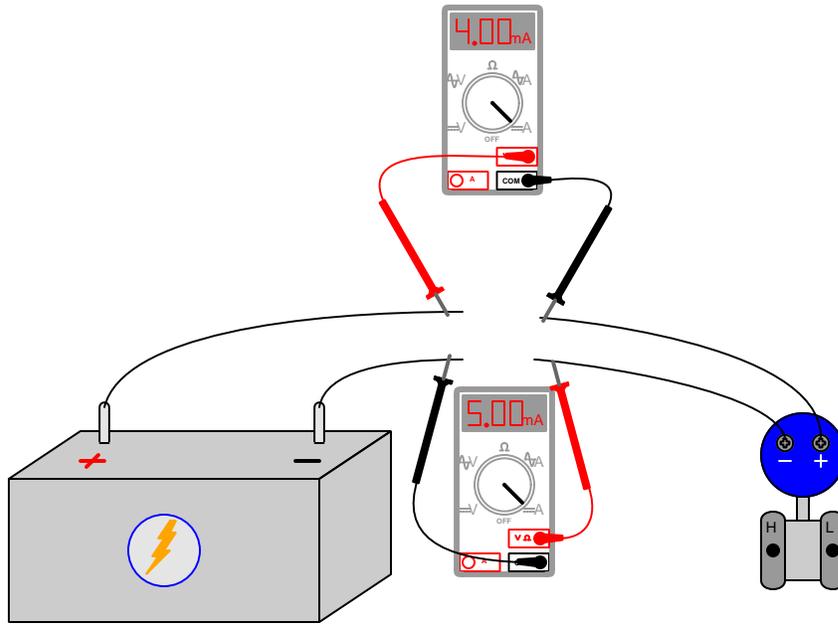
7.3.2 4-20 mA transmitter circuit fault

An electronic measurement device widely used in industrial applications is a *4-20 milliampere transmitter*. This type of device makes a physical measurement such as the temperature of fluid in a pipe, or the level of liquid in a vessel, and reports this measurement as an electric current signal ranging from 4 mA (meaning 0% of measurement range) to 20 mA (meaning 100% of measurement range). These “transmitters” rely on external energy sources, and function by *regulating* the amount of electric current passing through them according to the value of the measurement:



Since this is a series circuit, and we know current should be the same at all points of a series circuit at any given time, it should be possible to measure current *anywhere* in this circuit and interpret that current as a representation of the transmitter’s measurement (e.g. temperature, level).

However, suppose a technician happens to measure current through each of the two wires in this circuit, one at a time using the same ammeter, and obtains different readings:



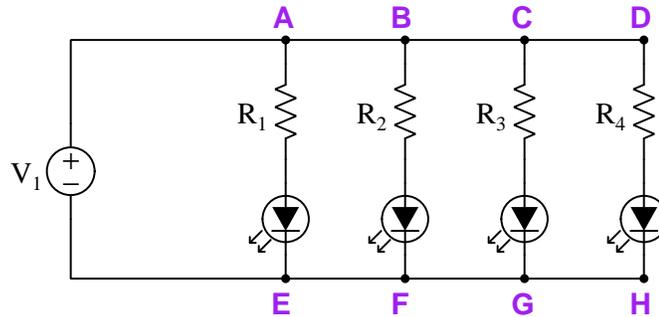
How is this possible?

Challenges

- Explain how Kirchhoff's Current Law relates to this scenario.
- Identify sources and loads in this circuit.

7.3.3 Multiple-LED circuit

This circuit uses an array of *Light-Emitting Diodes* (LEDs), each one with its own current-limiting resistor, to provide light from a single electrical source. Unfortunately, though, something in it has failed:



Instead of all four LEDs energizing, only the two LEDs on the left-hand side of the circuit emit light. The two LEDs on the right-hand side are dark. A technician measures good supply voltage between terminals A and E in the circuit.

Identify the likelihood of each specified fault for this circuit, declaring if each one is either “Possible” or “Impossible” based on the given symptoms. Consider each fault one at a time (i.e. no coincidental faults), determining whether or not each fault on its own would explain *all* measurements and symptoms in this circuit.

- R_1 failed open:
- R_2 failed open:
- R_3 failed open:
- R_4 failed open:
- Wire E-F failed open:
- Wire C-D failed open:
- Wire F-G failed open:
- Wire G-H failed open:
- Wire A-B failed open:

Challenges

- For each of the proposed fault, explain *why* you believe it is either possible or impossible.

- Identify at least one other fault *not* shown in the list, which could adequately explain all symptoms in this failed circuit.
- Was the technician's voltage test between points A and E diagnostically useful, after we already knew which LEDs were energized and which were not? Explain why or why not.

Chapter 8

Projects and Experiments

The following project and experiment descriptions outline things you can build to help you understand circuits. With any real-world project or experiment there exists the potential for physical harm. *Electricity can be very dangerous in certain circumstances, and you should follow proper safety precautions at all times!*

8.1 Recommended practices

This section outlines some recommended practices for all circuits you design and construct.

8.1.1 Safety first!

Electricity, when passed through the human body, causes uncomfortable sensations and in large enough measures¹ will cause muscles to involuntarily contract. The overriding of your nervous system by the passage of electrical current through your body is particularly dangerous in regard to your heart, which is a vital muscle. Very large amounts of current can produce serious internal burns in addition to all the other effects.

Cardio-pulmonary resuscitation (CPR) is the standard first-aid for any victim of electrical shock. This is a very good skill to acquire if you intend to work with others on dangerous electrical circuits. You should never perform tests or work on such circuits unless someone else is present who is proficient in CPR.

As a general rule, any voltage in excess of 30 Volts poses a definitive electric shock hazard, because beyond this level human skin does not have enough resistance to safely limit current through the body. “Live” work of any kind with circuits over 30 volts should be avoided, and if unavoidable should only be done using electrically insulated tools and other protective equipment (e.g. insulating shoes and gloves). If you are unsure of the hazards, or feel unsafe at any time, stop all work and distance yourself from the circuit!

A policy I strongly recommend for students learning about electricity is to *never come into electrical contact² with an energized conductor, no matter what the circuit’s voltage³ level!* Enforcing this policy may seem ridiculous when the circuit in question is powered by a single battery smaller than the palm of your hand, but it is precisely this instilled habit which will save a person from bodily harm when working with more dangerous circuits. Experience has taught me that students who learn early on to be careless with safe circuits have a tendency to be careless later with dangerous circuits!

In addition to the electrical hazards of shock and burns, the construction of projects and running of experiments often poses other hazards such as working with hand and power tools, potential

¹Professor Charles Dalziel published a research paper in 1961 called “The Deleterious Effects of Electric Shock” detailing the results of electric shock experiments with both human and animal subjects. The threshold of perception for human subjects holding a conductor in their hand was in the range of 1 milliampere of current (less than this for alternating current, and generally less for female subjects than for male). Loss of muscular control was exhibited by half of Dalziel’s subjects at less than 10 milliamperes alternating current. Extreme pain, difficulty breathing, and loss of all muscular control occurred for over 99% of his subjects at direct currents less than 100 milliamperes and alternating currents less than 30 milliamperes. In summary, it doesn’t require much electric current to induce painful and even life-threatening effects in the human body! Your first and best protection against electric shock is maintaining an insulating barrier between your body and the circuit in question, such that current from that circuit will be unable to flow through your body.

²By “electrical contact” I mean either directly touching an energized conductor with any part of your body, or indirectly touching it through a conductive tool. The only physical contact you should ever make with an energized conductor is via an electrically insulated tool, for example a screwdriver with an electrically insulated handle, or an insulated test probe for some instrument.

³Another reason for consistently enforcing this policy, even on low-voltage circuits, is due to the dangers that even some low-voltage circuits harbor. A single 12 Volt automobile battery, for example, can cause a surprising amount of damage if short-circuited simply due to the high current levels (i.e. very low internal resistance) it is capable of, even though the voltage level is too low to cause a shock through the skin. Mechanics wearing metal rings, for example, are at risk from severe burns if their rings happen to short-circuit such a battery! Furthermore, even when working on circuits that are simply too low-power (low voltage and low current) to cause any bodily harm, touching them while energized can pose a threat to the circuit components themselves. In summary, it generally wise (and *always* a good habit to build) to “power down” *any* circuit before making contact between it and your body.

contact with high temperatures, potential chemical exposure, etc. You should never proceed with a project or experiment if you are unaware of proper tool use or lack basic protective measures (e.g. personal protective equipment such as safety glasses) against such hazards.

