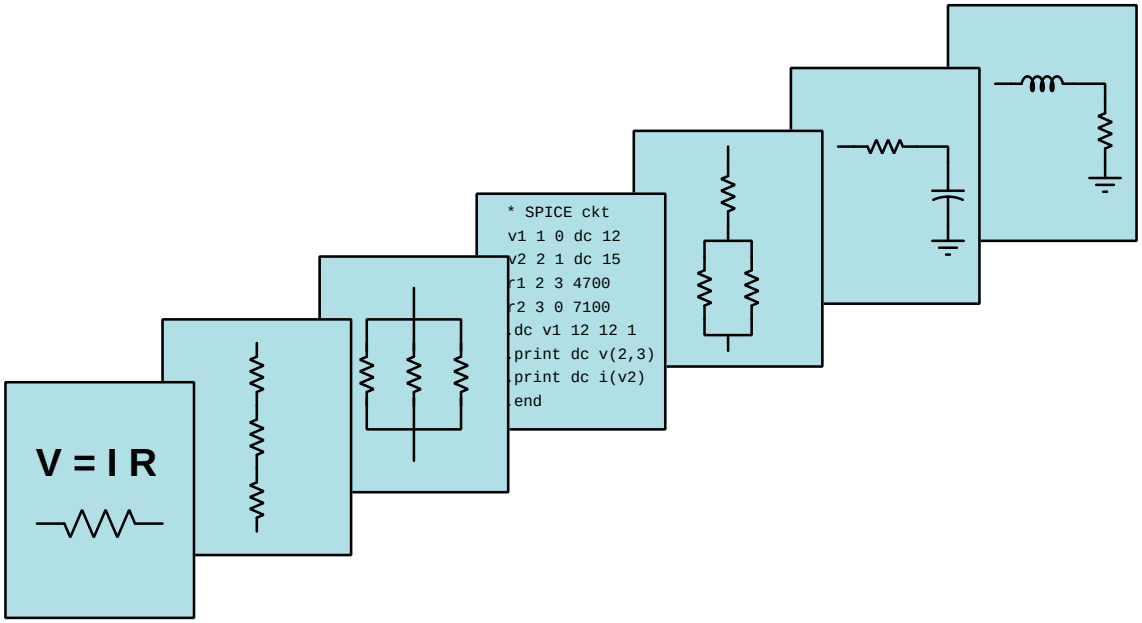
# Modular Electronics Learning (ModEL) project




## Kirchhoff's Voltage Law

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Last update = 17 September 2026

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# Chapter 1

# Introduction

## 1.1 Recommendations for students

Two extremely important principles of electric circuits are Kirchhoff's Voltage Law and Kirchhoff's Current Law, both named after Gustav Robert Kirchhoff who formulated them in the middle of the 19th century. These principles are every bit as important as Ohm's Law and Joule's Law, as they aid our analysis of all kinds of circuits. They are also pedagogically important because they clarify the essential natures of voltage and current, respectively.

All practitioners of electricity and electronics owe it to themselves to master Kirchhoff's Laws in order to build a firm conceptual foundation for later studies. This particular learning module focuses on the former of Kirchhoff's Laws (voltage), beginning with a review of basic concepts as well as series and parallel circuit connections. Mastery of Kirchhoff's Voltage Law enables the determination of voltages in any multi-component electrical circuit based on the magnitudes and polarities of other voltages in the same circuit.

Important concepts related to Kirchhoff's Voltage law include the relationship between **voltage** and **energy**, **conservation** of energy, the energy possessed by mass with certain types of **fields**, voltage **polarity** and its relationship to values displayed by **voltmeters**, **conductors** versus **insulators**, electrical **resistance**, **Ohm's Law**, **Joule's Law**, **sources** versus **loads**, **series** versus **parallel** networks, **ground** and **bus** connections, and electrical **shock hazard**.

The Tutorials apply *thought experiments* as a means of explaining Kirchhoff's Voltage Law, and it is highly recommended for you to perform your own (different) thought experiments on the same circuits. You will get much more understanding out of the text by approaching it in this proactive manner rather than simply reading the examples it gives. On a similar note, a good resource for you is the *Case Tutorial* chapter where you will find example circuits to apply and test your understanding of Kirchhoff's Voltage Law. The *aluminum smelting pots* example in particular is very practical as it directly relates to electrical safety and the placement of "ground" connections in a dangerous circuit.

When reading any mathematically-based presentation, a useful habit for effective learning is to actually perform the mathematics being shown in the text. Don't just passively read what the text tells you and trust that the math works – try the math for yourself. Not only will this serve

to confirm what you are reading, but it is also an excellent way to practice those mathematical techniques.

Here are some good questions to ask of yourself while studying this subject:

- How might an experiment be designed and conducted to demonstrate the principle of Kirchhoff's Voltage Law? What hypothesis (i.e. prediction) might you pose for that experiment, and what result(s) would either support or disprove that hypothesis?
- How does voltage in a circuit relate analogously to the potential energy of a mass carried to different altitudes?
- Why is Kirchhoff's Voltage Law true – i.e. what is the fundamental principle it is based on?
- What does the mathematical sign of a voltmeter's measurement indicate about voltage between two given points within a circuit?
- What exactly do the "+" and "−" polarity symbols mean in an electric circuit?
- Why is current the same everywhere in a series network – what is the fundamental principle at work here?
- Why is voltage the same across all components within a parallel network – what is the fundamental principle at work here?
- Why do we connect voltmeters and ammeters in very specific ways to circuits being tested?
- What conditions must be met to make a circuit "open" or "shorted"?
- Why is electrical power a function of both voltage and current?
- How is Kirchhoff's Voltage Law useful in ways that Ohm's Law and Joule's Law are not? In other words, how is Kirchhoff's Voltage Law uniquely useful?
- Why does Kirchhoff's Voltage Law still hold true even if the loop we happen to trace runs against the flow of current?
- Why does Kirchhoff's Voltage Law still hold true even if the loop we happen to trace does not follow real wire connections?
- How might circuits behave if Kirchhoff's Voltage Law were not true?
- Why is it important to *annotate* a circuit's schematic diagram with symbols showing voltage polarity when applying Kirchhoff's Voltage Law?
- How do "ground" symbols simplify schematic diagrams?
- What does it mean in a schematic diagram to have a "+V" line and a ground symbol connected to various components, with no electrical source explicitly drawn?
- Why is voltage always measured between *two* points?
- If we know that voltage always exists as a quantity between *two* points, how is it possible to legitimately quantify voltage at a single point in a circuit (e.g. $V_F$ = voltage at test point "F")?
- Why is grounding important to electrical safety?

## 1.2 Challenging concepts related to Kirchhoff's Voltage Law

The following list cites concepts related to this module's topic that are easily misunderstood, along with suggestions for properly understanding them:

- **Defining a series network** – series networks are defined by having a single path for current between the components. From this definition, and from the Conservation of Electric Charge, follows the conclusion that series-connected components must experience the same amount of continuous current. One challenge is that sometimes series-connected components do not physically or pictorially align to suggest a single path, even though they have but one *electrical* path for current. Another challenge is reasoning logically from this definition of "one path" to the necessary conclusion of "same current", as many students are so accustomed to memorizing facts without seeing the logical connections between those facts.

- **Definition versus properties of series networks** – again, series networks are defined by having just one pathway for current between components. Everything else we know about series networks *derives* from this definition but are not *synonymouns* with this definition. For example, the fact that voltages add in a series network is a consequence of series networks but not a defining quality. The fact that sereis-connected components necessarily share the same current is another consequence of series networks but not a defining quality. All we need to consider in order to understand this distinction is a counter-example of some electrical network exhibiting either summed voltages or equal currents without being series. A good counter-example of the former misconception is a five-resistor bridge network where all five resistor values are unique and non-proportionate, this being a clear example of a network where voltages across resistors on either side of the network indeed add up to the source's voltage value despite none of the resistors being in series with any other. A good counter-example of the latter misconception is any network that just happens to contain two or more components having the same current that are not series to each other (e.g. two identical resistors in parallel with each other). Applying counter-examples to disprove this and similar fallacies is a rhetorical technique called *reductio ad absurdum*, and the habit of looking out for counter-examples when testing claims is a powerful tool for critical thinking.

- **Defining a parallel network** – parallel networks are defined by all components spanning the same two sets of electrically-common points. From this definition, and from the Conservation of Energy, follows the conclusion that parallel-connected components must experience the same amount of continuous voltage. One challenge is that sometimes parallel-connected components do not physically or pictorially align in a parallel fashion, even though they are in fact parallel in the *electrical* sense. Another challenge is reasoning logically from this definition of "between the same two sets of electrically-common points" to the necessary conclusion of "same voltage", as many students are so accustomed to memorizing facts without seeing the logical connections between those facts. A helpful way to both identify parallel connections as well as to conceptualize the commonality of voltage is to use color-highlighting to mark all electrically-common points with the same color. This way, all points in one set have one color, all points in the other set have another color, and all components connecting to points with those two colors must be connected in parallel with each other. This coloring approach is used in the Tutorial chapters where points A-B-C-D are all colored blue and points E-F-G-H are all colored green. The *Case Tutorial* section "Example: potential in open and shorted circuits" is a

helpful reference for using color-highlighting to denote different amounts of electrical potential in a circuit.

- **Definition versus properties of parallel networks** – again, parallel networks are defined by all components spanning the same two sets of electrically-common points. Everything else we know about parallel networks *derives* from this definition but are not *synonymouns* with this definition. For example, the fact that currents divide in a parallel network is a consequence of parallel networks but not a defining quality. The fact that parallel-connected components necessarily share the same voltage is another consequence of parallel networks but not a defining quality. All we need to consider in order to understand this distinction is a counter-example of some electrical network exhibiting either divided currents or equal voltages without being parallel. A good counter-example of the former misconception is a five-resistor bridge network where all five resistor values are unique and non-proportionate, this being a clear example of a network where currents divide despite none of the resistors being in parallel with any other. A good counter-example of the latter misconception is any network that just happens to contain two or more components having the same voltage that are not parallel to each other (e.g. two identical resistors in series with each other). Applying counter-examples to disprove this and similar fallacies is a rhetorical technique called *reductio ad absurdum*, and the habit of looking out for counter-examples when testing claims is a powerful tool for critical thinking.

- **Voltage polarity symbols** – the "+" and "−" symbols used to denote voltages in circuits only have meaning as *pairs* and do not mean anything on their own.

- **Voltage as a relative quantity** – this is arguably one of the most difficult concepts students first encounter when studying electricity: understanding that voltage is a *difference* in energy levels between two locations, and not something that can exist at any single point. In this respect, voltage is similar to *distance* which is always something existing *between* two specified points.

- **Ground** – unfortunately the word "ground" has more than one accepted meaning in the field of electricity and electronics, and so one must be careful to define what is meant by "ground" before freely using this word. In this module the word "ground" primarily means an arbitrary point of reference for measuring electrical potentials, analogous to "sea level" as an arbitrary elevation from which to specify the height of a mountain peak or any other elevation we might wish to state.

- **Schematic annotation as a problem-solving tool** – as an instructor you will find most students new to the study of electronics attempt to solve circuit-analysis problems by inspection, or by plugging given values into familiar equations until something resembling an answer emerges. Such efforts are usually fruitless because it is difficult to track the real-world context of every circuit parameter doing things this way. You will need to demonstrate, encourage, and at times *require* your students to apply the alternative strategy of first sketching a schematic diagram of the circuit and then annotating it with all known values. Once the given information gains context through this annotation, appropriate principles (e.g. Ohm's Law, KVL, KCL) are much easier to identify and then properly apply for that circuit. Then, as calculated values get annotated on the same schematic diagram the next logical steps to follow become easier and easier to identify. I recommend using different colors to label different

parameters (e.g. blue for voltage, red for current, black for component values, green for power). Students should have blank "scratch" paper and writing instruments with them at all times, even during discussions, so as to neatly organize information and to articulate and focus their thoughts!

- **Choosing KVL loops** – the choice of which pathway to "step around" in any electrical network while applying Kirchhoff's Voltage Law is completely arbitrary so long as the path ends where it began. The loop need not follow in the same direction as current, nor need it even follow conductors, nor need it include a source! What you are doing when tracing a KVL loop is taking an imaginary "test charge" on a journey to various points in an electrical network and tallying how much energy that charge would have to gain or lose if it moved from one point to another. The path that imaginary "test charge" takes is utterly irrelevant to the truth of KVL, as is the means by which it might move from point to point along that path. To have practical value, any KVL loop we trace should have but one unknown voltage in it (i.e. one interval between points where we do not yet know the voltage) with all the other voltage values known.
- **Assuming zero for unknown voltages** – a common mistake among students new to this topic is to assume an unknown voltage must be *zero*. This, of course, is not true. There is a vast difference between "zero knowledge" of a voltage versus a voltage known to be zero!

The *Derivations and Technical References* chapter contains a section called "Ground in electric and electronic circuits" describing the various circuit-related meanings of the term "ground".

## 1.3 Recommendations for instructors

This section lists realistic student learning outcomes supported by the content of the module as well as suggested means of assessing (measuring) student learning. The outcomes state what learners should be able to do, and the assessments are specific challenges to prove students have learned.

- **Outcome** – Demonstrate effective technical reading and writing

  Assessment – Students present their outlines of this module's instructional chapters (e.g. Case Tutorial, Tutorial, Historical References, etc.) ideally as an entry to a larger Journal document chronicling their learning. These outlines should exhibit good-faith effort at summarizing major concepts explained in the text.

  Assessment – Students show how quantitative KVL results were obtained by the author in the Tutorial chapter's examples.

  Assessment – Students present their "Conceptual Toolbox" Theory Journal entries listing important concepts applied in the Tutorial text(s). A "Conceptual Toolbox" serves as inspiration when solving problems, each concept suggesting a potential problem-solving action. Challenge students to collaboratively generate a Conceptual Toolbox for the day's topic during the class session and then use it when solving problems together, thus fostering the habit of reasoning from principles rather than recalling procedures!

- **Outcome** – Apply the concept of Kirchhoff's Voltage Law to solve voltages in any circuit

  Assessment – Solve for an unknown voltage in a circuit where the schematic diagram shows other known voltages; e.g. pose problems in the form of the "Tracing KVL loops in a multi-source circuit" Quantitative Reasoning question.

  Assessment – Solve for both differential and ground-referenced voltages in a circuit where the schematic diagram shows all resistor and source values; e.g. pose problems in the form of the "Series resistor network with common-mode source" Quantitative Reasoning question.

- **Outcome** – Independent research

  Assessment – Translate[1] Gustav Robert Kirchhoff's own explanation of his voltage law from German into English and then summarize in your own words portions of it. A recommended source is pages 501-502 of *Annalen der Physik und Chemie*, No. 4, Band LXIV, published in Germany in the year 1845 by Johann Christian Poggendorff.

[1] If you aren't fluent in German, feel free to use language-translation software to do this for you!

# Chapter 2

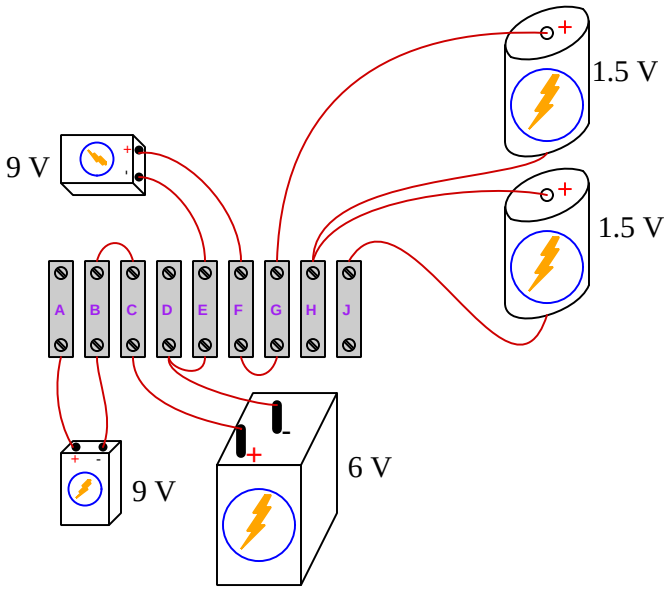
# Case Tutorial

The idea behind a *Case Tutorial* is to explore new concepts by way of example. In this chapter you will read less presentation of theory compared to other Tutorial chapters, but by close observation and comparison of the given examples be able to discern patterns and principles much the same way as a scientific experimenter. Hopefully you will find these cases illuminating, and a good supplement to text-based tutorials.

These examples also serve well as challenges following your reading of the other Tutorial(s) in this module – can you explain *why* the circuits behave as they do?

## 2.1 Example: multiple-battery array

Here, several batteries are provided for experimentation. No complete path exists for current (i.e. there is no "circuit" technically speaking), but we may still measure voltage between pairs of points. We will use *terminal blocks* to neatly organize all wire connections between batteries:



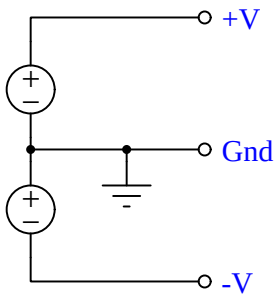

Rather than visually show the placement of test equipment in the diagram, measurements will be documented in text form as $V$ values[1]. With the circuit constructed as shown in the above illustration, we obtain the following measurements:

- $V_{CE} = 6$ Volts
- $V_{JG} = -3$ Volts
- $V_{DA} = -15$ Volts
- $V_{HD} = 7.5$ Volts
- $V_{BC} = 0$ Volts
- $V_{FB} = 3$ Volts
- $V_{EJ} = -6$ Volts
- $V_{AF} = 6$ Volts
- $V_{CG} = -3$ Volts

[1] Double-lettered subscripts for $V$ denote the placement of voltmeter test leads, with the first and second letters always representing the red and black test leads respectively. For example, $V_{BC}$ means the voltage measured between terminals B and C with the red test lead touching B and the black test lead touching C. Single-lettered subscripts for $V$ represent the voltmeter's red test lead, with the black test lead touching a defined "ground" point in the circuit.

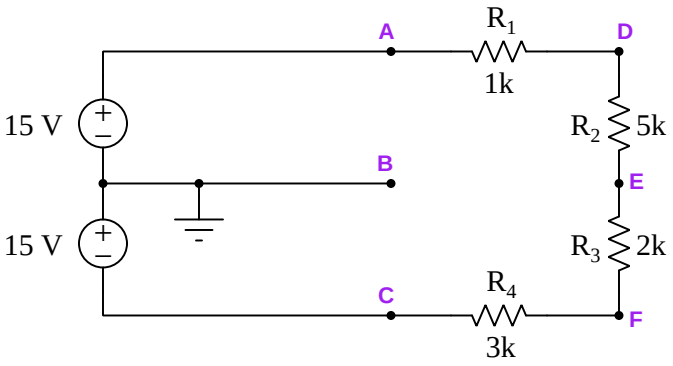
## 2.2 Example: split DC power supply

Many electronic circuits require what is called a *split* power supply, providing electrical power via three terminals: +V, −V, and "Ground":




From +V to Ground we measure the voltage of the upper source; from −V to ground we measure the negative voltage of the lower source; between +V and −V we measure those sources' voltage sum. Typical split DC supplies are symmetrical, meaning that +V and −V have the same absolute values.

These circuits also provide a great application for Kirchhoff's Voltage Law. Consider the following series circuit energized by a split power supply:



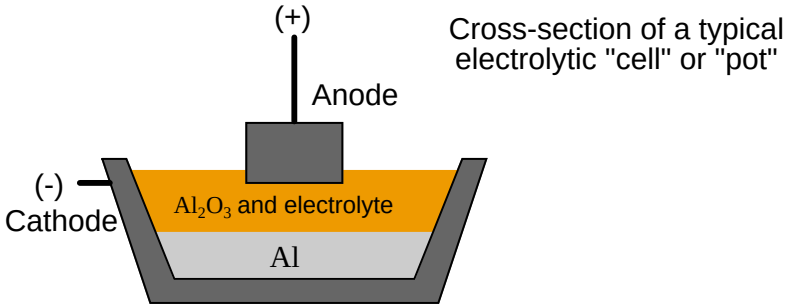

Voltages specified by double-letter subscripts imply the voltmeter's red test lead on the first point and the black test lead on the second; voltages specified by single-letter subscripts imply the voltmeter's red test lead on the specified point and the black test lead on ground:

- $V_{AD} = V_{R1} = +2.7273$ Volts
- $V_{DE} = V_{R2} = +13.636$ Volts
- $V_{EF} = V_{R3} = +5.4545$ Volts
- $V_{FC} = V_{R4} = +8.1818$ Volts
- $V_A = +15$ Volts

- $V_B = 0$ Volts
- $V_C = -15$ Volts
- $V_D = +12.273$ Volts
- $V_E = -1.3636$ Volts
- $V_F = -6.8182$ Volts

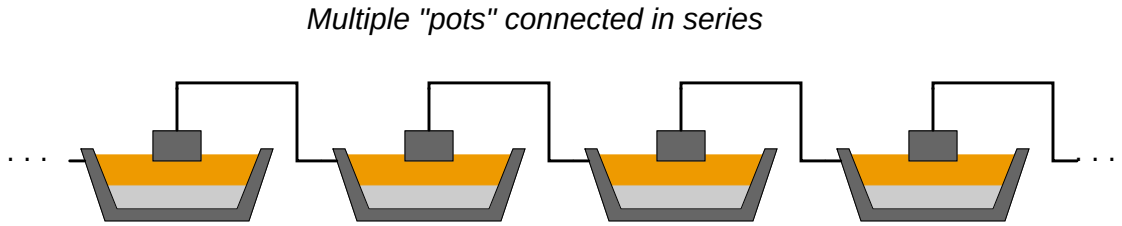
## 2.3 Example: aluminum smelting pots

Many manufacturing processes are *electrochemical* in nature, meaning that electricity is used to promote or force chemical reactions to occur. One such industry is aluminum smelting, where large amounts of DC current (typically *several hundred thousand* amperes!) is used to turn alumina ($Al_2O_3$) into pure metallic aluminum (Al):



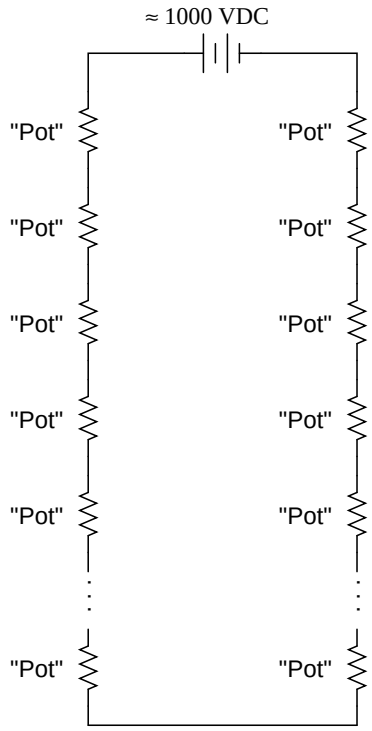

The alumina/electrolyte[2] solution is a molten bath of chemical compounds, lighter than pure aluminum itself. Molecules of pure aluminum precipitate out of this solution and settle at the bottom of the "pot" where the molten metal is periodically pumped out for further refining and processing. Fresh alumina powder is periodically dropped into the top of the pot to replenish what is converted into aluminum metal.

Although the amount of current necessary to smelt aluminum in this manner is huge, the voltage drop across each pot is only about 4 Volts. In order to keep the voltage and current levels reasonable in a large smelting facility, many of these pots are connected in series, where they act somewhat like resistors (being energy *loads* rather than energy *sources*):

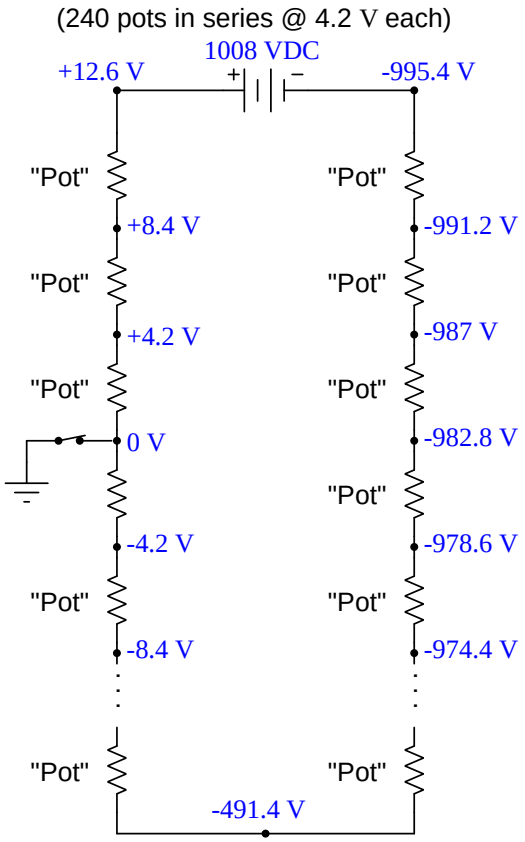


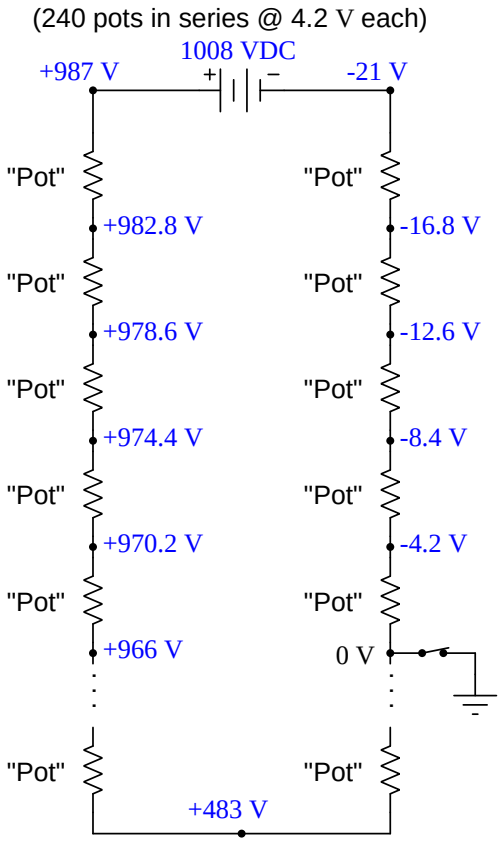

[2] An *electrolyte* is a liquid containing mobile ions capable of acting as electrical charge carriers. When a DC voltage is impressed across some distance within an electrolyte, the positively-charged "cations" drift toward the negative electrode (cathode) while the negatively-charged "anions" drift toward the positive electrode (anode). Electric current within a liquid solution, therefore, is a practical example where *both* the "electron-flow" and "conventional-flow" models of electricity are physically true. Within the electrolyte you actually have different types of electrical charge carriers flowing in opposite directions!

A typical "pot-line" might have 240 pots connected in series. With a voltage drop of just over 4 Volts apiece, the total voltage powering this huge series circuit averages around 1000 Volts DC:

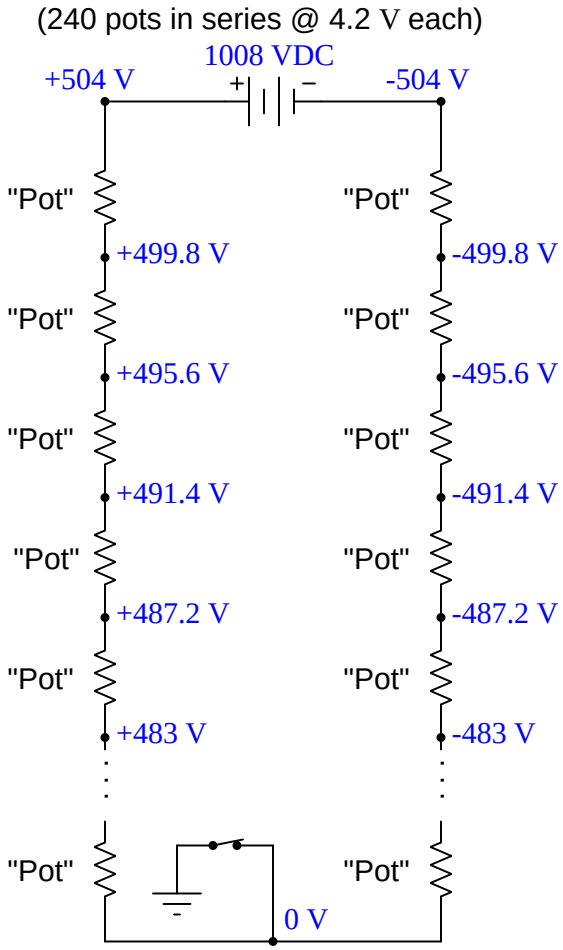


With these voltage levels, electrical safety is an obvious concern! To ensure the safety of personnel performing maintenance work around a specific pot, the system is equipped with a "movable ground" consisting of a large switch on wheels that may be connected to the steel frame of the shelter (with concrete pilings penetrating deep into the soil) and to the desired pot. Connecting this switch between the pot and Earth ground, then closing that switch, makes those two points electrically common to each other and therefore equipotential. Anyone physically contacting the grounded point and Earth ground will experience practically zero voltage between those two points, thus reducing shock hazard. Assuming a voltage drop of exactly 4.2 Volts across each pot, note what effect the ground's position has on the voltages around the circuit measured with respect to ground:



During typical operation the ground point is set mid-way along the series pot-line, resulting in the following voltages (all measured with respect to ground):



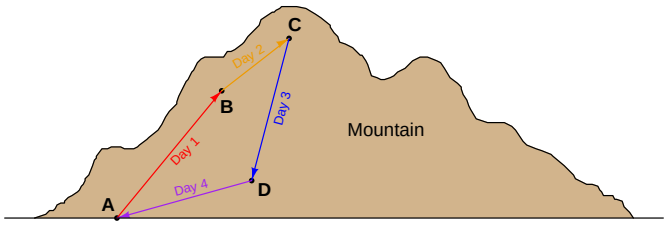
# Chapter 3

# Simplified Tutorial

*Energy* is what sets matter into motion, and it has many different forms. Two major categories of energy are *kinetic energy* and *potential energy*, the former being energy expressed in motion and the latter being energy having the *possibility* of motion. For example, a moving vehicle has kinetic energy, while the fuel stored in that vehicle has potential energy.