Some other safety-related practices should be followed as well:

- All power conductors extending outward from the project must be *firmly* strain-relieved (e.g. “cord grips” used on line power cords), so that an accidental tug or drop will not compromise circuit integrity.
- All electrical connections must be sound and appropriately made (e.g. soldered wire joints rather than twisted-and-taped; terminal blocks rather than solderless breadboards for high-current or high-voltage circuits). Use “touch-safe” terminal connections with recessed metal parts to minimize risk of accidental contact.
- Always provide overcurrent protection in any circuit you build. *Always*. This may be in the form of a fuse, a circuit breaker, and/or an electronically current-limited power supply.
- Always ensure circuit conductors are rated for more current than the overcurrent protection limit. *Always*. A fuse does no good if the wire or printed circuit board trace will “blow” before it does!
- Always bond metal enclosures to Earth ground for any line-powered circuit. *Always*. Ensuring an equipotential state between the enclosure and Earth by making the enclosure electrically common with Earth ground ensures no electric shock can occur simply by one’s body bridging between the Earth and the enclosure.
- Avoid building a high-energy circuit when a low-energy circuit will suffice. For example, I always recommend beginning students power their first DC resistor circuits using small batteries rather than with line-powered DC power supplies. The intrinsic energy limitations of a dry-cell battery make accidents highly unlikely.
- Use line power receptacles that are GFCI (Ground Fault Current Interrupting) to help avoid electric shock from making accidental contact with a “hot” line conductor.
- Always wear eye protection when working with tools or live systems having the potential to eject material into the air. Examples of such activities include soldering, drilling, grinding, cutting, wire stripping, working on or near energized circuits, etc.
- Always use a step-stool or stepladder to reach high places. Never stand on something not designed to support a human load.
- When in doubt, *ask an expert*. If anything even seems remotely unsafe to you, do not proceed without consulting a trusted person fully knowledgeable in electrical safety.

8.1.2 Other helpful tips

Experience has shown the following practices to be very helpful, especially when students make their own component selections, to ensure the circuits will be well-behaved:

- Avoid resistor values less than $1\text{ k}\Omega$ or greater than $100\text{ k}\Omega$, unless such values are definitely necessary⁴. Resistances below $1\text{ k}\Omega$ may draw excessive current if directly connected to a voltage source of significant magnitude, and may also complicate the task of accurately measuring current since any ammeter's non-zero resistance inserted in series with a low-value circuit resistor will significantly alter the total resistance and thereby skew the measurement. Resistances above $100\text{ k}\Omega$ may complicate the task of measuring voltage since any voltmeter's finite resistance connected in parallel with a high-value circuit resistor will significantly alter the total resistance and thereby skew the measurement. Similarly, AC circuit impedance values should be between $1\text{ k}\Omega$ and $100\text{ k}\Omega$, and for all the same reasons.
- Ensure all electrical connections are low-resistance and physically rugged. For this reason, one should avoid *compression splices* (e.g. "butt" connectors), solderless breadboards⁵, and wires that are simply twisted together.
- Build your circuit with **testing** in mind. For example, provide convenient connection points for test equipment (e.g. multimeters, oscilloscopes, signal generators, logic probes).
- Design permanent projects with **maintenance** in mind. The more convenient you make maintenance tasks, the more likely they will get done.
- **Always document and save your work.** Circuits lacking schematic diagrams are more difficult to troubleshoot than documented circuits. Similarly, circuit construction is simpler when a schematic diagram precedes construction. Experimental results are easier to interpret when comprehensively recorded. Consider modern videorecording technology for this purpose where appropriate.
- **Record your steps** when troubleshooting. **Talk to yourself** when solving problems. These simple steps clarify thought and simplify identification of errors.

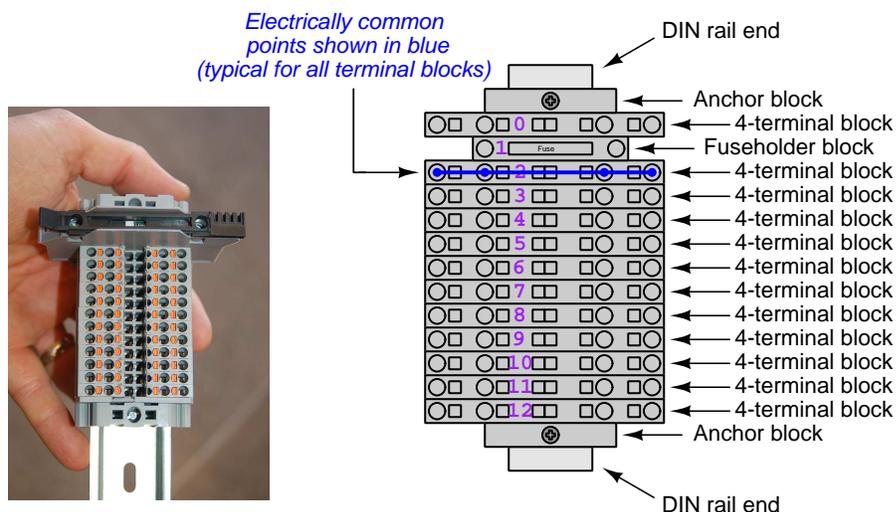
⁴An example of a necessary resistor value much less than $1\text{ k}\Omega$ is a *shunt resistor* used to produce a small voltage drop for the purpose of sensing current in a circuit. Such shunt resistors must be low-value in order not to impose an undue load on the rest of the circuit. An example of a necessary resistor value much greater than $100\text{ k}\Omega$ is an electrostatic *drain resistor* used to dissipate stored electric charges from body capacitance for the sake of preventing damage to sensitive semiconductor components, while also preventing a path for current that could be dangerous to the person (i.e. shock).

⁵Admittedly, solderless breadboards are very useful for constructing complex electronic circuits with many components, especially DIP-style integrated circuits (ICs), but they tend to give trouble with connection integrity after frequent use. An alternative for projects using low counts of ICs is to solder IC sockets into prototype printed circuit boards (PCBs) and run wires from the soldered pins of the IC sockets to terminal blocks where reliable temporary connections may be made.

8.1.3 Terminal blocks for circuit construction

Terminal blocks are the standard means for making electric circuit connections in industrial systems. They are also quite useful as a learning tool, and so I highly recommend their use in lieu of solderless breadboards⁶. Terminal blocks provide highly reliable connections capable of withstanding significant voltage and current magnitudes, and they force the builder to think very carefully about component layout which is an important mental practice. Terminal blocks that mount on standard 35 mm DIN rail⁷ are made in a wide range of types and sizes, some with built-in disconnecting switches, some with built-in components such as rectifying diodes and fuseholders, all of which facilitate practical circuit construction.

I recommend every student of electricity build their own terminal block array for use in constructing experimental circuits, consisting of several terminal blocks where each block has at least 4 connection points all electrically common to each other⁸ and at least one terminal block that is a fuse holder for overcurrent protection. A pair of anchoring blocks hold all terminal blocks securely on the DIN rail, preventing them from sliding off the rail. Each of the terminals should bear a number, starting from 0. An example is shown in the following photograph and illustration:



Screwless terminal blocks (using internal spring clips to clamp wire and component lead ends) are preferred over screw-based terminal blocks, as they reduce assembly and disassembly time, and also minimize repetitive wrist stress from twisting screwdrivers. Some screwless terminal blocks require the use of a special tool to release the spring clip, while others provide buttons⁹ for this task which may be pressed using the tip of any suitable tool.

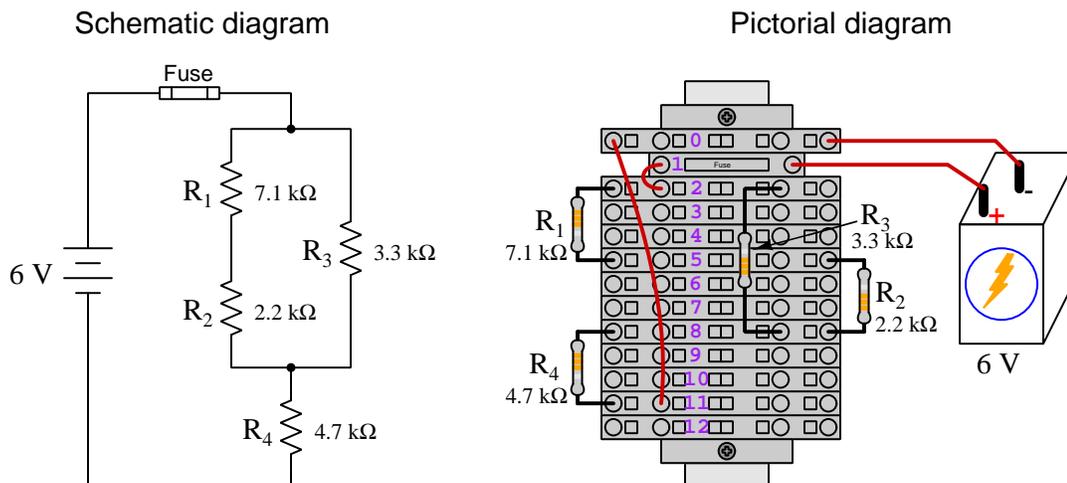
⁶Solderless breadboard are preferable for complicated electronic circuits with multiple integrated “chip” components, but for simpler circuits I find terminal blocks much more practical. An alternative to solderless breadboards for “chip” circuits is to solder chip sockets onto a PCB and then use wires to connect the socket pins to terminal blocks. This also accommodates *surface-mount* components, which solderless breadboards do not.

⁷DIN rail is a metal rail designed to serve as a mounting point for a wide range of electrical and electronic devices such as terminal blocks, fuses, circuit breakers, relay sockets, power supplies, data acquisition hardware, etc.

⁸Sometimes referred to as *equipotential*, *same-potential*, or *potential distribution* terminal blocks.

⁹The small orange-colored squares seen in the above photograph are buttons for this purpose, and may be actuated by pressing with any tool of suitable size.

The following example shows how such a terminal block array might be used to construct a series-parallel resistor circuit consisting of four resistors and a battery:



Numbering on the terminal blocks provides a very natural translation to SPICE¹⁰ netlists, where component connections are identified by terminal number:

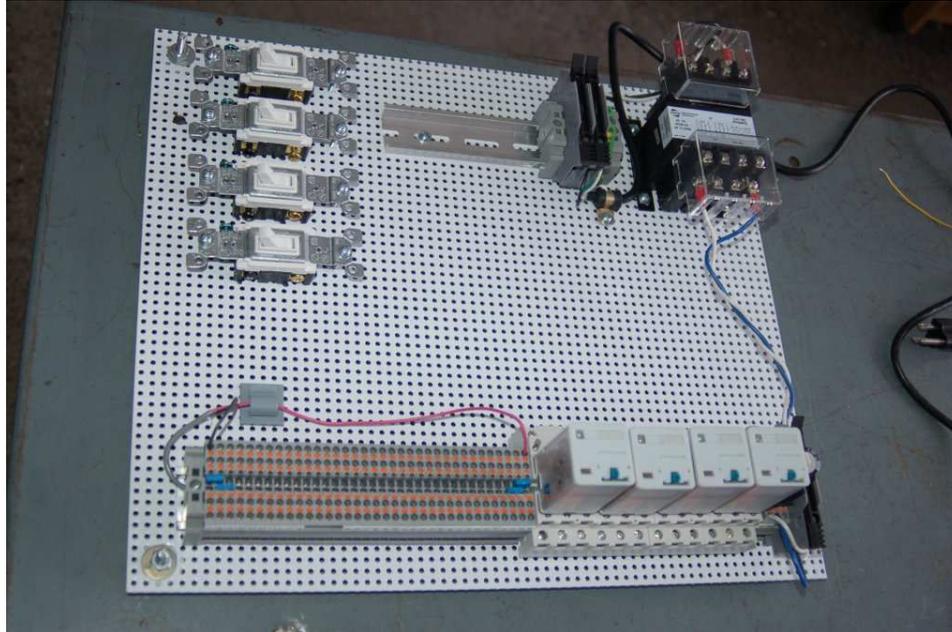
```
* Series-parallel resistor circuit
v1 1 0 dc 6
r1 2 5 7100
r2 5 8 2200
r3 2 8 3300
r4 8 11 4700
rjmp1 1 2 0.01
rjmp2 0 11 0.01
.op
.end
```

Note the use of “jumper” resistances `rjmp1` and `rjmp2` to describe the wire connections between terminals 1 and 2 and between terminals 0 and 11, respectively. Being resistances, SPICE requires a resistance value for each, and here we see they have both been set to an arbitrarily low value of 0.01 Ohm realistic for short pieces of wire.

Listing all components and wires along with their numbered terminals happens to be a useful documentation method for any circuit built on terminal blocks, independent of SPICE. Such a “wiring sequence” may be thought of as a *non-graphical description* of an electric circuit, and is exceptionally easy to follow.

¹⁰SPICE is computer software designed to analyze electrical and electronic circuits. Circuits are described for the computer in the form of *netlists* which are text files listing each component type, connection node numbers, and component values.

An example of a more elaborate terminal block array is shown in the following photograph, with terminal blocks and “ice-cube” style electromechanical relays mounted to DIN rail, which is turn mounted to a perforated subpanel¹¹. This “terminal block board” hosts an array of thirty five undedicated terminal block sections, four SPDT toggle switches, four DPDT “ice-cube” relays, a step-down control power transformer, bridge rectifier and filtering capacitor, and several fuses for overcurrent protection:



Four plastic-bottomed “feet” support the subpanel above the benchtop surface, and an unused section of DIN rail stands ready to accept other components. Safety features include electrical bonding of the AC line power cord’s ground to the metal subpanel (and all metal DIN rails), mechanical strain relief for the power cord to isolate any cord tension from wire connections, clear plastic finger guards covering the transformer’s screw terminals, as well as fused overcurrent protection for the 120 Volt AC line power and the transformer’s 12 Volt AC output. The perforated holes happen to be on $\frac{1}{4}$ inch centers with a diameter suitable for tapping with 6-32 machine screw threads, their presence making it very easy to attach other sections of DIN rail, printed circuit boards, or specialized electrical components directly to the grounded metal subpanel. Such a “terminal block board” is an inexpensive¹² yet highly flexible means to construct physically robust circuits using industrial wiring practices.

¹¹An electrical *subpanel* is a thin metal plate intended for mounting inside an electrical enclosure. Components are attached to the subpanel, and the subpanel in turn bolts inside the enclosure. Subpanels allow circuit construction outside the confines of the enclosure, which speeds assembly. In this particular usage there is no enclosure, as the subpanel is intended to be used as an open platform for the convenient construction of circuits on a benchtop by students. In essence, this is a modern version of the traditional *breadboard* which was literally a wooden board such as might be used for cutting loaves of bread, but which early electrical and electronic hobbyists used as platforms for the construction of circuits.

¹²At the time of this writing (2019) the cost to build this board is approximately \$250 US dollars.

8.1.4 Conducting experiments

An *experiment* is an exploratory act, a test performed for the purpose of assessing some proposition or principle. Experiments are the foundation of the *scientific method*, a process by which careful observation helps guard against errors of speculation. All good experiments begin with an *hypothesis*, defined by the American Heritage Dictionary of the English Language as:

An assertion subject to verification or proof, as (a) A proposition stated as a basis for argument or reasoning. (b) A premise from which a conclusion is drawn. (c) A conjecture that accounts, within a theory or ideational framework, for a set of facts and that can be used as a basis for further investigation.

Stated plainly, an hypothesis is an *educated guess* about cause and effect. The correctness of this initial guess matters little, because any well-designed experiment will reveal the truth of the matter. In fact, *incorrect* hypotheses are often the most valuable because the experiments they engender lead us to surprising discoveries. One of the beautiful aspects of science is that it is more focused on the process of *learning* than about the status of *being correct*¹³. In order for an hypothesis to be valid, it must be testable¹⁴, which means it must be a claim possible to refute given the right data. Hypotheses impossible to critique are useless.

Once an hypothesis has been formulated, an experiment must be designed to test that hypothesis. A well-designed experiment requires careful regulation of all relevant variables, both for personal safety and for prompting the hypothesized results. If the effects of one particular variable are to be tested, the experiment must be run multiple times with different values of (only) that particular variable. The experiment set up with the “baseline” variable set is called the *control*, while the experiment set up with different value(s) is called the *test* or *experimental*.