Energy is a *conserved* quantity, which means it can neither be created nor destroyed but the sum total in the universe always remains constant. This fact is called the *Law of Energy Conservation*, and it is one of the most fundamental principles in all of physics.

An object resting in an elevated location has *potential energy*, which is released if the object falls. So long as gravity and object mass are constant, the amount of potential energy possessed by any object is proportional to its height. If the object rises to a higher elevation, it gains (i.e. stores) potential energy; if it falls to a lower elevation, it loses (i.e. releases) potential energy. If an object undergoes a series of rises and falls only to return to its starting location, it finishes with the exact same amount of potential energy as it began. Consider the example of someone hiking up and down a mountain, beginning at location A, then hiking up to B, then up again to C, then down to D, and finally returning down to A. Every ascent represents a gain in potential energy, while every descent represents a loss:

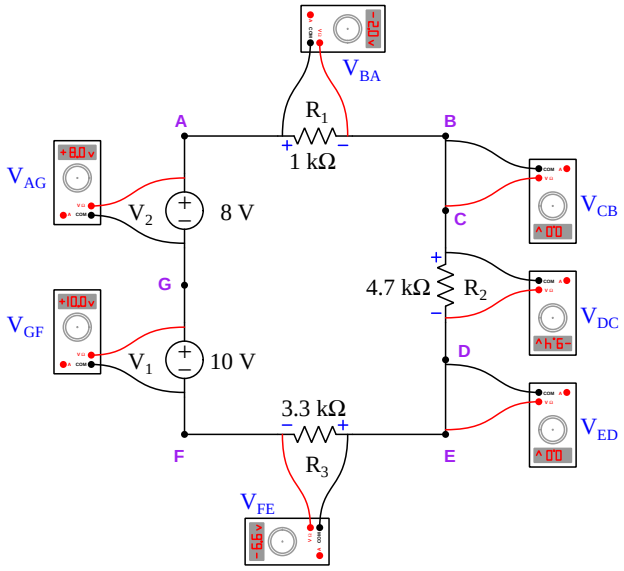



No matter the path taken or the mode of transport, though, the hiker's potential energy at location A at the end of the trip must be the same as their potential energy was at location A at the start of the trip, because it's same exact elevation with the same hiker mass and the same gravitational field. For the sake of argument, the hiker could have made the trip in reverse order (i.e. A-D-C-B-A) and this would still be true. They could have even skipped point C (the summit)

and their potential energy at A would still be the same at start and finish. They could have even flown in a helicopter to the summit rather than hike up the mountain, and their potential energy would still be the same as when they started once then return to A.

Electric charge carriers within an electric field are analogous to masses in a gravitational field, in that they also experience gains and losses in potential energy as they "rise" and "fall" within that field. If a charge carrier begins at some point in a circuit and returns to that same point, its potential energy will be the same at both times just as the hiker who begins and ends at the same elevation. As with the hiker, this conservation of potential energy at any given location is true regardless of the path taken by the electric charge carrier: *a return to the starting location guarantees zero net gain or loss in potential energy.* When applied to electricity, this fact is called *Kirchhoff's Voltage Law*, typically abbreviated as *KVL*.

Recall that the definition of *voltage* is the gain or loss of potential energy for an electric charge carrier between two locations. Applying Kirchhoff's Voltage Law to an electrical system means that when we measure all the voltages between successive pairs of points in any loop, the sum of those voltage measurements must be zero: in other words, every voltage rise must be balanced by an equal voltage drop in any loop where we return to the starting point.




In this example circuit where two series-aiding voltage sources power three resistors, the sum of all voltmeter measurements is zero. Resistor voltage drops were calculated by applying Ohm's Law to each resistor ($V = IR$) after solving for circuit current $I = \frac{V_{total}}{R_{total}}$ (18 Volts from the two sources divided by 9 kiloOhms of total series resistance). To test the principle of Kirchhoff's Voltage Law, we will start at point A and step around the loop clockwise tallying each voltmeter's measurement:

$$(-2.0\text{ V}) + (0.0\text{ V}) + (-9.4\text{ V}) + (0.0\text{ V}) + (-6.6\text{ V}) + (10.0\text{ V}) + (8.0\text{ V}) = 0\text{ V}$$

If we wish to determine the amount of voltage between any two points in this circuit not lying in the path of current, for example $V_{DG}$[1], neither Ohm's Law nor the properties of series networks would be of any help. However, we could solve for this unknown voltage using Kirchhoff's Voltage Law, tracing a loop that includes a leap between points D and G, then writing an equation with $V_{DG}$ as the only unknown voltage in that loop, and finally computing $V_{DG}$. For example, tracing loop D-E-F-G-D in our example circuit and writing a KVL equation where all voltages sum to zero:

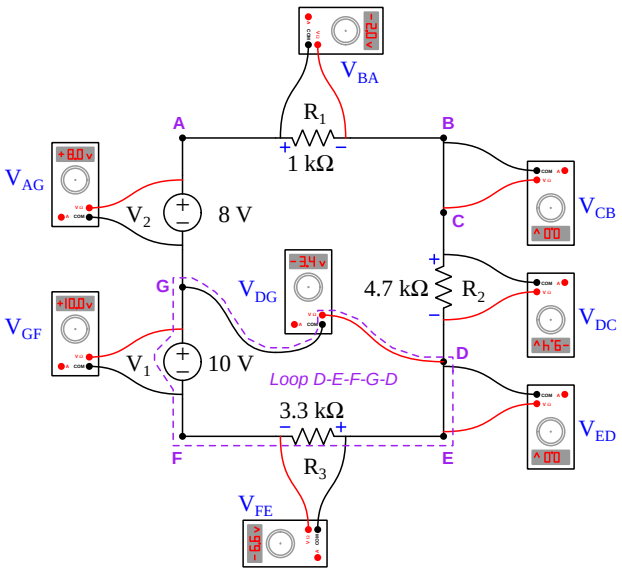
$$V_{ED} + V_{FE} + V_{GF} + V_{DG} = 0 \text{ V}$$

$$(0 \text{ V}) + (-6.6 \text{ V}) + (10.0 \text{ V}) + V_{DG} = 0 \text{ V}$$

$$3.4 \text{ V} + V_{DG} = 0 \text{ V}$$

$$V_{DG} = -3.4 \text{ V}$$

The negative value of $V_{DG}$ tells us charge carriers at point D each have a lower potential energy than charge carriers at point G; i.e. a voltmeter with red lead touching D and black lead touching G will register $-3.4$ Volts, as shown below:




[1]There is a convention for specifying voltmeter connections between two test points, which is that the first subscript letter denotes the connection point for the voltmeter's red-colored test lead, while the second subscript letter denotes the connection point for the black lead. Therefore, $V_{DG}$ means voltage read by a voltmeter with its red lead on D and its black lead on G. If a voltage is *ground-referenced* (also known as *single-ended*), it means that the voltage is taken between a single test point and some other location in the circuit called "ground", in which case only a single subscript is used to denote the red test lead because it is assumed the black lead will contact ground. Dual-subscripted voltages are called *differential*, as opposed to ground-referenced/single-ended.

As we can see, Kirchhoff's Voltage Law holds true for *any* loop we might happen to choose in an electrical network. This is true for the same reason that a mountain climber's net change in potential energy is zero so long as they always return to their starting point regardless of the path they took when hiking or even the mode of transport (e.g. hiking, skiing, rock climbing, helicopter ride).

Thus, Kirchhoff's Voltage Law is a useful mathematical tool for determining unknown voltages between points where other electrical laws and principles do not apply. This principle holds true for *any* electrical network, no matter how complicated, because the underlying principle of energy conservation is universal.

# Chapter 4

# Full Tutorial

## 4.1 Concept review

First, a review of some foundational principles.

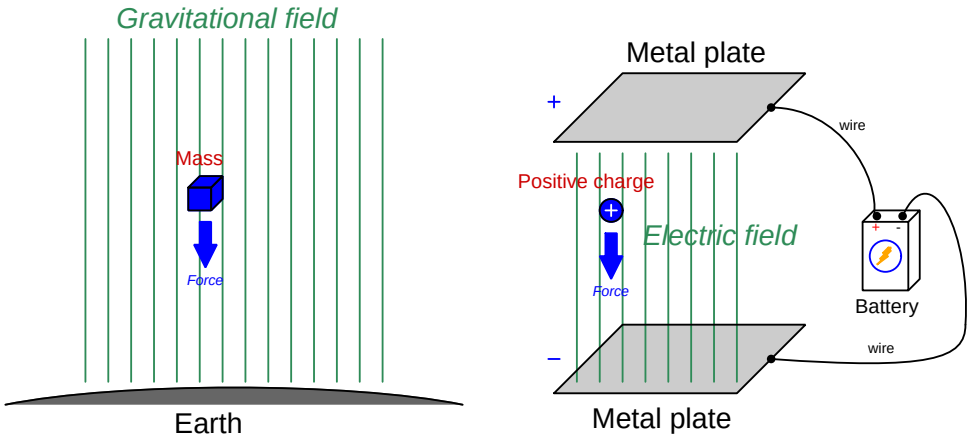
*Energy* is the ability to set matter into motion, and it may be found in a variety of different forms. Two major categories of energy are *kinetic energy* and *potential energy*, the former representing energy as a function of existing motion and the latter representing energy as a function of *possible* motion. For example, a moving vehicle possesses kinetic energy, while the fuel stored in that vehicle's tank possesses potential energy.

Energy is a *conserved* quantity, which means it can neither be created or destroyed, but the sum total always remains constant. This fact is codified as the *Law of Energy Conservation*, and so far as anyone knows there are no exceptions to this law.

The classic example of potential energy is a suspended mass within a gravitational field such as what we all experience living on planet Earth. The amount of potential energy possessed by any suspended mass is a function of the mass's magnitude, the strength of the gravitational field, and the height of the suspension. Mathematically, this form of potential energy is defined as $E_p = mgh$, where $E_p$ is the amount of potential energy in Joules, $m$ is the amount of mass in kilograms, $g$ is the acceleration of gravity[1] in meters per second per second, and $h$ is the amount of height in meters. Note that *height*, unlike the other variables in this formula, is a *relative* quantity between two locations. A mass only has potential energy with regard to how far it *might* fall. A mass resting at the top of a staircase has a certain amount of potential energy relative to a point half-way down the staircase, and twice that much potential energy relative to the very bottom of the staircase. It is impossible to determine exactly how much potential energy that mass resting at the top of the staircase possesses, but only how much it could potentially release if it falls to some pre-defined point. In other words, potential energy only has meaning when we compare *two* different locations or states-of-being.

[1] Acceleration is an expression of how rapidly an object's velocity changes, and as such may seem to be a strange way to define the effect of gravity. However, it actually is quite simple to understand: $g$ is the rate at which any object will accelerate in that gravitational field if allowed to free-fall (i.e. falling with no force to oppose its motion).

Masses suspended in gravitational fields are not the only objects possessing potential energy. Electric charges "suspended" in electric fields also possess potential energy, and just like gravitational potential energy, this *electric potential energy* is also relative between two different locations. The following illustration compares electric potential energy with gravitational potential energy:




*An electric charge experiences a force when exposed to an electric field, just as a mass experiences force (called "weight") when exposed to a gravitational field*

It is more convenient in the study of electric circuits to refer to the amount of electric potential energy possessed per unit of electric charge than it is to refer to the total potential energy held by any collection of charges. *Potential* is the term used to describe this "potential energy per charge", and it is analogous to expressing gravitational potential energy in terms of energy per unit of mass[2]. *Voltage* is the amount of potential difference between two points, and is measured in the unit of the Volt (V), a Volt defined as one Joule of electric potential energy per Coulomb of electric charge[3].

Electric charge, like energy and also like mass, is a *conserved* quantity. Just as the Law of Energy Conservation states that energy is eternal and therefore cannot be created or destroyed, the Law of Charge Conservation tells us electric charges are likewise eternal and can neither appear nor disappear but must always remain.

Since energy is conserved, and electric charge is conserved, it should come as no surprise that voltage (the difference of potential energy per charge between two different locations) is also a conserved quantity. This means voltage cannot simply appear or disappear, but must always be accounted for. Electric charges may gain potential energy by moving against the pull of the electric field (like a mass being hoisted to a higher elevation) or lose potential energy by moving with the pull of the electric field (like a mass allowed to fall to a lower elevation), but in both cases this represents a *transfer* of energy either to or from that charge, respectively. If an electric charge gains potential, that energy must have come from somewhere. If an electric charge loses potential, that energy must go somewhere.

[2] If gravitational potential energy is defined by the formula $E_p = mgh$, then gravitational *potential* (i.e. potential energy per unit mass, or $\frac{E_p}{m}$) is simply $gh$.

[3] One Coulomb of charge is equal to $6.2415 \times 10^{18}$ electrons' worth of electric charge.

A very interesting and important quality of electric charges is their ability to move through some types of matter. Metals, for example, contain a multitude of "free" electrons (a type of subatomic particle) which act as electric charge carriers which may move anywhere within a metal object's volume when exposed to an electric field. Certain liquids contain *ions* (atoms or molecules containing an unequal number of electrons and protons, and therefore an imbalanced charge state) which likewise are free to drift within the liquid's volume when exposed to an electric field. We classify substances possessing free charge carriers as electrical *conductors*, and the bulk motion of charge carriers in any particular direction through a conductor as an electric *current* ($I$). The coordinated motion of charge carriers may occur as far as the conductive material extends, which means we may use materials such as copper or aluminum (both excellent metallic conductors) to convey charge carriers across huge distances. Substances lacking free charge carriers are called *insulators*, this classification of materials including all gases[4], most plastics, most oils, glass, dry wood, and pure water.

Not all conductors are equally conductive – that is to say, not all conductive objects share the same mobility of charge carriers. The degree to which an object opposes the free motion of electric charge carriers through it is called *resistance* ($R$). A perfect conductor and a perfect insulator are just limiting cases (i.e. low and high extremes) of electrical resistance, which is a continuum.

When electric charge carriers drift through a conductive substance, we may quantify their rate of motion in terms of how many Coulombs of charges move past a particular point per second of time. This is the definition of the *Ampere* (A), the basic unit of measurement for electric current. Any resistance encountered by the moving charge carriers of an electric current results in some energy lost by those charge carriers in the form of heat. This means a *voltage* will develop across any resistance as a current passes through it, that voltage being the amount of energy lost[5] per unit charge due to resistance.

The mathematical relationship between voltage ($V$), current ($I$), and resistance ($R$) is called Ohm's Law, shown here in three algebraically equivalent forms:

$$V = IR \qquad I = \frac{V}{R} \qquad R = \frac{V}{I}$$

Resistance is measured in the unit of the Ohm ($\Omega$). By applying Ohm's Law ($R = \frac{V}{I}$), we may define one Ohm as one Volt of potential difference per Ampere of current.

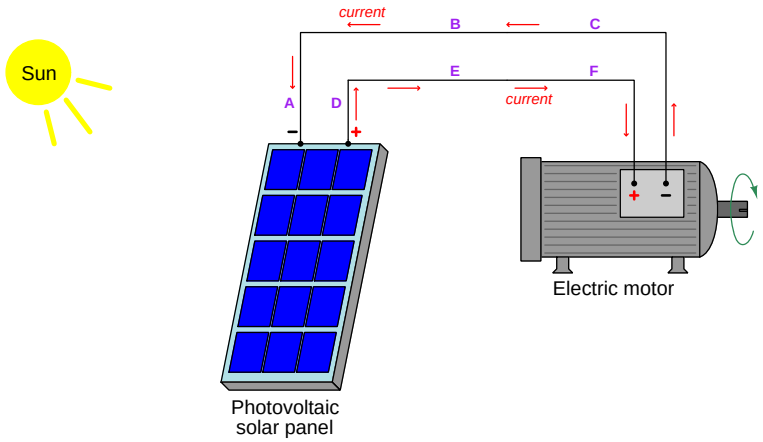
The mathematical relationship between *power* ($P$) which is the rate of energy transfer measured in Joules of energy per second of time (i.e. Watts, W) and the above-listed variables is called Joule's Law, shown here in three equivalent forms:

$$P = IV \qquad P = \frac{V^2}{R} \qquad P = I^2R$$

[4]Gases may be made electrically conductive, but only by infusing sufficient energy into the gas molecules to cause the atoms' electrons to be forcibly removed. Such ionization of gas molecules may be initiated by the application of highly-concentrated electric fields or by heating the gas to "glowing" temperatures. Gases outside of these extreme conditions are insulators.

[5]Since energy is always conserved, it is never really "lost" in the absolute sense of the word. What we mean here is that energy is *lost by* (i.e. given up by) each charge passing through, and that energy is then manifest as heat in the conductive substance.

Electric charges, with their ability to absorb and release potential energy, have the ability to act as *couriers of energy*. This is perhaps the single most significant reason why electricity is such a useful phenomenon in modern society: it is possible to build *circuits* made of metal wire where charge carriers are allowed to circulate, with devices infusing energy into the charge carriers at one location and other devices extracting energy from the charge carriers at another location. The electric charges moving around the circuit function as a sort of "conveyor belt" picking up energy at one location and transporting it to be used at another location. The following illustration shows such a circuit, with a photovoltaic panel transferring energy from photons[6] of sunlight to the charge carriers in the metal wires, and an electric motor transferring energy from the charge carriers to a rotating shaft which may turn a mechanism to do useful work:



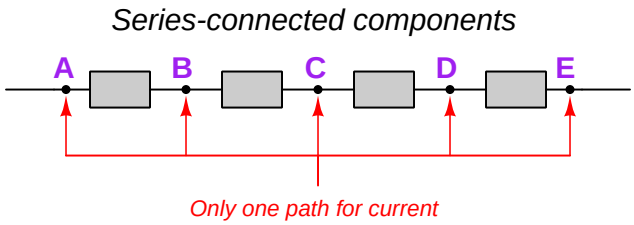

Here we see an electric current symbolized by red-colored arrows circulating counter-clockwise from the "+" terminal of the photovoltaic panel through points D, E, and F to the "+" terminal of the electric motor, through the motor to its "−" terminal, and then through points C, B, and A back to the "−" terminal of the photovoltaic panel where they pass through the panel and re-emerge at its "+" terminal. The + and − *polarity* symbols are intended to represent the relative potential energy states of charge carriers passing between two terminals on an electrical device. We see current entering the photovoltaic panel's "−" (i.e. low energy) terminal and exiting the photovoltaic panel's "+" (i.e. high energy) terminal because the panel transfers energy from the incident sunlight to the electric charge carriers passing through. We see current entering the electric motor's "+" (i.e. high energy) terminal and exiting the electric motor's "−" (i.e. low energy) terminal because the motor transfers potential energy away from the passing charge carriers and uses that energy to turn a mechanism. The orientation of the polarity symbol pairs with regard to the current direction tells us whether the device in question is a *source* of energy to the passing charges or a *load* drawing energy away from the passing charges.

Polarity marks only have meaning as *pairs*, because voltage is a function of potential energy, and potential energy is only quantifiable between *two* different points or conditions. Note that the wires themselves in the above circuit have no polarity marks shown across their ends, because we assume

[6] A "photon" is the particle form of light, possessing energy in the form of oscillating electric and magnetic fields but lacking mass.

the electrical resistance of those wires is negligible and therefore the voltage manifest between each of the wire's two ends is likewise negligible.

When electrical components are connected together such that they form a chain providing only one path for current, they are said to be connected in *series* with each other:




From this defining characteristic of only one path for current, and from the Law of Charge Conservation which states electric charges are eternal, we may conclude that any continuous current through a series network[7] must result in exactly same rate of charge carrier motion at every point in the network; i.e. *current must be equal at all series-connected points.*

Incorporating the Law of Energy Conservation, we may also conclude that voltages across a set of series-connected components algebraically add to equal the series network's total voltage: as electric charge carriers move from one end of the series chain to the other, their overall gain or loss of energy must be the sum of all the individual gains and losses of energy along the way since energy can neither be created nor destroyed. It also stands to reason that the total resistance ($R$) of a series network is equal to the sum of all the individual component and wire resistances. Mathematically stating each of these series network properties:

$$I_{total} = I_1 = I_2 \cdots = I_n$$

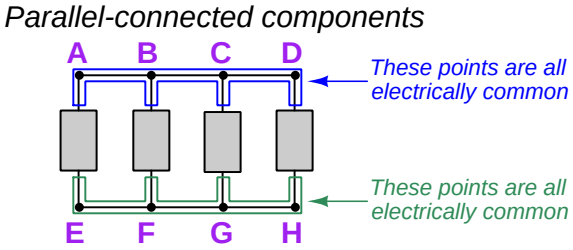
$$V_{total} = V_1 + V_2 \cdots + V_n$$

$$R_{total} = R_1 + R_2 \cdots + R_n$$

One practical application of series connections is in the measurement of electric current. Instruments designed to measure electric current are called *ammeters* and are always connected in series with the component whose current we intend to measure in order to guarantee the ammeter experiences the exact same amount of current as the component in question.

[7] A "network" defined here as any set of interconnected wires, devices, etc.

An alternative network form, called *parallel*, is where all components connect between the same two sets of conductors. Points directly connected together by a conductor of negligible resistance are said to be *electrically common* to one another, and the relative absence of resistance between those points ensures charge carriers may freely move between the connected points without losing energy. This equivalence of energy states between charge carriers is called *equipotentiality*:




The equipotentiality of charge carriers along each of the two sets of points is important to understand, as it explains the behavior of parallel-connected components. If charge carriers at points A through D all exist at the same potential energy level, and charge carriers at points E through H likewise all exist at the same potential energy level, then the difference of potential *between* any point A-D and any point E-H (e.g. from A to E, from D to H, from B to G, etc.) must be the same. *Therefore voltage across every component in a parallel network is the same.*

Current values through each component in a parallel network, however, may differ substantially. In fact, Ohm's Law ($I = \frac{V}{R}$) guarantees this will be the case if the components in question happen to have differing resistance values ($R$) but all share the same voltage value ($V$). Given the Law of Charge Conservation, we may conclude that the total current for a parallel network must be equal to the algebraic sum of all component ("branch") currents. Another property of parallel networks is that their total resistance is less than the resistance of the lowest-resistance branch. Mathematically stating each of these parallel network properties:

$$V_{total} = V_1 = V_2 \cdots = V_n$$

$$I_{total} = I_1 + I_2 \cdots + I_n$$

$$R_{total} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} \cdots + \frac{1}{R_n}}$$

One practical application of parallel connections is in the measurement of voltage. Instruments designed to measure electric potential difference are called *voltmeters* and are always connected in parallel with the component whose voltage we intend to measure in order to guarantee the voltmeter experiences the exact same amount of voltage as the component in question.

*Switches* are electrical components designed to deliberately establish ("make") or sever ("break") conductive pathways. Switches are usually designed such that a movable piece of metal arbitrarily contacts a stationary piece of metal, typically by the actuation of a pushbutton, lever, or some other mechanism. A switch positioned to make contact and permit the flow of electric charge carriers through it is said to be *closed* or *shorted*. A switch positioned to break contact and forbid the flow of current is said to be *open*. The following illustration shows a switch in both states:

A closed (shorted) switch guarantees negligible voltage drop across its terminals because the metallic bridge formed by the switch makes those two points electrically common, and therefore equipotential with each other. By extension, any component connected in parallel with a closed switch will also experience negligible voltage because paralleled components must share the same voltage. A closed (shorted) switch *may* conduct current, but this is not a guaranteed outcome because it depends on the presence of an external source to motivate that current.

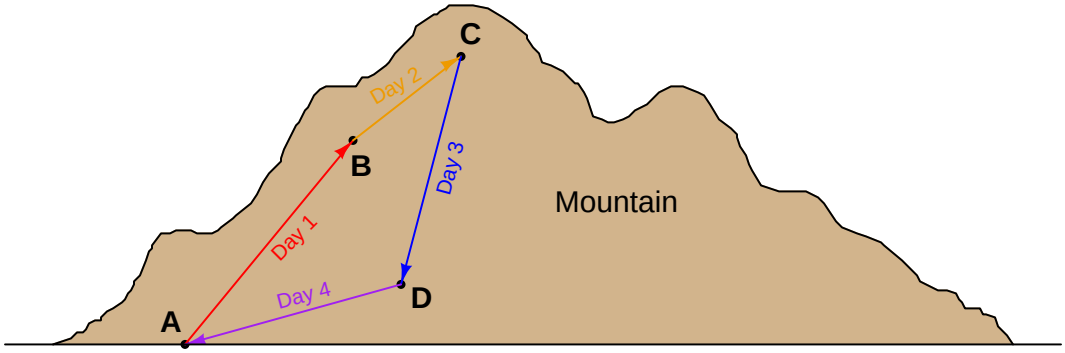
An open switch guarantees negligible current through it, because the air gap between the switch's metal pieces is electrically insulating. By extension, any component connected in series with an open switch will also experience negligible current, because series components must share the same current. An open switch *may* experience a voltage between its terminals, but this is not a guaranteed outcome because it depends on the presence of an external source to create that voltage.

Switches are used to insert controlled "open" and "short" conditions into electrical circuits, but sometimes these conditions appear without our consent. A metal wire that accidently detaches from a terminal, for example, is an unintentional *open*, otherwise known as an open *fault*. A metal wire that accidently comes into contact with another conductor, for example, is an unintentional *short*, otherwise known as a shorted *fault*. A thorough and accurate understanding of foundational electric circuit principles such as voltage, current, resistance, opens and shorts, equipotentiality, Ohm's Law, Joule's Law, series networks, and parallel networks is necessary to quickly identify the nature and location of faults in circuits, which is why this learning module (and others in the collection) devote so much text to the explanation and application of these concepts.

## 4.2 Energy conservation in closed loops

Now that we have reviewed some fundamental terms and principles of electric circuits, let us move on to the topic at hand: *Kirchhoff's Voltage Law*, often abbreviated as *KVL*. As with so many other characteristics of electricity, KVL is really nothing more than an expression of fundamental conservation laws. In this case, the principal conservation law expressed by KVL is the *Conservation of Energy*.

To begin our explanation of this law, I will present an analogy based on mountain-climbing. Suppose we wished to climb a mountain, and chose to equip ourselves with a hand-held *altimeter* to track our altitude. During the first day of our hike we start at the base of a mountain and climb up 1000 meters to set up camp for an overnight stay. Then, the next day we set off from this camp and climb another 400 meters. Deciding we've climbed high enough for two days, we set up camp again and stay the night. The next day we descend 1100 meters to a third location and camp once again. On the fourth day we hike back to our original starting point at the base of the mountain. We may summarize our hiking adventure as a sequence of rises and falls like this:



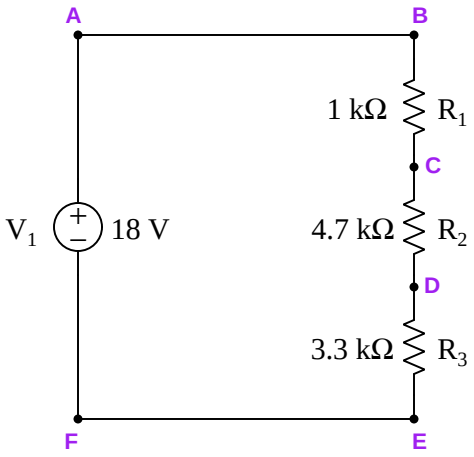

| Day | Path | Altitude gain/loss |
|---|---|---|
| Day 1 | A → B | +1000 m |
| Day 2 | B → C | +400 m |
| Day 3 | C → D | −1100 m |
| Day 4 | D → A | −300 m |
| (Total) | A → B → C → D → A | 0 m |

Of course, no one tells their friends they spent multiple days hiking a total altitude of zero, and so people generally speak in terms of the *highest* point reached when they recount mountain-climbing expeditions: in this case a peak altitude of 1400 meters. However, if we track each day's gain or loss in algebraic terms (maintaining the mathematical sign, either positive or negative), we see that the end sum *must always be zero* if we finish at our starting point. Our hand-held altimeter would confirm this fact by reading zero once again when we returned to the mountain's base, assuming the last time we reset it to zero was at the very start of our trek. We may summarize this observation as follows: *the sum of all altitude gains and losses in any closed loop equals zero.*

If we view this scenario from the perspective of gravitational potential energy as we carry a mass on this hike up and down the mountain, we would conclude that the mass gained (absorbed) potential energy on days 1 and 2, but lost (released) potential energy on days 3 and 4. After the four-day hike, the net potential energy imparted to the mass is zero, because it ends up at the exact same altitude where it began. It is worth noting that the particular path taken in each step is quite irrelevant to the mass's potential energy, because gravitational potential energy is strictly a function of mass ($m$), gravity ($g$), and height ($h$). We may summarize this observation as follows: *the sum of all gravitational potential energy gains and losses for a mass traveling along any closed loop equals zero.*

Now let us apply this same principle to an electric circuit. Instead of considering a mass elevating or descending within a gravitational field, we will consider an electric charge moving to different locations within an electric field[8]. Suppose we could manipulate a single electric charge, placing that one charge at various locations within a circuit and monitoring its potential energy at each location. We will use this "test charge" as a measurement tool for a set of *thought experiments* performed on circuits, moving the charge to different locations and comparing its relative potential energy levels as we do so.

Consider the following circuit, where a solitary voltage source ($V_1$) powers a set of three series-connected resistors ($R_1$ through $R_3$):

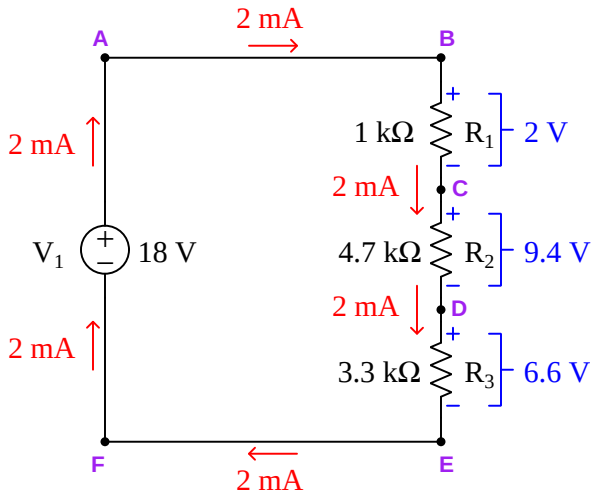



Being a series circuit, total circuit resistance will be the sum of the three resistors' individual values which is 9 kΩ. With a total (source) voltage of 18 Volts and a total resistance of 9 kΩ, Ohm's Law predicts a circuit current of 2 mA ($I = \frac{V}{R}$) flowing clockwise around the circuit (charge carriers exiting the source's + terminal and entering the source's − terminal). Since the current is equal for all series-connected components, each and every resistor in this circuit will experience the same

[8]Every energized circuit produces an electric field. Any time two different conductive surfaces exist at different potentials, an electric field exists between those conductors. This means that in any circuit where two or more conductors are not equipotential with each other, an electric field exists between those conductors. Charge carriers experience force when exposed to any electric field, and it is the relationship between the direction of this force and the direction of charge carrier displacement that determines whether the charge carrier's electric potential energy increases or decreases.

charge carrier flow rate of 2 mA (i.e. $2.0 \times 10^{-3}$ Coulombs of charge flowing through per second). The amount of voltage dropped across each resistor may also be predicted using Ohm's Law, by multiplying this common current value by each resistance value in turn ($V = IR$). This yields a voltage of 2 Volts for resistor $R_1$, 9.4 Volts for resistor $R_2$, and 6.6 Volts for resistor $R_3$. Resistors act as *loads*, which means the polarity of these voltages will be + where current enters each resistor and − where current exits each resistor.

Annotating the schematic diagram with this information will aid our analysis:




At this point we will begin our "thought experiment" using a single test charge to probe potential gains and losses throughout the circuit. To begin, we will place the test charge at point A and arbitrarily call that the "zero" potential point[9]. Next we will move our test charge to point B and assess its energy level. Note how points A and B are directly connected with wire, which we generally assume has negligible resistance. This means energy is neither gained nor lost by any charge carriers moving along the length of that wire (i.e. points A and B are *equipotential* with each other), and so our test charge still has a potential value of zero. As we move our test charge from point B to point C, however, it experiences a change in energy level. Note the polarity marks drawn next to resistor $R_1$, showing relative potentials for each terminal of that resistor: the upper terminal of $R_1$ is positive, while the lower terminal of $R_1$ is negative, which stands to reason because the charge carriers flowing clockwise through this circuit lose energy as they flow down through $R_1$. Likewise our test charge will go from a higher-energy level to a lower-energy level as we move it from point B to point C. If our test charge had a potential of zero at point B, it will now have a potential of −2 Volts at point C[10]. Moving our test charge from point C to point D incurs another energy loss,

[9]Since electric potential is always relative between two points, it really doesn't matter what numerical value we assign to the starting point because what we will be paying attention to is the relative gains or losses in potential as the charge gets moved to different points in the circuit. Like our mountain-climbing analogy where the starting point was defined as 0 meters altitude even though we were never informed if the base of the mountain was at sea level, our starting potential is arbitrary.

[10]This potential (voltage) value is purely relative to the point we arbitrarily deem zero, in this case point A. Our test charge has a potential of −2 Volts at point C only because it began with zero at point A. Had we chosen point

this time losing an additional 9.4 Volts, so that our test charge's potential at point D will be −11.4 Volts. Moving from point D to point E once again transitions from a higher-energy level (+) to a lower-energy level (−), this time losing another 6.6 Volts of potential to leave our charge with a −18 Volt potential at point E.

Moving our test charge from point E to point F is simply moving it along another equipotential path which means it neither gains nor loses energy along the way, still leaving it with −18 Volts of potential at point F. Returning to our starting location, moving the test charge from point F back to point A across the voltage source transitions it from a lower-energy level (−) to a higher-energy level (+) as indicated by the polarity marks on $V_1$. The potential increase from point F to point A is +18 Volts, which returns our test charge back to its original potential of zero at point A, just like the hiker whose altimeter registers zero again upon return to the starting location.

As mentioned previously, the path we choose to take is entirely arbitrary. So long as we return to the starting point, the net gain/loss in potential energy must always be zero whether we are dealing with the gravitational potential energy of a mass at different altitudes or the electric potential energy of a charge at different points in a circuit, because energy is a conserved quantity. Consider this alternative path, starting at test point A and moving counter-clockwise around the circuit rather than clockwise as before:

| **Step** | **Symbol** | **Voltage gain/loss** |
|---|---|---|
| A → F | $V_{FA}$ | −18 Volts |
| F → E | $V_{EF}$ | 0 Volts |
| E → D | $V_{DE}$ | +6.6 Volts |
| D → C | $V_{CD}$ | +9.4 Volts |
| C → B | $V_{BC}$ | +2 Volts |
| B → A | $V_{AB}$ | 0 Volts |
| *(Sum)* | $V_{AA}$ | *0 Volts* |

Even though we have moved our test charge "against the flow" of the charge carriers circulating in this circuit, the potential gains and losses still add up to zero at the end. Direction of current in a circuit entirely irrelevant to our choice of how we step around a loop when applying KVL, because energy is always conserved regardless of anything else. Summarizing: *the sum of all electric potential gains and losses in any closed loop equals zero.* Kirchhoff's Voltage Law may be mathematically expressed as follows:

$$V_{AB} + V_{BC} + V_{Cn} \cdots + V_{nA} = V_{AA} = 0$$

C as our zero reference point and moved the test charge up to point A, we could have to conclude it possessed +2 Volts of potential at A (with reference to C). The convention for expressing this +2 Volt *differential voltage* is $V_{AC}$, where the first subscript letter is the point of measurement (i.e. red voltmeter lead) and the second subscript letter is the point of reference (i.e. black voltmeter lead).

## 4.3 Choosing other loops

In fact, we don't even have to trace a conductive route with our imaginary test charge to apply KVL – this law works with literally *any* sequence of points beginning and ending at the same location. To illustrate this, we will perform a thought experiment where we move the charge from points D to C to B to A and then back again to D, with the last step from point A to point D through space not following any current pathway in the actual circuit. Referencing the hiking analogy, this is equivalent to a hiker taking a helicopter ride between points A and D rather than going from one to the other along an established trail:

| **Step** | **Symbol** | **Voltage gain/loss** |
|---|---|---|
| D $\rightarrow$ C | $V_{CD}$ | +9.4 Volts |
| C $\rightarrow$ B | $V_{BC}$ | +2 Volts |
| B $\rightarrow$ A | $V_{AB}$ | 0 Volts |
| A $\rightarrow$ D | $V_{DA}$ | ??? Volts |
| *(Sum)* | $V_{DD}$ | *0 Volts* |

The potential difference between points A and D in this circuit is not shown in the schematic diagram because it applies neither to the source nor to any one resistor. However, since we know KVL is true, and the algebraic sum of all potential gains and losses must be zero when the charge returns to its initial position, all we need to do to solve for the potential difference of the A $\rightarrow$ D step ($V_{DA}$, where D is the point of measurement and A is the point of reference) is solve for that voltage:

$$V_{CD} + V_{BC} + V_{AB} + V_{DA} = 0$$

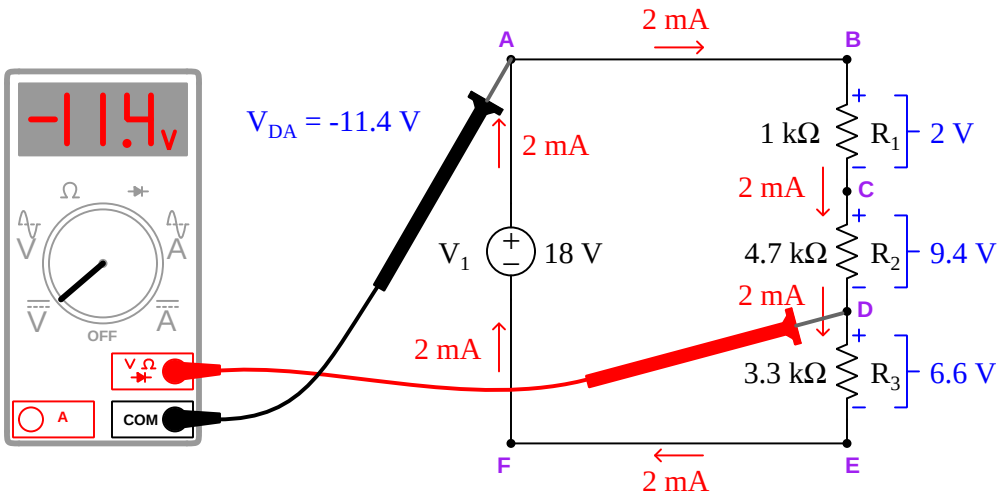
$$(+9.4 \text{ V}) + (+2 \text{ V}) + (0 \text{ V}) + V_{DA} = 0$$

$$11.4 \text{ V} + V_{DA} = 0$$

$$V_{DA} = -11.4 \text{ V}$$

The sign of the answer is important, as it tells us which of the two points is positive and which is negative with respect to the other. Since $V_{DA}$ means *"potential difference at D with reference to A"* or *"potential difference starting at A and stepping to D"*, we can tell that point D is a lower-energy ($-$) compared to point A's comparatively higher-energy ($+$) level. If we were to annotate these two points with their own dedicated $+/-$ polarity marks, we would place the "+" mark at A and the "$-$" mark at D.

Therefore, moving a test charge from point A to point D results in a loss of potential equal to −11.4 Volts. If a voltmeter were attached between points D and A, we could verify that point D is indeed negative with respect to point A:




If we were to reverse the direction of our imaginary test charge's path and travel DABCD instead, the unknown voltage we would be solving for would be $V_{AD}$ rather than $V_{DA}$ and its value would be positive rather than negative:

$$V_{AD} + V_{BA} + V_{CB} + V_{DC} = 0$$

$$V_{AD} + (0 \text{ V}) + (-2 \text{ V}) + (-9.4 \text{ V}) = 0$$

$$V_{AD} - 11.4 \text{ V} = 0$$

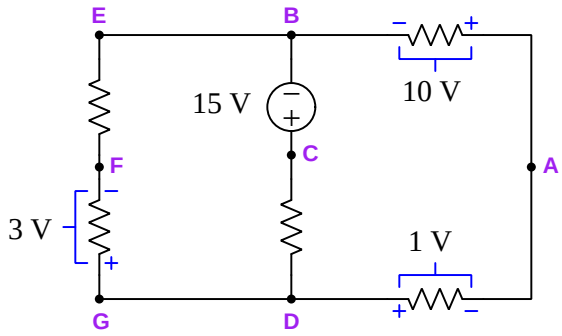
$$V_{AD} = 11.4 \text{ V}$$

Regardless of how we decide to apply KVL, the result is the same in this circuit: 11.4 Volts of potential difference between points A and D with point A having more potential energy (+) and point D having less potential energy (−).

These latter examples illustrate one of the most important uses of KVL: as a tool for calculating unknown voltages based on other (known) voltages in a loop.

## 4.4 Circuits with unknown currents

Consider the following circuit where we happen to know some of the voltage values but none of the resistor values nor any current value. With all the unknown quantities, both Ohm's Law and Joule's Law are of no help in determining the rest of the voltage values, such as $V_{EF}$ or $V_{CD}$:




However, KVL will suffice quite nicely in determining these unknown resistor voltage values: simply pick a loop where all but one voltage is known, and calculate that voltage necessary to make the loop's total equal zero. For example, we could solve for $V_{CD}$[11] by summing up all the potential gains and losses in loop BADCB:

$$V_{AB} + V_{DA} + V_{CD} + V_{BC} = 0$$

$$(+10 \text{ V}) + (+1 \text{ V}) + V_{CD} + (-15 \text{ V}) = 0 \text{ V}$$

$$V_{CD} - 4 \text{ V} = 0 \text{ V}$$

$$V_{CD} = 4 \text{ V}$$

The positive 4 Volt answer we get for $V_{CD}$ tells us our "test charge" gains in energy moving from point D to point C, which means C is positive (+) and D is negative (−) relative to each other.

[11] Remember that the convention for voltage subscripts is that the first subscript (C) is the point where we would connect our red voltmeter lead while the second subscript (D) is where we would connect our black voltmeter lead. Therefore $V_{CD}$ means the amount of voltage that would register on a voltmeter with red on C and black on D; or, the relative potential at point C compared to reference point D.

Likewise we could solve for $V_{EF}$ by stepping our imaginary test charge around a loop containing it (and no other unknown voltages). Since both the starting point and direction of our thought experiment doesn't matter, let's use loop DGFEBAD:

$$V_{GD} + V_{FG} + V_{EF} + V_{BE} + V_{AB} + V_{DA} = 0$$

$$(0 \text{ V}) + (-3 \text{ V}) + V_{EF} + (0 \text{ V}) + (+10 \text{ V}) + (+1 \text{ V}) = 0 \text{ V}$$

$$V_{EF} + 8 \text{ V} = 0 \text{ V}$$

$$V_{EF} = -8 \text{ V}$$

Therefore, the upper-left resistor drops 8 Volts, with point E at a lower electrical potential ($-$) compared to point F ($+$).

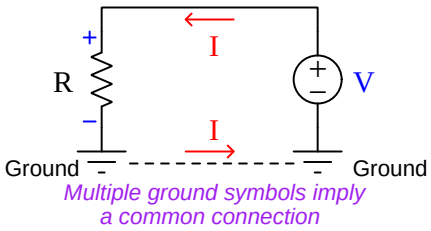
In these examples we see how Kirchhoff's Voltage Law may allow us to solve for unknown voltages in circumstances where Ohm's Law would be unhelpful to do the same, not knowing any of the resistor or current values. If ever you find yourself in a situation where you need to solve for some unknown voltage in an electrical network but there isn't enough information to apply Ohm's Law to the task, try Kirchhoff's Voltage Law instead!

It is worth reiterating that Kirchhoff's Voltage Law applies to any and all electrical networks because it is really nothing more than an expression of the Law of Energy Conservation as it applies to electric charge potential. This means KVL works just as well to analyze series networks as it does to analyze parallel networks, as it does to analyze series-parallel combination networks, and even as it does to analyze networks defying the conventions of series or parallel (e.g. bridge networks). *KVL is universal*, which is why it is so important to master for the study of electricity and electronics.

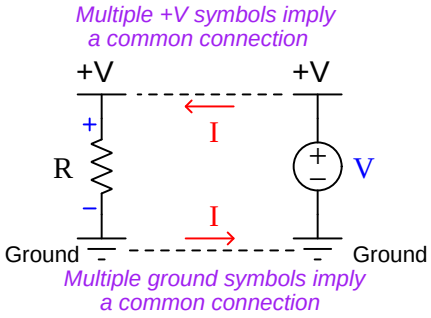
## 4.5 Referencing voltage to ground

An important concept in electric circuits is that of *ground*. This term is used in slightly different ways, which tends to be confusing[12]. In all its uses, though, "ground" refers to a point in a circuit deemed to be at zero potential. Sometimes this ground point will be electrically connected to the Earth by wire connection to a metal stake pounded into the soil called a *grounding rod*, and other times the "ground" point of a circuit is nothing more than a common reference point for taking voltage measurements which may or may not have any connection to the Earth.

The symbol for a ground connection in a circuit is a set of three parallel lines of progressively increasing length. If multiple ground symbols are shown in a circuit, it is assumed those points are all directly connected together by a low-resistance conductor and as such are electrically common and thus equipotential with each other:




A popular symbol used for common power source connections in electronic circuits is similar to the ground symbol, but consists of just one line segment instead of three, and is generally labeled with either a letter designation such as "+V" or a voltage value (with reference to ground) to signify a *power bus*: a common connection leading to one (ungrounded) terminal of a voltage source. The simple one-source, one resistor circuit shown previously appears here with this new symbol in addition to the ground symbol:

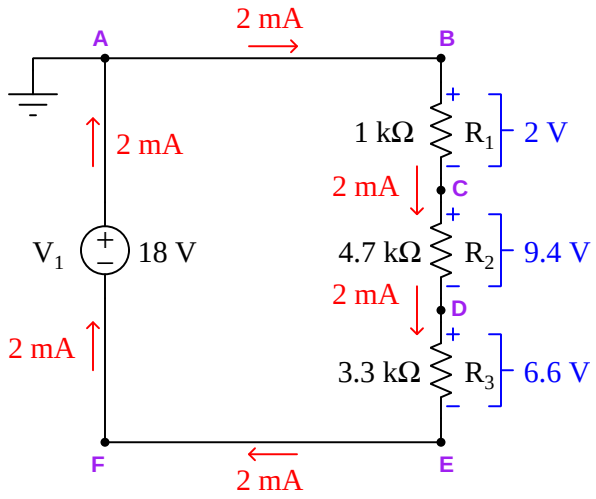



Taking this concept one step further, we could eliminate the schematic symbol for the voltage source entirely and simply show the one resistor with a "+V" symbol on top and a ground symbol on bottom. Many complicated electronic schematic diagrams do just this with all their load components, thus eliminating the need to draw additional source symbols or additional wires necessary to connect source(s) to loads.

[12] If different types of "grounds" are used in the same system, slightly different symbols will be used for each, and a legend provided to differentiate the symbols' meanings.

Circuit grounds are especially useful as reference points for voltage measurements, allowing one to meaningfully refer to voltages at single points. Remember, *voltage is fundamentally a difference of electrical potential, and as such is always relative between two points*, but if one of those two points is understood to be a common reference point (ground) then we may specify voltages at other points in the circuit without having to explicitly identify the common "ground" point every time. This is analogous to specifying the height of various mountain peaks by implicit reference to sea level which functions as a universally accepted "zero" elevation point around the planet.

We may illustrate this concept of "ground" as a reference point for voltages by returning to our three-resistor series circuit, this time showing a ground symbol at one of the labeled points:



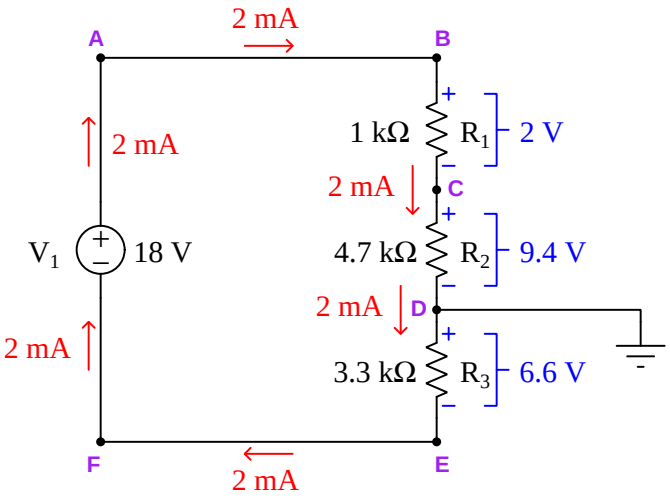

With the ground located at point A, we could identify voltages for each of the other (single) test points, each one symbolized by $V$ (for Voltage) and a single subscript letter identifying the point. The voltage values shown in the following table are representative of a test charge's potential at each of those lettered points assuming that test charge started at point A with zero potential:

| Symbol | Voltage value |
|---|---|
| $V_A$ | 0 Volts |
| $V_B$ | 0 Volts |
| $V_C$ | −2 Volts |
| $V_D$ | −11.4 Volts |
| $V_E$ | −18 Volts |
| $V_F$ | −18 Volts |

KVL still holds true for this circuit regardless of where ground is located, because KVL is based on the Law of Energy Conservation: no matter where ground is located, placing an imaginary "test charge" at that ground point and then marching the test charge around the circuit at each of the

other points until we arrive at ground again will always give us an algebraic sum of zero. Every step up or step down in that charge's potential energy level while hopping around to different points in the circuit must cancel out at the end, just like the net potential energy change of a traveling mass must return to its original value when that mass returns to its original location (height).

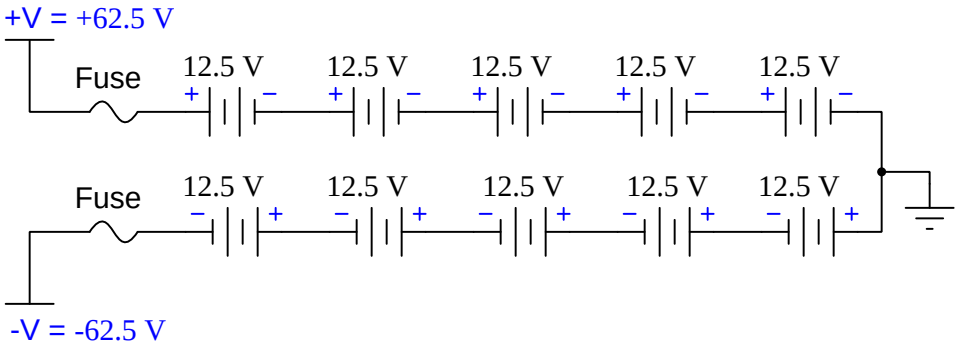
Relocating the ground in this same circuit merely re-sets our reference point, which is equivalent to relocating the black test lead of our voltmeter as we probe the other points with the red test lead, or equivalent to performing another "test charge" experiment with that charge's potential arbitrarily set to zero Volts at the new ground point. Grounding point D in this circuit for illustration:




| Symbol | Voltage value |
|---|---|
| $V_A$ | +11.4 Volts |
| $V_B$ | +11.4 Volts |
| $V_C$ | +9.4 Volts |
| $V_D$ | 0 Volts |
| $V_E$ | −6.6 Volts |
| $V_F$ | −6.6 Volts |

Please note that relocating the ground point does not affect the pathway for current in this circuit, nor the amount of current that circulates within it, nor any of the resistors' individual voltage drops, nor the source's voltage value, nor any power dissipations. No current flows into or out of this one ground connection, because there is no *other* ground in this circuit for that path to be complete. In this circuit, the ground symbol functions solely as a common reference point so we may legitimately speak of voltage values existing at other single points in the circuit.

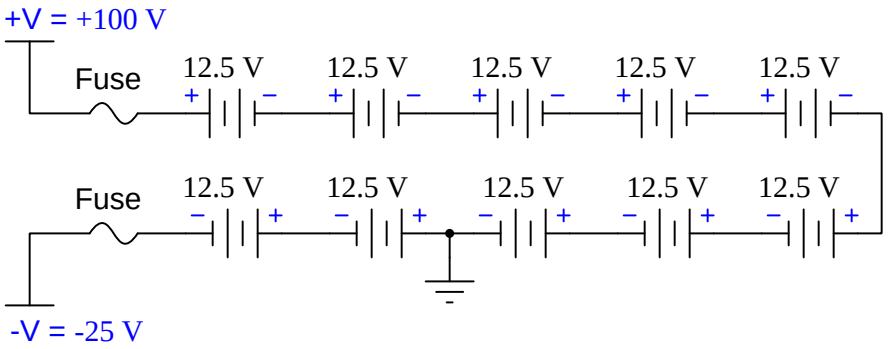
The existence of a single ground point in a circuit may seem academic, if all it does is provide a common reference for voltage measurements. However, there may actually be *safety* implications of the Earth ground's location in circuits harboring dangerous voltage levels. Consider the following battery bank for a 125 Volt DC power system, the "bank" consisting of ten series-connected 12.5 Volt secondary-cell batteries:




A relevant question to ask from the perspective of electrical safety is *"How much voltage might a person be exposed to if they happened to touch any of the conductors in this battery bank while standing on Earth ground?"* In this particular battery bank, there are two points of extreme potential: the +V power bus at +62.5 Volts and the −V power bus at −62.5 Volts. Therefore, the maximum voltage anyone could possibly experience between the point of circuit contact and their feet (on the ground) would be 62.5 Volts, with either terminal +V or −V being equally dangerous to touch[13]. This is considerably safer than if the ground happened to be located at any other point in the battery bank.

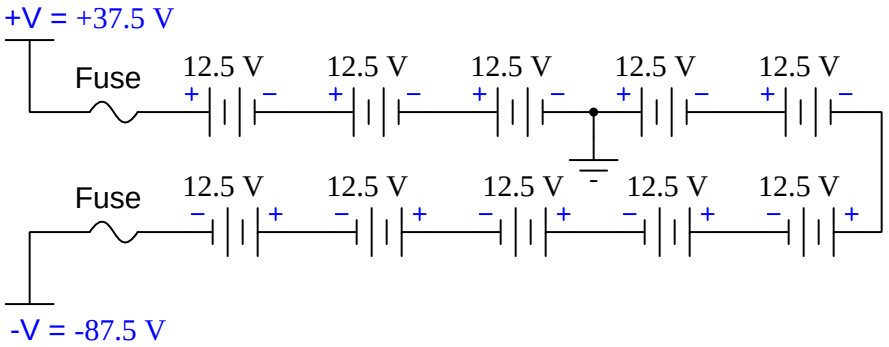
[13] It would be a mistake to think that the −V bus would be safer to touch than the +V just because −62.5 is a lower number than +62.5. Whether charge carriers enter the person's hand at +62.5 Volts potential and exit the feet at 0 Volts potential, or they enter the feet at 0 Volts and exit the hand at −62.5 Volts, each Coulomb of charge carriers passing through that person's body still delivers 62.5 Joules of energy, and it is this energy transfer that does bodily damage. Similarly, a mass falling to ground level from a height of 62.5 meters delivers just as much of a blow as if it fell from ground level to the bottom of a pit 62.5 meters deep: it is the distance the mass *falls* that dictates how much energy it loses (i.e. delivers in the blow), not the height it happens to start at.

For contrast, consider this alternative grounding location. The +V and −V bus voltages with reference to ground are determined the same way as we proved KVL, by taking an imaginary "test charge" beginning with zero potential at the ground location and moving that test charge to each of the far busses, noting how much potential it either gains or loses crossing each 12.5 Volt battery:




With this Earth ground location, the −V bus at −25 Volts potential would be much safer for anyone standing on ground to touch, but the +V bus at +100 Volts would be much more dangerous.

One more grounding location in this battery bank, just for illustrative purposes:

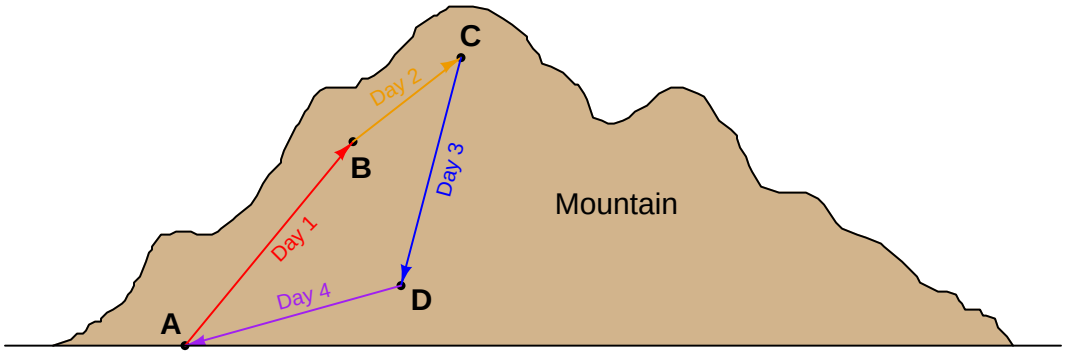



Here, it is the +V bus that is relatively safe to touch at +37.5 Volts with reference to Earth in comparison to the −V bus at −87.5 Volts with reference to Earth.

Note that in all cases, the potential difference *between* +V and −V is always 125 Volts regardless of the ground's location, and that anyone who happens to contact both of these points with their body (e.g. a hand-to-hand contact, with each hand touching a different bus) would be exposed to that full voltage. Therefore, the judicious placement of Earth ground in a circuit harboring dangerous voltage levels merely mitigates *some* of the electrical shock risk and not *all* of it.

## 4.6 A simplified application of Kirchhoff's Voltage Law

Kirchhoff's Voltage Law is based on the Law of Energy Conservation, which states that energy cannot be created or destroyed but must always be accounted in any system. For electrical charges this means any electrical charge that makes a circuitous journey must arrive at its starting point with the same amount of energy as it had at the beginning of that journey. This is similar to gains and losses in elevation during a hike where a person begins and ends their hike at one location:




| Day | Path | Altitude gain/loss |
|---|---|---|
| Day 1 | A → B | +1000 m |
| Day 2 | B → C | +400 m |
| Day 3 | C → D | −1100 m |
| Day 4 | D → A | −300 m |
| (Total) | A → B → C → D → A | 0 m |

We often use Kirchhoff's Voltage Law to determine unknown voltages in a circuit given the known values of other voltages in that same circuit. Analogously, we could apply a similar "Elevation Law" to determine the elevation difference between points D and A on this example hike if we did not already know it was −300 meters. Using the variable $h$ to represent each change in elevation, we may write a "loop" equation as follows and then algebraically solve for the unknown elevation difference between D and A:

$$h_{A \to B} + h_{B \to C} + h_{C \to D} + h_{D \to A} = 0$$

$$1000 \text{ m} + 400 \text{ m} + (-1100 \text{ m}) + h_{D \to A} = 0$$

$$300 \text{ m} + h_{D \to A} = 0$$

$$h_{D \to A} = -300 \text{ m}$$

This formalized, algebraic approach of writing a complete loop equation and then solving for the unknown variable is certainly valid, and for many applications in circuit analysis (especially when the loop equation must be combined with other equations for that circuit to solve for some quantity) is truly necessary. However, such a strict formalized approach is not always necessary, and in this simple hiking analogy it most certainly is not.