For some hypotheses a viable alternative to a physical experiment is a *computer-simulated experiment* or even a *thought experiment*. Simulations performed on a computer test the hypothesis against the physical laws encoded within the computer simulation software, and are particularly useful for students learning new principles for which simulation software is readily available¹⁵.

¹³Science is more about clarifying our view of the universe through a systematic process of error detection than it is about proving oneself to be right. Some *scientists* may happen to have large egos – and this may have more to do with the ways in which large-scale scientific research is *funded* than anything else – but *scientific method* itself is devoid of ego, and if embraced as a practical philosophy is quite an effective stimulant for humility. Within the education system, scientific method is particularly valuable for helping students break free of the crippling fear of *being wrong*. So much emphasis is placed in formal education on assessing correct retention of facts that many students are fearful of saying or doing anything that might be perceived as a mistake, and of course making mistakes (i.e. having one’s hypotheses disproven by experiment) is an indispensable tool for learning. Introducing science in the classroom – *real* science characterized by individuals forming actual hypotheses and testing those hypotheses by experiment – helps students become self-directed learners.

¹⁴This is the principle of *falsifiability*: that a scientific statement has value only insofar as it is liable to disprove given the requisite experimental evidence. Any claim that is unfalsifiable – that is, a claim which can *never* be disproven by any evidence whatsoever – could be completely wrong and we could never know it.

¹⁵A very pertinent example of this is learning how to analyze electric circuits using simulation software such as SPICE. A typical experimental cycle would proceed as follows: (1) Find or invent a circuit to analyze; (2) Apply your analytical knowledge to that circuit, predicting all voltages, currents, powers, etc. relevant to the concepts you are striving to master; (3) Run a simulation on that circuit, collecting “data” from the computer when complete; (4) Evaluate whether or not your hypotheses (i.e. predicted voltages, currents, etc.) agree with the computer-generated results; (5) If so, your analyses are (provisionally) correct – if not, examine your analyses and the computer simulation again to determine the source of error; (6) Repeat this process as many times as necessary until you achieve mastery.

Thought experiments are useful for detecting inconsistencies within your own understanding of some subject, rather than testing your understanding against physical reality.

Here are some general guidelines for conducting experiments:

- The clearer and more specific the hypothesis, the better. Vague or unfalsifiable hypotheses are useless because they will fit *any* experimental results, and therefore the experiment cannot teach you anything about the hypothesis.
- Collect as much data (i.e. information, measurements, sensory experiences) generated by an experiment as is practical. This includes the time and date of the experiment, too!
- *Never* discard or modify data gathered from an experiment. If you have reason to believe the data is unreliable, write notes to that effect, but never throw away data just because you think it is untrustworthy. It is quite possible that even “bad” data holds useful information, and that someone else may be able to uncover its value even if you do not.
- Prioritize *quantitative* data over *qualitative* data wherever practical. Quantitative data is more specific than qualitative, less prone to subjective interpretation on the part of the experimenter, and amenable to an arsenal of analytical methods (e.g. statistics).
- Guard against your own bias(es) by making your experimental results available to others. This allows other people to scrutinize your experimental design and collected data, for the purpose of detecting and correcting errors you may have missed. Document your experiment such that others may independently replicate it.
- Always be looking for sources of error. No physical measurement is perfect, and so it is impossible to achieve *exact* values for any variable. Quantify the amount of uncertainty (i.e. the “tolerance” of errors) whenever possible, and be sure your hypothesis does not depend on precision better than this!
- Always remember that scientific confirmation is provisional – no number of “successful” experiments will prove an hypothesis true for all time, but a single experiment can disprove it. Put into simpler terms, *truth is elusive but error is within reach*.
- Remember that scientific method is about *learning*, first and foremost. An unfortunate consequence of scientific triumph in modern society is that science is often viewed by non-practitioners as an unerring source of truth, when in fact science is an ongoing process of challenging existing ideas to probe for errors and oversights. This is why it is perfectly acceptable to have a failed hypothesis, and why the only truly failed experiment is one where nothing was learned.

The following is an example of a well-planned and executed experiment, in this case a physical experiment demonstrating Ohm's Law.

Planning Time/Date = 09:30 on 12 February 2019

HYPOTHESIS: the current through any resistor should be exactly proportional to the voltage impressed across it.

PROCEDURE: connect a resistor rated 1 k Ohm and 1/4 Watt to a variable-voltage DC power supply. Use an ammeter in series to measure resistor current and a voltmeter in parallel to measure resistor voltage.

RISKS AND MITIGATION: excessive power dissipation may harm the resistor and/or pose a burn hazard, while excessive voltage poses an electric shock hazard. 30 Volts is a safe maximum voltage for laboratory practices, and according to Joule's Law a 1000 Ohm resistor will dissipate 0.25 Watts at 15.81 Volts ($P = V^2 / R$), so I will remain below 15 Volts just to be safe.

Experiment Time/Date = 10:15 on 12 February 2019

DATA COLLECTED:

(Voltage)	(Current)	(Voltage)	(Current)
0.000 V	= 0.000 mA	8.100	= 7.812 mA
2.700 V	= 2.603 mA	10.00 V	= 9.643 mA
5.400 V	= 5.206 mA	14.00 V	= 13.49 mA

Analysis Time/Date = 10:57 on 12 February 2019

ANALYSIS: current definitely increases with voltage, and although I expected exactly one milliAmpere per Volt the actual current was usually less than that. The voltage/current ratios ranged from a low of 1036.87 (at 8.1 Volts) to a high of 1037.81 (at 14 Volts), but this represents a variance of only -0.0365% to +0.0541% from the average, indicating a very consistent proportionality -- results consistent with Ohm's Law.

ERROR SOURCES: one major source of error is the resistor's value itself. I did not measure it, but simply assumed color bands of brown-black-red meant exactly 1000 Ohms. Based on the data I think the true resistance is closer to 1037 Ohms. Another possible explanation is multimeter calibration error. However, neither explains the small positive and negative variances from the average. This might be due to electrical noise, a good test being to repeat the same experiment to see if the variances are the same or different. Noise should generate slightly different results every time.

The following is an example of a well-planned and executed *virtual* experiment, in this case demonstrating Ohm's Law using a computer (SPICE) simulation.

Planning Time/Date = 12:32 on 14 February 2019

HYPOTHESIS: for any given resistor, the current through that resistor should be exactly proportional to the voltage impressed across it.

PROCEDURE: write a SPICE netlist with a single DC voltage source and single 1000 Ohm resistor, then use NGSPICE version 26 to perform a "sweep" analysis from 0 Volts to 25 Volts in 5 Volt increments.

```
* SPICE circuit
v1 1 0 dc
r1 1 0 1000
.dc v1 0 25 5
.print dc v(1) i(v1)
.end
```

RISKS AND MITIGATION: none.

DATA COLLECTED:

DC transfer characteristic Thu Feb 14 13:05:08 2019

Index	v-sweep	v(1)	v1#branch
0	0.000000e+00	0.000000e+00	0.000000e+00
1	5.000000e+00	5.000000e+00	-5.00000e-03
2	1.000000e+01	1.000000e+01	-1.00000e-02
3	1.500000e+01	1.500000e+01	-1.50000e-02
4	2.000000e+01	2.000000e+01	-2.00000e-02
5	2.500000e+01	2.500000e+01	-2.50000e-02

Analysis Time/Date = 13:06 on 14 February 2019

ANALYSIS: perfect agreement between data and hypothesis -- current is precisely 1/1000 of the applied voltage for all values. Anything other than perfect agreement would have probably meant my netlist was incorrect. The negative current values surprised me, but it seems this is just how SPICE interprets normal current through a DC voltage source.

ERROR SOURCES: none.

As gratuitous as it may seem to perform experiments on a physical law as well-established as Ohm's Law, even the examples listed previously demonstrate opportunity for real learning. In the physical experiment example, the student should identify and explain why their data does not perfectly agree with the hypothesis, and this leads them naturally to consider sources of error. In the computer-simulated experiment, the student is struck by SPICE's convention of denoting regular current through a DC voltage source as being *negative* in sign, and this is also useful knowledge for future simulations. Scientific experiments are most interesting when things *do not* go as planned!

Aside from verifying well-established physical laws, simple experiments are extremely useful as educational tools for a wide range of purposes, including:

- Component familiarization (e.g. *Which terminals of this switch connect to the NO versus NC contacts?*)
- System testing (e.g. *How heavy of a load can my AC-DC power supply source before the semiconductor components reach their thermal limits?*)
- Learning programming languages (e.g. *Let's try to set up an "up" counter function in this PLC!*)

Above all, the priority here is to inculcate the habit of hypothesizing, running experiments, and analyzing the results. This experimental cycle not only serves as an excellent method for self-directed learning, but it also works exceptionally well for troubleshooting faults in complex systems, and for these reasons should be a part of every technician's and every engineer's education.

8.1.5 Constructing projects

Designing, constructing, and testing projects is a very effective means of practical education. Within a formal educational setting, projects are generally chosen (or at least vetted) by an instructor to ensure they may be reasonably completed within the allotted time of a course or program of study, and that they sufficiently challenge the student to learn certain important principles. In a self-directed environment, projects are just as useful as a learning tool but there is some risk of unwittingly choosing a project beyond one's abilities, which can lead to frustration.

Here are some general guidelines for managing projects:

- Define your goal(s) before beginning a project: what do you wish to achieve in building it? What, exactly, should the completed project *do*?
- Analyze your project prior to construction. Document it in appropriate forms (e.g. schematic diagrams), predict its functionality, anticipate all associated risks. In other words, *plan ahead*.
- Set a reasonable budget for your project, and stay within it.
- Identify any deadlines, and set reasonable goals to meet those deadlines.
- Beware of *scope creep*: the tendency to modify the project's goals before it is complete.
- Document your progress! An easy way to do this is to use photography or videography: take photos and/or videos of your project as it progresses. Document failures as well as successes, because both are equally valuable from the perspective of learning.

8.2 Experiment: Confirming Kirchhoff's Voltage Law

Conduct an experiment to confirm Kirchhoff's Current Law (i.e. that the algebraic sum of currents at any node equals zero). Your experiment should include *multiple* nodes in order to be comprehensive.

EXPERIMENT CHECKLIST:

- Prior to experimentation:
 - Write an hypothesis (i.e. a detailed description of what you expect will happen) unambiguous enough that it could be disproven given the right data.
 - Write a procedure to test the hypothesis, complete with adequate controls and documentation (e.g. schematic diagrams, programming code).
 - Identify any risks (e.g. shock hazard, component damage) and write a mitigation plan based on best practices and component ratings.
- During experimentation:
 - Safe practices followed at all times (e.g. no contact with energized circuit).
 - Correct equipment usage according to manufacturer's recommendations.
 - All data collected, ideally quantitative with full precision (i.e. no rounding).
- After each experimental run:
 - If the results fail to match the hypothesis, identify the error(s), correct the hypothesis and/or revise the procedure, and re-run the experiment.
 - Identify any uncontrolled sources of error in the experiment.
- After all experimental re-runs:
 - Save all data for future reference.
 - Write an analysis of experimental results and lessons learned.

Challenges

- Identify unique properties of *parallel*-connected components, and then use your experiment to prove this property is true either by appealing to empirical data you have recorded or by demonstrating those measurements live.
- Predict the effects of a particular resistor failing open.
- Predict the effects of a particular resistor failing shorted.
- Explain why you cannot obtain a reliable resistance measurement using an ohmmeter when the component in question is energized by a source.

- Identify any data points (empirical) that disagree with theory (prediction), and suggest sources of error.
- Explain why connecting an ammeter directly to the terminals of a voltage source such as a primary-cell battery is a bad idea.

8.3 Project: (first project)

This is a description of the project!

Challenges

- ???.
- ???.
- ???.

Appendix A

Problem-Solving Strategies

The ability to solve complex problems is arguably one of the most valuable skills one can possess, and this skill is particularly important in any science-based discipline.

- Study principles, not procedures. Don't be satisfied with merely knowing how to compute solutions – learn *why* those solutions work.
- Identify what it is you need to solve, identify all relevant data, identify all units of measurement, identify any general principles or formulae linking the given information to the solution, and then identify any “missing pieces” to a solution. Annotate all diagrams with this data.
- Sketch a diagram to help visualize the problem. When building a real system, always devise a plan for that system and analyze its function *before* constructing it.
- Follow the units of measurement and meaning of every calculation. If you are ever performing mathematical calculations as part of a problem-solving procedure, and you find yourself unable to apply each and every intermediate result to some aspect of the problem, it means you don't understand what you are doing. Properly done, every mathematical result should have practical meaning for the problem, and not just be an abstract number. You should be able to identify the proper units of measurement for each and every calculated result, and show where that result fits into the problem.
- Perform “thought experiments” to explore the effects of different conditions for theoretical problems. When troubleshooting real systems, perform *diagnostic tests* rather than visually inspecting for faults, the best diagnostic test being the one giving you the most information about the nature and/or location of the fault with the fewest steps.
- Simplify the problem until the solution becomes obvious, and then use that obvious case as a model to follow in solving the more complex version of the problem.
- Check for exceptions to see if your solution is incorrect or incomplete. A good solution will work for *all* known conditions and criteria. A good example of this is the process of testing scientific hypotheses: the task of a scientist is not to find support for a new idea, but rather to *challenge* that new idea to see if it holds up under a battery of tests. The philosophical

principle of *reductio ad absurdum* (i.e. disproving a general idea by finding a specific case where it fails) is useful here.

- Work “backward” from a hypothetical solution to a new set of given conditions.
- Add quantities to problems that are qualitative in nature, because sometimes a little math helps illuminate the scenario.
- Sketch graphs illustrating how variables relate to each other. These may be quantitative (i.e. with realistic number values) or qualitative (i.e. simply showing increases and decreases).
- Treat quantitative problems as qualitative in order to discern the relative magnitudes and/or directions of change of the relevant variables. For example, try determining what happens if a certain variable were to increase or decrease before attempting to precisely calculate quantities: how will each of the dependent variables respond, by increasing, decreasing, or remaining the same as before?
- Consider limiting cases. This works especially well for qualitative problems where you need to determine which direction a variable will change. Take the given condition and magnify that condition to an extreme degree as a way of simplifying the direction of the system’s response.
- Check your work. This means regularly testing your conclusions to see if they make sense. This does *not* mean repeating the same steps originally used to obtain the conclusion(s), but rather to use some other means to check validity. Simply repeating procedures often leads to *repeating the same errors* if any were made, which is why alternative paths are better.

Appendix B

Instructional philosophy

“The unexamined circuit is not worth energizing” – Socrates (if he had taught electricity)

These learning modules, although useful for self-study, were designed to be used in a formal learning environment where a subject-matter expert challenges students to digest the content and exercise their critical thinking abilities in the answering of questions and in the construction and testing of working circuits.

The following principles inform the instructional and assessment philosophies embodied in these learning modules:

- The first goal of education is to enhance clear and independent thought, in order that every student reach their fullest potential in a highly complex and inter-dependent world. Robust reasoning is *always* more important than particulars of any subject matter, because its application is universal.
- Literacy is fundamental to independent learning and thought because text continues to be the most efficient way to communicate complex ideas over space and time. Those who cannot read with ease are limited in their ability to acquire knowledge and perspective.
- Articulate communication is fundamental to work that is complex and interdisciplinary.
- Faulty assumptions and poor reasoning are best corrected through challenge, not presentation. The rhetorical technique of *reductio ad absurdum* (disproving an assertion by exposing an absurdity) works well to discipline student’s minds, not only to correct the problem at hand but also to learn how to detect and correct future errors.
- Important principles should be repeatedly explored and widely applied throughout a course of study, not only to reinforce their importance and help ensure their mastery, but also to showcase the interconnectedness and utility of knowledge.

These learning modules were expressly designed to be used in an “inverted” teaching environment¹ where students first read the introductory and tutorial chapters on their own, then individually attempt to answer the questions and construct working circuits according to the experiment and project guidelines. The instructor never lectures, but instead meets regularly with each individual student to review their progress, answer questions, identify misconceptions, and challenge the student to new depths of understanding through further questioning. Regular meetings between instructor and student should resemble a Socratic² dialogue, where questions serve as scalpels to dissect topics and expose assumptions. The student passes each module only after consistently demonstrating their ability to logically analyze and correctly apply all major concepts in each question or project/experiment. The instructor must be vigilant in probing each student’s understanding to ensure they are truly *reasoning* and not just *memorizing*. This is why “Challenge” points appear throughout, as prompts for students to think deeper about topics and as starting points for instructor queries. Sometimes these challenge points require additional knowledge that hasn’t been covered in the series to answer in full. This is okay, as the major purpose of the Challenges is to stimulate analysis and synthesis on the part of each student.