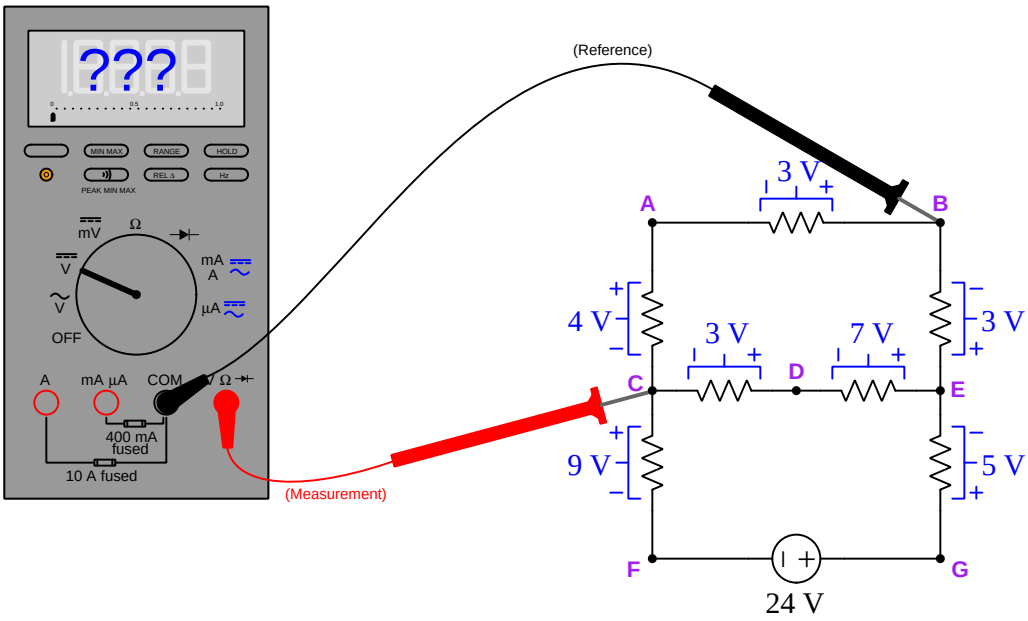
Consider the following alternative approach for determining the elevation gain/loss starting at point D and ending at point A. First, we arbitrarily define point D as our point of reference from which all other locations' elevations will be measured. In other words, we will consider point D to be "ground level" for the sake of our analysis. Then, starting at that arbitrary reference point, we will hike along the path to point A following all the known elevation gains and losses in the path while maintaining a running total of those gains and losses. Whatever value that running total happens to be when we reach our destination point (A) is therefore the elevation gain or loss with respect to our reference point D. In this example, it means starting at point D, hiking up to point C, then hiking down to points B and finally A. Whatever total value we get when summing all of those elevation changes must be $h_{D \to A}$:

$$h_{D \to C} + h_{C \to B} + h_{B \to A} = h_{D \to A}$$

$$1100 \text{ m} + (-400 \text{ m}) + (-1000 \text{ m}) = h_{D \to A} = -300 \text{ m}$$

We clearly arrive at the same correct result (300 meters of elevation drop from point D to point A) without having to algebraically manipulate a full loop equation equaling zero. In other words, *instead of plotting a full loop and then using algebra to solve for the missing piece of that complete loop, we instead plot a partial loop lacking the unknown quantity, with the total sum of that partial loop necessarily equaling the unknown quantity.*

This technique works exceptionally well for cases where we must determine the voltage indicated by a DC voltmeter with specified red and black test-lead locations. Consider this circuit with all resistor voltages pre-calculated and a digital voltmeter measuring the electrical potential at point C with reference to point B:




To very simply and easily compute this digital voltmeter's display value, all we need to do is start at the black test lead's location (point B) and "hike" our way to the red test lead's location (point C) following any path with known voltages along every step, tallying those voltage gains and/or losses along the way. The sum computed when we arrive at the red lead's location must be the voltmeter's indication. Examining this schematic, we find three different paths of varying complexity all yielding the same result:

$$(\text{B} \rightarrow \text{A}) + (\text{A} \rightarrow \text{C}) = (-3\ \text{V}) + (-4\ \text{V}) = -7\ \text{V}$$

$$(\text{B} \rightarrow \text{E}) + (\text{E} \rightarrow \text{D}) + (\text{D} \rightarrow \text{C}) = (+3\ \text{V}) + (-7\ \text{V}) + (-3\ \text{V}) = -7\ \text{V}$$

$$(\text{B} \rightarrow \text{E}) + (\text{E} \rightarrow \text{G}) + (\text{G} \rightarrow \text{F}) + (\text{F} \rightarrow \text{C}) = (+3\ \text{V}) + (+5\ \text{V}) + (-24\ \text{V}) + (+9\ \text{V}) = -7\ \text{V}$$

Therefore, this digital voltmeter connected with black lead on point B and red lead on point C should display *negative seven Volts* ($-7$ Volts) of potential difference.

# Chapter 5

# Historical References

This chapter is where you will find mathematical derivations too detailed to include in the tutorial, and/or tables and other technical reference material.

## 5.1 Gustav Robert Kirchhoff in his own words

Gustav Robert Kirchhoff formulated his voltage and current laws while still a university student. A reference to his voltage law is found on pages 501 and 502 of the German scientific journal *Annalen der Physik und Chemie* (*Annals of Physics and Chemistry*) published in 1845:

**Wir wollen zuerst den Fall näher betrachten, wenn die Scheibe unbegränzt ist. Es seyen $A_1$, $A_2$..$A_n$ die Einströmungspunkte (ich will der Kürze halber jetzt unter diesem Namen sowohl die Einströmungspunkte im engern Sinne als auch die Ausströmungspunkte verstehen, und will die Elektricitätsmengen, die durch die letzteren abfliefsen, als negative einströmende bezeichnen); die Elektricitätsmengen, die durch diese in der Zeiteinheit in die Scheibe treten, seyen $E_1$, $E_2$..$E_n$, wobei**

[page 501]

**$E_1 + E_2 + .. + E_n = o$ seyn mufs; bezeichnen wir dann die Entfernungen eines Punktes in dieser von $A_1$, $A_2$.. durch $r_1$, $r_2$..$r_n$, so läfst sich leicht zeigen, dafs wir al-**

[page 502]

Here we see his statement that the algebraic sum of voltages in a circuit equals zero ($E_1 + E_2 + \cdots + E_n = 0$).

# Chapter 6

# Derivations and Technical References

This chapter is where you will find mathematical derivations too detailed to include in the tutorial, and/or tables and other technical reference material.

## 6.1 Ground in electric and electronic circuits

*Ground* and *grounding* within both electric power and electronic circuits are commonly misunderstood topics, largely because these terms can often have very different meaning depending on context. Here we will explore some of these different meanings and hopefully eliminate some of the confusion surrounding them.

The very first concept we must understand in order to comprehend what "ground" is in any circuit is that *voltage* is fundamentally a *relative* measurement of potential energy different *between two points*. This is really true of all forms of potential energy, because "potential" means energy that has not yet been put to work setting matter into motion, implying a final state yet to be realized. When we say an electrical charge carrier possesses potential energy (i.e. electrical potential), this means it *could* (potentially) release energy to do work if it were to fall into a lower state of energy. The difference in energy held by that electric charge carrier from its original (high-potential) state to some other lower-potential state will be the amount of energy released by that charge carrier to do work, with that amount of work done dependent on both the charge carrier's initial energy level and its final energy level. *Voltage* is the term we use to describe such a difference in electrical potential.

We say that voltage always exists "between two points" or "between two locations" in a circuit because voltage is the amount of potential energy either lost or gained by electric charge carriers from one position in a circuit to another. It is impossible to quantify how much energy will be gained or lost by charge carriers unless we know both the starting point and the ending point for those charge carriers. In order to quantify the absolute amount of potential energy possessed by anything, we would have to know where or how it could lose every bit of its stored energy to be left with zero capacity to do work[1]. This is really unknowable from a practical perspective, and so the best we can do is quantify gains or losses in potential energy between different states.

This fact about voltage is also why we label it with *pairs* of "+" and "−" symbols, the "+" symbol representing the point of *higher potential* and the "−" symbol representing the point of *lower potential*. We cannot place an absolute value on electrical potential at any single location because no one can say where the location of zero energy is in the universe for electric charges. The best we can do is to rate one location as having higher or lower electrical potential than another, quantifying only the difference in potential between those two locations.

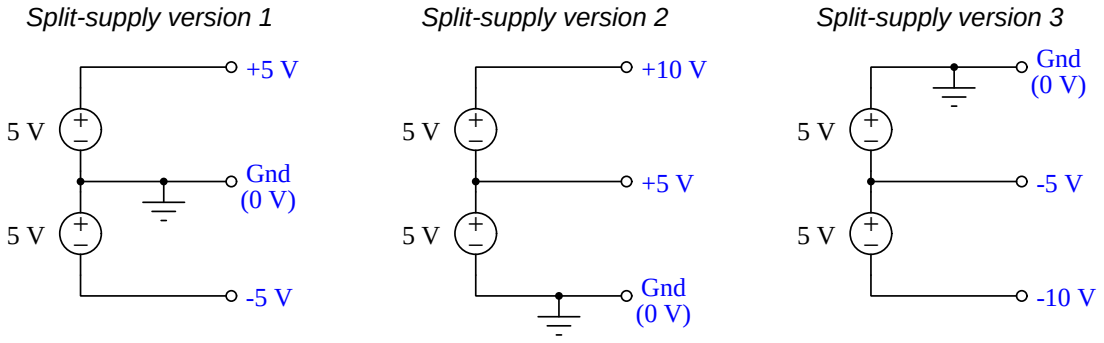
Now that we have reviewed this property of voltage and of potential energy in general, we are ready to explore what is meant by *ground* and *grounding*.

[1] Using gravitational potential energy as an example, a weight hoisted above the Earth's surface certainly possesses potential energy, and we might be tempted to quantify this potential energy by simply measuring its weight and its height above ground level and then multiplying those two quantities (i.e. potential energy equals force times distance). However, there are more places this weight could *potentially* fall than just ground level. That same weight, hoisted above the Earth's surface to the same height, will be assessed to have a far greater amount of potential energy if we consider it potentially falling down a deep hole rather than just falling to ground level. For that matter, one could even argue the weight has a *negative* quantity of potential energy with respect to what it would possess at the summit of the highest mountain! Potential energy, defined as the amount of work that *could* be done, necessarily depends on a system's initial condition *and* any number of different final conditions of which none are known to be truly zero-energy.

How do we rate the height of mountain peaks? Certainly, we may rate one mountain peak as being some number of meters higher or lower than another, but generally what people do instead is measure a mountain's peak height compared to *Mean Sea Level*. Since water is a liquid, and liquid tend to settle at equal heights under the influence of gravity when they are free to move about, the level of oceans throughout the world provide a relatively stable reference point for measurements of altitude. If we imagine a world without oceans, perhaps one with nothing but dry land and no open bodies of water anywhere, people living at different locations in this world would likely choose some flat-land reference points local (to them) for their measurements of nearby mountains, but these localized altitude measurements would be confusing if not meaningless for world-wide comparisons of mountain peaks.

Much the same is true of electrical and electronic circuits, where voltage is an important measure of electrical potential energy as well as a measure of signal strength where electricity is used as an analogue for representing other things[2]. One way to conveniently rate electrical potentials at different points in a circuit is by arbitrarily choosing one of those points to be the "zero-energy" reference point, and calling it *ground*. Ground in an electrical or electronic circuit is analogous to sea level for rating mountain peak heights or a *datum* or *benchmark* used for surveying land: simply a point of reference agreed-upon to be "zero" for the sake of ranking all other potentials to it.

A good example of this is an electrical network popularly known as a *split DC power supply*. Three different versions of this power supply network are shown with the same pair of 5-Volt DC sources, the only difference being the arbitrary location of the "Ground" (Gnd) point:

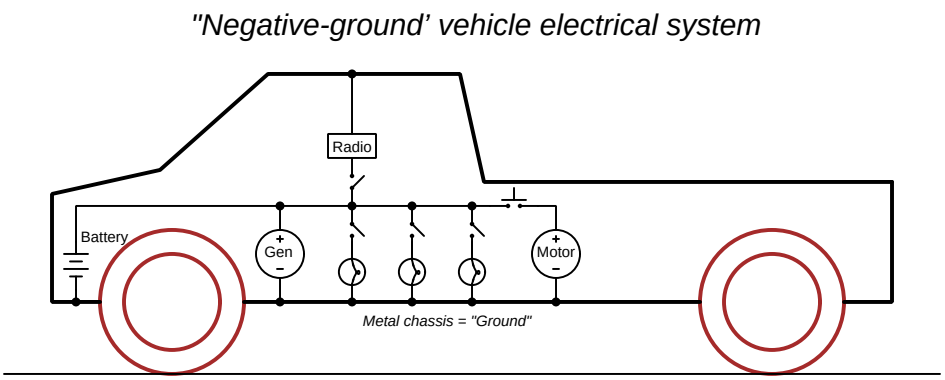



Each terminal's voltage label represents the amount of electrical potential there *relative to the "Ground" terminal*; i.e. "Ground" is simply a "sea-level" reference point against which all other potentials may be measured. The three different versions shown here illustrate just how arbitrary the location of "Ground" really is. Any load connected between the same pairs of terminals on each power supply would experience the same amount of voltage regardless of the "Ground" terminal assignment.

This is one meaning of the word "ground" in electric and electronic circuits: an arbitrary point of reference for the measurement and comparison of electrical power and signal potentials at different locations in the circuit.

[2]Many, many examples exist of voltage being used to represent non-electrical quantities. In digital computer systems, DC voltage levels represent binary "1" and "0" number states. In analog electronic sensor systems, voltage may represent temperature, speed, force, weight, sound pressure, or any number of other physical quantities.

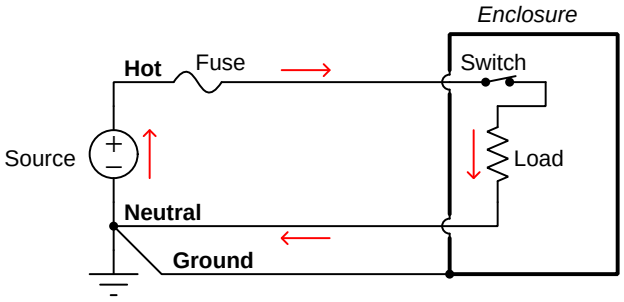
Another meaning of the word "ground" in circuits is a metallic surface used as a common conductor throughout some device or system for reasons of wiring economy. Automobiles with metal frames and chassis typically use their metallic bodies as an electrical conductor for nearly all circuits contained within. Doing so eliminates the need to install separate copper-wire conductors for every source and every load, thereby saving expense and weight:




Modern automotive electrical systems are "negative-ground" which means the vehicle chassis is always the lowest-potential point in any portion of the circuit. Interestingly, some early automobiles used "positive-ground" wiring which simply had the reverse polarity on all sources and loads: the vehicle chassis being the highest-potential point and all other points being either equal to it or at lower (−) potentials. This choice of polarity is really arbitrary, as electric circuits may be designed and built to function just as well either way. The "negative-ground" standardization we see today is really nothing more than automobile manufacturers agreeing to wiring their different vehicles the same way in order to enjoy interoperability of components.

This usage of the word *ground* – describing the metal chassis of some complex system exploited as a common electrical conductor for multiple circuits – is not incompatible with the previous usage of *ground*: a point of reference in a circuit arbitrarily defined to be "zero" potential. An automotive technician troubleshooting circuit faults in a vehicle often connects the black test lead of their voltmeter to the metal chassis, expecting to measure positive or zero voltage values on that meter when probing circuit test points with the red test lead. In a negative-ground electrical system, the chassis' electrical potential is *defined* as zero for the purpose of specifying voltages at other points in the circuit(s).

Yet another electrical meaning of the word "ground" is associated with *safety*, in which one metallic conductor of a power circuit is made electrically common to the Earth through a *grounding rod* buried in the soil. In the following circuit, the ground symbol in the lower-left corner is not merely a reference marker intended for anyone viewing the schematic diagram, but actually represents a metal rod driven deep into the soil and connected to one terminal of the voltage source with intent to achieve equipotentiality with the Earth:




Here the purpose of "grounding" is more important than merely establishing a reference point for voltage measurements. Instead, designating the conductor attached to this metal rod as "Ground" and attaching it to any conductive surface on or around an otherwise energized electrical appliance forces that surface to be equipotential to the same Earth that any humans and animals are standing on. This greatly mitigates the risk of electric shock should there be an internal fault within that appliance by making it impossible to have a difference of potential (i.e. voltage) between the grounded surface and the soil.

The following photograph shows the grounding rod for a commercial building, a large-gauge bare copper conductor clamped to the top of the rod which leads into the building's electrical room:

Ground wires in electric power systems should never carry current during normal operation. In fact, the presence of current through a ground conductor indicates a problem in the system! Note the two current-bearing power conductors in the previous schematic, specifically how the current-carrying conductor made common to Earth ground is called the *neutral*, while the other (non-grounded) power conductors is called *hot* by comparison. The Hot/Neutral designations always refer to which is grounded and which is not, rather than polarity. For example, if we were to reverse

the polarity of the DC voltage source shown in that schematic, the upper conductor (now −) would still be considered “hot” and the lower conductor (now +) would still be considered “neutral”.

# Chapter 7

# Questions

This learning module, along with all others in the ModEL collection, is designed to be used in an inverted instructional environment where students independently read[1] the tutorials and attempt to answer questions on their own *prior* to the instructor's interaction with them. In place of lecture[2], the instructor engages with students in Socratic-style dialogue, probing and challenging their understanding of the subject matter through inquiry.

Answers are not provided for questions within this chapter, and this is by design. Solved problems may be found in the Tutorial and Derivation chapters, instead. The goal here is *independence*, and this requires students to be challenged in ways where others cannot think for them. Remember that you always have the tools of *experimentation* and *computer simulation* (e.g. SPICE) to explore concepts!

The following lists contain ideas for Socratic-style questions and challenges. Upon inspection, one will notice a strong theme of *metacognition* within these statements: they are designed to foster a regular habit of examining one's own thoughts as a means toward clearer thinking. As such these sample questions are useful both for instructor-led discussions as well as for self-study.

[1] Technical reading is an essential academic skill for any technical practitioner to possess for the simple reason that the most comprehensive, accurate, and useful information to be found for developing technical competence is in textual form. Technical careers in general are characterized by the need for continuous learning to remain current with standards and technology, and therefore any technical practitioner who cannot read well is handicapped in their professional development. An excellent resource for educators on improving students' reading prowess through intentional effort and strategy is the book textitReading For Understanding – How Reading Apprenticeship Improves Disciplinary Learning in Secondary and College Classrooms by Ruth Schoenbach, Cynthia Greenleaf, and Lynn Murphy.

[2] Lecture is popular as a teaching method because it is easy to implement: any reasonably articulate subject matter expert can talk to students, even with little preparation. However, it is also quite problematic. A good lecture always makes complicated concepts seem easier than they are, which is bad for students because it instills a false sense of confidence in their own understanding; reading and re-articulation requires more cognitive effort and serves to verify comprehension. A culture of teaching-by-lecture fosters a debilitating dependence upon direct personal instruction, whereas the challenges of modern life demand independent and critical thought made possible only by gathering information and perspectives from afar. Information presented in a lecture is ephemeral, easily lost to failures of memory and dictation; text is forever, and may be referenced at any time.

General challenges following tutorial reading

- Summarize as much of the text as you can in one paragraph of your own words. A helpful strategy is to explain ideas as you would for an intelligent child: as simple as you can without compromising too much accuracy.
- Simplify a particular section of the text, for example a paragraph or even a single sentence, so as to capture the same fundamental idea in fewer words.
- Where did the text make the most sense to you? What was it about the text's presentation that made it clear?
- Identify where it might be easy for someone to misunderstand the text, and explain why you think it could be confusing.
- Identify any new concept(s) presented in the text, and explain in your own words.
- Identify any familiar concept(s) such as physical laws or principles applied or referenced in the text.
- Devise a proof of concept experiment demonstrating an important principle, physical law, or technical innovation represented in the text.
- Devise an experiment to disprove a plausible misconception.
- Did the text reveal any misconceptions you might have harbored? If so, describe the misconception(s) and the reason(s) why you now know them to be incorrect.
- Describe any useful problem-solving strategies applied in the text.
- Devise a question of your own to challenge a reader's comprehension of the text.

General follow-up challenges for assigned problems

- Identify where any fundamental laws or principles apply to the solution of this problem, especially before applying any mathematical techniques.
- Devise a thought experiment to explore the characteristics of the problem scenario, applying known laws and principles to mentally model its behavior.
- Describe in detail your own strategy for solving this problem. How did you identify and organized the given information? Did you sketch any diagrams to help frame the problem?
- Is there more than one way to solve this problem? Which method seems best to you?
- Show the work you did in solving this problem, even if the solution is incomplete or incorrect.
- What would you say was the most challenging part of this problem, and why was it so?
- Was any important information missing from the problem which you had to research or recall?
- Was there any extraneous information presented within this problem? If so, what was it and why did it not matter?
- Examine someone else's solution to identify where they applied fundamental laws or principles.
- Simplify the problem from its given form and show how to solve this simpler version of it. Examples include eliminating certain variables or conditions, altering values to simpler (usually whole) numbers, applying a limiting case (i.e. altering a variable to some extreme or ultimate value).
- For quantitative problems, identify the real-world meaning of all intermediate calculations: their units of measurement, where they fit into the scenario at hand. Annotate any diagrams or illustrations with these calculated values.
- For quantitative problems, try approaching it qualitatively instead, thinking in terms of "increase" and "decrease" rather than definite values.
- For qualitative problems, try approaching it quantitatively instead, proposing simple numerical values for the variables.
- Were there any assumptions you made while solving this problem? Would your solution change if one of those assumptions were altered?
- Identify where it would be easy for someone to go astray in attempting to solve this problem.
- Formulate your own problem based on what you learned solving this one.

General follow-up challenges for experiments or projects

- In what way(s) was this experiment or project easy to complete?
- Identify some of the challenges you faced in completing this experiment or project.

- Show how thorough documentation assisted in the completion of this experiment or project.
- Which fundamental laws or principles are key to this system's function?
- Identify any way(s) in which one might obtain false or otherwise misleading measurements from test equipment in this system.
- What will happen if (component $X$) fails (open/shorted/etc.)?
- What would have to occur to make this system unsafe?

## 7.1 Conceptual reasoning

These questions are designed to stimulate your analytic and synthetic thinking[3]. In a Socratic discussion with your instructor, the goal is for these questions to prompt an extended dialogue where assumptions are revealed, conclusions are tested, and understanding is sharpened. Your instructor may also pose additional questions based on those assigned, in order to further probe and refine your conceptual understanding.

Questions that follow are presented to challenge and probe your understanding of various concepts presented in the tutorial. These questions are intended to serve as a guide for the Socratic dialogue between yourself and the instructor. Your instructor's task is to ensure you have a sound grasp of these concepts, and the questions contained in this document are merely a means to this end. Your instructor may, at his or her discretion, alter or substitute questions for the benefit of tailoring the discussion to each student's needs. The only absolute requirement is that each student is challenged and assessed at a level equal to or greater than that represented by the documented questions.

It is far more important that you convey your *reasoning* than it is to simply convey a correct answer. For this reason, you should refrain from researching other information sources to answer questions. What matters here is that *you* are doing the thinking. If the answer is incorrect, your instructor will work with you to correct it through proper reasoning. A correct answer without an adequate explanation of how you derived that answer is unacceptable, as it does not aid the learning or assessment process.

You will note a conspicuous lack of answers given for these conceptual questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your conceptual answers, where applicable, is to use circuit simulation software to explore the effects of changes made to circuits. For example, if one of these conceptual questions challenges you to predict the effects of altering some component parameter in a circuit, you may check the validity of your work by simulating that same parameter change within software and seeing if the results agree.

[3] *Analytical* thinking involves the "disassembly" of an idea into its constituent parts, analogous to dissection. *Synthetic* thinking involves the "assembly" of a new idea comprised of multiple concepts, analogous to construction. Both activities are high-level cognitive skills, extremely important for effective problem-solving, necessitating frequent challenge and regular practice to fully develop.

### 7.1.1 Reading outline and reflections

*"Reading maketh a full man; conference a ready man; and writing an exact man"* – Francis Bacon

Francis Bacon's advice is a blueprint for effective education: reading provides the learner with knowledge, writing focuses the learner's thoughts, and critical dialogue equips the learner to confidently communicate and apply their learning. Independent acquisition and application of knowledge is a powerful skill, well worth the effort to cultivate. To this end, students should read these educational resources closely, journal their own reflections on the reading, and discuss in detail their findings with classmates and instructor(s). You should be able to do all of the following after reading any instructional text:

☑ Briefly SUMMARIZE THE TEXT in the form of a journal entry documenting your learning as you progress through the course of study. Share this summary in dialogue with your classmates and instructor. Journaling is an excellent self-test of thorough reading because you cannot clearly express what you have not read or did not comprehend.

☑ Demonstrate ACTIVE READING STRATEGIES, including verbalizing your impressions as you read, simplifying long passages to convey the same ideas using fewer words, annotating text and illustrations with your own interpretations, working through mathematical examples shown in the text, cross-referencing passages with relevant illustrations and/or other passages, identifying problem-solving strategies applied by the author, etc. Technical reading is a special case of problem-solving, and so these strategies work precisely because they help solve any problem: paying attention to your own thoughts (metacognition), eliminating unnecessary complexities, identifying what makes sense, paying close attention to details, drawing connections between separated facts, and noting the successful strategies of others.

☑ Identify IMPORTANT THEMES, especially GENERAL LAWS and PRINCIPLES, expounded in the text and express them in the simplest of terms as though you were teaching an intelligent child. This emphasizes connections between related topics and develops your ability to communicate complex ideas to anyone.

☑ Form YOUR OWN QUESTIONS based on the reading, and then pose them to your instructor and classmates for their consideration. Anticipate both correct and incorrect answers, the incorrect answer(s) assuming one or more plausible misconceptions. This helps you view the subject from different perspectives to grasp it more fully.

☑ Devise EXPERIMENTS to test claims presented in the reading, or to disprove misconceptions. Predict possible outcomes of these experiments, and evaluate their meanings: what result(s) would confirm, and what would constitute disproof? Running mental simulations and evaluating results is essential to scientific and diagnostic reasoning.

☑ Specifically identify any points you found CONFUSING. The reason for doing this is to help diagnose misconceptions and overcome barriers to learning.

### 7.1.2 Foundational concepts

Correct analysis and diagnosis of electric circuits begins with a proper understanding of some basic concepts. The following is a list of some important concepts referenced in this module's full tutorial. Define each of them in your own words, and be prepared to illustrate each of these concepts with a description of a practical example and/or a live demonstration.

Energy

Conservation of Energy

Conservation of Electric Charge

Thought experiments as a problem-solving strategy

Voltage

Conductors versus Insulators

Resistance

Current

Ohm's Law

Joule's Law

Electrical source

Electrical load

Series

Parallel

Equipotential points

Electrically common points

Switch

Open

Short

Kirchhoff's Voltage Law

Ground

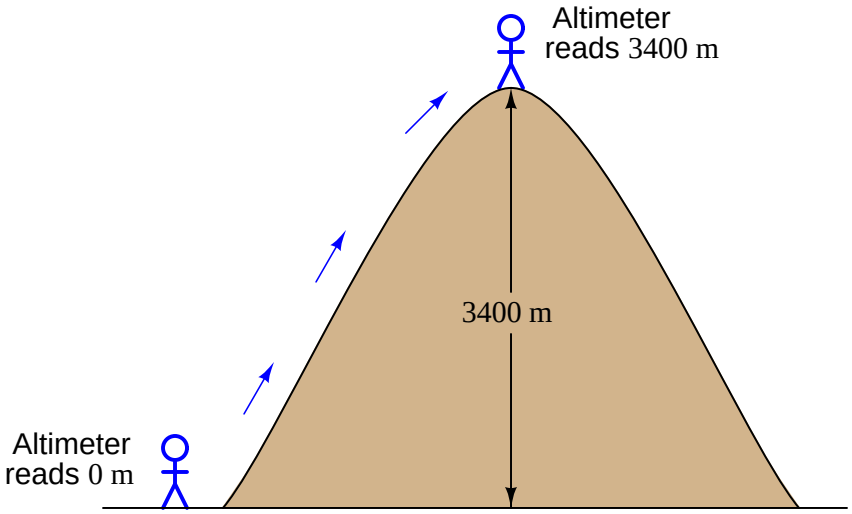
Electrical safety

### 7.1.3 Altimeter readings on a mountain

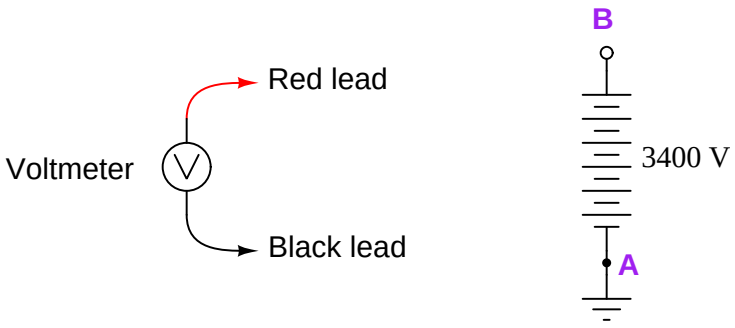
A *barometric altimeter* is a device used to measure altitude (height) by means of atmospheric pressure. The higher you go up from sea level, the less air pressure you encounter. This decrease in air pressure is closely correlated with height, and thus can be used to infer altitude.

This type of altimeter usually comes equipped with a "zero" adjustment, so that the instrument's indication may be offset to compensate for changes in air pressure resulting from different weather conditions. This same "zero" adjustment may also be used to establish the altimeter's zero indication at any arbitrary height, analogous to pressing the *tare* button on a digital weigh-scale.