The instructor must possess enough mastery of the subject matter and awareness of students’ reasoning to generate their own follow-up questions to practically any student response. Even completely correct answers given by the student should be challenged by the instructor for the purpose of having students practice articulating their thoughts and defending their reasoning. Conceptual errors committed by the student should be exposed and corrected not by direct instruction, but rather by reducing the errors to an absurdity³ through well-chosen questions and thought experiments posed by the instructor. Becoming proficient at this style of instruction requires time and dedication, but the positive effects on critical thinking for both student and instructor are spectacular.

An inspection of these learning modules reveals certain unique characteristics. One of these is a bias toward thorough explanations in the tutorial chapters. Without a live instructor to explain concepts and applications to students, the text itself must fulfill this role. This philosophy results in lengthier explanations than what you might typically find in a textbook, each step of the reasoning process fully explained, including footnotes addressing common questions and concerns students raise while learning these concepts. Each tutorial seeks to not only explain each major concept in sufficient detail, but also to explain the logic of each concept and how each may be developed

¹In a traditional teaching environment, students first encounter new information via *lecture* from an expert, and then independently apply that information via *homework*. In an “inverted” course of study, students first encounter new information via *homework*, and then independently apply that information under the scrutiny of an expert. The expert’s role in lecture is to simply *explain*, but the expert’s role in an inverted session is to *challenge, critique*, and if necessary *explain* where gaps in understanding still exist.

²Socrates is a figure in ancient Greek philosophy famous for his unflinching style of questioning. Although he authored no texts, he appears as a character in Plato’s many writings. The essence of Socratic philosophy is to leave no question unexamined and no point of view unchallenged. While purists may argue a topic such as electric circuits is too narrow for a true Socratic-style dialogue, I would argue that the essential thought processes involved with scientific reasoning on *any* topic are not far removed from the Socratic ideal, and that students of electricity and electronics would do very well to challenge assumptions, pose thought experiments, identify fallacies, and otherwise employ the arsenal of critical thinking skills modeled by Socrates.

³This rhetorical technique is known by the Latin phrase *reductio ad absurdum*. The concept is to expose errors by counter-example, since only one solid counter-example is necessary to disprove a universal claim. As an example of this, consider the common misconception among beginning students of electricity that voltage cannot exist without current. One way to apply *reductio ad absurdum* to this statement is to ask how much current passes through a fully-charged battery connected to nothing (i.e. a clear example of voltage existing without current).

from “first principles”. Again, this reflects the goal of developing clear and independent thought in students’ minds, by showing how clear and logical thought was used to forge each concept. Students benefit from witnessing a model of clear thinking in action, and these tutorials strive to be just that.

Another characteristic of these learning modules is a lack of step-by-step instructions in the Project and Experiment chapters. Unlike many modern workbooks and laboratory guides where step-by-step instructions are prescribed for each experiment, these modules take the approach that students must learn to closely read the tutorials and apply their own reasoning to identify the appropriate experimental steps. Sometimes these steps are plainly declared in the text, just not as a set of enumerated points. At other times certain steps are implied, an example being assumed competence in test equipment use where the student should not need to be told *again* how to use their multimeter because that was thoroughly explained in previous lessons. In some circumstances no steps are given at all, leaving the entire procedure up to the student.

This lack of prescription is not a flaw, but rather a feature. Close reading and clear thinking are foundational principles of this learning series, and in keeping with this philosophy all activities are designed to *require* those behaviors. Some students may find the lack of prescription frustrating, because it demands more from them than what their previous educational experiences required. This frustration should be interpreted as an unfamiliarity with autonomous thinking, a problem which must be corrected if the student is ever to become a self-directed learner and effective problem-solver. Ultimately, the need for students to read closely and think clearly is more important both in the near-term and far-term than any specific facet of the subject matter at hand. If a student takes longer than expected to complete a module because they are forced to outline, digest, and reason on their own, so be it. The future gains enjoyed by developing this mental discipline will be well worth the additional effort and delay.

Another feature of these learning modules is that they do not treat topics in isolation. Rather, important concepts are introduced early in the series, and appear repeatedly as stepping-stones toward other concepts in subsequent modules. This helps to avoid the “compartmentalization” of knowledge, demonstrating the inter-connectedness of concepts and simultaneously reinforcing them. Each module is fairly complete in itself, reserving the beginning of its tutorial to a review of foundational concepts.

This methodology of assigning text-based modules to students for digestion and then using Socratic dialogue to assess progress and hone students’ thinking was developed over a period of several years by the author with his Electronics and Instrumentation students at the two-year college level. While decidedly unconventional and sometimes even unsettling for students accustomed to a more passive lecture environment, this instructional philosophy has proven its ability to convey conceptual mastery, foster careful analysis, and enhance employability so much better than lecture that the author refuses to ever teach by lecture again.

Problems which often go undiagnosed in a lecture environment are laid bare in this “inverted” format where students must articulate and logically defend their reasoning. This, too, may be unsettling for students accustomed to lecture sessions where the instructor cannot tell for sure who comprehends and who does not, and this vulnerability necessitates sensitivity on the part of the “inverted” session instructor in order that students never feel discouraged by having their errors exposed. *Everyone* makes mistakes from time to time, and learning is a lifelong process! Part of the instructor’s job is to build a culture of learning among the students where errors are not seen as shameful, but rather as opportunities for progress.

To this end, instructors managing courses based on these modules should adhere to the following principles:

- Student questions are always welcome and demand thorough, honest answers. The only type of question an instructor should refuse to answer is one the student should be able to easily answer on their own. Remember, *the fundamental goal of education is for each student to learn to think clearly and independently*. This requires hard work on the part of the student, which no instructor should ever circumvent. Anything done to bypass the student's responsibility to do that hard work ultimately limits that student's potential and thereby does real harm.
- It is not only permissible, but encouraged, to answer a student's question by asking questions in return, these follow-up questions designed to guide the student to reach a correct answer through their own reasoning.
- All student answers demand to be challenged by the instructor and/or by other students. This includes both correct and incorrect answers – the goal is to practice the articulation and defense of one's own reasoning.
- No reading assignment is deemed complete unless and until the student demonstrates their ability to accurately summarize the major points in their own terms. Recitation of the original text is unacceptable. This is why every module contains an "Outline and reflections" question as well as a "Foundational concepts" question in the Conceptual reasoning section, to prompt reflective reading.
- No assigned question is deemed answered unless and until the student demonstrates their ability to consistently and correctly apply the concepts to *variations* of that question. This is why module questions typically contain multiple "Challenges" suggesting different applications of the concept(s) as well as variations on the same theme(s). Instructors are encouraged to devise as many of their own "Challenges" as they are able, in order to have a multitude of ways ready to probe students' understanding.
- No assigned experiment or project is deemed complete unless and until the student demonstrates the task in action. If this cannot be done "live" before the instructor, video-recordings showing the demonstration are acceptable. All relevant safety precautions must be followed, all test equipment must be used correctly, and the student must be able to properly explain all results. The student must also successfully answer all Challenges presented by the instructor for that experiment or project.

Students learning from these modules would do well to abide by the following principles:

- No text should be considered fully and adequately read unless and until you can express every idea *in your own words, using your own examples*.
- You should always articulate your thoughts as you read the text, noting points of agreement, confusion, and epiphanies. Feel free to print the text on paper and then write your notes in the margins. Alternatively, keep a journal for your own reflections as you read. This is truly a helpful tool when digesting complicated concepts.
- Never take the easy path of highlighting or underlining important text. Instead, *summarize* and/or *comment* on the text using your own words. This actively engages your mind, allowing you to more clearly perceive points of confusion or misunderstanding on your own.
- A very helpful strategy when learning new concepts is to place yourself in the role of a teacher, if only as a mental exercise. Either explain what you have recently learned to someone else, or at least *imagine* yourself explaining what you have learned to someone else. The simple act of having to articulate new knowledge and skill forces you to take on a different perspective, and will help reveal weaknesses in your understanding.
- Perform each and every mathematical calculation and thought experiment shown in the text on your own, referring back to the text to see that your results agree. This may seem trivial and unnecessary, but it is critically important to ensuring you actually understand what is presented, especially when the concepts at hand are complicated and easy to misunderstand. Apply this same strategy to become proficient in the use of *circuit simulation software*, checking to see if your simulated results agree with the results shown in the text.
- Above all, recognize that learning is hard work, and that a certain level of frustration is unavoidable. There are times when you will struggle to grasp some of these concepts, and that struggle is a natural thing. Take heart that it will yield with persistent and varied⁴ effort, and never give up!