For example, if a mountain climber sets her barometric altimeter to zero meters at the base of a mountain, then hikes to the summit of that mountain (3400 meters higher than the base), the altimeter should register +3400 meters at the summit:




While at the summit, the climber may re-set the altimeter's "zero" adjustment to register 0 meters once again. If the climber then descends to the base of the mountain, the altimeter will then register −3400 meters. Explain how this scenario of hiking with an altimeter relates to the measurement of voltage between points A and B in the following circuit:

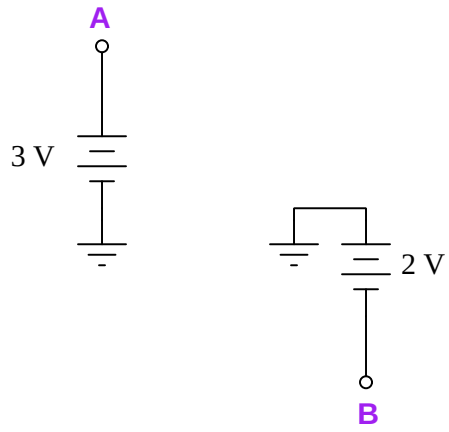


Challenges

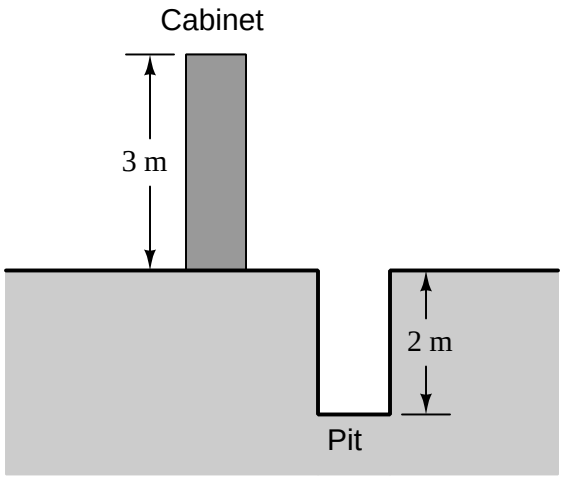
- Suppose the 3400 Volt battery were grounded at point B rather than point A. Describe a hiking scenario that would analogous to this revised schematic.

### 7.1.4 Incomplete KVL analogy

A student is puzzled by a problem given by his instructor. The task is to determine the differential voltage $V_{AB}$:




The student has already figured out the values of the ground-referenced voltages ($V_A = +3$ V and $V_B = -2$ V), but does not know for certain how to calculate the differential voltage between points A and B. Asking his instructor for an explanation, the instructor begins to draw this illustration:




Before the instructor begins to explain what the illustration means, however, she receives a call on her telephone and must leave the student momentarily. The student then asks you to help explain what the instructor's illustration might mean, and how it applies to the problem of determining voltage $V_{AB}$ in the original circuit.

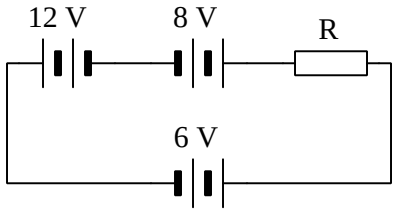
What would your explanation be? Where do you think the instructor was headed with her illustration, and how it might relate to voltages in a circuit?

Challenges

- Suppose the original schematic showed the two batteries with the 2-Volt battery's polarity reversed (i.e. negative grounded and positive on point B). Sketch an illustrative analogy of this network.
- Suppose the original schematic showed the two batteries with the 3-Volt battery's polarity reversed (i.e. positive grounded and negative on point A). Sketch an illustrative analogy of this network.

### 7.1.5 Three-battery circuit

Determine the direction of current and the resistor's voltage polarity in the following circuit, drawn using European component symbols:



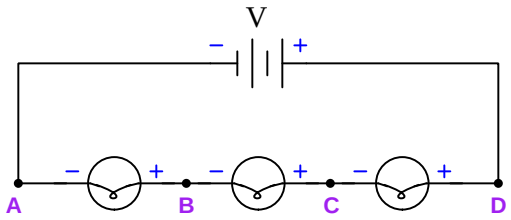

Also, determine the identity of each component in this circuit as either a *source* or a *load*.

Challenges

- Identify where energy is being transferred to and from in this circuit.
- Predict the effects of the resistor's resistance value increasing.
- Predict the effects of replacing the 8 Volt battery with a length of wire.
- Identify where a voltmeter could be connected in this circuit to obtain the *greatest possible* voltage reading.
- Identify where a voltmeter could be connected in this circuit to obtain the *least possible* voltage reading.
- Suppose none of the battery voltages were known to us, although their terminal polarities were clearly labeled. Explain how you could use an ammeter to identify whether the battery was acting as a source or as a load.

### 7.1.6 Polarities in a multi-lamp circuit

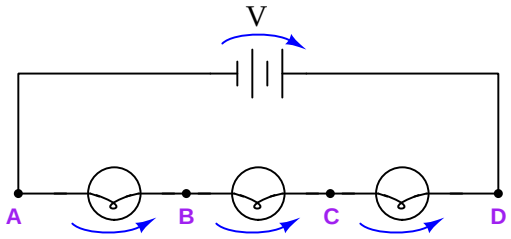
Consider this circuit comprised of three lamps and one battery, with points labeled A through D and polarity marks shown:




Suppose a student approached you with a question about this circuit. They seem confused on the identification of polarity within this circuit, asking you to explain how it is possible that the points lying between two lamps (e.g. point B and point C) seem to bear both positive and negative polarity labels, while point A is consistently negative and point D is consistently positive. *"How is it possible that point C is both positive and negative, while point D is just positive?"* asks the student.

Answer this student's question as accurately as you can, identifying and correcting any misconceptions along the way.

An alternative, though seldom used, method of denoting polarity is to use curved arrows instead of + and − symbols. In this symbology, the tip of the arrow represents "+" while the tail of the arrow represents "−" as shown here:



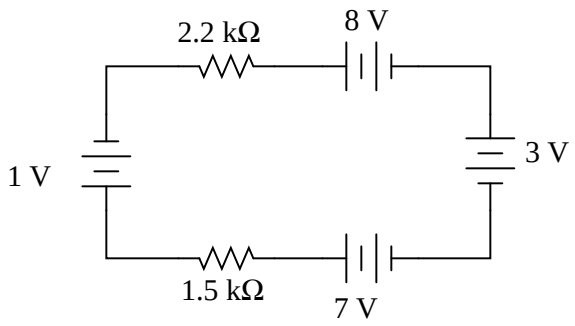

Explain how this alternative method of denoting voltage polarity may help the confused student.

Challenges

- Sketch straight arrows showing the direction of current (using conventional flow) in this circuit, and then explain the relationship between these straight current arrows and the curved voltage arrows. Do you see a pattern?
- Suppose the middle lamp fails shorted. Re-draw curved arrows showing voltage polarity across each of the three lamps in this new condition.
- Suppose the middle lamp fails open. Re-draw curved arrows showing voltage polarity across each of the three lamps in this new condition.

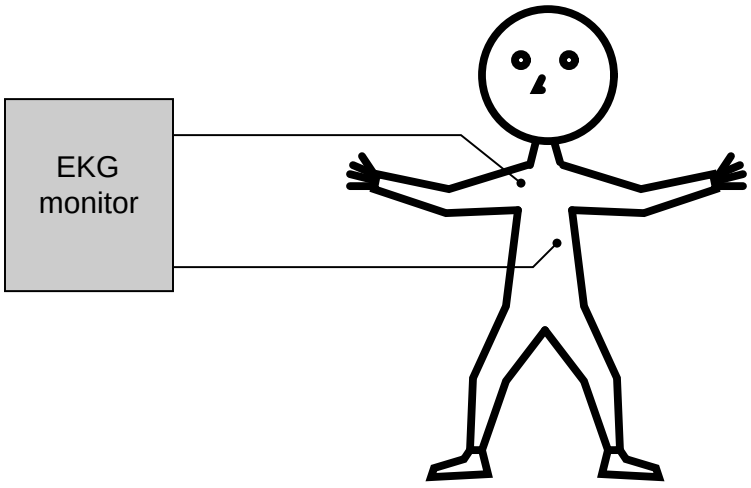
### 7.1.7 EKG monitor

When students begin learning about series circuits and Kirchhoff's Voltage Law, they are often mercilessly subjected to examples like this:

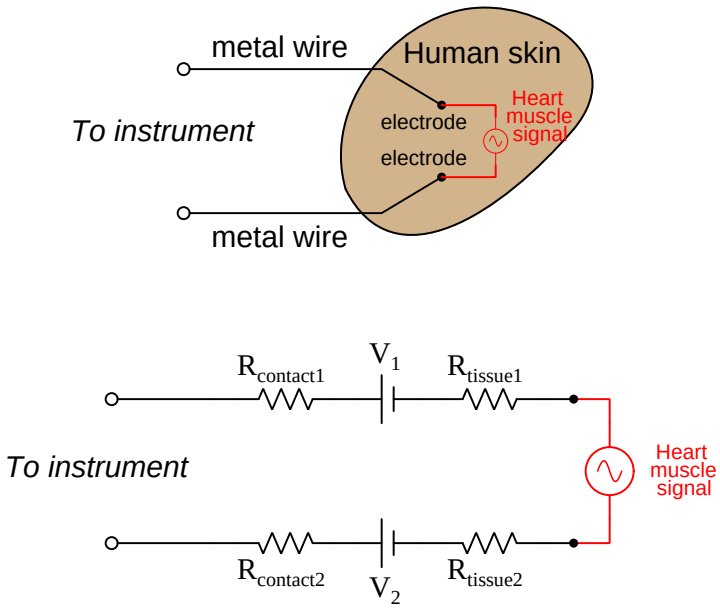



A common (and legitimate!) question asked by students is, *"Where would we ever encounter a circuit such as this, with batteries actually opposing each other?"* In practice, it is rare to find electrochemical batteries intentionally connected in such a manner, with some aiding and some opposing. However, there are many practical applications where the voltages involved are not intentional, but rather are unavoidable potentials created by junctions of dissimilar materials.

One practical example of opposed voltage sources is the application of EKG (electrocardiogram) measurements, where metal electrodes must be placed in contact with a human body to intercept tiny voltage signals from the contracting heart muscles:




A junction of metal wire to human skin is surprisingly complex from an electrical perspective, and may be approximately modeled by the following collection of idealized components:




Resistors $R_{contact1}$ and $R_{contact2}$ represent electrical resistance between the metal electrodes and skin at the point of contact. Resistors $R_{tissue1}$ and $R_{tissue2}$ represent electrical resistance of human tissue between the points of electrode contact and the actual heart muscle. The two series-opposing potentials ($V_1$ and $V_2$) are not intentional, but rather the result of electrochemical action between the metal electrode surfaces and human skin and therefore cannot be eliminated.

Explain how Kirchhoff's Voltage Law applies to this equivalent circuit, especially how the biomedical instrument "sees" only the heart muscle voltage signal and neither of the skin-contact voltages.

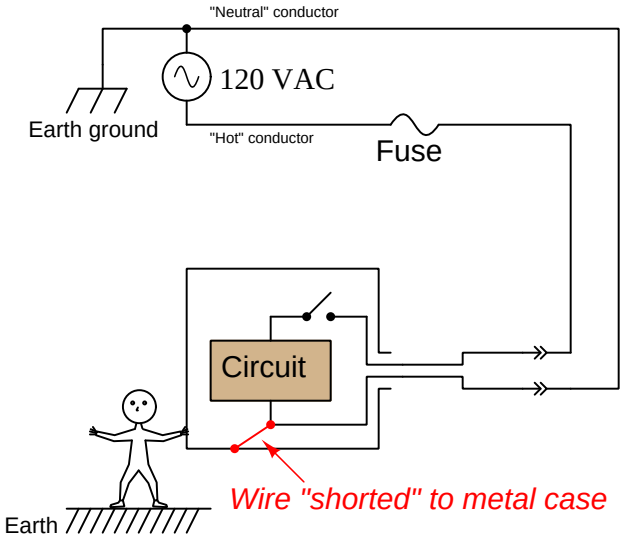
Challenges

- Identify a scenario where the skin-contact voltages *could* become problematic.
- Note that the heart muscle's voltage is modeled here as an $AC$ source. Does KVL apply to AC voltages as well as to DC voltages?

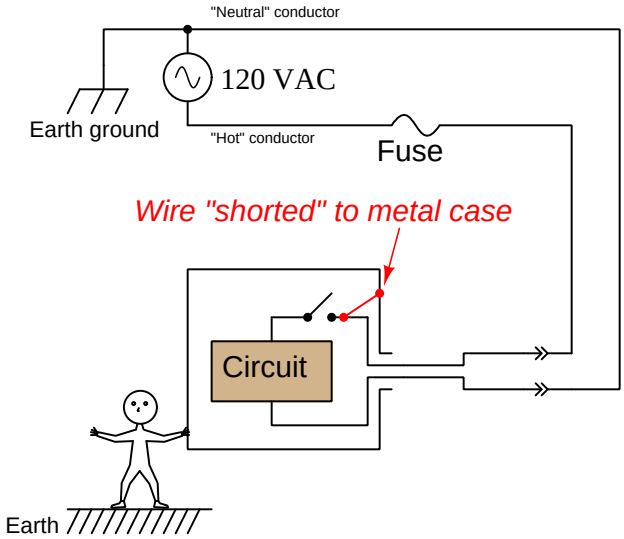
### 7.1.8 Shock hazard assessment

The following series of schematic diagrams show a 120 Volt AC household electrical appliance with various wiring faults.

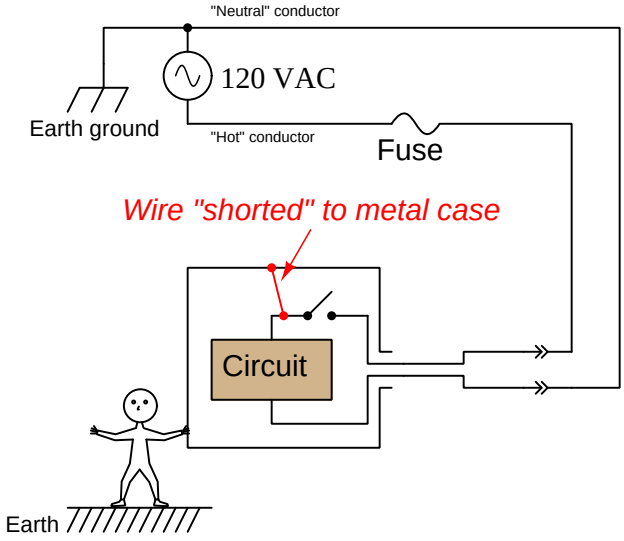
Is there any shock hazard posed to a person touching the metal case of this appliance as a result of the fault shown in the diagram? Explain your answer in detail:



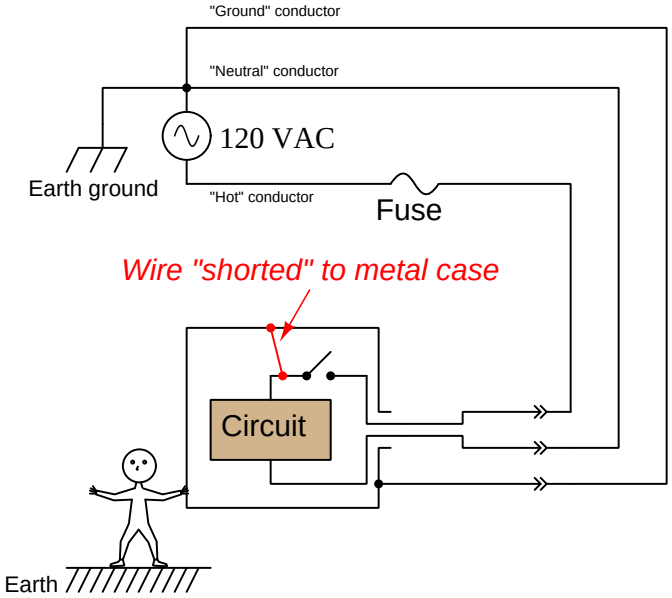
Re-assess the shock hazard to the person illustrated with the fault shown in the diagram. Once again, explain your answer in detail:




Re-assess the shock hazard to the person illustrated with the fault shown in the diagram. Once again, explain your answer in detail:



This version of the circuit includes a *ground wire* between the power source and the metal case of the appliance. Re-assess the shock hazard with the fault shown in the diagram. As always, explain your answer in detail:

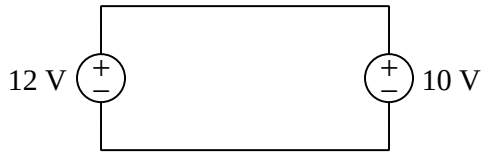



Challenges

- Based on a close analysis of the circuit, why do you suppose the "hot" conductor is given that name?
- Based on a close analysis of the circuit, why do you suppose the "neutral" conductor is given that name?
- The placement of the fuse in this circuit is not arbitrary. Explain why the fuse is always located between the source and the "hot" conductor, and never on the neutral conductor.

### 7.1.9 Paralleled voltage sources

Does the following circuit violate Kirchhoff's Voltage Law? Explain why or why not.




Describe what you think would happen if you were to connect two sources of unequal voltage together like this.

Challenges

- Modify the circuit so that it is no longer problematic, without altering either source's voltage.
- Describe a scenario where it would be useful to parallel two or more batteries.

## 7.2 Quantitative reasoning

These questions are designed to stimulate your computational thinking. In a Socratic discussion with your instructor, the goal is for these questions to reveal your mathematical approach(es) to problem-solving so that good technique and sound reasoning may be reinforced. Your instructor may also pose additional questions based on those assigned, in order to observe your problem-solving firsthand.

Mental arithmetic and estimations are strongly encouraged for all calculations, because without these abilities you will be unable to readily detect errors caused by calculator misuse (e.g. keystroke errors).

You will note a conspicuous lack of answers given for these quantitative questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. My advice is to use circuit simulation software such as SPICE to check the correctness of quantitative answers. Refer to those learning modules within this collection focusing on SPICE to see worked examples which you may use directly as practice problems for your own study, and/or as templates you may modify to run your own analyses and generate your own practice problems.

Completely worked example problems found in the Tutorial may also serve as "test cases[4]" for gaining proficiency in the use of circuit simulation software, and then once that proficiency is gained you will never need to rely[5] on an answer key!

[4] In other words, set up the circuit simulation software to analyze the same circuit examples found in the Tutorial. If the simulated results match the answers shown in the Tutorial, it confirms the simulation has properly run. If the simulated results disagree with the Tutorial's answers, something has been set up incorrectly in the simulation software. Using every Tutorial as practice in this way will quickly develop proficiency in the use of circuit simulation software.

[5] This approach is perfectly in keeping with the instructional philosophy of these learning modules: *teaching students to be self-sufficient thinkers*. Answer keys can be useful, but it is even more useful to your long-term success to have a set of tools on hand for checking your own work, because once you have left school and are on your own, there will no longer be "answer keys" available for the problems you will have to solve.

### 7.2.1 Miscellaneous physical constants

Note: constants shown in **bold** type are *exact*, not approximations. Values inside of parentheses show one standard deviation ($\sigma$) of uncertainty in the final digits: for example, the magnetic permeability of free space value given as $1.25663706212(19) \times 10^{-6}$ H/m represents a center value (i.e. the location parameter) of $1.25663706212 \times 10^{-6}$ Henrys per meter with one standard deviation of uncertainty equal to $0.0000000000019 \times 10^{-6}$ Henrys per meter.

Avogadro's number ($N_A$) = **6.02214076** $\times 10^{23}$ **per mole** ($mol^{-1}$)

Boltzmann's constant ($k$) = **1.380649** $\times 10^{-23}$ **Joules per Kelvin** (J/K)

Electronic charge ($e$) = **1.602176634** $\times 10^{-19}$ **Coulomb** (C)

Faraday constant ($F$) = **96,485.33212...** $\times 10^{4}$ **Coulombs per mole** (C/mol)

Magnetic permeability of free space ($\mu_0$) = $1.25663706212(19) \times 10^{-6}$ Henrys per meter (H/m)

Electric permittivity of free space ($\epsilon_0$) = $8.8541878128(13) \times 10^{-12}$ Farads per meter (F/m)

Characteristic impedance of free space ($Z_0$) = 376.730313668(57) Ohms ($\Omega$)

Gravitational constant ($G$) = $6.67430(15) \times 10^{-11}$ cubic meters per kilogram-seconds squared ($m^3/kg\text{-}s^2$)

Molar gas constant ($R$) = **8.314462618... Joules per mole-Kelvin** (J/mol-K) = 0.08205746(14) liters-atmospheres per mole-Kelvin

Planck constant ($h$) = **6.62607015** $\times 10^{-34}$ **joule-seconds** (J-s)

Stefan-Boltzmann constant ($\sigma$) = **5.670374419...** $\times 10^{-8}$ **Watts per square meter-Kelvin**$^4$ ($W/m^2 \cdot K^4$)

Speed of light in a vacuum ($c$) = **299,792,458 meters per second** (m/s) = 186282.4 miles per second (mi/s)

Note: All constants taken from NIST data "Fundamental Physical Constants – Complete Listing", from `http://physics.nist.gov/constants`, National Institute of Standards and Technology (NIST), 2018 CODATA Adjustment.

### 7.2.2 Introduction to spreadsheets

A powerful computational tool you are encouraged to use in your work is a *spreadsheet*. Available on most personal computers (e.g. Microsoft Excel), *spreadsheet* software performs numerical calculations based on number values and formulae entered into cells of a grid. This grid is typically arranged as lettered columns and numbered rows, with each cell of the grid identified by its column/row coordinates (e.g. cell `B3`, cell `A8`). Each cell may contain a string of text, a number value, or a mathematical formula. The spreadsheet automatically updates the results of all mathematical formulae whenever the entered number values are changed. This means it is possible to set up a spreadsheet to perform a series of calculations on entered data, and those calculations will be re-done by the computer any time the data points are edited in any way.

For example, the following spreadsheet calculates average speed based on entered values of distance traveled and time elapsed:

| | A | B | C | D |
|---|---|---|---|---|
| 1 | Distance traveled | 46.9 | Kilometers | |
| 2 | Time elapsed | 1.18 | Hours | |
| 3 | Average speed | = B1 / B2 | km/h | |
| 4 | | | | |
| 5 | | | | |

Text labels contained in cells `A1` through `A3` and cells `C1` through `C3` exist solely for readability and are not involved in any calculations. Cell `B1` contains a sample distance value while cell `B2` contains a sample time value. The formula for computing speed is contained in cell `B3`. Note how this formula begins with an "equals" symbol (`=`), references the values for distance and speed by lettered column and numbered row coordinates (`B1` and `B2`), and uses a forward slash symbol for division (`/`). The coordinates `B1` and `B2` function as *variables*[6] would in an algebraic formula.

When this spreadsheet is executed, the numerical value `39.74576` will appear in cell `B3` rather than the formula `= B1 / B2`, because 39.74576 is the computed speed value given 46.9 kilometers traveled over a period of 1.18 hours. If a different numerical value for distance is entered into cell `B1` or a different value for time is entered into cell `B2`, cell `B3`'s value will automatically update. All you need to do is set up the given values and any formulae into the spreadsheet, and the computer will do all the calculations for you.

Cell `B3` may be referenced by other formulae in the spreadsheet if desired, since it is a variable just like the given values contained in `B1` and `B2`. This means it is possible to set up an entire chain of calculations, one dependent on the result of another, in order to arrive at a final value. The arrangement of the given data and formulae need not follow any pattern on the grid, which means you may place them anywhere.

[6] Spreadsheets may also provide means to attach text labels to cells for use as variable names (Microsoft Excel simply calls these labels "names"), but for simple spreadsheets such as those shown here it's usually easier just to use the standard coordinate naming for each cell.

Common[7] arithmetic operations available for your use in a spreadsheet include the following:

- Addition (`+`)
- Subtraction (`-`)
- Multiplication (`*`)
- Division (`/`)
- Powers (`^`)
- Square roots (`sqrt()`)
- Logarithms (`ln()` , `log10()`)

Parentheses may be used to ensure[8] proper order of operations within a complex formula. Consider this example of a spreadsheet implementing the *quadratic formula*, used to solve for roots of a polynomial expression in the form of $ax^2 + bx + c$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

| | A | B |
|---|---|---|
| 1 | x_1 | = (-B4 + sqrt((B4^2) - (4*B3*B5))) / (2*B3) |
| 2 | x_2 | = (-B4 - sqrt((B4^2) - (4*B3*B5))) / (2*B3) |
| 3 | a = | 9 |
| 4 | b = | 5 |
| 5 | c = | -2 |

This example is configured to compute roots[9] of the polynomial $9x^2 + 5x - 2$ because the values of 9, 5, and $-2$ have been inserted into cells `B3`, `B4`, and `B5`, respectively. Once this spreadsheet has been built, though, it may be used to calculate the roots of *any* second-degree polynomial expression simply by entering the new $a$, $b$, and $c$ coefficients into cells `B3` through `B5`. The numerical values appearing in cells `B1` and `B2` will be automatically updated by the computer immediately following any changes made to the coefficients.

[7] Modern spreadsheet software offers a bewildering array of mathematical functions you may use in your computations. I recommend you consult the documentation for your particular spreadsheet for information on operations other than those listed here.

[8] Spreadsheet programs, like text-based programming languages, are designed to follow standard order of operations by default. However, my personal preference is to use parentheses even where strictly unnecessary just to make it clear to any other person viewing the formula what the intended order of operations is.

[9] Reviewing some algebra here, a *root* is a value for $x$ that yields an overall value of zero for the polynomial. For this polynomial ($9x^2 + 5x - 2$) the two roots happen to be $x = 0.269381$ and $x = -0.82494$, with these values displayed in cells `B1` and `B2`, respectively upon execution of the spreadsheet.

Alternatively, one could break up the long quadratic formula into smaller pieces like this:

$$y = \sqrt{b^2 - 4ac} \qquad z = 2a$$

$$x = \frac{-b \pm y}{z}$$

| | A | B | C |
|---|---|---|---|
| 1 | x_1 | = (-B4 + C1) / C2 | = sqrt((B4^2) - (4*B3*B5)) |
| 2 | x_2 | = (-B4 - C1) / C2 | = 2*B3 |
| 3 | a = | 9 | |
| 4 | b = | 5 | |
| 5 | c = | -2 | |

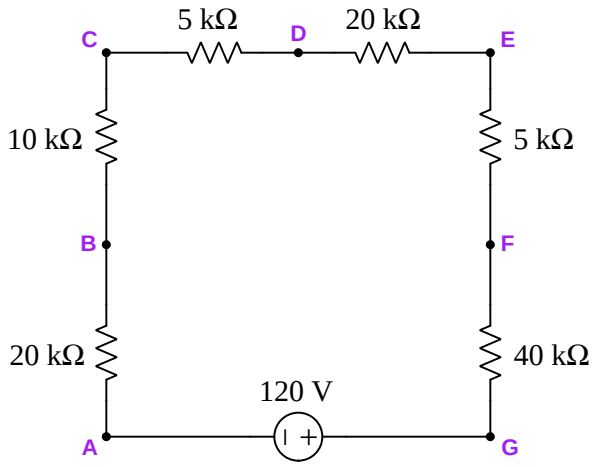
Note how the square-root term ($y$) is calculated in cell `C1`, and the denominator term ($z$) in cell `C2`. This makes the two final formulae (in cells `B1` and `B2`) simpler to interpret. The positioning of all these cells on the grid is completely arbitrary[10] – all that matters is that they properly reference each other in the formulae.

Spreadsheets are particularly useful for situations where the same set of calculations representing a circuit or other system must be repeated for different initial conditions. The power of a spreadsheet is that it automates what would otherwise be a tedious set of calculations. One specific application of this is to simulate the effects of various components within a circuit failing with abnormal values (e.g. a shorted resistor simulated by making its value nearly zero; an open resistor simulated by making its value extremely large). Another application is analyzing the behavior of a circuit design given new components that are out of specification, and/or aging components experiencing drift over time.

[10] My personal preference is to locate all the "given" data in the upper-left cells of the spreadsheet grid (each data point flanked by a sensible name in the cell to the left and units of measurement in the cell to the right as illustrated in the first distance/time spreadsheet example), sometimes coloring them in order to clearly distinguish which cells contain entered data versus which cells contain computed results from formulae. I like to place all formulae in cells below the given data, and try to arrange them in logical order so that anyone examining my spreadsheet will be able to figure out *how* I constructed a solution. This is a general principle I believe all computer programmers should follow: *document and arrange your code to make it easy for other people to learn from it.*

### 7.2.3 Tracing KVL loops in a series circuit

Calculate voltage drops in this series circuit, being sure to annotate each resistor with its own voltage drop and polarity markings (+ and − symbols):




Once all the resistor voltage drops have been calculated and annotated (*hint: you should not require a calculator to do the arithmetic!*), this circuit will prove useful as a test-case for Kirchhoff's Voltage Law. We may step around this circuit in *any* sequence we desire, and the algebraic sum of all the voltage gains and losses must be zero once we return to our starting point.

First, write the sum of all voltage gains and losses in the loop ABCDEFGA.

While Kirchhoff's Voltage Law is easiest to demonstrate in a loop over-tracing current pathways in a circuit, it is often most useful when applied to loops *jumping between points* not directly connected to each other. For example, write the sum of all voltage gains and losses in the loop BCDEB being equal to zero, and use this fact to calculate $V_{BE}$ (i.e. the voltage across the space between test points B and E).

Now, calculate the same $V_{BE}$ tracing a different loop in this circuit: EBAGFE.

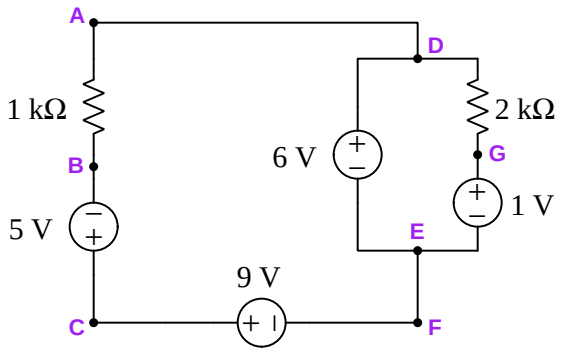
Finally, calculate $V_{FB}$ using Kirchhoff's Voltage Law.