Students interested in using these modules for self-study will also find them beneficial, although the onus of responsibility for thoroughly reading and answering questions will of course lie with that individual alone. If a qualified instructor is not available to challenge students, a workable alternative is for students to form study groups where they challenge⁵ one another.

To high standards of education,

Tony R. Kuphaldt

⁴As the old saying goes, “Insanity is trying the same thing over and over again, expecting different results.” If you find yourself stumped by something in the text, you should attempt a different approach. Alter the thought experiment, change the mathematical parameters, do whatever you can to see the problem in a slightly different light, and then the solution will often present itself more readily.

⁵Avoid the temptation to simply share answers with study partners, as this is really counter-productive to learning. Always bear in mind that the answer to any question is far less important in the long run than the method(s) used to obtain that answer. The goal of education is to empower one’s life through the improvement of clear and independent thought, literacy, expression, and various practical skills.

Appendix C

Tools used

I am indebted to the developers of many open-source software applications in the creation of these learning modules. The following is a list of these applications with some commentary on each.

You will notice a theme common to many of these applications: a bias toward *code*. Although I am by no means an expert programmer in any computer language, I understand and appreciate the flexibility offered by code-based applications where the user (you) enters commands into a plain ASCII text file, which the software then reads and processes to create the final output. Code-based computer applications are by their very nature *extensible*, while WYSIWYG (What You See Is What You Get) applications are generally limited to whatever user interface the developer makes for you.

The GNU/Linux computer operating system

There is so much to be said about Linus Torvalds' `Linux` and Richard Stallman's `GNU` project. First, to credit just these two individuals is to fail to do justice to the *mob* of passionate volunteers who contributed to make this amazing software a reality. I first learned of `Linux` back in 1996, and have been using this operating system on my personal computers almost exclusively since then. It is *free*, it is completely *configurable*, and it permits the continued use of highly efficient `Unix` applications and scripting languages (e.g. shell scripts, Makefiles, `sed`, `awk`) developed over many decades. `Linux` not only provided me with a powerful computing platform, but its open design served to inspire my life's work of creating open-source educational resources.

Bram Moolenaar's Vim text editor

Writing code for any code-based computer application requires a *text editor*, which may be thought of as a word processor strictly limited to outputting plain-ASCII text files. Many good text editors exist, and one's choice of text editor seems to be a deeply personal matter within the programming world. I prefer `Vim` because it operates very similarly to `vi` which is ubiquitous on `Unix/Linux` operating systems, and because it may be entirely operated via keyboard (i.e. no mouse required) which makes it fast to use.

Donald Knuth's \TeX typesetting system

Developed in the late 1970's and early 1980's by computer scientist extraordinaire Donald Knuth to typeset his multi-volume magnum opus *The Art of Computer Programming*, this software allows the production of formatted text for screen-viewing or paper printing, all by writing plain-text code to describe how the formatted text is supposed to appear. \TeX is not just a markup language for documents, but it is also a Turing-complete programming language in and of itself, allowing useful algorithms to be created to control the production of documents. Simply put, *\TeX is a programmer's approach to word processing*. Since \TeX is controlled by code written in a plain-text file, this means anyone may read that plain-text file to see exactly how the document was created. This openness afforded by the code-based nature of \TeX makes it relatively easy to learn how other people have created their own \TeX documents. By contrast, examining a beautiful document created in a conventional WYSIWYG word processor such as Microsoft **Word** suggests nothing to the reader about *how* that document was created, or what the user might do to create something similar. As Mr. Knuth himself once quipped, conventional word processing applications should be called WYSIAYG (What You See Is *All* You Get).

Leslie Lamport's \LaTeX extensions to \TeX

Like all true programming languages, \TeX is inherently extensible. So, years after the release of \TeX to the public, Leslie Lamport decided to create a massive extension allowing easier compilation of book-length documents. The result was \LaTeX , which is the markup language used to create all ModEL module documents. You could say that \TeX is to \LaTeX as **C** is to **C++**. This means it is permissible to use any and all \TeX commands within \LaTeX source code, and it all still works. Some of the features offered by \LaTeX that would be challenging to implement in \TeX include automatic index and table-of-content creation.

Tim Edwards' **Xcircuit** drafting program

This wonderful program is what I use to create all the schematic diagrams and illustrations (but not photographic images or mathematical plots) throughout the ModEL project. It natively outputs PostScript format which is a true vector graphic format (this is why the images do not pixellate when you zoom in for a closer view), and it is so simple to use that I have never had to read the manual! Object libraries are easy to create for **Xcircuit**, being plain-text files using PostScript programming conventions. Over the years I have collected a large set of object libraries useful for drawing electrical and electronic schematics, pictorial diagrams, and other technical illustrations.

Gimp graphic image manipulation program

Essentially an open-source clone of Adobe's `PhotoShop`, I use `Gimp` to resize, crop, and convert file formats for all of the photographic images appearing in the `MODEL` modules. Although `Gimp` does offer its own scripting language (called `Script-Fu`), I have never had occasion to use it. Thus, my utilization of `Gimp` to merely crop, resize, and convert graphic images is akin to using a sword to slice bread.

SPICE circuit simulation program

`SPICE` is to circuit analysis as `TEX` is to document creation: it is a form of markup language designed to describe a certain object to be processed in plain-ASCII text. When the plain-text "source file" is compiled by the software, it outputs the final result. More modern circuit analysis tools certainly exist, but I prefer `SPICE` for the following reasons: it is *free*, it is *fast*, it is *reliable*, and it is a fantastic tool for *teaching* students of electricity and electronics how to write simple code. I happen to use rather old versions of `SPICE`, version 2g6 being my "go to" application when I only require text-based output. `NGSPICE` (version 26), which is based on Berkeley `SPICE` version 3f5, is used when I require graphical output for such things as time-domain waveforms and Bode plots. In all `SPICE` example netlists I strive to use coding conventions compatible with all `SPICE` versions.

Andrew D. Hwang's ePiX mathematical visualization programming library

This amazing project is a `C++` library you may link to any `C/C++` code for the purpose of generating PostScript graphic images of mathematical functions. As a completely free and open-source project, it does all the plotting I would otherwise use a Computer Algebra System (CAS) such as `Mathematica` or `Maple` to do. It should be said that `ePiX` is *not* a Computer Algebra System like `Mathematica` or `Maple`, but merely a mathematical *visualization* tool. In other words, it won't determine integrals for you (you'll have to implement that in your own `C/C++` code!), but it can graph the results, and it does so beautifully. What I really admire about `ePiX` is that it is a `C++` programming library, which means it builds on the existing power and toolset available with that programming language. Mr. Hwang could have probably developed his own stand-alone application for mathematical plotting, but by creating a `C++` library to do the same thing he accomplished something much greater.

gnuplot mathematical visualization software

Another open-source tool for mathematical visualization is `gnuplot`. Interestingly, this tool is *not* part of Richard Stallman’s GNU project, its name being a coincidence. For this reason the authors prefer “gnu” *not* be capitalized at all to avoid confusion. This is a much “lighter-weight” alternative to a spreadsheet for plotting tabular data, and the fact that it easily outputs directly to an X11 console or a file in a number of different graphical formats (including PostScript) is very helpful. I typically set my `gnuplot` output format to default (X11 on my Linux PC) for quick viewing while I’m developing a visualization, then switch to PostScript file export once the visual is ready to include in the document(s) I’m writing. As with my use of `Gimp` to do rudimentary image editing, my use of `gnuplot` only scratches the surface of its capabilities, but the important points are that it’s *free* and that it *works well*.

Python programming language

Both Python and C++ find extensive use in these modules as instructional aids and exercises, but I’m listing Python here as a *tool* for myself because I use it almost daily as a *calculator*. If you open a Python interpreter console and type `from math import *` you can type mathematical expressions and have it return results just as you would on a hand calculator. Complex-number (i.e. *phasor*) arithmetic is similarly supported if you include the complex-math library (`from cmath import *`). Examples of this are shown in the Programming References chapter (if included) in each module. Of course, being a fully-featured programming language, Python also supports conditionals, loops, and other structures useful for calculation of quantities. Also, running in a console environment where all entries and returned values show as text in a chronologically-ordered list makes it easy to copy-and-paste those calculations to document exactly how they were performed.

Appendix D

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Appendix E

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Appendix F

Version history

This is a list showing all significant additions, corrections, and other edits made to this learning module. Each entry is referenced by calendar date in reverse chronological order (newest version first), which appears on the front cover of every learning module for easy reference. Any contributors to this open-source document are listed here as well.

9-10 February 2025 – edited the “Generator failure” Conceptual Reasoning question, and also made minor edits to the “Multiple-LED circuit” Diagnostic Reasoning question.

7 February 2025 – added another Quantitative Reasoning question, this one requiring both KVL and KCL to solve.

2 October 2024 – added another “Challenging concepts” bullet-item to the list in the Introduction chapter.

12 September 2024 – minor edits to the Full Tutorial chapter.

3 September 2024 – added another section to the Introduction chapter specifically for challenges related to this module’s topic.

1 July 2024 – divided the Introduction chapter into two sections, one for students and one for instructors, and added content to the instructor section recommending learning outcomes and measures.

6 July 2023 – added numerical example of KCL to the Simplified Tutorial.

25 June 2023 – added Technical Reference section on grounding.

27 November 2022 – placed questions at the top of the itemized list in the Introduction chapter prompting students to devise experiments related to the tutorial content.

29 June 2022 – divided the Full Tutorial into sections. Also substituted “node” in place of “point” in many places within the Full Tutorial.

- 15 February 2022** – edited the “Mysterious fuse failure” Diagnostic Reasoning problem to say that BOTH fuses blow, not just one.
- 7 February 2022** – added a new sub-question to “Battery currents” Quantitative Reasoning problem challenging students to double-check their work.
- 18 March 2021** – corrected multiple instances of “volts” that should have been capitalized “Volts”.
- 23 August 2020** – significantly edited the Introduction chapter to make it more suitable as a pre-study guide and to provide cues useful to instructors leading “inverted” teaching sessions.
- 14 May 2020** – added Conceptual Question on generator failure in a DC system.
- 24 April 2020** – added an instructor note about NFPA 70 article 240.8 (NEC 2017 edition) with regard to paralleled fuses and circuit breakers.
- 19 September 2019** – added a Case Tutorial chapter, and also some questions.
- 3 June 2019** – edited KCL confirmation experiment to make it less prescriptive.
- 23 May 2019** – added some Challenge questions.
- 20 May 2019** – changed “Amps” to “Amperes”.
- 14 May 2019** – added Challenge question to “Multiple-LED circuit” in Diagnostic Reasoning section. Also added a Challenge question to “Three-wire electrical power distribution” in Conceptual Reasoning section.
- 5 November 2018** – retitled Historical References section(s) so as to not be redundant to the “Historical References” chapter.
- September 2018** – renamed “Derivations and Technical References” chapter to “Historical References”.
- August 2018** – added historical reference to Kirchhoff’s publishing of KCL in the *Annals of Physics and Chemistry* in 1845, as a screenshot from a page of this journal. Also added content to the Introduction chapter.
- July 2018** – minor edits to Simplified Tutorial chapter. Added Historical references section.
- May 2018** – added a Simplified Tutorial chapter.
- August 2017** – minor edits, added a new conceptual question on 3-wire DC power circuits.
- January 2017** – minor clarifications made to the concepts of electrically common points, equipotential points, and parallel networks.
- October 2016** – created changelog for future use.

September 2016 – document first published.

Index

- Adding quantities to a qualitative problem, 88
- Ammeter, 21
- Ampere, 19
- Annotating diagrams, 87

- Benchmark, 33
- Branch, parallel circuit, 22
- Breadboard, solderless, 76, 77
- Breadboard, traditional, 79

- Cardio-Pulmonary Resuscitation, 74
- Checking for exceptions, 88
- Checking your work, 88
- Closed, 23
- Code, computer, 95
- Conductor, 19, 24
- Conservation of Charge, 18
- Conservation of Electric Charge, 13, 15, 24
- Conservation of Energy, 17, 18
- Conservation of Mass, 13
- Coulomb, 18
- CPR, 74
- Current, 19

- Dalziel, Charles, 74
- Datum, 33
- Dimensional analysis, 87
- DIN rail, 77
- DIP, 76
- Dynamo, 46

- Edwards, Tim, 96
- Electric potential, 18, 32
- Electric shock, 74
- Electrically common, 22, 23
- Electrically common points, 75
- Enclosure, electrical, 79
- Energy, 17

- Equipotential, 22, 23
- Equipotential points, 75, 77
- Experiment, 80
- Experimental guidelines, 81

- Fault, 23

- Graph values to solve a problem, 88
- Gravitational potential energy, 18
- Greenleaf, Cynthia, 37
- Ground, 33
- Ground, different definitions of, 32

- Hot, 36
- How to teach with these modules, 90
- Hwang, Andrew D., 97

- IC, 76
- Identify given data, 87
- Identify relevant principles, 87
- Instructions for projects and experiments, 91
- Insulators, 19
- Intermediate results, 87
- Inverted instruction, 90

- Joule, 17

- KCL, 14, 24, 25
- Kilogram, 17
- Kinetic energy, 17
- Kirchhoff's Current Law, 3, 14, 24, 25
- Kirchhoff's Voltage Law, 3
- Kirchhoff, Gustav Robert, 3
- Knuth, Donald, 96

- Lamport, Leslie, 96
- LED, 70
- Light-Emitting Diode, 70

- Limiting cases, 88
- Load, 20
- Maxwell, James Clerk, 29
- Mean Sea Level, 33
- Metacognition, 42
- Meter, 17
- Moolenaar, Bram, 95
- Murphy, Lynn, 37
- Netlist, SPICE, 63
- Network, 21
- Neutral, 36
- Node, 14, 25
- Ohm's Law, 19
- Open, 23
- Open fault, 23
- Open-source, 95
- Parallel, 14, 22
- Polarity symbols, 20
- Potential, 18
- Potential distribution, 77
- Potential energy, 17
- Potential energy, gravitational, 18
- Potential, electric, 32
- Problem-solving: annotate diagrams, 87
- Problem-solving: check for exceptions, 88
- Problem-solving: checking work, 88
- Problem-solving: dimensional analysis, 87
- Problem-solving: graph values, 88
- Problem-solving: identify given data, 87
- Problem-solving: identify relevant principles, 87
- Problem-solving: interpret intermediate results, 87
- Problem-solving: limiting cases, 88
- Problem-solving: qualitative to quantitative, 88
- Problem-solving: quantitative to qualitative, 88
- Problem-solving: reductio ad absurdum, 88
- Problem-solving: simplify the system, 87
- Problem-solving: thought experiment, 81, 87
- Problem-solving: track units of measurement, 87
- Problem-solving: visually represent the system, 87
- Problem-solving: work in reverse, 88
- Project management guidelines, 84
- Qualitatively approaching a quantitative problem, 88
- Reading Apprenticeship, 37
- Reductio ad absurdum, 88–90
- Resistance, 19
- Safety, electrical, 74
- Schoenbach, Ruth, 37
- Scientific method, 42, 80
- Scope creep, 84
- Sea Level, Mean, 33
- Series, 14, 21
- Short, 23
- Shorted fault, 23
- Shunt resistor, 76
- Simplifying a system, 87
- Socrates, 89
- Socratic dialogue, 90
- Solderless breadboard, 76, 77
- Source, 20
- SPICE, 37, 63, 81
- SPICE netlist, 78
- Stallman, Richard, 95
- Subpanel, 79
- Surface mount, 77
- Switch, 23
- Terminal block, 75–79
- Thought experiment, 81, 87
- Three-wire power system, 8, 46
- Torvalds, Linus, 95
- Units of measurement, 87
- Visualizing a system, 87
- Volt, 18
- Voltage, 18, 32
- Voltmeter, 22
- Wiring sequence, 78
- Work in reverse to solve a problem, 88
- WYSIWYG, 95, 96