Challenges

- Choose your own two points in this circuit separated by more than one component, and use Kirchhoff's Voltage Law to calculate voltage between those two points. Double-check your work by tracing some *alternative* loop using KVL to prove the same.

### 7.2.4 Tracing KVL loops in a multi-source circuit

Apply Kirchhoff's Voltage Law to the following loops in this multi-source circuit to determine voltages across the two resistors:




Loop BCFEDAB =

Loop EDGE =

Next, identify loops (by sequences of test-point letters) useful for determining the following voltages, and calculate those voltages according to the convention of the first subscript being the red-colored voltmeter lead and the second subscript being the black:

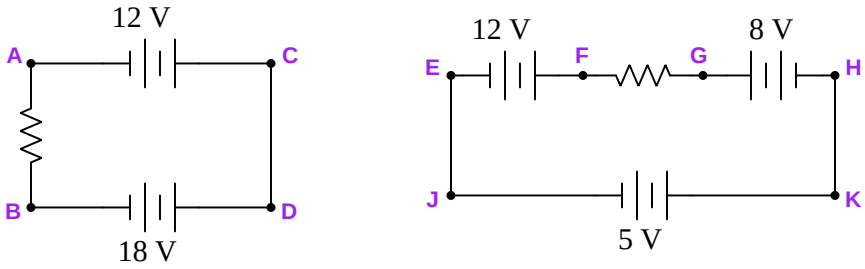
- $V_{BE}$ =
- $V_{CA}$ =
- $V_{GB}$ =

Challenges

- If the 1 kΩ resistor fails open, will its voltage drop be affected?
- If the 2 kΩ resistor fails open, will its voltage drop be affected?
- Identify *alternative* loops over which to apply KVL when solving for any of the requested voltages in this circuit.

### 7.2.5 One resistor, multiple batteries

Determine the magnitude and polarity of the resistor's voltage in each of these circuits:




Next, solve for the voltage between points **J** and **G** in the right-hand circuit, being sure to specify the loop you used for KVL (i.e. the specific sequence of test-points that includes a "leap" between **J** and **G**).

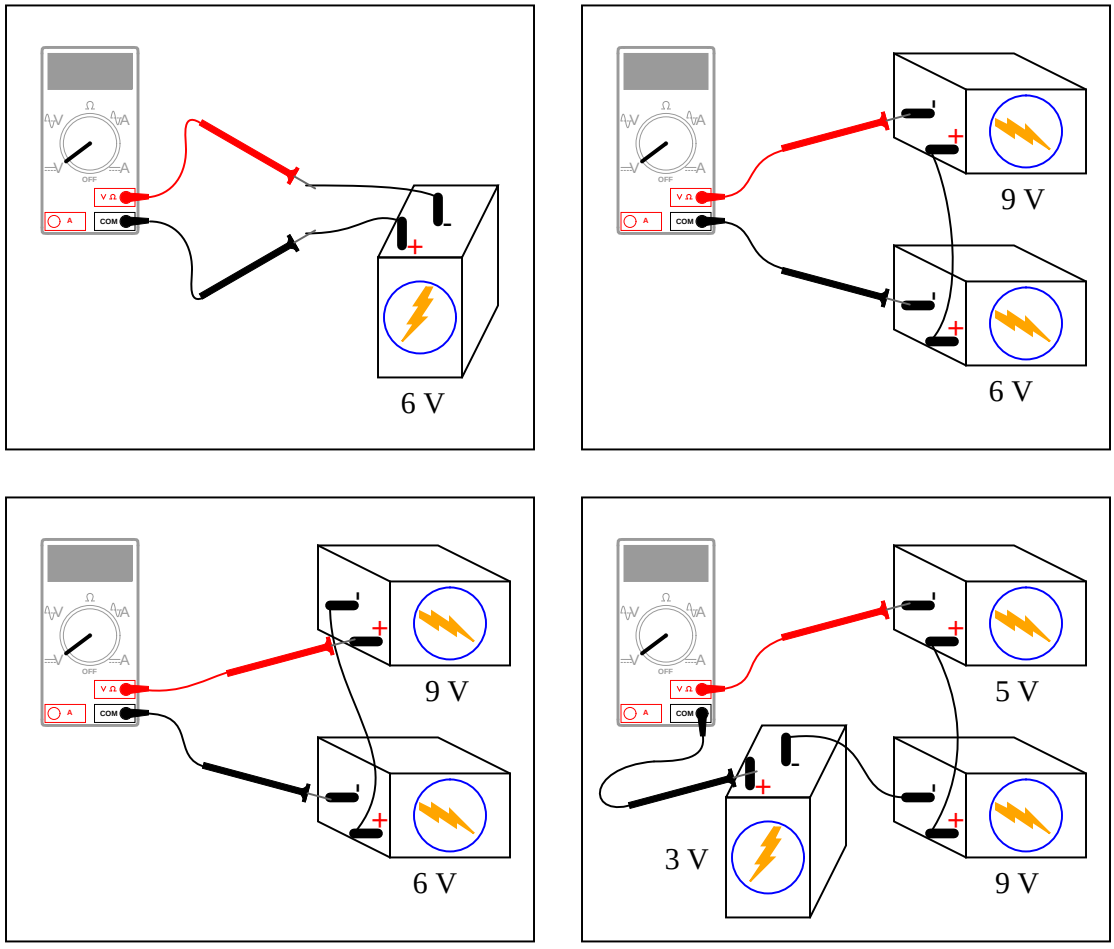
Next, solve for the voltage between points **F** and **H** in the right-hand circuit, also specifying the loop you used for KVL.

Challenges

- If the resistor fails open, will its voltage drop be affected?
- Identify *alternative* loops over which to apply KVL when solving for $V_{JG}$ and $V_{FH}$.

### 7.2.6 Series-connected batteries

Determine what each digital multimeter (DMM) will register in the following scenarios, being careful to express both the magnitude of the measurement as well as its mathematical sign (either + or −):

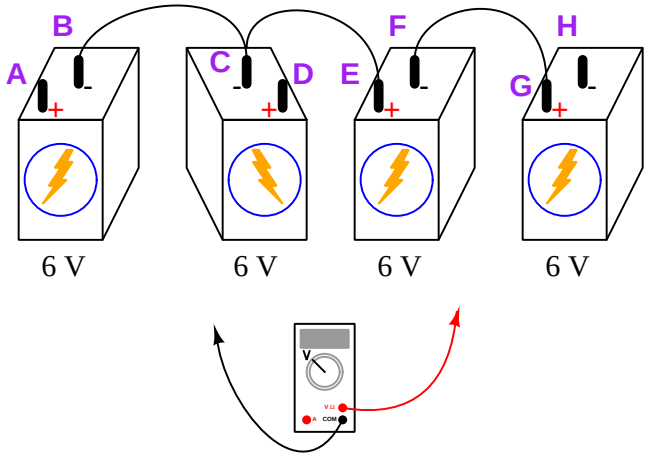



Challenges

- Explain the consequences of someone accidently configuring the DMM to measure *current* instead of voltage before connecting the meter to the batteries as shown.

### 7.2.7 More connected batteries

Determine what a digital multimeter (DMM) configured to measure voltage would indicate if connected between the following points in this battery network:



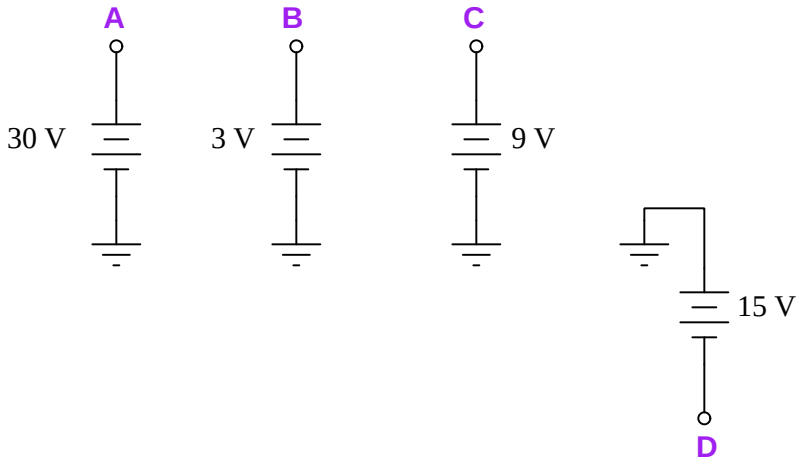

- Red test lead on F, black test lead on B ($V_{FB}$):
- Red test lead on F, black test lead on A ($V_{FA}$):
- Red test lead on C, black test lead on G ($V_{CG}$):
- Red test lead on A, black test lead on H ($V_{AH}$):
- Black test lead on C, red test lead on A ($V_{AC}$):
- Black test lead on D, red test lead on H ($V_{HD}$):
- Black test lead on A, red test lead on D ($V_{DA}$):

Challenges

- Explain the consequences of someone accidently configuring the DMM to measure *current* instead of voltage before connecting the meter to the batteries as shown.
- Suppose the polarity of the second battery (from the left) were reversed. How many of these voltage measurements would be affected by this reversal?

### 7.2.8 Grounded batteries

Determine the voltages registered by a voltmeter between the following points. Note that some of these are *ground-referenced* (i.e. *single-ended*) voltages with only one test point specified while others are *differential* voltages with two test points specified. Be sure to specify the proper mathematical sign for each voltage as it would be shown by a voltmeter with red test lead on the measurement point and black test lead on the reference point:



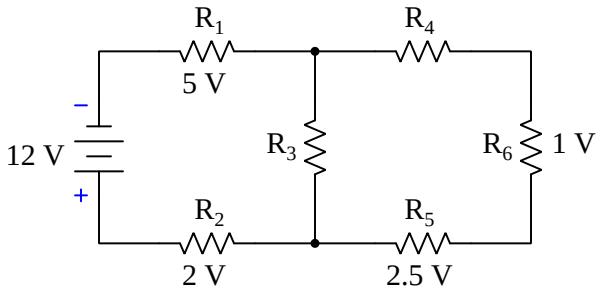

- $V_A =$
- $V_B =$
- $V_C =$
- $V_D =$
- $V_{AC} =$
- $V_{DB} =$
- $V_{BC} =$
- $V_{CD} =$
- $V_{BA} =$

Challenges

- Explain the consequences of someone accidently configuring the DMM to measure *current* instead of voltage before connecting the meter to the batteries as shown.
- Identify a pair of points having $-21$ Volts between them.
- Identify a pair of points having $+45$ Volts between them.

### 7.2.9 Voltages in a resistor network

Calculate the magnitudes and polarities of the voltages across resistors $R_3$ and $R_4$ in this resistor network:

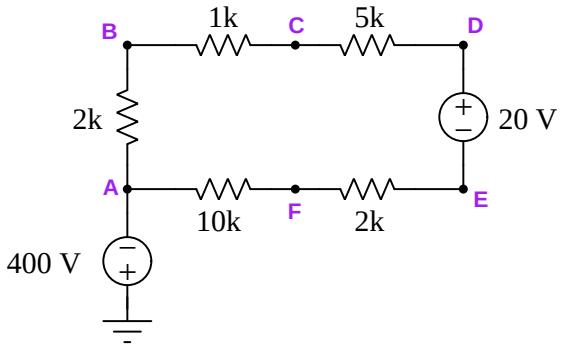



Challenges

- There is definitely more than one way to solve for the voltage drops in this circuit. After explaining how you solved for these values, identify and explain a different way.
- Identify the polarities of all other resistor voltage drops.
- Identify the effects of resistor $R_2$ failing open.
- Identify the effects of resistor $R_3$ failing shorted (e.g. a solder bridge across its terminals).

### 7.2.10 Series resistor network with common-mode source

Calculate each of the voltages listed below. Double-subscript voltages are polarized with the voltmeter's red test lead on the first subscript point and the black lead on the second. Single-subscript voltages are polarized with the voltmeter's red test lead on the specified point and the black lead on ground:



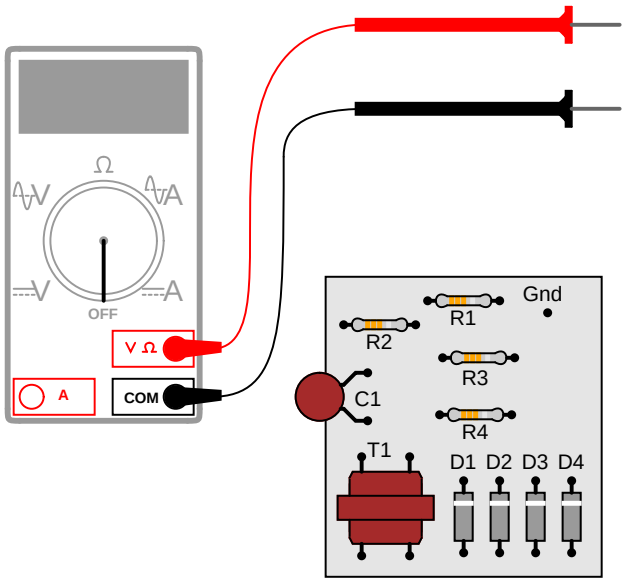

- $V_B =$
- $V_{FD} =$
- $V_{AE} =$
- $V_{DB} =$
- $V_C =$
- $V_{FC} =$
- $V_E =$

Challenges

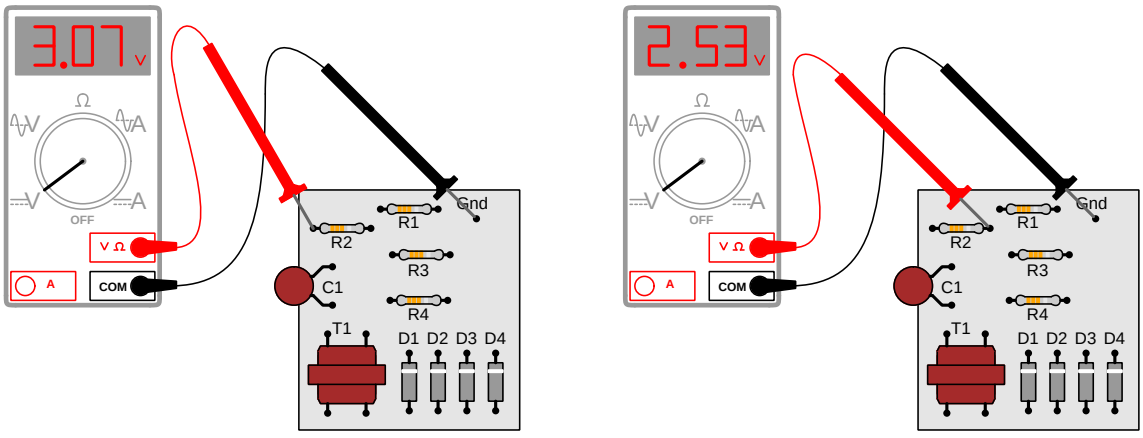
- Which of these resistors dissipates the greatest amount of power?

### 7.2.11 Printed circuit board measurements

Suppose you wanted to measure the amount of current going through resistor $R_2$ on this printed circuit board (PCB), but did not have the luxury of breaking the circuit to do so (unsoldering one end of the resistor, detaching it from the PCB, and connecting an ammeter in series). All you can do while the circuit is powered is measure voltage with a voltmeter:




So, you decide to touch the black probe of the voltmeter to the circuit's "Gnd" (ground) test point, and measure the voltage with reference to ground on both sides of $R_2$. The results are shown here:




$R_2$'s color code is Orange, Orange, Red, Gold. Based on this information, determine both the

direction and the magnitude of DC current through this resistor, and explain how you did so.

Challenges

- Devise a simpler way to determine $R_2$'s current, using the same test equipment.
- Identify possible sources of error in this procedure, where we infer current without taking a direct current measurement.
- Suppose the meter's red test lead were the one connected to ground. What effect would this have on the results, and would it impact the determination of $R_2$'s current?

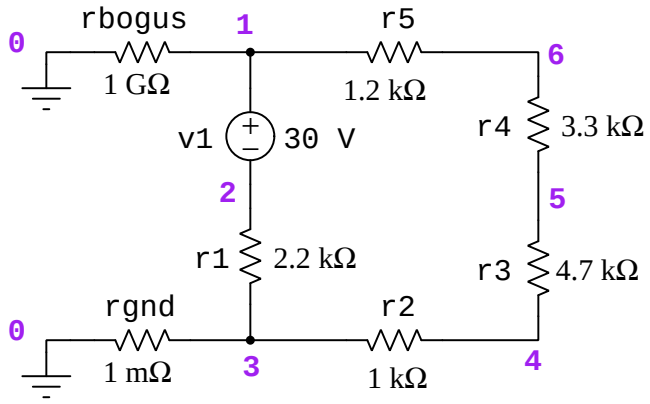
### 7.2.12 Interpreting a SPICE analysis

A computer program called SPICE was developed in the early 1970's, whereby a text-based description of an electric circuit (called a *netlist*) could be entered into a computer, and then computer would be directed to apply fundamental laws of electric circuits to the netlist circuit description according to algorithms coded in SPICE.

Here is a netlist text for a five-resistor series circuit with an easily-moved ground connection, powered by a DC voltage source:

```
* Five-resistor circuit with movable ground
v1 1 2 dc 30
r1 2 3 2200
r2 3 4 1000
r3 4 5 4700
r4 5 6 3300
r5 6 1 1200
rgnd 3 0 1m
rbogus 1 0 1e9
.op
.width out=80
.end
```

The "movable ground" consists of resistor `rgnd` which is only 1 milliOhm ($1 \times 10^{-3}$ Ω) and therefore is comparable to a short length of copper wire, connecting between nodes 3 and 0, node zero of course being SPICE's default definition of ground in any circuit. I refer to this ground as being "movable" because one need only edit the first node number of `rgnd` to connect any other point of this circuit to ground, leaving the rest of the netlist unedited. Another "extra" resistor in this circuit (`rbogus`) is there only to placate SPICE, which *demands* there be more than one connection to node 0 in the circuit. This `rbogus` resistor has been given an enormously high value (1 *giga*Ohm!) so as to practically be an "open" and therefore not substantially affect the circuit's operation.



When SPICE (software version 2G6) processes the contents of this "netlist" file, it outputs a text description of the analysis. The following text has been edited for clarity (e.g. blank lines, extraneous characters, and statistical data removed):

**Ground connected to node 3:**

```
 node   voltage     node   voltage     node   voltage     node   voltage
(  1)   24.6774    (  2)   -5.3226    (  3)    0.0000    (  4)    2.4194
(  5)   13.7903    (  6)   21.7742
```

Interpret the output of SPICE to the best of your ability. Identify all parameters that you can, and see if the results of the computer's analysis agree with your own calculations.

Now, here is an example of the output from SPICE after the ground resistor `rgnd` has been moved from node 3 to node 5:

**Ground connected to node 5:**

```
 node   voltage     node   voltage     node   voltage     node   voltage
(  1)   10.8871    (  2)  -19.1129    (  3)  -13.7903    (  4)  -11.3710
(  5)    0.0000    (  6)    7.9838
```

Interpret the output of SPICE to the best of your ability. Identify all parameters that you can, and see if the results of the computer's analysis agree with your own calculations.

Once you become familiar with the analysis data format of SPICE, you will be able to use the "Gallery" of SPICE simulations found in the *"SPICE Modeling of Resistor Circuits"* module as practice problems for developing your own circuit analysis skills.

Challenges

- Modify this SPICE netlist to utilize a voltage source with a value of 6 Volts instead of 30 Volts.
- Modify this SPICE netlist to specify resistor $R_1$'s resistance using scientific (power-of-ten) notation.
- Modify this SPICE netlist to specify resistor $R_1$'s resistance using metric prefix ("kilo") notation.

## 7.3 Diagnostic reasoning

These questions are designed to stimulate your deductive and inductive thinking, where you must apply general principles to specific scenarios (deductive) and also derive conclusions about the failed circuit from specific details (inductive). In a Socratic discussion with your instructor, the goal is for these questions to reinforce your recall and use of general circuit principles and also challenge your ability to integrate multiple symptoms into a sensible explanation of what's wrong in a circuit. Your instructor may also pose additional questions based on those assigned, in order to further challenge and sharpen your diagnostic abilities.

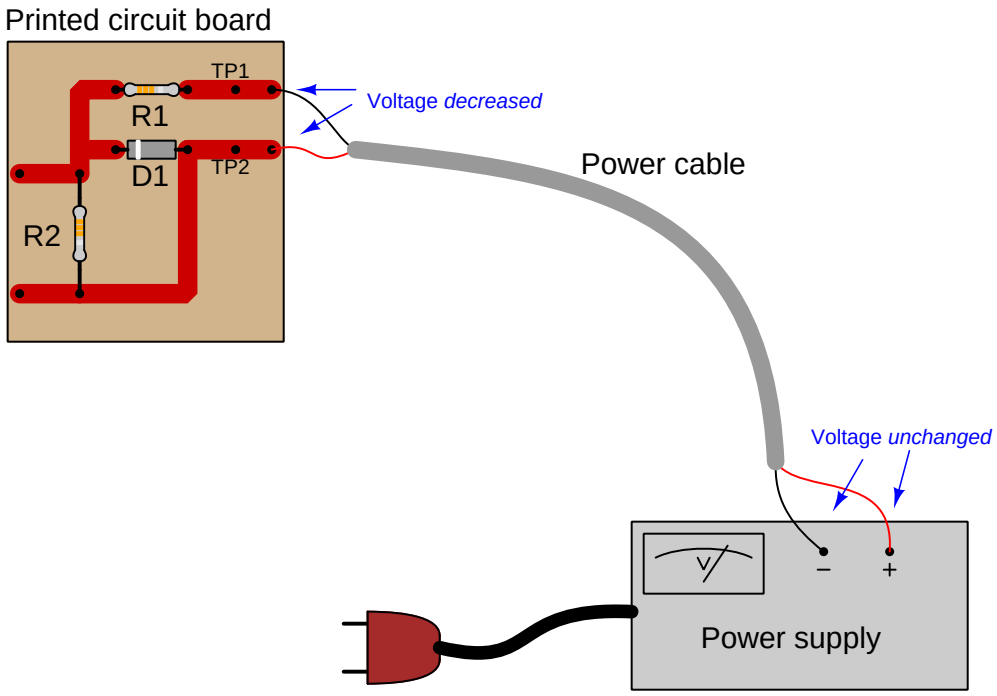
As always, your goal is to fully *explain* your analysis of each problem. Simply obtaining a correct answer is not good enough – you must also demonstrate sound reasoning in order to successfully complete the assignment. Your instructor's responsibility is to probe and challenge your understanding of the relevant principles and analytical processes in order to ensure you have a strong foundation upon which to build further understanding.

You will note a conspicuous lack of answers given for these diagnostic questions. Unlike standard textbooks where answers to every other question are given somewhere toward the back of the book, here in these learning modules students must rely on other means to check their work. The best way by far is to debate the answers with fellow students and also with the instructor during the Socratic dialogue sessions intended to be used with these learning modules. Reasoning through challenging questions with other people is an excellent tool for developing strong reasoning skills.

Another means of checking your diagnostic answers, where applicable, is to use circuit simulation software to explore the effects of faults placed in circuits. For example, if one of these diagnostic questions requires that you predict the effect of an open or a short in a circuit, you may check the validity of your work by simulating that same fault (substituting a very high resistance in place of that component for an open, and substituting a very low resistance for a short) within software and seeing if the results agree.

### 7.3.1 Sudden change in voltage

Suppose you were measuring the voltage between "test points" TP1 and TP2 on this printed circuit board, where the circuit receives DC power from a voltage source. Suddenly, this voltage decreases. Wondering if perhaps the power supply voltage itself decreased, you go back to measure its output voltage and find that it is unchanged from before:



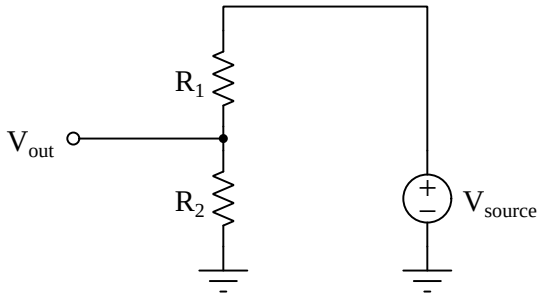

Explain what must have happened to cause the voltage between TP1 and TP2 on the circuit board to suddenly decrease, given the power supply's stable voltage output. Hint: the components on the printed circuit board are irrelevant to the answer!

Challenges

- Identify the next diagnostic test(s) you might perform on this circuit to further identify the nature and/or location of the fault.
- Explain why the PCB's components are irrelevant to this problem.

### 7.3.2 Diagnosing a fault type

Suppose a voltage divider circuit is faulted, but we don't know exactly what the fault is (yet). The output voltage (measured in reference to ground) should be a precise fraction of $V_{source}$, but instead $V_{out}$ is zero all the time:




This could be caused by a failed-open $R_1$ or a shorted $R_2$. Visual inspection of the resistors doesn't reveal anything unusual, and so a fellow technician suggests taking a voltage measurement across $R_1$. "If you measure full voltage across resistor $R_1$," he says, "then you know $R_1$ is failed open."

First, explain what is incorrect about this advice.

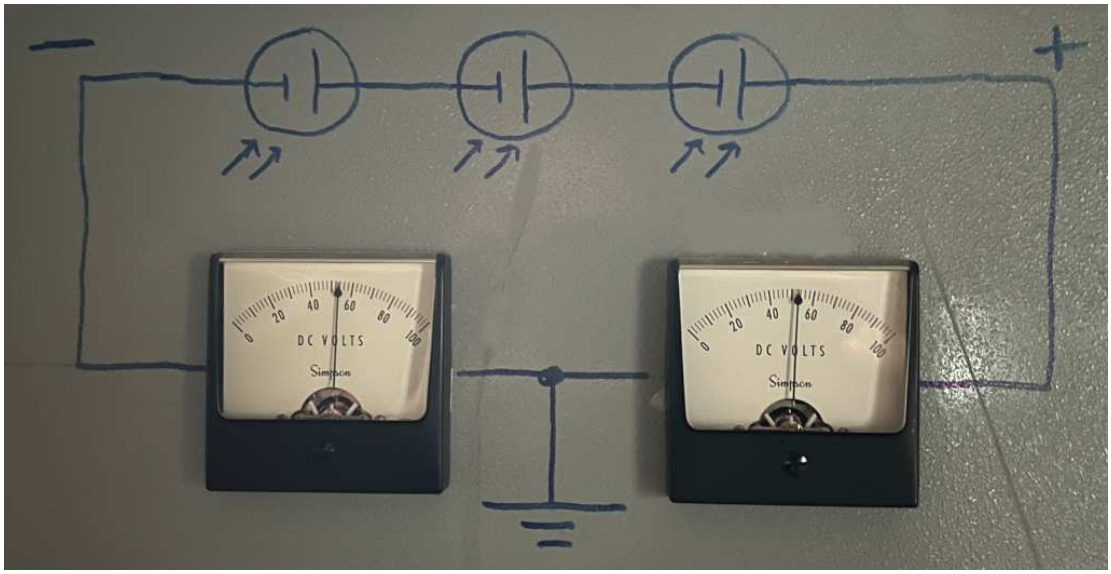
Next, identify a better diagnostic test you could perform to discern whether $R_1$ is open or $R_2$ is shorted.

Challenges

- Identify a fault *other* than an open or shorted resistor that could cause $V_{out}$ to be zero.
- Identify a pair of resistance values for $R_1$ and $R_2$ resulting in a voltage division ratio of $\frac{1}{3}$.
- Would the voltage division ratio of this divider circuit be affected by reversing the source's polarity? Why or why not?

### 7.3.3 Ground fault detection for a solar panel array

The following photograph shows a pair of analog voltmeters sensing voltage from an array of three identical photovoltaic (solar electric) panels connected in series. One connection to Earth ground exists, and that is at the point between the two voltmeters:




These two voltmeters comprise a *ground fault* detection system by which we may discern if there happen to be any unintentional connections between any other point in the circuit and the Earth. At the time this photograph was taken there were no ground faults in the system, so what we see here is a healthy (unfaulted) circuit.

Identify what these two voltmeters would do if there happened to be a ground fault at the left-most point in the circuit (at the most negative point).

Identify what these two voltmeters would do if there happened to be a ground fault at the right-most point in the circuit (at the most positive point).

Identify what these two voltmeters would do if there happened to be a ground fault at any point between any two of the solar panels.

Explain why these two voltmeters indicate precisely the same amount of voltage in the photograph, appealing to trusted principles of electricity.

What is the total voltage of this solar panel array?

What is the output voltage of each solar panel individually?

Challenges

- Assuming the same total array voltage as shown in the photograph, what would it mean if the left-hand voltmeter registered 40 Volts?

# Chapter 8

# Projects and Experiments

The following project and experiment descriptions outline things you can build to help you understand circuits. With any real-world project or experiment there exists the potential for physical harm. *Electricity can be very dangerous in certain circumstances, and you should follow proper safety precautions at all times!*

## 8.1 Recommended practices

This section outlines some recommended practices for all circuits you design and construct.

### 8.1.1 Safety first!

Electricity, when passed through the human body, causes uncomfortable sensations and in large enough measures[1] will cause muscles to involuntarily contract. The overriding of your nervous system by the passage of electrical current through your body is particularly dangerous in regard to your heart, which is a vital muscle. Very large amounts of current can produce serious internal burns in addition to all the other effects.

Cardio-pulmonary resuscitation (CPR) is the standard first-aid for any victim of electrical shock. This is a very good skill to acquire if you intend to work with others on dangerous electrical circuits. You should never perform tests or work on such circuits unless someone else is present who is proficient in CPR.

As a general rule, any voltage in excess of 30 Volts poses a definitive electric shock hazard, because beyond this level human skin does not have enough resistance to safely limit current through the body. "Live" work of any kind with circuits over 30 volts should be avoided, and if unavoidable should only be done using electrically insulated tools and other protective equipment (e.g. insulating shoes and gloves). If you are unsure of the hazards, or feel unsafe at any time, stop all work and distance yourself from the circuit!

A policy I strongly recommend for students learning about electricity is to *never come into electrical contact*[2] *with an energized conductor, no matter what the circuit's voltage*[3] *level!* Enforcing this policy may seem ridiculous when the circuit in question is powered by a single battery smaller than the palm of your hand, but it is precisely this instilled habit which will save a person from bodily harm when working with more dangerous circuits. Experience has taught me that students who learn early on to be careless with safe circuits have a tendency to be careless later with dangerous circuits!

In addition to the electrical hazards of shock and burns, the construction of projects and running of experiments often poses other hazards such as working with hand and power tools, potential

[1] Professor Charles Dalziel published a research paper in 1961 called "The Deleterious Effects of Electric Shock" detailing the results of electric shock experiments with both human and animal subjects. The threshold of perception for human subjects holding a conductor in their hand was in the range of 1 milliampere of current (less than this for alternating current, and generally less for female subjects than for male). Loss of muscular control was exhibited by half of Dalziel's subjects at less than 10 milliamperes alternating current. Extreme pain, difficulty breathing, and loss of all muscular control occurred for over 99% of his subjects at direct currents less than 100 milliamperes and alternating currents less than 30 milliamperes. In summary, it doesn't require much electric current to induce painful and even life-threatening effects in the human body! Your first and best protection against electric shock is maintaining an insulating barrier between your body and the circuit in question, such that current from that circuit will be unable to flow through your body.

[2] By "electrical contact" I mean either directly touching an energized conductor with any part of your body, or indirectly touching it through a conductive tool. The only physical contact you should ever make with an energized conductor is via an electrically insulated tool, for example a screwdriver with an electrically insulated handle, or an insulated test probe for some instrument.

[3] Another reason for consistently enforcing this policy, even on low-voltage circuits, is due to the dangers that even some low-voltage circuits harbor. A single 12 Volt automobile battery, for example, can cause a surprising amount of damage if short-circuited simply due to the high current levels (i.e. very low internal resistance) it is capable of, even though the voltage level is too low to cause a shock through the skin. Mechanics wearing metal rings, for example, are at risk from severe burns if their rings happen to short-circuit such a battery! Furthermore, even when working on circuits that are simply too low-power (low voltage and low current) to cause any bodily harm, touching them while energized can pose a threat to the circuit components themselves. In summary, it's generally wise (and *always* a good habit to build) to "power down" *any* circuit before making contact between it and your body.

contact with high temperatures, potential chemical exposure, etc. You should never proceed with a project or experiment if you are unaware of proper tool use or lack basic protective measures (e.g. personal protective equipment such as safety glasses) against such hazards.

Some other safety-related practices should be followed as well:

- All power conductors extending outward from the project must be *firmly* strain-relieved (e.g. "cord grips" used on line power cords), so that an accidental tug or drop will not compromise circuit integrity.
- All electrical connections must be sound and appropriately made (e.g. soldered wire joints rather than twisted-and-taped; terminal blocks rather than solderless breadboards for high-current or high-voltage circuits). Use "touch-safe" terminal connections with recessed metal parts to minimize risk of accidental contact.
- Always provide overcurrent protection in any circuit you build. *Always.* This may be in the form of a fuse, a circuit breaker, and/or an electronically current-limited power supply.
- Always ensure circuit conductors are rated for more current than the overcurrent protection limit. *Always.* A fuse does no good if the wire or printed circuit board trace will "blow" before it does!
- Always bond metal enclosures to Earth ground for any line-powered circuit. *Always.* Ensuring an equipotential state between the enclosure and Earth by making the enclosure electrically common with Earth ground ensures no electric shock can occur simply by one's body bridging between the Earth and the enclosure.
- Avoid building a high-energy circuit when a low-energy circuit will suffice. For example, I always recommend beginning students power their first DC resistor circuits using small batteries rather than with line-powered DC power supplies. The intrinsic energy limitations of a dry-cell battery make accidents highly unlikely.
- Use line power receptacles that are GFCI (Ground Fault Current Interrupting) to help avoid electric shock from making accidental contact with a "hot" line conductor.
- Always wear eye protection when working with tools or live systems having the potential to eject material into the air. Examples of such activities include soldering, drilling, grinding, cutting, wire stripping, working on or near energized circuits, etc.
- Always use a step-stool or stepladder to reach high places. Never stand on something not designed to support a human load.
- When in doubt, *ask an expert.* If anything even seems remotely unsafe to you, do not proceed without consulting a trusted person fully knowledgeable in electrical safety.

### 8.1.2 Other helpful tips

Experience has shown the following practices to be very helpful, especially when students make their own component selections, to ensure the circuits will be well-behaved:

- Avoid resistor values less than 1 kΩ or greater than 100 kΩ, unless such values are definitely necessary[4]. Resistances below 1 kΩ may draw excessive current if directly connected to a voltage source of significant magnitude, and may also complicate the task of accurately measuring current since any ammeter's non-zero resistance inserted in series with a low-value circuit resistor will significantly alter the total resistance and thereby skew the measurement. Resistances above 100 kΩ may complicate the task of measuring voltage since any voltmeter's finite resistance connected in parallel with a high-value circuit resistor will significantly alter the total resistance and thereby skew the measurement. Similarly, AC circuit impedance values should be between 1 kΩ and 100 kΩ, and for all the same reasons.

- Ensure all electrical connections are low-resistance and physically rugged. For this reason, one should avoid *compression splices* (e.g. "butt" connectors), solderless breadboards[5], and wires that are simply twisted together.

- Build your circuit with **testing** in mind. For example, provide convenient connection points for test equipment (e.g. multimeters, oscilloscopes, signal generators, logic probes).

- Design permanent projects with **maintenance** in mind. The more convenient you make maintenance tasks, the more likely they will get done.

- **Always document and save your work**. Circuits lacking schematic diagrams are more difficult to troubleshoot than documented circuits. Similarly, circuit construction is simpler when a schematic diagram precedes construction. Experimental results are easier to interpret when comprehensively recorded. Consider modern videorecording technology for this purpose where appropriate.

- **Record your steps** when troubleshooting. **Talk to yourself** when solving problems. These simple steps clarify thought and simplify identification of errors.

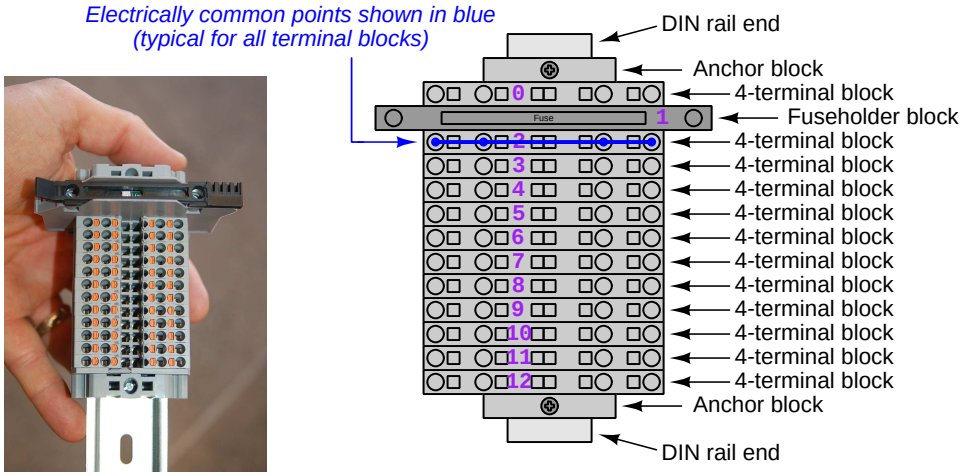
[4] An example of a necessary resistor value much less than 1 kΩ is a *shunt resistor* used to produce a small voltage drop for the purpose of sensing current in a circuit. Such shunt resistors must be low-value in order not to impose an undue load on the rest of the circuit. An example of a necessary resistor value much greater than 100 kΩ is an electrostatic *drain resistor* used to dissipate stored electric charges from body capacitance for the sake of preventing damage to sensitive semiconductor components, while also preventing a path for current that could be dangerous to the person (i.e. shock).

[5] Admittedly, solderless breadboards are very useful for constructing complex electronic circuits with many components, especially DIP-style integrated circuits (ICs), but they tend to give trouble with connection integrity after frequent use. An alternative for projects using low counts of ICs is to solder IC sockets into prototype printed circuit boards (PCBs) and run wires from the soldered pins of the IC sockets to terminal blocks where reliable temporary connections may be made.

### 8.1.3 Terminal blocks for circuit construction

Terminal blocks are the standard means for making electric circuit connections in industrial systems, and are also quite useful for student-built experiments, providing a robust alternative to solderless breadboards[6]. Terminal blocks provide highly reliable connections rated for significant voltage and current magnitudes, and they force the builder to think carefully about component placement which is an important mental practice. Terminal blocks designed to mount on standard 35 mm DIN rail[7] come in many types and capabilities.

Every student of electricity should build their own terminal block array consisting of several terminal blocks where each block has at least 4 connection points all electrically common to each other[8] and at least one terminal block that is a fuse holder for overcurrent protection. A pair of anchoring blocks hold all terminal blocks securely on the DIN rail, preventing them from sliding off the rail. Each of the terminals should bear a unique number starting from 0. An example is shown in the following photograph and illustration:




Screwless terminal blocks (using internal spring clips to clamp wire and component lead ends) are preferred over screw-based terminal blocks, as they reduce assembly and disassembly time, and also minimize repetitive wrist stress from twisting screwdrivers. Some screwless terminal blocks require the use of a special tool to release the spring clip, while others such as the Phoenix Contact model 3211019 blocks shown in the photograph provide buttons[9] for this task which may be pressed using the tip of any suitable tool such as a small screwdriver or multimeter test probe tip.

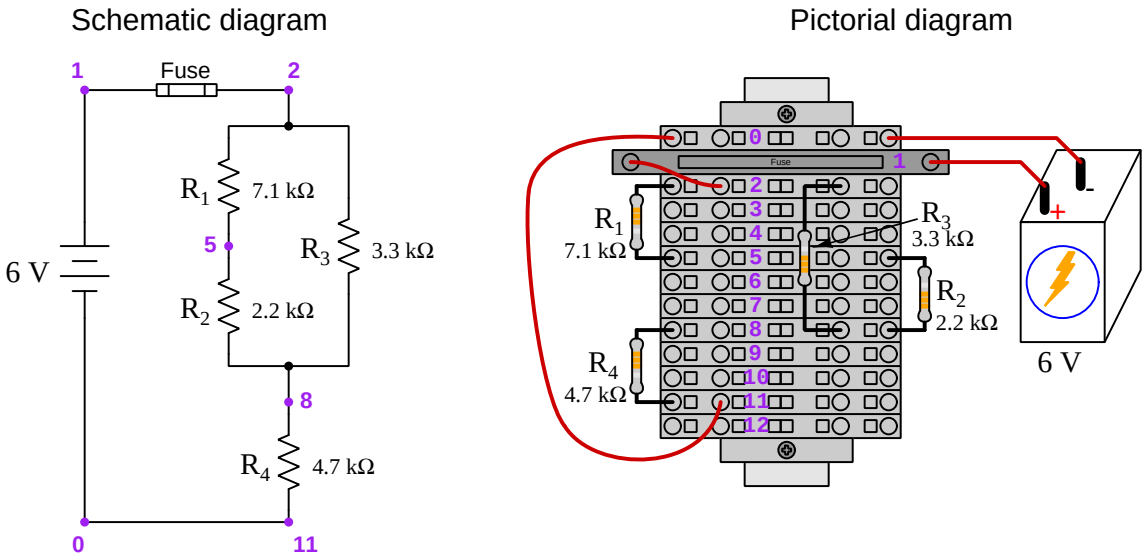
[6] Solderless breadboard are preferable for complicated electronic circuits with multiple integrated "chip" components, but for simpler circuits I find terminal blocks much more practical. An alternative to solderless breadboards for "chip" circuits is to solder chip sockets onto a PCB and then use wires to connect the socket pins to terminal blocks. This also accommodates *surface-mount* components, which solderless breadboards do not.

[7] DIN rail is a metal rail designed to serve as a mounting point for a wide range of electrical and electronic devices such as terminal blocks, fuses, circuit breakers, relay sockets, power supplies, data acquisition hardware, etc.

[8] Sometimes referred to as *equipotential*, *same-potential*, or *potential distribution* terminal blocks.

[9] The small orange-colored squares seen in the above photograph are buttons for this purpose, and may be actuated by pressing with any tool of suitable size.

The following example shows how such a terminal block array might be used to construct a series-parallel resistor circuit consisting of four resistors and a battery:




Numbering on the terminal blocks provides a very natural translation to SPICE[10] netlists, where component connections are identified by terminal number:

```
* Series-parallel resistor circuit
v1 1 0 dc 6
r1 2 5 7100
r2 5 8 2200
r3 2 8 3300
r4 8 11 4700
rfuse 1 2 0.01
rwire 0 11 0.01
.op
.end
```

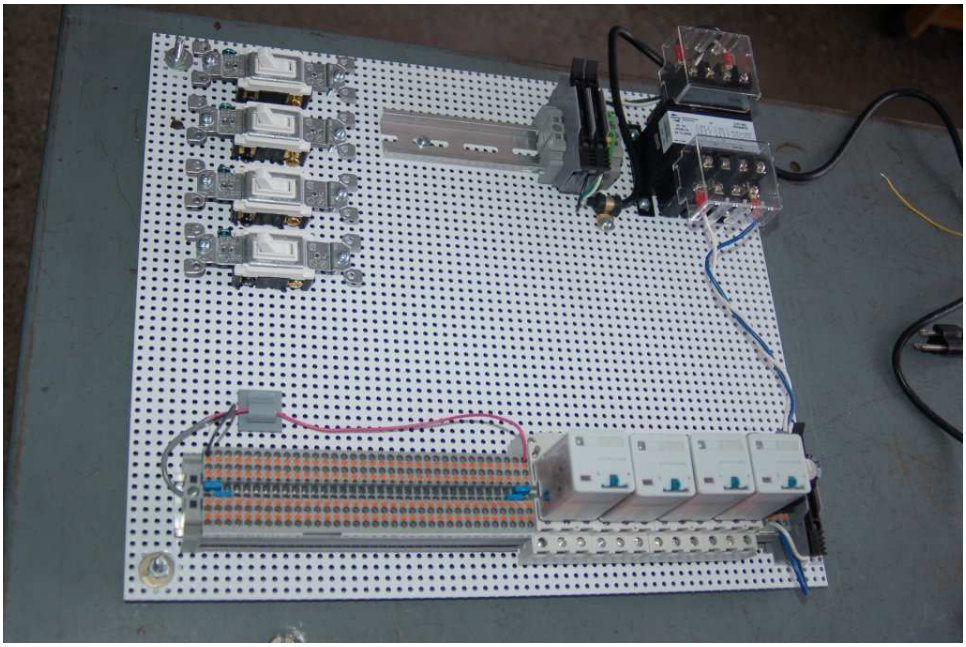
Note the use of two resistances `rfuse` and `rwire` to describe fuse resistance between terminals 1 and 2 and "jumper" wire resistance between terminals 0 and 11, respectively. Being resistances, SPICE requires a resistance value for each, and here we see they have both been set to an arbitrarily low value of 0.01 Ohm appropriate to their real-life resistance values.

Listing all components and wires along with their numbered terminals happens to be a useful documentation method for any circuit built on terminal blocks, independent of SPICE. Such a

[10] SPICE is computer software designed to analyze electrical and electronic circuits. Circuits are described for the computer in the form of *netlists* which are text files listing each component type, connection node numbers, and component values.

"wiring sequence" may be thought of as a *non-graphical description* of an electric circuit, and is exceptionally easy to follow.

An example of a more elaborate terminal block array is shown in the following photograph, with terminal blocks and "ice-cube" style electromechanical relays mounted to DIN rail, which is turn mounted to a perforated subpanel[11]. This "terminal block board" hosts an array of thirty five undedicated terminal block sections, four SPDT toggle switches, four DPDT "ice-cube" relays, a step-down control power transformer, bridge rectifier and filtering capacitor, and several fuses for overcurrent protection:



Four plastic-bottomed "feet" support the subpanel above the benchtop surface, and an unused section of DIN rail stands ready to accept other components. Safety features include electrical bonding of the AC line power cord's ground to the metal subpanel (and all metal DIN rails), mechanical strain relief for the power cord to isolate any cord tension from wire connections, clear plastic finger guards covering the transformer's screw terminals, as well as fused overcurrent protection for the 120 Volt AC line power and the transformer's 12 Volt AC output. The perforated holes happen to be on $\frac{1}{4}$ inch centers with a diameter suitable for tapping with 6-32 machine screw threads, their presence making it very easy to attach other sections of DIN rail, printed circuit boards, or specialized electrical components directly to the grounded metal subpanel. Such a "terminal block board" is an inexpensive[12] yet highly flexible means to construct physically robust circuits using industrial wiring practices.

[11] An electrical *subpanel* is a thin metal plate intended for mounting inside an electrical enclosure. Components are attached to the subpanel, and the subpanel in turn bolts inside the enclosure. Subpanels allow circuit construction outside the confines of the enclosure, which speeds assembly. In this particular usage there is no enclosure, as the subpanel is intended to be used as an open platform for the convenient construction of circuits on a benchtop by students. In essence, this is a modern version of the traditional *breadboard* which was literally a wooden board such as might be used for cutting loaves of bread, but which early electrical and electronic hobbyists used as platforms for the construction of circuits.

[12] At the time of this writing (2019) the cost to build this board is approximately $250 US dollars.

### 8.1.4 Conducting experiments

An *experiment* is an exploratory act, a test performed for the purpose of assessing some proposition or principle. Experiments are the foundation of the *scientific method*, a process by which careful observation helps guard against errors of speculation. All good experiments begin with an *hypothesis*, defined by the American Heritage Dictionary of the English Language as:

> An assertion subject to verification or proof, as (a) A proposition stated as a basis for argument or reasoning. (b) A premise from which a conclusion is drawn. (c) A conjecture that accounts, within a theory or ideational framework, for a set of facts and that can be used as a basis for further investigation.

Stated plainly, an hypothesis is an *educated guess* about cause and effect. The correctness of this initial guess matters little, because any well-designed experiment will reveal the truth of the matter. In fact, *incorrect* hypotheses are often the most valuable because the experiments they engender lead us to surprising discoveries. One of the beautiful aspects of science is that it is more focused on the process of *learning* than about the status of *being correct*[13]. In order for an hypothesis to be valid, it must be testable[14], which means it must be a claim possible to refute given the right data. Hypotheses impossible to critique are useless.

Once an hypothesis has been formulated, an experiment must be designed to test that hypothesis. A well-designed experiment requires careful regulation of all relevant variables, both for personal safety and for prompting the hypothesized results. If the effects of one particular variable are to be tested, the experiment must be run multiple times with different values of (only) that particular variable. The experiment set up with the "baseline" variable set is called the *control*, while the experiment set up with different value(s) is called the *test* or *experimental*.

For some hypotheses a viable alternative to a physical experiment is a *computer-simulated experiment* or even a *thought experiment*. Simulations performed on a computer test the hypothesis against the physical laws encoded within the computer simulation software, and are particularly useful for students learning new principles for which simulation software is readily available[15].

[13] Science is more about clarifying our view of the universe through a systematic process of error detection than it is about proving oneself to be right. Some scient*ists* may happen to have large egos – and this may have more to do with the ways in which large-scale scientific research is *funded* than anything else – but scientific *method* itself is devoid of ego, and if embraced as a practical philosophy is quite an effective stimulant for humility. Within the education system, scientific method is particularly valuable for helping students break free of the crippling fear of *being wrong*. So much emphasis is placed in formal education on assessing correct retention of facts that many students are fearful of saying or doing anything that might be perceived as a mistake, and of course making mistakes (i.e. having one's hypotheses disproven by experiment) is an indispensable tool for learning. Introducing science in the classroom – *real* science characterized by individuals forming actual hypotheses and testing those hypotheses by experiment – helps students become self-directed learners.

[14] This is the principle of *falsifiability*: that a scientific statement has value only insofar as it is liable to disproof given the requisite experimental evidence. Any claim that is unfalsifiable – that is, a claim which can *never* be disproven by any evidence whatsoever – could be completely wrong and we could never know it.

[15] A very pertinent example of this is learning how to analyze electric circuits using simulation software such as SPICE. A typical experimental cycle would proceed as follows: (1) Find or invent a circuit to analyze; (2) Apply your analytical knowledge to that circuit, predicting all voltages, currents, powers, etc. relevant to the concepts you are striving to master; (3) Run a simulation on that circuit, collecting "data" from the computer when complete; (4) Evaluate whether or not your hypotheses (i.e. predicted voltages, currents, etc.) agree with the computer-generated results; (5) If so, your analyses are (provisionally) correct – if not, examine your analyses and the computer simulation again to determine the source of error; (6) Repeat this process as many times as necessary until you achieve mastery.

Thought experiments are useful for detecting inconsistencies within your own understanding of some subject, rather than testing your understanding against physical reality.

Here are some general guidelines for conducting experiments:

- The clearer and more specific the hypothesis, the better. Vague or unfalsifiable hypotheses are useless because they will fit *any* experimental results, and therefore the experiment cannot teach you anything about the hypothesis.
- Collect as much data (i.e. information, measurements, sensory experiences) generated by an experiment as is practical. This includes the time and date of the experiment, too!
- *Never* discard or modify data gathered from an experiment. If you have reason to believe the data is unreliable, write notes to that effect, but never throw away data just because you think it is untrustworthy. It is quite possible that even "bad" data holds useful information, and that someone else may be able to uncover its value even if you do not.
- Prioritize *quantitative* data over *qualitative* data wherever practical. Quantitative data is more specific than qualitative, less prone to subjective interpretation on the part of the experimenter, and amenable to an arsenal of analytical methods (e.g. statistics).
- Guard against your own bias(es) by making your experimental results available to others. This allows other people to scrutinize your experimental design and collected data, for the purpose of detecting and correcting errors you may have missed. Document your experiment such that others may independently replicate it.
- Always be looking for sources of error. No physical measurement is perfect, and so it is impossible to achieve *exact* values for any variable. Quantify the amount of uncertainty (i.e. the "tolerance" of errors) whenever possible, and be sure your hypothesis does not depend on precision better than this!
- Always remember that scientific confirmation is provisional – no number of "successful" experiments will prove an hypothesis true for all time, but a single experiment can disprove it. Put into simpler terms, *truth is elusive but error is within reach.*
- Remember that scientific method is about *learning*, first and foremost. An unfortunate consequence of scientific triumph in modern society is that science is often viewed by non-practitioners as an unerring source of truth, when in fact science is an ongoing process of challenging existing ideas to probe for errors and oversights. This is why it is perfectly acceptable to have a failed hypothesis, and why the only truly failed experiment is one where nothing was learned.

The following is an example of a well-planned and executed experiment, in this case a physical experiment demonstrating Ohm's Law.

```
Planning Time/Date = 09:30 on 12 February 2019

HYPOTHESIS: the current through any resistor should be exactly proportional
to the voltage impressed across it.

PROCEDURE: connect a resistor rated 1 k Ohm and 1/4 Watt to a variable-voltage
DC power supply.  Use an ammeter in series to measure resistor current and
a voltmeter in parallel to measure resistor voltage.

RISKS AND MITIGATION: excessive power dissipation may harm the resistor and/
or pose a burn hazard, while excessive voltage poses an electric shock hazard.
30 Volts is a safe maximum voltage for laboratory practices, and according to
Joule's Law a 1000 Ohm resistor will dissipate 0.25 Watts at 15.81 Volts
(P = V^2 / R), so I will remain below 15 Volts just to be safe.

Experiment Time/Date = 10:15 on 12 February 2019

DATA COLLECTED:
  (Voltage)    (Current)         (Voltage)   (Current)
   0.000 V   =  0.000 mA          8.100    =  7.812 mA
   2.700 V   =  2.603 mA          10.00 V  =  9.643 mA
   5.400 V   =  5.206 mA          14.00 V  =  13.49 mA

Analysis Time/Date = 10:57 on 12 February 2019

ANALYSIS: current definitely increases with voltage, and although I expected
exactly one milliAmpere per Volt the actual current was usually less than
that.  The voltage/current ratios ranged from a low of 1036.87 (at 8.1 Volts)
to a high of 1037.81 (at 14 Volts), but this represents a variance of only
-0.0365% to +0.0541% from the average, indicating a very consistent
proportionality -- results consistent with Ohm's Law.

ERROR SOURCES: one major source of error is the resistor's value itself.  I
did not measure it, but simply assumed color bands of brown-black-red meant
exactly 1000 Ohms.  Based on the data I think the true resistance is closer
to 1037 Ohms.  Another possible explanation is multimeter calibration error.
However, neither explains the small positive and negative variances from the
average.  This might be due to electrical noise, a good test being to repeat
the same experiment to see if the variances are the same or different.  Noise
should generate slightly different results every time.
```

The following is an example of a well-planned and executed *virtual* experiment, in this case demonstrating Ohm's Law using a computer (SPICE) simulation.

```
Planning Time/Date = 12:32 on 14 February 2019

HYPOTHESIS: for any given resistor, the current through that resistor should be
exactly proportional to the voltage impressed across it.

PROCEDURE: write a SPICE netlist with a single DC voltage source and single
1000 Ohm resistor, then use NGSPICE version 26 to perform a "sweep" analysis
from 0 Volts to 25 Volts in 5 Volt increments.

    * SPICE circuit
    v1 1 0 dc
    r1 1 0 1000
    .dc v1 0 25 5
    .print dc v(1) i(v1)
    .end

RISKS AND MITIGATION: none.

DATA COLLECTED:
       DC transfer characteristic  Thu Feb 14 13:05:08  2019
    ------------------------------------------------------------
    Index   v-sweep         v(1)            v1#branch
    ------------------------------------------------------------
    0       0.000000e+00    0.000000e+00    0.000000e+00
    1       5.000000e+00    5.000000e+00    -5.00000e-03
    2       1.000000e+01    1.000000e+01    -1.00000e-02
    3       1.500000e+01    1.500000e+01    -1.50000e-02
    4       2.000000e+01    2.000000e+01    -2.00000e-02
    5       2.500000e+01    2.500000e+01    -2.50000e-02

Analysis Time/Date = 13:06 on 14 February 2019

ANALYSIS: perfect agreement between data and hypothesis -- current is precisely
1/1000 of the applied voltage for all values.  Anything other than perfect
agreement would have probably meant my netlist was incorrect.  The negative
current values surprised me, but it seems this is just how SPICE interprets
normal current through a DC voltage source.

ERROR SOURCES: none.
```

As gratuitous as it may seem to perform experiments on a physical law as well-established as Ohm's Law, even the examples listed previously demonstrate opportunity for real learning. In the physical experiment example, the student should identify and explain why their data does not perfectly agree with the hypothesis, and this leads them naturally to consider sources of error. In the computer-simulated experiment, the student is struck by SPICE's convention of denoting regular current through a DC voltage source as being *negative* in sign, and this is also useful knowledge for future simulations. Scientific experiments are most interesting when things *do not* go as planned!

Aside from verifying well-established physical laws, simple experiments are extremely useful as educational tools for a wide range of purposes, including:

- Component familiarization (e.g. *Which terminals of this switch connect to the NO versus NC contacts?*)
- System testing (e.g. *How heavy of a load can my AC-DC power supply source before the semiconductor components reach their thermal limits?*)
- Learning programming languages (e.g. *Let's try to set up an "up" counter function in this PLC!*)

Above all, the priority here is to inculcate the habit of hypothesizing, running experiments, and analyzing the results. This experimental cycle not only serves as an excellent method for self-directed learning, but it also works exceptionally well for troubleshooting faults in complex systems, and for these reasons should be a part of every technician's and every engineer's education.

### 8.1.5 Constructing projects

Designing, constructing, and testing projects is a very effective means of practical education. Within a formal educational setting, projects are generally chosen (or at least vetted) by an instructor to ensure they may be reasonably completed within the allotted time of a course or program of study, and that they sufficiently challenge the student to learn certain important principles. In a self-directed environment, projects are just as useful as a learning tool but there is some risk of unwittingly choosing a project beyond one's abilities, which can lead to frustration.

Here are some general guidelines for managing projects:

- Define your goal(s) before beginning a project: what do you wish to achieve in building it? What, exactly, should the completed project *do*?
- Analyze your project prior to construction. Document it in appropriate forms (e.g. schematic diagrams), predict its functionality, anticipate all associated risks. In other words, *plan ahead.*
- Set a reasonable budget for your project, and stay within it.
- Identify any deadlines, and set reasonable goals to meet those deadlines.
- Beware of *scope creep*: the tendency to modify the project's goals before it is complete.
- Document your progress! An easy way to do this is to use photography or videography: take photos and/or videos of your project as it progresses. Document failures as well as successes, because both are equally valuable from the perspective of learning.

## 8.2 Experiment: Confirming Kirchhoff's Voltage Law

Conduct an experiment to confirm Kirchhoff's Voltage Law (i.e. that the algebraic sum of voltages in any loop equals zero). Your experiment should include *multiple* loops in order to be comprehensive.

**EXPERIMENT CHECKLIST:**

- Prior to experimentation:
  - ☑ Write an hypothesis (i.e. a detailed description of what you expect will happen) unambiguous enough that it could be disproven given the right data.
  - ☑ Write a procedure to test the hypothesis, complete with adequate controls and documentation (e.g. schematic diagrams, programming code).
  - ☑ Identify any risks (e.g. shock hazard, component damage) and write a mitigation plan based on best practices and component ratings.
- During experimentation:
  - ☑ Safe practices followed at all times (e.g. no contact with energized circuit).
  - ☑ Correct equipment usage according to manufacturer's recommendations.
  - ☑ All data collected, ideally quantitative with full precision (i.e. no rounding).
- After each experimental run:
  - ☑ If the results fail to match the hypothesis, identify the error(s), correct the hypothesis and/or revise the procedure, and re-run the experiment.
  - ☑ Identify any uncontrolled sources of error in the experiment.
- After all experimental re-runs:
  - ☑ Save all data for future reference.
  - ☑ Write an analysis of experimental results and lessons learned.

Challenges

- Identify unique properties of *series*-connected components, and then use your experiment to prove this property is true either by appealing to empirical data you have recorded or by demonstrating those measurements live.
- Predict the effects of a particular resistor failing open.
- Predict the effects of a particular resistor failing shorted.
- Explain why you cannot obtain a reliable resistance measurement using an ohmmeter when the component in question is energized by a source.

- Identify any data points (empirical) that disagree with theory (prediction), and suggest sources of error.
- Explain why connecting an ammeter directly to the terminals of a voltage source such as a primary-cell battery is a bad idea.

# Appendix A

# Problem-Solving Strategies

The ability to solve complex problems is arguably one of the most valuable skills one can possess, and this skill is particularly important in any science-based discipline.

- Study principles, not procedures. Don't be satisfied with merely knowing how to compute solutions – learn *why* those solutions work.
- Identify what it is you need to solve, identify all relevant data, identify all units of measurement, identify any general principles or formulae linking the given information to the solution, and then identify any "missing pieces" to a solution. Annotate all diagrams with this data.
- Sketch a diagram to help visualize the problem. When building a real system, always devise a plan for that system and analyze its function *before* constructing it.
- Follow the units of measurement and meaning of every calculation. If you are ever performing mathematical calculations as part of a problem-solving procedure, and you find yourself unable to apply each and every intermediate result to some aspect of the problem, it means you don't understand what you are doing. Properly done, every mathematical result should have practical meaning for the problem, and not just be an abstract number. You should be able to identify the proper units of measurement for each and every calculated result, and show where that result fits into the problem.
- Perform "thought experiments" to explore the effects of different conditions for theoretical problems. When troubleshooting real systems, perform *diagnostic tests* rather than visually inspecting for faults, the best diagnostic test being the one giving you the most information about the nature and/or location of the fault with the fewest steps.
- Simplify the problem until the solution becomes obvious, and then use that obvious case as a model to follow in solving the more complex version of the problem.
- Check for exceptions to see if your solution is incorrect or incomplete. A good solution will work for *all* known conditions and criteria. A good example of this is the process of testing scientific hypotheses: the task of a scientist is not to find support for a new idea, but rather to *challenge* that new idea to see if it holds up under a battery of tests. The philosophical

principle of *reductio ad absurdum* (i.e. disproving a general idea by finding a specific case where it fails) is useful here.

- Work "backward" from a hypothetical solution to a new set of given conditions.
- Add quantities to problems that are qualitative in nature, because sometimes a little math helps illuminate the scenario.
- Sketch graphs illustrating how variables relate to each other. These may be quantitative (i.e. with realistic number values) or qualitative (i.e. simply showing increases and decreases).
- Treat quantitative problems as qualitative in order to discern the relative magnitudes and/or directions of change of the relevant variables. For example, try determining what happens if a certain variable were to increase or decrease before attempting to precisely calculate quantities: how will each of the dependent variables respond, by increasing, decreasing, or remaining the same as before?
- Consider limiting cases. This works especially well for qualitative problems where you need to determine which direction a variable will change. Take the given condition and magnify that condition to an extreme degree as a way of simplifying the direction of the system's response.
- Check your work. This means regularly testing your conclusions to see if they make sense. This does *not* mean repeating the same steps originally used to obtain the conclusion(s), but rather to use some other means to check validity. Simply repeating procedures often leads to *repeating the same errors* if any were made, which is why alternative paths are better.

# Appendix B

# Instructional philosophy

## B.1 First principles of learning

- **Anyone can learn anything** given appropriate time, effort, resources, challenges, encouragement, and expectations. Dedicating time and investing effort are the student's responsibility; providing resources, challenges, and encouragement are the teacher's responsibility; high expectations are a responsibility shared by both student and teacher.

- **Transfer is not automatic.** The human mind has a natural tendency to compartmentalize information, which means the process of taking knowledge learned in one context and applying it to another usually does not come easy and therefore should never be taken for granted.

- **Learning is iterative.** The human mind rarely learns anything perfectly on the first attempt. Anticipate mistakes and plan for multiple tries to achieve full understanding, using the lessons of those mistakes as feedback to guide future attempts.

- **Information is absorbed, but understanding is created.** Facts and procedures may be memorized easily enough by repeated exposure, but the ability to reliably apply principles to novel scenarios only comes through intense personal effort. This effort is fundamentally creative in nature: casting new concepts in one's own words, running mental experiments to perceive patterns and trends, etc. These acts of making knowledge "one's own" need not be perfect in order to be effective, as the value lies in the activity and not necessarily the finished product. Depth of understanding and permanence of memory are both directly related to mental effort during the learning process – i.e. *easy learning is shallow and fleeting.*

- **Education trumps training.** There is no such thing as an entirely isolated subject, as all fields of knowledge are connected. Training is narrowly-focused and task-oriented. Education is broad-based and principle-oriented. When preparing for a life-long technical career, education beats training every time.

- **Character matters.** Poor habits are more destructive than deficits of knowledge or skill. This is especially true in collective endeavors, where a team's ability to function depends on trust between its members. Simply put, no one wants an untrustworthy person on their team. An essential component of education then, is character development.

- **People learn to be responsible by bearing responsibility.** An irresponsible person is someone who has never *had* to be responsible for anything that mattered enough to them. Just as anyone can learn anything, anyone can become responsible if the personal cost of irresponsibility becomes high enough.

- **What gets measured, gets done.** Accurate and relevant assessment of learning is key to ensuring all students learn. Therefore, it is imperative to measure what matters.

- **Failure is nothing to fear.** Every human being fails, and fails in multiple ways at multiple times. Eventual success only happens when we don't stop trying.

## B.2 Proven strategies for instructors

- Assume every student is capable of learning anything they desire given the proper conditions. Treat them as capable adults by granting real responsibility and avoiding artificial incentives such as merit or demerit points.
- Create a consistent culture of high expectations across the entire program of study. Demonstrate and encourage patience, persistence, and a healthy sense of self-skepticism. Anticipate and de-stigmatize error. Teach respect for the capabilities of others as well as respect for one's own fallibility.
- Replace lecture with "inverted" instruction, where students first encounter new concepts through reading and then spend class time in Socratic dialogue with the instructor exploring those concepts and solving problems individually. There is a world of difference between observing someone solve a problem versus actually solving a problem yourself, and so the point of this form of instruction is to place students in a position where they *cannot* passively observe.
- Require students to read extensively, write about what they learn, and dialogue with you and their peers to sharpen their understanding. Apply Francis Bacon's advice that "reading maketh a full man; conference a ready man; and writing an exact man". These are complementary activities helping students expand their confidence and abilities.
- Use artificial intelligence (AI) to challenge student understanding rather than merely provide information. Find productive ways for AI to critique students' clarity of thought and of expression, for example by employing AI as a Socratic-style interlocutor or as a reviewer of students' journals. Properly applied, AI has the ability to expand student access to critical review well outside the bounds of their instructor's reach.
- Build frequent and rapid feedback into the learning process so that students know at all times how well they are learning, to identify problems early and fix them before they grow. Model the intellectual habit of self-assessing and self-correcting your own understanding (i.e. a cognitive *feedback loop*), encouraging students to do the same.
- Use "mastery" as the standard for every assessment, which means the exam or experiment or project must be done with 100% competence in order to pass. Provide students with multiple opportunity for re-tries (different versions of the assessment every time).
- Require students to devise their own hypotheses and procedures on all experiments, so that the process is truly a scientific one. Have students assess their proposed experimental procedures for risk and devise mitigations for those risks. Let nothing be pre-designed about students' experiments other than a stated task (i.e. what principle the experiment shall test) at the start and a set of demonstrable knowledge and skill objectives at the end.
- Have students build as much of their lab equipment as possible: building power sources, building test assemblies[1], and building complete working systems (no kits!). In order to provide

[1] In the program I teach, every student builds their own "Development Board" consisting of a metal chassis with DIN rail, terminal blocks, and an AC-DC power supply of their own making which functions as a portable lab environment they can use at school as well as take home.

this same "ground-up" experience for every new student, this means either previous students take their creations with them, or the systems get disassembled in preparation for the new students, or the systems grow and evolve with each new student group.

- Incorporate external accountability for you and for your students, continuously improving the curriculum and your instructional methods based on proven results. Have students regularly network with active professionals through participation in advisory committee meetings, service projects, tours, jobshadows, internships, etc. Practical suggestions include requiring students to design and build projects for external clients (e.g. community groups, businesses, different departments within the institution), and also requiring students attend all technical advisory committee meetings and to dialogue with the industry representatives at those meetings.
- Repeatedly explore difficult-to-learn concepts across multiple courses, so that students have multiple opportunities to build their understanding.
- Relate all new concepts, whenever possible, to previous concepts and to relevant physical laws. Challenge each and every student, every day, to *reason* from concept to concept and to explain the logical connections between. Challenge students to verify their conclusions by multiple approaches (e.g. double-checking their work using different methods). Ask *"Why?"* often.
- Maintain detailed records on each student's performance and share these records privately with them. These records should include academic performance as well as professionally relevant behavioral tendencies.
- Hold mandatory "check-in" meetings between all program faculty and each new student during their first term. Offer these to all other students as an option, except for any students continuing to manifest unprofessional behaviors, poor academic performance, or who have some other need for a face-to-face meeting with faculty.
- Address problems while they are small, before they grow larger. This is equally true for tutoring technical concepts as it is for helping students build professional habits.
- Build rigorous quality control into the curriculum to ensure every student masters every important concept, and that the mastery is retained over time. This includes (1) review questions added to every exam to re-assess knowledge taught in previous terms, (2) cumulative exams at the end of every term to re-assess all important concepts back to the very beginning of the program, and (3) review assessments in practical (hands-on) coursework to ensure critically-important skills were indeed taught and are still retained. What you will find by doing this is that it actually boosts retention of students by ensuring that important knowledge gets taught and is retained over long spans of time. In the absence of such quality control, student learning and retention tends to be spotty and this contributes to drop-out and failure rates later in their education.
- Finally, *never rush learning.* Education is not a race. Give your students ample time to digest complex ideas, as you continually remind yourself of just how long it took you to achieve mastery! Long-term retention and the consistently correct application of concepts are always the result of *focused effort over long periods of time* which means there are no shortcuts to learning.

## B.3 Proven strategies for students

The single most important piece of advice I have for any student of any subject is to take responsibility for your own development in all areas of life including mental development. Expecting others in your life to entirely guide your own development is a recipe for disappointment. This is just as true for students enrolled in formal learning institutions as it is for auto-didacts pursuing learning entirely on their own. Learning to think in new ways is key to being able to gainfully use information, to make informed decisions about your life, and to best serve those you care about. With this in mind, I offer the following advice to students:

- **Approach all learning as valuable**. No matter what course you take, no matter who you learn from, no matter the subject, there is something useful in every learning experience. If you don't see the value of every new experience, you are not looking closely enough!
- **Continually challenge yourself**. Let other people take shortcuts and find easy answers to easy problems. The purpose of education is to stretch your mind, in order to shape it into a more powerful tool. This doesn't come by taking the path of least resistance. An excellent analogy for an empowering education is productive physical exercise: becoming stronger, more flexible, and more persistent only comes through intense personal effort.
- **Master the use of language**. This includes reading extensively, writing every day, listening closely, and speaking articulately. To a great extent language channels and empowers thought, so the better you are at wielding language the better you will be at grasping abstract concepts and articulating them not only for your benefit but for others as well.
- **Do not limit yourself to the resources given to you**. Read books that are not on the reading list. Run experiments that aren't assigned to you. Form study groups outside of class. Take an entrepreneurial approach to your own education, as though it were a business you were building for your future benefit.
- **Express and share what you learn**. Take every opportunity to teach what you have learned to others, as this will not only help them but will also strengthen your own understanding[2].
- Realize that **no one can give you understanding**, just as no one can give you physical fitness. These both must be *built*.
- **Above all, recognize that learning is hard work, and that a certain level of frustration is unavoidable**. There are times when you will struggle to grasp some of these concepts, and that struggle is a natural thing. Take heart that it will yield with persistent and varied[3] effort, and never give up! That concepts don't immediately come to you is not a sign of something wrong, but rather of something right: that you have found a worthy challenge!

[2] On a personal note, I was surprised to learn just how much my own understanding of electronics and related subjects was strengthened by becoming a teacher. When you are tasked every day with helping other people grasp complex topics, it catalyzes your own learning by giving you powerful incentives to study, to articulate your thoughts, and to reflect deeply on the process of learning.

[3] As the old saying goes, "Insanity is trying the same thing over and over again, expecting different results." If you find yourself stumped by something in the text, you should attempt a different approach. Alter the thought experiment, change the mathematical parameters, do whatever you can to see the problem in a slightly different light, and then the solution will often present itself more readily.

## B.4 Design of these learning modules

*"The unexamined circuit is not worth energizing"* – Socrates (if he had taught electricity)

These learning modules, although useful for self-study, were designed to be used in a formal learning environment where a subject-matter expert challenges students to digest the content and exercise their critical thinking abilities in the answering of questions and in the construction and testing of working circuits. Every effort has been made to embed the following instructional and assessment philosophies within:

- The first goal of education is to enhance clear and independent thought, in order that every student reach their fullest potential in a highly complex and inter-dependent world. Robust reasoning is *always* more important than particulars of any subject matter, because its application is universal.
- Literacy is fundamental to independent learning and thought because text continues to be the most efficient way to communicate complex ideas over space and time. Those who cannot read with ease are limited in their ability to acquire knowledge and perspective.
- Articulate communication is fundamental to work that is complex and interdisciplinary.
- Faulty assumptions and poor reasoning are best corrected through challenge, not presentation. The rhetorical technique of *reductio ad absurdum* (disproving an assertion by exposing an absurdity) works well to discipline student's minds, not only to correct the problem at hand but also to learn how to detect and correct future errors.
- Important principles should be repeatedly explored and widely applied throughout a course of study, not only to reinforce their importance and help ensure their mastery, but also to showcase the interconnectedness and utility of knowledge.

These learning modules were expressly designed to be used in an "inverted" teaching environment[4] where students first read the introductory and tutorial chapters on their own, then individually attempt to answer the questions and construct working circuits according to the experiment and project guidelines. The instructor never lectures, but instead meets regularly with each individual student to review their progress, answer questions, identify misconceptions, and challenge the student to new depths of understanding through further questioning. Regular meetings between instructor and student should resemble a Socratic[5] dialogue, where questions serve as scalpels to dissect topics and expose assumptions. The student passes each module only after consistently demonstrating their ability to logically analyze and correctly apply all major concepts in each question or project/experiment. The instructor must be vigilant in probing each student's understanding to ensure they are truly *reasoning* and not just *memorizing*. This is why "Challenge" points appear throughout, as prompts for students to think deeper about topics and as starting points for instructor queries. Sometimes these challenge points require additional knowledge that hasn't been covered in the series to answer in full. This is okay, as the major purpose of the Challenges is to stimulate analysis and synthesis on the part of each student.

The instructor must possess enough mastery of the subject matter and awareness of students' reasoning to generate their own follow-up questions to practically any student response. Even completely correct answers given by the student should be challenged by the instructor for the purpose of having students practice articulating their thoughts and defending their reasoning. Conceptual errors committed by the student should be exposed and corrected not by direct instruction, but rather by reducing the errors to an absurdity[6] through well-chosen questions and thought experiments posed by the instructor. Becoming proficient at this style of instruction requires time and dedication, but the positive effects on critical thinking for both student and instructor are spectacular.

An inspection of these learning modules reveals certain unique characteristics. One of these is a bias toward thorough explanations in the tutorial chapters. Without a live instructor to explain concepts and applications to students, the text itself must fulfill this role. This philosophy results in lengthier explanations than what you might typically find in a textbook, each step of the reasoning process fully explained, including footnotes addressing common questions and concerns students raise while learning these concepts. Each tutorial seeks to not only explain each major concept in sufficient detail, but also to explain the logic of each concept and how each may be developed

[4] In a traditional teaching environment, students first encounter new information via *lecture* from an expert, and then independently apply that information via *homework*. In an "inverted" course of study, students first encounter new information via *homework*, and then independently apply that information under the scrutiny of an expert. The expert's role in lecture is to simply *explain*, but the expert's role in an inverted session is to *challenge*, *critique*, and if necessary *explain* where gaps in understanding still exist.

[5] Socrates is a figure in ancient Greek philosophy famous for his unflinching style of questioning. Although he authored no texts, he appears as a character in Plato's many writings. The essence of Socratic philosophy is to leave no question unexamined and no point of view unchallenged. While purists may argue a topic such as electric circuits is too narrow for a true Socratic-style dialogue, I would argue that the essential thought processes involved with scientific reasoning on *any* topic are not far removed from the Socratic ideal, and that students of electricity and electronics would do very well to challenge assumptions, pose thought experiments, identify fallacies, and otherwise employ the arsenal of critical thinking skills modeled by Socrates.

[6] This rhetorical technique is known by the Latin phrase *reductio ad absurdum*. The concept is to expose errors by counter-example, since only one solid counter-example is necessary to disprove a universal claim. As an example of this, consider the common misconception among beginning students of electricity that voltage cannot exist without current. One way to apply *reductio ad absurdum* to this statement is to ask how much current passes through a fully-charged battery connected to nothing (i.e. a clear example of voltage existing without current).

from "first principles". Again, this reflects the goal of developing clear and independent thought in students' minds, by showing how clear and logical thought was used to forge each concept. Students benefit from witnessing a model of clear thinking in action, and these tutorials strive to be just that.

Another feature of these learning modules is that they do not treat topics in isolation. Rather, important concepts are introduced early in the series, and appear repeatedly as stepping-stones toward other concepts in subsequent modules. This helps to avoid the "compartmentalization" of knowledge, demonstrating the inter-connectedness of concepts and simultaneously reinforcing them. Each module is fairly complete in itself, reserving the beginning of its tutorial to a review of foundational concepts.

To high standards of education,

Tony R. Kuphaldt

# Appendix C

# Tools used

I am indebted to the developers of many open-source software applications in the creation of these learning modules. The following is a list of these applications with some commentary on each.

You will notice a theme common to many of these applications: a bias toward *code*. Although I am by no means an expert programmer in any computer language, I understand and appreciate the flexibility offered by code-based applications where the user (you) enters commands into a plain ASCII text file, which the software then reads and processes to create the final output. Code-based computer applications are by their very nature *extensible*, while WYSIWYG (What You See Is What You Get) applications are generally limited to whatever user interface the developer makes for you.

The `GNU/Linux` computer operating system

> There is so much to be said about Linus Torvalds' `Linux` and Richard Stallman's `GNU` project. First, to credit just these two individuals is to fail to do justice to the *mob* of passionate volunteers who contributed to make this amazing software a reality. I first learned of `Linux` back in 1996, and have been using this operating system on my personal computers almost exclusively since then. It is *free*, it is completely *configurable*, and it permits the continued use of highly efficient `Unix` applications and scripting languages (e.g. shell scripts, Makefiles, `sed`, `awk`) developed over many decades. `Linux` not only provided me with a powerful computing platform, but its open design served to inspire my life's work of creating open-source educational resources.

Bram Moolenaar's `Vim` text editor

> Writing code for any code-based computer application requires a *text editor*, which may be thought of as a word processor strictly limited to outputting plain-ASCII text files. Many good text editors exist, and one's choice of text editor seems to be a deeply personal matter within the programming world. I prefer `Vim` because it operates very similarly to `vi` which is ubiquitous on `Unix`/`Linux` operating systems, and because it may be entirely operated via keyboard (i.e. no mouse required) which makes it fast to use.

Donald Knuth's TEX typesetting system

Developed in the late 1970's and early 1980's by computer scientist extraordinaire Donald Knuth to typeset his multi-volume magnum opus *The Art of Computer Programming*, this software allows the production of formatted text for screen-viewing or paper printing, all by writing plain-text code to describe how the formatted text is supposed to appear. TEX is not just a markup language for documents, but it is also a Turing-complete programming language in and of itself, allowing useful algorithms to be created to control the production of documents. Simply put, TEX *is a programmer's approach to word processing*. Since TEX is controlled by code written in a plain-text file, this means anyone may read that plain-text file to see exactly how the document was created. This openness afforded by the code-based nature of TEX makes it relatively easy to learn how other people have created their own TEX documents. By contrast, examining a beautiful document created in a conventional WYSIWYG word processor such as Microsoft `Word` suggests nothing to the reader about *how* that document was created, or what the user might do to create something similar. As Mr. Knuth himself once quipped, conventional word processing applications should be called WYSIAYG (What You See Is *All* You Get).

Leslie Lamport's LATEX extensions to TEX

Like all true programming languages, TEX is inherently extensible. So, years after the release of TEX to the public, Leslie Lamport decided to create a massive extension allowing easier compilation of book-length documents. The result was LATEX, which is the markup language used to create all ModEL module documents. You could say that TEX is to LATEX as `C` is to `C++`. This means it is permissible to use any and all TEX commands within LATEX source code, and it all still works. Some of the features offered by LATEX that would be challenging to implement in TEX include automatic index and table-of-content creation.

Tim Edwards' `Xcircuit` drafting program

This wonderful program is what I use to create all the schematic diagrams and illustrations (but not photographic images or mathematical plots) throughout the ModEL project. It natively outputs PostScript format which is a true vector graphic format (this is why the images do not pixellate when you zoom in for a closer view), and it is so simple to use that I have never had to read the manual! Object libraries are easy to create for `Xcircuit`, being plain-text files using PostScript programming conventions. Over the years I have collected a large set of object libraries useful for drawing electrical and electronic schematics, pictorial diagrams, and other technical illustrations.

`Gimp` graphic image manipulation program

Essentially an open-source clone of Adobe's `PhotoShop`, I use `Gimp` to resize, crop, and convert file formats for all of the photographic images appearing in the ModEL modules. Although `Gimp` does offer its own scripting language (called `Script-Fu`), I have never had occasion to use it. Thus, my utilization of `Gimp` to merely crop, resize, and convert graphic images is akin to using a sword to slice bread.

`SPICE` circuit simulation program

`SPICE` is to circuit analysis as TEX is to document creation: it is a form of markup language designed to describe a certain object to be processed in plain-ASCII text. When the plain-text "source file" is compiled by the software, it outputs the final result. More modern circuit analysis tools certainly exist, but I prefer `SPICE` for the following reasons: it is *free*, it is *fast*, it is *reliable*, and it is a fantastic tool for *teaching* students of electricity and electronics how to write simple code. I happen to use rather old versions of `SPICE`, version 2g6 being my "go to" application when I only require text-based output. `NGSPICE` (version 26), which is based on Berkeley `SPICE` version 3f5, is used when I require graphical output for such things as time-domain waveforms and Bode plots. In all `SPICE` example netlists I strive to use coding conventions compatible with all `SPICE` versions.

Andrew D. Hwang's `ePiX` mathematical visualization programming library

This amazing project is a `C++` library you may link to any `C/C++` code for the purpose of generating PostScript graphic images of mathematical functions. As a completely free and open-source project, it does all the plotting I would otherwise use a Computer Algebra System (CAS) such as `Mathematica` or `Maple` to do. It should be said that `ePiX` is *not* a Computer Algebra System like `Mathematica` or `Maple`, but merely a mathematical *visualization* tool. In other words, it won't determine integrals for you (you'll have to implement that in your own `C/C++` code!), but it can graph the results, and it does so beautifully. What I really admire about `ePiX` is that it is a `C++` programming library, which means it builds on the existing power and toolset available with that programming language. Mr. Hwang could have probably developed his own stand-alone application for mathematical plotting, but by creating a `C++` library to do the same thing he accomplished something much greater.

`gnuplot` mathematical visualization software

Another open-source tool for mathematical visualization is `gnuplot`. Interestingly, this tool is *not* part of Richard Stallman's GNU project, its name being a coincidence. For this reason the authors prefer "gnu" *not* be capitalized at all to avoid confusion. This is a much "lighter-weight" alternative to a spreadsheet for plotting tabular data, and the fact that it easily outputs directly to an X11 console or a file in a number of different graphical formats (including PostScript) is very helpful. I typically set my `gnuplot` output format to default (X11 on my Linux PC) for quick viewing while I'm developing a visualization, then switch to PostScript file export once the visual is ready to include in the document(s) I'm writing. As with my use of `Gimp` to do rudimentary image editing, my use of `gnuplot` only scratches the surface of its capabilities, but the important points are that it's *free* and that it *works well*.

`Python` programming language

Both Python and C++ find extensive use in these modules as instructional aids and exercises, but I'm listing Python here as a *tool* for myself because I use it almost daily as a *calculator*. If you open a Python interpreter console and type `from math import *` you can type mathematical expressions and have it return results just as you would on a hand calculator. Complex-number (i.e. *phasor*) arithmetic is similarly supported if you include the complex-math library (`from cmath import *`). Examples of this are shown in the Programming References chapter (if included) in each module. Of course, being a fully-featured programming language, Python also supports conditionals, loops, and other structures useful for calculation of quantities. Also, running in a console environment where all entries and returned values show as text in a chronologically-ordered list makes it easy to copy-and-paste those calculations to document exactly how they were performed.

# Appendix D

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# Appendix E

# References

Poggendorff, Johann C., *Annalen der Physik und Chemie*, No. 4, Band LXIV, Leipzig, Germany, 1845.

# Appendix F

# Version history

This is a list showing all significant additions, corrections, and other edits made to this learning module. Each entry is referenced by calendar date in reverse chronological order (newest version first), which appears on the front cover of every learning module for easy reference. Any contributors to this open-source document are listed here as well.

**17 September 2026** – added some instructor notes.

**7 September 2026** – added a new Diagnostic Reasoning question, this one on two voltmeters applied to a solar panel array.

**10-14 September 2025** – minor edits to the Full Tutorial chapter.

**24 August 2025** – added new bullet-points to the list in the "Challenging concepts related to parallel circuits" section of the Introduction chapter regarding the confusion of definitions versus properties of both series and parallel networks.

**6 February 2025** – added a comment to the Full Tutorial mentioning how moving a test charge between two points not directly connected by any conductive path in a circuit is analogous to a hiker taking a helicopter from one location to another instead of hiking along a trail.

**2 October 2024** – added another "Challenging concepts" bullet-item to the list in the Introduction chapter.

**11-12 September 2024** – fixed a minor typographical error: "once gain" should have been "once again". Other minor edits to the Full Tutorial chapter as well, including a typographical error correction that was in the tutorial for an embarrasingly long amount of time (the first table showing voltage gains and losses had Step C to B as +1 Volt rather than +2 Volts as it should have been)! Jordan Wiggins caught this error for me!

**3 September 2024** – added another section to the Introduction chapter specifically for challenges related to this module's topic.

**1 July 2024** – divided the Introduction chapter into two sections, one for students and one for instructors, and added content to the instructor section recommending learning outcomes and measures.

**18 October 2023** – added a new Tutorial section showing a "shortcut" method of applying KVL.

**25 June 2023** – added Technical Reference section on grounding.

**27 November 2022** – placed questions at the top of the itemized list in the Introduction chapter prompting students to devise experiments related to the tutorial content.

**15 September 2022** – minor edits to the Full Tutorial.

**5 September 2022** – added another Quantitative Reasoning question.

**29 June 2022** – divided the Full Tutorial into sections.

**10 February 2022** – minor edit to the Full Tutorial explaining why the path chosen is irrelevant. Also edited open terminals in several questions, from solid dots to open circles.

**7 February 2022** – minor edit to "Tracing KVL loops in a series circuit" Quantitative Reasoning problem challenging students to use mental math rather than their calculators.

**25 June 2021** – added a Case Tutorial section on split power supplies.

**18 March 2021** – corrected multiple instances of "volts" that should have been capitalized "Volts".

**11 February 2021** – minor additions to the Introduction chapter.

**18 November 2020** – minor additions to the Introduction chapter.

**17 September 2020** – minor additions to the Introduction chapter.

**23 August 2020** – significantly edited the Introduction chapter to make it more suitable as a pre-study guide and to provide cues useful to instructors leading "inverted" teaching sessions.

**27 January 2020** – corrected KVL loop typo in one of the Quantitative Reasoning questions ("Tracing KVL loops in a multi-source circuit".

**18 September 2019** – added a Case Tutorial chapter.

**26 August 2019** – added mention of difficult concepts to the Introduction.

**3 June 2019** – edited KVL confirmation experiment to make it less prescriptive, but also to ensure that students actually apply Kirchhoff's Voltage Law (i.e. proving that the sum of voltages measured in a loop equals zero) rather than some facsimile of it (e.g. the sum of load voltages must equal the source). Also, added more questions.

**30 May 2019** – added "single-ended" as a synonym for "ground-referenced" when speaking of voltage measurements made in reference to some established "ground" point.

**25 May 2019** – clarified the point that the direction one chooses when applying KVL around a loop has nothing to do with the direction of current flow in the circuit, but rather is an arbitrary choice on the part of the person analyzing the circuit. Also added explanations for differential versus ground-referenced voltage measurements. Also modified the "Incomplete KVL analogy" Conceptual Reasoning question to be in meters rather than feet.

**14 May 2019** – retitled "Non-connected batteries" to be "Grounded batteries" in the Quantitative Reasoning section.

**5 November 2018** – retitled Historical References section(s) so as to not be redundant to the "Historical References" chapter.

**September 2018** – renamed "Derivations and Technical References" chapter to "Historical References".

**August 2018** – added historical reference to Kirchhoff's publishing of KVL in the *Annals of Physics and Chemistry* in 1845, as screenshots of pages from this journal. Also added content to the Introduction chapter.

**July 2018** – minor edits to Simplified Tutorial chapter, and also correct a typographical error where both "meters" and "feet" were applied as units to the same number value.

**May 2018** – added a Simplified Tutorial chapter.

**August 2017** – added a conceptual question applying curved-arrow notation for representing voltage polarity.

**January 2017** – minor clarifications made to the concepts of electrically common points, equipotential points, and parallel networks.

**October 2016** – created changelog for future use.

**September 2016** – document first published.

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